Mechanisms to Appoint Arbitrator Panels or Sets of Judges by Compromise Between Concerned Parties

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Mechanisms to Appoint Arbitrator Panels or Sets of Judges by Compromise Between Concerned Parties

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Abstract

We propose mechanisms for two parties with potentially conflicting objectives to jointly select a predetermined number of candidates to occupy decision-making positions. Two leading examples of these situations are: i) the selection of an arbitrator panel by two conflicting firms, and ii) the bipartisan coalition's selection of a set of judges to occupy court vacancies. We analyze the efficiency, fairness, and simplicity of equilibrium outcomes in strategic games induced by these mechanisms. Their effectiveness hinges on the parties' preferences over the sets containing the required number of candidates to be chosen.

Keywords: Appointing Arbitrators, Appointing Judges, Rule of k Name, Split Appointment Rules, Compromise, Unanimity Compromise Set, Top Compromise Set. **JEL classification:** D02, D71, D72.

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1 Introduction

In this paper, we propose different mechanisms through which two parties in litigation can participate in the selection of a fixed number of candidates, guided by their expectations about decisions that these candidates if selected, may adopt in the future. We model the game that each mechanism induces as a sequential game of complete information and solve for its subgame perfect equilibria. This is a reasonable starting point because our main motivation is to study the selection of sets of professionals, such as arbitrators or judges, with a priori well-known abilities, by parties who clearly understand their conflicting interests and how these will be affected by the choice of candidates[1].

Our analysis here is part of a sequence of works that started with the theoretical analysis of the Rules of k Names and their use in different contexts. The closest reference in that line is a recent paper (Barberà and Coelho, 2022) where we address a similar problem, where the objective is to select a single arbitrator or judge, rather than a committee.

Even if the case where only one candidate must be selected is relevant for different applications, there are also many instances in which the concerned parties must end up choosing a larger and fixed number of candidates.

For example, in the case of arbitration, when high amounts are at stake, the claims of two opponents are heard by a panel composed of three arbitrators instead of a sole one. In such cases, many arbitral institutions stipulate that if a panel of three is to be selected, each party shall choose one member, and then the two party-appointed arbitrators shall agree on a third one, who will act as president. In other cases, it is required that the three arbitrators should be jointly selected by the two parties. For instance, one of the two main providers of arbitration services in the US, JAMS (JAMS Arbitration, Mediation and ADR services), proceeds as follows: It sends to both parties in dispute a list of at least ten candidates in the case of a tripartite panel. Each party may then strike three names and shall rank the remaining ones in order of preference. Finally, the three candidates with the highest composite ranking are appointed. In the literature, the procedure used by JAMS is called the veto-rank method.

Our proposals are also applicable to solve issues that arise when several judges must be chosen. A recent event in Spanish parliamentary life underscores the necessity of

[1] Of course, other interpretations are possible and may suggest extensions in different directions, which are not considered in this paper.

reevaluating the current methods used by the Parliament for the total or partial renewal of various judicial bodies. These processes were frozen for years because the dominant parties could not find a way to combine their interests and attain the required two-thirds majority in parliament to appoint new members to these bodies. Another instance where our methods may apply is the appointment of judges to the Brazilian Superior Court of Justice. Although that country's constitution establishes how to choose one judge when there is one vacancy, it is not unusual for several vacancies to be open at the same time, and then all of them are filled simultaneously. In such situations, the Brazilian court presents a list of size equal to the number of vacant positions plus two, for the President of the Republic to appoint, out of this list, the needed number of candidates for the vacant positions.

Most of the methods used in practice in these and other similar situations are far from satisfactory. In particular, one reason to explore the possibility of using alternative methods is that the current ones may often lead to an inefficient composition of the resulting panels if put into practice. This is the case, for instance, of the JAMS method described above[2]. Hence, finding efficient methods is one of our concerns. In addition, the interested parties may refuse to participate in the process or try to delay it if the rules for participation are not attractive to them. One of the potential reasons leading to stalemate is the claim of unfairness that may arise if one of the parties feels disadvantaged by the proposed selection mechanisms. This is why, in this paper, we also focus on the design of procedures that are not only efficient but also as fair as possible, in the precise ways defined below.

It is not surprising that JAMS, among other methods, is not efficient, since Hurwicz and Schmeidler (1978) and Maskin (1999) proved that there exists no deterministic mechanism with two players playing simultaneously, except for dictatorship, guaranteeing that every Nash equilibrium is Pareto efficient. We will circumvent this impossibility result by using procedures that involve sequential movements and eventually restricting the universe of admissible preference profiles.

Let us describe the mechanisms we propose.

First, note that, although the objective of our proposed mechanisms is to select sets of candidates, we base their description and analysis on the characteristics of the parties'

[2] See in Appendix 1 an example of this inefficiency.

preferences over individual candidates. This is to keep the size of the mechanisms at a reasonable level. An alternative would be to consider from the beginning that the basic alternatives are the sets of a given fixed size, but then the number of alternatives becomes very large and the number of potential preferences of agents even more so. Because we rely on preferences over singletons, it is important to control for the connection between the parties' preferences over individual candidates and their preferences over sets of the required size, because, after all, these preferences will drive their strategic actions and we should use them to evaluate the efficiency of our mechanisms.

There is extensive literature regarding the extension of preference relations defined on singletons to preferences defined on their power set. Our characterizations of equilibria are valid for a large class of such extensions satisfying a natural axiom, whereas the results regarding the efficiency of equilibrium outcomes are dependent on the further requirement that the extensions be based on the leximin or leximax principles[3].

We start by presenting our mechanisms, first introducing two procedures that will be used as building blocks, yet still suffer from some flaws that we will address later. They are called the $((\theta, h)-$**Split Appointment Rule** and $\theta-$**Rule of k Names**[4].

The $(\theta, h)-$**Split Appointment Rule**, where h is an integer in $[0, \theta]$, works as follows: the first mover (nominator 1) nominates h candidates from $\mathbf{C}$, and then the second mover (nominator 2) completes the set of θ winning candidates by nominating the necessary additional candidates among those not previously selected.

Notice that the passage from a singleton to a set of chosen candidates makes it possible for each party to select part of the needed candidates and opens the way to define this simple and natural mechanism.

The second procedure is based on a method that we have studied extensively, and we call it the Rule of k Names.

The $\theta-$**Rule of k Names**, where k is an integer in $[\theta, \mathbf{c}]$, where $\mathbf{c}$ stands for the number of candidates, works as follows: the first mover (the proposer) selects k candidates out of those in an original list $\mathbf{C}$, and then the other party (the chooser) selects θ winners out of those proposed by her opponent.

We can characterize the unique subgame perfect Nash equilibrium (SPNE) outcome

[3]See Barberà, Bossert, and Pattanaik (2004) for an extensive survey.

[4]Strictly speaking, we describe families of rules sharing the same principles but varying in their parameters and the positions of the players. However, in what follows, we refer to the class in singular.

of the games induced by each of these methods and establish conditions under which it is efficient.

Under the $(\theta, h)-$**Split Appointment Rule**, in equilibrium, nominator 1 always selects her preferred candidates among the nominator 2's $(\mathbf{c}-(\theta-h))$-bottom candidates. This selection occurs because nominator 2 will invariably opt for her $(\theta-h)$ most preferred candidates if they have not already been chosen by her opponent. This unique equilibrium outcome is efficient if at least one of the parties' preferences over sets are leximax extensions of her preferences over alternatives

By contrast, the unique equilibrium outcome induced by the use of the $\theta-$**Rule of k Names** method is such that the proposer selects her θ best candidates among the $\mathbf{c}-(k-\theta)$ best candidates for the chooser. This is because the chooser will never select any of her $(k-\theta)$ worst candidates, regardless of the proposal it faces. And then the equilibrium outcome is proven to be efficient if the preferences of at least one of the parties' preferences are leximin extensions of her preferences over alternatives.

Despite their eventual efficiency, both mechanisms that we started with as building blocks give a systematic advantage to one of the parties over the other, depending on the value of the parameters and the order of choice. *Specifically, we prove that regardless of their preferences, both parties would prefer the same player position (whether first or second mover) due to the outcomes that this implies for each one.* Because of that, we propose two extensions for each one of them and come up with mechanisms that reduce the gap in the treatment of parties while still keeping the efficiency properties of their predecessors.

The two mechanisms that improve upon the $(\theta, h)-$Split Appointment Rule are called the $\theta-$**Compromise Split Appointment Rule** and the $\theta-$**Splitting Appointment Contest**. Those that improve upon the $\theta-$Rule of k Names method are called the $\theta-$**Compromise Rule of k Names** and the $\theta-$**Rule of k Names Contest**. What these four mechanisms have in common is that, before putting into practice one of the methods that we propose as building blocks, parties are allowed to be active participants in determining the value of parameters and the order of play necessary to fully define the procedure. The main difference between the two variants of each method lies in the length and complexity of the games they induce.

Before defining the new mechanisms and describing their properties, let us present the criteria by which we shall evaluate their fairness. We define, for each preference profile, two sets of candidates that express notions of compromise from different perspectives. This allows us, then, to discuss whether the SPNE outcomes of the games induced by our four mechanisms are either contained in one or the other.

The first of these two sets expressing the idea of fair treatment of the parties is the **Unanimity Compromise Set**, defined as the first intersection of the parties' top-ranked candidates with at least one candidate as we consider sequentially lower and lower levels in their rankings.

This set is extensively studied in the existing literature and can be obtained as the outcome of the Fallback Bargaining procedure proposed and studied by Hurwicz and Sertel (1997) and Brams and Kilgour (2001) (see Cailloux, Napolitano, and Sanver, 2023).

The first solution is a generalization of the Unanimity Compromise Set to select θ alternatives. The **θ-Unanimity Compromise Set** is the first intersection of the parties' top-ranked candidates with at least θ candidates as we consider sequentially lower and lower levels in their rankings.

The second compromise solution, called **θ-Top Compromise Set**, is new, as far as we know. It is the first union of the parties' top-ranked candidates with at least θ candidates as we consider sequentially lower and lower levels in their rankings.

These compromise sets are never empty and have θ or $\theta + 1$ elements.

Notice that the θ-Unanimity Compromise Set aims at equalizing the opportunities of players to avoid their worst outcomes, while the θ-Top Compromise Set pays special attention to their shared opportunities to attain the best-ranked ones.

Let us now be more specific about our new mechanisms.

The **$\theta-$Compromise Rule of k Names** works as follows: the first mover chooses any integer k in $[\theta, \mathbf{c}]$. Once this choice is made public, the second mover decides whether to play as the proposer or the chooser. Then the proposer selects k candidates out of those in an original list $\mathbf{C}$, and the chooser selects θ candidates out of those proposed by the opponent.[5]

The **$\theta-$Compromise Split Appointment Rule** works as follows: the first mover chooses any integer h in $\left[0, \left\lceil \frac{\theta}{2} \right\rceil\right]$. Once this choice is made public, the second mover

[5] Compromise Rule of k Names for selecting a single alternative was first proposed by Hervé Moulin in his book *The Strategy of Social Choice* (Example 3, page 82).

decides whether to play as the first nominator or the second one. The first one selects h candidates out of those in $\mathbf{C}$, and then the second selects $\theta - h$ among the remaining candidates not yet chosen from $\mathbf{C}$.

Under these two mechanisms, we will demonstrate that both parties weakly prefer to play as the second mover. However, the generic advantage for the second mover is very limited, as it only matters when the compromise solution is not unique. Our next two mechanisms eliminate this asymmetry through simultaneous movement in the first stage, determining both the set to be offered and by whom.

The $\theta-$**Rule of k Names Contest** works as follows: Both parties simultaneously propose a non-empty subset of $\mathbf{C}$ with cardinality greater than or equal to θ. The subset with the highest cardinality prevails and whoever proposed the discarded subset shall select the θ winning candidates from the prevailing subset. If the cardinalities are the same and odd, the parties know that Party 1's proposed subset prevails, otherwise, Party 2's proposed subset prevails.[6]

The $\theta-$**Splitting Appointment Contest** works as follows: Both parties simultaneously propose a subset of $\mathbf{C}$ with a cardinality smaller than or equal to θ. Denote by h the cardinality of the subset with the smallest cardinality. The candidates of this subset are selected. Whoever proposed the discarded subset shall select $\theta - h$ candidates among the remaining candidates not yet chosen from $\mathbf{C}$. If the cardinalities are the same and odd, the parties know that Party 1's proposed subset prevails; otherwise, Party 2's proposed subset prevails.

The unique SPNE outcomes under the $\theta-$Compromise Rule of k Names and $\theta-$Rule of k Names Contest are always contained in the $\theta-$Unanimity Compromise Set. If at least one of the parties has leximin extension preferences, they are always Pareto efficient. Conversely, the unique SPNE outcomes under the $\theta-$Compromise Split Appointment Rule and $\theta-$Splitting Appointment Contest are always contained in the θ-Top Compromise Set. If at least one of the parties has leximax extension preferences, they are always Pareto efficient.

Let us develop an example where parties are required to select four out of six candidates, illustrating how the mechanisms work and induce the parties to adopt a compromise

[6] Barberà and Coelho (2022) proposed to apply this mechanism for the selection of a single alternative and referred to it as the Shortlisting Contest method.

solution. Assume that Party 1 prefers the candidates in the order, $c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6$, and Party 2's preferences are in the order $c1 \succ_2 c2 \succ_2 c6 \succ_2 c4 \succ_2 c5 \succ_2 c3$, and that both parties know the preferences of each other.

The 4-Unanimity Compromise Solution is $\{c1, c2, c4, c5\}$ and the depth of the profile (d^*) is 5 (the ranking of the worst alternative in this set according to the parties' preferences).

Let us start with the 4-Compromise Rule of k Names, and assume the preferences of both parties on four element sets are leximin extensions. The last two stages of this mechanism consist of subgames characterized by a value of k and by who submits the shortlist (the proposer). Party 2 has the advantage of choosing whether to be the proposer or not. Party 1 can minimize this advantage by choosing the value of k.

Let's analyze the outcome if Party 2 acted as the proposer for each possible value of k:

$k = 4$: $\{c1, c2, c4, c6\}$

$k = 5$: $\{c1, c2, c4, c5\}$

$k = 6$: $\{c1, c2, c3, c4\}$

If the proposer was Party 1, the outcome would be:

$k = 4$: $\{c1, c2, c3, c4\}$

$k = 5$: $\{c1, c2, c4, c5\}$

$k = 6$: $\{c1, c2, c4, c6\}$

Therefore, Party 2's best strategy is to act as the proposer only if $k \leq 5$. In this paper, we provide a formula for this threshold which is $\mathbf{c} - d^* + \theta$. Consequently, Party 1's best strategy is to choose $k = 5$ to ensure the election of $\{c1, c2, c4, c5\}$. Under the game induced by the 4$-$Rule of k Names Contest mechanism, in equilibrium, we show that both parties propose subsets with the same cardinality, equal to $\mathbf{c} - d^* + \theta$. Party 1 proposes $\{c1, c2, c3, c4, c5\}$ and Party 2 proposes $\{c1, c2, c4, c5, c6\}$, so $\{c1, c2, c3, c4, c5\}$ prevails, and then in the second stage, Party 2 chooses $\{c1, c2, c4, c5\}$.

The outcome turns out to be the same in both cases and it is Pareto efficient under our assumption on extended preferences. It is also contained in the 4-Unanimity Compromise Set. Notice, however, that if both parties' preferences over sets were leximax extensions, the outcome would be the same but it would be Pareto dominated by $\{c1, c2, c3, c6\}$.

Let's now consider the 4$-$Compromise Split Appointment Rule. The 4-Top Compromise Set is $\{c1, c2, c3, c6\}$ and the union depth of the profile (d_u^*) is 3 (ranking of the worst alternatives in this set according to parties' preferences). Suppose that both parties' preferences over sets are leximax extensions of the preferences over candidates.

The last two stages of this mechanism consist of different subgames that are characterized by a value of h and by who will nominate in the first place. Given this preference profile, if Party 2 played as the first nominator, the equilibrium outcome would be:

$h = 0$: $\{c1, c2, c3, c4\}$

$h = 1$: $\{c1, c2, c3, c6\}$

$h = 2$: $\{c1, c2, c4, c6\}$

If the first nominator was Party 1, the equilibrium outcome would be:

$h = 0$: $\{c1, c2, c4, c6\}$

$h = 1$: $\{c1, c2, c3, c6\}$

$h = 2$: $\{c1, c2, c3, c4\}$

Consequently, the second mover's best strategy is to choose to act as the second nominator only if $h \leq 1$. In this paper, we provide a formula for this threshold which is $\theta - d_u^*$. Given this strategy, the first mover chooses $h = 1$ to ensure the election of $\{c1, c2, c3, c6\}$ which is Pareto efficient and is contained in the 4-Top Compromise Set. Under the game induced by the $\theta-$Splitting Appointment Contest mechanism, in equilibrium, we show that the party whose subset prevails when both announced subsets have a cardinality equal to $\theta - d_u^*$, proposes a subset with this cardinality, while her opponent proposes a subset with a cardinality one unit higher. Thus, in the example at hand, Party 1 proposes $\{c3\}$ and Party 2 proposes $\{c4, c6\}$, so $\{c3\}$ prevails, and then in the second stage, Party 2 chooses $\{c1, c2, c6\}$. Thus, the SPNE outcome is $\{c1, c2, c3, c6\}$. Notice that if both parties' preferences over sets were leximin extensions, the outcome would be the same but it would be Pareto dominated by $\{c1, c2, c4, c5\}$.

Let us finish this introduction with references to the related literature, Echenique and Núñez (2022), Laslier, Núñez, and Sanver (2021), Núñez and Laslier (2015), de Clippel, Eliaz and Knight (2014) and Anbarci (1993 and 2006) propose methods to achieve compromise between two concerned parties. These are designed to select only a single alternative. Refer to Barberà and Coelho (2022) for a comprehensive review of most of these methods related to the idea of Rules of k Names. Regarding the selection of a group of fixed size, as far as we know, the closest papers to ours are Felsenthal and Machover

(1992) and Van der Linden (2018) who also study the $\theta-$Rule of k Names.

Felsenthal and Machover (1992) generalize the sequential voting by veto (studied by Mueller (1978) and Moulin (1983)) to select a subset with a fixed size. According to this procedure, a sequence of n voters must select θ alternatives. The ith voter, when her turn comes, vetoes v_i alternatives. The θ remaining non-vetoed alternatives are selected. So, the $\theta-$Rule of k Names is a special case of this method when $n = 2$, $v_1 = \mathbf{c} - k$ and $v_2 = k - \theta$. They assume the same set of assumptions that we use such as complete information and the Responsiveness axiom of the preferences. They provide an algorithm to compute the unique backward induction outcome of this method but do not provide results regarding Pareto optimality or fairness.

Van der Linden (2018) introduces a new measure of strategic complexity to compare jury selection preemptory-challenge procedures used in practice. One such procedure is the Rule of k Names, which he terms the "One-shotQ procedure" and identifies as having the lowest complexity among them. His metric of strategic complexity, which he calls the "Dominance Threshold," represents the number of rounds required to eliminate strategies that are never best responses for the parties to have a dominant strategy. In contrast to our work, he does not offer findings concerning subgame Nash equilibria, Pareto optimality, or compromise.

Several papers also study the strategic aspects of voting rules in multi-winner elections with multiple voters, with many focusing on approval balloting. See, for instance, Meir et al. (2008), Aleskerov et al. (2012), and Laslier and Van der Straete (2016).

The remainder of this paper is organized as follows: Section 2 presents the formal model and the characterization of the equilibria of the two auxiliary rules whose parameters are endogenized by our four mechanisms. Section 3 presents the characterizations of the unique SPNE outcome of our proposed compromise-seeking mechanisms and their properties. Section 4 concludes with final comments and suggested lines for further work. Appendix 1 demonstrates that efficiency can be violated by using the rule proposed by JAMS, even when the conditions under which we achieve efficiency through our methods hold. Appendix 2 contains proofs of theorems and propositions that are only outlined in the main text.

2 The Model and Results

Consider any finite set of candidates, C. There are two parties, 1 and 2. Let $\mathbf{P}$ be the set of all strict orders on C.[7] Preference profiles are elements of $\mathbf{P} \times \mathbf{P}$, denoted as $(\succ_1, \succ_2)$. These two components are interpreted to be the preferences of parties 1 and 2, respectively.

Let $\mathbf{W}$ be the set of all strict orders on all subsets of C with cardinality θ to be interpreted as one of the parties' preferences over subsets. We assume that these preferences respect the following axiom, which establishes a condition under which a given party prefers one subset of candidates over another given her preferences over individual candidates.

Definition 1 *Responsiveness Axiom. For any $Y \subseteq \mathbf{C}$ and any $a, b \in \mathbf{C}$, we have if $a \succ b$, $b \in Y$, $a \notin Y$ then $\{a\} \cup Y \backslash \{b\}$ is preferred to Y.*[8]

It is worth noting that the Responsiveness Axiom indicates a lack of complementarity among candidates. As demonstrated in Bossert (1995), this axiom and the Fixed-Cardinality Neutrality axiom characterize the class of rank-ordered lexicographic ordering. This class encompasses both leximin and leximax orderings.[9]

2.1 Characterizing the equilibria under the $\theta-$Rules of k Names

Our first proposition provides a characterization of the SPNE outcome of the game among two parties induced by the use of the $\theta-$Rules of k Names.

Proposition 1 *The unique SPNE outcome of the game induced by the $\theta-$Rule of k Names is the set of proposer's preferred θ candidates among the chooser's $(c-k+\theta)$-top candidates. There may be several SPNE strategy profiles leading to the unique common outcome.*

[7]Transitive: For all $x, y, z \in \mathbf{C}$: ($x \succ y$ and $y \succ z$) implies that $x \succ z$. Asymmetric: For all $x, y \in \mathbf{C}$: $x \succ y$ implies that $\neg(y \succ x)$. Irreflexive: For all $x \in \mathbf{C}$,$\neg(x \succ x)$. Complete: For all $x, y \in \mathbf{C}$: $x \neq y$ implies that ($y \succ x$ or $x \succ y$).

[8]The Responsiveness Axiom, a modified version of the monotonicity axiom defined by Kannai and Peleg (1984), is used by various authors including Kaymak and Sanver (2003) and Barberà and Coelho (2008).

[9]Lainé, Lang, Aziz, Özkal-Sanver, and Sanver (2024) examine the Responsiveness axiom and lexicographic extensions to explore the Pareto efficiency of ordinal multiwinner voting rules in the context of nonstrategic voters.

A strategy profile is an SPNE of this game if and only if its strategies satisfy the following two conditions:
C1. The chooser always selects her θ preferred candidates in any subset submitted by the proposer.
C2. The proposer always submits a subset that contains her θ preferred candidates among the chooser j ´s $(c-k+\theta)$-top candidates and any other $k-\theta$ lower ranked candidates than those θ candidates according to chooser's preferences.

Proof. At equilibrium, the chooser must select a best response to each of the proposer's actions. Our assumption that parties' preferences are strict and satisfy the Responsiveness Axiom ensures, respectively, that the chooser's best response to each action of her opponent is unique and that the corresponding best response set is the union of the chooser's best ranked candidates among those offered by the proposer. This proves statement C1.
Given the above, a best response of the proposer to the chooser's equilibrium action must conform to the description in C2. This is because, under this rule, the chooser will never select any of her $(k-\theta)$ worst candidates, regardless of the proposal it faces. Hence, the proposer can select her best θ candidates among the $\mathbf{c}-(k-\theta)$ best candidates for the chooser. Given this rule, the unique most desirable choice that the proposer can aim at equilibrium is the one formed by her θ best alternatives among those that might be selected by the chooser in response, again using our hypothesis regarding parties' preferences. To obtain this result, the proposer must complete her list of k candidates by adding to the above θ ones another $k-\theta$ that the chooser ranks below them.
Finally, observe that even if these remaining $k-\theta$ candidates may be selected in a non-unique way, and hence the best strategies of the proposer may not be unique, the equilibrium outcome will be the same under all of them. This remark about uniqueness completes the proof. ■

Corollary 1 *If at least one of the parties has preferences over sets that are leximin extensions of the preferences over candidates then the unique SPNE outcome under the $\theta-$Rule of k Names is Pareto Efficient.*

Proof. Suppose that X, the SPNE outcome, is Pareto dominated by another set of candidates Y of the same size. Given our assumption of strict preferences, this means that both the proposer and the chooser prefer Y to X. For a party whose preferences over sets are leximin extensions of her preferences over candidates to prefer Y to X, there must be two distinct candidates, y in Y and x in X such that all candidates in both sets that are ranked below them in the party's preferences are the same and y is preferred to x.

First, assume that the chooser's preferences are the leximin extension of her preferences over alternatives and let x and y be as above. Then, if the elements of X are among the $(\mathbf{c} - k + \theta)$ top candidates of the chooser, so are also those of Y. This contradicts the assumption that X is an equilibrium because that would require it to be the proposer's best set of candidates out of these top ones for the chooser, according to Proposition 1, while Y is better than X for the proposer.

Now consider the case where the proposer's preferences over sets are the leximin extension of those about candidates. By Proposition 1, X is the set of the proposer's preferred candidates among the chooser' $(\mathbf{c} - k + \theta)$ top candidates. Take any $b \in Y \backslash X$, this candidate exists by the Responsiveness Axiom, and since the chooser prefers Y to X. Note that b is among the chooser' $(\mathbf{c} - k + \theta)$ top candidates. Additionally, by leximin assumption and proposer's preference for Y over X, proposer prefers b to x. Let Z be $Z \equiv X \cup \{b\} \backslash \{x\}$, which proposer prefers to X according to the leximin assumption. Note that all elements of Z are among the chooser´s $(\mathbf{c} - k + \theta)$-top candidates. This is a contradiction since X is the set of proposer's preferred candidates among the chooser´s $(\mathbf{c} - k + \theta)$-top candidates. ■

2.2 Characterizing the equilibria under the $(\theta, k)-$Split Appointment Rule

Our second proposition provides a characterization of the SPNE outcome of the game among two parties induced by the use of the $(\theta, h)-$Split Appointment Rule.

Proposition 2 *The unique SPNE outcome of the game induced by the $(\theta, h)-$Split Appointment Rule is the set formed by nominator 2's $(\theta - h)$-top candidates jointly with nominator 1's h preferred candidates among nominator 2' s $(\mathbf{c} - (\theta - h))$-bottom candidates. There may be several SPNE strategy profiles leading to the unique common outcome. A strategy profile is an SPNE of this game if and only if its strategies satisfy the following* *two conditions:*

(i) *In the first stage, nominator 1 nominates a subset formed by her h preferred candidates among nominator 2's $(\mathbf{c}+h-\theta)$-bottom candidates.*

(ii) *In the second stage, given any subset S nominated by nominator 1, her opponent picks her $(\theta-h)$ preferred candidates from the set difference of $\boldsymbol{C}$ and S.*

Proof. Let (X_1, X_2) represent the sets proposed by the parties at equilibrium, such that $X_1 \cup X_2$ constitutes the equilibrium outcome of the game induced by the (θ, h)-Split Appointment Rule. For X_2 to be a best response to the choice of X_1 by Party 1, it must be the best subset of size $\theta - h$, according to Party 2's preferences, among those subsets that can be formed with the candidates in the set $\mathbf{C}\backslash X_1$. This best subset X_2 is unique, because of our assumption that preferences over sets are strict, and formed by Party 2's highest ranked elements, because we assume that the Responsiveness Axiom holds. Knowing that 2 will choose her $\theta - h$ best elements not yet chosen by 1, Party 1 will concentrate on selecting her best set X_1 of candidates out of 2's $(\mathbf{c}-(\theta-h))$ bottom candidates. This set, given the Responsiveness Axiom, will be formed by 1's best elements in that subset according to the preferences of the first party, and will be unique, because of the assumption of strict preferences over sets.
Finally, note that the uniqueness of the optimal strategy of Party 1 at equilibrium carries with it the uniqueness of Party 2's response. ■

Corollary 2 *If at least one of the parties has preferences over sets that are leximax extensions of the preferences over candidates then the unique SPNE outcome under the $(\theta, h)-$Split Appointment Rule is Pareto Efficient.*

Proof. Suppose that Party 2's preferences are leximax. Then, the set of 2's $\theta - h$ best candidates coincides with the unconditional best set of that size. Hence, since at equilibrium, 1 will give 2 the chance to select her absolute best while having to accept 1's choices, there is no room for any further improvement of 2's position, and this proves the equilibrium to be Pareto efficient as long as preferences are strict.
Suppose now that Party 1's preferences are leximax. Since 2's preferences satisfy the Responsiveness Axiom, whatever strategy 1 uses, the optimal choice of 2 will contain 2's best candidates out of those that have not been already proposed by 1, and the outcome

will always contain all of 2's best $(\theta - h)$ candidates. Given that constraint, the absolute best action by 1 consists in letting 2 make the choice of these candidates in her entirety and to select the best h ones out of the rest of the candidates. This choice, conditional to the unavoidable selection of 2's $\theta - h$ candidates, is 1's absolute best under the assumption of leximax preferences, and cannot be improved upon. This proves that the equilibrium will be Pareto efficient. ■

3 Characterizing the equilibria under our Mechanisms

3.1 Compromise Solutions and Their Parameters

3.1.1 The $\theta-$Unanimity Compromise Set, $\theta-$depth, and $\theta-$mirrored depth

As defined in the introduction, the $\theta-$Unanimity Compromise Set is the first intersection of the parties' top-ranked candidates with at least θ candidates as we consider sequentially lower and lower levels in their rankings. Now let us define two parameters derived from this set.

Definition 2 *The $\theta-$depth of a preference profile over candidates is denoted by $d^*(\succ_1, \succ_2, \theta)$ and it is the smallest value of q in [1,c] such that the intersection between the parties' q-top candidates has at least θ candidates.*

Definition 3 *The $\theta-$mirrored depth of the preference profile is denoted by $k^*(\succ_1, \succ_2, \theta)$ and it is defined as follows:*

$k^*(\succ_1, \succ_2, \theta) \equiv c - d^*(\succ_1, \succ_2, \theta) + \theta.$

Example 1 *Consider this preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6$ and $c1 \succ_2 c2 \succ_2 c6 \succ_2 c4 \succ_2 c3 \succ_2 c1$) and $\theta = 3$. Note that 3-Unanimity Compromise Solution is $\{c1, c2, c4\}$. Applying the definitions 2 and 3, we have that $d^*(\succ_1, \succ_2, \theta) = 4$ and $k^*(\succ_1, \succ_2, \theta) \equiv \mathbf{c} - d^*(\succ_1, \succ_2, \theta) + \theta = 6 - 4 + 3 = 5$.*

Intuitively, the $\theta-$depth of a preference profile provides the ranking of the worst candidate in that intersection according to the parties' preferences. The $\theta-$mirrored depth of the preference profile is equal to θ plus the number of candidates ranked below that profile's depth. Setting a value of k equal to this parameter ensures that candidates within the equilibrium outcome are positioned above the profile's depth.

Remark 1 *The intersection between the parties' $d^*(\succ_1, \succ_2, \theta)$-top candidates is equal to the θ-Unanimity Compromise Solution. In this set, there are θ or $\theta+1$ candidates. Note that the $\theta \le d^*(\succ_1, \succ_2, \theta) \le \frac{\mathbf{c}+\theta}{2}$ if c+θ is even, $\theta \le d^*(\succ_1, \succ_2, \theta) \le \frac{\mathbf{c}+\theta+1}{2}$, otherwise. This parameter attains its maximum value when the parties have opposite preferences over individual candidates. Conversely, it reaches its minimum value when the parties share the same $\theta-$top candidates. This implies that $\frac{\mathbf{c}+\theta}{2} \le k^*(\succ_1, \succ_2, \theta) \le \mathbf{c}$ if c+θ is even and $\frac{\mathbf{c}+\theta-1}{2} \le k^*(\succ_1, \succ_2, \theta) \le \mathbf{c}$, otherwise.*

3.1.2 The θ-Top Compromise Set and $\theta-$union_depth

As defined in the introduction, the $\theta-$Top Compromise Set is the first union of the parties' top-ranked candidates with at least θ candidates as we consider sequentially lower and lower levels in their rankings. Now we define the $\theta-$union_depth derived from this set.

Definition 4 *The $\theta-$union_ depth of a preference profile over candidates is denoted by $d_u^*(\succ_1, \succ_2, \theta)$ and it is the smallest value of q in* [1,c] *in which the union of parties' q-top candidates has at least θ candidates.*

Example 2 *Consider this preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6$ and $c1 \succ_2 c2 \succ_2 c6 \succ_2 c4 \succ_2 c3 \succ_2 c4$) and $\theta = 3$. Note that $3-$Top Compromise Set is $\{c1, c2, c3, c6\}$. Applying Definition 4, we have that $d_u^*(\succ_1, \succ_2, \theta) = 3$.*

Intuitively, the $\theta-$union_depth of a preference profile provides the ranking of the worst candidate in the $\theta-$Top Compromise Set according to the parties' preferences.

Our next remark can clarify Definition 4.

Remark 2 *Thus, the union of parties' $d_u^*(\succ_1, \succ_2, \theta)$-top candidates is equal to θ-Top Compromise Set. In this set, there are θ or $\theta+1$ candidates. Note also that $\lceil \frac{\theta}{2} \rceil \le d_u^*(\succ_1, \succ_2, \theta) \le \theta$. This parameter attains its minimum value when the parties have opposite preferences over individual candidates. Conversely, it reaches its maximum value when the parties share the same $\theta-$top candidates.*

In the following two subsections, we present our theorems concerning the four mechanisms that we propose. To facilitate reading, we provide intuitive and quite detailed outlines of proofs in the form of remarks, while reserving the more detailed arguments and formal proofs for Appendix 2.

3.2 Characterizing the equilibria under the $\theta-$Compromise Rule of k Names and the $\theta-$Rule of k Names Contest

We turn now to characterize, by means of theorems 1 and 2, a subgame perfect equilibrium of the games induced by the use of the $\theta-$Compromise Rule of k Names and the $\theta-$Rule of k Names Contest. The proofs of these theorems rely on the next proposition, which delineates the preferences of agents regarding the roles of proposer and chooser under the $\theta-$Rule of k Names. It asserts that regardless of the specific value of k and the preference profile, both parties hold identical preferences regarding the roles of proposer and chooser.

Proposition 3 *Consider any integer k' in $[\theta, c]$. Under any SPNE strategy profile, if k' is not greater (greater) than the mirrored depth, $k^*(\succ_1, \succ_2, \theta)$, then both parties are weakly better off when playing as the proposer (chooser) under the $\theta-$Rule of $k^*(\succ_1, \succ_2, \theta)$ Names than playing as the chooser (proposer) under the $\theta-$Rule of k' Names.*

The proof of Proposition 3 is partially based on our next remark.

Remark 3 *Note that, by the Responsiveness Axiom, Party $i \in \{1, 2\}$'s best θ candidates among her opponent's $d^*(\succ_1, \succ_2, \theta)$ top candidates are also Party i's best θ candidates in the intersection of parties' $d^*(\succ_1, \succ_2, \theta)$ top candidates. This implies, jointly with Proposition 1 and Definition 2, that the SPNE outcome under the $\theta-$Rule of $k^*(\succ_1, \succ_2, \theta)$ Names is the proposer's best θ candidates in the θ-Unanimity Compromise Solution. Thus, it also implies that under $\theta-$Rule of $k^*(\succ_1, \succ_2, \theta)$ Names, both parties weakly prefer playing as the proposer than playing as the chooser.*

We use Proposition 3 in the proof of Theorem 1.

Theorem 1 *The game induced by the $\theta-$Compromise Rule of k Names method has a subgame perfect equilibrium such that:*

(i) *In the first stage, Party 1 chooses $k = k^*(\succ_1, \succ_2, \theta)$.*

(ii) *In the second stage, for any integer value of k in $[\theta, c]$ chosen by Party 1, Party 2 opts to be the proposer if $k \leq k^*(\succ_1, \succ_2, \theta)$, and to be the chooser otherwise.*

(iii) *In the third stage, for any value of* k *in* $[\theta, c]$ *chosen by Party 1, whoever is the proposer proposes a subset that contains her* θ *preferred candidates among the chooser's* $(c - k + \theta)$ *top candidates, plus the chooser's* $k - \theta$ *worst candidates.*

(iv) *In the fourth stage, for any subset* S*, whoever is the chooser picks her* θ *preferred candidates out of the opposing party's proposed subset.*

As a consequence, the unique SPNE outcome of $\theta-$*Compromise Rule of k Names method is the set of Party 2's preferred* θ *candidates among Party 1'* $d^*(\succ_1, \succ_2, \theta)-$*top candidates.*

Example 3 *Consider this preference profile (*$c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6$ *and* $c1 \succ_2 c2 \succ_2 c6 \succ_2 c5 \succ_2 c3 \succ_2 c4$*) and the* $4-$*Compromise Rule of k Names. Note that according to definitions 2 and 3, we have that* $d^*(\succ_1, \succ_2, \theta) = 5$ *and* $k^*(\succ_1, \succ_2, \theta) = 6 - 5 + 4 = 5$*. By Theorem 1, the unique SPNE outcome under the* $4-$*Compromise Rule of k Names method is the set of Party 2's preferred four candidates among Party 1's* 5*-top candidates, which is* $\{c1, c2, c3.c5\}$*.*

Remark 4 provides an intuition for Theorem 1.

Remark 4 *The strategy profiles of the subgames formed by the last two stages of the* $\theta-$*Compromise Rule of k Names have a similar format to those characterized by Proposition 1. The first two stages of this mechanism determine which one of these subgames the parties will play. By Proposition 1, if Party 1 chooses a value of k equal to* $k^*(\succ_1, \succ_2, \theta)$*, it will induce an outcome that is contained in the* $\theta-$*Unanimity Compromise Solution. Proposition 3 helps us map out the most effective strategies that parties may employ in these initial stages. It explains why there is no profitable deviation under the SPNE described in Theorem 1. Under that strategy profile, if Party 1 deviates by choosing a* k *greater than* $k^*(\succ_1, \succ_2, \theta)$*, this party would assume the role of proposer as long as her opponent's strategy remains unchanged. However, Proposition 3 shows that playing as the chooser under* $k^*(\succ_1, \succ_2, \theta)$ *is at least as good as playing as the proposer under a* k *greater than* $k^*(\succ_1, \succ_2, \theta)$*. Additionally, Party 2 has also no incentive to deviate, given that being the proposer under* $k^*(\succ_1, \succ_2, \theta)$ *is at least as good as being the chooser under any value of* k *greater than* $k^*(\succ_1, \succ_2, \theta)$*.*

Corollary 3 *The unique SPNE outcome under the $\theta-$Compromise Rule of k Names method is the subset of Party 2's preferred θ candidates within the $\theta-$Unanimity Compromise Solution.*

The following result about efficiency is a consequence of Theorem 1 and Corollary 1.

Corollary 4 *If the preferences over sets of at least one of the parties are leximin extensions of the preferences over individual candidates then the SPNE outcome under the $\theta-$Compromise Rule of k Names is always Pareto Efficient.*

We now state the characterization of a subgame perfect equilibrium of the game induced by the $\theta-$Rule of k Names Contest method.

Theorem 2 *The game induced by the $\theta-$Rule of k Names Contest has a subgame perfect equilibrium such that*

(i) *In the first stage, each party proposes a subset with cardinality equal to $k^*(\succ_1, \succ_2, \theta)$ that contains its preferred candidate among the opposing party's $(c - k^*(\succ_1, \succ_2, \theta) + \theta)$ top candidates plus the $k^*(\succ_1, \succ_2, \theta) - 1$ worst candidates according to the opposing party's preference.*

(ii) *In the second stage, whoever party is the chooser picks its θ preferred candidates out of the opposing party's proposed subset.*

As a consequence, a set is a subgame perfect equilibrium outcome when $k^*(\succ_1, \succ_2, \theta)$ is odd (respectively, $k^*(\succ_1, \succ_2, \theta)$ is even) if and only if it is Party 1's (respectively Party2's) θ best candidates among Party 2's (respectively Party1's) $d^*(\succ_1, \succ_2, \theta)-$top candidates.

Example 4 *Consider this preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c1 \succ_2 c2 \succ_2 c4 \succ_2 c3 \succ_2 c5$) and the $3-$Rule of k Names Contest. Note that according to definitions 2 and 3, we have that $d^*(\succ_1, \succ_2, \theta) = 4$ and $k^*(\succ_1, \succ_2, \theta) = 5 - 4 + 3 = 4$. By Theorem 2, as $k^*(\succ_1, \succ_2, \theta)$ is even, the unique SPNE outcome under the $3-$Rule of k Names Contest is the set of Party 2's preferred three candidates among Party 1's top four candidates, which is $\{c1, c2, c4\}$.*

Remark 5 *The subset size contest of the $\theta-$Rule of k Names Contest prompts both parties to propose subsets with the same size, discouraging the other from proposing a subset of a different size. These subsets, with a cardinality of $k^*(\succ_1, \succ_2, \theta)$, have compositions similar to those induced by the $\theta-$Rule of $k^*(\succ_1, \succ_2, \theta)$ Names, as described in Proposition 1. If $k^*(\succ_1, \succ_2, \theta)$ is odd, Party 1's subset prevails; otherwise, Party 2's does. According to Proposition 3, the party whose subset prevails (does not prevail) lacks the incentive to deviate by proposing another subset with larger (smaller) cardinality, as this would result in them becoming the chooser (the proposer) if her opponent's strategy remains unchanged. The subset prevailing in equilibrium leads to an outcome contained within the $\theta-$ Unanimity Compromise Set.*

Corollary 5 *If $k^*(\succ_1, \succ_2, \theta)$ is odd (respectively, $k^*(\succ_1, \succ_2, \theta)$ is even) then the unique SPNE outcome under the $\theta-$Rule of k Names Contest is the subset of Party2's preferred θ candidates (respectively, Party 1's preferred θ candidates) within the $\theta-$ Unanimity Compromise Solution.*

The following result about efficiency is a consequence of Theorem 2 and Corollary 1.

Corollary 6 *If the preferences over sets of at least one of the parties are leximin extensions of the preferences over individual candidates then the SPNE outcome under the $\theta-$Rule of k Names Contest is always Pareto Efficient.*

3.2.1 Characterizing the equilibria under the $\theta-$Compromise Split Appointment Rule and the $\theta-$Splitting Appointment Contest

The next proposition addresses the preferences of agents regarding their possible roles within the $\theta-$Compromise Split Appointment Rule and $\theta-$Splitting Appointment Contest. In a similar spirit to Proposition 3, which addressed the same issue for the two previous mechanisms.

Proposition 4 *Under any SPNE strategy profile, if $\acute{h}$ is not greater than $\theta - d_u^*(\succ_1, \succ_2, \theta)$, then both parties are weakly better off when playing as the second nominator under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule than playing as the first nominator under the $(\theta, \acute{h})-$Split Appointment Rule. Otherwise, both parties are weakly better off playing as the first nominator under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule than playing as the second nominator under the $(\theta, \acute{h})-$Split Appointment Rule.*

The proof of Proposition 4 is partially based on our next remark and Claim 1.

Remark 6 *Note that, by Proposition 2, the SPNE outcome under $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule is the set formed by nominator 2's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates jointly with nominator 1's $(\theta - d_u^*(\succ_1, \succ_2, \theta))$ preferred candidates among nominator 2' s $(\mathbf{c} - d_u^*(\succ_1, \succ_2, \theta))$-bottom candidates. This implies, joint with Definition 4, that this SPNE outcome is contained in the union of parties' $d_u^*(\succ_1, \succ_2, \theta)$-top candidates (the θ-Top Compromise Set). According to Remark 2, the θ-Top Compromise Set has θ or $\theta + 1$ candidates. Suppose that this set has θ candidates. Thus, this SPNE outcome is equal to the θ-Top Compromise Set. Suppose that this set has $\theta + 1$ candidates. Thus, this SPNE outcome contains the first nominator's $d_u^*(\succ_1, \succ_2, \theta) - 1$-top candidates and the second nominator's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates. Therefore, under $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule, both parties weakly prefer playing as the nominator 2 than playing as the nominator 1.*

Claim 1 *Under any SPNE strategy profile, both parties are weakly better off when playing as the first nominator under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule than playing as the second nominator under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1)-$Split Appointment Rule.*

The next two examples illustrate Claim 1. In the first example, the θ-Top Compromise Set has θ candidates.

Example 5 *Consider the $(\theta = 3, h)-$Compromise Split Appointment Rule and this preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c1 \succ_2 c4 \succ_2 c5 \succ_2 c2 \succ_2 c3$). First notice that $d_u^*(\succ_1, \succ_2, \theta) = 2$ and the θ-Top Compromise Set is $\{c1, c2, c4\}$. If Party 1 is the first nominator then the SPNE outcome is $\{c1, c2, c4\}$ when $h = \theta - d_u^*(\succ_1, \succ_2, \theta) = 1$. Conversely, if Party 1 is the second nominator and $h = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1 = 2$ then the SPNE outcome is $\{c1, c4, c5\}$. In this case, Party 1 prefers the first SPNE outcome.*

In our next example, the θ-Top Compromise Set has $\theta + 1$ candidates.

Example 6 *Consider the $(\theta = 2, h)-$Compromise Split Appointment Rule and this preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c1 \succ_2 c4 \succ_2 c5 \succ_2 c2 \succ_2 c3$). First notice that $d_u^*(\succ_1, \succ_2, \theta) = 2$ and the θ-Top Compromise Set is $\{c1, c2, c4\}$. If Party 1 is* *the first nominator then the SPNE outcome is* $\{c1, c4\}$ *when* $h = \theta - d^*_u(\succ_1, \succ_2, \theta) = 0$*. Conversely, if Party 1 is the second nominator and* $h = \theta - d^*_u(\succ_1, \succ_2, \theta) + 1 = 1$ *then the SPNE outcome is* $\{c1, c4\}$*. In this case, the two SPNE outcomes are equal.*

We use Proposition 4 in the next theorem to build a subgame perfect equilibrium of the game induced by the $\theta-$Compromise Split Appointment Rule.

Theorem 3 *The game induced by the* $\theta-$*Compromise Split Appointment Rule has a subgame perfect equilibrium such that:*

(i) *In the first stage, Party 1 chooses* $h = \theta - d^*_u(\succ_1, \succ_2, \theta)$*.*

(ii) *In the second stage, for any integer value of* h in $\left[0, \left\lceil \frac{\theta}{2} \right\rceil\right]$*, Party 2 opts to be the second nominator unless* $h > \theta - d^*_u(\succ_1, \succ_2, \theta)$*.*

(iii) *In the third stage, for any value of* h in $\left[0, \left\lceil \frac{\theta}{2} \right\rceil\right]$*, whoever is the first nominator nominates a subset that contains her* h *preferred candidates among her opponent's* $(\mathbf{c} + h - \theta)$*-bottom candidates.*

(iv) *In the fourth stage, for any choice of the first nominator* S*, whoever is the second nominator picks her* $\theta - \#S$ *preferred candidates from the set difference of* $\boldsymbol{C}$ *and the opposing party's proposed subset.*

As a consequence, the unique SPNE outcome of the $\theta-$Compromise Split Appointment Rule is the union of the set of *Party 2's* $d^*_u(\succ_1, \succ_2, \theta)$*-top candidates and the set of Party 1's* $(\theta - d^*_u(\succ_1, \succ_2, \theta))$*-top candidates among Party 2's* $d^*_u(\succ_1, \succ_2, \theta)$*-bottom candidates.*

Example 7 *Consider this preference profile (*$c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6$ *and* $c1 \succ_2 c2 \succ_2 c6 \succ_2 c5 \succ_2 c4 \succ_2 c3$*) and the* $5-$*Compromise Split Appointment Rule. Applying Theorem 3, in equilibrium, Party 1 chooses* $h = \theta - d^*_u(\succ_1, \succ_2, \theta) = 5 - 4 = 1$ *and Party 2 opts to be the second nominator unless* $h > 1$*. Thus, in the third stage, Party 1 selects* $\{c3\}$ *and in the fourth stage Party 2 selects* $\{c1, c2, c6, c5\}$*. Therefore, the unique SPNE outcome of this method is* $\{c1, c2, c3, c5, c6\}$*.*

Remark 7 provides an intuition for Theorem 3.

Remark 7 *The strategy profiles of the subgames formed by the last two stages of the $\theta-$Compromise Split Appointment Rule have a similar format to those characterized by Proposition 2. The first two stages of this mechanism determine which one of these subgames the parties will play. By Proposition 2, if Party 1 chooses a value of h equal to $\theta - d_u^*(\succ_1, \succ_2, \theta)$, it will induce an outcome that is contained in the θ-Top Compromise Solution. Proposition 4 helps us map out the most effective strategies that parties may employ in these initial stages. It explains why there is no profitable deviation under the SPNE described in Theorem 2. If Party 1 chooses a value of h larger than $\theta - d_u^*(\succ_1, \succ_2, \theta)$, it will assume the role of the second nominator if her opponent's strategy remains unchanged. According to Proposition 4, it is not a profitable deviation. Additionally, Party 2 has no incentive to deviate from her strategy, as this proposition states that being the second nominator is at least as good as being the first nominator for any value of h that is less than or equal to $\theta - d_u^*(\succ_1, \succ_2, \theta)$.*

Corollary 7 *The unique SPNE outcome under the $\theta-$Compromise Split Appointment Rule is a subset of the θ-Top Compromise Solution. If the cardinality of the θ-Top Compromise Solution is $\theta+1$ then the SPNE outcome is the set formed by the union of Party 2's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates and Party 1's $(d_u^*(\succ_1, \succ_2, \theta) - 1)$ -top candidates. Otherwise, it is equal to the θ-Top Compromise Solution.*

The following result about efficiency is a consequence of Theorem 3 and Corollary 2.

Corollary 8 *If the preferences over sets of at least one of the parties are leximax extensions of the preferences over individual candidates then the SPNE outcome under the $\theta-$Compromise Split Appointment Rule is always Pareto Efficient.*

We now state the characterization of a subgame perfect equilibrium of the game induced by the $\theta-$Splitting Appointment Contest method.

Theorem 4 *The game induced by the $\theta-$Splitting Appointment Contest has a subgame perfect equilibrium such that*

(i) *If $\theta - d_u^*(\succ_1, \succ_2, \theta)$ is odd (respectively, if $\theta - d_u^*(\succ_1, \succ_2, \theta)$ is even), then, in the first stage, Party 1 proposes a subset with cardinality equal to $h_1 = \theta - d_u^*(\succ_1, \succ_2, \theta)$ (respectively, $h_1 = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1$) that contains his h_1 preferred candidates* *among her opponent's $(c+h_1-\theta)$-bottom candidates, and Party 2 proposes a subset with cardinality equal to $h_2 = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1$ (respectively, $h_2 = \theta - d_u^*(\succ_1, \succ_2, \theta)$) that contains his h_2 preferred candidates among her opponent's $(c + h_2 - \theta)$-bottom candidates.*

(ii) *In the second stage, for any subset S that prevails in the first stage, whoever proposed the discarded subset picks her $\theta - \#S$ preferred candidates from the set difference of C and the opposing party's proposed subset.*

As a consequence, if $\theta - d_u^*(\succ_1, \succ_2, \theta)$ is odd (respectively, if $\theta - d_u^*(\succ_1, \succ_2, \theta)$ is even) then the unique SPNE outcome of the $\theta-$Splitting Appointment Contest method is the union of the set of Party 2's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates (respectively, Party 1's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates) and the set of Party 1's $(\theta - d_u^*(\succ_1, \succ_2, \theta))$-top candidates (respectively, Party 2's $(\theta - d_u^*(\succ_1, \succ_2, \theta))$-top candidates) among Party 2's $d_u^*(\succ_1, \succ_2, \theta)$-bottom candidates (respectively, Party 1's $d_u^*(\succ_1, \succ_2, \theta)$-bottom candidates).

Example 8 *Consider this preference profile ($c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5$ and $c1 \succ_2 c2 \succ_2 c5 \succ_2 c4 \succ_2 c3$) and the 3−Splitting Appointment Contest. Notice that $\theta - d_u^*(\succ_1, \succ_2, \theta) = 3 - 3 = 0$ is even. Applying Theorem 4, in equilibrium, Party 1 proposes $\{c3\}$ and Party 2 proposes the empty set. Thus, Party 2's proposed subset prevails. Thus, in the second stage, Party 1 selects $\{c1, c2, c3\}$. Therefore, the unique SPNE outcome of this method is $\{c1, c2, c3\}$.*

Remark 8 *The nomination size contest of the $\theta-$Splitting Appointment Contest compels at least one of the parties to submit a subset with a cardinality small enough to dissuade the other from submitting an even smaller one, while their opponent selects another subset with a cardinality just one unit higher. The party whose subset has a cardinality equal to $\theta - d_u^*(\succ_1, \succ_2, \theta)$ is the one whose subset prevails given the tie-breaking criterion. According to Proposition 4, the party that submits a subset with the smallest (largest) cardinality does not have an incentive to deviate by proposing another with a larger (smaller) cardinality, as she would become the second nominator (first nominator). The subset that prevails in equilibrium induces an outcome contained in the $\theta-$ Top Compromise Set.*

Corollary 9 *The unique SPNE outcome under the $\theta-$Splitting Appointment Contest is a subset of the θ-Top Compromise Solution. If $h = \theta - d_u^*(\succ_1, \succ_2, \theta)$ is even (respectively,*

$h = \theta - d_u^*(\succ_1, \succ_2, \theta)$ is odd) and the cardinality of the θ-Top Compromise Solution is $\theta + 1$ then the SPNE outcome is the set formed by the union of Party 1's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates (respectively, Party 2's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates) and Party 2's $(d_u^*(\succ_1, \succ_2, \theta) - 1)$ -top candidates (respectively, Party 1's $(d_u^*(\succ_1, \succ_2, \theta) - 1)$-top candidates). Otherwise, it is equal to the θ-Top Compromise Solution.

The following result about efficiency is a consequence of Theorem 4 and Corollary 2.

Corollary 10 *If the preferences over sets of at least one of the parties are leximax extensions of the preferences over individual candidates then the SPNE outcome under the $\theta-$Splitting Appointment Contest is always Pareto Efficient.*

4 Final Remarks

Our paper introduces four mechanisms that show promise in resolving decision-making scenarios involving the selection of committees of fixed size. We demonstrate that these mechanisms can induce compromise solutions as long as the parties' preferences over sets satisfy the Responsiveness axiom. Additionally, their efficiency depends on whether at least one of the agents has leximax or leximin preferences over sets.

Looking towards future research, a clear avenue involves investigating whether their SPNE outcomes are efficient for the cases where the parties' preferences over sets belong to other classes of lexicographic orders different from leximin and leximax extensions. On this issue, we can anticipate that the $\theta-$Compromise Split Appointment Rule and the $\theta-$Split Appointment Contest are efficient for the case where the extensions of the parties' preferences to sets are lexicographic with respect to orders that give priority to candidates equal to or above the median candidate in the individual preferences, over those candidates that are ranked worse than the median.

Another very natural step would be to conduct laboratory experiments to evaluate these mechanisms in line with theoretical predictions and to compare their respective advantages and disadvantages in practice. For those interested in conducting such experiments, we can anticipate the release of our forthcoming working paper, joint with Carlos Alós-Ferrer and Matías Núñez, which presents the results of an experiment comparing the Compromise Rule of k Names with de Clippel, Eliaz, and Knight's (2014) shortlist method.

Since our mechanisms induce compromise outcomes by granting the parties the flexibility to choose the parameters of the θ-Rule of k Names and the θ-Split Appointment Rule, an attractive question to investigate is the characterization of the optimal choice of these parameters, in line with the methodology proposed by Barberà and Coelho (2017 and 2018). In the context of selecting a single alternative using the Rule of k Names, their ex-ante perspective recommends the use of $k(\mathbf{c}) = \mathbf{c} + 2 - \sqrt{2\mathbf{c} + 2}$ to equalize the parties' expected utilities.

Moreover, it would be worthwhile to investigate whether a generalized version of the Voting Alternating Offers and Vetoes, proposed by Anbarci (1993 and 2006), may exhibit similar properties to our mechanisms, as observed in the case of selecting a single alternative.

Last, but not least, it would be relevant to assess the level of strategic complexity of our mechanisms and explore the potential for their use in jury selection, along lines similar to those proposed in Van der Linden (2018) and Moro and Van der Linden (2023), and to study their impact on the composition of juries and the outcomes of jury trials.

5 References

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APPENDIX 1

The next example shows that the method recommended by JAMS (JAMS Arbitration, Mediation and ADR services) may induce a Pareto inefficient outcome.

Example 9 *Consider the set of candidates* $\{c1, c2, c3, c4, c5, c6, c7, c8, c9, c10\}$*. JAMS's method works as follows: each party may then strike three names and shall rank the remaining ones in order of preference. Finally, the three candidates with the highest Borda score are appointed. Consider this preference profile over alternatives (*$c1 \succ_1 c2 \succ_1 c3 \succ_1 c4 \succ_1 c5 \succ_1 c6 \succ_1 c7 \succ_1 c8 \succ_1 c9 \succ_1 c10$ *and* $c1 \succ_2 c2 \succ_2 c7 \succ_2 c4 \succ_2 c5 \succ_2 c6 \succ_2 c3 \succ_2 c8 \succ_2 c9 \succ_2 c10$*).*
Under the assumption that the parties' preferences over sets are leximin or leximax extensions of the preferences over the individual candidates, the following Nash strategy equilibrium induces the election of $\{c1, c2, c5\}$ *which is Pareto dominated by* $\{c1, c2, c4\}$*: Party 2 vetoes* $c3, c4$ *and* $c10$ *and ranks* $c1 > c2 > c5 > c6 > c7 > c8 > c9$ *and Party 1 vetoes* $c4, c7$ *and* $c10$ *and ranks* $c1 > c2 > c3 > c5 > c6 > c8 > c9$*. So,* $c1, c2$ *and* $c5$ *are alternatives with the highest Borda scores.*

APPENDIX 2

Proof of Proposition 3. Denote by X the subgame equilibrium outcome when Party i is the proposer under Rule of $k^*(\succ_1, \succ_2, \theta)$ Names. Denote by Y the subgame equilibrium outcome when Party i is the chooser under the Rule of k' Names.

Suppose that $k^*(\succ_1, \succ_2, \theta) \geq k'$. We need to prove that Party i prefers X to Y. The proof is by contradiction. Suppose otherwise that Party i prefers Y to X.

Firstly, note that according to Proposition 1 and Definition 3, the candidates of X are Party i's best θ candidates among Party j's $d^*(\succ_1, \succ_2, \theta)$ top candidates. This implies that the candidates of X are Party i's best θ candidates in the intersection of parties' $d^*(\succ_1, \succ_2, \theta)$ top candidates. Consequently, the candidates of X are also among Party i's $d^*(\succ_1, \succ_2, \theta)$ top candidates. This implies that X is also included in Party i's $(c-k'+\theta)$ top candidates, as $k^*(\succ_1, \succ_2, \theta) \geq k'$. Thus, by the Responsiveness Axiom, we have that Party j prefers Y to X. This conclusion is derived from Proposition 1, which establishes that the candidates of Y are Party j's θ best candidates among Party i's $(c-k'+\theta)$ top candidates. Notice that Y is not contained in Party j's $d^*(\succ_1, \succ_2, \theta)$ top candidates, otherwise, X would not be an SPNE outcome. Let Z be the set formed by Party j's θ best candidates among the union of X and Y. It implies that Z is among Party i's $(c-k'+\theta)$ top candidates. By the Responsiveness Axiom, it also implies that Z is preferred to Y according to Party j's preference relation. Thus, we reached a contradiction, since Y is the set of Party j's θ best candidates among Party i's $(c-k'+\theta)$ top candidates.

Now, suppose that $k' > k^*(\succ_1, \succ_2, \theta)$. Denote by X the subgame equilibrium outcome when Party i acts as the chooser under the Rule of $k^*(\succ_1, \succ_2, \theta)$ Names. Denote by Y the subgame equilibrium outcome when Party i acts as the proposer under Rule of k' Names. Suppose by contradiction that Party i prefers Y to X.

By Proposition 1, the candidates of Y are Party i's θ best candidates of Party j's $(c-k'+\theta)$ top candidates. Given $k' > k^*(\succ_1, \succ_2, \theta)$, this implies that the candidates of Y are also among Party j's $(c-k^*(\succ_1, \succ_2, \theta)+\theta)$ top candidates.

Notice also by Proposition 1, the candidates of X are Party j's θ best candidates of Party i's $d^*(\succ_1, \succ_2, \theta)$ top candidates. By Proposition 1 and Definition 3, the candidates of X are also among Party j's $d^*(\succ_1, \succ_2, \theta)$ top candidates. Let Z be the set formed by Party i's θ best candidates among the union of X and Y. This would imply that Z is among Party j's $d^*(\succ_1, \succ_2, \theta)$ top candidates. By the Responsiveness Axiom, it would also imply that Z is preferred to X or $Z = X$ according to Party i's preference relation. If Party i's prefers Z to X, we reached a contradiction, since X is the set of Party i's θ best candidates among Party j's $d^*(\succ_1, \succ_2, \theta)$ top candidates. If $Z = X$, by the Responsiveness Axiom, then Party i prefers X to Y which is a contradiction. ■

Proof of Theorem 1. First let us prove that the strategy profile stated in Theorem 1 is a subgame perfect equilibrium. Notice that the strategies adopted in the second, third and fourth stages are direct consequences of propositions 1 and 3.

Now, let us prove that Party 1 does not have a profitable deviation. Given Party 2's strategy, it is enough to consider only $k' > k^*(\succ_1, \succ_2, \theta)$. If $k' > k^*(\succ_1, \succ_2, \theta)$, Party 1 will become the proposer. It follows by Proposition 3 that it would be not a profitable deviation. Therefore, our initial strategy profile is a subgame perfect equilibrium.

Now let us argue that the equilibrium outcome is unique. Under this mechanism, only one player moves at each stage. Hence, subgame perfect equilibria and backward induction equilibria coincide, and any backward induction equilibrium outcome is unique as long as the parties' preferences over sets of size θ are strict. ■

Proof of Theorem 2. Without loss of generality suppose that $k^*(\succ_1, \succ_2, \theta)$ is odd. First, let us prove that the strategy profile stated in Theorem 2 is a subgame perfect equilibrium. Denote by X its outcome and by Z^i the subset proposed by Party $i \in \{1, 2\}$ under this strategy profile. First, notice that given $k^*(\succ_1, \succ_2, \theta) = \mathbf{c} - d^*(\succ_1, \succ_2, \theta) + \theta$ and Corollary 1, X is the set formed by Party i's θ best candidates in the $\theta-$Unanimity Compromise Set.

We need only prove that for each $i' \in \{1, 2\}$ there exists no subset $S \subset \mathbf{C}$, such that $\#|S| \geq \theta$ and $S \neq Z^{i'}$, that would make Party i' better off by choosing S instead of $Z^{i'}$, while the other player's strategy remains unchanged. Proposition 1 implies that it is enough to consider only deviations with subsets S such that: $S \subset \mathbf{C}$, with $\#|S| \geq \theta$, that contains Party $i' \in \{1, 2\}$ θ preferred candidates among the opposing party's $(\mathbf{c} - \#|S| + \theta)$ top candidates plus the opposing party's $\#|S| - \theta$ worst candidates.

Given the rules of the mechanism and the other player strategy, if Party i deviates by choosing a subset with cardinality smaller than $k^*(\succ_1, \succ_2, \theta)$, it will pick the winning candidates out of the subset proposed by its opponent. And if it deviates by choosing a subset with cardinality higher than $k^*(\succ_1, \succ_2, \theta)$, its opponent will pick the winning candidates out of its subset. It follows from Proposition 3 that neither of these two possible types of deviations would be profitable.

Having proved that our proposed strategy profile is a subgame perfect equilibrium, let us show that X is the unique subgame perfect equilibrium outcome of the game.

Given that fact, we suppose by contradiction that besides the equilibrium outcome described in Theorem 2 there is another one. Now, we will prove that no strategy profile could sustain it.

Let us denote by SX the strategy profile described in Theorem 2 that sustains X as an equilibrium outcome and Party 1's proposed subset is the one that prevails. Suppose by contradiction that X is not unique. Let $Y \neq X$ be another subgame perfect equilibrium outcome. Denote by SY the strategy profile that sustains Y as a subgame perfect equilibrium outcome and by k' the cardinality of the subset from which one of the parties picks Y on its equilibrium path.

Suppose that $k' < k^*(\succ_1, \succ_2, \theta)$ and Party 2's subset prevails under SY. This would imply that the cardinality of the other subset proposed by the opponent is equal to or smaller than k'. Notice that Party 2 prefers X to Y, otherwise, under SX, this party would have a profitable deviation by proposing a subset with cardinality k' with the same composition as the one in SY. By Proposition 1, as $k' < k^*(\succ_1, \succ_2, \theta)$, the candidates of X and Y are contained among Party 1's $(\mathbf{c} - k' + \theta)$ top candidates. Thus, Party 2 prefers Y to X. Therefore, we reached a contradiction. Suppose that $k' < k^*(\succ_1, \succ_2, \theta)$ and Party 1's subset prevails under SY. This would imply that Party 1 prefers X to Y, otherwise, under SX, this party would have a profitable deviation by proposing a subset with cardinality k' with the same composition as the one in SY. By Proposition 1, the candidates of Y are Party 1's θ best candidates among Party 2's $(\mathbf{c} - k' + \theta)$ top candidates. By definition of $k^*(\succ_1, \succ_2, \theta)$, the candidates of X are among Party 2's $(\mathbf{c} - k^*(\succ_1, \succ_2, \theta) + \theta)$ top candidates. As $k' < k^*(\succ_1, \succ_2, \theta)$, the candidates of X and Y are contained among Party 2's $(\mathbf{c} - k' + \theta)$ top candidates. It implies that Party 1 prefers Y to X and this is a contradiction. Suppose now that $k' > k^*(\succ_1, \succ_2, \theta)$ and Party 2's subset prevails under SY. This would imply that the cardinality of Party 1's proposed subset is equal to or larger than k'. Notice that Party 2 prefers Y to X, otherwise, under SY, this party would have a profitable deviation by proposing a subset with cardinality $k^*(\succ_1, \succ_2, \theta)$ with the same composition as the one in SX and the outcome would be weakly preferred to X according to Proposition 3. By Proposition 1, as $k' > k^*(\succ_1, \succ_2, \theta)$, the candidates of X and Y are contained among Party 1's $(\mathbf{c} - k^*(\succ_1, \succ_2, \theta) + \theta)$ top candidates. Thus, Party 2 prefers X to Y. Therefore, we reached a contradiction. Suppose now that $k' > k^*(\succ_1, \succ_2, \theta)$ and Party 1's subset prevails under **SY**. This would imply that the cardinality of Party 2's proposed subset is equal to or larger than k'. Notice that Party 1 prefers Y to X, otherwise, under SY, this party would have a profitable deviation by proposing a subset with cardinality $k^*(\succ_1, \succ_2, \theta)$ with the same composition as the one in SX. By Proposition 1, as $k' > k^*(\succ_1, \succ_2, \theta)$, the candidates of X and Y are contained among Party 2's $(\mathbf{c} - k^*(\succ_1, \succ_2, \theta) + \theta)$ top candidates. Thus, Party 1 prefers X to Y. Therefore, we reached a contradiction. ■

Proof of Claim 1. Denote by X the SPNE outcome under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule when Party i acts as the first nominator. Denote by Y the SPNE outcome under $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1)-$Split Appointment Rule when Party i acts as the second nominator. Suppose that the θ-Top Compromise Solution has θ candidates. According to Proposition 2 and Definition 4, X is equal to the union of parties' $d_u^*(\succ_1, \succ_2, \theta)$-top candidates. In addition, Y is equal to the union of the union of Party j' $d_u^*(\succ_1, \succ_2, \theta) + 1$-top candidates and Party i's $d_u^*(\succ_1, \succ_2, \theta) - 1$-top candidates. Thus, Party i weakly prefers X to Y. Now suppose that θ-Top Compromise Solution has $\theta + 1$ candidates. Again, according to Proposition 2 and Definition 4, X is equal to the union of Party j' $d_u^*(\succ_1, \succ_2, \theta)$-top candidates and Party i's $d_u^*(\succ_1, \succ_2, \theta) - 1$-top candidates. In addition, Y is equal to the union of the union of Party j' $d_u^*(\succ_1, \succ_2, \theta)$-top candidates and Party i's $d_u^*(\succ_1, \succ_2, \theta) - 1$-top candidates. Thus, Party i is indifferent between X and Y as $X = Y$. ■

Proof of Proposition 4. Consider $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule. By Remark 6, both parties weakly prefer playing as the second nominator when $h = \theta - d_u^*(\succ_1, \succ_2, \theta)$. Therefore, if h' is not greater than $\theta - d_u^*(\succ_1, \succ_2, \theta)$, then both parties are weakly better off when playing as the second nominator under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule than playing as the first nominator under the $(\theta, h')-$Split Appointment rule.

Now let us prove that if $h' > \theta - d_u^*(\succ_1, \succ_2, \theta)$ then both parties are weakly better off playing as the first nominator under the $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule than playing as the second nominator under the $(\theta, h')-$Split Appointment Rule. Note that it is sufficient to prove for the case where $h' = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1$. Denote by X the SPNE outcome under $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta))-$Split Appointment Rule when Party i is the first nominator. Denote by Y the SPNE outcome under $(\theta, h = \theta - d_u^*(\succ_1, \succ_2, \theta) + 1)-$Split Appointment Rule when Party i is the second nominator. According to Claim 1, Party i weakly prefers X to Y. ■

Proof of Theorem 3. First, let us prove that the strategy profile stated in Theorem 3 is a subgame perfect equilibrium. Notice that the strategies adopted in the second, third and fourth stages are direct consequences of Proposition 2.

Now, let us prove that Party 1 does not have a profitable deviation. Given Party 2's strategy, it is enough to consider only $h' > \theta - d_u^*(\succ_1, \succ_2, \theta)$. If $h' > \theta - d_u^*(\succ_1, \succ_2, \theta)$, Party 1 will become the second nominator. It follows by Proposition 4 that it would be not a profitable deviation. Therefore, our initial strategy profile is a subgame perfect equilibrium.

Now let us argue that the equilibrium outcome is unique. Under this mechanism, only one player moves at each stage. Hence, subgame perfect equilibria and backward induction equilibria coincide, and any backward induction equilibrium outcome is unique as long as the parties' preferences over sets of size θ are strict. ■

Proof of Theorem 4. First, let us prove that the strategy profile stated in Theorem 4 is a subgame perfect equilibrium. Without loss of generality suppose that $h = \theta - d_u^*(\succ_1, \succ_2, \theta)$ is odd. This implies that Party 1 is the one who submits the smallest subset, so this party acts as the first nominator. According to Proposition 4, Party 1 (Party 2) does not have an incentive to deviate by proposing another subset with a larger (smaller) cardinality, as this party would become the second nominator (first nominator). Denote by X its SPNE outcome under this strategy profile. Notice that given the composition of Party 1's proposed subset and Remark 2, X is contained in the union of the set of Party 2's $d_u^*(\succ_1, \succ_2, \theta)$-top candidates and the set of Party 1's $(\theta - d_u^*(\succ_1, \succ_2, \theta))$-top candidates among Party 2's $d_u^*(\succ_1, \succ_2, \theta)$-bottom candidates.

Having proved that our proposed strategy profile is a subgame perfect equilibrium, let us show that X is the unique subgame perfect equilibrium outcome of the game.

The proof is by contradiction. We suppose otherwise that besides the equilibrium outcome described in Theorem 4 there was another one. We will prove that no strategy profile could sustain it.

Let us denote by SX the strategy profile described in Theorem 4 that sustains X as an equilibrium outcome and Party 1's proposed subset is the one that prevails. Suppose by contradiction that X is not unique. Let $Y \neq X$ be another subgame perfect equilibrium outcome. Denote by SY the strategy profile that sustains Y as a subgame perfect equilibrium outcome and by h_y the cardinality of the subset that prevails under this strategy. Proposition 2 implies that it is enough to consider, in the first stage, only profitable deviations with subsets S such that: $S \subset \mathbf{C}$, that contains Party $\acute{i} \in \{1, 2\}$ $\#|S|$ preferred candidates among her opponent's $(\mathbf{c} + \#|S| - \theta)$-bottom candidates. Additionally, in the second stage, whoever proposed the discarded subset shall select $\theta - \#|S|$ candidates among the remaining candidates not yet chosen from $\mathbf{C}$.

Suppose that $h_y < \theta - d^*_u(\succ_1, \succ_2, \theta)$, as a consequence of Proposition 4, this strategy cannot be sustained at equilibrium because whoever proposed the prevailed subset would have an incentive in deviating by proposing another subset with cardinality equal to $\theta - d^*_u(\succ_1, \succ_2, \theta)$. Because if that party continues to be the first nominator or becomes the nominator 1, it will be strictly better off, as the new outcome would be at least as good as X. Suppose that $h_y \geq \theta - d^*_u(\succ_1, \succ_2, \theta)$, as a consequence of Proposition 4, this strategy cannot be sustained at equilibrium because whoever proposed the discarded subset would have an incentive in deviating by proposing another subset with cardinality equal to $\theta - d^*_u(\succ_1, \succ_2, \theta)$. Because if that party continues to be the second nominator or becomes the first nominator, it will be strictly better off, as the new outcome would be at least as good as X. ■