Asymmetric Monetary Policy Tradeoffs

BSE Working Paper 1404| September 2023

Davide Debortoli, Mario Forni, Luca Gambetti, Luca Sala

bse.eu/research

Asymmetric Monetary Policy Tradeoffs*

Davide Debortoli
ICREA-UPF, CREi, BSE and CEPR

Mario Forni
Università di Modena e Reggio Emilia, CEPR and RECent

Luca Gambetti
Universitat Autonoma de Barcelona, BSE, Università di Torino and Collegio Carlo Alberto

Luca Sala
Università Bocconi, IGIER and Baffi Carefin

September 2023

Abstract

We measure the inflation-unemployment tradeoff associated with monetary easing and tightening, during booms and recessions, using a novel nonlinear Proxy-SVAR approach. We find evidence of significant non-linearities for the U.S. economy (1973:M1 - 2019:M6): stimulating economic activity during recessions is associated with minimal costs in terms of inflation, and reducing inflation during booms delivers small costs in terms of unemployment. Overall, these results provide support for countercyclical monetary policies, in contrast with what predicted by a flat Phillips curve, or previous studies on nonlinear effects of monetary policy. Our results can be rationalized by a simple model with downward nominal wage rigidity, which is also used to assess the validity of our empirical approach.

JEL classification: C32, E32.
Keywords: monetary policy, inflation-unemployment tradeoff, structural VAR models, proxy-SVAR.

*We gratefully acknowledge the financial support from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (Barcelona School of Economics CEX2019-000915-S) and through grants PID2020-116268GB-I00 (Debortoli) and PGC2018-094364-B-I00 (Gambetti), from the FAR 2017 Department of Economics "Marco Biagi" (Forni) and from the Italian Ministry of Research and University, PRIN 2017 grant J44I20000180001 (Forni, Gambetti and Sala).

1 Introduction

The presence of an inflation-unemployment tradeoff —or for short the "monetary policy tradeoff"— is at the heart of monetary policymaking. How much inflation is needed to stimulate economic activity? What are the costs of reducing inflation in terms of unemployment? These long-lasting questions became particularly relevant in recent times, as the US and European economy faced the deepest postwar crisis in 2008-09, and more recently the highest surge in inflation since the 1970's.

The magnitude of the monetary policy tradeoff is traditionally measured as the (inverse) slope of a Phillips curve, i.e. a relationship linking inflation and unemployment. The corresponding estimates are usually obtained within *linear* settings, where it is implicitly assumed that the inflation-unemployment tradeoff is constant, and independent from the sign of the monetary intervention, the underling economic conditions, or other factors. However, there are several reasons to question the validity of that assumption. On the one hand, at least since the Great Depression (Keynes, 1936, Chapter 21) it has been argued that monetary tightening is more powerful than monetary easing, due to their potentially different effects on prices, wages, credit conditions, etc.[1] On the other hand, since the late 1980's the inflation rate appears to be largely insensitive to movements in the unemployment rate —as if the Phillips curve had flattened, or disappeared. In this context, a constant inflation-unemployment tradeoff would have the following implications: (i) central banks could stimulate economic activity with minimal costs in terms of inflation; and (ii) reducing inflation would be associated with a very large increase in the unemployment rate. Both implications are clearly extreme, and of doubtful relevance for policymaking purposes.

In this paper, we provide new evidence on the nature of the monetary policy tradeoff for the US economy, and study, in particular, whether the size of the tradeoff depends on the sign of the monetary intervention (easing vs tightening) and the state of the economy (booms vs recessions).

The contribution of the paper is twofold. First, from a methodological viewpoint, building on the work of Mertens and Ravn (2013), Stock and Watson (2018) and Plagborg-Møller and Wolf (2021) we extend the Proxy-VAR-IV approach to a nonlinear context. The economy is described by a Vector Moving Average (VMA) augmented with *nonlinear* functions of the monetary policy shock. These nonlinear functions give rise to a nonlinear dynamic transmission of monetary policy. The model admits a simple VARX representation where the shock and its nonlinear function represent two exogenous variables. Even though the exogenous shock is not observed, under mild assumptions —i.e. the existence of a valid instrument for the shock and the existence of a monetary policy rule— it can be estimated as the fitted value of the regression of the instrument on the residuals of a (misspecified) *linear* VAR, where the nonlinear functions of the shock are neglected. The monetary tradeoff is then calculated as the ratio of the (cumulative) impulse responses of inflation and unemployment (or viceversa), *conditional* to identified monetary shocks,

[1] Some examples in that regard are models with asymmetric price adjustments (e.g., Ball and Mankiw (1994)), or occasionally binding financial constraints (e.g. Bernanke, 1993, and De Long and Summers, 1998).

distinguishing the effects of positive and negative monetary shocks, during booms and recessions.

Second, from an economic viewpoint, we find that the inflation-unemployment tradeoff varies substantially, depending on the sign of the monetary intervention and the state of the economy, thus calling into question typical predictions associated with linear —and possibly nearly flat— Phillips curves. In particular, the inflation costs of stimulating economic activity are found to be small (and insignificant) during recessions. At the same time, the employment costs of lowering inflation are found to be moderate, especially during economic expansions —e.g. we find that reducing inflation by 1 percentage point requires an increase in the unemployment rate of roughly 0.5 percentage points. In other words, our results suggest that central banks can engage into disinflationary policies without necessarily incurring very large unemployment costs.

For policymaking purposes, our results provide support to the use of countercyclical monetary policies —both monetary easing in recessions and tightening in expansions— as those policies are associated with relatively favorable tradeoffs. The tradeoff worsens dramatically for other types of policies (e.g. tightening in recessions). In this respect, our analysis provides a different perspective relative to previous results on the nonlinear effects of monetary policy shocks. The recent works of Tenreyro and Thwaites (2016) and Barnichon and Matthes (2018) suggest that monetary policy is not very effective at stimulating economic activity, especially during recessions —as if the central bank was "pushing on a string". Yet, we also show that a monetary easing during recessions has a moderate effect on prices, so that the central bank faces a relatively favorable inflation-unemployment tradeoff. Thus, monetary policy can be a useful tool, even during recessions: it is possible to achieve a favorable balance between inflation and unemployment, as long as the central bank's interventions are sufficiently aggressive to achieve the desired economic stimulus.

We also show that a sign- and state-dependent monetary policy tradeoff can arise in a simple model economy with labor market frictions, in the form of downward nominal wage rigidities.[2] The model gives rise to inflation and output dynamics that are qualitatively very similar to their empirical counterparts. We then apply our empirical approach to artificial data generated by the model, and show that the resulting estimates capture entirely the non-linearities of the underlying economy.

The remainder of the paper is organized as follows. Section 2 contains a brief review of the related literature. Section 3 formally defines our measure of the monetary policy tradeoff. Section 4 discusses the econometric approach. Section 5 presents the empirical evidence, Section 6 presents a model with downward nominal wage rigidities, and Section 7 concludes.

[2] Similar models can be found in Kim and Ruge-Murcia (2009), Benigno and Ricci (2011) and Schmitt-Grohé and Uribe (2016), among others.

2 Related literature

This paper contributes to the vast empirical literature about the inflation-unemployment tradeoff, starting from the original evidence of Phillips (1958) and Samuelson and Solow (1960), and followed by the empirical works on the New-Keynesian Phillips curve of Roberts (1995), Fuhrer and Moore (1995), Galí and Gertler (1999) and Sbordone (2002), among others.[3] More specifically, our paper is related to the recent body of that literature proposing novel approaches to identify the empirical relationship between measures of inflation and economic activity. A number of authors (e.g. McLeay and Tenreyro, 2020, Beraja, Hurst and Ospina, 2019, and Hazell et. al., 2022) exploit variations at the regional level to overcome the simultaneity problem of distinguishing between demand and supply shocks. Similarly to us, Ball (1994) proposes a non-parametric estimate of the output-inflation tradeoff using the (cumulative) trend deviations of output during disinflationary episodes, for a sample of OECD countries. More recently, Barnichon and Mesters (2020, 2021) and Galí and Gambetti (2020) exploit identified monetary shocks to obtain *conditional* estimates of the inflation-unemployment relationship. A common finding in this literature is that the Phillips curve has not flattened over time (see also Stock and Watson, 2020, and Del Negro et. al., 2020). Our results are consistent with that view, but additionally uncover that the relationship between inflation and unemployment is sign- and state-dependent. In this respect our results are consistent with the evidence in Daly and Hobijn (2014) and Gagnon and Collins (2020), among others, who document the presence of important non-linearities in the Phillips curve, using different approaches.

Our work is also related to the large literature studying the effects of monetary shocks. Most studies in this literature have relied on linear SVARs.[4] Few recent studies looked at the nonlinear effects of monetary shocks. Tenreyro and Thwaites (2016) find that monetary policy is less powerful during recessions, while Barnichon and Matthes (2018) find that monetary tightening is more powerful than monetary easing.[5] Our contribution relative to those works is twofold. First, we propose a novel empirical approach to study the effects of both sign- and state-dependence, within a single framework.[6] Second, we estimate the inflation-unemployment tradeoff, rather than focusing on macroeconomic variables in isolation, which provides a different perspective about the desirability of countercyclical monetary policies.

[3] See Mavroeidis, Plagborg-Møller and Stock (2014) for a survey of earlier works.

[4] A partial list of early contributions studying the effects of monetary policy shocks includes Bernanke and Blinder (1992), Bernanke and Gertler (1995), Bernanke and Mihov (1998), Christiano, Eichenbaum and Evans (1999), Cochrane (1994), Leeper, Sims and Zha (1996), Sims and Zha (2006) and Strongin (1995). Several advances have been made in recent years, especially in terms of shock identification, as for instance in the works of Romer and Romer (2004), Uhlig (2005), Gertler and Karadi (2015), Arias, Caldara and Rubio-Ramírez (2019), Jarocinsky and Karadi (2020), Caldara and Herbst (2019) and Miranda-Agrippino and Ricco (2021).

[5] Early contributions on the topic include Cover (1992), Karras (1996) and Weise (1999). More recently, Santoro et. al. (2014) find that the output-gap responds more during recessions than during booms.

[6] Tenreyro and Thwaites (2016) use instead nonlinear local projections identifying the policy shocks as in Romer and Romer (2004), while Barnichon and Matthes (2018) estimate a non-linear Vector Moving Average representation using Functional Approximation of Impulse Response (FAIR) approach.

3 Defining the Monetary Policy Tradeoff

Our goal is to measure the inflation cost of reducing unemployment —the tradeoff for monetary easing— and viceversa the unemployment cost of reducing inflation —the tradeoff for monetary tightening. In linear settings, the two measure are tightly related, as one measure is simply the inverse of the other. This is no longer the case in a nonlinear setting. It is therefore necessary to treat the two cases separately.

We define the tradeoff for monetary easing (respectively, tightening) as the average change in inflation (unemployment) forecasts induced by a monetary shock that causes a 1 percentage point change in average forecast unemployment (inflation), and where averages are taken over an horizon of H periods.[7] The tradeoffs can be calculated using the average impulse responses of unemployment (y) and inflation (π) to a one-unit monetary shock.

Formally, the tradeoffs for monetary easing ($\mathcal{T}_H^+$) and tightening ($\mathcal{T}_H^-$) are defined as

$$\mathcal{T}_H^+(s_{t-1}) \equiv \frac{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{\pi,+}(s_{t-1})}{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{y,+}(s_{t-1})} \qquad \mathcal{T}_H^-(s_{t-1}) \equiv \frac{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{y,-}(s_{t-1})}{\frac{1}{H}\sum_{h=0}^{H}\mathcal{R}_h^{\pi,-}(s_{t-1})}, \tag{1}$$

where $\mathcal{R}_h^{x,+}(s_{t-1})$ denotes the impulse response at horizon h of a generic variable x to a monetary easing, $s_{t-1} \in 0,1$ is a recession indicator, and $\mathcal{R}_h^{x,-}(s_{t-1})$ denotes instead the corresponding impulse responses for a monetary tightening.

Following this procedure allows us to mitigate well-known challenges associated with typical Phillips curve estimates (see e.g. Mavroeidis et. al., 2014), for the following reasons: (i) a measure of the tradeoff can be obtained under minimal assumptions about the structure of the underlying economy, e.g. without postulating a specific Phillips curve, or other structural equations, which may lead to misspecification problems; (ii) there is no need to rely on data on inflation expectations or the “natural" rate of unemployment, which are not directly observable, and may lead to additional biases and uncertainties in coefficient estimates due to measurement error; and (iii) we obtain a measure of the tradeoff *caused* by exogenous monetary interventions which is immune from typical endogeneity problems of Phillips curve estimates.[8]

4 Methodology: a Nonlinear Proxy-SVAR

In this section we present our empirical model, the identification assumptions, and the estimation approach. We consider a model economy where macroeconomic variables react (linearly) both to the monetary shock and to nonlinear functions of that shock. We show that, under suitable

[7] In this respect, the measure of the monetary tradeoff resembles the concept of government spending multiplier in the fiscal literature, which is calculated as the ratio of the cumulative response of output and government spending, see e.g. Mountford and Uhlig (2009) and Ramey and Zubairy (2018).

[8] More specifically, as is common in the monetary policy literature, we are assuming that the “natural” rate of unemployment (or output) is orthogonal to monetary shocks, so that a measure of the unemployment (or output) gap is not needed to calculate the implied monetary tradeoff.

conditions, such a nonlinear model admits a VARX representation that can be estimated using an external instrument, analogously to what is usually done in linear settings. To do so, we build on Forni, Gambetti and Ricco (2023), who show that in invertible linear models the shock of interest can be obtained as the projection of the instrument onto the vector of reduced form residuals of a VAR. Here we extend that result to our nonlinear context.

4.1 Representation assumptions

Let $\mathbf{x}_t$ be an n-dimensional vector of observable stationary macroeconomic variables. We assume the following structural representation.

Assumption A0 (*The structural representation*).

$$\mathbf{x}_t \quad = \quad \boldsymbol{\nu} + \boldsymbol{\Gamma}(L)\mathbf{u}_t + \boldsymbol{\alpha}(L)u_t^r + \boldsymbol{\Phi}(L)\mathbf{g}(u_t^r) \tag{2}$$

where $\boldsymbol{\nu}$ is a vector of constants, u_t^r is the monetary policy shock, $\mathbf{g}(u_t^r)$ is a k-dimensional vector of nonlinear static functions of the shock, and $\mathbf{u}_t$ is a m-dimensional vector of additional structural shocks, other than monetary policy.[9] We further assume that the vector $[u_t^r \ \mathbf{u}_t']'$ is i.i.d. zero mean, with an identity covariance matrix.

The serial and mutual independence assumption implies that all structural shocks, including u_t^r and $\mathbf{g}(u_t^r)$, are uncorrelated with the lags of $\mathbf{g}(u_t^r)$ and $\mathbf{x}_t$. The vector $\boldsymbol{\alpha}(L) = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 L + \boldsymbol{\alpha}_2 L^2 ...$ represents the impulse responses functions to the monetary policy shock and $\boldsymbol{\Phi}(L) = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 L + \boldsymbol{\Phi}_2 L^2 + ...$ is an $n \times k$ matrix of impulse response functions to the nonlinear functions of the monetary shock $\mathbf{g}(u_t^r)$. For example, in our application below we set $\mathbf{g}(u_t^r) = [|u_t^r| \ \ s_{t-1}u_t^r]'$ where s_{t-1} represents a dummy variable capturing the state of the economy before the shocks hits. Finally $\boldsymbol{\Gamma}(L) = \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 L + \boldsymbol{\Gamma}_2 L^2 + ...$ is a $n \times m$ matrix of impulse response functions to the remaining structural shocks. Equation (2) can be seen as a Vector Moving Average, augmented with nonlinear functions of the monetary policy shock.

The total effects of a monetary policy shock $u_t^r = \bar{u}^r$ are then given by the sum of the linear and nonlinear terms:

$$\boldsymbol{\mathcal{R}}(L, \bar{u}^r) \equiv \boldsymbol{\alpha}(L)\bar{u}^r + \boldsymbol{\Phi}(L)\mathbf{g}(\bar{u}^r). \tag{3}$$

The total responses defined in eq. (3) simply correspond, in this nonlinear context, to the Generalized Impulse Response Functions defined as $E(\mathbf{x}_{t+h}|u_t^r = \bar{u}^r) - E(\mathbf{x}_{t+h}|u_t^r = 0)$, $h = 0, 1, ...$. We discuss below how to estimate the model and the implied impulse response functions.

Stationarity of $\boldsymbol{\Gamma}(L)\mathbf{u}_t$ ensures the existence of the following representation:

$$\mathbf{x}_t = \boldsymbol{\nu} + \boldsymbol{\Psi}(L)\mathbf{e}_t + \boldsymbol{\alpha}(L)u_t^r + \boldsymbol{\Phi}(L)\mathbf{g}(u_t^r), \tag{4}$$

[9] Notice that the number of shocks can be different (larger or smaller) than the number of variables. Moreover, $\mathbf{u}_t$ could also include nonlinear functions of other shocks.

where $\mathbf{\Psi}(L)\mathbf{e}_t$ is the Wold representation of $\mathbf{\Gamma}(L)\mathbf{u}_t$.[10] We further characterize the process $\mathbf{\Psi}(L)\mathbf{e}_t$ by making the following assumption:

Assumption A1 (*Finite-order VARX representation*). We assume that

(a) $\mathbf{\Psi}(L) = \mathbf{A}(L)^{-1}$

(b) $\mathbf{\Phi}(L) = \mathbf{A}(L)^{-1}\tilde{\mathbf{\Phi}}(L)$,

(b) $\boldsymbol{\alpha}(L) = \mathbf{A}(L)^{-1}\tilde{\boldsymbol{\alpha}}(L)$,

where $\mathbf{A}(L) = \mathbb{I}_n - \mathbf{A}_1 L - \cdots - \mathbf{A}_p L^p$ is a matrix of polynomials of degree p, and $\tilde{\boldsymbol{\alpha}}(L)$ and $\tilde{\mathbf{\Phi}}(L)$ are polynomials of degree $q \leq p$.

Assumption A1 (a) imposes that the inverse of $\mathbf{\Psi}(L)$ exists (i.e. its determinant does not vanish on the unit circle) and is a finite order polynomial matrix. This is a standard assumption in SVAR analysis. We further require that $\tilde{\boldsymbol{\alpha}}(L)$ and $\tilde{\mathbf{\Phi}}(L)$ are polynomials of order $q \leq p$. Such assumption is needed to avoid collinearity problems since, as we will discuss below, the monetary policy shock is obtained as a combination of the current value and p lags of $\mathbf{x}_t$.

Under Assumption A1, eq. (4) can be rewritten as

$$\mathbf{A}(L)\mathbf{x}_t = \boldsymbol{\mu} + \tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\mathbf{\Phi}}(L)\mathbf{g}(u_t^r) + \mathbf{e}_t, \tag{5}$$

or equivalently,

$$\mathbf{x}_t = \boldsymbol{\mu} + \tilde{\mathbf{A}}(L)\mathbf{x}_{t-1} + \tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\mathbf{\Phi}}(L)\mathbf{g}(u_t^r) + \mathbf{e}_t \tag{6}$$

where $\boldsymbol{\mu} \equiv \mathbf{A}(1)\boldsymbol{\nu}$, $\tilde{\mathbf{A}}(L) \equiv \mathbf{A}_1 + \mathbf{A}_2 L + \cdots + \mathbf{A}_p L^{p-1}$. Eq. (6) is a VARX model where the monetary policy shock and its nonlinear functions are the exogenous variables.[11]

Finally, we assume the existence of a finite VAR representation for $\mathbf{x}_t$, i.e. we require that the Wold representation of $\mathbf{x}_t$ is invertible.

Assumption A2 (*VAR representation*). The vector $\mathbf{x}_t$ admits the VAR representation

$$\mathbf{x}_t = \boldsymbol{\vartheta} + \mathbf{B}(L)\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t = \boldsymbol{\vartheta} + \sum_{j=1}^{\infty} \mathbf{B}_j \mathbf{x}_{t-j} + \boldsymbol{\varepsilon}_t \tag{7}$$

where $\boldsymbol{\varepsilon}_t$ is orthogonal to $\mathbf{x}_{t-j}$, $j = 1, \ldots, \infty$.

[10] If the structural representation $\mathbf{\Gamma}(L)\mathbf{u}_t$ is invertible, then $\mathbf{\Psi}(L) = \mathbf{\Gamma}(L)\mathbf{\Gamma}_0^{-1}$ and $\mathbf{e}_t = \mathbf{\Gamma}_0\mathbf{u}_t$, $\mathbf{\Gamma}_0^{-1}$ being either the inverse of $\mathbf{\Gamma}_0$, if $m = n$, or a left inverse of $\mathbf{\Gamma}_0$, if $m < n$. If the structural representation $\mathbf{\Gamma}(L)\mathbf{u}_t$ is not invertible (e.g. when $m > n$ or $m = n$ but $\mathbf{\Gamma}(L)$ vanishes within the unit disk), then $\mathbf{e}_t$ is a linear combination of the present *and past* values of $\mathbf{u}_t$ and $\mathbf{\Psi}(L)\mathbf{e}_t$ is just a statistical represention, devoid of economic meaning.

[11] Notice that $\mathbf{e}_t$ is orthogonal to the regressors. For, being a linear combination of the present and past values of $\mathbf{u}_t$ (we have $\mathbf{e}_t = \mathbf{A}(L)\mathbf{\Gamma}(L)\mathbf{u}_t$), it is orthogonal to both u_t^r and $g(u_t^r)$ at all leads and lags. Moreover, it is orthogonal to the past of $\mathbf{\Gamma}(L)\mathbf{u}_t$ by construction, so that it is orthogonal to $\mathbf{x}_{t-k}$, $k > 0$, see equation (2).

Assumption A2 is made for convenience. The cointegration case can be treated as usual by considering a VAR in the levels of the variables, rather than in first differences.

What is the relation between the VAR representation above and the VARX representation in (6)? Let us start from eq. (6) and consider the linear projection of $\tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\boldsymbol{\Phi}}(L)\mathbf{g}(u_t^r)$ onto the constant and the past history of $\mathbf{x}_t$, i.e.

$$\tilde{\boldsymbol{\alpha}}(L)u_t^r + \tilde{\boldsymbol{\Phi}}(L)\mathbf{g}(u_t^r) = \boldsymbol{\theta} + \mathbf{C}(L)\mathbf{x}_{t-1} + \boldsymbol{w}_t.$$

It is easily seen that $\boldsymbol{\vartheta} = \boldsymbol{\mu} + \boldsymbol{\theta}$, $\mathbf{B}(L) = \tilde{\mathbf{A}}(L) + \mathbf{C}(L)$ and $\boldsymbol{\varepsilon}_t = \mathbf{e}_t + \boldsymbol{w}_t$.[12]

If $\tilde{\boldsymbol{\Phi}}(L) = 0$, the structural representation (2) reduces to a linear model and standard SVAR analysis can be conducted using representation (7). Hence the linear model is nested in our model. We can test for linearity by testing either for the null $\tilde{\boldsymbol{\Phi}}(L) = 0$ in equation (6) or for the null $\boldsymbol{\Phi}(L) = 0$ in the impulse-response functions (3).

4.2 Identification assumptions

In the previous subsection we have shown conditions under which our nonlinear economy admits a VARX representation. Unfortunately, direct estimation of the VARX (6) is unfeasible, because in this case the exogenous variables are not observable.[13] We discuss below how to obtain a valid measure of the exogenous shock that can be used to estimate the VARX.

Our procedure is an extension of the proxy-SVAR identification (Mertens and Ravn, 2013 and Stock and Watson, 2018) to our nonlinear framework and involves two main steps: (i) estimating the shock by regressing a valid external instrument of the monetary policy shock onto the vector of the VAR residuals $\boldsymbol{\varepsilon}_t$; and (ii) using the estimated shock and its nonlinear function as regressors in model (6) to estimate the nonlinear impulse response functions (3).

The identification procedure relies on two assumptions. The first assumption is standard in the proxy-SVAR literature and requires the existence of a valid instrument, as specified below.

Assumption A3 (*Proxy*). The proxy z_t is given by

$$z_t = a + bu_t^r + \delta(L)'\mathbf{x}_{t-1} + v_t = a + bu_t^r + \sum_{j=1}^{\infty} \delta_j'\mathbf{x}_{t-j} + v_t, \tag{8}$$

[12] An interesting special case is $\tilde{\boldsymbol{\alpha}}(L) = \tilde{\boldsymbol{\alpha}}_0$ and $\tilde{\boldsymbol{\Phi}}(L) = \tilde{\boldsymbol{\Phi}}_0$, i.e. the lags of the exogenous variables do not appear in (6). Since both u_t^r and $\mathbf{g}(u_t^r)$ are orthogonal to the past history of $\mathbf{x}_t$, in this case we have $\mathbf{C}(L) = 0$ and $\boldsymbol{w}_t = \tilde{\boldsymbol{\alpha}}_0 u_t^r + \tilde{\boldsymbol{\Phi}}_0\mathbf{g}(u_t^r)$. Hence the VAR dynamics concide with the VARX dynamics, $\mathbf{B}(L) = \tilde{\mathbf{A}}(L)$, and the terms driven by the exogenous variables enter the VAR residual, $\boldsymbol{\varepsilon}_t = \tilde{\boldsymbol{\alpha}}_0 u_t^r + \tilde{\boldsymbol{\Phi}}_0\mathbf{g}(u_t^r) + \mathbf{e}_t$. This special case shows clearly that standard proxy-SVAR identification does not work properly to estimate the linear component of the IRFs, unless u_t^r is orthogonal to $\mathbf{g}(u_t^r)$. For, the covariances of the VAR residuals $\boldsymbol{\varepsilon}_t$ with the proxy are not proportional in general to the covariances of the shock u_t^r with the proxy. Below we show that, despite this, the policy shock itself can be consistently estimated.

[13] If the monetary shock was perfectly observable, then eq. (6) or a local projection version of it could be estimated by OLS. In section 4.5 we discuss why such a procedure could be problematic if only an imperfect measures of the shock is available.

where $b \neq 0$ and v_t is an error independent of the structural shocks at all leads and lags. Notice that under Assumption A3, the standard conditions for a valid instrument, i.e. $\text{cov}(z_t, u_t^r) = b \neq 0$ (relevance) and $\text{cov}(z_t, \mathbf{u}_t) = 0$ (exogeneity), are satisfied.

The second assumption ensures that the monetary shock can be estimated as a combination of current and past data. In particular, we assume the existence of a monetary policy rule, i.e. an equation where the interest rate reacts to current and past values of $\mathbf{x}_t$, as well as to the monetary policy shock, but not on its nonlinear functions. This is stated formally in the following assumption.

Assumption A4 (*Monetary Policy Rule*). The central bank follows the monetary policy rule

$$r_t = \left[\psi + \boldsymbol{\xi}' \mathbf{x}_t^{-r} + \sum_{j=1}^{\infty} \boldsymbol{\varphi}_j' \mathbf{x}_{t-j} \right] + \sigma_r u_t^r. \quad (9)$$

The term in square brackets in eq. (9) represents the "systematic" component of the rule, where $\mathbf{x}_t^{-r}$ is the vector containing all variables in $\mathbf{x}_t$ but the interest rate, ψ is a constant scalar, while $\boldsymbol{\xi}$ and $\boldsymbol{\varphi}_j$ denote vectors of parameters with dimension $(n-1) \times 1$ and $n \times 1$, respectively. The residual $\sigma_r u_t^r$ is the non-normalized monetary policy shock.

A monetary policy rule like (9) is standard in the monetary policy literature, and is implied by any structural VAR model used to identify monetary policy shocks. According to the rule in A4, the central bank may react to all variables contemporaneously. For this reason, our rule is more general than the one implied by standard recursive (Cholesky) identification schemes, where it is assumed that the central bank does not react contemporaneously to a subset of the structural shocks —see e.g. Christiano et al. (1996, 1999).[14]

Assumption A4 can be rephrased by saying that the variables in $\mathbf{x}_t$ are informationally sufficient[15] for the monetary policy shock, or, in other words, the monetary policy shock is fundamental for $\mathbf{x}_t$. In our setting, such assumption imposes special restrictions on eq. (6).[16] Notice that equation (9) cannot be estimated directly to get the policy shock, since the residual is not orthogonal to $\mathbf{x}_t^{-r}$.[17]

[14] Earlier empirical studies on the nonlinear effects of monetary policy (see e.g. Cover, 1992 and Karras, 1996) directly estimated a monetary rule like (9), and treated the residual as a measure of the monetary shock. As is well known, that procedure is only valid under the assumption that monetary shocks have no contemporaneous effects on macroeconomic variables other than the interest rate —a restriction that we do not impose here.

[15] On the concept of informational sufficiency see Forni and Gambetti (2014) and Forni, Gambetti and Sala (2019).

[16] In particular, there exists a linear combination of the variables, namely $\gamma' \mathbf{x}_t \equiv r_t - \boldsymbol{\xi}' \mathbf{x}_t^{-r}$ that depends neither on the nonlinear term, i.e. $\gamma' \tilde{\boldsymbol{\Phi}}(L) = 0$, nor on $\mathbf{e}_t$, i.e. $\gamma' \mathbf{e}_t = 0$ (so that the variance-covariance matrix of $\mathbf{e}_t$ must be singular).

[17] Othogonality with respect to $\mathbf{x}_t^{-r}$ would be equivalent to a Choleski identification scheme with r_t ordered last.

4.3 The key result

We are now ready to present the main result underpinning our empirical approach. To that end, let us consider the VAR representation (7). This representation is, in a sense, misspecified, since it does not take into account the nonlinear term. Despite this, the following Proposition shows that the VAR residuals in $\boldsymbol{\varepsilon}_t$ can be combined with the external instrument to recover the monetary policy shock.

Proposition. *Under Assumptions A0 to A4 the monetary policy shock is equal, up to a multiplicative constant, to the orthogonal projection of the instrument z_t onto the VAR innovations $\boldsymbol{\varepsilon}_t$.*

Proof. Let us assume, without loss of generality, that r_t is ordered first in the vector $\mathbf{x}_t$ and let $\gamma \equiv [1 \quad -\boldsymbol{\xi}']'$. Rearranging the monetary rule (9) we get

$$\gamma' \mathbf{x}_t = \psi + \sum_{j=1}^{\infty} \boldsymbol{\varphi}_j' \mathbf{x}_{t-j} + \sigma_r u_t^r, \tag{10}$$

where u_t^r is orthogonal to $\mathbf{x}_{t-j}$, $j = 1, \ldots, \infty$ by Assumption A0. On the other hand, pre-multiplying the linear VAR (7) by γ' we obtain

$$\gamma' \mathbf{x}_t = \gamma' \boldsymbol{\vartheta} + \gamma' \sum_{j=1}^{\infty} \mathbf{B}_j \mathbf{x}_{t-j} + \gamma' \boldsymbol{\varepsilon}_t. \tag{11}$$

Subtracting (11) from (10) and reordering terms we get

$$\psi - \gamma' \boldsymbol{\vartheta} + \sum_{j=1}^{\infty} \boldsymbol{\varphi}_j' \mathbf{x}_{t-j} - \gamma' \sum_{j=1}^{\infty} \mathbf{B}_j \mathbf{x}_{t-1} = \gamma' \boldsymbol{\varepsilon}_t - \sigma_r u_t^r. \tag{12}$$

Now, the right side of (12) is orthogonal to the left side, because both u_t^r and $\boldsymbol{\varepsilon}_t$ are zero-mean and orthogonal to $\mathbf{x}_{t-j}$, $j = 1, \ldots, \infty$ by Assumptions A0 and A2. Then, the term $\sigma_r u_t^r - \gamma' \boldsymbol{\varepsilon}_t$ is orthogonal to itself and therefore is null, implying that

$$\sigma_r u_t^r = \gamma' \boldsymbol{\varepsilon}_t. \tag{13}$$

Eq. (13) indicates that, at any given point in time, the monetary shock must be equal to a linear combination of the innovations of the linear VAR. Now let us show that such linear combination can be estimated using the external instrument z_t.[18] Let $\mathbb{P}$ denote the linear projection operator. By projecting both sides of equation (8) onto the entries of $\boldsymbol{\varepsilon}_t$ we get $\mathbb{P}(z_t|\boldsymbol{\varepsilon}_t) = \mathbb{P}(a|\boldsymbol{\varepsilon}_t) + \mathbb{P}(bu_t^r|\boldsymbol{\varepsilon}_t) + \mathbb{P}(\delta(L)'\mathbf{x}_{t-1}|\boldsymbol{\varepsilon}_t) + \mathbb{P}(v_t|\boldsymbol{\varepsilon}_t) = \mathbb{P}(bu_t^r|\boldsymbol{\varepsilon}_t)$, by the orthogonality properties in Assumptions A2 and A3. But $bu_t^r = (b/\sigma_r)\gamma' \boldsymbol{\varepsilon}_t$. It then follows that $\mathbb{P}(z_t|\boldsymbol{\varepsilon}_t) = (b/\sigma_r)\gamma' \boldsymbol{\varepsilon}_t = bu_t^r$.■

[18] The recursive (Cholesky) identification often used in the literature can be viewed as a special case of our procedure where the vector γ, rather than being estimated using an external instrument, is assumed to satisfy specific restrictions, e.g. $\gamma_i = 1$ and $\gamma_j = 0$ for $j \neq i$ where i denotes the position of the interest rate in the vector $\mathbf{x}_t$.

To sum up, if there is a combination of variables that delivers the structural shock and a valid instrument is available, the structural shock can be estimated as the projection of the instrument on the VAR residuals. In this respect, unlike the standard proxy-SVAR approach, we use the proxy to find the shock, rather than the impulse response functions. The shock and its nonlinear function can then be used as regressors to estimate the nonlinear VARX in eq. (6), as described below.

4.4 Estimation

The above result justifies the following estimation procedure.

I. Estimate the VAR (7) to obtain consistent estimates of the residual $\boldsymbol{\varepsilon}_t$, say $\hat{\varepsilon}_t$.[19]

II. Estimate the linear regression

$$z_t = \hat{\boldsymbol{\lambda}}'\hat{\boldsymbol{\varepsilon}}_t + \hat{\eta}_t. \tag{14}$$

where $\boldsymbol{\lambda} = (b/\sigma_r)\boldsymbol{\gamma}$, as derived in the Proposition. An estimate of the normalized shock is obtained by standardizing the fitted value of the above regression, i.e. $\hat{u}_t^r = \hat{\boldsymbol{\lambda}}'\hat{\boldsymbol{\varepsilon}}_t/\text{std}(\hat{\boldsymbol{\lambda}}'\hat{\boldsymbol{\varepsilon}}_t)$.

III. Estimate equation (6) using as regressors the current value and the lags of the the estimated shock $\hat{u}_t^r$ and its nonlinear functions $\mathbf{g}(\hat{u}_t^r)$. This gives the estimates $\widehat{\mathbf{A}(L)}$, $\widehat{\tilde{\boldsymbol{\Phi}}(L)}$ and $\widehat{\tilde{\boldsymbol{\alpha}}(L)}$. Finally, according to Assumption A1, one can estimate the IRFs of the linear and the nonlinear terms as $\widehat{\boldsymbol{\alpha}(L)} = \widehat{\mathbf{A}(L)}^{-1}\widehat{\tilde{\boldsymbol{\alpha}}(L)}$ and $\widehat{\boldsymbol{\Phi}(L)} = \widehat{\mathbf{A}(L)}^{-1}\widehat{\tilde{\boldsymbol{\Phi}}(L)}$.

IV. Compute the impulse response functions according to equation (3).

In Appendix A.1 we describe in details how to build confidence intervals using a bootstrapping procedure, and in Appendix A.2 we assess the validity of our empirical approach on artificial data from the VARX model (6).

4.5 Aside: a word of caution on local projections

A natural alternative approach when an instrument of the shock is available, is to use Local Projections (LP). That approach is perfectly valid when the underlying model is linear, i.e. the nonlinear term $\mathbf{g}(u_t^r)$ is not present in the model equations (see e.g. Stock and Watson, 2018). However, when the term $\mathbf{g}(u_t^r)$ is present, using the external instrument in a LP (or VARX) setting becomes more problematic. Indeed, that approach would only be valid if the external instrument were a perfect measure of the shock, which is arguably not the case in practice.

To illustrate the nature of the problem, we make use of an elementary example. Consider the following simplified model

$$x_t = \alpha_x u_t^r + \phi_x g(u_t^r) + e_t \tag{15}$$

[19] Of course we have to approximate the VAR(∞) with a finite-order VAR.

which can be viewed as a single equation of model (6), and where for simplicity we are abstracting from the lag-dependence, i.e. we assume that $\tilde{\mathbf{A}}(L) = 0$.

As discussed above, the nonlinear impulse response functions are given by $\alpha_x \bar{u}^r + \phi_x g(\bar{u}^r)$. If u_t^r were perfectly observable, then a simple OLS estimation would deliver the two coefficients and the implied impulse response functions. Alternatively, suppose that the shock u_t^r is not perfectly observable and the econometrician only observes a noisy proxy of the shock, $z_t = u_t^r + v_t$, where v_t can be interpreted as a noise shock, which is assumed to be independent at all leads and lags to u_t^r. For simplicity of exposition let us also assume that $\mathbb{E}(g(z_t)u_t^r) = \mathbb{E}(g(u_t^r)z_t) = 0$, $Cov(g(u_t^r), g(z_t)) \neq 0$ and $Cov(u_t^r, g(u_t^r)) = 0$.[20] When estimating the above model using $g(z_t)$ and z_t as regressors, the OLS estimators are

$$\tilde{\alpha}_x = \frac{Cov(x_t, z_t)}{Var(z_t)} = \alpha_x \frac{Cov(u_t^r, z_t)}{Var(z_t)} \neq \alpha_x$$

and

$$\tilde{\phi}_x = \frac{Cov(x_t, g(z_t))}{Var(g(z_t))} = \phi_x \frac{Cov(g(u_t^r), g(z_t))}{Var(g(z_t))} \neq \phi_x.$$

The previous equations shows that, as it is well known, the presence of a measurement error in the regressors leads to a bias in OLS estimates – even after correcting for the denominators which are known. In a linear setting (when $\phi_x = 0$), this problem can be easily solved by appropriately rescaling the impulse response functions. In practice, it suffices dividing the estimated impulse response $\tilde{\alpha}_x$ by the estimated coefficient for another variable (for instance the policy instrument) α_r. As shown in Stock and Watson (2018), following this procedure delivers an unbiased estimate of the normalized (or relative) impulse response, i.e. $\tilde{\alpha}_x/\tilde{\alpha}_r = \alpha_x/\alpha_r$.

Such a rescaling procedure, however, is not successful in a nonlinear setting. Indeed, in the above simple case, it is possible to obtain the unbiased coefficients of the two terms separately, i.e. α_x/α_r and ϕ_x/ϕ_r, but it is not possible in general to combine them in order to obtain the total nonlinear response. To see this, let us rewrite the above model as $x_t = \frac{\alpha_x}{\alpha_r}\alpha_r u_t^r + \frac{\phi_x}{\phi_r}\phi_r g(u_t^r) + e_t$. Since α_r and ϕ_r are unknown, the rescaled responses cannot be combined. What can be computed is $\frac{\alpha_x}{\alpha_r}\alpha_r \bar{u}^r + \frac{\phi_x}{\phi_r} g(\alpha_r \bar{u}^r)$. This however will corresponds to the true response only if $\phi_r g(\bar{u}^r) = g(\alpha_r \bar{u}^r)$, a restriction which of course will not hold in general. On the contrary, when $\phi_x = 0$ the procedure yields the correct linear responses $\frac{\alpha_x}{\alpha_r}$ to an α_r-standard deviation shock $\alpha_r u_t^r$.

In other words, the source of the problem is that the unbiased rescaled responses to the linear and nonlinear components must be combined using parameters which are unknown and cannot, in general, be obtained. Without having the correct parameters the total nonlinear response cannot be consistently estimated.

This simple example calls for some caution in using nonlinear local projections or VARX with external instruments which are noisy measures of the underlying structural shock. Appendix A.2 provides a quantitative illustration of the problem in the context of a simulated example.

[20] Relaxing this assumption would exacerbate the problem under consideration.

The same argument illustrates the importance of the first step in the procedure proposed here. Estimating the VARX model directly with the instrument in place of the estimated shock would produce the difficulty discussed above.

5 Empirical Analysis

In this section we present our main empirical results about the nonlinear transmission of monetary policy shocks and present some robustness checks.

For the nonlinear proxy SVAR we use a specification very similar to that used in Miranda-Agrippino and Ricco (2021). The VAR includes five variables, namely the 1-year Treasury bond rate (1YB), the growth rate of industrial production (IP), the excess bond premium (EBP), the unemployment rate (UR) and CPI inflation (CPI LE). All data are for the U.S. economy, at a monthly frequency for the period from 1973:M1 to 2019:M6, and we include three lags for each variable. The instrument to recover the monetary shock is taken from Degasperi and Ricco (2022), which is an extended version of Miranda-Agrippino and Ricco (2021).

In order to study the sign- and state-dependence of monetary shocks we set the nonlinear functions $\mathbf{g}(u_t^r) \equiv [|u_t^r| \quad s_{t-1}u_t^r]'$, s_t being an indicator of the state of the economy such that $s_{t-1} = 1$ when the average GDP growth over the previous 12 months is negative, and $s_{t-1} = 0$ otherwise.[21] Thus, according to eq. (2), the nonlinear effects of monetary shocks are captured by the polynomial matrix $\mathbf{\Phi(L)} \equiv [\boldsymbol{\phi}^1(L) \quad \boldsymbol{\phi}^2(L)]$, with the first column $\boldsymbol{\phi}^1(L)$ capturing sign-dependence, and the second column $\boldsymbol{\phi}^2(L)$ capturing state-dependence.

Denoting with $\boldsymbol{\alpha}_h$ and $\boldsymbol{\phi}_h^i$, $i = 1, 2$ the h-th lag in the polynomials $\boldsymbol{\alpha}(L)$ and $\boldsymbol{\phi}^i(L)$, and normalizing the shock size to unity, the impulse responses during booms ($s_{t-1} = 0$) and recessions ($s_{t-1} = 1$), at horizon h, are given for monetary easing ($u_t^r = -1$) by

$$\mathcal{R}_h^+(s_{t-1} = 0) \equiv -\boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 \qquad \mathcal{R}_h^+(s_{t-1} = 1) \equiv -\boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 - \boldsymbol{\phi}_h^2$$

while for a monetary tightening ($u_t^r = 1$) we have

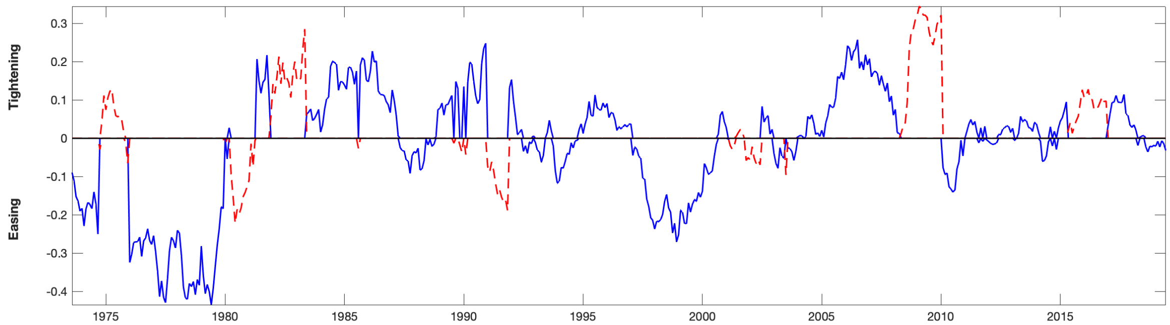
$$\mathcal{R}_h^-(s_{t-1} = 0) \equiv \boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 \qquad \mathcal{R}_h^-(s_{t-1} = 1) \equiv \boldsymbol{\alpha}_h + \boldsymbol{\phi}_h^1 + \boldsymbol{\phi}_h^2$$

Given these impulse responses, the corresponding monetary tradeoffs can be easily calculated according to eq. (1).

As explained in Section 4.3, our measure of the monetary shock is obtained by regressing the external instrument on the VAR residuals. Figure 1 reports the resulting series for the monetary shock, smoothed by using a moving average of 12 months. Notably, the largest monetary easing shocks (a negative shock) is identified during 1975-1979, a period with relatively high GDP

[21] We also analyzed the effects of sign- and state-dependence in isolation, and considered the unemployment rate as business cycle indicator. The corresponding results are reported in an online appendix.

Figure 1: Identified Monetary Shocks



Notes: Time-series of identified monetary shocks, for monetary easing (negative values), tightening (positive values), during booms (blue solid line) and recessions (red dashed line).

growth and high inflation; instead, the largest tightening (positive shock) occurred during the recession of 2008-09, which is consistent with the view that US Federal Reserve had to abandon its conventional policy rule, due to a binding zero lower bound constraint.[22] Instead, during the Great Moderation period, the identified monetary shocks are smaller, and predominantly countercyclical (with the period 1998-99 being the main exception). Interestingly during the 1982 recession, during the Volcker mandate, monetary policy is estimated to be contractionary.

5.1 Results

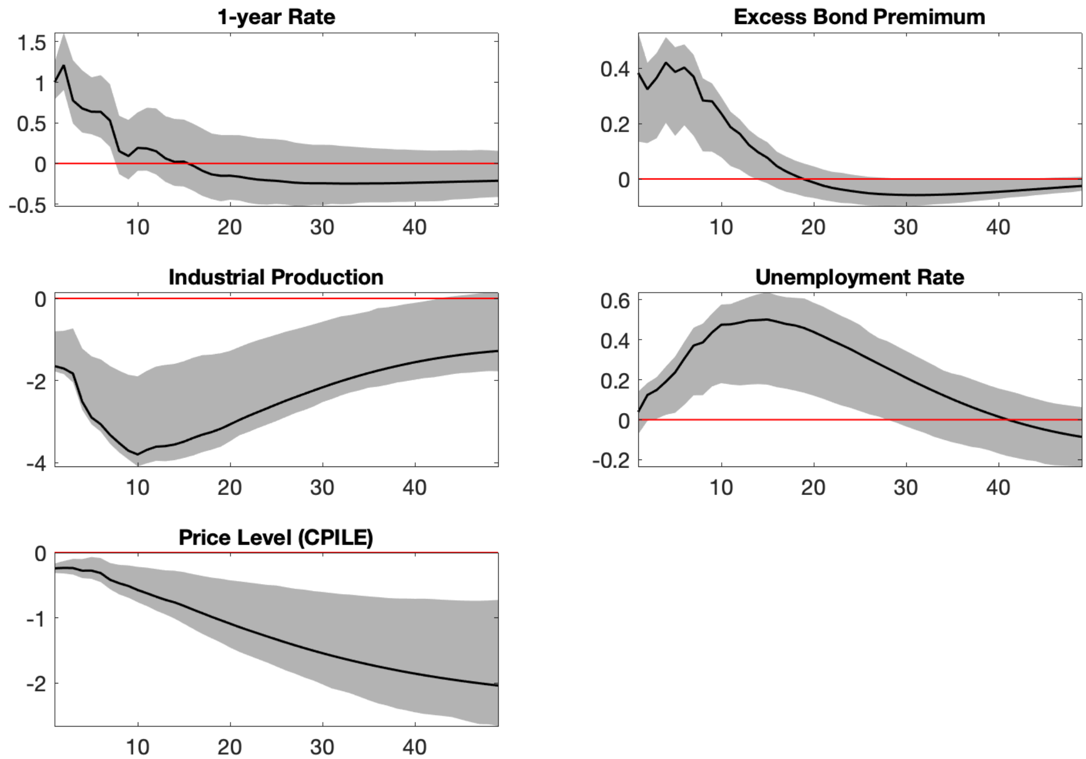
As a preliminary step, we estimate a *linear* proxy-SVAR, thus ignoring the presence of nonlinear terms. Results are summarized in Figure 2, which plots the point estimates of the impulse response functions (solid lines) together with their 68% confidence bands (gray area).

Consistently with conventional results in the literature, we find that an increase in interest rate is associated with a significant decline in inflation and industrial production, and a significant increase in the unemployment rate and in the excess bond premium.

Figure 3 plots the impulse responses for the *nonlinear* model, distinguishing between the effects of the linear component $\boldsymbol{\alpha}(L)$ (first column), the sign component $\boldsymbol{\phi}^1(L)$ (second column), and the state component $\boldsymbol{\phi}^2(L)$ (third column). The responses of the linear term are similar to those displayed in Figure 2 for the linear model. More interestingly, for all variables the responses of the nonlinear components are similar in magnitude to those of the linear counterparts and significant in most cases, thus rejecting the null of linearity and suggesting that non-linearities play a substantial role in shaping the overall responses to a monetary shock. In particular, both the sign (second column) and the state (third column) components have persistent and significant positive effects on unemployment (fourth row), implying that monetary policy leads to larger

[22]When the zero lower bound is binding, since the interest rate cannot be lowered in response to the falling inflation and output, there must be an increase of the discretionary component in order to keep the interest rate at zero.

Figure 2: Impulse response functions in the linear Proxy-SVAR

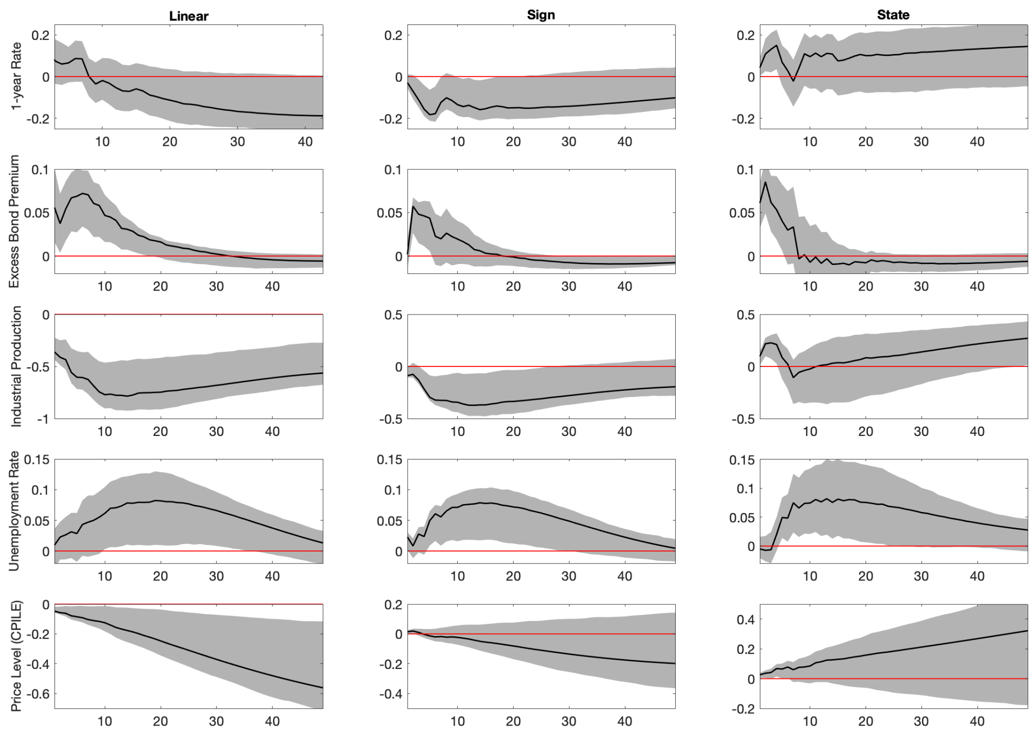


Notes: Impulse responses to a monetary shocks. Solid lines represent point estimates, the gray areas are 68% confidence bands.

changes in unemployment if a tightening is implemented during a recession. Instead, for the case of prices (last column) the sign and state components operate in opposite directions, implying that the largest changes in inflation are associated with monetary tightening during an expansion.

This can be seen more clearly in Figure 4, which compares the total effects of monetary easing and tightening during booms (first two columns) and recessions (last two columns). For real variables (unemployment and industrial production), a monetary tightening always has large and significant effects. Instead, a monetary easing has much smaller effects. In this respect, our result are broadly consistent with the findings of Tenreyro and Thwaites (2016) and Barnichon and Matthes (2018), and could lead to the conclusion that monetary policy is not a very powerful tool to stimulate economic activity. Yet, it could be objected that a weak response of economic activity, as long as statistically significant, does not invalidate *per se* the effectiveness of monetary policy. It rather implies that the central bank should adopt more aggressive measures to fight a recession. The desirability of a more aggressive stance, however, crucially depends on the inflation-unemployment tradeoff facing the central bank, as more aggressive measures would

Figure 3: Impulse response functions in the nonlinear model



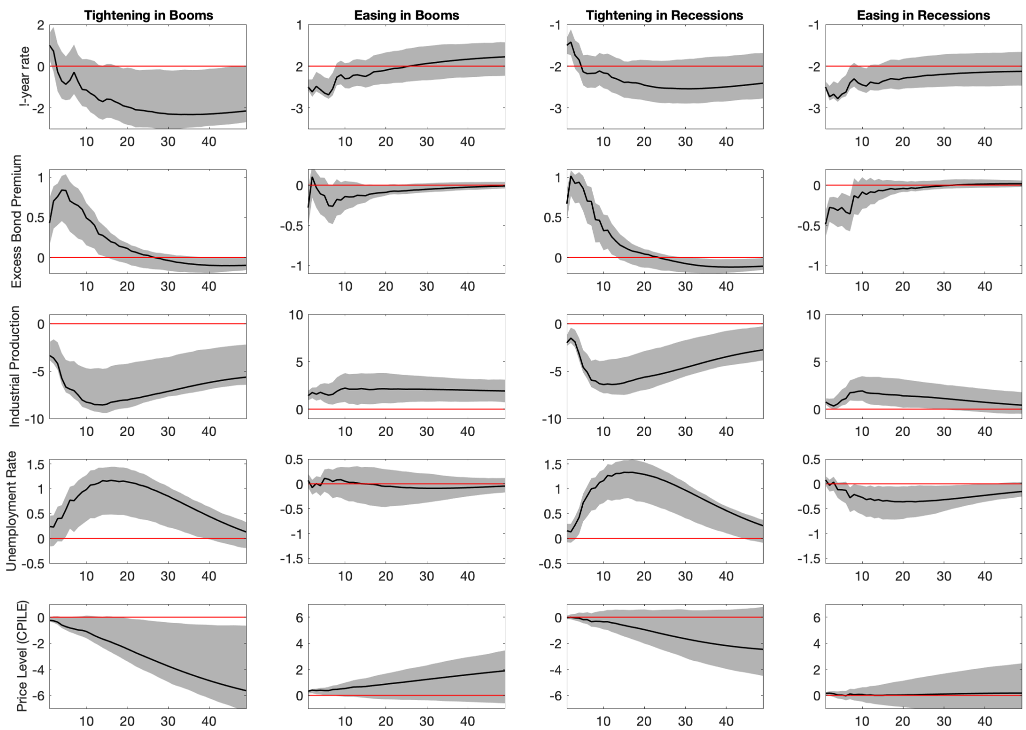
Notes: Impulse response to a monetary shock of the linear (first column), sign (second column) and state (third column) components. Solid lines represent point estimates, the gray areas are 68% confidence bands.

likely result into larger costs in terms of inflation.

To get a sense of the inflation-unemployment tradeoff, Figure 5 displays a scatterplot of the (cumulative) impulse responses of inflation (horizontal axis) and unemployment (vertical axis), where each point corresponds to the cumulative effect over alternative horizons (e.g., $H = 12, 24, 36$ months).

A few interesting results stand out. First a monetary easing during recessions (red solid line) leads to a protracted and significant decline of the unemployment rate, but no significant change in inflation. That result squares with the implication of a flat Phillips curve, thus leading to the opposite conclusion to the one reached in Tenreyro and Thwaites (2016) and Barnichon and Matthes (2018): a monetary easing could be an effective tool to stimulate the economy in recessionary periods since even large policy interventions are associated to very modest costs in terms of inflation. Second, deflationary policies during expansions (blue dashed) have sizable costs in terms of unemployment. Those costs, however, are substantially smaller than the costs associated with contractionary policies during recessions (red dashed) or those implied by a flat Phillips curve (which would be infinite). Third, a policy easing in an expansion is extremely

Figure 4: The Effects of Monetary Easing and Tightening, during Booms and Recessions



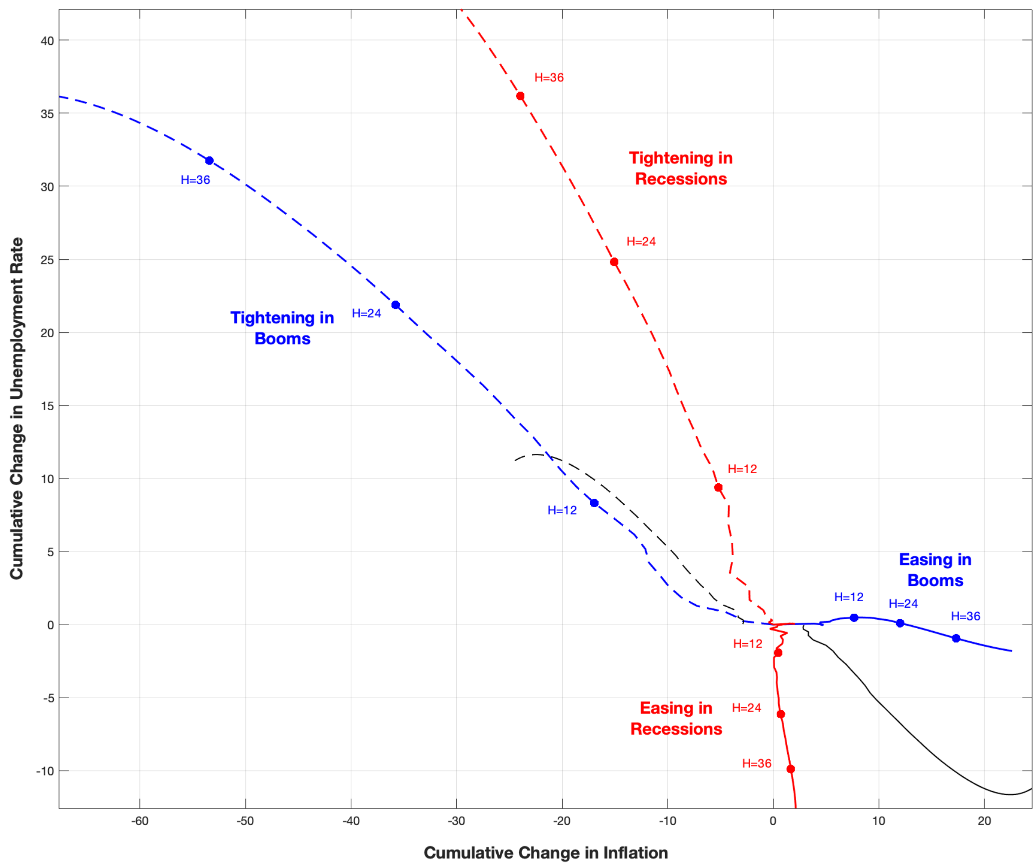
Notes: Impulse response to a monetary shock during booms (first two columns) and recessions (last two columns), for monetary tightening (first and third column) and easing (second and fourth column).Solid lines represent point estimates, the gray areas are 68% confidence bands.

inflationary with virtually no effects on the unemployment rate –i.e. an extremely large inflation-unemployment tradeoff.

Table 1 reports the estimated values of the monetary tradeoffs, together with the corresponding 68% confidence intervals. The inflation cost of reducing unemployment during a recession (second column) is small, ranging between 0.03 and 0.17 (in absolute value) depending on the horizon considered, and generally insignificant. In a linear model, this would imply an unemployment cost between 6 and 33 percentage points for each percentage point reduction in inflation (i.e. the inverse of 0.17 and 0.03, respectively). Instead, we find that during a boom (third column), the unemployment cost of reducing inflation by 1 percentage point is an order of magnitude smaller, and ranges between 0.5 and 0.6 percentage points. Similar results are obtained if we consider the pre-2009 sample (last two columns), to exclude the period when the zero-lower bound was binding. The tradeoff is much bigger in other situations – e.g. a tightening during recessions – as could be seen in Figure 5.[23]

[23]The corresponding values are not reported in Table 1, since in many instances calculating the tradeoff would require dividing by values close to zero, giving rise to uninformative results.

Figure 5: The Effects of Monetary Shocks on Inflation and Unemployment



Notes: The figure plots the relationship between the cumulative change in inflation (x-axis) and in the unemployment rate (y-axis) at different horizons (H={12,24,36}), in response to monetary easing (solid lines) and tightening (dashed lines), during booms (blue lines) and recessions (red lines).

All in all, the results provide support to the use of countercyclical monetary policies —both monetary easing in recessions and tightening in expansions— as those policies are associated with relatively favorable inflation-unemployment tradeoffs.

6 A model with downward nominal wage rigidities

This section illustrates a simple theoretical model with downward nominal wage rigidities that gives rise to a sign- and state-dependent monetary tradeoff.

This model is used for two main purposes: first, to interpret the evidence discussed in the previous section; second, to assess, by means of a Monte Carlo simulation, whether our empirical approach is able capture the nonlinearities featured by the theoretical model.

Table 1: Monetary Tradeoffs

Horizon	Full-Sample (1973:M1-2019:M6) Easing in Recession $\mathcal{T}_H^+(s_{t-1}=1)$	Full-Sample (1973:M1-2019:M6) Tightening in Boom $\mathcal{T}_H^-(s_{t-1}=0)$	Pre-2009 Easing in Recession $\mathcal{T}_H^+(s_{t-1}=1)$	Pre-2009 Tightening in Boom $\mathcal{T}_H^-(s_{t-1}=0)$
$H=12$	-0.03	-0.51	0.14	-0.72
	(-2.93, 4.28)	(-1.12, 0.003)	(-4.51, 4.16)	(-1.14, 0.37)
$H=24$	-0.12	-0.61	-0.29	-0.82
	(-2.49, 2.41)	(-1.25, 0.004)	(-2.95, 2.19)	(-1.61, 0.44)
$H=36$	-0.17	-0.59	-0.46	-0.77
	(-2.48, 2.20)	(-1.29, -0.05)	(-2.90, 1.63)	(-1.46, 0.56)
$H=48$	-0.17	-0.53	-0.51	-0.69
	(-2.41, 2.33)	(-1.20, -0.006)	(-3.07, 1.79)	(-1.20, 0.65)

Notes: The table reports the size of the monetary tradeoff at different horizons (H=12,24,36,58 months) associated with monetary easing during recessions —i.e. the inflation cost of reducing unemployment ($\mathcal{T}_H^+(s_{t-1}=1)$— and monetary tightening during a boom —i.e. the unemployment cost of reducing inflation ($\mathcal{T}_H^+(s_{t-1}=1)$). Column 2 and 3 refers to the full-sample (1973:M1-2019:M6), while the last two columns refer to the pre-2009 sample.

6.1 Preferences, Technology and Monetary Policy

The economy is populated by a large number of identical households with preferences described by the objective function $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$, where C_t denotes consumption and $\beta \in (0,1)$ is the subjective discount factor. The household budget constraint is given by

$$P_t C_t + B_t = W_t N_t + r_{t-1} B_{t-1}, \tag{16}$$

where P_t denotes the price level, B_t denote nominal one-period riskless bonds, and r_t is the gross nominal interest rate between period t and $t+1$.

Each household supplies inelastically one unit of labor $\bar{N}=1$. However, the labor market features downward nominal wage rigidities, so that $W_t \geq \phi W_{t-1}$, where $\phi \leq 1$ is a parameter measuring the severity of the rigidity. Whenever the latter constraint is binding, only a fraction $N_t \leq \bar{N} = 1$ of households is employed, and the remaining $1 - N_t$ households remain unemployed. In other words, the presence of downward nominal wage rigidities may give rise to “involuntary" unemployment.

Output of the single good (Y_t) is produced by perfectly competitive firms using labor as the only input according to the linear technology $Y_t = \exp\{a_t\} N_t$, where a_t denotes total factor productivity, which is assumed to follow the exogenous random-walk process $a_t = a_{t-1} + u_t^a$, with $u_t^a \sim N\left(-\sigma_a^2/2, \sigma_a^2\right)$. Firms' profit maximization implies that real wages $W_t/P_t = \exp\{a_t\}$ in every period. Also, it follows that the “natural” level of output (i.e. the level of output prevailing when the economy operates at full employment) is given by $Y_t^n \equiv \exp\{a_t\}$.

Monetary policy is conducted according to a Taylor-type interest rate rule

$$r_t = \bar{R}\Pi_t^{\phi_\pi} \exp\{m_t\} \tag{17}$$

where $\bar{R}$ is the steady-state interest rate, $\phi_\pi > 1$ is a parameter measuring the central bank's response to inflation, and m_t is a monetary policy shocks, following the AR(1) process $m_t = \rho_m m_{t-1} + u_t^r$, where $u_t^r \sim N\left(-\sigma_r^2/2, \sigma_r^2\right)$.

6.2 Equilibrium

The competitive equilibrium of this economy is fully characterized by the following two equations, summarizing the relationship between output and inflation:

$$1 = \Pi_t^{\phi_\pi} \exp\{m_t\}\mathbb{E}_t \left\{ \left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma} \Pi_{t+1}^{-1} \right\} \tag{18}$$

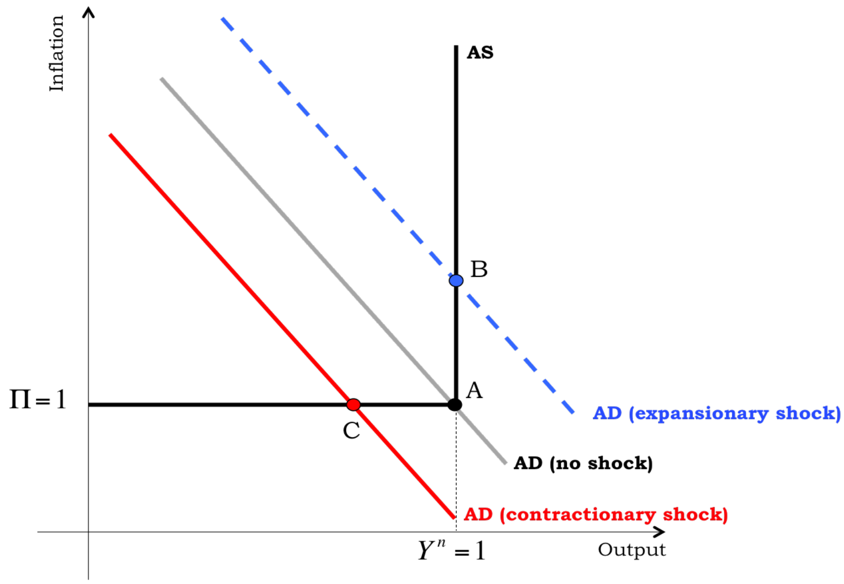
$$(Y_t/\exp\{a_t\} - 1)\left(\exp\{u_t^a\} - \phi\Pi_t^{-1}\right) = 0 \tag{19}$$

Equation (18) is an aggregate demand (AD) relationship, and is obtained combining the consumption Euler equation from the household's optimal consumption/savings decision with the monetary policy rule (17) and the market clearing condition $Y_t = C_t$. Equation (19) describes instead an aggregate supply (AS) relationship, and is obtained combining the production function, the household's labor supply subject to the downward nominal wage rigidity, and the firms' labor demand implying that the real wage $W_t/P_t = \exp\{a_t\}$.

Figure 6 provides a graphical illustration of the main mechanism of the model. It plots the aggregate demand (AD) and aggregate supply (AS) curves, for a given level of expected output and inflation. Note that the presence of downward wage rigidities introduce a "kink" in the aggregate supply relationship, and for this reason the real effects of monetary policy shocks are asymmetric. Suppose for instance that the economy is initially in a situation where technology is at its steady state level, the economy is at full-employment, i.e. $Y_t^n = \exp a_t = 1$, and (gross) inflation $\Pi = 1$, so that the downward wage rigidity is not binding (point A in the graph). Starting from that situation, an expansionary monetary shock stimulates aggregate demand (i.e. the AD shifts to the right, to point B) putting upward pressures on nominal wages and prices, meaning that the downward wage rigidity is not binding (i.e. the economy lies in the vertical portion of the AS curve). Thus, the only effect of the monetary shock is an increase in inflation, with no effect on output. On the contrary, a contractionary monetary shock that reduces aggregate demand (the AD shifts to the left) makes the downward wage rigidity binding (i.e. the economy moves to the horizontal part of the AS curve), which implies a reduction in output, with no effect on inflation (point C).

More generally, within this model the effects of monetary easing and tightening depends on whether the economy is or not at full-employment. Thus, conditional on economic activity re-

Figure 6: A Simple Model with Downward Wage Rigidities



Notes: The figure show the Aggregate Supply (AS) and Aggregate Demand (AD) curves. Point (A) denotes the steady state equilibrium (no monetary shock). Point (B) denotes the equilibrium with an expansionary monetary shock, and point (C) is the equilibrium with a contractionary shock.

maining below full-employment, the effects of monetary policies would be completely symmetric, as the economy moves along a flat portion of the supply curve, with no effect on prices. Yet, when looking at the *average* effects of monetary shocks across periods with full-employment and periods with "involuntary" unemployment, a monetary tightening has larger effects on output and weaker effects on prices than monetary easing. This is because, other things equal, in response to a monetary tightening the economy remains below full-employment for a longer period of time than in response to a monetary easing.[24]

6.3 Quantitative results

In order to provide a quantitative illustration of the described asymmetries, we adopt a quarterly calibration of the model, where the discount factor $\beta = 0.99$, the intertemporal elasticity of substitution $\sigma = 1$, the downward wage rigidity parameter $\phi = 1$, the monetary policy coefficient $\phi_\pi = 1.5$. Regarding the two shock processes, in line with existing empirical estimates (see e.g. Smets and Wouters, 2007), we set the autocorrelation of the monetary shock $\rho_m = 0.5$ and the standard deviations $\sigma_r = 0.25$ percent, while the standard deviation of (permanent) innovations to technology is $\sigma_a = 0.45$. The model is solved and simulated using a (non-linear) global projection method, where the expectation term in the aggregate demand (18) is approximated with

[24] For this reason, monetary tightening has larger real effects also when controlling for the state of the economy before the monetary shock hits s_{t-1}, as we did in the empirical exercise.

a Chebyshev polynomial on a coarse grid for the monetary policy shock (see Appendix A.3 for more details).

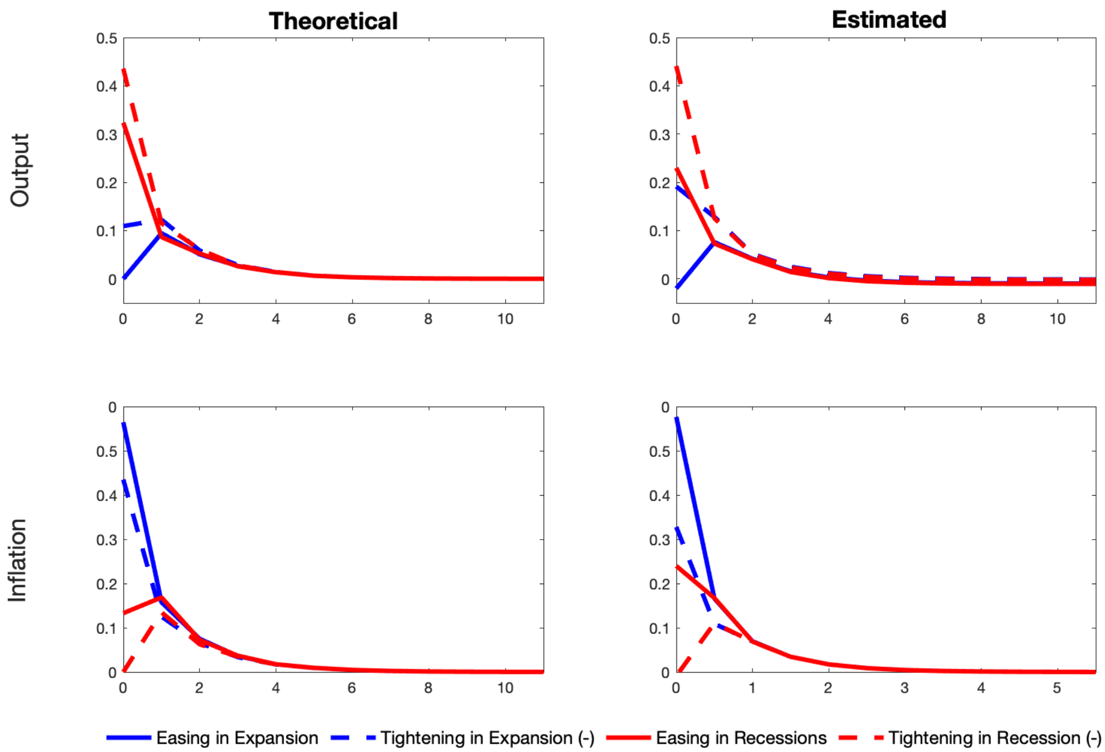
We then perform a Monte Carlo simulation using the model discussed above to validate our empirical procedure. This exercise is particularly important since the empirical specification (6) is possibly misspecified when the data are generated by a nonlinear model, but still could represent a good approximation of the nonlinear dynamics embedded in the DSGE. In particular, we generate 1000 realizations of the technology shock and the monetary policy shock from the model, and calculate the implied series for output, inflation and the interest rate. For every realization of the monetary policy shock, we also construct an instrument which is equal to the shock plus an independent measurement error with standard deviation equal to 0.025 (the same standard deviation of the monetary policy shock). We then apply our econometric procedure using output, prices and the interest rate. First, we estimate the monetary policy shock (the VAR used to estimate the residuals has two lags and includes the three variables output, inflation and the interest rate). Second, we estimate the nonlinear impulse response functions from equation (6) (the VARX is estimated with two lags for both endogenous and exogenous variables). The exogenous variables in the VARX are: the estimated shock itself, its absolute value, and the interaction between the shock and a dummy taking value one if the technology shock in the previous time period was negative and one if positive. Then, we compute the average impulse response functions across the 1000 realizations.

Figure 7 displays the average impulse responses to a monetary shock, where averages are taken across different histories of technology shocks. The first column displays the theoretical impulse responses from the model, and shows that downward nominal wage rigidities can rationalize (at least qualitatively) the sign- and state-dependence of the effects of monetary shocks found in the data. For instance, monetary easing during a recession (red solid line) leads on impact to a 0.3 percent increase in output, at a cost of only a 0.1 percentage point increase on inflation. Instead, an equal size monetary tightening during a boom (blue dotted line) reduces inflation by roughly 0.4 percentage points, with essentially no cost in terms of output. The second column displays the impulse responses obtained by applying our econometric procedure on the artificial data generated by the model. Such responses are very similar to their theoretical counterparts. This result suggests that the empirical nonlinear representation (6) together with the proxy-SVAR identification works remarkably well in approximating the nonlinearities arising from the theoretical model. We believe this is an important result since it sheds some light on the linkages between DSGE models and empirical models with relevant nonlinearities. This is a relatively unexplored issue in the literature which we plan to study further in future research.

7 Conclusions

We propose a novel empirical approach to show that, for the US economy, the inflation-unemployment tradeoff varies substantially depending on the sign of the intervention (easing and tightening),

Figure 7: Monte-Carlo Exercise



Notes: The figure show the average impulse responses of output (first row) and annualized inflation (second row) to a monetary shock. The first column reports the average generalized impulse responses from the theoretical model, calculated as the difference between the path of a variable in the presence of a monetary shock, and the corresponding path without a monetary shock, and where averages are taken across 1000 histories of technology shocks. The second column correspond to the estimated impulse responses using the empirical approach described in Section 4, and where averages are taken with respect to 1000 histories of technology and monetary shocks. To facilitate the comparison, the responses to a monetary tightening are multiplied by minus 1.

and that state of the economy (booms and recessions). In particular, we find small (or no) inflation costs of reducing unemployment during recessions, and moderate unemployment costs of reducing inflation during booms, while the tradeoff is much larger in other cases. We also show that the empirical findings can be rationalized by a simple model with downward nominal wage rigidities.

Overall, our results provide support to the use of countercyclical monetary policies, both during booms and recessions. This conclusion is subject to two main caveats. On the one hand, since the inflation-unemployment tradeoff changes dramatically depending on the state of the economy, the use of countercyclical policies could be risky is situations of high uncertainty regarding the the underlying economic conditions —e.g. a disinflationary policy could be very costly if output growth is weaker than expected. On the other hand, our measure of the tradeoff corresponds to the effects of *average* monetary interventions during the sample period. Clearly, the tradeoff may vary if the central bank adopts unusual policies, in terms of size, persistence, or types of intervention (conventional vs unconventional), and may also depend on the accompanying fiscal policy measures. In this respect, our empirical approach may constitute a useful tool to study additional sources of nonlinearities, and explore the link between nonlinear theoretical models and empirical evidence.

References

Arias, J.E., D. Caldara and J.F Rubio-Ramírez (2019), The Systematic Component of Monetary Policy in SVARs: An Agnostic Identification Procedure, Journal of Monetary Economics, Elsevier, 101(C), 1-13.

Ball, L., and N.G. Mankiw (1994), A sticky-price manifesto, Carnegie-Rochester Conference Series on Public Policy, Elsevier, 41(1), 127-151, December.

Ball, L. (1994). What determines the sacrifice ratio?, in Monetary Policy (pp. 155-193). The University of Chicago Press.

Barnichon, R., and C. Matthes (2018), Functional Approximation of Impulse Responses, Journal of Monetary Economics, Elsevier, 99(C), 41-55.

Barnichon, R. and G. Mesters (2020), Identifying modern macro equations with old shocks, The Quarterly Journal of Economics, 135(4), pp.2255-2298.

Barnichon, R. and G. Mesters (2021), The Phillips multiplier, Journal of Monetary Economics, 117, pp.689-705.

Beraja, M., Hurst, E. and J. Ospina (2019), The aggregate implications of regional business cycles, Econometrica, 87(6), pp.1789-1833.

Bernanke, B.S. (1993), Credit in the macroeconomy, Quarterly Review, Federal Reserve Bank of New York, 18, pp. 50-70.

Bernanke, B.S., and A. Blinder (1992), The Federal Funds Rate and the Channels of Monetary Transmission, American Economic Review, 82, 901-921.

Bernanke, B.S., M. Gertler, (1995), Inside the Black Box: The Credit Channel of Monetary Policy Transmission, Journal of Economic Perspectives, Vol. 9, No. 4, pages 27 - 48.

Bernanke, B.S., I. Mihov (1998), Measuring Monetary Policy, The Quarterly Journal of Economics, Oxford University Press, vol. 113(3), pages 869-902.

Benigno, P., and L. A. Ricci (2011), The Inflation-Output Trade-Off with Downward Wage Rigidities. American Economic Review, 101 (4): 1436-66.

Caldara, D., and E. Herbst (2019), Monetary Policy, Real Activity, and Credit Spreads: Evidence from Bayesian Proxy SVARs, American Economic Journal: Macroeconomics, 11 (1): 157-92.

Christiano, L., M. Eichenbaum, and C. Evans (1996), The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds, The Review of Economics and Statistics, MIT Press, vol. 78(1), 16-34.

Christiano, L., M. Eichenbaum, and C. Evans (1999), Monetary Policy Shocks: What Have We Learned and to What End?, in: J.B Taylor and M. Woodford (eds.), Handbook of Macroeconomics, 1, Elsevier: North Holland, 65-148.

Cochrane, J.H. (1994), Permanent and Transitory Components of GNP and Stock Prices, Quarterly Journal of Economics, 109, 241-265.

Cover, J.P. (1992), Asymmetric Effects of Positive and Negative Money-Supply Shocks, Quarterly Journal of Economics, November 1992, 107(4), pp. 1261-82.

Degasperi, R. and G. Ricco (2021), Information and Policy Shocks in Monetary Surprises, Working Paper.

Daly, M.C. and B. Hobijn (2014), Downward nominal wage rigidities bend the Phillips curve. Journal of Money, Credit and Banking, 46(S2), pp.51-93.

DeLong, J. B., and L.H. Summers (1988), How Does Macroeconomic Policy Affect Output?, Brookings Papers on Economic Activity, 1988, (2), pp. 433-94.

Del Negro, M., G.E. Primiceri, M. Lenza, and A. Tambalotti (2020), What's up with the Phillips curve?, Brookings Papers on Economic Activity, pp. 301-373.

Forni, M., and L. Gambetti (2014), Sufficient information in structural VARs, Journal of Monetary Economics, 66(C), pp. 124-136.

Forni, M., Gambetti, L., and G. Ricco (2023), External Instrument SVAR Analysis for Noninvertible Shocks, CEPR Discussion Papers 17886.

Forni, M., Gambetti, L., and L. Sala (2019), Structural VARs and noninvertible macroeconomic models, Journal of Applied Econometrics, 34(2), pp 221-246.

Forni, M., Gambetti, L., Maffei-Faccioli, N. and L. Sala (2023), The Impact of Financial Shocks on the Forecast Distribution of Output and Inflation, CEPR Discussion Papers 18076, forthcoming in Journal of Business and Economics Statistics.

Fuhrer J. C., and G. Moore (1995), Inflation Persistence, Quarterly Journal of Economics, 110 (1995), 127-159.

Gagnon, J. and C.G. Collins (2019), Low inflation bends the Phillips curve. Peterson Institute for International Economics Working Paper, (19-6).

Galí J., and M. Gertler (1999), Inflation Dynamics: A Structural Econometrics Analysis, Journal of Monetary Economics, 44, 195-222.

Galí J. and L. Gambetti (2020), Has the U.S. Wage Phillips Curve Flattened? A Semi-Structural Exploration, in G. Castex, J. Gali and D. Saravia (eds.) Changing Inflation Dynamics, Evolving Monetary Policy, pp. 149-172, Central Bank of Chile.

Gertler, M., and P. Karadi (2015), Monetary Policy Surprises, Credit Costs, and Economic Activity, American Economic Journal: Macroeconomics, 2015, 7(1): 44-76.

Hazell, J., Herreno, J., Nakamura, E. and J. Steinsson (2022), The slope of the Phillips Curve: evidence from US states, The Quarterly Journal of Economics, 137(3), pp.1299-1344.

Jarocinski, M., and P. Karadi (2020), Deconstructing Monetary Policy Surprises - The Role of Information Shocks, American Economic Journal: Macroeconomics, 12(2), 1-43.

Karras, Georgios (1996), Are the Output Effects of Monetary Policy Asymmetric? Evidence from a Sample of European Countries, Oxford Bulletin of Economics and Statistics, May 1996, 58(2), pp. 267-78.

Keynes, J.M. (1936), The General Theory of Employment, Interest, and Money, Palgrave Macmillan.

Kim, J. and F.J. Ruge-Murcia (2009), How much inflation is necessary to grease the wheels?, Journal of Monetary Economics, 56(3), 365-377.

Leeper, E.M., C.A. Sims and T. Zha (1996), What Does Monetary Policy Do? Brookings Papers on Economic Activity, 2, 1-63.

Mavroeidis S., Plagborg-Møller M., and J.H. Stock James (2014), Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve,? Journal of Economic Literature, 52, 124-188 393-413.

Mertens, K., and M. O. Ravn (2013), The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States,American Economic Review, 103 (4), 1212-47.

McLeay, M. and S. Tenreyro (2020), Optimal inflation and the identification of the Phillips curve, NBER Macroeconomics Annual, 34(1), 199-255.

Miranda-Agrippino, S. and G. Ricco (2021), The Transmission of Monetary Policy Shocks, American Economic Journal: Macroeconomics, 13(3), pp.74-107.

Mountford, A. and H. Uhlig, H. (2009), What are the effects of fiscal policy shocks?, Journal of applied econometrics, 24(6), pp.960-992.

Phillips A. W. (1958), The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861?1957,? Economica, 25, 283?299.

Plagborg-Møller, M. and Christian K. Wolf, (2021), Local Projections and VARs Estimate the Same Impulse Responses, Econometrica, 89 (2), 955?980.

Ramey, V. A., and S. Zubairy (2018), Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data, Journal of Political Economy, University of Chicago Press, 126(2), 850-901.

Roberts J. M. (1995), New Keynesian Economics and the Phillips Curve, Journal of Money, Credit and Banking, 27, 975?984.

Romer, C., and D. Romer (2004), A New Measure of Monetary Shocks: Derivation and Implications, American Economic Review, American Economic Association, vol. 94(4), pages 1055-1084, September.

Samuelson P. and R. Solow (1960), Analytical Aspects of Anti-Inflation Policy,? American Economic Review, 50, 177?194.

Santoro, E., I. Petrella, D. Pfajfar and E. Gaffeo, (2014), Loss aversion and the asymmetric transmission of monetary policy, Journal of Monetary Economics, Vol. 68, 19-36.

Sbordone A. M. (2002), ?Prices and Unit Labor Costs: A New Test of Price Stickiness,? Journal of Monetary Economics, 49, 265?292.

Schmitt-Grohe, S., and M. Uribe (2016), Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment, Journal of Political Economy 124, October 2016, 1466-1514.

Sims, C. A., and T. Zha, (2006), Does monetary policy generate recessions?, Macroeconomic Dynamics, 10(02), pp. 231-272.

Smets, F., and R. Wouters (2007), Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, American Economic Review, 97(3), 586-606.

Stock, J.H. and M.W. Watson (2018), Identification and estimation of dynamic causal effects in macroeconomics using external instruments. The Economic Journal, 128(610), pp.917-948.

Stock, J.H. and M.W. Watson (2020), Slack and cyclically sensitive inflation, Journal of Money, Credit and Banking, 52(S2), pp.393-428.

Strongin, S. (1995), The Identification of Monetary Policy Disturbances: Explaining the Liquidity Puzzle, Journal of Monetary Economics, 35, 463-498.

Tenreyro, S., and G. Thwaites (2016). Pushing on a String: US Monetary Policy Is Less Powerful in Recessions, American Economic Journal: Macroeconomics 8(4), 43-74.

Uhlig, H. (2005), What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure, Journal of Monetary Economics, 52, 381-419.

Weise, C. (1999), The Asymmetric Effects of Monetary Policy: A non-linear Vector Autoregression Approach, Journal of Money, Credit and Banking, 31(1), 85-108.

Appendices

A.1 Inference

To draw the confidence bands we use a bootstrap procedure that takes into account the fact that we have generated regressors. The bootstrap works as follows:

1. Draw with replacement T integers $i(t)$, $t = 1, \dots, T$, uniformly distributed between $p+1$ and T, and construct the artificial sequences $\mathbf{u}_t^1 = \hat{\mathbf{u}}_{i(t)}$, $g(u_t^{1,r}) = g(\hat{u}_{i(t)}^r)$, $\mathbf{e}_t^1 = \hat{\mathbf{e}}_{i(t)}$, and $z_t^1 = z_{i(t)}$, for $t = 1, \dots, T$.

2. With the sequences obtained in step 1 compute an artificial dataset $\mathbf{x}_t^1$, $t = 1, ..., T$, using equation (6) with the initial conditions, $\mathbf{x}_t$, $t = 1, ..., p$, and possibly, if $q > 0$, $g(u_t^{1,r})$, $t = p - q + 1, \dots, p$.

3. With the new dataset, repeat the estimation procedure. In particular:

   (a) Estimate the VAR of equation (7) and get the new residuals.

   (b) Estimate the new shock from equation (14), by regressing the bootstrapped instrument obtained in Step 1 onto the residuals obtained in Step 3(a).

   (c) Use the dataset obtained in Step 2, the shock estimated in Step 3(b) and its nonlinear function to estimate equation (6) and the impulse response functions.

4. Repeat steps 1-3 $J-1$ times to obtain $J-1$ datasets $\mathbf{x}_t^j$, $j = 2, ..., J$ and the related impulse response functions. Compute the confidence band as usual, by taking appropriate pointwise percentiles.

A.2 Simulations

In this Appendix we run a simulation to assess the validity of the empirical procedure when the data generating process is given by eq. (6). To keep things tractable we consider the following simplified version of the model:

$$\mathbf{x}_t = \mathbf{A}_1 \mathbf{x}_{t-1} + \boldsymbol{\mu} + \tilde{\boldsymbol{\alpha}} u_t^r + \tilde{\boldsymbol{\Phi}} \mathbf{g}(u_t^r) + \mathbf{e}_t. \tag{A.1}$$

We set

$$\mathbf{A}_1 = \begin{pmatrix} 0.3 & 0.7 & -0.1 \\ -0.2 & -0.4 & 0.2 \\ 0.3 & -0.2 & 0.6 \end{pmatrix}.$$

We fix the matrix coefficients (rather than randomly generating them) to ensure stability. We set $m = n - 1$ so that $\mathbf{e}_t = \boldsymbol{\Gamma}_0 \mathbf{u}_t$ where $\boldsymbol{\Gamma}_0$ is 3×2 matrix whose coefficients are randomly generated from a uniform distribution in $[-1, 1]$ and $\mathbf{u}_t$ is a 2×1 vector with distribution $N(0, I)$. The monetary policy shock is also generated from a standardized Normal, $u_t^r \sim N(0,1)$. We consider a single nonlinear function $g(u_t^r) = |u_t^1|$. In this case $\tilde{\boldsymbol{\Phi}} \equiv [\tilde{\boldsymbol{\phi}}^1]$ is just a column vector. The elements of $\tilde{\boldsymbol{\alpha}}$ and the elements two and three of $\tilde{\boldsymbol{\phi}}^1$. i.e. $\tilde{\boldsymbol{\phi}}_2^1$ and $\tilde{\boldsymbol{\phi}}_3^1$, are also uniform in $[-1,1]$. The element on the other hand $\tilde{\boldsymbol{\phi}}_1^1$ is obtained by imposing that the first entry of $\boldsymbol{\phi}^1$ is zero, i.e $\boldsymbol{\phi}_1^1 = 0$, so that assumption A4 is satisfied. We generate 1000 dataset of length $T = 300$ observations as the length of the instrument in the empirical application. For each dataset the econometrican observes $z_t = u_t^r + v_t$ where $v_t \sim N(0,1)$.

In a second simulation we use the same setting but assuming an asymmetric distribution for u_t^r. Indeed we set $u_t^r = u_t^1$ and $u_t^1 \sim \chi_2^2$.

Panel (a) of Figure A.1 plots the results of the first simulation. The black lines are the average across the point estimates, the gray areas are the 68% and 95% bands. The blue dotted lines are the true impulse response functions. The black and blue line are essentially identical suggesting that the approach succeeds at estimating the true responses. Panel (b) plots the results of the second simulation. As before the average response and the true responses are almost identical confirming the validity of the procedure even the distribution of the policy shock is not normal. In the last section of the paper we preform a simulation using a DSGE model.

We repeat the first simulation, but now using the local projection approach to estimate the impulse response functions. We make two exercises.[25] In the first exercise we assume that the shock is observable so that the shock and its absolute value are the regressors. We control for two lags of the three variables. In the second exercise we use $z_t = u_t^r + v_t$ as exogenous variable and its absolute value.[26] The goal of this second experiment is to understand what are the problem one faces working with local projections but with a shock contaminated by noise.

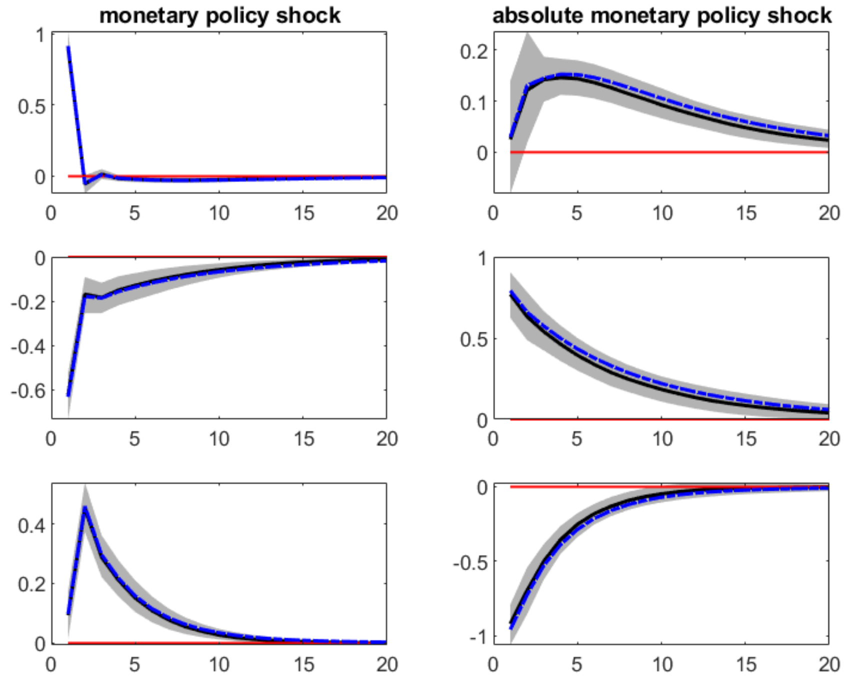
Figure A.2 report the results. Panel (a) refers to the case of perfectly observable shock and Panel (b) to the shock plus noise case. When the shock is observable, the local projection approach performs very well and the average responses overlap with the theoretical responses. As expected, the bands are larger than in the proxy-SVAR case. When the shock is observed with noise, local projections do a terrible job. As discussed before all the responses are biased downward because of the presence of measurement error. It is true that the sign of the responses is correctly captured, but the magnitudes are completely different. As a consequence the nonlinear responses would be misleading.

In another simulation, not reported here, we split the shocks into negative and positive since this is a strategy often followed in the literature to estimate asymmetries in terms of sign. Again if the shock is perfectly measured the estimated impulse response functions are identical to the true ones. However, if the shock is measured with noise, the responses (even those normalized by some impact effect) are distorted and different from the true ones.

[25] The estimated equation is $x_{t+j}^i = c + \tilde{\alpha}_j u_t^r + \tilde{\beta}_j |u_t^r| + \boldsymbol{\psi}_J'(L)\mathbf{x}_{t-1} + \xi_t$, where $j = 0, 1, ...$ and $\psi_j'(L) = \psi_{0j}' + \psi_{1j}' L$ and ψ_{ij} is a n-dimensional column vector. The responses are $\tilde{\alpha}_j$ and $\tilde{\beta}_j$.

[26] The estimated equation is $\mathbf{x}_{t+j}^i = c + \tilde{\alpha}_j z_{rt} + \tilde{\beta}_j |z_{rt}| + \psi_j'(L)\mathbf{x}_{t-1} + \xi_t$, where $j = 0, 1, ...$ and $\psi_j'(L) = \psi_{0j}' + \psi_{1j}' L$ and $\boldsymbol{\psi}_{ij}$ is a n-dimensional column vector.. The responses are $\tilde{\alpha}_j$ and $\tilde{\beta}_j$.

Panel (a)



Panel (b)

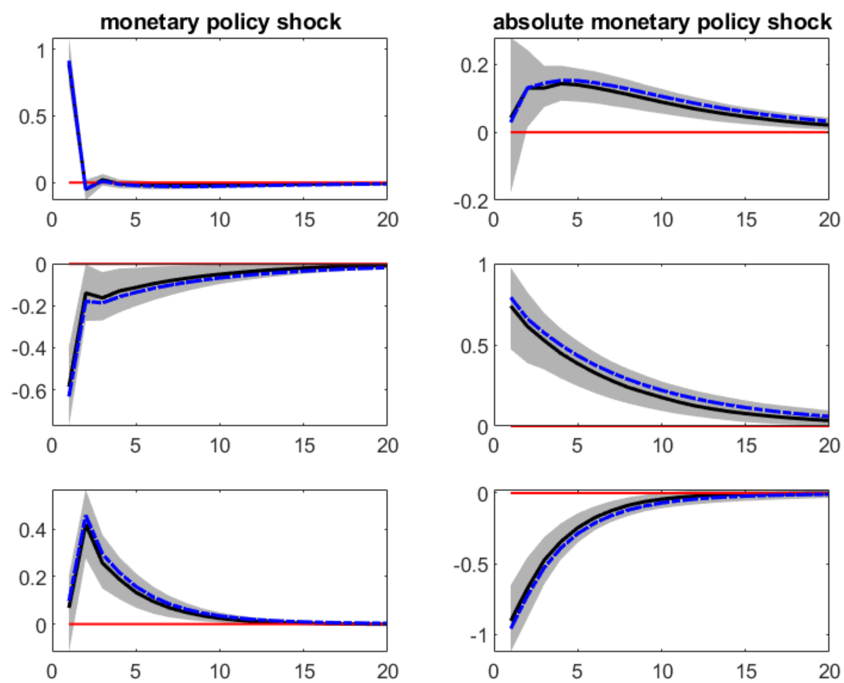
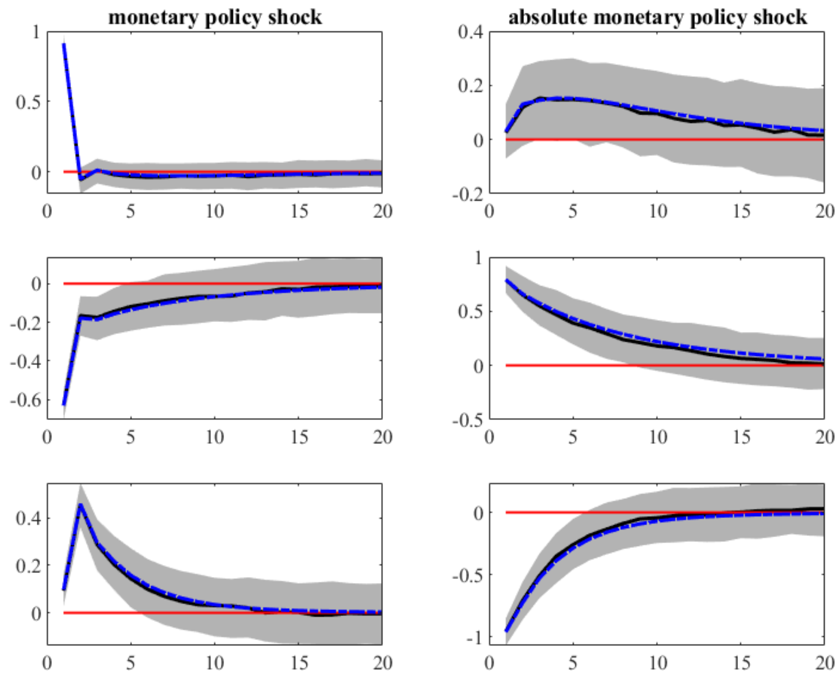


Figure A.1: Impulse response functions from the proxy-SVAR using 1000 data sets generated from (6) . Panel (a) simulation with standardized Normal structural shocks. Panel (b) the policy shock has a chi-square distrubution with two degrees of freedom.

Panel (a)



Panel (b)

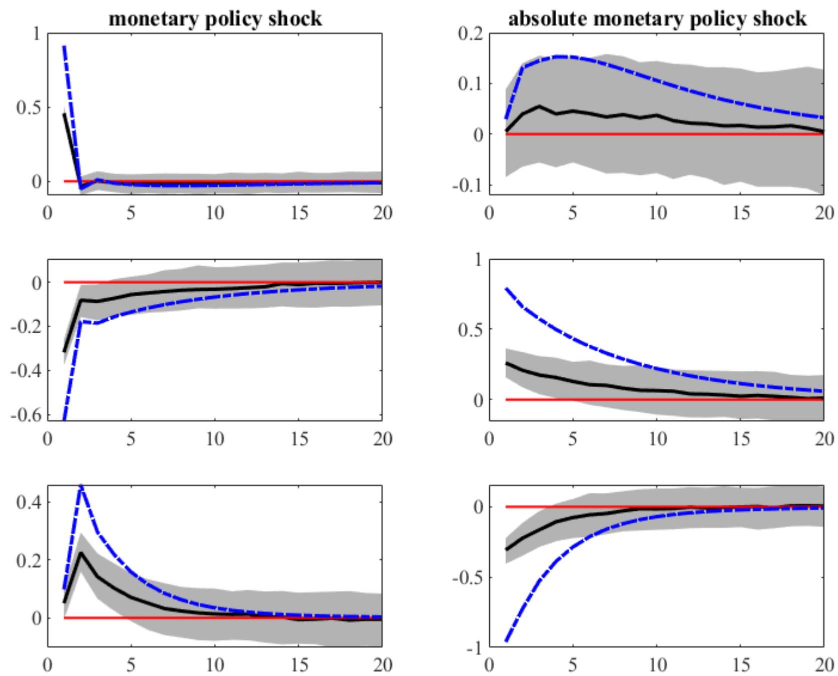


Figure A.2: Impulse response functions from the local projection approach using 1000 data sets generated from (6). Panel (a) shock itself is used in the local projection. Panel (b) the instrument (shock plu noise) is used in the local projection.

A.3 Solution of the Theoretical Model

Solving the theoretical model of Section 6 amounts to solve the following system

$$1 = \beta \bar{R} \Pi_t^{\phi_\pi} \tilde{Y}_t^{\sigma} \exp\{m_t\} \mathbb{E}_t \left\{ \exp\{-\sigma u_{a,t+1}\} \tilde{Y}_{t+1}^{-\sigma} \Pi_{t+1}^{-1} \right\} \tag{A.2}$$

$$\left(\tilde{Y}_t - 1\right) \left(\exp\{u_t^a\} - \Pi_t^{-1} \right) = 0 \tag{A.3}$$

where $\tilde{Y}_t \equiv Y_t / \exp\{a_t\}$ is detrended output. To solve the model, we approximate the expectation term on the RHS of the aggregate demand through a Chebyshev polynomial on a coarse grid for the monetary shocks, i.e. we approximate the function

$$X(m_t) \equiv \mathbb{E}_t \left\{ \exp\{-\sigma u_{t+1}^a\} \left(\tilde{Y}_{t+1}\right)^{-\sigma} \Pi_{t+1}^{-1} \right\}.$$

Note that, since the technology innovation u_t^a is assumed to be *i.i.d.*, it does not affect future expectations and thus it does not constitute an argument of the function $X(\cdot)$. The advantage of this procedure is that the expectation function $X(\cdot)$ is a smooth function of the monetary shock, while the policy functions of inflation and output are not, due to the "kink" related to downward wage rigidities.

For a given guess of the function $X(m_t)$, the solution of the model can be obtained analytically as

$$\begin{aligned}
&\tilde{y}_t = 0,\ \pi_t = -\frac{1}{\phi_\pi}\left[x_t + m_t + (\bar{r} - \rho)\right] && \text{if } m_t \le \phi_\pi u_t^a - x_t - (\bar{r} - \rho) \\
&\tilde{y}_t = -\frac{1}{\sigma}\left[x_t + m_t - \phi_\pi u_t^a + (\bar{r} - \rho)\right],\ \pi_t = -u_t^a && \text{if } m_t > \phi_\pi u_t^a - x_t - (\bar{r} - \rho)
\end{aligned}$$

where lower-case variables denote the log of upper case variables, which can be used to calculate $\tilde{y}_{t+1}$ and π_{t+1} for all realizations of future shocks. The initial guess constitutes an equilibrium if it satisfies

$$X(m_t) = \mathbb{E}_t \left[\exp\{-\sigma\left(u_{a,t+1} + \tilde{y}_{t+1}\right) - \pi_{t+1}\}\right].$$

Online Appendix (not for publication)

B.1 Model with Only Sign-Dependence

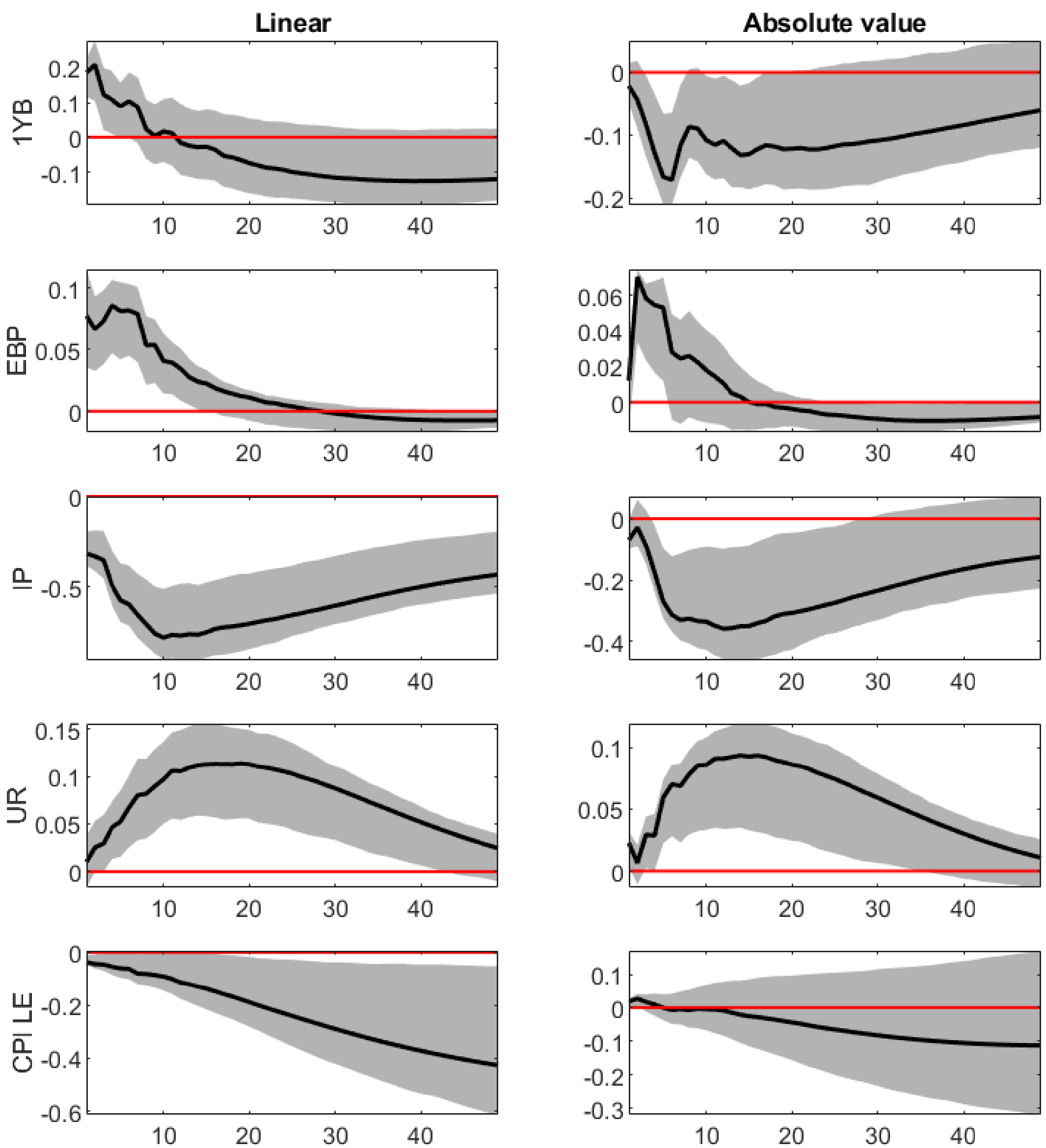


Figure B.1: Only sign-dependence. Impulse response functions. Solid lines are the point estimates, the gray areas are 68% confidence bands.

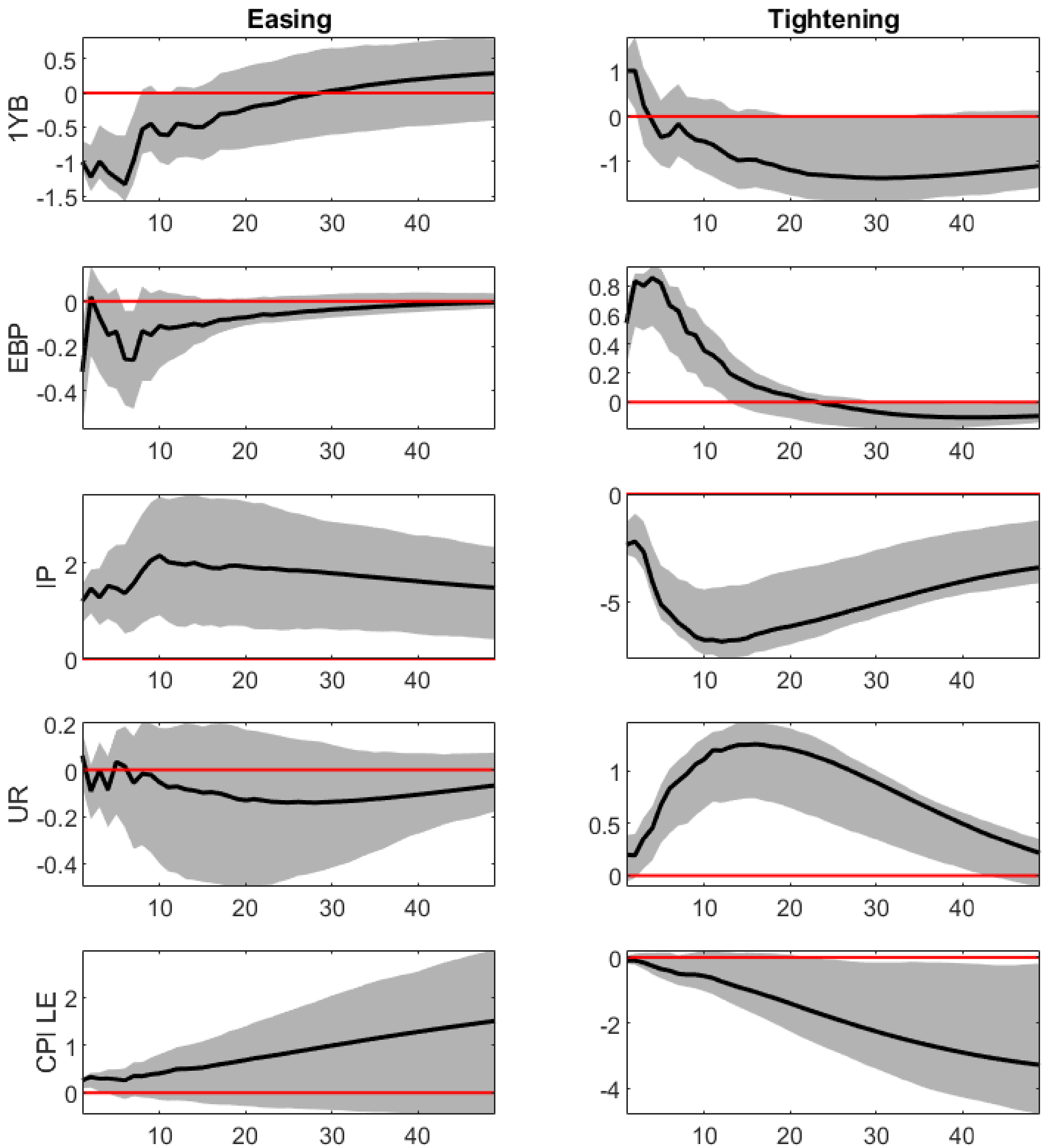


Figure B.2: Only sign-dependence. Impulse response functions. Solid lines are the point estimates, the gray areas are 68% confidence bands.

B.2 Model with only State-Dependence

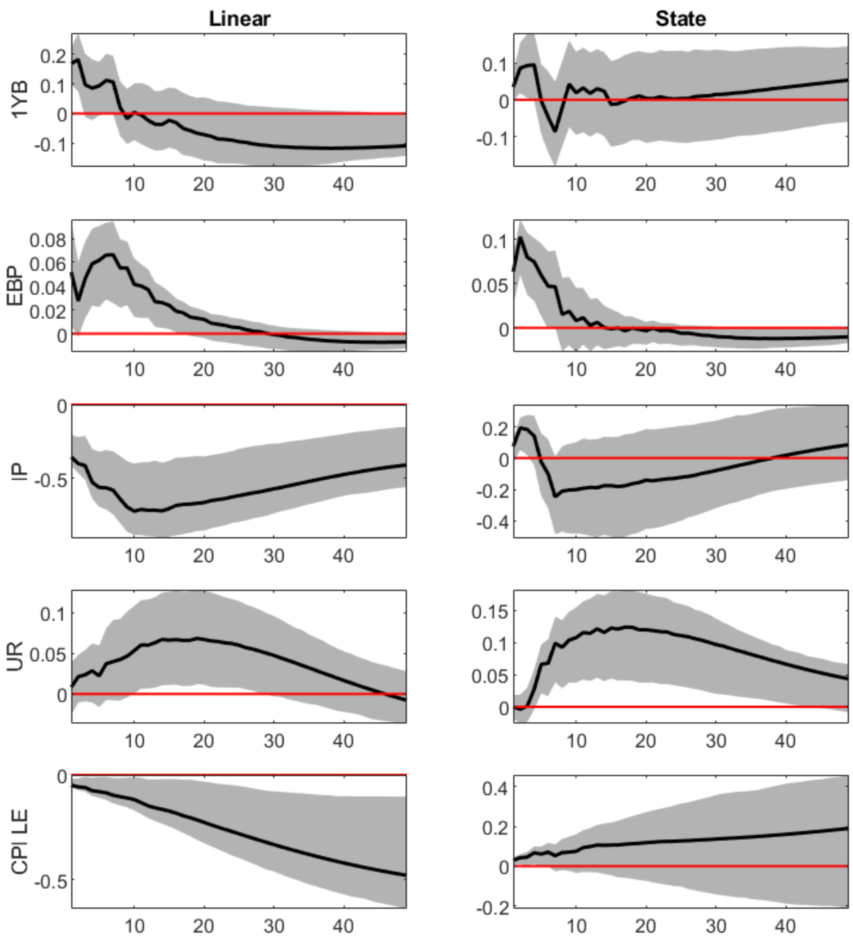


Figure B.3: Only state-dependence. Impulse response functions. Solid lines are the point estimates, the gray areas are 68% confidence bands.

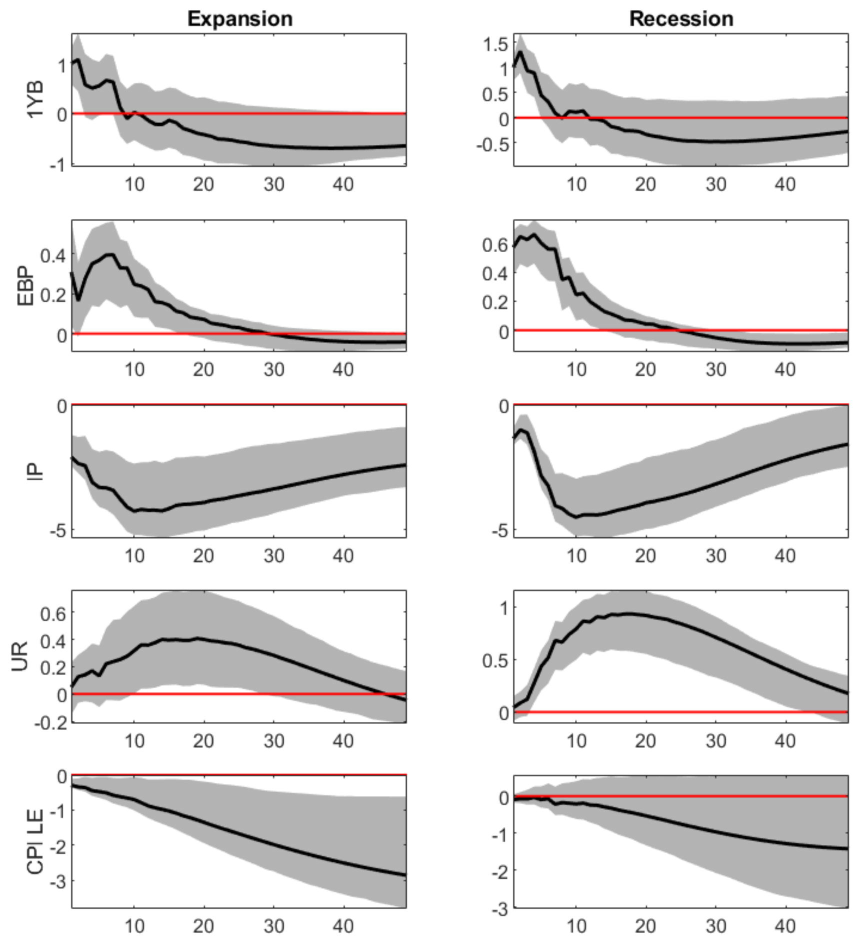


Figure B.4: Only state-dependence. Impulse response functions. Solid lines are the point estimates, the gray areas are 68% confidence bands.

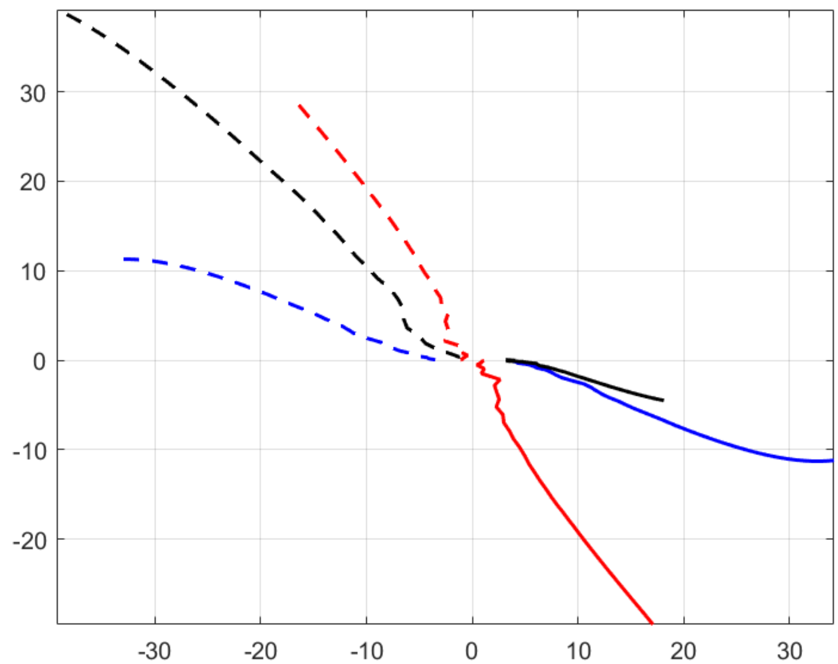


Figure B.5: Tradeoff curves. Blue expansions; red recessions; black single regime; dotted contractionary shock, solid expansionary shock.

B.3 Unemployment Rate as a State Variable

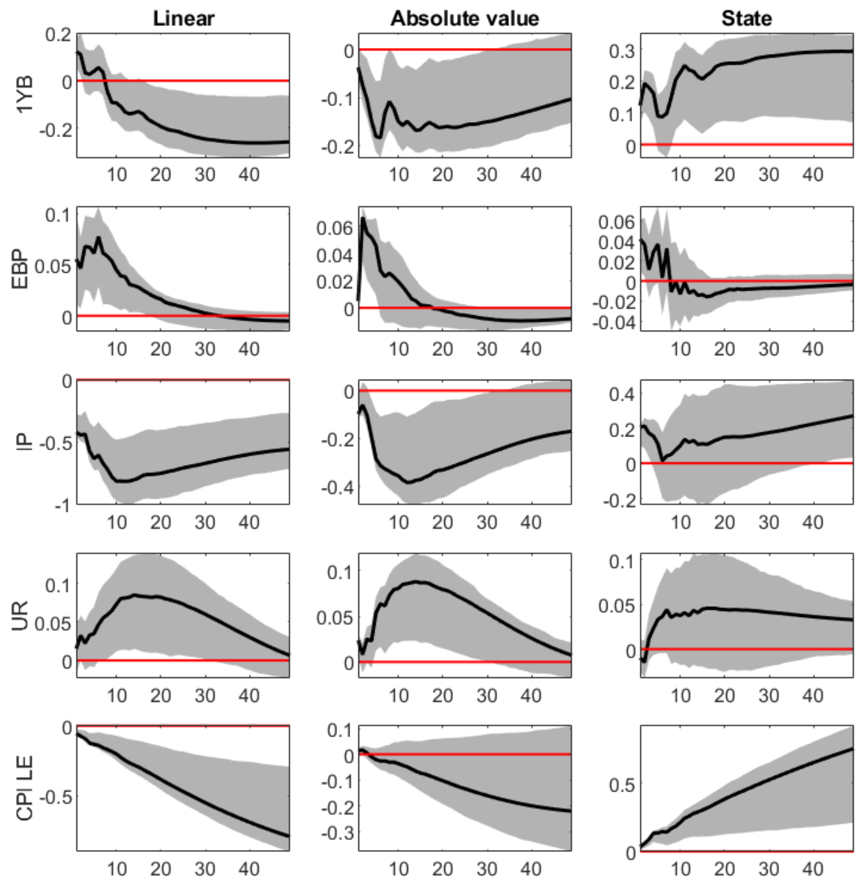


Figure B.6: Unemployment rate as state variable, long sample. Impulse response functions of linear, absolute value and state components. Solid lines are the point estimates, the gray areas are 68% confidence bands.

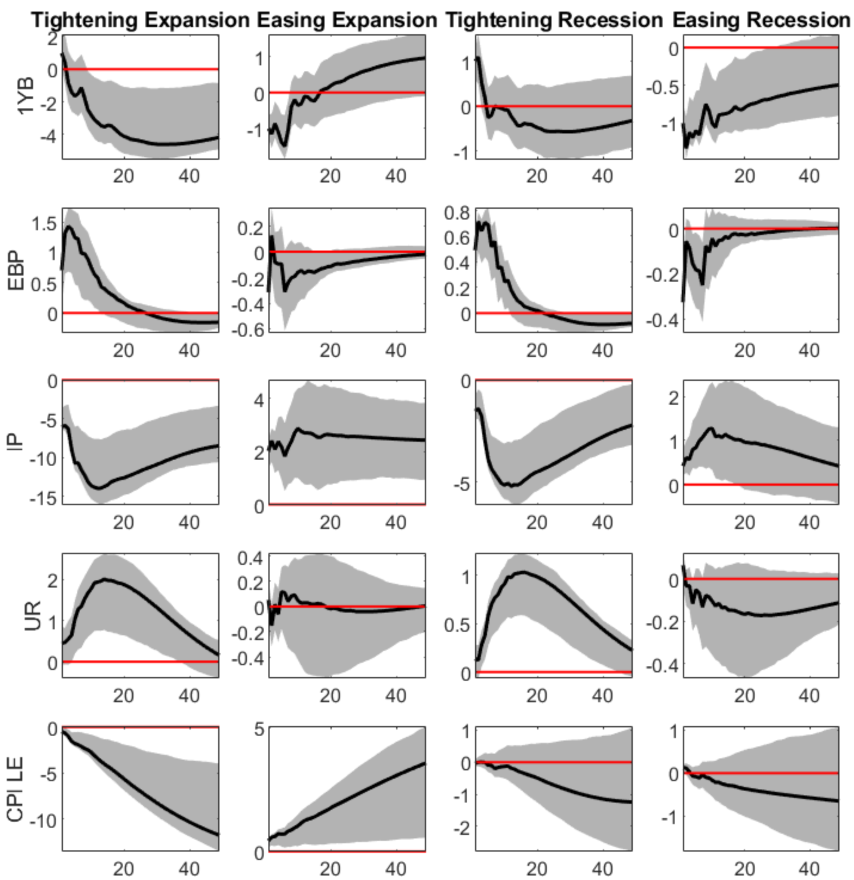


Figure B.7: Unemployment rate as state variable, long sample. Nonlinear proxy-SVAR. Impulse response functions of easing and contractions in expansions and recessions. Solid lines are the point estimates, the gray areas are 68% confidence bands.

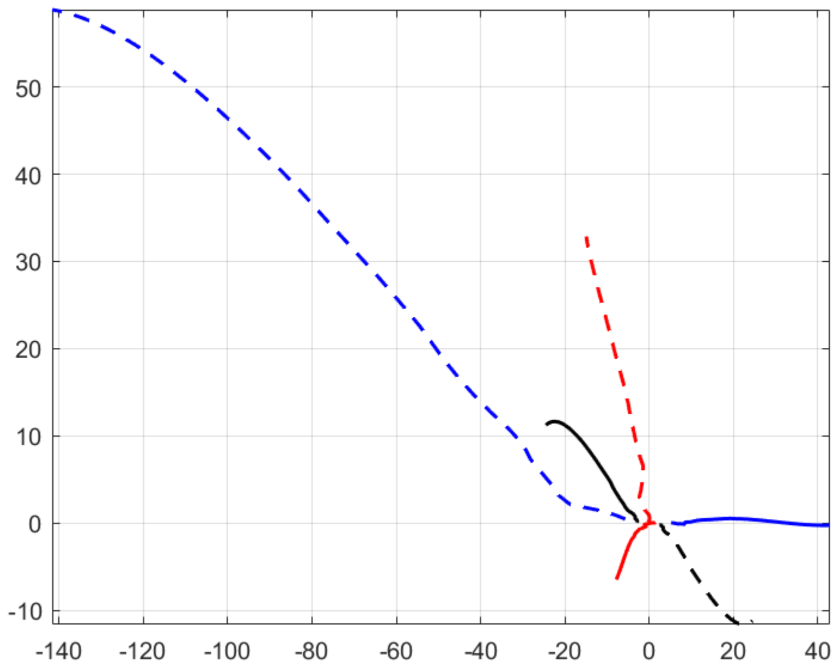


Figure B.8: Unemployment rate as state variable, long sample. Tradeoff curves. Blue expansions, red recessions, black linear, dotted contractionary shock, solid expansionary shock.