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Abstract

We propose a simple three-stage model where heterogeneous schools compete via tuition fees, individuals with the ex-ante unknown ability make their education choices to (eventually) get a diploma and reveal their ability, and finally the job market determines the assignment of workers to firms and the equilibrium wages. In equilibrium, wages in the labor market and schools' fees and individuals' school choices are strongly related. We also analyze the effects of the existence of a public school or a subsidy on social welfare.

Keywords: Education choices, skills, job market.

JEL Classification: I26, C78.

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1 Introduction

Education decisions determine the workers' characteristics in the labor market and the economy's output. Also, future wages influence the schools' competition and the individuals' education choices.

We propose a three-stage model. First, two schools differing in terms of how demanding they are, compete for students through their tuition fees. Then individuals with ex-ante unknown ability select which school to attend, if any. Finally, a one-to-one labor market matching between firms and workers occurs. The equilibrium wages (hence, the gains from education) are a function of the workers' and the firms' productivity.

Among our results, we highlight that subsidizing high-demanding schools is especially helpful when the workers' ability is very valuable and the dispersion of skills in the population is large. Introducing a public school can also increase welfare.

We contribute to the literature that relates education decisions and job market conditions. Finn and Mullins (2015) study the effects of minimum wages and college costs in a search and matching model where heterogeneous individuals make ex-ante schooling choices. MacLeod and Urquiola (2019) consider two identical schools and study the school choice when each firm recruits from only one school or schools face capacity constraints. Boleslavsky and Cotton (2015) consider one firm and two schools of a given capacity, competing through investment and grading systems. Hatfield et al. (2014) show how the labor market's design and frictions affect incentives for human capital acquisition. We complement the literature by considering heterogeneous schools competing via tuition fees and the relationship between schools' competition and job market outcomes.

2 Model

We consider three sets of (risk-neutral) players: individuals, schools, and firms.

There is a continuous of individuals of size one. They have the same ex-ante ability to perform a specialized task if they obtain a diploma. It is public knowledge that ex-post a proportion α has an ability $k > 0$ in the specialized task, and the remaining individuals have ability 0. Hence, an individual's ex-ante expected ability is αk. Individuals can pay for education, or there is a perfect credit market.

Two schools train and provide the diploma to perform the specialized task. The schools' type is public information; they have no capacity constraints and bear no costs. An individual graduates at the high-demanding school S_H if (and only if) he has ability k. Hence, the ability of S_H's graduates is $k_h = k$. An individual always graduates at the low-demanding school S_L, with the expected ability $k_e = \alpha k$. We denote type $k_\emptyset$ an individual without a degree.

There is a continuum of firms. Firm i is characterized by its productivity γ_i, uniformly distributed in $[0, 1]$. We assume that there is a mas-point of firms with $\gamma_i = 0$, so that the population of firms is "larger" than that of individuals.[1]

Each firm needs one worker. A contract between firm i and worker j specifies the wage w_{ij}. Individuals without a degree can only perform routine tasks. Their output in firm i is $R_{i\emptyset} = \beta$. The outcome of a graduated worker of ability k_j, with $j = h, e$, is $R_{ij} = \beta + \gamma_i k_j$.[2]

Decisions are made in three stages. First, schools S_H and S_L compete in fees, F_H and F_L. Second, individuals decide their education, given (F_H, F_L). We denote Q_H (resp., Q_L) the amount of individuals attending school S_H (resp., S_L), and $Q_\emptyset$ the amount not attending any school. The school selection and the education outcome are public information.

In the third stage, individuals (workers) and firms match in the labor market. We formalize it as a one-to-one assignment game with transferable utility. The equilibrium determines the matching between firms and workers and wages.

3 Equilibrium decisions

We solve by backward induction.

[1] This assumption is made only for convenience. It allows us to identify the salary of the lowest-ability workers straightforwardly.

[2] The main characteristic of this production function is that there is complementarity between firms' productivity and workers' ability. Any function that exhibits this complementary leads to similar results.

Stage 3: Labor market

We denote workers by their productivity k_j, with $j = h, e, \emptyset$. If the distribution of the individuals in schools at $T = 2$ is $(Q_H, Q_L, Q_\emptyset)$, with $Q_H + Q_L + Q_\emptyset = 1$, then there are αQ_H k_h-workers, Q_L k_e-workers, and $Q_\emptyset + (1-\alpha)Q_H$ $k_\emptyset$ workers.

The total surplus generated by the partnership (i, j) coincides with the outcome R_{ij}. Given the complementary between firms' and workers' productivity, the equilibrium matching is positive assortative. This implies two cut-offs, γ_e and γ_h, with $0 \leq \gamma_e \leq \gamma_h \leq 1$, such that (a) firms in $[\gamma_h, 1]$ match with the k_h-workers, (b) firms in $[\gamma_e, \gamma_h]$ hire k_e-workers, (c) firms in $(0, \gamma_e]$ hire $k_\emptyset$-workers, and (d) firms with $\gamma = 0$ are unmatched. The equilibrium cut-offs are $\gamma_e = Q_\emptyset + (1-\alpha)Q_H$ and $\gamma_h = 1 - \alpha Q_H$.

In equilibrium, workers with the same productivity get the same wage. To explain the wages, suppose that the three types of workers are in the market. First, the competition of firms with $\gamma_i = 0$ to hire $k_\emptyset$-workers leads to $w_\emptyset = \beta$. Second, the marginal firm γ_e pays a k_e-worker a salary that leaves it indifferent to hiring a $k_\emptyset$-worker: $w_e = \beta + \gamma_e \alpha k$. Finally, k_h-workers obtain the most firm γ_h is ready to pay for them (the alternative is a k_e-worker): $w_h = \beta + \gamma_h k - (\gamma_h - \gamma_e)\alpha k$. A similar argument leads to the equilibrium wages for all possible configurations:

Lemma 1. *Given* $(\alpha Q_H, Q_L, Q_\emptyset + (1-\alpha)Q_H)$*:*

1) If $Q_H > 0$ *and* $Q_L > 0$*:* $w_\emptyset = \beta$, $w_e = \beta + \gamma_e \alpha k$, $w_h = \beta + \gamma_h k - (\gamma_h - \gamma_e)\alpha k$.

2) If $Q_H > 0$ *and* $Q_L = 0$*:* $w_\emptyset = \beta$, $w_h = \beta + \gamma_h k$.

3) If $Q_H = 0$, $Q_L > 0$, *and* $Q_\emptyset > 0$*:* $w_\emptyset = \beta$, $w_e = \beta + \gamma_e \alpha k$.

4) If $Q_H = 0$, $Q_L = 1$ *or* $Q_\emptyset = 0$*:* $w_e = \beta$.

Stage 2: Individual's education decision

An individual's utility is the difference between his expected salary and the school's fee: $U(\emptyset) = \beta$, $U(S_L) = w_e - F_L$, and $EU(S_H) = \alpha w_h + (1-\alpha)\beta - F_H$. The individual selects the school solving $Max\{U(\emptyset), U(S_L), EU(S_H)\}$.

Stage 1: Schools choose F_H and F_L

Anticipating the effect on the individuals' choice, the revenue-maximizing schools set their fees simultaneously and non-cooperatively. Proposition 1 states, in particular, that all individuals attend school at equilibrium.

Proposition 1. *In equilibrium,*

i) schools' fees: $F_H = \frac{2}{3}\alpha(1-\alpha)k$, $F_L = \frac{1}{3}\alpha(1-\alpha)k$,

ii) demands for schools: $Q_H = \frac{2}{3}$, $Q_L = \frac{1}{3}$,

iii) wages: $w_\emptyset = \beta$, $w_e = \beta + \frac{2}{3}\alpha(1-\alpha)k$, $w_h = \beta + (1-\alpha)k$.

Corollary 1. *In equilibrium,*

i) schools' total revenue: $R = \frac{5}{9}\alpha(1-\alpha)k$,

ii) individuals' total surplus: $U = \beta + \frac{1}{3}\alpha(1-\alpha)k$,

iii) firms' total profit: $\Pi = \frac{1}{18}\alpha(8\alpha+1)k$,

iv) total welfare: $W = \beta + \frac{1}{18}\alpha(17-8\alpha)k$.

Education fulfills two roles in our model. First, graduating from any school allows performing the specialized task. Second, school S_H reveals an individual's productivity on this task. The market compensates this second role with higher expected salaries to students attending S_H: $\alpha w_h + (1-\alpha)w_\emptyset > w_e$. It also explains that $F_H > F_L$.

As expected, a higher k leads to higher wages, fees, and participants' surplus. The influence of α is more complex. A higher α reflects a better pool of individuals. However, the population heterogeneity depends on α: it increases until $\alpha = 1/2$ and then decreases. This explains the non-monotonicity of the equilibrium fees (F_H, F_L) (and schools' revenue) in α. For $\alpha = 0$ and $\alpha = 1$, schools have similar education systems; competition leads to $F_H = F_L = 0$. The maximum schools' differentiation occurs at $\alpha = \frac{1}{2}$, where fees are maximum.

The individuals' surplus is increasing (resp., decreasing) for $\alpha < \frac{1}{2}$ (resp., $\alpha > \frac{1}{2}$). An increase in α makes k_h-workers more abundant and decreases w_h. In contrast, w_e first increases and then decreases in α. Indeed, for low α, k_e-individuals are less productive but also rarer, so the firm γ_e is more productive and pays them more. For large α, k_e-workers are more abundant and productive, but γ_e is smaller, which induces a lower wage. Still, firms' and total welfare increase with α.

4 Improving the market outcome

We have assumed that there are two schools. However, the social welfare is maximum when all individuals attend school S_H. Nevertheless, if S_H is the only school, it can use its monopoly power. Proposition 2 states its choice and total welfare under monopoly.

Proposition 2. *Under monopoly:*

a) For $\alpha \geq \frac{1}{2}$*:* $F_H = \frac{\alpha k}{2}$, $Q_H = \frac{1}{2\alpha}$*. Total welfare is* $W_H^T = \beta + \frac{3k}{8}$.

b) For $\alpha \leq \frac{1}{2}$*:* $F_H = \alpha(1-\alpha)k$, $Q_H = 1$*. Total welfare is* $W_H^T = \beta + \frac{\alpha(2-\alpha)k}{2}$.

Corollary 2. *Total welfare is higher when* S_H *is the only school in the market (i.e.,* $W_H^T \geq W^T$*) iff* $\alpha \leq \frac{17-\sqrt{73}}{16} \approx 0.53$.

When S_H is a monopoly and the proportion of high-ability students is low ($\alpha \leq \frac{1}{2}$), then their identification is essential for the top firms. Thus, all the students attend S_H. This is the first-best. A social planner also benefits from preventing the entrance of S_L if α is not too high. Otherwise, a monopoly S_H sets too high a fee; consequently, too few individuals attend, and welfare is lower.

Subsidizing the cost of attending S_H can also improve welfare with two schools. Consider a subsidy of δ to school S_H for each student attending.[3] Proposition 3 states the equilibrium, where λ denotes the shadow cost of the public funds.

Proposition 3. *In equilibrium with two schools and a subsidy* δ *per student to* S_H*:*

i) schools' fees: $F_H(\delta) = \frac{2}{3}(\alpha(1-\alpha)k - \delta)$, $F_L(\delta) = \frac{1}{3}(\alpha(1-\alpha)k - \delta)$,

ii) demands: $Q_H(\delta) = \frac{2}{3} + \frac{1}{3\alpha(1-\alpha)k}\delta$, $Q_L(\delta) = \frac{1}{3} - \frac{1}{3\alpha(1-\alpha)k}\delta$,

iii) cost of the subsidy: $C(\delta) = \frac{2}{3}\delta + \frac{1}{3\alpha(1-\alpha)k}\delta^2$,

iv) total welfare: $W^T(\delta) = \beta + \frac{1}{18}\left(\alpha(17-8\alpha)k + 2\delta - \frac{1}{\alpha(1-\alpha)k}\delta^2\right) - \lambda C(\delta)$.

A subsidy δ leads to a decrease of $\frac{2}{3}\delta$ in F_H and only $\frac{1}{3}\delta$ for F_L. Hence, demand Q_H is higher, which is the policy's objective. Of course, the planner's cost $C(\delta)$ increases with δ.

[3] This policy is equivalent to giving a subsidy of δ to each student attending S_H.

When $\lambda = 0$, total welfare $W^T(\delta)$ increases in δ until $\delta = \alpha(1-\alpha)k$, where all students attend S_H. When $\lambda > 0$, the optimal subsidy is an interior solution.

Corollary 3. *The optimal subsidy δ when $\lambda > 0$ is (i) increasing in k, and (ii) increasing (decreasing) in α for $\alpha < 1/2$ (resp., $\alpha > 1/2$).*

Corollary 3 suggests that subsidizing high-demanding schools is especially helpful in economies where graduates' ability is crucial for firms (a high k) and the dispersion of individuals' skills is large.

Finally, consider that one of the two schools is public and the other private. A public school can be defined by: (a) it sets its fee to maximize total welfare; (b) it is free. Under definition (a), the social optimum ($Q_H = 1, Q_L = 0$) is reached. If school S_H is public, it sets $F_H = 0$, and all individuals attend S_H. If S_L is public, then the Nash equilibrium is $F_L = F_H = \alpha(1-\alpha)k$, which implies $Q_H = 1, Q_L = 0$.

Under definition (b), if S_H is public and $F_H = 0$, then the social optimum is also reached. However, if school S_L is public and $F_L = 0$, then S_H sets $F_H = \frac{\alpha(1-\alpha)k}{2}$, leading to $Q_H = \frac{1}{2}$ and $Q_L = \frac{1}{2}$. Total welfare is $\beta + \frac{1}{8}\alpha(7 - 3\alpha)k$, which is lower than in the two-private school case because the public school attracts too many students.

5 Final comments

Our model opens the door to many potential extensions. We find it particularly enriching to consider differences in the schools' cost structure and capacity or the abilities they teach to students. These and other extensions can shed further light on the implications of the labor market characteristics on a country's school system.

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Appendix

Proof of Proposition 1: First, notice that since all individuals are ex-ante identical, an equilibrium where they choose different education strategies requires that they are indifferent between these alternatives.

Second, we can restrict attention to situations where F_H, F_L induce $Q_H > 0$ and $Q_L > 0$. Let us consider $Q_H = 1$ and $Q_L = 0$ with $F_L = 0$ (otherwise S_L could decrease its fee to attract students). In such a situation, an individual obtains $EU(S_H) = \beta + \alpha\gamma_h k - F_H$ (see case 2 of Lemma 1). If he switches to S_L, then we are in case 1 of Lemma 1; hence, $U(S_L) = \beta + \gamma_e \alpha k$. Moreover, if only one individual selects S_L, $\gamma_e = \gamma_h = 1-\alpha$. Then, $EU(S_H) \geq U(S_L)$ implies $F_H \leq 0$. This cannot be an equilibrium: S_H would increase its profits by setting a positive and not too large F_H because some individuals would still attend S_H. Similarly, if $Q_L = 1$, $Q_H = 0$, and $F_H = 0$, then $U(S_L) = \beta - F_L$ (case 1 of Lemma 1) whereas an individual who would switch to S_H would obtain $EU(S_H) = \beta + \alpha(\gamma_h k - (\gamma_h - \gamma_e)\alpha k) = \beta + \alpha(1-\alpha)k$ since in this case, $\gamma_h = 1$ and $\gamma_e = 1$. Hence, $EU(S_H) > U(S_L)$, which is not possible in equilibrium.

Therefore, there are two potential equilibria, depending on whether $Q_\emptyset > 0$ or $Q_\emptyset = 0$. In both cases, the expressions for the salaries are given in case 1 of Lemma 1. Moreover, $EU(S_H) = U(S_L)$ implies that $\gamma_h = \gamma_e + \frac{F_H - F_L}{\alpha(1-\alpha)k}$.

(a) If $Q_\emptyset > 0$, then $U(S_L) = U(\emptyset) = \beta$. This implies $\gamma_e = \frac{F_L}{\alpha k}$ and, using $\gamma_h = \gamma_e + \frac{F_H - F_L}{\alpha(1-\alpha)k}$, we also have $\gamma_h = \frac{(F_H/\alpha) - F_L}{(1-\alpha)k}$. The conditions $0 \leq \gamma_e \leq \gamma_h \leq 1$ require:

$$F_H \geq F_L \tag{1}$$

$$F_H \leq \alpha F_L + \alpha(1-\alpha)k. \tag{2}$$

Moreover, $Q_\emptyset \geq 0$ asks for $\gamma_e = Q_\emptyset + (1-\alpha)Q_H \geq \frac{(1-\alpha)}{\alpha}(1-\gamma_h)$, that is,

$$F_H \geq \alpha(1-\alpha)\alpha. \tag{3}$$

(b) If $Q_\emptyset = 0$ then all the workers above γ_h and below γ_e come from S_H, hence $\alpha\gamma_e = (1-\alpha)(1-\gamma_h)$. Therefore, $\gamma_e = (1-\alpha) - \frac{(F_H - F_L)}{\alpha k}$ and $\gamma_h = (1-\alpha) + \frac{(F_H - F_L)}{(1-\alpha)k}$. We have

$\gamma_e \leq \gamma_h$ iff (1) holds. In addition, $U^{t=2}(S_L) \geq U^{t=2}(\emptyset)$ requires

$$F_H \leq (1-\alpha)\alpha k. \tag{4}$$

Claim 1. *The best response of S_H is:*

$$F_H(F_L) = \begin{cases} F_L & \text{if } F_L \geq \alpha(1-\alpha)k \\ \frac{1}{2}(\alpha(1-\alpha)k + F_L) & \text{if } F_L \leq \alpha(1-\alpha)k. \end{cases}$$

Proof of Claim 1: Consider first Region 1, where $F_H \geq \alpha(1-\alpha)k$. In this case, $Q_\emptyset \geq 0$ and $Q_H(F_H, F_L) = \frac{(1-\gamma_h)}{\alpha} = \frac{(1-\alpha)k - \frac{F_H}{\alpha} + F_L}{\alpha(1-\alpha)k}$. School S_H maximizes

$$\max_{F_H} F_H \left(\frac{(1-\alpha)k - \frac{F_H}{\alpha} + F_L}{\alpha(1-\alpha)k} \right) \text{ s.t. } (1), (2), (3). \tag{5}$$

Solving by Kuhn-Tucker, we obtain the solution

$$F_H(F_L) = \begin{cases} F_L & \text{if } F_L \geq \alpha(1-\alpha)k \\ \alpha(1-\alpha)k & \text{if } F_L \leq \alpha(1-\alpha)k. \end{cases}$$

Region 2. If $F_H \leq \alpha(1-\alpha)k$ then, $Q_\emptyset = 0$ and $Q_H = \frac{\alpha(1-\alpha)k - F_H + F_L}{\alpha(1-\alpha)k}$. S_H solves:

$$\max_{F_H} F_H \left(\frac{\alpha(1-\alpha)k - F_H + F_L}{\alpha(1-\alpha)k} \right) \text{ s.t. } (1), (4). \tag{6}$$

In this region, it is necessarily the case that $F_L \leq \alpha(1-\alpha)k$ because $F_L \leq F_H \leq \alpha(1-\alpha)k$. Solving by Kuhn-Tucker, we obtain:

$$F_H = \frac{\alpha(1-\alpha)k + F_L}{2}. \tag{7}$$

When $F_L \leq \alpha(1-\alpha)k$, the candidate in Region 1 ($F_H = \alpha(1-\alpha)k$) is feasible also in Region 2. Therefore, the candidate we found in Region 2 is the optimum when $F_L \leq \alpha(1-\alpha)k$. QED

The analysis of Claim 1 also implies that, in equilibrium, it is necessarily the case that $F_L \leq \alpha(1-\alpha)k$, that is, $F_H \leq \alpha(1-\alpha)k$. Otherwise, S_H's best response leads to $Q_L = 0$, and this cannot be a Nash equilibrium since a lower F_L, and attracting some students, is superior for S_L. Hence, we look for $F_L(F_H)$ only for $F_H \leq \alpha(1-\alpha)k$.

Claim 2. *School's S_L best response is $F_L(F_H) = \frac{F_H}{2}$ if $F_H \leq \alpha(1-\alpha)k$.*

Proof of Claim 2: We ignore $F_L \leq F_H$ (equation (1)) and check that the solution satisfies it. The demand for S_L is $Q_L(F_H, F_L) = \frac{F_H - F_L}{\alpha(1-\alpha)k}$. School S_L maximizes $F_L Q_L(S_H, S_L)$, whose solution is $F_L(F_H) = \frac{F_H}{2}$. QED

The equilibrium fees in Proposition 1 follow from the best response in Claims 1 and 2. Using the fees, the demands for the schools and the salaries also follow directly. □

Proof of Proposition 1: The proof derives from the the fact that (see the Online Appendix for details)

(a) The best response of S_H is:

$$F_H(F_L) = \begin{cases} F_L & \text{if } F_L \geq \alpha(1-\alpha)k \\ \frac{1}{2}(\alpha(1-\alpha)k + F_L) & \text{if } F_L \leq \alpha(1-\alpha)k. \end{cases}$$

(b) School's S_L best response is $F_L(F_H) = \frac{F_H}{2}$ if $F_H \leq \alpha(1-\alpha)k$.

The equilibrium fees in Proposition 1 follow from the best response in (a) and (b). Using the fees, the demands for the schools and the salaries follow. □

Proof of Corollary 1: The firms' equilibrium total profits are

$$\Pi^T = \int_{\frac{2(1-\alpha)}{3}}^{\frac{3-2\alpha}{3}} \left(\alpha\gamma - \frac{2}{3}\alpha(1-\alpha)\right) k d\gamma + \int_{\frac{3-2\alpha}{3}}^{1} (\gamma - (1-\alpha))k d\gamma = \frac{1}{18}\alpha(8\alpha+1)k. \quad (8)$$

The other expressions are immediate. □

Proof of Proposition 2: Since $Q_L = 0$, from case 2) of Lemma 1 we obtain $EU(S_H) = \alpha\gamma_h k + \beta - F_H$. If $Q_H < 1$, then $EU(S_H) = U(\emptyset)$ leads to $\gamma_h = \frac{F_H}{\alpha k}$; hence, $Q_H = \frac{(1-\gamma_h)}{\alpha} = \frac{(1-\frac{F_H}{\alpha k})}{\alpha}$. The optimal fee is $F_H = \frac{\alpha k}{2}$ and the demand $Q_H = \frac{1}{2\alpha}$. Thus, $Q_H < 1$ if $\alpha \geq \frac{1}{2}$. Otherwise, $Q_H = 1$ and $\gamma_h = (1-\alpha)$. In this region, the equilibrium fee is $F_H = \alpha(1-\alpha)k$. □

Proof of Corollary 2: Immediate. □

Proof of Proposition 3: It is identical to the proof of Proposition 1, just taking into account that the per-student income of S_H is $F_H + \delta$ instead of F_H. □

Proof of Corollary 3: The optimal δ is characterized by

$$\frac{\partial W^T}{\partial \delta}(\delta^*) = \frac{2}{18}\left(1 - \frac{1}{\alpha(1-\alpha)k}\delta\right) - \lambda\frac{2}{3}\left(1 + \frac{1}{\alpha(1-\alpha)k}\delta\right) = 0.$$

Moreover, $\frac{\partial^2 W^T}{\partial^2 \delta} < 0$. The corollary follows from $\frac{\partial^2 W^T}{\partial \delta \partial k} > 0$ and $\frac{\partial^2 W^T}{\partial \delta \partial \alpha} > 0$ iff $\alpha < 1/2$. □