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Information vs Competition: How Platform Design Affects Profits and Surplus

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Information vs Competition: How Platform Design Affects Profits and Surplus[a]

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Abstract

We study the design of online platforms that aggregate information and facilitate transactions. Two different designs can be observed in the market: revealing platforms that disclose the identity of transaction partners (e.g. Booking) and anonymous platforms that do not (e.g. Hotwire). To analyse the implications of this design choice for profits and surplus, we develop a model in which consumers differ in their location as well as their preferred product variety. Sellers offer their products for sale both directly ('offline') and indirectly via the platform ('online') but are unable to credibly disclose the product variety they offer when selling offline. The model gives rise to a novel trade-off associated with the anonymous platform design: offline, consumers observe location but not variety; online, they observe variety but not location. While the revealing design leads to more informed consumers and better matches, the anonymous design allows sellers to price discriminate and introduces competition between sellers whose markets would otherwise be segmented. We show that the comparison between the designs depends crucially on the relative importance of information about location vis-à-vis information about variety. For an intermediate range, the anonymous design outperforms the revealing design in terms of both profits and welfare.

JEL Classification: D02, D21, D43, D61, D83, L11, L13, L15

Keywords: *anonymous* information platforms; opaque products; horizontal competition; experience goods; mismatch costs.

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1 Introduction

Online platforms that aggregate information and facilitate transactions play an important role in many markets. A leading example is the hotel industry, where platforms like Booking.com and HRS.com allow consumers to search and directly make reservations for hotel rooms. The success of these platforms owes in large part to the fact that they make it much easier for consumers to find products that match their preferences. The platforms reduce search costs by aggregating information on the characteristics and prices of the products available in the market. In addition, platforms may be able to disclose certain types of product information more credibly than the sellers themselves. For example, whether a hotel's rooms are well insulated against noise, whether the hotel's internet connection is fast and reliable, or whether the hotel offers a nice breakfast buffet, is information that is difficult for hotels to credibly disclose themselves, and which some consumers care more about than others.

Two alternative platform designs can be observed in the market: *anonymous* and *revealing*. Platforms with an anonymous design keep the identity of the transaction partners hidden until after the transaction has been concluded on the platform. By contrast, platforms with a revealing design disclose the identity of at least one side of the transaction (usually, the seller's) from the outset. Examples of the revealing platform design include Booking.com or Expedia.com, while examples of the anonymous one include Hotwire.com.[1]

Ostensibly, the anonymous design seeks to prevent buyers and sellers from transacting outside the platform. While it certainly achieves this, in the process it may also undermine what is perhaps the main reason for the existence of online platforms – namely, to ensure better matches. The platform cannot hide a seller's identity without also hiding certain relevant product characteristics, such as the seller's precise location, so the anonymous design results in a loss of information. In this paper we ask whether this information loss implies that the anonymous design necessarily hurts platform users. Are buyers and sellers better or worse off when the platform reveals the seller's identity? While revealing platforms are the subject of a large literature and their implications are well understood, anonymous platforms remain largely understudied. This is particularly timely since several platforms using a revealing design have recently been challenged by antitrust authorities in Europe and Asia (e.g. for their of use price-parity clauses), arguably putting pressure on the business model underlying this design.

We develop a model in which two sellers offer their products for sale both directly and through a platform. To ease the exposition, we refer to direct sales as *offline* and to sales via the platform as *online*.[2] Each seller offers a single product. The products differ along

[1]While data on the market shares of revealing and anonymous platforms are hard to come by, Internet traffic data suggest that both are important. For example, the Expedia Group reports that, in January 2020, their revealing platform 'Expedia' received 48 million monthly unique visitors, while their anonymous platform 'Hotwire' received 8.5 million monthly unique visitors; see `https://advertising.expedia.com/getting-started/brands/expedia/` and `https://advertising.expedia.com/getting-started/brands/hotwire/` (both last accessed on 3 February 2022).

[2]Despite this terminology, it does not matter for our results whether direct sales truly occur offline, or

two dimensions: their *location*, at either end of a Hotelling line, and a second feature we refer to as the *variety* of the product. Consumers are distributed along the Hotelling line and incur transport costs to travel to a seller. In addition, each consumer has a preferred variety and derives no utility from the other.

The platform is informed about both the location and the variety of the product sold by each seller. It always discloses the varieties of the products available for purchase online. However, depending on its design, it may or may not reveal the locations of the products. By contrast, when buying offline, consumers observe a seller's location but the seller cannot credibly communicate whether it sells a consumer's preferred variety.

The sellers determine the prices of their products both online and offline. To focus on the information effects of platform design, we abstract from platform pricing and assume that sellers appropriate the full price regardless of the channel through which the product is sold.[3] This assumption implies that, with a revealing platform, online and offline prices must be the same: consumers can find out about both location and variety of each seller online and then buy through whichever sales channel is cheaper. With an anonymous platform, the information available online and offline is different. Offline, consumers can observe location but not variety; online, they observe variety but not location. Accordingly, sellers can set different prices on the two sales channels.

Comparing the two platform designs, a first key difference we already hinted at above is that the revealing platform leads to better matches because it provides better information about product characteristics. The anonymous platform sometimes leads online buyers to travel further than they should; moreover, it sometimes leads offline buyers to obtain a variety they dislike even though their preferred variety is available elsewhere. The revealing platform thus saves on transportation costs and prevents consumers from purchasing products that do not fit their tastes.

Our analysis also identifies two subtler effects that work against the superiority of the revealing design. First, the availability of a second sales channel, where products can be priced differently and which attracts a specific subset of consumers (namely, those that do not care much about location), enables sellers to engage in *price discrimination*. They can sell to those that care little about location online, which enables them to raise the price offline to those that care strongly about location. As is well known, this can raise both profits and consumer surplus. Second, the fact that consumers cannot observe the varieties sold offline makes two sellers offering different varieties appear similar to consumers. This creates *competition*, putting pressure on prices and raising total surplus through market expansion.

We find that whether the information effect or the price-discrimination and competition effects prevail depends crucially on the relative importance of information about variety and location. When information about location is more important than information about variety, the revealing design results in higher profits and welfare. When information about

whether they occur via the seller's own website.

[3]Although we do not model platform pricing, we comment on the implications of our analysis for platforms' choice of business model below.

location is more important, both perform equally well. However, in an intermediate range, the anonymous design is superior to the revealing design in terms of profits, consumer surplus, and welfare.

One polar case arises when both sellers offer the same variety, so that only information about location matters. In that case, there is competition between sellers even with the revealing design. The anonymous design intensifies competition but, for most of the parameter space, the resulting price drop only redistributes surplus; and even when it results in market expansion, the extra surplus is not enough to offset the inefficiency from online buyers travelling further than necessary. As a result, when both sellers offer the same variety, the revealing design always dominates the anonymous design in terms of both profits and welfare.[4]

When, instead, sellers offer different varieties, each of them is the only game in town for the variety they sell. Hence, with the revealing platform, each of them has monopoly power, and for large parts of the parameter space, the market is not covered. The anonymous platform causes matching inefficiencies because offline buyers sometimes do not obtain their preferred variety and because online buyers sometimes buy even though the gains from trade are negative or do not buy even though they are positive. However, by allowing firms to price discriminate and by introducing competition, the anonymous platform draws additional consumers into the market. When the relative importance of information about variety and location, as measured by the ratio between the utility from purchasing the preferred variety and transport costs, is in an intermediate range, the additional surplus generated by market expansion outweighs the inefficiencies created by information loss. As a result, the anonymous platform outperforms the revealing platform in terms of both profits and total surplus.[5]

Our analysis has both managerial and policy implications. On the managerial side, our results suggest that sellers of differentiated products can benefit from using an anonymous rather than a revealing platform when their products differ in more than one dimension, one of which is difficult to credibly disclose. Although we do not explicitly model platform pricing, we conjecture that platforms themselves could also stand to benefit from using an anonymous rather than a revealing design, provided the sellers in their market are sufficiently differentiated. This conjecture is based on the observation that, regardless of their business model, platforms are typically able to extract more profit when their users have higher surplus.[6] This is particularly relevant because of two recent developments: the antitrust interventions against price-parity clauses by a number of competition authorities (mostly in Europe), and the proposed prohibition of such clauses for so-called "gatekeepers" as part of the Digital Markets Act in the European Union.[7] These developments may make

[4]Note that the same is not true for consumer surplus, which tends to be higher with the anonymous design.

[5]For a subset of this range, the anonymous design also raises consumer surplus.

[6]Platforms whose business model relies on advertising revenue would also benefit from the larger number of transactions that the anonymous design generates.

[7]Lawmakers in the United States are considering similar legislation.

it harder for platforms to earn money via the revealing design.[8]

On the policy side, our analysis can inform the debate on platform regulation. It identifies conditions under which a shift to an anonymous design benefits platform users and increases welfare, and others under which it hurts users and decreases welfare. This may be of particular importance for regulators contemplating the ban of price-parity clauses, which, if imposed on a global scale, may prompt platforms to rely more heavily on the anonymous design.

In what follows, Section 1.1 briefly discusses how our paper relates to the existing literature. Section 2 then introduces the model. Section 3 derives the equilibrium, and Section 4 compares the welfare impact of anonymous and revealing platforms. Section 5 concludes. All proofs are relegated to Appendix A.

1.1 Related Literature

At a general level, we build on a large body of work on online intermediaries (i.e. platforms). Spulber (2019) and Jullien and Sand-Zantman (2021) provide reviews of the literature.[9] Our static model studies the case of a single platform. Readers interested in multi-homing and competition among platforms should consult Casadesus-Masanell and Campbell (2019), Halaburda and Yehezkel (2019), and Karle et al. (2020) as well as the literature cited therein. Those interested in dynamic models of platforms should consult Cabral (2019) and Kanoria and Saban (2021).

Information plays a crucial role in our model. By definition, anonymous platforms differ from direct sales or revealing platforms in the information provided to potential buyers. In that sense, our approach is related to the literature on obfuscation, where the seller optimally decides the amount of information to reveal.[10] Contrary to the obfuscation literature, we take the amount of hidden information as exogenous and we interpret it as a design choice of the platform motivated to guarantee anonymity. Also, we let consumers choose the sales channel - online or offline - and, hence, consumers select their preferred pair of price and product information.

Our work relates to the literature on firms selling 'opaque products', i.e. products for which some characteristics are voluntarily withheld by the seller. Anderson and Celik (2020); Balestrieri et al. (2021), among others, focus on the case of a monopolist selling an opaque product and show that opacity can raise profits by enabling sellers to price discriminate. Fay (2008) and Shapiro and Shi (2008) allow for competition across sellers and are, perhaps, the contributions that are closest to ours. There are, however, crucial

[8]For more on price-parity (also known as MFN) clauses, see Johnson (2017); Wang and Wright (2020); Calzada et al. (In Press); Ronayne and Taylor (In Press) and the literature review by Argenton and Geradin (2021).

[9]Spulber (2019) offers insights on how the economics of platforms differs from the standard partial and general equilibrium literature. Jullien and Sand-Zantman (2021) focus instead on competition and competition policy.

[10]See, among others, Ellison and Ellison (2009); Celik (2014); Janssen and Teteryatnikova (2016); Petrikaitė (2018); Jullien and Pavan (2019); Romanyuk and Smolin (2019); Armstrong and Zhou (In Press).

differences between our model and theirs. Overall, our richer design allows to unveil a larger set of channels through which platforms operate.[11]

First, both Fay (2008) and Shapiro and Shi (2008) guarantee by design that a subset of consumers always buys from each firm and competition takes place only for the residual demand. Within the contestable part of the market, consumers are only heterogeneous in one dimension. [12] In our setting every consumer, irrespective of their location, may buy from either seller. Furthermore, we allow for two dimensions of heterogeneity.[13] Finally, we do not assume that the market is covered. These elements are crucial, for they allow us to study the trade-off between offline and online sales that is typical of anonymous platforms, where additional information is available online but, meanwhile, some information is hidden online (while the same is available offline). They also allow us to appreciate the role of the platform in determining the speed at which the market is covered: indeed, we show that the number of transactions taking place in equilibrium is substantially different between revealing and anonymous platforms.[14]

Despite having similar appearance, our model is intrinsically different from the literature in which platforms are used as a search device (Baye and Morgan, 2001; Dinerstein et al., 2018; Ronayne, 2021; Ronayne and Taylor, In Press). Indeed, prices in our model are ex-ante observable and consumers' misinformation is about the product characteristics.

The literature provides support for two features embedded in our model. First, we assume that platforms are able to convey relevant information to potential consumers. The empirical literature has tested such claim in various ways and, overall, there is a consensus that platforms are able to transmit valuable information even if a share of the reviews is fake.[15] Second, products in our model are only horizontally differentiated. The literature (Cabral and Hortaçsu, 2010; Hossain et al., 2011; Klein et al., 2016; Vial and Zurita, 2017), both theoretical and empirical, suggests that the vertical component on platforms may be secondary. The intuition behind this result is that poorly ranked products either disappear or converge to their competitors' quality.

[11] Fay (2008) and Shapiro and Shi (2008) are designed to study of how opaqueness may facilitate market segmentation and price discrimination. We also embed those in our model.

[12] Fay (2008) assumes that a fixed share of the population is loyal to a brand, while Shapiro and Shi (2008) assume that a share of the population has prohibitive transport costs and always buys from the closest firm. In the contestable part of the market, heterogeneity is defined spatially through a linear (Fay, 2008) or circular (Shapiro and Shi, 2008) city.

[13] Strictly speaking, Shapiro and Shi (2008) introduce two dimensions of heterogeneity, but the one on transport costs (low or high) only defines the preference over selling channels without affecting the preferences over whom to buy from.

[14] A few additional features distinguish them from us. Shapiro and Shi (2008) allow for N active firms and are able, therefore, to study the impact of a change in the number of available varieties. In Fay (2008), firms choose the quantity to be sold online but online prices are set by the platform: hence, firms fix the capacity of the platform (in exchange for a fee), then the platform sells the product, competing against the firms.

[15] Fradkin et al. (2021) establish this using data from the anonymous platform Airbnb, while the remaining literature (including Chevalier and Mayzlin, 2006; Dellarocas, 2006; Anderson and Magruder, 2012; Ghose et al., 2012; Mayzlin et al., 2013; Luca and Zervas, 2016) uses data from revealing platforms.

2 The model

Two firms selling a single good are exogenously located each at one end of a unitary-length Hotelling line. The good sold by the firms can be produced in two different varieties, A and B. Let $\kappa_j \in \{A, B\}$ denote the variety sold by firm $j = 0, 1$, and let $\kappa = (\kappa_0, \kappa_1)$ denote a market configuration. The set of market configurations from which κ is drawn is $K = \{(A, A), (A, B), (B, A), (B, B)\}$.

A mass 1 of consumers is uniformly distributed over the unit interval. A consumer located at $x \in [0, 1]$ can purchase from firm 0 at cost tx or from firm 1 at cost $t(1 - x)$, where $t > 0$ is a parameter measuring transport costs. Each consumer always likes only one of the two varieties, denoted $k \in \{A, B\}$. The probability of liking one or the other is ex-ante the same. The value for an agent of consuming variety κ_j is then $\overline{v} = \begin{cases} v > 0 & \text{if } \kappa_j = k \\ 0 & \text{if } \kappa_j \neq k \end{cases}$. Transport cost and the value of the good are independent. We interpret them as generic attributes of the good over which agents have different preferences, but a spatial interpretation of the transport cost is also possible.

Firms can sell directly to consumers (e.g. through a brick and mortar store). We will refer to direct sales as offline and to offline consumers as *walkers*.[16] Firm j charges an offline price p_j^w, observable before the transport cost is realised. When agents buy offline, they observe the attribute that determines the transport cost but not the variety (κ_j). One natural interpretation is that the difference between varieties A, B comes from an experience component that cannot be conveyed through direct sales.

An online platform aggregates information on firms and facilitates transactions. Agents buying through the platform are referred to as *surfers*. The online price for variety κ_j is $p^s_{\kappa_j}$. We assume that the platform observes the market configuration κ while all other market participants (i.e. firms and consumers) observe which varieties are available for sale in the market, but not the precise market configuration. Formally, firms' and consumers' information partition is $H = \{\{(A, A)\}, \{(B, B)\}, \{(A, B), (B, A)\}\}$; that is, if the true market configuration is κ, they only know that the market configuration lies in $h(\kappa)$, with

$$h(\kappa) = \begin{cases} \kappa & \text{if } \kappa \in \{(A, A), (B, B)\} \\ \{(A, B), (B, A)\} & \text{if } \kappa \in \{(A, B), (B, A)\} \end{cases}$$

We consider two different platform designs: anonymous and revealing. A revealing platform discloses all the attributes of the good, including which variety κ_j each firm produces. Consumers can match offline and online information, so that the presence of a revealing platform unveils the variety κ_j also for *walkers*. In this full information setting, potential buyers can compute their transport cost; they know their valuation and prices both offline and online. This implies that arbitrage is costless and each firm must charge the same price online and offline.

[16]The model directly applies also to the case in which direct sales occur through a proprietary website of the firm.

Anonymous platforms differ from revealing ones in that they disclose the varieties for sale and their price, but they are able to hide the firm's identity. In other words, on an anonymous platform, consumers learn which varieties are for sale and their respective prices, but they ignore the identity of the seller, which is hidden until the transaction is concluded. Hence, consumers cannot match the information online with products sold offline, but they are also unable to compute the transport cost they will incur. In such an environment, firms can set different prices for platform and direct sales.

Formally, the platform posts a list L of product offers available via the platform, with $L \in \{(\ell_0, \ell_1), (\ell_1, \ell_0)\}$, where $\ell_j = (\kappa_j, p_j^s) \in \{A, B\} \times [0, \infty)$ is the variety and online price pair offered by firm j. A revealing platform always posts $L = (\ell_0, \ell_1)$; hence, consumers know that the first option in the list corresponds to firm 0's offer and the second to firm 1's offer. An anonymous platform posts

$$L = \begin{cases} (\ell_0, \ell_1) & \text{with probability } 1/2 \\ (\ell_1, \ell_0) & \text{with probability } 1/2; \end{cases}$$

hence, consumers do not know which option in the list corresponds to which firm.

The timing is as follows:

1. Nature draws a market configuration $\kappa \in K$ according to some commonly known probability distribution; for our purposes, all that matters is that $\Pr(\kappa = (A, B)) = \Pr(\kappa = (B, A))$.

2. Consumers and firms observe $h(\kappa) \in H$.

3. Firms set prices for online and offline sales, $p_j = (p_j^s, p_j^w)$.

4. The platform observes the market configuration κ and the prices p_j and posts a list of online options L.

5. Consumers decide whether to buy directly from firm $j \in \{0, 1\}$ at the offline price p_j^w, or whether to buy the first or second option in the platform's list L, or not to buy at all.

6. If a transaction takes place, consumers enjoy a value $\overline{v}$ and pay the transport cost (tx or $t(1 - x)$) and the price (p_j^s or p_j^w).

Our solution concept is Perfect Bayesian Equilibrium (PBE). When the platform is revealing, the game described above is one of complete information: market configuration and seller locations are common knowledge. In that case, PBE collapses to subgame-perfect Nash equilibrium. When the platform is anonymous, the game is one of incomplete information: although buyers and sellers are symmetrically informed about the market configuration, sellers know their locations while consumers who buy online only know the variety that is sold but not from which location. In the analysis that follows, we restrict attention to symmetric equilibria, so prices do not convey information about sellers' locations (i.e. on the equilibrium path consumers believe that a firm selling online is equally likely to be located at either end of the Hotelling line). However, if out-of-equilibrium beliefs about locations are allowed to depend on prices, many different prices can be supported as an equilibrium. To deal with this multiplicity, we impose the refinement that beliefs are *passive*: when observing an unexpected price, consumers do not revise their beliefs about the location of the deviating firm.[17]

3 Equilibrium analysis

Depending on the market configuration drawn by nature, there are two conceptually distinct cases: one in which both sellers offer the same variety and another in which each of them offers a different variety. We analyse the first case in Section 3.1 and the second in Section 3.2.

3.1 One variety

Consider first the case in which both firms sell the same variety. Due to our assumptions on the information structure of the game, consumers know which variety is for sale in the market. Thus, half of them are inactive (namely, those whose preferred variety is not offered). We study first the equilibrium when the platform uses the revealing design. Then, we turn to the case where the platform uses the anonymous design.

3.1.1 Revealing platform

By construction, the revealing platform allows any consumer to observe all the relevant characteristics of sellers and to match sellers on- and off-line. This has two consequences. First, consumers act in a full-information environment, so the model collapses to the standard Hotelling model. Second, firms set the same price on- and off-line: should prices differ, consumers would always buy through the cheapest channel, so only the lower of a firm's prices would matter for demand. We thus focus on a unique price p_j^r for each firm j.

The remainder of this section only considers the half of consumers whose preferred variety is for sale. For a consumer located at x, the utility of buying from firm j at price p_j^r is U_j, given by

$$U_j = \begin{cases} v - p_0^r - tx & \text{if } j = 0 \\ v - p_1^r - t(1-x) & \text{if } j = 1. \end{cases} \tag{1}$$

Consumers buy from firm j if their utility is positive and greater than what they can obtain if they buy from firm j's competitor.

Notice that it can never be optimal for a firm to set $p_j^r > v$, as otherwise nobody would buy. Similarly, it cannot be optimal to set $|p_1^r - p_0^r| > t$, which would lead to a case where one firm serves the whole market and leaves some surplus to all buyers, while the other

[17] Passive beliefs are common in the industrial-organisation literature, particularly (though not only) in the context of vertical contracts (see Rey and Tirole, 2007).

serves nobody. Should this be the case, the firm serving the whole market would have an incentive to increase its price, while the competitor would have an incentive to decrease it. In what follows we thus assume $p_j^r \leq v$ and $|p_1^r - p_0^r| \leq t$. Under those conditions, the share of consumers who buy from firm 0, for a given p_1^r, is:

$$q_0 = \begin{cases} \dfrac{v - p_0^r}{t} & \text{if } p_0^r > 2v - t - p_1^r \\[2ex] \dfrac{1}{2}\left(1 + \dfrac{p_1^r - p_0^r}{t}\right) & \text{if } p_0^r \leq 2v - t - p_1^r. \end{cases} \tag{2}$$

At the cutoff price between the demand regimes, when $p_0^r = 2v - t - p_1^r$, the consumer who is exactly indifferent between buying from firms 0 and 1, located at $\tilde{x}_{0,1} = {}^{(1+(p_1^r - p_0^r)/t)}/_2$, receives a utility of zero. For prices below the cutoff, the market is covered; for prices above the cutoff, the market is not covered.[18]

Lemma 1. *When both firms sell the same variety and the platform is revealing, there is a unique symmetric pure-strategy equilibrium in which the price and the quantity sold by each firm, (p^r, Q_j^r), are given by*

$$p^r = \begin{cases} \frac{v}{2} & \textit{for } \frac{v}{t} \leq 1 \\ v - \frac{t}{2} & \textit{for } \frac{v}{t} \in \left(1, \frac{3}{2}\right) \\ t & \textit{for } \frac{v}{t} \geq \frac{3}{2} \end{cases} ; \quad Q_j^r = \begin{cases} \frac{v}{4t} & \textit{if } \frac{v}{t} \leq 1 \\ \frac{1}{4} & \textit{if } \frac{v}{t} > 1 \end{cases} \tag{3}$$

Profits, consumer surplus and total welfare are

$$\pi_j^r = \begin{cases} \frac{v^2}{8t} & \textit{if } \frac{v}{t} < 1 \\ \frac{2v-t}{8} & \textit{if } \frac{v}{t} \in \left[1, \frac{3}{2}\right) \\ \frac{t}{4} & \textit{if } \frac{v}{t} \geq \frac{3}{2} \end{cases} ; \quad S^r = \begin{cases} \frac{v^2}{8t} & \textit{if } \frac{v}{t} < 1 \\ \frac{t}{8} & \textit{if } \frac{v}{t} \in \left[1, \frac{3}{2}\right) \\ \frac{4v-5t}{8} & \textit{if } \frac{v}{t} \geq \frac{3}{2} \end{cases} ; \quad W^r = \begin{cases} \frac{3v^2}{8t} & \textit{if } \frac{v}{t} < 1 \\ \frac{4v-t}{8} & \textit{if } \frac{v}{t} \geq 1 \end{cases} \tag{4}$$

Proof. See Appendix A. □

When the product value is small relative to transport costs ($v/t < 1$), only consumers that are located close to the firm are interested in buying. Then, firms are local monopolists and can charge the monopoly price. As the value increases, the threat of competition pushes firms to extract less surplus from consumers. For $v/t \in [1, {}^3/_2)$, firms avoid competition by setting a sub-optimal price (i.e. below the monopoly level) that guarantees that their marginal buyer does not buy from their competitor. This solution arises as a result of the kink in the demand function (which creates a discontinuity in the firm's marginal revenue) and is somewhat reminiscent of the behaviour of a monopolist setting a limit price to deter entry. When $v/t \geq {}^3/_2$, the cost of avoiding competition would be too large; firms start competing.

[18]Note that we do not have to condition on whether or not $p_1 \geq v - t$. Even though, for $p_1 < v - t$, all consumers receive strictly positive utility when buying from firm 1, so that the market is necessarily covered, our earlier argument that we can restrict attention to prices $p_j \leq v$ implies that, when $p_1 < v - t$ and hence $2v - t - p_1 > v$, we cannot have $p_0 > 2v - t - p_1$.

3.1.2 Anonymous platform

Consumers who like the variety that is for sale will choose the selling channel that maximises their net surplus, as long as their expected utility is positive.

Because everyone knows which variety is for sale, there is no uncertainty offline. Walkers buying from firm j enjoy utility $U_j^w = \begin{cases} v - p_0^w - tx & \text{if } j = 0 \\ v - p_1^w - t(1-x) & \text{if } j = 1 \end{cases}$.

Let $\tilde{x}_j^w$ be the consumer indifferent between buying from firm j and not buying ($U_j^w = 0$), and $\tilde{x}_{0,1}^w$ be the consumer indifferent between buying from firm 0 and 1 ($U_0^w = U_1^w$). Then,

$$\tilde{x}_0^w \equiv \frac{v - p_0^w}{t}, \quad \tilde{x}_1^w \equiv 1 - \frac{v - p_1^w}{t}, \quad \tilde{x}_{0,1}^w \equiv \frac{1}{2}\left(1 - \frac{p_0^w - p_1^w}{t}\right) \tag{5}$$

Surfers are unable to anticipate the location of the seller: if they buy, they have no specific bias in favour of either seller and, hence, they simply select the cheapest. Let $p_k^s = \min\{p_0^s, p_1^s\}$ denote the lowest online price when variety k is for sale. Then, the expected utility of buying variety k online is $EU_k^s = v - p_k^s - {}^t\!/_2$, which is positive as long as $v > p_k^s + {}^t\!/_2$. The consumer located at $\tilde{x}_{k,j}^s$ is indifferent between buying variety k online from the cheapest seller or offline from firm j. We have:

$$\tilde{x}_{k,0}^s \equiv \frac{1}{t}\left(p_k^s - p_0^w + \frac{t}{2}\right); \ \tilde{x}_{k,1}^s \equiv \frac{1}{t}\left(-p_k^s + p_1^w + \frac{t}{2}\right). \tag{6}$$

Under those conditions, the share of consumers who buy offline from firm 0, for a given p_0^w, is:

$$q_0 = \begin{cases} \tilde{x}_0^w & \text{if } p_0^w \in [2v - t - p_1^w, v] \text{ and } p_k^s > v - \frac{t}{2} \\ \tilde{x}_{0,1}^w & \text{if } p_0^w < \min\{2v - t - p_1^w, 2p_k^s - p_1^w, t + p_1^w\} \\ \tilde{x}_{k,0}^s & \text{if } p_0^w \in [2p_k^s - p_1^w, v] \text{ and } p_k^s < v - \frac{t}{2} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The share of consumers who buy online from firm 0, for a given p_0^s, is:

$$q_0^s = \begin{cases} \frac{\tilde{x}_{k,0}^s - \tilde{x}_{k,1}^s}{2} & \text{if } p_0^s = p_1^s \text{ and } \max\{p_0^w - \frac{t}{2}, p_1^w - \frac{t}{2}\} < p_0^s < \frac{p_0^w + p_1^w}{2} \\ \tilde{x}_{k,0}^s - \tilde{x}_{k,1}^s & \text{if } p_0^s < p_1^s \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Lemma 2. *When both firms sell the same variety and the platform is anonymous, there is a unique symmetric pure-strategy equilibrium in which the prices (on- and offline) and the quantities sold by each firm (on- and offline), (p^w, p^s, Q_j^w, Q_j^s), are given by*

$$(p^w, p^s) = \begin{cases} \left(\frac{v}{2}, 0\right) & \textit{if } \frac{v}{t} \leq \frac{1}{2} \\ \left(\frac{t}{4}, 0\right) & \textit{if } \frac{v}{t} > \frac{1}{2} \end{cases}; \quad (Q_j^w, Q_j^s) = \begin{cases} \left(\frac{v}{4t}, 0\right) & \textit{if } \frac{v}{t} \leq \frac{1}{2} \\ \left(\frac{1}{8}, \frac{1}{8}\right) & \textit{if } \frac{v}{t} > \frac{1}{2} \end{cases} \tag{9}$$

The associated equilibrium profit, consumer surplus and total welfare are given by

$$\pi_j^a = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{t}{32} & \text{if } \frac{v}{t} \geq \frac{1}{2} \end{cases}; \; S^a = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{3}{8}v - \frac{5}{32}t & \text{if } \frac{v}{t} \geq \frac{1}{2}. \end{cases}; \; W^a = \begin{cases} \frac{3v^2}{8t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{3}{8}v - \frac{3}{32}t & \text{if } \frac{v}{t} \geq \frac{1}{2} \end{cases} \tag{10}$$

Proof. See Appendix A. □

When $v/t \leq 1/2$, the market is not covered: agents located at $x \in [0, v/2t]$ or $x \in [1 - v/2t, 1]$ buy offline (walkers), while nobody buys online. Instead, when $v/t > 1/2$, the market is covered: agents located at $x \in [0, 1/4]$ or $x \in [3/4, 1]$ buy offline (walkers) and those at $x \in [1/4, 3/4]$ buy online (surfers).

Lemma 2 proves that firms sell online at their marginal cost (assumed to be 0). This can be understood as the combination of two effects: i) firms compete on prices and buyers consider online products to be homogeneous, given the information set according to which one cannot compute transport costs ex-ante for online purchases, ii) firms would benefit from an equilibrium where online prices are high enough to push everyone to buy offline, however both firms have an incentive to undercut their competitor. This happens because any marginal reduction in the online price leads to a small reduction in offline profits due to self-cannibalisation, but it also generates an online market-share expansion (at the expense of the competitor) that outweighs the reduction in offline profits.

3.2 Both varieties

We move now to the analysis of the case when each firm sells a different variety. In this case, each consumer is only interested in the product sold by one firm, which becomes a monopolist for the (mass $1/2$) consumers whose preferred variety they sell. However, as we show below, competition between firms arises when consumers do not have full information about product characteristics.

We start again by considering the case of a revealing platform and turn to the case of the anonymous platform later.

3.2.1 Revealing platform

Whenever firm 0 produces variety $\kappa_0 = k$, the utility of agents who like variety k and buy from firm 0 is $U_0 = v - p_0^r - tx$. Out of the mass $1/2$ interested in variety k, the fraction that buys is $q_0 = \min\{1, \max\{(v-p_0^r)/t, 0\}\}$. Firm 0's profit-maximisation problem is $\max_{p_0^r} p_0^r q_0/2$.

Lemma 3. *When the firms sell different varieties and the platform is revealing, there is a unique symmetric pure-strategy equilibrium in which the price and the quantity sold by each firm, (p^r, Q_j^r), are given by*

$$p^r = \begin{cases} \frac{v}{2} & \text{if } \frac{v}{t} \leq 2 \\ v - t & \text{if } \frac{v}{t} > 2 \end{cases}; \; Q_j^r = \begin{cases} \frac{v}{4t} & \text{if } \frac{v}{t} \leq 2 \\ \frac{1}{2} & \text{if } \frac{v}{t} > 2. \end{cases} \tag{11}$$

The associated equilibrium profit, consumer surplus and total welfare are

$$\pi_j^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} \leq 2 \\ \frac{v-t}{2} & \text{if } \frac{v}{t} > 2. \end{cases} ; \quad S^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} \leq 2 \\ \frac{t}{2} & \text{if } \frac{v}{t} > 2. \end{cases} ; \quad W^r = \begin{cases} \frac{3v^2}{8t} & \text{if } \frac{v}{t} < 2 \\ \frac{2v-t}{2} & \text{if } \frac{v}{t} \geq 2. \end{cases} \tag{12}$$

Proof. See Appendix A. □

The revealing platform guarantees that all agents are fully informed, hence, it guarantees full efficiency in terms of buyer-seller matching, but it also segments the market in such a way that each seller becomes a local monopolist. Hence, we find here the typical welfare loss that comes from market power.

Note that condition $v/t = 2$ in Eqs. (11) and (12) corresponds to the point where the market is covered. Each firm reaches agents located over the whole (unit-length) Hotelling line, focusing only on the mass $1/2$ of consumers that like the variety that the firm produces.

3.2.2 Anonymous platform

When two varieties are for sale and the platform is anonymous, consumers face several information issues. When buying via the platform, they are able to observe the variety that they purchase, but they do not observe the firm's location; hence, they cannot anticipate their transport cost. In expectation, the transport cost is $t/2$ for all, so the expected utility of buying online does not depend on a buyer's location. When buying directly from the seller, consumers know their transport cost but do not observe the variety κ_j that j sells; hence, they cannot anticipate the value of consumption. In expectation, it is $v/2$ regardless of the seller from which they buy.

The expected utility of a walker is then $EU_0^w = v/2 - p_0^w - tx$ if buying from firm 0, or $EU_1^w = v/2 - p_1^w - t(1-x)$ if buying from firm 1. The expected utility of a surfer buying variety κ_j is $EU_{\kappa_j}^s = v - p_j^s - t/2$, where p_j^s denotes the online price for variety κ_j.

Let $\tilde{x}_j^w$ again denote the consumer who is indifferent between buying offline from firm j and not buying, and let $\tilde{x}_{0,1}^w$ denote the consumer who is indifferent between buying offline from firm 0 or 1. Then,

$$\tilde{x}_0^w = \frac{1}{t}\left(\frac{v}{2} - p_0^w\right) ; \ \tilde{x}_1^w = 1 - \frac{1}{t}\left(\frac{v}{2} - p_1^w\right) ; \ \tilde{x}_{0,1}^w = \frac{1}{2}\left(1 - \frac{p_0^w - p_1^w}{t}\right) \tag{13}$$

Finally, the consumer located at $\tilde{x}_{\kappa_j,0}^s$ is indifferent between buying variety κ_j online (without knowing which seller provides that variety) or to purchase offline from firm 0 (without knowing which variety they will receive), and similarly for $\tilde{x}_{\kappa_j,1}^s$. Then,

$$\tilde{x}_{\kappa_j,0}^s \equiv \frac{1}{t}\left(-\frac{v}{2} + p_j^s - p_0^w + \frac{t}{2}\right) ; \ \tilde{x}_{\kappa_j,1}^s \equiv \frac{1}{t}\left(\frac{v}{2} - p_j^s + p_1^w + \frac{t}{2}\right) \tag{14}$$

The information asymmetry that characterises this setting leads to several interesting features. First, buyers are unable to tell apart sellers, which introduces some competition between sellers. Second, buyers self-select into sales channels, which sellers can use to price-discriminate across types of consumers.

We start by determining offline demand. For that, notice that the outside option for walkers can be either not to buy (if $p^s_j > v - t/2$), or to buy online (if $p^s_j \leq v - t/2$). We consider the two cases separately.

If $p^s_j > v - t/2$, online purchases are unattractive compared to not buying and the setting corresponds to the standard Hotelling game, with the only exception that buyers do not know the valuation of the good for sale and, therefore, they act based on the expected value $v/2$. Without loss of generality, we restrict attention to offline prices such that $p^w_j \leq v/2$ for $j = 0, 1$ and $|p^w_1 - p^w_0| \geq t$. (As discussed in Section 3.1.1, it is never optimal to set $p^w_j > v/2$ or $|p^w_1 - p^w_0| > t$.)

For a given p^w_1, the fraction of consumers for whom $k = \kappa_j$ that firm 0 attracts offline is:

$$q_0^{\kappa_j} = \begin{cases} \tilde{x}^w_0 & \text{for } p^w_0 > v - t - p^w_1 \\ \tilde{x}^w_{0,1} & \text{for } p^w_0 \leq v - t - p^w_1. \end{cases} \tag{15}$$

The cutoff price between the demand regimes, $p^w_0 = v - t - p^w_1$, is such that the agent located at $\tilde{x}^w_{0,1}$ receives a utility of zero. For p^w_0 below the cutoff, the market is covered; otherwise, the market is not covered.[19]

If $p^s_j \leq v - t/2$, online purchases replace not buying as the consumer's outside option. Without loss of generality, we restrict attention to prices such that $p^w_j \leq p^s_k - {}^{(v-t)}/_2$ for all j and k, and $|p^w_1 - p^w_0| \leq t$. The first condition implies that at least the consumers located at the end points ($x = 0$ and $x = 1$) weakly prefer to buy offline rather than online; if it holds with equality the measure of consumers buying offline is zero, so there is no need to consider higher prices.[20] The fraction of consumers for whom $k = \kappa_j$ that firm 0 attracts offline is:

$$q_0^{\kappa_j} = \begin{cases} \tilde{x}^s_{\kappa_j,0} & \text{if } p^w_0 > 2p^s_j - p^w_1 - v \\ \tilde{x}^w_{0,1} & \text{if } p^w_0 \leq 2p^s_j - p^w_1 - v \end{cases} \tag{16}$$

The cutoff price between the demand regimes, $p^w_0 = 2p^s_j - p^w_1 - v$, is such that the consumer located at $\tilde{x}^s_{\kappa_j,0}$ is indifferent between buying online, buying offline from firm 0, and buying offline from firm 1. For p^w_0 below the cutoff, all consumers buy offline, while for p^w_0 above the cutoff some consumers buy offline and others online.[21]

[19]Note that, like in Section 3.1, we do not have to condition on whether or not $p^w_1 \geq v/2 - t$. Even though, for $p^w_1 < v/2 - t$, all consumers receive strictly positive utility when buying from firm 1, so that the market is necessarily covered, our earlier argument that we can restrict attention to prices $p^w_j \leq v/2$ implies that, when $p^w_1 < v/2 - t$ and hence $v - t - p^w_1 > v/2$, we cannot have $p^w_0 > v - t - p^w_1$.

[20]Note that $p^s_j \leq v - t/2$ implies $p^s_j - {}^{(v-t)}/_2 \leq v/2$, which ensures that the consumers at the end points receive positive utility from buying offline.

[21]Note that we do not have to condition on whether or not $p_1 \geq p^s_j - {}^{(v+t)}/_2$. Even though, for $p_1 < p^s_j - {}^{(v+t)}/_2$, all consumers (including the one at $x = 0$) are better off buying offline from firm 1 than buying online, so that there are never any online sales, our earlier argument that we can restrict attention to prices $p^w_j \leq p^s_k - {}^{(v-t)}/_2$ implies that, when $p^w_1 < p^s_j - {}^{(v+t)}/_2$ and hence $2p^s_j - p^w_1 - v > p^s_j - {}^{(v+t)}/_2$, we cannot have $p^w_0 > 2p^s_j - p^w_1 - v$.

We now turn to online demand. The fraction of consumers who prefer variety κ_0 that buys from firm 0 online is

$$q_0^s = \begin{cases} 0 & \text{if } p_0^s > \min\{\frac{2v-t}{2}, \frac{v+p_0^w+p_1^w}{2}\} \\ \tilde{x}_{\kappa_0,1}^s - \tilde{x}_{\kappa_0,0}^s & \text{if } \max\{p_0^w, p_1^w\} + \frac{v-t}{2} < p_0^s \leq \min\{\frac{2v-t}{2}, \frac{v+p_0^w+p_1^w}{2}\} \\ \min\{1-\tilde{x}_{\kappa_0,0}^s, \tilde{x}_{\kappa_0,1}^s\} & \text{if } \begin{cases} p_0^s > \min\{p_0^w, p_1^w\} + \frac{v-t}{2} \\ \text{and} \\ p_0^s \leq \min\{\frac{2v-t}{2}, \frac{v+p_0^w+p_1^w}{2}, \frac{\max\{2p_0^w, 2p_1^w\}+v-t}{2}\} \end{cases} \\ 1 & \text{if } p_0^s \leq \min\{\frac{2v-t}{2}, \frac{\min\{2p_0^w, 2p_1^w\}+v-t}{2}\}. \end{cases} \tag{17}$$

On top of the two extreme cases in which p_0^s is either so large that nobody buys from firm 0 online or so small that everybody interested in variety κ_0 does, there are two intermediate cases.[22] The second line of Eq. (17) corresponds to a situation in which consumers in the middle of the line buy online while those towards the two ends of the line buy offline from the seller located close by, i.e. $0 < \tilde{x}_{\kappa_0,0}^s < \tilde{x}_{\kappa_0,1}^s < 1$. The third line corresponds to a situation in which consumers located close to the seller with the lower offline price buy from this seller offline while all others buy online, i.e. (assuming $p_0^w < p_1^w$) $0 < \tilde{x}_{\kappa_0,0}^s < 1$ while $\tilde{x}_{\kappa_0,1}^s \geq 1$.

Firm 0's profit is $\pi_0 = [p_0^w(q_0^A+q_0^B)+p_0^s q_0^s]/2$. The following lemma states equilibrium prices in a symmetric equilibrium, where $p_0^w = p_1^w = p^w$ and $p_0^s = p_1^s = p^s$.

Lemma 4. *When the firms sell different varieties and the platform is anonymous, Eq. (18) describes the unique symmetric pure-strategy equilibrium, where p^w, p^s, Q_j^w, Q_j^s represent respectively the equilibrium prices (off- and online) and the quantities sold (off- and online) by each firm.*

$$(p^w, p^s) = \begin{cases} \left(\frac{v}{4}, 0\right) & \textit{if } \frac{v}{t} < \frac{1}{2} \\ \left(\frac{4v-t}{8}, \frac{2v-t}{2}\right) & \textit{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \left(\frac{4t-v}{7}, \frac{3t+v}{7}\right) & \textit{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \left(\frac{v-t}{2}, v-t\right) & \textit{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right] \end{cases}; \quad (Q_j^w, Q_j^s) = \begin{cases} \left(\frac{v}{4t}, 0\right) & \textit{if } \frac{v}{t} < \frac{1}{2} \\ \left(\frac{1}{8}, \frac{3}{8}\right) & \textit{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \left(\frac{5t-3v}{14t}, \frac{3v+2t}{14t}\right) & \textit{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \left(0, \frac{1}{2}\right) & \textit{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right]. \end{cases} \tag{18}$$

For $v/t > 3$, there exists no symmetric equilibrium in pure strategies.
The associated equilibrium profit, consumer surplus and total welfare are given by

$$\begin{cases} \pi_j^a = \frac{v^2}{16t}, & S^a = \frac{v^2}{16t}, & W^a = \frac{3v^2}{16t}, & \textit{if } \frac{v}{t} < \frac{1}{2} \\ \pi_j^a = \frac{(28v-13t)}{64}, & S^a = \frac{t}{64}, & W^a = \frac{7}{8}v - \frac{25t}{64}, & \textit{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \pi_j^a = \frac{13t^2-3tv+3v^2}{49t}, & S^a = \frac{9v^2+138tv-157t^2}{196t}, & W^a = \frac{33v^2+114tv-53t^2}{196t}, & \textit{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \pi_j^a = \frac{v-t}{2}, & S^a = \frac{t}{2}, & W^a = v - \frac{t}{2}, & \textit{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right] \end{cases} \tag{19}$$

Proof. See Appendix A. □

[22] Note that in the last line we do not need to condition on $p_0^s \leq (v+p_0^w+p_1^w)/2$ since this condition is implied by $p_0^s \leq \min\{p_0^w, p_1^w\} + (v-t)/2$.

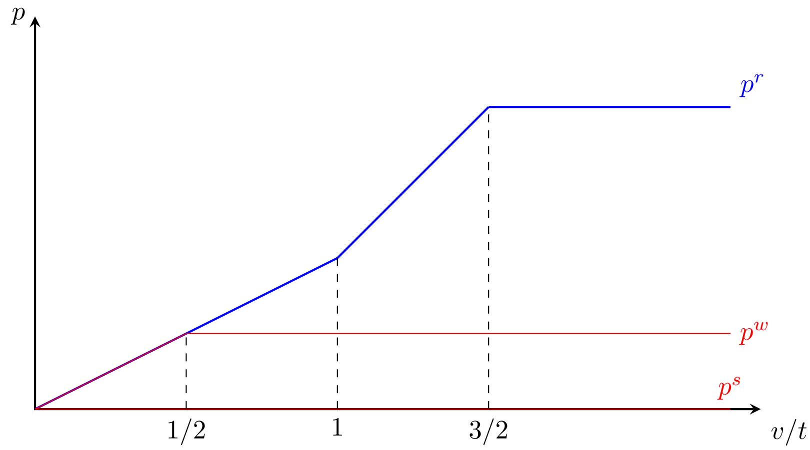
4 Welfare

We now use the results from Sections 3.1 and 3.2 to compare the outcomes with the revealing and anonymous platform. We start in Section 4.1 by looking at the case where only one variety is sold. In Section 4.2 we consider the case where two varieties are for sale. We will use the notation $\Pi^r = 2\pi^r$ and $\Pi^a = 2\pi^a$ to denote the sellers' joint profit under the revealing and anonymous platform, respectively. All figures are drawn for $t = 1$.

4.1 One variety

When one variety is available, Fig. 1 depicts prices as defined in Eqs. (3) and (9). Their relationship is described in Lemma 5.

Figure 1: Prices, 1-variety setting



Notes: for the plot, we assumed $t = 1$. p^r, p^w, p^s represent respectively the unique price under the revealing platform, the *walkers* and the *surfers* price under the anonymous platform.

Lemma 5. *Suppose a single variety is for sale. The price under the revealing design is always weakly greater than the offline price under the anonymous design ($p^r \geq p^w$), with strict inequality for $v/t > 1/2$. Both p^r and p^w are always strictly greater than the online price under the anonymous design (p^s).*

Proof. See Appendix A.

□

Notice that the revealing platform guarantees full information to buyers, who always buy from the closest seller. Interestingly, this is also the case under the anonymous design as long as all transactions only occur offline ($v/t \leq 1/2$), in which case prices are the same irrespective of the platform design.

However, as soon as the anonymous online market becomes active ($v/t > 1/2$), surfers cannot observe ex-ante the location of the seller from whom they buy. One consequence of this is that sellers are ex-ante identical in the eyes of surfers, which leads to Bertrand competition online. Online competition indirectly pushes offline prices down too. This explains why the walkers' price is smaller than the price on the revealing platform for $v/t > 1/2$.

Prices are only partially indicative of welfare, because the expected value of consumption changes across types of platforms. In particular, with the anonymous platform half of surfers buy from the seller that is farther away from them. Using Eqs. (4) and (10), Proposition 1 compare surplus, profits and total welfare under the two platform designs. Fig. 2 graphically summarises Proposition 1.[23]

Proposition 1 (Welfare comparison with one variety)**.** *For $v/t \leq 1/2$, the two designs are equivalent in terms of consumer surplus, profits and total welfare.*
For $v/t \in (1/2, 15/4]$, the revealing design generates greater profits and total welfare, while the anonymous design generates greater consumer surplus.
For $v/t > 15/4$, the revealing design is superior to the anonymous one in all three dimensions.

Proof. See Appendix A. □

The difference in prices across platforms (as shown in Fig. 1) explains why consumer surplus is greater under the anonymous platform and why profit is smaller. Our results on total welfare are driven by two main forces. On the one hand, while offline purchases are equally efficient with both designs, the fact that prices are lower with the anonymous platform (for $v/t > 1/2$) leads to a larger number of valuable transactions.

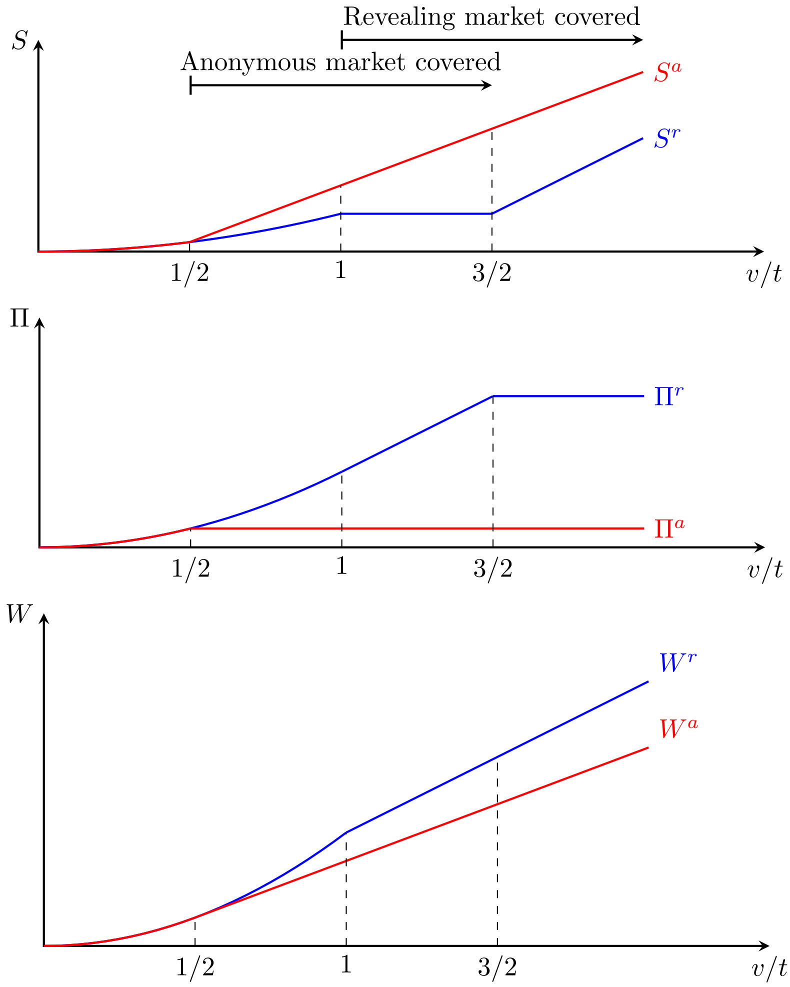
On the other hand, the anonymous platform induces the above-mentioned mismatch cost for surfers, who purchase from the closest seller only with probability $1/2$. This leads to a welfare loss proportional to transport costs. The relevance of this effect is magnified by the fact that $p^s = 0$, meaning the online market is particularly attractive for buyers and a large share of transactions take place there.

For $v/t \in (1/2, 1)$ more units are exchanged with the anonymous platform, but only offline sales are as efficient as with the revealing platform. For v/t close to $1/2$, the difference in sales volumes is substantial but each extra unit that is sold online produces very little welfare, for v/t is close to the online average transport cost ($1/2$). Any increase in v (up to $v/t = 1$) implies an increase in the benefits from trade but, at the same time, the difference in trading volumes between platform designs is shrinking. Indeed, notice that the value of a sale (v) is the same regardless of the sales channel (revealing/anonymous and on- or offline) but average transport costs are larger for online purchases through an anonymous platform.[24]

[23]Note that Fig. 2 does not show the full range of v/t covered by Proposition 1. In particular, $v/t \geq 15/4$, where consumer surplus is larger under the revealing design, is missing.

[24]The average transport cost is always $t/2$ when buying online on the anonymous platform. Instead, on the revealing platform it ranges between $t/8$ (when $v/t = 1/2$) and $t/4$ (when the market is fully covered at v/t=1).

Figure 2: Surplus, Profit and Welfare, 1-variety setting



Notes: for the plot, we assumed $t = 1$. S^r and S^a represent total consumer surplus respectively under the revealing and anonymous platform; Π^r and Π^a represent total market profit respectively under the revealing and anonymous platform. Finally, W^r and W^a represent total consumer surplus respectively under the revealing and anonymous platform.

4.2 Two varieties

For the two-variety case, a symmetric pure-strategy equilibrium exists under the anonymous platform design only if $v/t \leq 3$, as established in Section 3.2. Accordingly, our welfare analysis will be restricted to the case where $v/t \in (0, 3]$.

When two varieties are sold, prices are defined by Eqs. (11) and (18) and described in Lemma 6 and Fig. 3.

Lemma 6. *When two varieties are for sale, the following relationships hold among equilibrium prices:*

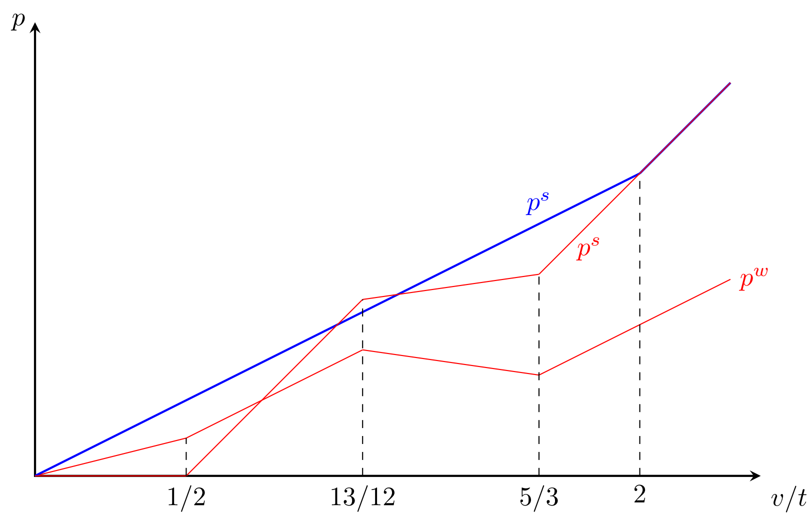
1. *under the anonymous platform design, the online price is larger than the offline price ($p^s > p^w$) if and only if $v/t > 3/4$;*

2. *the price under the revealing design is always greater than the offline price under the anonymous design ($p^r > p^w$);*

3. *the price under the revealing design is identical to the online price under the anonymous design for $v/t \in [2, 3]$. The online price is greater for $v/t \in (1, 6/5)$. Finally, the price under the revealing design is greater for $v/t < 1$ and $v/t \in (6/5, 2)$.*

Proof. See Appendix A. □

Figure 3: Prices, 2-variety setting



Notes: for the plot, we assumed $t = 1$. p^r, p^w, p^s represent respectively the unique price under the revealing platform, the *walkers* and the *surfers* price under the anonymous platform.

There are three main factors that explain the differences in pricing. First, offline prices under the anonymous design are lower because walkers obtain their preferred variety only with probability 1/2, so that their expected value is $v/2$ rather than v. Second, the anonymous design allows sellers to engage in price discrimination. By selling online to those consumers who are insensitive to location, they can raise prices to location-sensitive buyers offline without losing location-insensitive buyers. Third, the anonymous design introduces competition between sellers, whose markets are segmented under the revealing design. Because variety is not observable offline, sellers appear similar to consumers although they sell different varieties. This puts pressure on offline prices, which in turn holds online prices in check as well.

Proposition 2 compares total welfare, consumer surplus and profits across platform designs (revealing versus anonymous). Fig. 4 graphically represents the results described in Proposition 2.

Proposition 2 (Welfare comparison with two varieties)**.** *For each of profits, consumer surplus, and welfare, there exists an intermediate range of v/t in which the anonymous design is strictly superior to the revealing design.*

Specifically, for $v/t \in [2,3]$ the two designs are equivalent in all three dimensions, while for $v/t < 2$:

a) consumer surplus is larger under the anonymous design if and only if $v/t > \underline{s}$;

b) profit is larger under the anonymous design if and only if $v/t \in (\underline{\pi}, \overline{\pi})$;

c) welfare is larger under the anonymous design if and only if $v/t > \underline{w}$;

where $\underline{s} \approx 1.34$, $\underline{\pi} \approx 0.55$, $\overline{\pi} \approx 1.62$, $\underline{w} \approx 0.6$.

Proof. See Appendix A. □

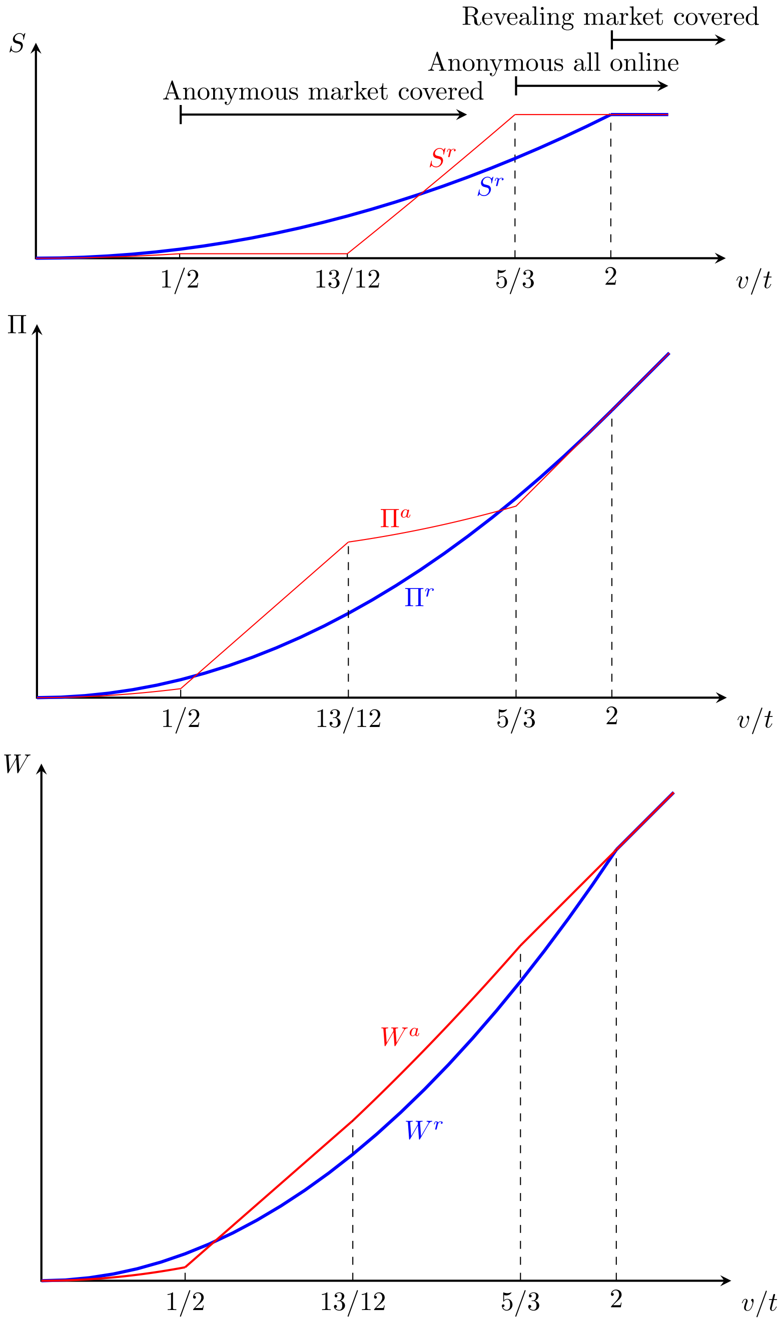
Proposition 2 shows that total welfare is larger with the anonymous platform for any value of $v/t > \underline{w}$. This may seem surprising. The revealing platform provides buyers with full information, thereby preventing any matching inefficiencies. By contrast, the anonymous platform generates inefficiencies both on- and off-line. Surfers always buy their preferred variety from the closest possible seller.[25] However, they do not observe the location where their preferred variety is sold and base their decision on the expected transport cost. As a result, they sometimes buy even though actual transport costs exceed v and sometimes refrain from buying even though transport costs are lower than v. Walkers do not observe the variety that is sold at the closest firm and hence sometimes mistakenly buy a product that is not valuable to them. The inefficiencies caused by the anonymous platform only disappear when all transactions take place online and the value is large enough to guarantee that all transactions are socially desirable (i.e. v exceeds transport costs).

To understand why the anonymous platform can generate more welfare, despite the inefficiencies, notice that under the revealing platform there is monopoly pricing while under the anonymous platform, the price-discrimination and competition effects discussed above ensure a larger volume of transactions. The market is fully covered as soon as $v/t = 1/2$, as opposed to $v/t = 2$ with the revealing platform.

The increase in transactions the anonymous platform generates can compensate for the matching inefficiencies. This happens as soon as $v/t > \underline{w}$. As v/t increases further, eventually (for $v/t > 2$) the anonymous market is fully covered online and transactions are always efficient, even for agents located at the extremes ($x = 0, x = 1$). At that point, matching inefficiencies disappear.

[25] Each variety is sold by one seller only; therefore, unlike in the case of one variety, everyone will always buy their preferred variety from the closest seller.

Figure 4: Surplus, Profit and Welfare, 2-variety setting



Notes: for the plot, we assumed $t = 1$. S^r and S^a represent total consumer surplus respectively under the revealing and anonymous platform; Π^r and Π^a represent total market profit respectively under the revealing and anonymous platform. Finally, W^r and W^a represent total consumer surplus respectively under the revealing and anonymous platform.

5 Conclusion

Online platforms that aggregate information and intermediate between buyers and sellers have become key actors in many markets. In this paper we study how profits and welfare depend on the platform design (anonymous versus revealing). We develop a model in which consumers are heterogeneous in two dimensions: their location and their preferences over the two varieties that could be for sale. Sellers are active both offline (direct sales) and online (platform sales). However, they are unable to credibly disclose the product variety they offer when selling offline. By contrast, the platform can disclose the variety but - if anonymous - in order to hide the identity of the seller it cannot disclose location.

The model features a novel trade-off associated with the anonymous platform design: offline, consumers observe the product location but not its variety; online, the opposite occurs. This contrasts with the revealing platform which discloses both elements and thus ensures better consumer-product matches.

Interestingly, despite the information loss associated with the anonymous design, we show that, for a wide range of parameter values, anonymity increases profits, consumer surplus or total welfare. Even more surprisingly, the anonymous design is, at times, superior to the revealing design in all three dimensions simultaneously. This is possible because the reduced information caused by the anonymous design fosters competition because it decreases the degree of perceived differentiation across products. This puts downward pressure on prices and leads to an increase in the number of transactions. This increase in transactions can compensate for the matching inefficiencies.

The revealing design tends to perform better when, for consumers, being informed about sellers' location is relatively more important than being informed about the variety they sell. This includes the case where both firms sell the same variety so that there is no uncertainty in this dimension. Conversely, the anonymous design outperforms the revealing one when consumers care less about learning the location.

References

Anderson, M., Magruder, J., 2012. Learning from the crowd: Regression discontinuity estimates of the effects of an online review database. Economic Journal 122, 957–989.

Anderson, S.P., Celik, L., 2020. Opaque selling. Information Economics and Policy 52, 100869.

Argenton, C., Geradin, D., 2021. What the economic literature says about MFN clauses in the online travel industry. Mimeo .

Armstrong, M., Zhou, J., In Press. Consumer information and the limits to competition. American Economic Review .

Balestrieri, F., Izmalkov, S., Leao, J., 2021. The market for surprises: Selling substitute goods through lotteries. Journal of the European Economic Association 19, 509–535.

Baye, M., Morgan, J., 2001. Information gatekeepers on the internet and the competitiveness of homogeneous product markets. American Economic Review 91, 454–474.

Cabral, L., 2019. Towards a theory of platform dynamics. Journal of Economics & Management Strategy 28, 60–72.

Cabral, L., Hortaçsu, A., 2010. The dynamics of seller reputation: evidence from ebay. Journal of Industrial Economics 58, 54–78.

Calzada, J., Manna, E., Mantovani, A., In Press. Platform price parity clauses and segmentation. Journal of Economics & Management Strategy .

Casadesus-Masanell, R., Campbell, N., 2019. Platform competition: Betfair and the uk market for sports betting. Journal of Economics & Management Strategy 28, 29–40.

Celik, L., 2014. Information unraveling revisited: disclosure of horizontal attributes. Journal of Industrial Economics LXII, 113–136.

Chevalier, J.A., Mayzlin, D., 2006. The effect of word of mouth on sales: Online book reviews. Journal of Marketing Research XLIII, 345–354.

Dellarocas, C., 2006. Strategic manipulation of internet opinion forums: Implications for consumers and firms. Management Science 52, 1577–1593.

Dinerstein, M., Einav, L., Levin, J., Sundaresan, N., 2018. Consumer price search and platform design in internet commerce. American Economic Review 108, 1820–1859.

Ellison, G., Ellison, S.F., 2009. Search, obfuscation, and price elasticities on the internet. Econometrica 77, 427–452.

Fay, S., 2008. Selling an opaque product through an intermediary: The case of disguising one's product. Journal of Retailing 84, 59–75.

Fradkin, A., Grewal, E., Holtz, D., 2021. Reciprocity and unveiling in two-sided reputation systems: Evidence from an experiment on Airbnb. Marketing Science 40, 1013–1029.

Ghose, A., Ipeirotis, P.G., Li, B., 2012. Designing ranking systems for hotels on travel search engines by mining user-generated and crowdsourced content. Marketing Science 31, 493–520.

Halaburda, H., Yehezkel, Y., 2019. Focality advantage in platform competition. Journal of Economics & Management Strategy 28, 49–59.

Hossain, T., Minor, D., Morgan, J., 2011. Competing matchmakers: An experimental analysis. Management Science 57, 1913–1925.

Janssen, M., Teteryatnikova, M., 2016. Horizontal product differentiation: Disclosure and competition. Journal of Industrial Economics 64, 589–620.

Johnson, J., 2017. The agency model and MFN clauses. Review of Economic Studies 84, 1151–1185.

Jullien, B., Pavan, A., 2019. Information management and pricing in platform markets. Review of Economic Studies 86, 1666–1703.

Jullien, B., Sand-Zantman, W., 2021. The economics of platforms: A theory guide for competition policy. Information Economics and Policy 54, 100880.

Kanoria, Y., Saban, D., 2021. Facilitating the search for partners on matching platforms. Management Science 67, 5990–6029.

Karle, H., Peitz, M., Reisinger, M., 2020. Segmentation versus agglomeration: Competition between platforms with competitive sellers. Journal of Political Economy 128, 2329–2374.

Klein, T.J., Lambertz, C., Stahl, K.O., 2016. Market transparency, adverse selection, and moral hazard. Journal of Political Economy 124, 1677–1713.

Luca, M., Zervas, G., 2016. Fake it till you make it: Reputation, competition, and yelp review fraud. Management Science 62, 3412–3427.

Mayzlin, D., Dover, Y., Chevalier, J., 2013. Promotional reviews: An empirical investigation of online review manipulation. American Economic Review 104, 2421–55.

Petrikaitė, V., 2018. Consumer obfuscation by a multiproduct firm. RAND Journal of Economics 49, 206–223.

Rey, P., Tirole, J., 2007. A primer on foreclosure, in: Armstrong, M., Porter, R. (Eds.), Handbook of Industrial Organization. Elsevier, North Holland. volume 3, pp. 2145–2220.

Romanyuk, G., Smolin, A., 2019. Cream skimming and information design in matching markets. American Economic Journal: Microeconomics 11, 250–76.

Ronayne, D., 2021. Price comparison websites. International Economic Review 62, 1081–1110.

Ronayne, D., Taylor, G., In Press. Competing sales channels with captive consumers. Economic Journal .

Shapiro, D., Shi, X., 2008. Market segmentation: The role of opaque travel agencies. Journal of Economics & Management Strategy 17, 803–837.

Spulber, D.F., 2019. The economics of markets and platforms. Journal of Economics & Management Strategy 28, 159–172.

Vial, B., Zurita, F., 2017. Entrants' reputation and industry dynamics. International Economic Review 58, 529–559.

Wang, C., Wright, J., 2020. Search platforms: showrooming and price parity clauses. The RAND Journal of Economics 51, 32–58.

Appendix A Proofs

Proof of Lemma 1. The first-order condition of the firm's profit-maximisation problem is

$$p_0^r \frac{\partial q_0}{\partial p_0^r} + q_0 = 0, \tag{20}$$

with

$$\frac{\partial q_0}{\partial p_0^r} = \begin{cases} -\frac{1}{2t} & \text{if } p_1^r < v - t \text{ or } p_1^r \leq 2v - t - p_0^r \\ -\frac{1}{t} & \text{if } p_1^r \geq v - t \text{ and } p_0^r > 2v - t - p_1^r. \end{cases}$$

In a symmetric equilibrium, $p_0^r = p_1^r = p^r$. There are two candidates for an interior solution, solving (20), namely, $p^r = t$ and $p^r = v/2$:

- $p^r = t$ is an equilibrium if $p^r < v - t$ or $p^r \leq 2v - t - p^r$ at $p^r = t$, i.e. if $v/t \geq 3/2$;
- $p^r = v/2$ is an equilibrium if $p^r \geq v - t$ and $p^r > 2v - t - p^r$ at $p^r = v/2$, i.e. if $v/t < 1$.

For $1 \leq v/t < 3/2$, neither of them is an equilibrium. In that case, the equilibrium is the corner solution $p^r = v - t/2$: for $v/t \in [1, 3/2)$ the marginal profit, $p_0^r(\partial q_0/\partial p_0^r) + q_0$, is positive for $p_0^r < v - t/2$ and negative for $p_0^r > v - t/2$ (evaluated at $p_1^r = v - t/2$).

Firm 0's profits are $\pi_0^r = p_0^r q_0/2$. Each firm's equilibrium profit is:

$$\pi_j^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < 1 \\ \frac{v}{4} - \frac{t}{8} & \text{if } \frac{v}{t} \in \left[1, \frac{3}{2}\right) \\ \frac{t}{4} & \text{if } \frac{v}{t} \geq \frac{3}{2}. \end{cases} \tag{21}$$

Recalling that consumers who prefer variety B are inactive, consumers surplus for firm 0's buyers is $S_0^r = 1/2 \int_0^{q_0} (v - p_0^r - tx)\, dx$. By symmetry, $S_1^r = S_0^r$. Let $S^r = S_0^r + S_1^r$, then:

$$S^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < 1 \\ \frac{t}{8} & \text{if } \frac{v}{t} \in \left[1, \frac{3}{2}\right) \\ \frac{v}{2} - \frac{5t}{8} & \text{if } \frac{v}{t} \geq \frac{3}{2}. \end{cases} \tag{22}$$

Total welfare is $W^r = 2\pi^r + S^r$. We have:

$$W^r = \begin{cases} \frac{3v^2}{8t} & \text{if } \frac{v}{t} < 1 \\ \frac{v}{2} - \frac{t}{8} & \text{if } \frac{v}{t} \geq 1. \end{cases} \tag{23}$$

□

Proof of Lemma 2. We consider separately the case for $v < t/2$ and $v \geq t/2$.

Suppose that $v/t < 1/2$**.** Nobody buys online, for the expected utility online would be negative. The offline market is covered if and only if $v \geq t/2 + (p_0^w + p_1^w)/2$, however, this can never be the case when $v/t < 1/2$. Hence, $v \leq t/2 + (p_0^w + p_1^w)/2$ and profits are defined as:

$$\begin{cases} \pi_0^a = p_0^w \frac{v - p_0^w}{2t} \\ \pi_1^a = p_0^w \frac{t - v + p_1^w}{2t} \end{cases}$$

By the FOC, the optimal price is $p_0^w = p_1^w = v/2$. At $p_j^w = v/2$, walkers located at $x \in [0, v/2t]$ buy from firm 0, while those located at $x \in [1 - v/2t, 1]$ buy from firm 1.

The equilibrium profit is $\pi_j^a = v^2/8t$ and it all comes from offline sales. Total consumers' surplus is $S^a = v^2/8t$ (remember only one variety is sold, hence half of agents is not buying). Total welfare is $W^a = \pi_0^a + \pi_1^a + S^a = 3v^2/8t$.

Suppose that $v/t \geq 1/2$**.** Let's focus now on the case when the online market is nonempty. The expected utility is positive for surfers if $v > p_k^s + t/2$. Furthermore, some agents buy online only if $\tilde{x}_{k,0}^s < \tilde{x}_{k,1}^s$, hence, $2p_k^s < p_0^w + p_1^w$.

Suppose, for now, that both conditions are satisfied. Profits for firm 0 are defined as:

$$\pi_0^a(p_0^w, p_0^s) = \begin{cases} \frac{1}{2} p_0^w \tilde{x}_{k,0}^s & \text{if } p_0^s > p_1^s \\ \frac{1}{2}\left(p_0^w \tilde{x}_{k,0}^s + \frac{1}{2} p_0^s (\tilde{x}_{k,1}^s - \tilde{x}_{k,0}^s)\right) & \text{if } p_0^s = p_1^s \\ \frac{1}{2}\left(p_0^w \tilde{x}_{k,0}^s + p_0^s (\tilde{x}_{k,1}^s - \tilde{x}_{k,0}^s)\right) & \text{if } p_0^s < p_1^s \end{cases} \tag{24}$$

We focus here on equilibria that are online-symmetric, i.e. $p_0^s = p_1^s$. We start by showing that it is never profitable to deviate from such equilibrium by rising the own price. Suppose that a candidate equilibrium is such that $p_0^s = p_1^s$. Since $\tilde{x}_{k,0}^s$ and $\tilde{x}_{k,1}^s$ only depend on the cheapest online price (p_k^s) but not on the specific price posted by firm j (p_j^s), any deviation implying an increase in p_j^s would weakly reduce profits, because firm j would stop selling online. Hence, such deviation cannot be optimal.

Consider now a deviation implying a reduction in p_j^s. This would instead make the firm's online market share double, however this would have a cost associated: the decrease in p_j^s corresponds to a decrease in p_k^s, which means the locations of the indifferent agents shift and, in particular, it would make some agents shift from off- to on-line, jeopardising the offline profit. A deviation is profitable only if so is the net effect. An ϵ-decrease of p_0^s would lead to a change in profits of $((p_k^s/2 - \epsilon)(v - 2p_k^s + p_0^w + p_1^w) + 2\epsilon(p_k^s - \epsilon) - \epsilon p_0^w)/2t$, which is positive as long as $4\epsilon^2 + 2\epsilon(v - 4p_k^s + 2p_0^w + p_1^w) - p_k^s(v - 2p_k^s + p_0^w + p_1^w) < 0$. Solving the quadratic equation, it follows immediately that profit increases if a firm cuts prices by any ϵ smaller than some strictly positive value. The threshold for ϵ decreases in p_k^s, reaching 0 at $p_k^s = 0$.

Therefore, undercutting the online price is always profitable as a deviation, as long as $p_k^s > 0$ and the initial conditions are satisfied.

Hence, an online-symmetric equilibrium with positive prices ($p_1^s = p_0^s > 0$) cannot exist. The only online-symmetric candidate equilibrium that remains requires that $p_j^s = 0$. This reduces our initial existence conditions to $p_0^w + p_1^w > 0$ (in order to have online sales, some of the offline prices must be strictly positive) and $v > t/2$ (in order to have online sales, the expected utility of buying online must be positive).

With $p_j^s = 0$, firms earn nothing from online sales and their profit is

$$\begin{cases} \pi_0^a = \frac{p_0^w \tilde{x}_{k,0}^s}{2} \\ \pi_1^a = \frac{p_1^w (1-\tilde{x}_{k,1}^s)}{2} \end{cases} \tag{25}$$

From the FOC, it follows that the offline profit maximising symmetric price is $p^w = t/4$ and condition $p_0^w + p_1^w > 0$ is satisfied. It also follows that $\tilde{x}_{k,0}^s = 1/4$ and $\tilde{x}_{k,1}^s = 3/4$. Quantities sold follow immediately.

Profit, in equilibrium, is $\pi_j^a = t/32$ and it is all obtained from offline sales. Only one variety is for sale and it's observable, hence only half of agents may consume and there's no mismatch cost in this setting.

If variety κ is for sale, walkers' surplus is $S^w = \left(\left(\tilde{x}_{k,0}^s + 1 - \tilde{x}_{k,1}^s\right)(v - p^w) - 2t \int_0^{1/4} x dx\right)/2 = (4v-t)/32$. Surfers' expected transport cost is always $\frac{t}{2}$, hence, their surplus is $S^s = (\tilde{x}_{k,1}^s - \tilde{x}_{k,0}^s)(v - t/2)/2 = (2v-t)/8$. Total consumer surplus is $S^a = 3v/8 - 5t/32$ and total welfare is $W^a = 2t/32 + (3v/8 - 5t/32) = 3v/8 - 3t/32$. □

Proof of Lemma 3. The maximisation problem is well-defined and the equilibrium price directly follows from the first order condition. Consumer surplus for those who buy from firm 0 is again $S_0^r = 1/2 \int_0^{q_0} (v - p_0^r - tx)\, dx$, with $S_1^r = S_0^r$ by symmetry. The total consumer surplus is $S^r = S_0^r + S_1^r$. Total welfare is simply given by $W^r = 2\pi_j^r + S^r$. □

Proof of Lemma 4. Combining Eqs. (13) and (14) with Eqs. (15) to (17), we obtain:

$$q_0^{\kappa_j} = \begin{cases} \dfrac{v/2 - p_0^w}{t} & \text{if } p_0^w > v - t - p_1^w \\ \dfrac{1}{2}\left(1 + \dfrac{p_1^w - p_0^w}{t}\right) & \text{if } p_0^w \leq v - t - p_1^w. \end{cases}$$

$$q_0^{\kappa_j} = \begin{cases} \dfrac{1}{2}\left(1 + \dfrac{2(p_j^s - p_0^w) - v}{t}\right) & \text{if } p_0^w > 2p_j^s - p_1^w - v \\ \dfrac{1}{2}\left(1 + \dfrac{p_1^w - p_0^w}{t}\right) & \text{if } p_0^w \leq 2p_j^s - p_1^w - v. \end{cases}$$

$$q_0^s = \begin{cases} 0 & \text{if } p_0^s > \min\{\frac{2v-t}{2}, \frac{v+p_0^w+p_1^w}{2}\} \\ \dfrac{v+p_0^w+p_1^w-2p_0^s}{t} & \text{if } \max\{p_0^w, p_1^w\} + \frac{v-t}{2} < p_0^s \le \min\{\frac{2v-t}{2}, \frac{v+p_0^w+p_1^w}{2}\} \\ \dfrac{v+t+2(\min\{p_0^w, p_1^w\} - p_0^s)}{2t} & \text{if } \begin{cases} p_0^s > \min\{p_0^w, p_1^w\} + \frac{v-t}{2} \\ \text{and} \\ p_0^s \le \min\{\frac{2v-t}{2}, \frac{v+p_0^w+p_1^w}{2}, \frac{\max\{2p_0^w, 2p_1^w\}+v-t}{2}\} \end{cases} \\ 1 & \text{if } p_0^s \le \min\{\frac{2v-t}{2}, \frac{\min\{2p_0^w, 2p_1^w\}+v-t}{2}\}. \end{cases}$$

The rest of the proof proceeds as follows. First, we identify equilibrium candidates. Then, we check for each of them whether there are profitable deviations. Finally, using the equilibrium prices we obtain, we compute the equilibrium quantities.

To systematically draw up the list of candidate equilibria (in pure strategies), we distinguish two types of solutions for online and offline prices: interior solutions and corner solutions. The first-order conditions for interior solutions for p^w and p^s are, from firm 0's perspective:

$$q_0^{\kappa_0} + q_0^{\kappa_1} + p_0^w \frac{\partial (q_0^{\kappa_0} + q_0^{\kappa_1})}{\partial p_0^w} + p_0^s \frac{\partial q_0^s}{\partial p_0^w} = 0 \tag{26}$$

$$q_0^s + p_0^w \frac{\partial q_0^{\kappa_0}}{\partial p_0^s} + p_0^s \frac{\partial q_0^s}{\partial p_0^s} = 0. \tag{27}$$

We consider the following categories of equilibrium candidates:

(i) Both the online and offline price are interior solutions.

(ii) The offline price is interior while the online price is a corner solution.

(iii) The online price is interior while the offline price is a corner solution.

(iv) Both the online and offline price are corner solutions.

Category (i). Both prices being interior requires that offline and online demand for firm 0 be $q_0^{\kappa_j} = \frac{1}{2}\left(1 + \frac{2(p_j^s - p_0^w) - v}{t}\right)$, $j = 0, 1$, and $q_0^s = \frac{v + p_0^w + p_1^w - 2p_0^s}{t}$, respectively. We then have $\partial(q_0^{\kappa_0} + q_0^{\kappa_1})/\partial p_0^w = -2/t$, $\partial q_0^s/\partial p_0^w = 1/t$, $\partial q_0^{\kappa_0}/\partial p_0^s = 1/t$, and $\partial q_0^s/\partial p_0^s = -2/t$. Using $p_0^w = p_1^w = p^s$ and $p_0^s = p_1^s = p^s$, the first-order conditions become

$$1 + \frac{2(p^s - p^w) - v}{t} - \frac{2p^w}{t} + \frac{p^s}{t} = 0 \tag{28}$$

$$\frac{p^w}{t} - \frac{2p^s}{t} + \frac{2(p^w - p^s) + v}{t} = 0. \tag{29}$$

Solving yields $p^w = (4t - v)/7$ and $p^s = (3t + v)/7$.

A necessary condition for this to be an equilibrium is that the prices satisfy the conditions for demand to be as assumed. For offline demand, this requires $p^w \geq p^s - (v + t)/2$ and $p^w > p^s - v/2$, which simplifies to $3v + 2t \geq 0$ and is always satisfied. For online demand, it requires $p^w + (v - t)/2 < p^s \leq v - t/2$, which requires $^{13}/_{12} \leq v/t < {}^5/_3$.

Equilibrium profit for each firm is $\pi^* = (3v^2 - 3vt + 13t^2)/49t$. Since both prices are interior and thus satisfy the first-order conditions for a local maximum, there cannot be a profitable deviation locally. However, one of the firms, say firm 0, could deviate non-locally by raising its online price to extract all surplus, $p_0^s = v - t/2$, and choosing p_0^w to maximise profits. The best such deviation solves

$$\max_{p_0^w} \frac{1}{2}\left[p_0^w\left(\frac{v/2 - p_0^w}{t} + \frac{1}{2}\left(\frac{1 + 2(p_1^s - p_0^w) - v}{t}\right)\right) + p_0^s\frac{v + p_0^w + p_1^w - 2p_0^s}{t}\right],$$

with $p_0^s = v - t/2$, $p_1^w = (4t - v)/7$, and $p_1^s = (3t + v)/7$. The solution is $p_0^w = (8v + 3t)/28$, yielding a deviation profit of $\pi^D = (888vt - 384v^2 - 299t^2)/784t$, which is always less than the equilibrium profit.

Hence, the deviation is unprofitable and we conclude that $(p^w, p^s) = \left({}^{(4t-v)}/_7, {}^{(3t+v)}/_7\right)$ constitutes an equilibrium for $^{13}/_{12} \leq v/t < {}^5/_3$.

Category (ii). There are three cases to consider, depending on the online price: (a) $p^s > v - t/2$, (b) $p^s = v - t/2$, (c) $p^s < v - t/2$.

(a) If $p^s > v - t/2$, online purchases are unattractive, and offline demand is given by (15). There are two interior candidate equilibria: $p^w = v/4$ and $p^w = t$.
For $p^w = v/4$, under the relevant conditions, some consumers are not buying, and each firm could raise profit by selling to them online at price $p_j^s = v - t/2$ without affecting offline sales, so this cannot be an equilibrium.
For $p^w = t$, under the relevant conditions, firm 0's profit when decreasing p_0^s slightly below $v/2 + t$, at which consumers start buying online, is

$$\pi_0 = \frac{1}{2}\left[p_0^w\left(1 + \frac{p_0^s + p_1^s - 2p_0^w - v}{t}\right) + p_0^s\left(\frac{v + p_0^w + p_1^w - 2p_0^s}{t}\right)\right].$$

Differentiating with respect to p_0^s and evaluating at $p_0^w = p_1^w = t$ and $p_0^s = v/2 + t$, we obtain $\partial\pi_0/\partial p_0^s = -v/t < 0$. Hence, lowering its price slightly below $v/2 + t$ is a profitable deviation, so this cannot be an equilibrium either.

(b) If $p^s = v - t/2$, consumers obtain zero expected surplus from buying online, yet they will buy online if buying offline gives them negative surplus. A set of consumers for whom this is the case only exists if prices are such that offline demand is $q_0^{\kappa_j} = (v/2 - p_0^w)/t$ and online demand is $q_0^s = (v + p_0^w + p_1^w - 2p_0^s)/t$. The profit-maximising offline price solves

$$\max_{p_0^w} \frac{1}{2}\left[p_0^w\left(\frac{v - 2p_0^w}{t}\right) + \left(v - \frac{t}{2}\right)\frac{v + p_0^w + p_1^w - 2(v - t/2)}{t}\right],$$

yielding $p_0^w = v/2 - t/8$. The associated equilibrium profit is $\pi^* = (28v - 13t)/64$. Existence requires $p^w \geq 0 \Leftrightarrow v \geq t/4$ and $p^s \geq 0 \Leftrightarrow v \geq t/2$, with the latter implying the former. The best deviation is the combination of (p_0^w, p_0^s) that solves

$$\max_{p_0^w, p_0^s} \frac{1}{2} \left[p_0^w \left(1 + \frac{p_0^s + p_1^s - 2p_0^w - v}{t} \right) + p_0^s \left(\frac{v + p_0^w + p_1^w - 2p_0^s}{t} \right) \right]$$

for $p_1^w = v/2 - t/8$ and $p_1^s = v - t/2$. Straightforward computations yield $p_0^w = v/4 + 7t/48$ and $p_0^s = v/2 + t/24$. The associated deviation profit is $\pi^D = (144v^2 + 24vt + 13t^2)/768t$, which is always larger than π^*. However, the deviation is possible only if $p_0^s = v/2 + t/24 \leq v - t/2$ (otherwise buying online is unattractive), or $v/t \geq 13/12$. We conclude that $(p^w, p^s) = (v/2 - t/8, v - t/2)$ is an equilibrium for $1/2 \leq v/t < 13/12$.

(c) If $p^s < v - t/2$, online purchases give consumers positive expected surplus, so offline demand is given by (16). The potential corner solutions for the online price are $p^s \geq p^w + v/2$, so that nobody buys online, and $p^s = p^w + (v - t)/2$, so that everybody buys online. The former is equivalent to $p^s \geq v - t/2$ and has already been treated under (a) and (b). The latter requires that firms do not have an incentive to lower p^w and divert sales from online to offline. There are two such equilibrium candidates: (i) one in which the candidate p^w is such that it is a best response for each firm j to choose $p_j^w = p^w$, holding fixed p_j^s (as well as the rival's prices p_{-j}^w and p_{-j}^s), and (ii) one in which the candidate p^w is such that it is a best response for firm j to choose $p_j^w = p^w$ while adjusting $p_j^s = p_j^w + (v - t)/2$ (still holding fixed p_{-j}^w and p_{-j}^s). We now consider each candidate in turn.

Candidate (i): In this case, p^w must solve

$$\max_{p_0^w} \frac{1}{2} \left[p_0^w \left(1 + \frac{p_0^s + p_1^s - 2p_0^w - v}{t} \right) + p_0^s \left(\frac{v + p_0^w + p_1^w - 2p_0^s}{t} \right) \right],$$

the first-order condition of which is

$$\frac{1}{2} \left(1 + \frac{p_0^s + p_1^s - 2p_0^w - v}{t} - \frac{2p_0^w}{t} + \frac{p_0^s}{t} \right) = 0.$$

Evaluating the first-order condition at $p_0^w = p_1^w = p^w$ and $p_0^s = p_1^s = p^s$, we obtain a system of two equations in two unknowns given by

$$1 + \frac{3p^s - 4p^w - v}{t} = 0$$
$$p^s = p^w + \frac{v - t}{2},$$

the solution of which is $p^s = v - t$ and $p^w = (v - t)/2$. The associated equilibrium profit is $\pi^* = (v - t)/2$.

We now check deviations. Suppose firm 0 lowers its online price. This can be profitable only if the firm also lowers its offline price, as otherwise all consumers would continue buying online (at a a lower price). There are three cases for the offline price. First, firm 0 can deviate to an offline price such that some (but not all) consumers of each variety start buying offline. The prices p_0^w and p_0^s for the best such deviation solve

$$\max_{p_0^w, p_0^s} \frac{1}{2}\left[p_0^w\left(1+\frac{p_0^s+p_1^s-2p_0^w-v}{t}\right)+p_0^s\left(\frac{v+t+2(p_0^w-p_0^s)}{2t}\right)\right]$$

given $p_1^w=(v-t)/2$ and $p_1^s=v-t$. This yields $p_0^w=(v+t)/4$ and $p_0^s=(v+t)/2$. This deviation is possible if the deviation prices are smaller than the equilibrium prices, i.e. if $(v+t)/2<v-t$ and $(v+t)/4<(v-t)/2$, which is true if $v/t>3$. The condition for the online demand for variety κ_0 to be as specified is $\min\{p_0^w,p_1^w\}+(v-t)/2 = (v+t)/4+(v-t)/2<p_0^s=(v+t)/2$, or $v/t<3$. These conditions cannot hold at the same time, thus ruling out this deviation. Second, firm 0 can deviate to an offline price such that some consumers of κ_0 and all consumers of κ_1 switch to firm 0 offline. Then $p_0^w=(v-3t)/2$ while p_0^s solves

$$\max_{p_0^s} \frac{v-3t}{2}\left[\frac{1}{2}\left(1+\frac{2(p_0^s-(v-3t)/2)-v}{t}\right)+\frac{1}{2}\right]+p_0^s\frac{v+t+2((v-3t)/2-p_0^s)}{2t},$$

yielding $p_0^s=(3v-5t)/4$. However, $p_0^w\geq 0$ requires $v/t\geq 3$, while for online demand to be as specified we need $\min\{p_0^w,p_1^w\}+(v-t)/2<p_0^s$ or

$$\frac{v-3t}{2}+\frac{v-t}{2}<\frac{3v-5t}{4} \quad\Leftrightarrow\quad \frac{v}{t}<3,$$

which cannot hold simultaneously. Third, firm 0 can deviate to an offline price such that all consumers of κ_0 continue to buy online while *some* consumers of κ_1 switch from firm 1 online to firm 0 offline. Then, $p_0^s=p_0^w+(v-t)/2$ so that $q_0^s=1$, while p_0^w is chosen to solve

$$\max_{p_0^w} \frac{p_0^w}{2}\left(1+\frac{2(p_1^s-p_0^w)-v}{t}\right)+\left(p_0^w+\frac{v-t}{2}\right)\cdot 1,$$

yielding $p_0^w=(v+t)/4$ and $p_0^s=(3v-t)/4$. For this to constitute a price decrease, it must be that $(v+t)/4<(v-t)/2$, or $v/t>3$. The condition for offline demand to be as specified is $p_0^w>2p_1^s-p_1^w-v$, or $v/t<7$. The condition for online demand to be as specified is $p_0^s\leq\min\{v-t/2,\min\{p_0^w,p_1^w\}+(v-t)/2\}$. For $v/t>3$, we have $p_0^w<p_1^w$ and $p_0^w+(v-t)/2=(3v-t)/4\leq v-t/2$, so the condition always holds. Deviation profit is $\pi^D=(v^2+10tv-7t^2)/(32t)$, which is always greater than π^*, so the deviation is profitable. Fourth, firm 0 can deviate to an offline price such that all consumers of κ_0 continue to buy online while *all* consumers of κ_1 switch to firm 0 offline. This requires $p_0^w=(v-3t)/2$ and $p_0^s=p_0^w+(v-t)/2=v-2t$. The associated profit is $\pi^D=(1/2)[p_0^w+p_0^s]=(3v-7t)/4$, which exceeds the equilibrium profit for $v/t>5$. We conclude that there exist profitable deviations from this equilibrium if $v/t>3$.

Next, suppose firm 0 raises its online price, so that some consumers of variety κ_0 start buying offline, and adjusts its offline price such that some consumers of variety κ_1 also buy offline from firm 0. The optimal combination of p_0^w and p_0^s solves

$$\max_{p_0^w, p_0^s} \frac{1}{2}\left[p_0^w\left(1+\frac{p_0^s+p_1^s-2p_0^w-v}{t}\right)+p_0^s\frac{v+p_0^w+p_1^w-2p_0^s}{t}\right].$$

Given $p_1^w=(v-t)/2$ and $p_1^s=v-t$, this yields $p_0^w=(3v-t)/12$ and $p_0^s=(3v-t)/6$. These prices are positive provided $v/t \geq 1/3$. The associated deviation profit is $\pi^D=(3v-t)^2/48$, which is always greater than equilibrium profit, so the deviation is profitable. We now check the conditions for the deviation to satisfy the required conditions on demand. In order to have $p_0^w > 2p_0^s - p_1^w - v$ we need $(3v-t)/12 > (3v-t)/3-(v-t)/2-v$ or $v/t \geq 1/3$, which always holds for positive prices. To have $p_0^w > 2p_1^s - p_1^w - v$ we need $(3v-t)/12 > 2(v-t)-(v-t)/2-v$ or $v/t < 17/3$. To check the condition required for $\max\{p_0^w, p_1^w\}+(v-t)/2 < p_0^s$, notice first that $p_0^w < p_1^w$ if and only if $v/t > 5/3$. Suppose $v/t > 5/3$. Then, the condition becomes $v-t < (3v-t)/6$, which can only hold if $v/t < 5/3$, a contradiction. Suppose instead $v/t < 5/3$, so that the condition becomes $(3v-t)/12+(v-t)/2 < (3v-t)/6$, or $v/t < 5/3$, which is assumed to hold. Next, let us check the condition $p_0^s \leq \min\{v-t/2, (v+p_0^w+p_1^w)/2\}$. We have $v-t/2 < (v+p_0^w+p_1^w)/2$ if and only if $v/t < 5/3$. Given the result on the previous condition, we can thus restrict attention to the case where $\min\{v-t/2, (v+p_0^w+p_1^w)/2\} = v-t/2$. We then have $p_0^s < v-t/2$ if and only if $v/t > 2/3$. We conclude that the deviation is possible for $v/t \in (2/3, 5/3)$. Finally, since for $v/t \leq 2/3$, the condition that $p_0^s < v-t/2$ is violated, consider an alternative deviation whereby $p_0^s = v-t/2$ and p_0^w is adjusted optimally. Thus, p_0^w solves

$$\max_{p_0^w} p_0^w\left(\frac{v/2-p_0^w}{t}\right)+\left(v-\frac{t}{2}\right)\frac{v+t+p_0^w+(v-t)/2-2(v-t/2)}{t}$$

yielding $p_O^w=(3v-t)/4$. The associated deviation profit is $\pi^D=\frac{v^2+22tv-11t^2}{32t}$, which is always larger than the equilibrium profit π^*. In order for the online demand to be as required, we need $\max\{p_0^w, p_1^w\}+(v-t)/2 < p_0^s$. We have $p_0^w=(3v-t)/4 > (v-t)/2 = p_1^w$, so the relevant condition is $(3v-t)/4+(v-t)/2 < v-t/2$, or $v < t$. Hence, there are profitable deviations if $v/t < 5/3$. Putting both kinds of deviations (lowering and raising p_0^s) together, we conclude that $(p^w, p^s)=((v-t)/2, v-t)$ is an equilibrium if and only if $v/t \in [5/3, 3]$.

Candidate (ii): Because $p_0^s = p_0^w+(v-t)/2$ in this case, firm 0's offline demand from consumers whose preferred variety is κ_0 is zero while online demand from these consumers is one for any p_0^w. The equilibrium p^w is found by solving

$$\max_{p_0^w} \frac{1}{2}\left[\frac{p_0^w}{2}\left(1+\frac{2(p_1^s-p_0^w)-v}{t}\right)+\left(p_0^w+\frac{v-t}{2}\right)\cdot 1\right],$$

the first-order condition of which is

$$\frac{1}{2}\left(1+\frac{2(p_1^s-p_0^w)-v}{t}\right)-\frac{p_0^w}{t}+1=0.$$

Evaluating at $p_1^s = p_1^w + (v-t)/2$ and $p_0^w = p_1^w = p^w$ yields $p^w = t$, which implies $p^s = (v+t)/2$. The associated equilibrium profit is $\pi^* = (v+t)/4$. The required condition on demand is that $p^s \leq v - t/2$, or $v/t \geq 2$.

Consider a deviation whereby firm 0 raises its online price and lowers its offline price sufficiently for some (but not all) consumers of each variety to buy offline from firm 0. The optimal such deviation solves

$$\max_{p_0^w, p_0^s} \frac{1}{2}\left[p_0^w\left(1+\frac{p_0^s+p_1^s-2p_0^w-v}{t}\right)+p_0^s\frac{v+p_0^w+p_1^w-2p_0^s)}{t}\right]$$

with $p_1^w = t$ and $p_1^s = (v+t)/2$. This yields $p_0^w = 2t/3$ and $p_0^s = (3v+7t)/12$. Clearly, we have $p_0^w < p^w = t$. We have $p_0^s > p^s$ if and only if $v/t > 1/3$, which is true for the relevant range ($v/t \geq 2$). The deviation prices satisfy the conditions for online demand to be as specified if $\max\{p_0^w, p_1^w\} + (v-t)/2 < p_0^s$, or $v/t > 1/3$, and $p_0^s \leq \min\{v - t/2, (v + p_0^w + p_1^w)/2\}$. We have $v - t/2 < (v + p_0^w + p_1^w)/2$ if and only if $v/t < 8/3$. Thus for $v/t < 8/3$ the condition is $p_0^s \leq v - t/2$ or $v/t > 13/9$, while for $v/t \geq 8/3$ the condition is $p_0^s \leq (v + p_0^w + p_1^w)/2$ or $v + t \geq 0$, which is always satisfied. The conditions for offline demand to be as specified are $p_0^w > 2p_j^s - p_1^w - v$ for $j = 0, 1$. Since under this deviation $p_0^s > p_1^s$, it suffices that $p_0^w > 2p_0^s - p_1^w - v$ or $v + t > 0$, which is always satisfied. Deviation profit is $\pi^D = (3v^2 + 6tv + 19t^2)/(48t)$, which is always larger than equilibrium profit since

$$\pi^D > \pi^* \quad \Leftrightarrow \quad 3(v-t)^2 + 4t^2 > 0.$$

We conclude that this deviation is both possible and profitable for $v/t > 13/9$, hence ruling out this equilibrium over the entire relevant range ($v/t \geq 2$).

Category (iii). There is no such equilibrium candidate. A corner solution for offline sales means that either everybody or nobody buys offline. But since online all consumers obtain the same expected surplus, an interior online price cannot be part of an equilibrium in that case.

Category (iv). There are two cases to consider: (a) $p^s \geq v - t/2$ and $p^w = (v-t)/2$, so that nobody buys online and the offline price is at the kink of the demand; (b) $p^w \geq v/2$ and $p^s = v - t/2$, so that nobody buys offline and firms extract the full surplus online.

(a) Each firm's profit is $\pi^* = (v-t)/4$. Applying Lemma 1 and replacing v by $v/2$, we know that even in the absence of an online sales channel this can only be an equilibrium for $v/2 \in (t, 3t/2)$, or $2 < v/t < 3$. Suppose that v/t is in this range and consider a deviation by firm 0 to an online price p_0^s that is marginally below $v - t/2$. Firm 0's resulting profit is

$$\pi_0 = \frac{1}{2}\left[p_0^w\left(\frac{1}{2}\left(1 + \frac{2(p_0^s - p_0^w) - v}{t}\right) + \frac{1}{2}\right) + p_0^s\left(\frac{v + p_0^w + p_1^w - 2p_0^s}{t}\right)\right],$$

with $p_0^w = p_1^w = (v - t)/2$. Notice that for $p_0^s = v - t/2$, we have $\pi_0 = (v - t)/4 = \pi^*$. Hence, to rule out this equilibrium candidate, it suffices to show that firm 0's deviation profit is decreasing in p_0^s when evaluated at $p_0^s = v - t/2$, i.e.

$$\left.\frac{\partial \pi_0}{\partial p_0^s}\right|_{p_0^s = v - t/2} = \frac{1}{2}\left[-\frac{v - t}{t} - \frac{2(v - t/2)}{t}\right],$$

which is always negative in the relevant range of v/t.

(b) Each firm's profit is $\pi^* = (1/2)(v - t/2)$. A necessary condition for this to be an equilibrium is that firm 0 cannot gain by unilaterally lowering p_0^w marginally below $v/2$. Profit in the case of such a deviation would be

$$\pi_0 = \frac{1}{2}\left[p_0^w\left(\frac{v - 2p_0^w}{t}\right) + \left(v - \frac{t}{2}\right)\frac{v + p_0^w + v/2 - 2(v - t/2)}{t}\right].$$

Differentiating with respect to p_0^w and evaluating at $p_0^w = v/2$ yields

$$\left.\frac{\partial \pi_0}{\partial p_0^w}\right|_{p_0^w = v/2} = -\frac{v}{2t} + \frac{v - t/2}{2t} < 0.$$

Hence, this deviation is always profitable, ruling out this equilibrium candidate.

We conclude that for $v/t \leq 3$, there exists a unique symmetric equilibrium price pair, given by:

$$(p^w, p^s) = \begin{cases} \left(\frac{v}{4}, 0\right) & \text{if } \frac{v}{t} < \frac{1}{2} \\ \left(\frac{4v-t}{8}, \frac{2v-t}{2}\right) & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \left(\frac{4t-v}{7}, \frac{3t+v}{7}\right) & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \left(\frac{v-t}{2}, v - t\right) & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right]. \end{cases} \tag{30}$$

For $v/t > 3$, there exists no symmetric equilibrium in pure strategies.

Plugging Eq. (30) into Eqs. (15) to (17), we obtain the quantity sold.

$$(Q_j^w, Q_j^s) = \begin{cases} \left(\frac{v}{4t}, 0\right) & \text{if } \frac{v}{t} < \frac{1}{2} \\ \left(\frac{1}{8}, \frac{3}{8}\right) & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \left(\frac{5t-3v}{14t}, \frac{3v+2t}{14t}\right) & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \left(0, \frac{1}{2}\right) & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right]. \end{cases} \tag{31}$$

In particular, notice that, given the existence conditions for each equation:

- the first segment of Eq. (15) is used to obtain Q_j^w for $v/t < 1/2$
- the first segment of Eq. (16) is used to obtain Q_j^w for all $v/t \geq 1/2$
- the first segment of Eq. (17) is used to obtain Q_j^s for $v/t < 1/2$
- the second segment of Eq. (17) is used to obtain Q_j^s for $v/t \in [1/2, 5/3)$
- the fourth segment of Eq. (17) is used to obtain Q_j^s for $v/t \in [5/3, 3]$

Also, notice that $q_0^{\kappa_j}$ is the share of consumers of variety κ_j that buys offline from firm 0. Since walkers do not observe the variety sold by firm j, each firm attracts the same amount of walkers that like variety A or B. The total quantity sold offline by firm j is $Q_j^w = (q_0^A + \frac{1}{2} q_0^B)/2$.

Finally, notice that q_j^s is the share of consumers of variety κ_j that buys online from firm j. Nobody who likes variety κ_{-j} would buy from firm j. Therefore, $Q_j^s = q_j^s/2$.

Profits, surplus and total welfare follow immediately. In the case of the anonymous platform, it is also possible to compute separately the part of profit that is obtained off- and on-line. Eq. (32) summarises it, with π_j^w, π_j^s respectively denoting profit off- and on-line.

$$(\pi_j^w, \pi_j^s) = \begin{cases} \left(\frac{v^2}{16t}, 0\right) & \text{if } \frac{v}{t} < \frac{1}{2} \\ \left(\frac{4v-t}{64}, \frac{6v-3t}{16}\right) & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \left(\frac{(5t-3v)(4t-v)}{98t}, \frac{(3v+2t)(3t+v)}{98t}\right) & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \left(0, \frac{v-t}{2}\right) & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right]. \end{cases} \tag{32}$$

□

Proof of Lemma 5. Prices are

$$p^w = \begin{cases} \frac{v}{2} & \text{if } \frac{v}{t} \leq 1/2 \\ \frac{t}{4} & \text{if } \frac{v}{t} > 1/2 \end{cases}; \quad p^s = 0; \quad p^r = \begin{cases} \frac{v}{2} & \text{if } \frac{v}{t} \leq 1 \\ v - \frac{t}{2} & \text{if } \frac{v}{t} \in \left(1, \frac{3}{2}\right) \\ t & \text{if } \frac{v}{t} \geq \frac{3}{2} \end{cases}. \tag{33}$$

It is immediate to see that $p^r = p^w$ if $v/t \leq 1/2$. Also, for $v/t > 1/2$ it follows that $t/4 < v/2$, hence, the revealing price is greater than the walkers price, □

Proof of Proposition 1. From Eqs. (4) and (10) we have that

$$\pi_j^a = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{t}{32} & \text{if } \frac{v}{t} \geq \frac{1}{2} \end{cases}; \quad \pi_j^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} \leq 1 \\ \frac{v}{4} - \frac{t}{8} & \text{if } \frac{v}{t} \in \left(1, \frac{3}{2}\right) \\ \frac{t}{4} & \text{if } \frac{v}{t} \geq \frac{3}{2} \end{cases}. \tag{34}$$

$$S^a = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{3}{8}v - \frac{5}{32}t & \text{if } \frac{v}{t} \geq \frac{1}{2} \end{cases}; \quad S^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} \leq 1 \\ \frac{t}{8} & \text{if } \frac{v}{t} \in \left(1, \frac{3}{2}\right) \\ \frac{v}{2} - \frac{5t}{8} & \text{if } \frac{v}{t} \geq \frac{3}{2} \end{cases}. \tag{35}$$

$$W^a = \begin{cases} \frac{3v^2}{8t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{3}{8}v - \frac{3}{32}t & \text{if } \frac{v}{t} \geq \frac{1}{2} \end{cases}; \quad W^r = \begin{cases} \frac{3v^2}{8t} & \text{if } \frac{v}{t} \leq 1 \\ \frac{v}{2} - \frac{t}{8} & \text{if } \frac{v}{t} > 1 \end{cases}. \tag{36}$$

From a simple comparison of the values for each interval, results follow immediately. Profit are the same for $v/t \leq 1/2$. For $v/2 > 1/2$, profit is greater with the revealing platform.

Consumers' surplus is the same for $v/t \leq 1/2$. For $v/t > 1/2$, consumers' surplus is greater with the anonymous platform for $v/t \in (1/2, 15/4)$, while it is larger with the revealing platform if $v/t > 15/4$.

Total welfare is the same for $v/t \leq 1/2$. For $v/t > 1/2$, consumers' surplus is greater with the revealing platform. □

Proof of Lemma 6. Given the following prices,

$$\begin{cases} p^w = \frac{v}{4}, & p^s = 0 & \text{if } \frac{v}{t} < \frac{1}{2} \\ p^w = \frac{1}{2}v - \frac{1}{8}t, & p^s = v - \frac{t}{2} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ p^w = \frac{1}{7}(4t - v), & p^s = \frac{1}{7}(3t + v) & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ p^w = \frac{v-t}{2}, & p^s = v - t & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right] \end{cases}; \quad p^r = \begin{cases} \frac{v}{2} & \text{if } \frac{v}{t} \leq 2 \\ v - t & \text{if } \frac{v}{t} > 2 \end{cases}. \tag{37}$$

the results follow simply by pairwise comparison for each segment.

In particular, notice that $p^r > p^s$ when

$$\begin{cases} \frac{v}{2} > v - \frac{t}{2} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{v}{2} > \frac{1}{7}(3t + v) & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \frac{v}{2} > v - t & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases}; \quad \text{hence,} \quad \begin{cases} \frac{v}{t} < 1 & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{v}{t} > \frac{6}{5} & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \frac{v}{t} < 2 & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases} \tag{38}$$

Hence, $p^r > p^s$ if either $v/t < 1$ or $v/t \in (6/5, 2)$. Instead, $p^s > p^r$ if $v/t \in (1, 6/5)$. Finally $p^s = p^r$ for $v/t \in [2, 3]$.

□

Proof of Proposition 2. From Eqs. (12) and (19), we have

$$\pi^a \begin{cases} \frac{v^2}{16t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{(28v-13t)}{64} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{13t^2-3tv+3v^2}{49t} & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \frac{v-t}{2} & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right] \end{cases}; \quad \pi^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < 2 \\ \frac{v-t}{2} & \text{if } \frac{v}{t} \geq 2 \end{cases}. \tag{39}$$

Under the anonymous platform, total consumers' surplus (including agents buying from both firms) is $S^a = 2\int_0^{q_0}(v/2 - p^w - tx)dx + q^s(v - p^s - t/2)$

$$S^a = \begin{cases} \frac{v^2}{16t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{t}{64} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{9v^2+138tv-157t^2}{196t} & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \frac{t}{2} & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right] \end{cases}; \quad S^r = \begin{cases} \frac{v^2}{8t} & \text{if } \frac{v}{t} < 2 \\ \frac{t}{2} & \text{if } \frac{v}{t} \geq 2 \end{cases}. \tag{40}$$

Total welfare is computed as consumers surplus plus twice the profit and it is:

$$W^a = \begin{cases} \frac{3v^2}{16t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{7}{8}v - \frac{25t}{64} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{33v^2+114tv-53t^2}{196t} & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ v - \frac{t}{2} & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 3\right] \end{cases}; \quad W^r = \begin{cases} \frac{3v^2}{8t} & \text{if } \frac{v}{t} < 2 \\ v - \frac{t}{2} & \text{if } \frac{v}{t} \geq 2 \end{cases}. \tag{41}$$

Comparing profits, $\pi^r > \pi^a$ when

$$\begin{cases} \frac{v^2}{8t} > \frac{v^2}{16t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{v^2}{8t} > \frac{(28v-13t)}{64} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{v^2}{8t} > \frac{13t^2-3tv+3v^2}{49t} & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \frac{v^2}{8t} > \frac{v-t}{2} & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases}, \text{hence, } \begin{cases} always & \text{if } \frac{v}{t} < \frac{1}{2} \\ v^2 - \frac{7}{2}tv > -\frac{13}{8}t^2 & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ v^2 + \frac{24}{25}tv > \frac{104}{25}t^2 & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ v^2 - 4tv > -4t^2 & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases}$$

which finally simplifies into:

$$\begin{cases} always & \text{if } \frac{v}{t} < \frac{1}{2} \\ v \notin \left[\frac{t}{4}(7-\sqrt{23}), \frac{t}{4}(7+\sqrt{23})\right] & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ v \notin \left[-\frac{t}{25}(12+\sqrt{2744}), \frac{t}{25}(-12+\sqrt{2744})\right] & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ always & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases} \tag{42}$$

Therefore, $\pi^r \geq \pi^a$ if and only if $v/t < {}^{(7-\sqrt{23})}/_4$ or $v/t > {}^{(-12+\sqrt{2744})}/_{25}$, with $\pi^r = \pi^a$ when $v/t \in \left[\frac{5}{3}, 3\right]$. We define $\underline{\pi} \equiv {}^{(7-\sqrt{23})}/_4 \approx 0.55$ and $\overline{\pi} \equiv {}^{(-12+\sqrt{2744})}/_{25} \approx 1.62$.

Comparing surplus, it follows immediately that the two functions cross only once, when ${}^{(9v^2+138tv-157t^2)}/_{196t} = {}^{v^2}/_{8t}$. Hence, we obtain that, within the interval $v/t \in [{}^{13}/_{12}, {}^5/_3)$, surplus is larger with the anonymous platform if $v/t \in \left({}^{(138-7\sqrt{190})}/_{31}, {}^5/_3\right)$. For $v/t \in ({}^5/_3, 2)$, surplus under the anonymous platform is always larger. We define $\underline{s} \equiv {}^{(138-7\sqrt{190})}/_{31} \approx 1.34$.

Total welfare is larger under the revealing platform if

$$\begin{cases} \frac{3v^2}{8t} > \frac{3v^2}{16t} & \text{if } \frac{v}{t} < \frac{1}{2} \\ \frac{3v^2}{8t} > \frac{7}{8}v - \frac{25t}{64} & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ \frac{3v^2}{8t} > \frac{33v^2+114tv-53t^2}{196t} & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ \frac{3v^2}{8t} > v - \frac{t}{2} & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases}, \text{hence, } \begin{cases} always & \text{if } \frac{v}{t} < \frac{1}{2} \\ v^2 + \frac{25}{24}t^2 > \frac{7}{3}tv & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ v^2 + \frac{106}{81}t^2 > \frac{76}{27}tv & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ v^2 + \frac{4}{3}t^2 > \frac{8}{3}tv & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases}.$$

The latter equation boils down to

$$\begin{cases} always & \text{if } \frac{v}{t} < \frac{1}{2} \\ v < \frac{14-\sqrt{46}}{12}t & \text{if } \frac{v}{t} \in \left[\frac{1}{2}, \frac{13}{12}\right) \\ never & \text{if } \frac{v}{t} \in \left[\frac{13}{12}, \frac{5}{3}\right) \\ never & \text{if } \frac{v}{t} \in \left[\frac{5}{3}, 2\right) \end{cases}. \tag{43}$$

Therefore, we conclude that total welfare is larger under the revealing platform if and only if $v < {}^{(14-\sqrt{46})}/_{12}$. We define $\underline{w} \equiv {}^{(14-\sqrt{46})}/_{12} \approx 0.6$. $\square$

Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.