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An Experimental QRE Analysis

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Abstract

The theoretical literature on public school choice proposes centralized mechanisms that assign children to schools on the basis of parents' preferences and the priorities children have for different schools. The related experimental literature analyzes in detail how various mechanisms fare in terms of welfare and stability of the resulting matchings, yet often provides only aggregate statistics of the individual behavior that leads to these outcomes (i.e., the degree to which subjects tell the truth in the induced simultaneous move game). In this paper, we show that the quantal response equilibrium (QRE) adequately describes individual behavior and the resulting matching in three constrained problems for which the immediate acceptance mechanism and the student-optimal stable mechanism coincide. Specifically, the comparative statics of the logit-QRE with risk-neutral and expected-payoff-maximizing agents capture the directional changes of subject behavior and the prevalence of the different stable matchings when cardinal payoffs (i.e., relative preference intensities) are modified in the experiment.

Keywords: laboratory experiment; school choice; immediate acceptance mechanism; Boston mechanism; deferred acceptance mechanism; quantal response equilibrium.

JEL–Numbers: C78, C91, C92, D78, I20.

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1 Introduction

Motivation

In many public school choice programs, centralized mechanisms such as the student-optimal stable mechanism and the immediate acceptance mechanism are used to assign children to schools on the basis of parents' preferences and the priorities of children for different schools (based on, e.g., walking distance, siblings, etc.).[1] In a *constrained* setting where parents can only rank a limited number of schools, which happens in many real-life applications,[2] there may be incentives to behave strategically. Specifically, a very common situation is that the parents prefer their child to go to school x instead of school y but the child has higher priority at school y than at school x. The parents then face an important decision. Will they report their preferences on the pair (x, y) truthfully or will they misrepresent their preferences by ranking school y above school x? Our paper contributes to the experimental literature that analyzes the structure of manipulations in school choice problems by studying how cardinal payoffs affect subject behavior.

For the constrained setting, laboratory experiments have shown that subjects may fail to coordinate on a Nash equilibrium.[3] In fact, to the best of our knowledge, there are no studies that give clear-cut descriptions of subjects' behavior and resulting matchings. We consider three constrained problems for which the immediate acceptance mechanism and the student-optimal stable mechanism coincide and show that quantal response theory introduced by McKelvey and Palfrey (1995) is capable of describing the main behavioral patterns.[4] From a logit-QRE perspective, if the payoff of school x is much higher than the payoff of school y, there are incentives to play a strategy in which school x is ranked above school y. On the other hand, if the payoffs of the two schools are close to each other, strategies that rank school y above school x have the advantage that not much payoff is foregone and that the risk of getting an even worse outcome than school y diminishes. We next detail the experimental design and our main findings to clarify these points further.

Experiment and contribution

We consider a setting with three students and two schools (s_1 and s_2) such that each student can only apply to one school. The preferences of the students and the priorities students have at the schools remain the same throughout the experiment. Two parameters vary in our study. First, the number of seats schools offer changes each six rounds of an experimental session. School s_1 offers one and school s_2 offers two seats in the first six rounds of a session (we refer to this situation as

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[1]The student-optimal mechanism is the mechanism based on the deferred acceptance algorithm (Gale and Shapley, 1962). The immediate acceptance mechanism is also known as the "Boston mechanism." For further details we refer to Abdulkadiroğlu and Sönmez (2003) who initiated the market design literature on school choice.

[2]See Dur (2019), Dur and Morrill (2020), and Kojima and Ünver (2014) for details.

[3]See the subsection "Related literature" at the end of the Introduction for references.

[4]The quantal response equilibrium has been applied as an equilibrium notion in many experimental settings including all-pay auctions (Anderson, Goeree, and Holt, 1998), the traveler's dilemma (Capra et al., 1999), jury decision rules (Guarnaschelli, McKelvey, and Palfrey, 2000), alternating offer bargaining games (Goeree and Holt, 2000), coordination games (Anderson, Goeree, and Holt, 2001), first price auctions (Goeree, Holt, and Palfrey, 2002), generalized matching pennies games (Goeree, Holt, and Palfrey, 2003), capacity allocation games (Chen, Su, and Zhao, 2012), and two-sided matching (Echenique, Wilson, and Yariv, 2016).

problem $\mathcal{P}_1$), both schools offer one seat in the next six rounds (problem $\mathcal{P}_2$), and in the final six rounds s_1 offers two and s_2 offers one seat (problem $\mathcal{P}_3$). Our treatment variable is the material payoff subjects receive from their matches. A subject always receives 1 monetary unit if she is assigned to the school she likes most and nothing if she remains unmatched. In treatment L (low payoff), a subject receives 0.3 monetary units if she is matched to the school she likes least. This payoff equals 0.7 monetary units in treatment H (high payoff). In each of $\mathcal{P}_1$ and $\mathcal{P}_3$, there are two stable matchings: the student-optimal stable matching μ^I and the school-optimal stable matching μ^S. All students weakly prefer μ^I to μ^S. There is a unique stable matching in $\mathcal{P}_2$.[5]

Our experimental predictions are based on the logit version of the quantal response equilibrium (logit-QRE) with risk-neutral and expected-payoff-maximizing agents, which is parametrized by a "rationality parameter" $\lambda \geq 0$.[6] It is well-known that for large λ, the logit-QRE tends towards a Nash equilibrium in mixed strategies. For both $\mathcal{P}_1$ and $\mathcal{P}_3$, the resulting matching under this limiting logit-QRE is μ^I in treatment L but μ^S in treatment H. The underlying intuition is that at the pure strategy Nash equilibrium that yields μ^I some students take the risk of being unmatched. This risk comes with the potential benefit of being matched to the most preferred school. For it being worthwhile to take this risk (instead of applying to the worst school, which ensures a seat, but comes with a low payoff), the payoff difference between the two schools must be sufficiently high, which is the case in treatment L but not in treatment H. A similar argument applies in the general (non-limiting) case as well. In fact, we establish that independently of the actual rationality parameters in treatments L and H, μ^I is more likely to be obtained than μ^S in treatment L but μ^S is more prevalent than μ^I in treatment H. Our data supports this prediction, which we consider to be our main finding. In $\mathcal{P}_1$ of treatment L, μ^I is obtained in 30.2% and μ^S in 27.2% of the cases (the frequency of an unstable matching is thus 42.6%). On the other hand, in $\mathcal{P}_1$ of treatment H, μ^I is obtained in 19.7% and μ^S in 45.1% of the cases. The frequencies for $\mathcal{P}_3$ are 43.2% for μ^I and 37.0% for μ^S in treatment L, but 24.1% for μ^I and 59.9% for μ^S in treatment H. In each of the two problems, the frequency difference between μ^I and μ^S is significant at the 5-percent level for treatment H but not for treatment L.

The logit-QRE also permits us to derive hypotheses regarding student behavior. Point predictions indicate for a given problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$ and for a given treatment whether a student is more likely to apply to school s_1 or to school s_2. And treatment comparisons refer to how the probability with which a student applies to each of the two schools changes between the two treatments. The experimental data is in almost all instances consistent with these predictions. In particular, for $\mathcal{P}_1$ and $\mathcal{P}_3$, 4 out of a total of 5 statistical tests regarding treatment comparisons yield a one-sided p-value below 0.05. With respect to the point predictions, there is more noise in the data because "only" 3 out of a total of 12 comparisons are significant at the 5-percent level.

We also estimate the logit-QRE for the pooled data at the problem level via maximum

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[5]Problem $\mathcal{P}_2$ has been added to the design mainly for completeness reasons. We thus concentrate in the introduction on our two main problems $\mathcal{P}_1$ and $\mathcal{P}_3$. A complete set of predictions and the corresponding data analysis for problem $\mathcal{P}_2$ is presented alongside the other two problems in Sections 2 and 3, respectively.

[6]We concentrate on the logit-QRE because Haile, Hortaçsu, and Kosenok (2008) show that in its most general form, the quantal response equilibrium is not falsifiable in any game (any behavior can be rationalized). See also Goeree, Holt, and Palfrey (2005).

likelihood. The estimated λ is largest in $\mathcal{P}_2$, which is arguably the simplest setting because it has a unique Nash equilibrium outcome (supported by an infinite number of Nash equilibria in mixed strategies). In each of $\mathcal{P}_1$ and $\mathcal{P}_3$, the estimated λ is smaller in treatment L than in treatment H. Finally, we find that this discrepancy in the estimation of λ is reduced if agents are risk-averse but not if they have preferences for the student-optimal stable matching (the expected utility depends positively on the probability that this matching is reached) or the expected group payoff.

Related literature

There is a steadily increasing number of experimental studies that complement the theoretical literature on school choice (see Hakimov and Kübler, 2020, for an excellent overview). It is by now well understood that some subjects fail to report their true preferences under the unconstrained student-optimal stable mechanism. Under the immediate acceptance mechanism, subject behavior is even less in line with the true preferences (see, e.g., Calsamiglia, Haeringer, and Klijn, 2010; Chen and Kesten, 2019; Chen and Sönmez, 2006; Featherstone and Niederle, 2016; and Pais and Pinter, 2008). The kind of deviations subjects employ under the two mechanisms are very similar. In particular, the order of two schools at the top of their preference ranking might be switched if students have a higher priority in the less desired of the two schools.

The existing experiments on school choice mainly compare various mechanisms in terms of their outcome (stability and welfare of the resulting matching), but only offer a less detailed overview of the behavior that leads to these outcomes (i.e., often the focus is a comparison of truth-telling rates across mechanisms). It seems therefore necessary to develop alternative models that capture the observed behavioral patterns. The logit-QRE is a natural starting point because it takes relative preference intensities explicitly into account.[7] Apart from our study, only Echenique, Wilson, and Yariv (2016) and Dreyfuss, Heffetz, and Rabin (2021) estimate a logit-QRE in a two-sided matching market. The main difference between our study and Echenique, Wilson, and Yariv (2016) is that we specifically design the experiment in such a way that the predictions of the logit-QRE depend on the treatment condition. Furthermore, Echenique, Wilson, and Yariv (2016) do not derive testable hypotheses from the logit-QRE.

Using the data of Li (2017) on the (strategy-proof) random serial dictatorship mechanism, Dreyfuss, Heffetz, and Rabin (2021) suggest that expectation-based loss aversion can explain individuals' deviations from truth-telling. When the underlying mechanism is strategy-proof, the logit-QRE with expected-payoff-maximizing agents converges towards truthful preference revelation, which implies that deviations of rational subjects are likely to be caused by non-standard preferences.

Remainder

We proceed as follows. In Section 2, we detail the experimental design and procedures and we derive predictions from the logit-QRE. In Section 3, we present all experimental results. Section 4 concludes. The appendix contains the experimental instructions, formal proofs, additional data analysis, and theoretical predictions for lists of length 2 (i.e., the unconstrained mechanisms).

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[7]Abdulkadiroğlu, Che, and Yasuda (2011) highlight in a restricted environment that the immediate acceptance mechanism allows parents to express their relative preference intensities.

2 Laboratory experiment

2.1 Design and procedures

Our experiment is designed to analyze behavior in three school choice problems in which three students (labeled 1, 2, and 3) seek to obtain a seat at schools s_1 and s_2. The preferences of the students and the priorities of the schools are presented in Table 1 and are the same in all three problems. The number of seats schools offer, however, changes over the course of an experimental session and is given in Table 2.

	Preferences			Priorities	
	1	2	3	s_1	s_2
Best match:	s_1	s_1	s_2	3	2
Second best:	s_2	s_2	s_1	1	1
Third best:				2	3

Table 1: Preferences of students over schools and priorities of schools over students.

Problem	# seats	
	s_1	s_2
$\mathcal{P}_1$	1	2
$\mathcal{P}_2$	1	1
$\mathcal{P}_3$	2	1

Table 2: Number of seats at schools.

During the experiment, subjects assume the role of students. Schools are not strategic players. The information in Table 1 is common knowledge. The number of seats schools offer is made public at the beginning of each round of the experiment. Given this information, the subjects' task is to submit a *single* application to a school (not necessarily to the most preferred school) to be used by a central clearinghouse to assign students to schools. Specifically, we focus on the constrained immediate acceptance or, equivalently, the constrained deferred acceptance mechanism (the equivalence is due to the fact that lists are of length 1):

Constrained Immediate/Deferred Acceptance Mechanism

Step 1. Each student sends an application to exactly one school.

Step 2. Each school that has received at least one application accepts the application from the student with the highest priority (among all received applications). If a school with two seats has received at least two applications, then it also accepts the application from the student with the second highest priority (among all received applications). All other applications (if any) are rejected.

Each student is assigned to the school that she applied to provided that the school accepted her application. If the application of a student was rejected, then the student remains without a seat.

The experiment was programmed within the z–Tree toolbox provided by Fischbacher (2007) and carried out at Lineex (www.lineex.es) hosted at the University of Valencia. At the beginning of a session, subjects received written instructions that were read aloud by an instructor (see Appendix A). Participants were informed in particular that the experiment would take a total of 18 rounds and that the number of seats schools offer would change each six rounds. In all sessions, $\mathcal{P}_1$ was always played first (rounds 1–6), $\mathcal{P}_2$ always played second (rounds 7–12), and $\mathcal{P}_3$ always played third (13–18).[8] Then, the computer software started. The program matched participants anonymously into groups of three. Within each group, one subject was assigned the role of student 1, another subject the role of student 2, and a third subject the role of student 3. Groups and roles did not change over the course of the experiment. Before round 1, subjects first went individually over an illustrative example (to get used to the mechanism) and then played a trial round that was not taken into account for payment (to become familiar with the computer software). At the beginning of each round, the computer screen presented the preferences of the three group members, the priorities of the two schools, and the number of seats at each school. Subjects took then their respective decisions. At the end of each round, each subject was informed of her resulting match.

Our treatment variable is the payoff subjects receive. In each round, a subject received 1 ECU if she ended up at her most preferred school and 0 ECU if her application was rejected (in which case she ended up unmatched). In treatment L (low), a subject received 0.3 ECU if she ended up at her second most preferred (or equivalently, her least preferred) school. In treatment H (high), this payoff was 0.7 ECU. Each ECU was worth 1 €.

We ran two sessions (one with 42 and another one with 39 subjects) per treatment. In total, 162 undergraduates from various disciplines participated in the experiment. Each session lasted about 90 minutes. Apart from the payoff subjects accumulated during the session, they also received a show-up fee of 3€. Subjects earned on average 13.73€ in treatment L and 14.60€ in treatment H.

2.2 Predictions

In this subsection, we derive our experimental predictions. A *(school choice) problem* is a five-tuple $\mathcal{P} = \langle I, S, q, P_I, P_S \rangle$ where

- $I = \{1, 2, \ldots, n\}$ is a finite set of students (individuals);
- S is a finite set of schools;
- $q \equiv (q_s)_{s \in S}$ where for each $s \in S$, q_s is the number of available seats at school s;
- $P_I \equiv (P_i)_{i \in I}$ is a profile of strict preference relations of the students, where for each $i \in I$, P_i is a complete, irreflexive, and transitive binary relation over $S \cup \{i\}$; and
- $P_S \equiv (P_s)_{s \in S}$ is a profile of strict priority relations of the schools, where for each $s \in S$, P_s is a complete, irreflexive, and transitive binary relation over I.

We assume that for each student i, each school is preferred to her outside option (being unmatched), which is denoted by i. A *matching* of students to schools is a function $\mu : I \to S$ such that for

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[8]There is no need to counterbalance the order in which problems are played because we do not compare outcomes between problems.

each $i \in I$ and for each $s \in S$,

- $\mu(i) \in S \cup \{i\}$ and
- $|\mu^{-1}(s)| \le q_s$.

We will often write a matching μ as the vector $(\mu(1), \mu(2), \ldots, \mu(n))$, where for each i, $\mu(i)$ is called student i's *match*. Whenever $\mu(i) = i$, student i is unmatched, i.e., remains without seat. A pair $(i, s) \in I \times S$ *blocks* matching μ if $s\, P_i\, \mu(i)$ and

- $|\mu^{-1}(s)| < q_s$ or
- $|\mu^{-1}(s)| = q_s$ and there exists $k \in \mu^{-1}(s)$ such that $i\, P_s\, k$.

A matching is *stable* if no pair (i, s) blocks it. The set of stable matchings is non-empty (Gale and Shapley, 1962). Moreover, there exists a student-optimal stable matching μ^I which is weakly preferred by all students to all other stable matchings. Similarly, there exists a school-optimal stable matching μ^S which is student-pessimal, i.e., all students weakly prefer any other stable matching to μ^S. Table 3 shows all stable matchings in our three problems. In problems $\mathcal{P}_1$ and $\mathcal{P}_3$, there are two stable matchings (μ^I and μ^S). Since there is only one stable matching in problem $\mathcal{P}_2$, the side-optimal stable matchings μ^I and μ^S coincide in this problem.

Problem	μ^I		μ^S
$\mathcal{P}_1$	(s_1, s_2, s_2)	$\ne$	(s_2, s_2, s_1)
$\mathcal{P}_2$	$(1, s_2, s_1)$	$=$	$(1, s_2, s_1)$
$\mathcal{P}_3$	(s_1, s_1, s_2)	$\ne$	(s_1, s_2, s_1)

Table 3: Set of all stable matchings.

We consider the strategic game induced by the school choice mechanism. Let $I = \{1, 2, 3\}$ be the set of players. Each player $i \in I$ has a set of pure strategies $A_i = \{a_{i1}, a_{i2}\}$, where a_{ij} is player i's strategy of sending an application to school s_j. Let $A \equiv \times_{i \in I} A_i$ denote the set of strategy-profiles $a = (a_1, a_2, a_3)$.

Let $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$. For each $a \in A$ and each $i \in I$, let $\beta_i(a, \mathcal{P})$ denote the match of player i when players send applications a and schools have priorities and seat availability as given by $\mathcal{P}$. Fix a problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$ and a treatment $T \in \{L, H\}$. We assume that player i has a utility function $u_i : A \to \mathbb{R}$ that reflects the possible per-round payoffs in the experiment, i.e.,

$$u_i(a) \equiv \begin{cases} 0 & \text{if } \beta_i(a, \mathcal{P}) = i, \text{ i.e., player } i \text{ is unmatched;} \\ 0.3 & \text{if } \beta_i(a, \mathcal{P}) \text{ is player } i\text{'s bad match and } T = L; \\ 0.7 & \text{if } \beta_i(a, \mathcal{P}) \text{ is player } i\text{'s bad match and } T = H; \\ 1 & \text{if } \beta_i(a, \mathcal{P}) \text{ is player } i\text{'s good match.} \end{cases}$$

Here, good (bad) match refers to player i's most (least) preferred school at $\mathcal{P}$.

A strategy-profile $\hat{a} \in A$ is a Nash equilibrium (in pure strategies) if for each $i \in I$ and each $a_i \in A_i$, $u_i(\hat{a}) \ge u_i(a_i, (\hat{a}_j)_{j \ne i})$. For each problem, the set of Nash equilibrium outcomes of the

simultaneous-move game induced by the constrained immediate acceptance mechanism coincides with the set of stable matchings (Haeringer and Klijn, 2009). Hence, while there is a unique stable matching and therefore a unique Nash equilibrium outcome in problem $\mathcal{P}_2$, there are two Nash equilibrium outcomes in each of the other two problems.

Let Δ_i denote the set of mixed strategies for player i. More specifically, an element of Δ_i is a probability distribution $p_i : A_i \to \mathbb{R}$, i.e., $p_i(a_{i1}) + p_i(a_{i2}) = 1$ and for each $a_{ij} \in A_i$, $p_i(a_{ij}) \geq 0$. Let $\Delta \equiv \times_{i \in I} \Delta_i$. The domain of the utility function u is extended from A to Δ by defining for each $p \in \Delta$,

$$u_i(p) \equiv \sum_{a=(a_1,a_2,a_3)\in A} \left[\prod_{i\in I} p_i(a_i)\right] \cdot u_i(a).$$

Since $p_i(a_{i1})+p_i(a_{i2}) = 1$, we can denote $p_i(a_{i1})$ by p_i and $p_i(a_{i2})$ by $1-p_i$, and thus interchangeably refer to an element of Δ_i by $(p_i, 1-p_i)$ or simply by p_i. For each $p = (p_1, p_2, p_3) \in \Delta$ and each $i \in I$, we let $p_{-i} \equiv (p_j)_{j\neq i}$.

A strategy-profile $\hat{p} \in \Delta$ is a Nash equilibrium in mixed strategies if for each $i \in I$ and each $p_i \in \Delta_i$, $u_i(\hat{p}) \geq u_i(p_i, \hat{p}_{-i})$. The set of Nash equilibria in mixed strategies is computed and depicted for each problem in Figure 4 of Appendix B. The resulting probability distributions over matchings are presented in Table 4.

| Problem	Treatment	$(\hat{p}_1, \hat{p}_2, \hat{p}_3)$	μ^I	μ^S	unstable
$\mathcal{P}_1$	L, H	$(1, 0, 0)$	1	0	0
	L, H	$(0, 0, 1)$	0	1	0
	L	$(0.3, 0, 0.7)$	0.09	0.49	0.42
	L	$(0, 0.3, 0.7)$	0	0.49	0.51
	H	$(0.7, 0, 0.3)$	0.49	0.09	0.42
	H	$(0, 0.7, 0.3)$	0	0.21	0.79
$\mathcal{P}_2$	L, H	$(\hat{p}_1, 0, 1)$ for each $\hat{p}_1 \in [0,1]$	1	1	0
$\mathcal{P}_3$	L, H	$(1, 1, 0)$	1	0	0
	L, H	$(1, 0, 1)$	0	1	0
	L	$(1, 0.3, 0.7)$	0.09	0.49	0.42
	H	$(1, 0.7, 0.3)$	0.49	0.09	0.42

Table 4: Probability distribution over matchings induced by Nash equilibria in mixed strategies $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$. Note: $x = 0.3$ in treatment L and $x = 0.7$ in treatment H. Recall that in $\mathcal{P}_2$, $\mu^I = \mu^S$.

Table 4 highlights that there are multiple Nash equilibrium outcomes in problems $\mathcal{P}_1$ and $\mathcal{P}_3$. The notion of quantal response equilibrium for normal form games, introduced by McKelvey and Palfrey (1995), can be used as an equilibrium selection criterion. Quantal response equilibria in its logit form are defined by means of a non-negative parameter λ that is inversely related to the players' error level. Given $\lambda \geq 0$, a strategy-profile $p^* \in \Delta$ is a logit quantal response equilibrium (logit-QRE) if for each $i \in I$ and each $a_{ij} \in A_i$,

$$p_i^*(a_{ij}) = \frac{e^{\lambda \cdot u_i(a_{ij}, p^*_{-i})}}{e^{\lambda \cdot u_i(a_{i1}, p^*_{-i})} + e^{\lambda \cdot u_i(a_{i2}, p^*_{-i})}}. \tag{1}$$

If $\lambda = 0$, then players choose uniformly at random. By Theorem 2 in McKelvey and Palfrey (1995), if $\lambda \to \infty$, then p^* converges to a Nash equilibrium in mixed strategies. We refer to this Nash equilibrium as the *limiting logit-QRE*.

In each problem, since each of the three players has two pure strategies, the system of equations (1) reduces to three equations with three unknowns, i.e., p_1^*, p_2^*, and p_3^* (the probabilities with which the students apply to school s_1). Given $\lambda \geq 0$, we indicate this logit-QRE probability for student i in problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$ of treatment $T \in \{L, H\}$ by $p_i^*(\lambda \,|\, \mathcal{P}, T)$. Figure 1 depicts the logit-QRE probabilities for all instances (combinations of problems and treatments) of our experiment.
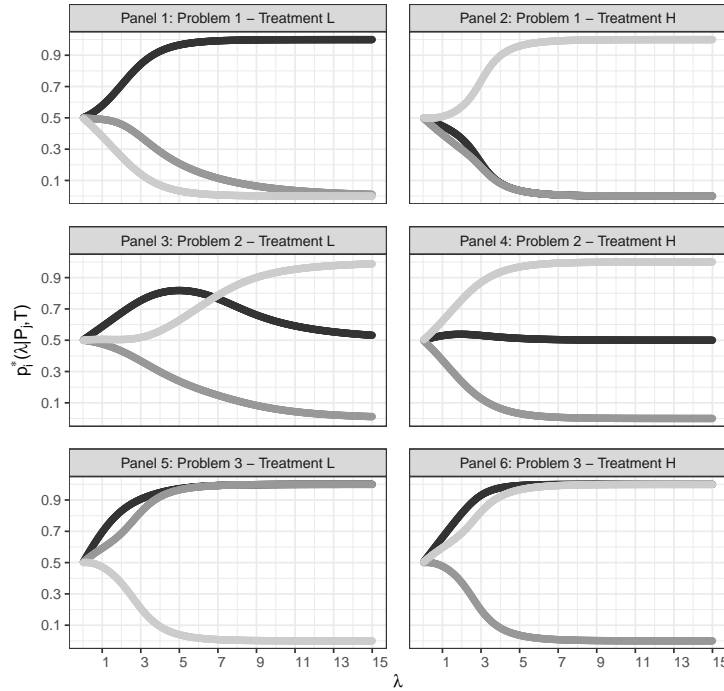
Figure 1: Logit-QRE probabilities. Color scheme: $(p_1^*, p_2^*, p_3^*) \to$ (black, dark-gray, light-gray).

Most importantly, in each of the problems $\mathcal{P}_1$ and $\mathcal{P}_3$, the limiting logit-QRE depends on the treatment. In particular, panel 1 of Figure 1 reveals that in $\mathcal{P}_1$ ($q_1 = 1$ and $q_2 = 2$) of treatment L, as λ grows large, the probability with which student 1 applies to school s_1 and students 2 and 3 apply to school s_2 approaches 1. So, all students are accepted with probability 1 for large λ and the resulting matching is (s_1, s_2, s_2), which corresponds to the student-optimal stable matching of $\mathcal{P}_1$. However, panel 2 shows that in $\mathcal{P}_1$ of treatment H, the resulting matching under the limiting logit-QRE is the school-optimal stable matching of $\mathcal{P}_1$, i.e., (s_2, s_2, s_1). A similar observation can be made for $\mathcal{P}_3$ ($q_1 = 2$ and $q_2 = 1$): for large λ, the resulting matching converges again to the student-optimal stable matching (s_1, s_1, s_2) in treatment L, while the school-optimal stable matching (s_1, s_2, s_1) is obtained in treatment H. Finally, while $(1, s_2, s_1)$ is the unique Nash equilibrium outcome in $\mathcal{P}_2$ ($q_1 = q_2 = 1$), Table 4 shows that it is sustained by an infinite number of Nash equilibria in mixed strategies. The probability distribution over matchings induced by the logit-QRE converges to the degenerate probability distribution where $(1, s_2, s_1)$ has probability 1.

This is because in the limiting logit-QRE of both treatments, the probability with which student 1 applies to school s_1 converges to 0.5, while the probability with which students 2 and 3 apply to schools s_2 and s_1, respectively, approaches 1. However, the trajectory of the logit-QRE clearly depends on the treatment.[9]

Figure 1 permits us to make point predictions and derive treatment effects regarding student behavior. *Point predictions* indicate (ranges of) probabilities that are consistent with logit-QRE behavior and analyze in particular whether a student is more likely to apply to school s_1 or to school s_2. Based on Figure 1 we see that for each $\lambda > 0$,

$\mathcal{P}_1:\ p_1^*(\lambda\,|\,\mathcal{P}_1,L) > \frac{1}{2} > p_1^*(\lambda\,|\,\mathcal{P}_1,H),\quad p_2^*(\lambda\,|\,\mathcal{P}_1,L), p_2^*(\lambda\,|\,\mathcal{P}_1,H) < \frac{1}{2},\quad \text{and } p_3^*(\lambda\,|\,\mathcal{P}_1,L) < \frac{1}{2} < p_3^*(\lambda\,|\,\mathcal{P}_1,H);$

$\mathcal{P}_2:\ \text{for each } T\in\{L,H\},\quad p_1^*(\lambda\,|\,\mathcal{P}_2,T) \geq \frac{1}{2},\quad p_2^*(\lambda\,|\,\mathcal{P}_2,T) < \frac{1}{2},\quad \text{and } p_3^*(\lambda\,|\,\mathcal{P}_2,T) > \frac{1}{2};$

$\mathcal{P}_3:\ p_1^*(\lambda\,|\,\mathcal{P}_3,L), p_1^*(\lambda\,|\,\mathcal{P}_3,H) > \frac{1}{2},\quad p_2^*(\lambda\,|\,\mathcal{P}_3,L) > \frac{1}{2} > p_2^*(\lambda\,|\,\mathcal{P}_3,H),\quad \text{and } p_3^*(\lambda\,|\,\mathcal{P}_3,L) < \frac{1}{2} < p_3^*(\lambda\,|\,\mathcal{P}_3,H).$

As a consequence we formulate the first part of our first hypothesis as follows.

Hypothesis 1.a (point predictions).

$\mathcal{P}_1:$ *In treatment L, student 1 is (students $\{2,3\}$ are) more (less) likely to apply to s_1 than to s_2. In treatment H, student 3 is (students $\{1,2\}$ are) more (less) likely to apply to s_1 than to s_2.*

$\mathcal{P}_2:$ *Student 1 is weakly more likely, student 2 is less likely, and student 3 is more likely to apply to s_1 than to s_2.*

$\mathcal{P}_3:$ *In treatment L, students $\{1,2\}$ are (student 3 is) more (less) likely to apply to s_1 than to s_2. In treatment H, students $\{1,3\}$ are (student 2 is) more (less) likely to apply to s_1 than to s_2.*

Treatment effects deal with comparative statics by studying how the probability with which a student applies to school s_1 is affected by the payoff $x \in \{0.3, 0.7\}$ (i.e., the two treatments L and H) that the student receives from being matched to her least preferred school. We distinguish between ambiguous and unambiguous effects. Given a problem and a student, the treatment effect is unambiguous if the logit-QRE probability in one treatment is always greater than in the other treatment, i.e., independently of the rationality parameter λ. Formally, given a problem $\mathcal{P}$ and a student i, the treatment effect is *unambiguous* if

$$\text{for all } \lambda, \lambda' > 0,\ p_i^*(\lambda\,|\,\mathcal{P},L) > p_i^*(\lambda'\,|\,\mathcal{P},H) \quad \text{or} \quad \text{for all } \lambda, \lambda' > 0,\ p_i^*(\lambda\,|\,\mathcal{P},L) < p_i^*(\lambda'\,|\,\mathcal{P},H).$$

We can observe from Figure 1 that there are four unambiguous effects.[10] Specifically, in $\mathcal{P}_1$, the probability with which student 1 (student 3) applies to s_1 in treatment L is always greater (smaller) than in treatment H. Similarly, in $\mathcal{P}_3$, the probability with which student 2 (student 3) applies to s_1 in treatment L is greater (smaller) than in treatment H.

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[9]The structure of the limiting logit-QRE in Figure 1 hinges on the fact that students submit a single application. We show in Appendix D that if students submit lists of length 2, then the limiting logit-QRE converges for all three problems of both treatments to the student-optimal stable matching. Thus, the robustness of our results cannot be analyzed with this alternative design.

[10]For all $\lambda, \lambda' > 0$, $p_1^*(\lambda\,|\,\mathcal{P}_1,L) > p_1^*(\lambda'\,|\,\mathcal{P}_1,H)$, $p_3^*(\lambda\,|\,\mathcal{P}_1,L) < p_3^*(\lambda'\,|\,\mathcal{P}_1,H)$, $p_2^*(\lambda\,|\,\mathcal{P}_3,L) > p_2^*(\lambda'\,|\,\mathcal{P}_3,H)$, and $p_3^*(\lambda\,|\,\mathcal{P}_3,L) < p_3^*(\lambda'\,|\,\mathcal{P}_3,H)$.

The treatment effect is *ambiguous* if the comparison depends on the actual values λ takes in the two treatments, i.e., if

there are $\lambda, \lambda', \tilde{\lambda}, \tilde{\lambda}' > 0$ such that $p_i^*(\lambda \,|\, \mathcal{P}, L) > p_i^*(\lambda' \,|\, \mathcal{P}, H)$ and $p_i^*(\tilde{\lambda} \,|\, \mathcal{P}, L) < p_i^*(\tilde{\lambda}' \,|\, \mathcal{P}, H)$.

Ambiguous treatment effects occur for student 2 in $\mathcal{P}_1$, all three students in $\mathcal{P}_2$, and student 1 in $\mathcal{P}_3$. We are able to make some further predictions in these cases if we assume that λ is treatment-independent. We first define

$$\delta_i(\lambda \,|\, \mathcal{P}) = p_i^*(\lambda \,|\, \mathcal{P}, L) - p_i^*(\lambda \,|\, \mathcal{P}, H),$$

i.e., the logit-QRE probability difference for student i between treatment L and treatment H in problem $\mathcal{P}$, as a function of $\lambda > 0$ (in both treatments). Figure 2 graphically presents this function for all students in all problems.
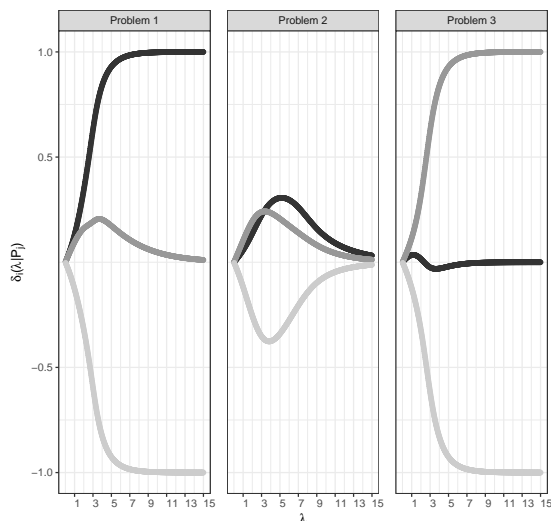
Figure 2: Logit-QRE probability difference $\delta_i(\lambda \,|\, \mathcal{P})$. Color scheme: (student 1, student 2, student 3) $\rightarrow$ (black, dark-gray, light-gray).

It can be observed that only for student 1 in $\mathcal{P}_3$ there are values $\lambda, \lambda' > 0$ such that $\delta_1(\lambda \,|\, \mathcal{P}_3) > 0$ and $\delta_1(\lambda' \,|\, \mathcal{P}_3) < 0$. So, for student 1 in $\mathcal{P}_3$ there is no clear treatment effect even under the additional assumption that λ is the same in both treatments. For the other four ambiguous effects we have that for all $\lambda > 0$, $\delta_2(\lambda \,|\, \mathcal{P}_1) > 0$, $\delta_1(\lambda \,|\, \mathcal{P}_2) > 0$, $\delta_2(\lambda \,|\, \mathcal{P}_2) > 0$, and $\delta_3(\lambda \,|\, \mathcal{P}_2) < 0$.

Hypothesis 1.b (treatment effects).

Unambiguous effects. Student 1 in $\mathcal{P}_1$ and student 2 in $\mathcal{P}_3$ apply to s_1 more often and student 3 in $\{\mathcal{P}_1, \mathcal{P}_3\}$ applies to s_1 less often in treatment L than in treatment H.

Ambiguous effects. Student 1 in $\mathcal{P}_2$ and student 2 in $\{\mathcal{P}_1, \mathcal{P}_2\}$ apply to s_1 more often and student 3 in $\mathcal{P}_2$ applies to s_1 less often in treatment L than in treatment H.

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Next, we derive predictions about the prevalence of the side-optimal stable matchings. Given λ, let $p^I(\lambda \,|\, \mathcal{P}, T)$ be the probability with which the student-optimal stable matching μ^I is obtained in problem $\mathcal{P}$ of treatment T. The probability $p^S(\lambda \,|\, \mathcal{P}, T)$ for the school-optimal stable matching is similarly defined. The four panels in the left part of Figure 3 depict for each problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_3\}$ and for each treatment $T \in \{L, H\}$, the trajectories of p^I and p^S as a function of λ. The two panels in the right part of Figure 3 depict the probability difference

$$d(\lambda \,|\, \mathcal{P}, T) \equiv p^I(\lambda \,|\, \mathcal{P}, T) - p^S(\lambda \,|\, \mathcal{P}, T).$$
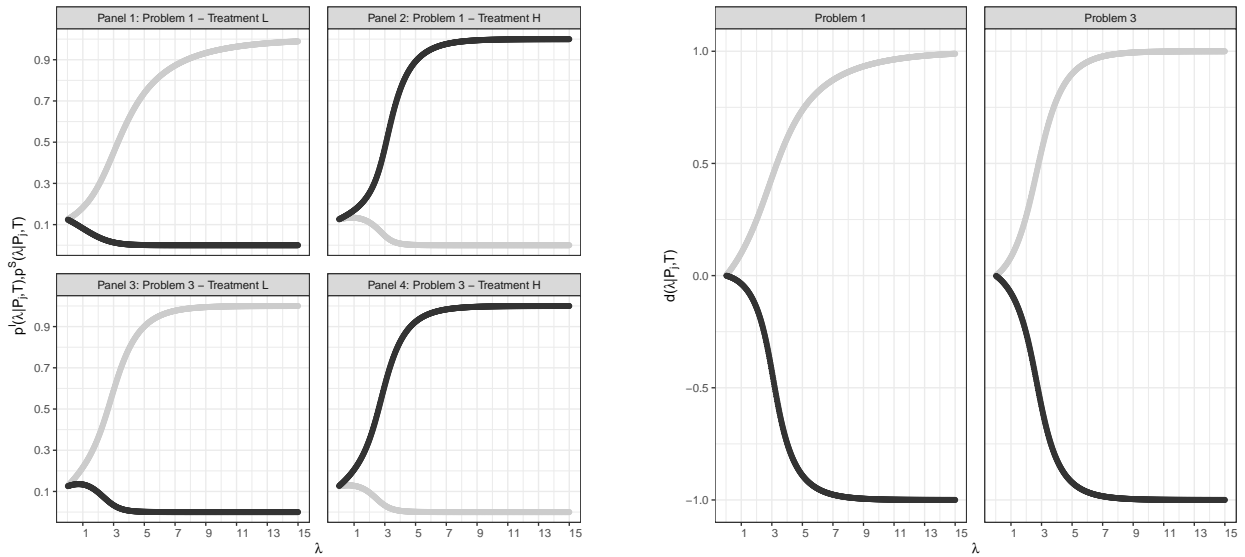
Figure 3: To the left: probabilities of the student-optimal stable matching $p^I(\lambda \,|\, \mathcal{P}, T)$ (light-gray) and the school-optimal stable matching $p^S(\lambda \,|\, \mathcal{P}, T)$ (black) under the logit-QRE in $\mathcal{P}_1$ and $\mathcal{P}_3$ for each of the two treatments. To the right: probability difference $d(\lambda \,|\, \mathcal{P}, T)$ in $\mathcal{P}_1$ and $\mathcal{P}_3$ for treatment $T = H$ (black) and treatment $T = L$ (light-gray).

In all four panels in the left part of Figure 3, if $\lambda = 0$, i.e, all three students apply to each of the two schools with probability $\frac{1}{2}$, then both μ^I and μ^S occur with the same probability (namely, $\frac{1}{8}$). Also, for each of the problems $\mathcal{P}_1$ and $\mathcal{P}_3$, p^I is strictly increasing over the whole domain of λ in treatment L but mostly decreases in λ in treatment H. The opposite happens for p^S: it mostly decreases in λ in treatment L but strictly increases over the whole domain of λ in treatment H. This is reflected in the right part of Figure 3 where it can be observed that in each problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_3\}$ and for all $\lambda > 0$, $d(\lambda \,|\, \mathcal{P}, L) > d(0 \,|\, \mathcal{P}, L) = 0 = d(0 \,|\, \mathcal{P}, H) > d(\lambda \,|\, \mathcal{P}, H)$. These inequalities allow us to formulate the following hypothesis.

Hypothesis 2 (side-optimal stable matchings). *In each $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_3\}$, it is more likely to obtain μ^I than μ^S in treatment L and it is less likely to obtain μ^I than μ^S in treatment H.*

3 Results

We next present the results of our experiment. First, we statistically test our two hypotheses in order to assess whether the logit-QRE makes sound directional predictions in our setting. Afterwards, we estimate λ.

Problem/Student		Treatment		
		L		H
	1	49% (0.600)	[0.062]	34% (0.263)
$\mathcal{P}_1$	2	18% (0.107)	[0.031]	8% (0.039)
	3	40% (0.424)	[0.024]	62% (0.243)
	1	72% (0.129)	[0.019]	62% (0.240)
$\mathcal{P}_2$	2	7% (0.032)	[0.069]	4% (0.012)
	3	84% (0.098)	[0.067]	94% (0.028)
	1	93% (0.006)		99% (0.001)
$\mathcal{P}_3$	2	57% (0.448)	[0.040]	34% (0.219)
	3	48% (0.551)	[0.025]	70% (0.209)

Table 5: Frequency that a student applies to s_1. In parenthesis [square brackets]: one-sided p-values of the Wilcoxon signed-rank [Mann-Whitney U] tests at the student level (pooled over all six rounds of a problem) corresponding to Hypothesis 1.

Problem/Treatment		Matching			
		μ^I		μ^S	unstable
$\mathcal{P}_1$	L	0.302	(0.353)	0.272	0.426
	H	0.197	(0.008)	0.451	0.352
$\mathcal{P}_2$	L		0.784		0.216
	H		0.901		0.099
$\mathcal{P}_3$	L	0.432	(0.363)	0.370	0.198
	H	0.241	(0.003)	0.599	0.160

Table 6: Probability distribution over matchings. In parenthesis: one-sided p-values of the Wilcoxon signed-rank tests at the group level (pooled over all six rounds of a problem) corresponding to Hypothesis 2.

The relevant data for the statistical analysis is displayed in Tables 5 and 6. Table 5 indicates the frequencies with which students apply to s_1 and, thus, analyzes how subjects behave in the different student roles. The corresponding predictions are in Hypothesis 1. We employ Wilcoxon signed-rank tests for the within-treatments comparisons (point predictions) and Mann-Whitney U tests for the between-treatments comparisons. Table 6 shows the probability distribution over matchings that we obtained in the experiment. We focus in Hypothesis 2 on the probability p^I that the student-optimal stable matching and the probability p^S that the school-optimal stable matching is reached. For this reason the probabilities of all other matchings are added up under the column "unstable."[11] For a visual overview of the per-round data and some additional statistical tests we refer to Appendix C.

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[11] Truly independent observations for all statistical tests are obtained by aggregating data over all six consecutive rounds in which a particular problem is played. As an example, consider the hypothesis for treatment L that the resulting matching in $\mathcal{P}_1$ is more likely to be μ^I than μ^S. For each of the six groups, we calculate the overall

Evidence on Hypothesis 1:

We show that the experimental data in Table 5 considerably supports Hypothesis 1.

Consider first $\mathcal{P}_1$. It is expected that in treatment L, student 1 is more and students 2 and 3 are less likely to apply to s_1 than to s_2. Also, in treatment H, student 3 is expected to be more and students 1 are 2 are expected to be less likely to apply to s_1 than to s_2. Only the data for student 1 in treatment L is outright against this prediction: the experimental frequency with which the student applies to s_1 is 49%, yet this frequency is expected to be greater than 50%. From a statistical point of view, only the comparison for student 2 in treatment H is significant at the 5% level (the one-sided p-value is 0.039). With respect to the treatment effects, students 1 and 2 are expected to be more and student 3 is expected to be less likely to apply to s_1 in treatment L than in treatment H. The experimental data goes in the correct direction. The one-sided p-values of the corresponding Mann-Whitney U tests are 0.062 for student 1, 0.031 for student 2, and 0.024 for student 3.

Consider next $\mathcal{P}_2$. It is expected that in both treatments, student 3 is more and student 2 is less likely to apply to s_1 than to s_2. It is evident from Table 5 that the experimental data is completely in line with these predictions even though the comparison for student 3 in treatment L is only significant at a one-sided p-value of 0.098. With respect to student 1, for whom the point prediction was expressed in a weak sense, the hypothesis that this student applies to school s_1 with probability $\frac{1}{2}$ cannot be rejected in either of the two treatments. The experimental data is also consistent with the ambiguous treatment effects in Hypothesis 1: Students 1 and 2 are more and student 3 is less likely to apply to s_1 in treatment L than in treatment H. The one-sided p-values of the corresponding Mann-Whitney U tests are 0.019 for student 1, 0.069 for student 2, and 0.067 for student 3.

Consider finally $\mathcal{P}_3$. It is expected that in treatment L, students 1 and 2 are more and student 3 is less likely to apply to s_1 than to s_2. Also, in treatment H, students 1 and 3 are expected to be more and student 2 is expected to be less likely to apply s_1 than to s_2. The experimental data is again consistent with these predictions. The comparisons for student 1 are in both treatments significant at the 1% level. Finally, student 2 (student 3) is supposed to apply to s_1 more (less) often in treatment L than in treatment H. The data confirms these predictions: the one-sided p-values of the Mann-Whitney U tests are 0.040 for student 2 and 0.025 for student 3. $\square$

A first observation from Table 6 is that even though our setting is highly stylized, it is not straightforward for subjects to coordinate in $\{\mathcal{P}_1, \mathcal{P}_3\}$ on a Nash equilibrium in pure strategies. A Nash equilibrium in pure strategies leads to a stable matching, however an unstable matching is reached in $\mathcal{P}_1$ in more than 35% and in $\mathcal{P}_3$ in more than 16% of the cases. Moreover, the probability distribution over matchings in these two problems does not coincide with the one induced by the non-degenerate mixed strategy Nash equilibria in Table 4. To see this, we employ Wilcoxon signed-rank tests at the group level to analyze whether the empirical distribution over

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empirical probabilities p^I and p^S with which μ^I and μ^S are obtained for this $\mathcal{P}_1$. For each group, we then take the difference $d = p^I - p^S$ between these empirical probabilities. Thus, we obtain for each group a sole number between -1 and 1. The associated Wilcoxon signed-rank test then analyzes whether the mean of these six numbers is significantly greater than 0.

matchings is different from the one that arises from the non-degenerate Nash equilibria in mixed strategies. The (truly independent) observation for a group is the probability with which a matching (μ^I, μ^S, or unstable) is reached over the course of the six rounds in which the problem is played. We find that in treatment L of $\mathcal{P}_1$, p^S is significantly different from 0.49 (two-sided $p = 0.0052$). Also, in treatment H of $\mathcal{P}_1$, p^S is significantly greater than 0.09 and 0.21 (one-sided $p \leq 0.0003$). And the probability of an unstable matching is significantly different from 0.42 in treatment L (two-sided $p < 0.0001$) and treatment H (two-sided $p = 0.0052$) of $\mathcal{P}_3$. Finally, in $\mathcal{P}_2$ the unique stable outcome is "only" obtained in 78% of the cases in treatment L and in 90% of the cases in treatment H. This hints at misplays, yet the per-round data at the bottom of Figure 5 in Appendix C provides evidence of learning and coordination effects because unstable matchings occur less often in later rounds of a problem.

Evidence on Hypothesis 2:
Hypothesis 2 states that for each problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_3\}$, it is more likely to obtain μ^I than μ^S in treatment L and it is less likely to obtain μ^I than μ^S in treatment H. We find for $\mathcal{P}_1$ that the probability difference between μ^I and μ^S is $0.302 - 0.272 = 0.030$ in treatment L and $0.197 - 0.451 = -0.254$ in treatment H. The one-sided p-values of the Wilcoxon signed-rank tests at the group level are 0.3528 in treatment L and 0.0078 in treatment H. Similar observations hold for problem $\mathcal{P}_3$: the probability difference between μ^I and μ^S is $0.432 - 0.370 = 0.062$ in treatment L (one-sided p=0.3625) and $0.241 - 0.599 = -0.358$ in treatment H (one-sided p=0.0033). We conclude that the experimental data is supportive of Hypothesis 2. $\square$

In the final part of this section, we estimate the logit-QRE via maximum likelihood. Suppose that a total of K subjects participate in a given treatment $T \in \{L, H\}$ in each student role i. Let $x_{l,i}^t$ be a particular observation for subject l in role i of round $t \in \{1, \ldots, 6\}$. We define $x_{l,i}^t = 1$ if student l in role i applies in round t to school s_1. Otherwise, $x_{l,i}^t = 0$. Under the logit-QRE, $p_i^*(\lambda)$ is the probability that $x_{l,i}^t = 1$ and $1 - p_i^*(\lambda)$ is the probability that $x_{l,i}^t = 0$. The joint likelihood of observing the data is then

$$L(\lambda) = \prod_{i=1}^{3} \prod_{l=1}^{K} \prod_{t=1}^{6} p_i^*(\lambda)^{x_{l,i}^t} \cdot (1 - p_i^*(\lambda))^{1 - x_{l,i}^t}.$$

The data can be pooled if it is assumed that subject behavior is time-independent, i.e., for all $t, t' \in \{1, \ldots, 6\}$, $x_{l,i}^t = x_{l,i}^{t'} \equiv x_{l,i}$.[12] Then,

$$\ln L(\lambda) = 6 \cdot \sum_{i=1}^{3} \sum_{l=1}^{K} x_{l,i} \cdot \ln(p_i^*(\lambda)) + (1 - x_{l,i}) \cdot \ln(1 - p_i^*(\lambda)).$$

Let $\tilde{p}_i = \sum_{l=1}^{K} x_{l,i}/K$ be the empirical probability from the experiment that subjects in student role i apply to school s_1. We finally obtain that

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[12]Time-independence is a restrictive condition, but we do not have sufficient data points to perform an estimation for each round.

$$\ln L(\lambda) = 6 \cdot K \cdot \sum_{i=1}^{3} \tilde{p}_i \cdot \ln(p_i^*(\lambda)) + (1 - \tilde{p}_i) \cdot \ln(1 - p_i^*(\lambda)).$$

In order to maximize this function, we calculate the equilibrium probabilities of the logit-QRE numerically on a fine grid —the unique model parameter λ is varied in steps of 0.01 between 0 and 10, which means that 1000 different values for λ are considered— and evaluate the objective function at these equilibrium values. For each of the 1000 estimations of λ we use a random sample that consists of 60% of the available data for each student.[13] This yields a distribution of estimates for λ and allows us to analyze treatment effects. We denote by $\widehat{\lambda}_j^T$ the mean of the estimated distribution for problem $\mathcal{P}_j$ of treatment T.

Table 7 presents the estimation results for three different models (utility functions). We first concentrate on the case when the subjects are risk-neutral expected-payoff maximizers (Model I) and the only parameter to be estimated is λ. The intuition of the logit-QRE is that larger values of λ imply that choices are closer to Nash equilibrium behavior. In this sense, λ measures the rationality of the observed behavior. In our experiment, $\mathcal{P}_2$ is arguably the simplest of the three problems. In this problem, the continuum of mixed strategy Nash equilibria leads to the same (stable) matching. In the other two problems, there are two stable matchings and a coordination problem arises because different Nash equilibria induce different stable (or even unstable) matchings. Our estimation results in Table 7 support this interpretation since in Model I, $\widehat{\lambda}$ is largest in $\mathcal{P}_2$ (for both treatments).[14] Furthermore, it is worth noting that in each of the problems $\mathcal{P}_1$ and $\mathcal{P}_3$, $\widehat{\lambda}$ is larger in treatment H than in treatment L. For a possible explanation, recall that for large λ, the logit-QRE converges to μ^S in treatment H and to μ^I in treatment L. At the pure strategy Nash equilibrium that induces μ^S, students apply to their "safety schools," i.e., they maximize the probability of being accepted. Yet, at the pure strategy Nash equilibrium that yields μ^I some students take the risk of remaining unmatched (if some of the other students deviates). This risk comes with the potential benefit of being matched to the good school. If the payoff of the (bad) safety school is close to the payoff of the good school, which is the case in treatment H, then a subject may prefer to send an application to her (bad) safety school, which would result in a relatively large λ. On the other hand, if there is a big payoff difference between the good and the (bad) safety school, which is the case in treatment L, the logit-QRE demands to make the risky choice but some subjects might still apply to their (bad) safety school due to risk preferences (see, e.g., Klijn, Pais, and Vorsatz, 2013[15]), which would result in a relatively small λ. As a consequence,

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[13]The log-likelihood function is strictly concave and we find an interior solution on the considered grid. Hence, even though the logit-QRE is defined for all $\lambda > 0$, there is no need to widen the grid.

[14]It might be surprising that $\widehat{\lambda}_2^L = 8.1266$ is substantially greater than $\widehat{\lambda}_2^H = 4.3436$ even though there are only minor treatment differences in Table 5. The logit-QRE trajectories in Figure 1 provide an explanation for this. According to the empirical data, there is a high likelihood that student 3 applies to school s_1. Yet, a high p_3^* requires a substantially greater λ in treatment L than in treatment H.

[15]Klijn, Pais, and Vorsatz (2013) show experimentally that the student-optimal stable mechanism is more robust to changes in the cardinal preference structure than the immediate acceptance mechanism and that subjects with a higher degree of risk aversion (measured through a Holt-Laury lottery task) are more likely to play a "protective strategy" under the student-optimal stable mechanism but not under the immediate acceptance mechanism.

one could expect to obtain larger estimates of λ in treatment H than in treatment L for both $\mathcal{P}_1$ and $\mathcal{P}_3$.

	$\mathcal{P}_1$		$\mathcal{P}_2$		$\mathcal{P}_3$	
	H	L	H	L	H	L
Model I						
λ	2.663	0.699	4.347	8.127	2.586	1.808
	(0.089)	(0.173)	(0.255)	(0.261)	(0.073)	(0.078)
Model II						
λ	1.309	0.404	0.917	0.989	2.566	1.794
	(0.024)	(0.107)	(0.048)	(0.007)	(0.071)	(0.078)
r	1.000	1.000	0.979	0.993	0.001	0.001
	(0.000)	(0.000)	(0.025)	(0.008)	(0.000)	(0.000)
Model III						
λ	4.764	1.390	4.346	2.280	11.372	1.798
	(0.097)	(0.093)	(0.258)	(0.104)	(2.597)	(0.078)
c	0.678	0.000	1.000	0.000	0.252	1.000
	(0.030)	(0.000)	(0.000)	(0.000)	(0.025)	(0.000)

Table 7: Maximum likelihood estimation results. Means and standard deviations (in parenthesis) are obtained via 1000 random samples with 60% of the observations each. For each problem, treatment effects are significant at two-sided $p < 0.0001$ (Mann-Whitney U test).

We address in Model II the question whether subjects' risk aversion indeed affects the estimation of λ by assuming a CARA utility function on expected payoffs.[16] Equation (1) then becomes

$$p_i^*(a_{ij}) = \frac{e^{\lambda \cdot [u_i(a_{ij}, p_{-i}^*)^r/(1-r)]}}{e^{\lambda \cdot [u_i(a_{i1}, p_{-i}^*)^r/(1-r)]} + e^{\lambda \cdot [u_i(a_{i2}, p_{-i}^*)^r/(1-r)]}},$$

where r is the Arrow-Pratt measure of risk aversion. If $r = 0$, subjects are risk-neutral; and if $r \to 1$, then $u_i(a_{ij}, p_{-i}^*)^r/(1-r)$ tends to $\ln(u_i(a_{ij}, p_{-i}^*))$.

Table 7 shows that $\widehat{r}$ is in all problems consistent across treatments, yet there are important differences between problems. Risk aversion is maximal in $\mathcal{P}_1$ and $\mathcal{P}_2$, while $\widehat{r} \to 0$ in $\mathcal{P}_3$. It is worth noting that player 1 has the dominant strategy $p_1 = 1$ in $\mathcal{P}_3$, which implies that the optimal behavior of this player is independent of her risk aversion. If one compares specifications under the premise that λ is exogenous and is "supposed to be" constant across treatments, Model II improves upon Model I. In Model I, the ratio $|\widehat{\lambda}_j^H/\widehat{\lambda}_j^L|$ is 3.81 in $\mathcal{P}_1$, 0.53 in $\mathcal{P}_2$, and 1.43 in $\mathcal{P}_3$. The ratios for Model II are closer to 1 than the ratios of Model I: 3.24 in $\mathcal{P}_1$, 0.92 in $\mathcal{P}_2$, and 1.43 in $\mathcal{P}_3$.

Finally, we study in Model III whether the systematic differences of the between-treatment estimates of λ in Model I are explained by subjects trying to coordinate on the student-optimal stable matching. For that it is assumed that the utility function is a linear combination of the expected payoffs and the probability that the student-optimal stable matching is reached. In particular,

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[16]We are very grateful to an anonymous referee for suggesting us to analyze the impact of risk preferences and preferences for the student-optimal stable matching on the estimates of λ.

$$p_i^*(a_{ij}) = \frac{e^{\lambda \cdot [c \cdot u_i(a_{ij}, p_{-i}^*) + (1-c) \cdot p(\mu^I)]}}{e^{\lambda \cdot [c \cdot u_i(a_{i1}, p_{-i}^*) + (1-c) \cdot p(\mu^I)]} + e^{\lambda \cdot [c \cdot u_i(a_{i2}, p_{-i}^*) + (1-c) \cdot p(\mu^I)]}},$$

where $c \in [0, 1]$. We find that the estimation outcome of this model is worse than that of Model II. First, $\widehat{c}$ varies substantially between treatments and between problems (and not only between problems as in Model II). And second, the ratio $|\widehat{\lambda}_j^H / \widehat{\lambda}_j^L|$ is 3.42 in $\mathcal{P}_1$, 1.90 in $\mathcal{P}_2$, and 6.32 in $\mathcal{P}_3$. For all problems, these ratios are further away from 1 than the ratios of Model II. One natural alternative to Model III is to assume that subjects have preferences for the expected group payoff instead of preferences for the student-optimal stable matching. We also estimate this alternative model and, perhaps surprisingly, no evidence of this type of preferences for efficiency is found. In all cases, $\widehat{c} = 1$.

4   Concluding remarks

An important part of the experimental literature on school choice focuses on comparing different centralized assignment mechanisms. The theoretical literature is sometimes able to provide very sharp predictions. For example, in an unconstrained setting in which subjects can rank all schools, both the student-optimal stable mechanism and the top trading cycles mechanism are strategy-proof, that is, in the induced simultaneous-move games subjects have an incentive to reveal their preferences truthfully. One can then use the truth-telling rates observed in a laboratory experiment to make inferences about the quality of the decisions. Since the immediate acceptance mechanism is manipulable, the comparison of subject behavior between the student-optimal stable mechanism and the immediate acceptance mechanism is not as straightforward, even in an unconstrained setting. Thus, comparing truth-telling rates between mechanisms is only useful in certain clearly defined instances. In general, there is a need to specify a behavioral model that permits the derivation of testable predictions and compare the quality of the decisions between treatments. The quantal response equilibrium is a natural tool in this respect.

With the help of a laboratory experiment we have analyzed the effects of changes in the cardinal payoffs (i.e., relative preference intensities) on subject behavior for three stylized school choice problems. Our main finding is that the logit-QRE correctly captures the qualitative changes of both subject behavior and the resulting matching. We note that the logit-QRE can also be used to derive predictions across mechanisms, which is a recurring topic in the experimental literature. Given the potential interest for future research, we briefly discuss an example. Consider the problem with three students (1, 2, and 3) and three schools (s_1, s_2, and s_3) in Table 8. Each school has 1 seat and gives a strictly positive payoff. Also assume that students submit lists of size 3.

	Preferences			Priorities		
	1	2	3	s_1	s_2	s_3
Best match:	s_1	s_2	s_3	2	3	1
Second best:	s_2	s_3	s_1	3	1	2
Third best:	s_3	s_1	s_2	1	2	3

Table 8: Preferences of students over schools and priorities of schools over students.

There are three stable matchings. In the student-optimal stable matching, each student is assigned to her best match. In the "median" stable matching, each student is assigned to her second best match. And in the school-optimal stable matching, each student is assigned to her worst match. The unconstrained student-optimal stable mechanism is strategy-proof, which implies that agents report their preferences truthfully in the limiting logit-QRE. Since students cannot remain unmatched under the immediate acceptance mechanism, the worst school must be ranked last (not doing so is a weakly dominated strategy). Whether students rank their best school or their second best school first in the limiting logit-QRE under the immediate acceptance mechanism is a function of the payoff structure. If the payoff of the best and the second best school are "close enough" to each other, the limiting logit-QRE is equal to the median stable matching. And if there is a "suficient" payoff difference for these two schools, the student-optimal stable matching is obtained.

Finally, part of the recent experimental literature on school choice studies mechanisms that incorporate concerns for affirmative action (i.e., Klijn, Pais, and Vorsatz, 2016; Kawagoe, Matsubae, and Takizawa, 2018), allow for information acquisition (Chen and He, 2021), or employ a dynamic assignment procedure (i.e., Klijn, Pais, and Vorsatz, 2019; Bó and Hakimov, 2020; and Dur, Hammond, and Kesten, 2021). Also, there are innovative experimental designs that analyze the roots of sub-optimal behavior in experimental matching markets (i.e., Guillen and Hakimov, 2017; Ding and Schotter, 2017; Dur, Hammond, and Morrill, 2018; and Guillen and Veszteg, 2019). It seems worthwhile to more closely study the predictive power of the quantal response equilibrium in these settings.

References

1. Abdulkadiroğlu, A. and T. Sönmez (2003). School choice: A mechanism design approach. *American Economic Review* 93(3): 729–747.

2. Abdulkadiroğlu, A., Y.-K. Che, and Y. Yasuda (2011). Resolving conflictive preferences in school choice: The Boston mechanism reconsidered. *American Economic Review* 101(1): 399–410.

3. Anderson, S.P., J.K. Goeree, and C.A. Holt (1998). Minimum-effort coordination games: An analysis of all-pay auctions. *Journal of Political Economy* 106(4): 828-853.

4. Anderson, S.P., J.K. Goeree, and C.A. Holt (2001). Rent seeking with bounded rationality: Stochastic potential and logit equilibrium. *Games and Economic Behavior* 34(2): 177-199.

5. Bó, I. and R. Hakimov (2020). Iterative versus standard deferred acceptance: Experimental evidence. *Economic Journal* 130(626): 356–392.

6. Bonkoungou, S. and A. Nesterov (2021). Comparing school choice and college admissions mechanisms by their strategic accessibility. *Theoretical Economics* 16(3): 881–909.

7. Calsamiglia, C., G. Haeringer, and F. Klijn (2010). Constrained school choice: An experimental study. *American Economic Review* 100(4): 1860–1874.

8. Capra, M.C., J.K. Goeree, R. Gomez, and C.A. Holt (1999). Anomalous behavior in a traveler's dilemma. *American Economic Review* 89(3): 678–690.

9. Chen, Y. and O. Kesten (2019). Chinese college admissions and school choice reforms: An experimental study. *Games and Economic Behavior* 115(1): 83–100.

10. Chen, Y. and T. Sönmez (2006). School choice: An experimental study. *Journal of Economic Theory* 127(1): 202–231.

11. Chen, Y. and Y. He (2021). Information acquisition and provision in school choice: An experimental study. *Journal of Economic Theory* 197: 105345.

12. Chen, Y., X. Su, and Z. Zhao (2012). Modeling bounded rationality in capacity allocation games with the quantal response equilibrium. *Management Science* 58(10): 1952–1962.

13. Ding, T. and A. Schotter (2019). Learning and mechanism design: An experimental test of school matching mechanisms with intergenerational advice. *Economic Journal* 129(623): 2779–2804.

14. Dreyfuss, B., O. Heffetz, and M. Rabin (2021). Expectations-based loss aversion may help explain seemingly dominated choices in strategy-proof mechanisms. *American Economic Journal: Microeconomics*, forthcoming.

15. Dur, U. (2019). The modified Boston mechanism. *Mathematical Social Sciences* 101: 31–40.

16. Dur, U. and T. Morrill (2020). What you don't know can help you in school assignment. *Games and Economic Behavior* 120: 246–256.

17. Dur, U, R.G. Hammond, and T. Morrill (2018). Identifying the harm of manipulable school-choice mechanisms. *American Economic Journal: Economic Policy* 10(1): 187–213.

18. Dur, U., R.G. Hammond, and O. Kesten (2021). Sequential school choice: Theory and evidence from the field. *Journal of Economic Theory* 198: 105344.

19. Echenique, F., A. Wilson, and L. Yariv (2016). Clearinghouse for two-sided matching: An experimental study. *Quantitative Economics* 7(2): 449–482.

20. Featherstone, C.R. and M. Niederle (2016). Boston versus deferred acceptance in an interim setting: An experimental investigation. *Games and Economic Behavior* 100(1): 353–375.

21. Fischbacher, U. (2007). Z-tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics* 10(2): 171–178.

22. Gale, D. and L.S. Shapley (1962). College admissions and the stability of marriage. *American Mathematical Monthly* 69(1): 9–15.

23. Goeree, J.K. and C.A. Holt (2000). Asymmetric inequality aversion and noisy behavior in alternating-offer bargaining games. *European Economic Review* 44(4): 1079–1089.

24. Goeree, J.K., C.A. Holt, and T.R. Palfrey (2002). Quantal response equilibrium and overbidding in private-value auctions. *Journal of Economic Theory* 104(1): 257–272.

25. Goeree, J.K., C.A. Holt, and T.R. Palfrey (2003). Risk averse behavior in generalized matching pennies games. *Games and Economic Behavior* 45(1): 97–113.

26. Goeree, J.K., C.A. Holt, and T.R. Palfrey (2005). Regular quantal response equilibrium. *Experimental Economics* 8: 347–367.

27. Guarnaschelli, S., R.D. McKelvey, and T.R. Palfrey (2000). An experimental study of jury decision rules. *American Political Science Review* 94(2): 407–423.

28. Guillen, P. and R. Hakimov (2017). Not quite the best response: truth-telling, strategy-proof matching, and the manipulation of others. *Experimental Economics* 20(3): 670–686.

29. Guillen, P. and R. Veszteg (2021). Strategy-proofness in experimental matching markets. *Experimental Economics* 24(2): 650–668.

30. Haeringer, G. and F. Klijn (2009). Constrained school choice. *Journal of Economic Theory* 144(5): 1921–1947.

31. Haile, P.A., A. Hortaçsu, and G. Kosenok (2008). On the empirical content of quantal response equilibrium. *American Economic Review* 98(1): 180–200.

32. Hakimov, R. and D. Kübler (2020). Experiments on centralized school choice and college admissions: A survey. *Experimental Economics* 24: 434–488.

33. Kawagoe, T., T. Matsubae, and H. Takizawa (2018). The skipping-down strategy and stability in school choice problems with affirmative action: Theory and experiment. *Games and Economic Behavior* 109: 212–239.

34. Klijn, F., J. Pais, and M. Vorsatz (2013). Preference intensities and risk aversion in school choice: A laboratory experiment. *Experimental Economics* 16: 1–23.

35. Klijn, F., J. Pais, and M. Vorsatz (2016). Affirmative action through minority reserves: An experimental study on school choice. *Economics Letters* 139: 72–75.

36. Klijn, F., J. Pais, and M. Vorsatz (2019). Static versus dynamic deferred acceptance in school choice: A laboratory experiment. *Games and Economic Behavior* 113: 147–163.

37. Kojima, F. and M.U. Ünver (2014). The "Boston" school-choice mechanism: An axiomatic approach. *Economic Theory* 55: 515–544.

38. Li, S. (2017). Obviously strategy-proof mechanisms. *American Economic Review* 107(11): 3257–3287.

39. McKelvey, R.D. and T.R. Palfrey (1995). Quantal response equilibria for normal form games. *Games and Economic Behavior* 10(1): 6–38.

40. Pais, J. and Á. Pintér (2008). School choice and information: An experimental study on matching mechanisms. *Games and Economic Behavior* 64(1): 303–328.

41. Pathak, P.A. and T. Sönmez (2013). School admissions reform in Chicago and England: Comparing mechanisms by their vulnerability to manipulation. *American Economic Review* 103(1): 80–106.

Appendix A: Instructions (translated from Spanish)

General Instructions

Dear participant, thank you for taking part in this experiment. The purpose of this session is to study how people make decisions. The session will last about 90 minutes. In addition to the 3 Euro show-up fee you can – depending on your decisions – earn some more money. In order to ensure that the experiment takes place in an optimal setting, we would like to ask you to abide to the following rules during the whole experiment:

- Please, do not communicate with other participants!

- Do not forget to switch off your mobile phone!

- Read the instructions carefully. If something is unclear or if you have any question now or at any time during the experiment, please ask one of the experimenters. However, do not ask out loud, raise your hand instead. We will answer questions privately.

If you do not obey the rules, the data becomes useless for us and in this case we will have to exclude you from this experiment and you will not receive any monetary compensation. Payoffs during the experiment are expressed in ECU (experimental currency units). At the end of the session you will receive 1 Euro for each ECU obtained in the course of the experiment.

Description

The basic decision environment in the experiment is as follows. There are three students —let us call them E_1, E_2, and E_3— that can be assigned to a school. There are two schools —denoted C_1 and C_2— and each school can have 1 or 2 available seats (we will specify this later on in these instructions).

Since the schools differ in their location and quality, students have different opinions regarding which school they would like to attend. The desirability of schools in terms of location and quality is expressed in a table such as the one reproduced below. Note: this is an illustrative *example* and hence any table you will see later in the experiment might be different.

	E_1	E_2	E_3
Most preferred school	C_2	C_1	C_1
Least preferred school	C_1	C_2	C_2

Table 9: Preferences of students E_1, E_2 and E_3 over schools

Each column gives the preferences of a particular student. Consider the column that is marked E_3. This column gives the preferences of student E_3. and tells us that he/she would most of all like to obtain a seat in school C_1. Therefore, the least preferred school of student E_3 is C_2. Finally,

not obtaining a seat at any of the schools is the worst possible outcome. The columns of E_1 and E_2 have similar interpretations.

Some students have already a brother or sister attending one of the schools. Also, the students differ in walking distance to the schools. The authorities use these and other factors to determine the schools' priorities over the students. Each school has a priority ordering where all students are ranked. The priority orderings of the schools can be summarized in a table such as the one below. Note: this is an illustrative *example* and hence any table you will see later in the experiment might be different.

	C_1	C_2
Highest priority student	E_1	E_2
Second highest priority school	E_2	E_3
Lowest priority school	E_3	E_1

Table 10: Priority orderings of schools C_1 and C_2

Each column gives the priority ordering of a particular school. Consider the column that is marked C_2. This column gives the priority ordering of school C_2 and tells us that this school gives the highest priority to receiving student E_2. If this is not possible, then school C_2 gives priority to student E_3 to be enrolled. The lowest priority student of school C_2 is student E_1. The column of C_1 has similar interpretations.

The Matching Procedure

To decide if and how students are assigned to schools, the following procedure is followed. It consists of two phases.

Phase 1. Students are asked to simultaneously and independently send an application to one school. For instance, it can happen that the students apply to schools as described by the table below. Here each column shows the application of a student.

	E_1	E_2	E_3
School applied to	C_2	C_2	C_1

Table 11: Applications of student E_1, E_2, and E_3

Note: this is an illustrative *example* of three applications. Each student is free to apply to the school that he/she thinks is appropriate. The application does not necessarily have to coincide with the most preferred. In fact, in our example students E_1 and E_3 apply to their most preferred school, but this is not the case for student E_2.

Phase 2. The students' *applications* together with the schools' priority orderings determine an assignment of students to schools in the following way.

- **Step 1:** Each school that has received at least one application accepts the application from the student with the highest priority (among all received applications). If a school with two seats has received at least two applications, then it also accepts the application from the student with the second highest priority (among all received applications). All other applications (if any) are rejected.

- **Step 2:** Each student is assigned to the school that she applied to provided that the school accepted her application. If the application of a student was rejected, then the student remains without a seat.

The Experiment

In the beginning of the experiment, the computer randomly divides the participants into groups of 3. The assignment process is random and anonymous, so no participant will know who is in which group. Then, each participant in a group gets randomly assigned the role of a student in such a way that one group member will be in the role of student E_1, another group member will be in the role of student E_2, and a third member will be in the role of student E_3.

You will play the basic decision situation explained above 18 times in total. The composition of the group and the roles of the participants within each group do not change over the course of the experiment (for example, if you are assigned the role of student E_2, then this will be your role until the end of the experiment; also, you will always be playing with the same participant in role E_1 and with the same participant in role E_3). Every 6 rounds the numbers of seats school offer change. The first table with student preferences and the second table with priority orderings of schools remain the same in all 18 rounds of the experiment.

In each of the 18 rounds, payoffs are such that you receive **1 ECU** if you end up at the **school you prefer most**, x **ECU** if you are assigned to your **second most preferred school**, and **0 ECU** if you end up **unassigned**. At the end of the experiment, we will sum up your payoffs over the 18 rounds. Your final payoff will be equivalent of the sum of the per-round ECUs and the 3 Euro show-up fee.

The first thing you will see when the computer program starts is an illustrative example. Then, there will be one trial round that does not count for your final payoff so that you can familiarize yourself with the computer program. Afterwards, the first of the 18 rounds that count for payment starts.

Note: in the experimental sessions of treatment H, the parameter x took the value 0.7*; in the experimental sessions of treatment L, the parameter x took the value* 0.3*.*

Appendix B: Nash equilibria

Let x be the per-round payoff for obtaining a seat at the second most preferred school, i.e., $x = 0.3$ in treatment L and $x = 0.7$ in treatment H. Recall that a mixed strategy of player/student i is completely described by the probability $p_i \in [0, 1]$ with which the student applies to school s_1 (so, $1 - p_i$ is the probability with which student i applies to school s_2).

The set of NE in mixed strategies of problem $\mathcal{P}_1$

One can easily verify that the normal-form game is as follows:

$1\backslash 2$	s_1	s_2
s_1	$0, 0, \boldsymbol{x}$	$0, \boldsymbol{x}, x$
s_2	$\boldsymbol{x}, 0, x$	$\boldsymbol{x}, \boldsymbol{x}, \boldsymbol{x}$

Table 12: Player 3 plays s_1

$1\backslash 2$	s_1	s_2
s_1	$\boldsymbol{1}, 0, \boldsymbol{x}$	$\boldsymbol{1}, \boldsymbol{x}, 1$
s_2	$x, \boldsymbol{1}, 1$	$x, x, 0$

Table 13: Player 3 plays s_2

Thus, the best response correspondences of the three students are as follows:[17]

$$br_1(p_2, p_3) = \begin{cases} 1 & \text{if} \quad 1 - p_3 > x \\ [0,1] & \text{if} \quad 1 - p_3 = x \\ 0 & \text{if} \quad 1 - p_3 < x \end{cases}$$

$$br_2(p_1, p_3) = \begin{cases} 1 & \text{if} \quad (1 - p_1)(1 - p_3) > x \\ [0,1] & \text{if} \quad (1 - p_1)(1 - p_3) = x \\ 0 & \text{if} \quad (1 - p_1)(1 - p_3) < x \end{cases}$$

$$br_3(p_1, p_2) = \begin{cases} 1 & \text{if} \quad x > p_1 + p_2 - p_1 p_2 \\ [0,1] & \text{if} \quad x = p_1 + p_2 - p_1 p_2 \\ 0 & \text{if} \quad x < p_1 + p_2 - p_1 p_2 \end{cases}$$

We compute the set of Nash equilibria by checking the three Cases I, II, and III below. Let (p_1, p_2, p_3) be a Nash equilibrium.

I: $1 - p_3 > x$. It follows from br_1 that $p_1 = 1$. Since $p_1 = 1$, $(1 - p_1)(1 - p_3) = 0 < x$. Therefore, $p_2 = 0$ by br_2. Hence, $x < 1 = p_1 + p_2 - p_1 p_2$. So, $p_3 = 0$ by br_3. Now one easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\mathbf{1, 0, 0})$ is indeed a Nash equilibrium.

II: $1 - p_3 < x$. It follows from br_1 that $p_1 = 0$. Since $p_1 = 0$, $(1 - p_1)(1 - p_3) = 1 - p_3 < x$. Therefore, $p_2 = 0$ by br_2. Hence, $x > 0 = p_1 + p_2 - p_1 p_2$. So, $p_3 = 1$ by br_3. Now one easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\mathbf{0, 0, 1})$ is indeed a Nash equilibrium.

III: $1 - p_3 = x$. We distinguish between two subcases.

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[17]For instance, given strategies (p_2, p_3) of students 2 and 3, the expected utility of student 1 from applying to s_1 is $1 \times (1 - p_3)$ because s_1 has only one seat and only student 3 has higher priority for the school than student 1. Similarly, applying to s_2 yields the expected utility $x \times 1$, because s_2 has two seats and student 1 has the second highest priority for the school (so, applying to the school guarantees entrance).

- Subcase $0 < p_1 \leq 1$. Then, since $x > 0$, $(1 - p_1)(1 - p_3) < x$. Then, $p_2 = 0$ by br_2. Since $p_3 = 1 - x \in \{0.3, 0.7\}$ and $p_3 = br_3(p_1, p_2)$, we have $x = p_1 + p_2 - p_1 p_2 = p_1$. Now one easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{x}, \boldsymbol{0}, \boldsymbol{1} - \boldsymbol{x})$ is indeed a Nash equilibrium.

- Subcase $p_1 = 0$. Then, $(1 - p_1)(1 - p_3) = x$. Since $p_3 = 1 - x \in \{0.3, 0.7\}$ and $p_3 = br_3(p_1, p_2)$, we have $x = p_1 + p_2 - p_1 p_2 = p_2$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{0}, \boldsymbol{x}, \boldsymbol{1} - \boldsymbol{x})$ is indeed a Nash equilibrium.

The set of NE in mixed strategies of problem $\mathcal{P}_2$

One can easily verify that the normal-form game is as follows:

1\2	s_1	s_2
s_1	$0, 0, \boldsymbol{x}$	$\boldsymbol{0, x, x}$
s_2	$\boldsymbol{x}, 0, \boldsymbol{x}$	$\boldsymbol{0, x, x}$

Table 14: Player 3 plays s_1

1\2	s_1	s_2
s_1	$\boldsymbol{1}, 0, \boldsymbol{x}$	$\boldsymbol{1, x}, 0$
s_2	$x, \boldsymbol{1}, 0$	$0, x, 0$

Table 15: Player 3 plays s_2

Thus, the best response correspondences are as follows:

$$br_1(p_2, p_3) = \begin{cases} 1 & \text{if } 1 - p_3 > p_2 x \\ [0,1] & \text{if } 1 - p_3 = p_2 x \\ 0 & \text{if } 1 - p_3 < p_2 x \end{cases}$$

$$br_2(p_1, p_3) = \begin{cases} 1 & \text{if } (1 - p_1)(1 - p_3) > x \\ [0,1] & \text{if } (1 - p_1)(1 - p_3) = x \\ 0 & \text{if } (1 - p_1)(1 - p_3) < x \end{cases}$$

$$br_3(p_1, p_2) = \begin{cases} 1 & \text{if } x > p_1 p_2 \\ [0,1] & \text{if } x = p_1 p_2 \\ 0 & \text{if } x < p_1 p_2 \end{cases}$$

We compute the set of Nash equilibria by checking the three cases I, II, and III below. Let (p_1, p_2, p_3) be a Nash equilibrium.

I: $1 - p_3 > p_2 x$. It follows from br_1 that $p_1 = 1$. Since $p_1 = 1$, $(1 - p_1)(1 - p_3) = 0 < x$. Therefore, $p_2 = 0$ by br_2. Hence, $x > 0 = p_1 p_2$. So, $p_3 = 1$ by br_3. But then $0 = 1 - p_3 > p_2 x = 0$. Hence, there is no Nash equilibrium in this case.

II: $1 - p_3 < p_2 x$. It follows from br_1 that $p_1 = 0$. Since $p_1 = 0$, $(1 - p_1)(1 - p_3) = 1 - p_3 < p_2 x \leq x$. Therefore, $p_2 = 0$ by br_2. Hence, $x > 0 = p_1 p_2$. So, $p_3 = 1$ by br_3. But then $0 = 1 - p_3 < p_2 x = 0$. Hence, there is no Nash equilibrium in this case.

III: $1 - p_3 = p_2 x$. We distinguish among three subcases.

- Subcase $(1 - p_1)(1 - p_3) > x$. Then, $p_2 = 1$ by br_2. Thus, $1 - p_3 = x$. From $(1 - p_1)(1 - p_3) > x$ and $1 - p_3 = x$ we obtain $(1 - p_1)x > x$, which, by $x > 0$, is equivalent to $p_1 < 0$. Hence, there is no Nash equilibrium in this subcase.

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- Subcase $(1 - p_1)(1 - p_3) = x$. Substituting $1 - p_3 = p_2 x$ yields $(1 - p_1)p_2 x = x$ or equivalently (because $x \neq 0$), $(1 - p_1)p_2 = 1$. Then, since $p_1, p_2 \in [0,1]$, $p_1 = 0$ and $p_2 = 1$. Since $1 - p_3 = p_2 x$, $p_3 = 1 - x < 1$. However, from $x > 0 = p_1 p_2$ and br_3 it follows that $p_3 = 1$. Hence, there is no Nash equilibrium in this subcase.

- Subcase $(1 - p_1)(1 - p_3) < x$. Then, $p_2 = 0$ by br_2. From $1 - p_3 = p_2 x$, $p_3 = 1$. Now one easily verifies that for **each $p_1 \in [0, 1]$** the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{p_1, 0, 1})$ is indeed a Nash equilibrium.

The set of NE in mixed strategies of problem $\mathcal{P}_3$

One can easily verify that the normal-form game is as follows:

$1\backslash2$	s_1	s_2
s_1	$\boldsymbol{1, 0}, x$	$\boldsymbol{1, x, 1}$
s_2	$x, \boldsymbol{1}, \boldsymbol{x}$	$0, x, \boldsymbol{x}$

Table 16: Player 3 plays s_1

| $1\backslash2$	s_1	s_2
s_1	$\boldsymbol{1, 1, 1}$	$\boldsymbol{1}, x, 0$
s_2	$x, \boldsymbol{1}, 0$	$0, x, 0$

Table 17: Player 3 plays s_2

In particular, player 1 has the dominant strategy $p_1 = 1$. The best response correspondences of the other two players are then as follows:

$$
br_2(1, p_3) = \begin{cases} 1 & \text{if } 1 - p_3 > x \\ [0,1] & \text{if } 1 - p_3 = x \\ 0 & \text{if } 1 - p_3 < x \end{cases}
$$

$$
br_3(1, p_2) = \begin{cases} 1 & \text{if } x > p_2 \\ [0,1] & \text{if } x = p_2 \\ 0 & \text{if } x < p_2 \end{cases}
$$

We compute the set of Nash equilibria by checking the three cases I, II, and III below. Let (p_1, p_2, p_3) be a Nash equilibrium.

I: $1 - p_3 > x$. It follows from br_2 that $p_2 = 1$. Then, from br_3 and $x < 1 = p_2$, $p_3 = 0$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{1, 1, 0})$ is indeed a Nash equilibrium.

II: $1 - p_3 < x$. It follows from br_2 that $p_2 = 0$. Then, from br_3 and $x > 0 = p_2$, $p_3 = 1$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{1, 0, 1})$ is indeed a Nash equilibrium.

III: $1 - p_3 = x$. Since $p_3 = 1 - x \in \{0.3, 0.7\}$ and $p_3 = br_3(1, p_2)$, $x = p_1 p_2 = p_2$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{1, x, 1 - x})$ is indeed a Nash equilibrium.
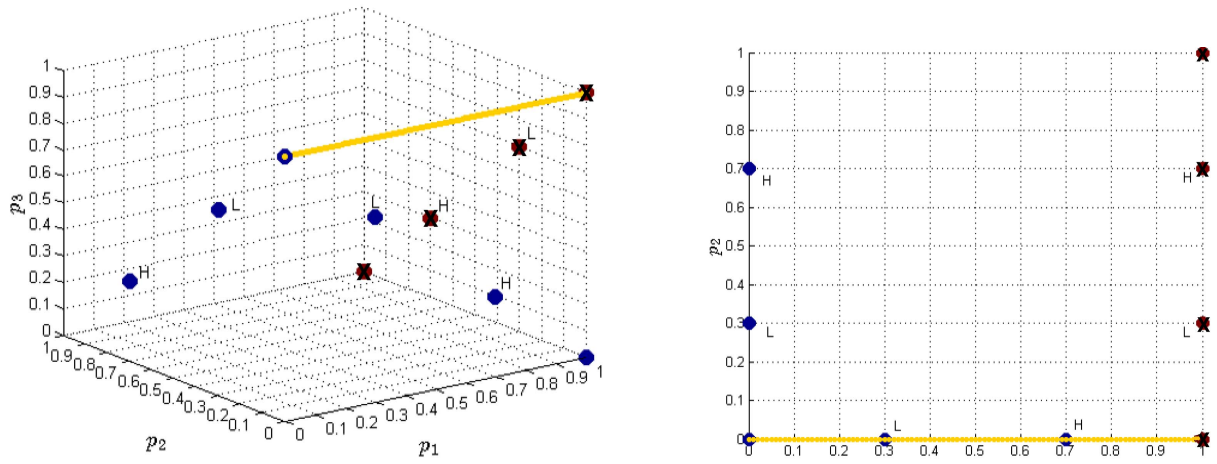
Figure 4: The set of Nash equilibria in mixed strategies for problem $\mathcal{P}_1$ (blue dots), for problem $\mathcal{P}_2$ (yellow line segment), and for problem $\mathcal{P}_3$ (red dots with cross). The strategy-profiles labeled L (H) are only Nash equilibria for treatment L (H).

Appendix C: Disaggregated data
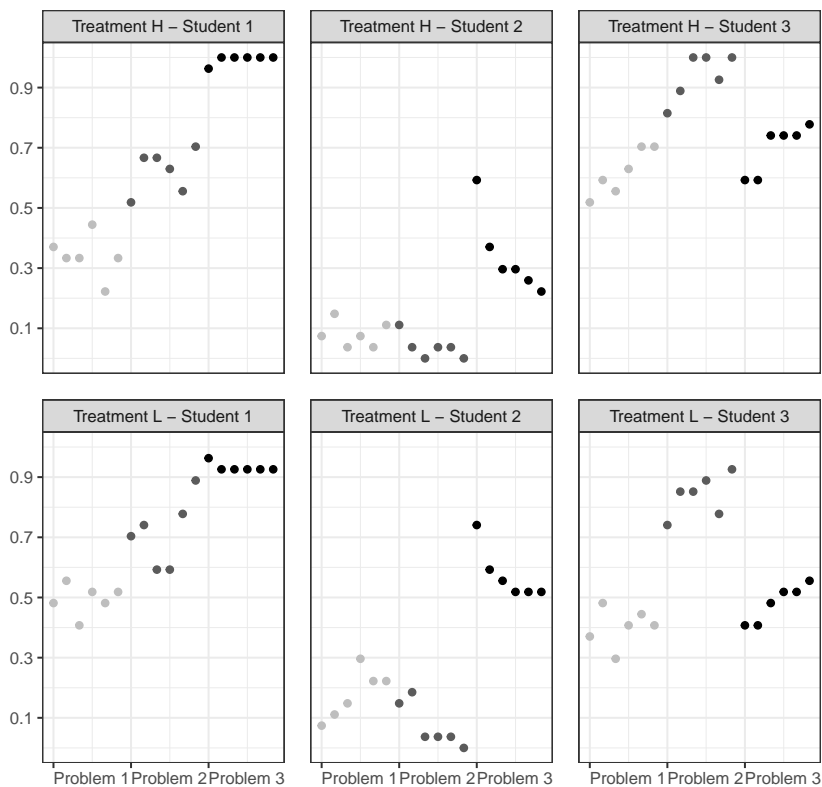
Figure 5: Probability that a student applies to school s_1 in a given round.

Figure 5 indicates how subject behavior changes over rounds. One natural hypothesis regarding learning is that repetitions bring subject behavior closer to the limiting logit-QRE. To statistically analyze this hypothesis, we divide the six rounds in which a particular problem is played into two subsets of three rounds each, namely the first three rounds and the last three rounds. Table 18 shows that all differences that are significant at the 5% level (4 comparisons out of a total of 18) support this hypothesis.

		Treatment L				Treatment H			
Problem	Student	First		Last	QRE	First		Last	QRE
	1	48%	(0.3998)	51%	100%	35%	(0.3199)	33%	0%
$\mathcal{P}_1$	2	11%	(0.1053)	25%	0%	9%	(0.7575)	7%	0%
	3	38%	(0.4014)	42%	0%	**56%**	**(0.0364)**	**68%**	**100%**
	1	68%	(0.0766)	75%	50%	62%	(0.3797)	63%	50%
$\mathcal{P}_2$	2	**12%**	**(0.0094)**	**2%**	**0%**	5%	(0.1999)	2%	0%
	3	81%	(0.1375)	86%	100%	**90%**	**(0.0193)**	**98%**	**100%**
	1	94%	(0.4924)	93%	100%	99%	(0.1678)	100%	100%
$\mathcal{P}_3$	2	63%	(0.1896)	52%	100%	**42%**	**(0.0124)**	**26%**	**0%**
	3	43%	(0.2513)	53%	0%	64%	(0.1461)	75%	100%

Table 18: Frequency that a student applies to school s_1 in the first three rounds (column "First") and in the last three rounds (column "Last") in which a problem is played. The limiting logit-QRE strategy is indicated in column "QRE." One-sided p-values of the Mann-Whitney U tests at the student level are presented in parenthesis. Significant differences at the 5%-level are indicated in boldface.

Figure 6 shows the distributions over matchings per round.
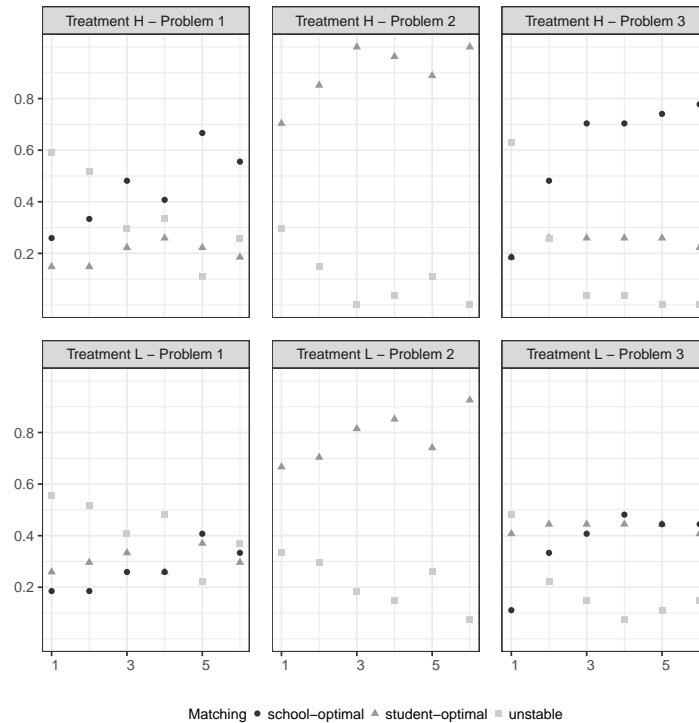
Figure 6: Probability distribution over matchings per round.

Table 19 analyzes the hypothesis that repetitions help reaching a stable matching. We divide the six rounds in which a particular problem is played again into two subsets of three rounds each (the

first three and the last three rounds, respectively). We observe that in all six instances in Table 19, the frequency of reaching an unstable matching is substantially lower in the last three than in the first three rounds. The effect is significant at the 5%-level in five out of the six instances.

Problem	Treatment L			Treatment H		
	First		Last	First		Last
$\mathcal{P}_1$	49%	(0.0550)	35%	47%	(0.0011)	23%
$\mathcal{P}_2$	27%	(0.0287)	16%	15%	(0.0094)	5%
$\mathcal{P}_3$	28%	(0.0074)	11%	30%	(0.0000)	1%

Table 19: Frequency of unstable matchings in the first three rounds (column "First") and in the last three rounds (column "Last") in which a problem is played. One-sided p-values of the Mann-Whitney U tests at the group level are presented in parenthesis.

Appendix D: Lists of length 2

We analyze the robustness of the theoretical results for the immediate acceptance and the student-optimal stable mechanism, respectively, when students submit lists of length 2. Note that the two mechanisms do not coincide in general in this setting.[18] For each problem $\mathcal{P} \in \{\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3\}$, $\mathcal{P}^{IA}$ and $\mathcal{P}^{DA}$ refer to problem $\mathcal{P}$ under the immediate acceptance and the student-optimal stable mechanism, respectively. A mixed strategy of player/student i is completely described by the probability $p_i \in [0,1]$ with which the student uses strategy s_1, s_2 (so, $1 - p_i$ is the probability with which student i uses strategy s_2, s_1). Again, x is the per-round payoff for obtaining a seat at the second most preferred school, i.e., $x = 0.3$ in treatment L and $x = 0.7$ in treatment H. Table 26 shows the probability distribution of matchings induced by the Nash equilibria. There are neither differences between treatments nor between mechanisms. Our most important findings are the logit-QRE predictions in Figure 7. Together with Table 26 they establish that in both treatments, the student-optimal stable matching is obtained in the limiting logit-QRE. Therefore, we expect that a similar experiment applied to this alternative setting would yield results that are different from our study.

The set of NE in mixed strategies of problems $\mathcal{P}_1^{IA}$ and $\mathcal{P}_1^{DA}$

One can easily verify that the normal-form game is as follows:

1\2	s_1, s_2	s_2, s_1
s_1, s_2	$\boldsymbol{x}, \boldsymbol{x}, x$	$\boldsymbol{x}, \boldsymbol{x}, x$
s_2, s_1	$\boldsymbol{x}, \boldsymbol{x}, x$	$\boldsymbol{x}, \boldsymbol{x}, \boldsymbol{x}$

Table 20: Player 3 plays s_1, s_2

1\2	s_1, s_2	s_2, s_1
s_1, s_2	$\boldsymbol{1}, \boldsymbol{x}, \boldsymbol{1}$	$\boldsymbol{1}, \boldsymbol{x}, \boldsymbol{1}$
s_2, s_1	$x, \boldsymbol{1}, \boldsymbol{1}$	$x, x, \boldsymbol{x}$

Table 21: Player 3 plays s_2, s_1

Thus, the best response correspondences are as follows:

$$br_1(p_2, p_3) = \begin{cases} 1 & \text{if } p_3 \neq 1 \\ [0,1] & \text{if } p_3 = 1 \end{cases}$$

$$br_2(p_1, p_3) = \begin{cases} 1 & \text{if } p_1 \neq 1 \text{ and } p_3 \neq 1 \\ [0,1] & \text{if } p_1 = 1 \text{ or } p_3 = 1 \end{cases}$$

$$br_3(p_1, p_2) = \begin{cases} 0 & \text{if } p_1 \neq 0 \text{ or } p_2 \neq 0 \\ [0,1] & \text{if } p_1 = 0 \text{ and } p_2 = 0 \end{cases}$$

We compute the set of Nash equilibria by checking the two cases I and II below. Let (p_1, p_2, p_3) be a Nash equilibrium.

I: $p_3 \neq 1$. From br_1, $p_1 = 1$. Then, from br_2 and br_3, $p_2 \in [0,1]$ and $p_3 = 0$, respectively. One easily verifies that for **each $\boldsymbol{p_2} \in [\boldsymbol{0,1}]$**, the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{1}, \boldsymbol{p_2}, \boldsymbol{0})$ is indeed a Nash equilibrium.

---

[18]For example, Pathak and Sönmez (2013) and Bonkoungou and Nesterov (2020) theoretically compare the strategic manipulability of several mechanisms, including the immediate acceptance mechanism and the student-optimal stable mechanism.

II: $p_3 = 1$. Then, from br_3, $p_1 = p_2 = 0$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = \mathbf{(0, 0, 1)}$ is indeed a Nash equilibrium.

The set of NE in mixed strategies of problems $\mathcal{P}_2^{IA}$

One can easily verify that the normal-form game is as follows:

1\2	s_1, s_2	s_2, s_1
s_1, s_2	$0, \boldsymbol{x}, x$	$\mathbf{0}, \boldsymbol{x}, \boldsymbol{x}$
s_2, s_1	$\boldsymbol{x}, 0, \boldsymbol{x}$	$\mathbf{0}, \boldsymbol{x}, \boldsymbol{x}$

Table 22: Player 3 plays s_1, s_2

1\2	s_1, s_2	s_2, s_1
s_1, s_2	$\mathbf{1}, 0, \mathbf{1}$	$\mathbf{1}, \boldsymbol{x}, 0$
s_2, s_1	$x, \mathbf{1}, 0$	$0, x, \boldsymbol{x}$

Table 23: Player 3 plays s_2, s_1

Thus, the best response correspondences are as follows:

$$
br_1(p_2, p_3) = \begin{cases} 1 & \text{if } 1 - p_3 > xp_2 \\ [0,1] & \text{if } 1 - p_3 = xp_2 \\ 0 & \text{if } 1 - p_3 < xp_2 \end{cases}
$$

$$
br_2(p_1, p_3) = \begin{cases} 1 & \text{if } (x+1)p_1p_3 > x - 1 + p_1 + p_3 \\ [0,1] & \text{if } (x+1)p_1p_3 = x - 1 + p_1 + p_3 \\ 0 & \text{if } (x+1)p_1p_3 < x - 1 + p_1 + p_3 \end{cases}
$$

$$
br_3(p_1, p_2) = \begin{cases} 1 & \text{if } x(p_1 + p_2) > (1+x)p_1p_2 \\ [0,1] & \text{if } x(p_1 + p_2) = (1+x)p_1p_2 \\ 0 & \text{if } x(p_1 + p_2) < (1+x)p_1p_2 \end{cases}
$$

We compute the set of Nash equilibria by checking the three Cases I, II, and III below. Let (p_1, p_2, p_3) be a Nash equilibrium.

I: $1 - p_3 > xp_2$. Since $p_2 \geq 0$ and $x > 0$, $p_3 < 1$. From br_1, $p_1 = 1$. Since $(x + 1)p_1p_3 = (x + 1)p_3 < x + p_3 = x - 1 + p_1 + p_3$, it follows from br_2 that $p_2 = 0$. Then, from br_3, $p_3 = 1$, which contradicts $p_3 < 1$. Hence, there is no equilibrium in this case.

II: $1 - p_3 < xp_2$. Since $p_3 \leq 1$, $p_2 > 0$. From br_1, $p_1 = 0$. From br_3, $p_3 = 1$. Since $(x + 1)p_1p_3 = 0 < x = x - 1 + p_1 + p_3$, it follows from br_2 that $p_2 = 0$, which contradicts $p_2 > 0$. Hence, there is no equilibrium in this case.

III: $1 - p_3 = xp_2$. We distinguish among three subcases.

- Subcase $p_2 = 0$. Since $1 - p_3 = xp_2$, $p_3 = 1$. One easily verifies that for **each $\boldsymbol{p_1} \in [\mathbf{0}, \mathbf{1}]$**, the strategy-profile $(p_1, p_2, p_3) = (\boldsymbol{p_1}, \mathbf{0}, \mathbf{1})$ is indeed a Nash equilibrium.

- Subcase $p_2 = 1$. Since $1 - p_3 = xp_2$, $p_3 \in (0, 1)$. Hence, from br_3, $x(p_1 + p_2) = (1+x)p_1p_2$, i.e., $p_1 = x \in (0, 1)$. Then, from br_2, $(x + 1)p_1p_3 = x - 1 + p_1 + p_3$. Thus, $(x + 1)xp_3 = p_3 - 1$. In the case that $x = 0.3$, we obtain $p_3 > 1$, and in the case that $x = 0.7$, we obtain $p_3 < 0$. In either case, this yields a contradiction with $p_3 \in (0, 1)$. Hence, there is no equilibrium in this subcase.

- Subcase $p_2 \in (0, 1)$. Since $1 - p_3 = xp_2$, $p_3 \in (0, 1)$. From br_3, $x(p_1 + p_2) = (1 + x)p_1p_2$. From br_2, $(x+1)p_1p_3 = x-1+p_1+p_3$. Using the latter equality, substituting $p_3 = 1-xp_2$, applying straightforward simplifications, and finally substituting $x(p_1+p_2) = (1+x)p_1p_2$ yields $p_1p_2 = \frac{x}{1-x^2}$. Therefore,

$$x(p_1 + p_2) = (1 + x)p_1p_2 = (1 + x)\frac{x}{1 - x^2} = (1 + x)\frac{x}{(1 - x)(1 + x)} = \frac{x}{(1 - x)}.$$

  Hence, $p_1 + p_2 = \frac{x}{(1-x)}$. Finally, it is easy to check that for each of $x = 0.3$ and $x = 0.7$, the unique solution (p_1, p_2) of the system $p_1p_2 = \frac{x}{1-x^2}$ and $p_1+p_2 = \frac{x}{(1-x)}$ satisfies $p_2 > 1$ which contradicts $p_2 \in (0, 1)$. Hence, there is no equilibrium in this subcase.

The set of NE in mixed strategies of problem $\mathcal{P}_2^{DA}$

One can easily verify that each profile of pure strategies induces payoffs $(0, x, x)$. Therefore, each profile of mixed strategies is a Nash equilibrium.

The set of NE in mixed strategies of problems $\mathcal{P}_3^{IA}$ and $\mathcal{P}_3^{DA}$

One can easily verify that the normal-form game is as follows:

1\2	s_1, s_2	s_2, s_1
s_1, s_2	$\mathbf{1}, \boldsymbol{x}, x$	$\mathbf{1}, \boldsymbol{x}, \boldsymbol{x}$
s_2, s_1	$x, \mathbf{1}, \boldsymbol{x}$	$\mathbf{1}, x, \boldsymbol{x}$

Table 24: Player 3 plays s_1, s_2

1\2	s_1, s_2	s_2, s_1
s_1, s_2	$\mathbf{1}, \mathbf{1}, \mathbf{1}$	$\mathbf{1}, x, \boldsymbol{x}$
s_2, s_1	$x, \mathbf{1}, \boldsymbol{x}$	$\mathbf{1}, x, \boldsymbol{x}$

Table 25: Player 3 plays s_2, s_1

Thus, the best response correspondences are as follows:

$$br_1(p_2, p_3) = \begin{cases} 1 & \text{if } p_2 \neq 0 \\ [0,1] & \text{if } p_2 = 0 \end{cases}$$

$$br_2(p_1, p_3) = \begin{cases} 1 & \text{if } p_1 \neq 1 \text{ or } p_3 \neq 1 \\ [0,1] & \text{if } p_1 = 1 \text{ and } p_3 = 1 \end{cases}$$

$$br_3(p_1, p_2) = \begin{cases} 0 & \text{if } p_1 \neq 0 \text{ and } p_2 \neq 0 \\ [0,1] & \text{if } p_1 = 0 \text{ or } p_2 = 0 \end{cases}$$

We compute the set of Nash equilibria by checking the two cases I and II below. Let (p_1, p_2, p_3) be a Nash equilibrium.

I: $p_2 \neq 0$. From br_1, $p_1 = 1$. Then, from br_3, $p_3 = 0$. Thus, from br_2, $p_2 = 1$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\mathbf{1}, \mathbf{1}, \mathbf{0})$ is indeed a Nash equilibrium.

II: $p_2 = 0$. Then, from br_2, $p_1 = p_3 = 1$. One easily verifies that the strategy-profile $(p_1, p_2, p_3) = (\mathbf{1}, \mathbf{0}, \mathbf{1})$ is indeed a Nash equilibrium.

Problem	Treatment	$(\hat{p}_1, \hat{p}_2, \hat{p}_3)$	μ^I	μ^S	unstable
$\mathcal{P}_1^{IA}, \mathcal{P}_1^{DA}$	L, H	$(1, \hat{p}_2, 0)$ for each $\hat{p}_2 \in [0,1]$	1	0	0
	L, H	$(1, 0, 1)$	0	1	0
$\mathcal{P}_2^{IA}$	L, H	$(\hat{p}_1, 0, 1)$ for each $\hat{p}_1 \in [0,1]$	1	1	0
$\mathcal{P}_2^{DA}$	L, H	$(\hat{p}_1, \hat{p}_2, \hat{p}_3)$ for each $\hat{p}_1, \hat{p}_2, \hat{p}_3 \in [0,1]$	1	1	0
$\mathcal{P}_3^{IA}, \mathcal{P}_3^{DA}$	L, H	$(1, 1, 0)$	1	0	0
	L, H	$(1, 0, 1)$	0	1	0

Table 26: Lists of length 2. Probability distribution over matchings induced by Nash equilibria in mixed strategies $(\hat{p}_1, \hat{p}_2, \hat{p}_3)$. Recall that in $\mathcal{P}_2$, $\mu^I = \mu^S$.
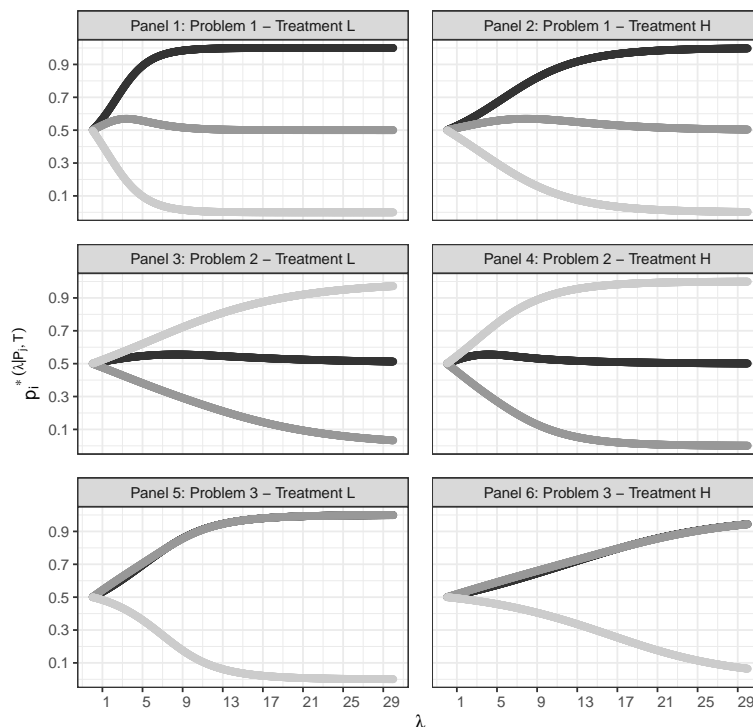
Figure 7: Lists of length 2. Logit-QRE probabilities under the immediate acceptance mechanism. Color scheme: $(p_1^*, p_2^*, p_3^*) \rightarrow$ (black, dark-gray, light-gray).