(Not) Addressing Issues in Electoral Campaigns

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Abstract

Two candidates competing for election may raise some issues for debate during the electoral campaign, while avoiding others. We present a model in which the decision to introduce an issue, or to reply to the opponent's position on one that she raised, may result in further additions to the list of topics that end up being discussed. Candidates' strategic decisions are driven by their appraisal of their expected vote share at the end of the campaign. Our analysis appeals to a protocol-free equilibrium concept, and predicts the list of topics that will be touched upon by each candidate, and the order in which they might be addressed. We show that important phenomena observed during campaigns, like the convergence of the parties to address the same issues, or else their diverging choice on which ones to treat, or the relevance of issue ownership can be explained within our stark basic model.

Keywords: Electoral campaigns, issues, equilibrium sets of continuation campaigns, issue convergence, issue divergence, issue ownership.
JEL-Classification: D72, P16.

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1 Introduction

Contenders in an electoral campaign may decide to raise some issues for controversy, while staying quiet about others. They may decide to confront their views on issues raised by their opponents, while ignoring the challenge in other cases. They may have a comparative advantage in dealing with issues that their parties have a good record on, and start with a handicap on other fronts. All of these are results of the strategic considerations that lead candidates to build campaigns through the choice of topics that they will address, conditional on what they think others will do to attract voters. A vast literature on electoral campaigns has considered, among other factors, the sort of decisions we just described, under the names of issue convergence (confronting opposite positions on the same issue), issue divergence (avoiding to address the same issues that others propose) or issue ownership (recognizing a priori advantages regarding certain issues when deciding which ones to debate).

The empirical literature is mixed and provides evidence for both issue convergence and divergence (see e.g. Petrocik, 1996; Spiliotes and Vavreck, 2003; Sigelman and Buell, 2004; Green-Pedersen, 2007). This entails the need to explain both outcomes, ideally with one single model which can produce both issue convergence and divergence and outcomes between these two extremes depending on the choice of parameters. The contribution of our paper is to deliver such a model.

We propose a stark model of campaign formation, where two candidates can independently determine what issues to address, and on which ones to remain silent, based on their estimated vote share in the election. Addressing an issue can be given two interpretations. One is that by addressing an issue a candidate announces her policy on that issue while staying silent on an issue means that the status quo policy on that issue will prevail in case the candidate is elected. Voters are then assumed to base their vote on the belief that candidates will keep their promises once elected. The other interpretation is that addressing an issue makes the candidate's position on that issue salient in the eyes of the voters. Here voters are assumed to base their vote on the salient positions of the candidates while they believe that the candidates will stick to the status quo policy for the issues they have not addressed during the campaign.

We consider a dynamic campaign formation but different from other authors (see, e.g. Colomer and Llavador, 2012; Egorov, 2015) we do not assume a given order of moves but rather derive the order endogenously as part of the equilibrium. To this end we adapt a protocol-free equilibrium concept that has been introduced by Dutta et al. (2004).

Our basic model can already explain all possible outcomes including the extreme cases of issue convergence and issue divergence. We also introduce some variants of our basic model to incorporate two relevant additional considerations. One is to recognize that limited resources may affect the form of equilibrium campaigns by limiting the number of issues that contenders can afford to productively address. Another is to take into account that campaigning costs may affect the electoral race by introducing additional considerations in defining the candidates' objectives, beyond the maximization of voting shares, and thus turn the game into one that is no longer zero sum.

Much of the previous theoretical literature on electoral campaigns has considered models where candidates or parties can allocate a given budget across different issues thereby increasing the salience of the issues which is assumed to affect voters' preferences (see, e.g., Amorós and Puy, 2013; Aragonès, Castanheira, and Giani, 2015; Dragu and Fan, 2016; Denter, 2020). In these models the salience of an issue is determined by the total investment of both candidates or parties into the issue while in our model a candidate can only affect the salience of her own position on an issue.

Admittedly, our model omits variables whose role has been analyzed by the literature on electoral campaigns, and even those that we explicitly consider are treated in a simplified matter. These additional variables include, for example, the intensity with which candidates treat each of the issues, or their ability to misrepresent their true objectives. Yet, our strategy of research here has been to concentrate on showing that the major phenomena we initially described can arise even in the absence of these additional qualifying considerations, as a result of a minimum of variables. Hence, the nature of our results, and our interpretation of their consequences. Our results are not to deny the relevance of additional considerations or complications, but point at the fact that these are not strictly needed to generate the basic phenomena we want to highlight.

The paper proceeds as follows. In section 2 we present our model and the equilibrium notion. Section 3 discusses three relevant special cases and also presents a simple example to analyze the effect of issue ownership on issue convergence and issue divergence. Section 4 considers the general case of an arbitrary number of issues. Section 5 studies the effect of campaigning costs. Section 6 concludes.

2 The Model

We consider two candidates, A and B, who compete in an electoral campaign. There is a finite set of policy issues $\mathcal{K}$ with cardinality $K \geq 1$. Examples for issues are social security, education, environmental protection, immigration etc. During the electoral campaign candidates address issues in $\mathcal{K}$. For $i = A, B$, let $z^i \in \{0,1\}^K$ be such that $z^i_k = 1$ if candidate i has addressed issue $k \in \mathcal{K}$ and $z^i_k = 0$, otherwise. Whenever convenient we write the vector z^i as a binary number $z^i = z^i_1 \dots z^i_K$.[1] We call (z^A, z^B) with $z^i \in \{0,1\}^K$ for $i = A, B$, a *state*.

Let $p(z^A, z^B)$ stand for the vote share of candidate A at state (z^A, z^B). We assume that these vote shares are different in each state, i.e.

$$p(z^A, z^B) \neq p(\hat{z}^A, \hat{z}^B) \text{ for } (z^A, z^B) \neq (\hat{z}^A, \hat{z}^B).$$

In most of the paper we retain the implicit assumption that both candidates assess in the same way the vote shares that each one will obtain. That implies that they play a zero-sum game. In section 4 we show that under the zero-sum assumption, our equilibrium payoffs are unique, and in section 5 we prove that otherwise multiple equilibria with different payoffs may exist. Hence, the zero-sum assumption ties our hands tighter, and makes our results more conclusive.

A protocol-free electoral campaign

During an electoral campaign, each candidate may decide to address some issues and not others, and to do it according to a given sequence, either raising a new one, responding to the opponent's previous mention of it, or ignoring it.

[1]For example, if $K = 3$, then $z^i = 101$ means that candidate i has addressed issues 1 and 3 but not issue 2.

Our choice of equilibrium notion is intended to endogenize the sequence of such choices, rather than imposing it in the form of a protocol. If candidate i addresses issue k, we denote this by the pair (k, i). At any point in time an electoral campaign then is characterized by a sequence of pairs $(k, i) \in \mathcal{K} \times \{A, B\}$ recording which candidate has addressed which issue and in what order. We call any such sequence a *campaign*. Formally, let $m \in \{1, \ldots, 2K\}$. A campaign of length m then is a sequence $s = (s_1, \ldots, s_m)$ with $s_l \in \mathcal{K} \times \{A, B\}$ for all $l = 1, \ldots, m$, and $s_l \neq s_{l'}$ for all $l \neq l'$. The empty campaign $\varnothing$, where no candidate has addressed any issue is defined to have length 0. By S^m we denote the set of campaigns of length m, where $0 \leq m \leq 2K$, and by $S = \bigcup_{m=0}^{2K} S^m$ we denote the set of all campaigns.

For a given campaign $s = (s_1, \ldots, s_m) \in S$ and $(k, i) \in \mathcal{K} \times \{A, B\}$ we write, for short, $(k, i) \in s$ whenever $(k, i) = s_l$ for some $l \in \{1, \ldots, m\}$, and $(k, i) \notin s$ whenever $(k, i) \neq s_l$ for all $l = 1, \ldots, m$. For $s \in S^m$, where $0 \leq m \leq 2K$, and $(k, i) \in \mathcal{K} \times \{A, B\}, (k, i) \notin s$, $(s, (k, i))$ denotes the campaign $s' \in S^{m+1}$ with $s'_l = s_l$ for $l = 1, \ldots, m$, and $s'_{m+1} = (k, i)$.

Each campaign $s \in S$ defines a state $(z^A(s), z^B(s))$, where $z^i(s) \in \{0, 1\}^K$ denotes which issues have been addressed by candidate $i \in \{A, B\}$ at campaign s, that is

$$z_k^i(s) = \begin{cases} 1, \text{ if } (k, i) \in s, \\ 0, \text{ if } (k, i) \notin s \end{cases}$$

for $i = A, B$.

The electoral campaign ends if at a given campaign $s \in S$ no candidate wants to address an issue that he or she has not addressed before. In particular, the electoral campaign ends at any full campaign $s \in S^{2K}$, when all issues have been addressed by both candidates. But note that the electoral campaign may end before a full campaign is reached. There may be issues that have only been addressed by one candidate but not by the other, and also ones not addressed by any of the two candidates.

Once the electoral campaign has ended there is an election and

$$P(s) = p(z^A(s), z^B(s))$$

denotes candidate A's vote share at the given campaign s.

We shall now define the notion of continuation campaigns, that is crucial in the analysis of our equilibrium concept. Let $m \in \{0, 1, \ldots, 2K\}$ and let $s \in S^m$. We say that s' is a *continuation campaign* at s if $s'_l = s_l$ for all $l = 1, \ldots, m$. Note that by definition s is a continuation campaign at s. By $S(s)$ we denote the set of continuation campaigns at $s \in S$. A *collection of sets of continuation campaigns* is a family of subsets of $S(s)$ for each $s \in S$.

We shall use a concept of equilibrium that adapts a proposal by Dutta et al. (2004). We have adopted this concept because it is very general and it allows candidates to choose the order in which they express themselves and also that in which they address the different issues. The notion of equilibrium is defined on collections of sets of campaigns, rather than on specific campaigns. It demands from equilibrium collections to satisfy three conditions, which together provide it a sense of consistency and rationality. The first condition just demands that each campaign in the collection should at least be followed by another, which may include the possibility of being its own successor. The second condition is also very mild and a stopping requirement. Candidates stop adding issues to the campaign if and only if they unanimously agree that this would hurt them. The third condition requires continuation equilibria to satisfy a minimum rationality requirement: If there are multiple continuation equilibria, then candidates should not build campaigns that are worse for them than any other equilibrium campaign. Moreover, if there is a unique continuation equilibrium that does not involve stopping, then the candidate who initiates the continuation must not be worse off by doing so than by stopping at the given campaign. Definition 2.1 formalizes these ideas.

Definition 2.1 (Equilibrium Continuation Campaigns)
A collection of sets of continuation campaigns $(CE(s))_{s\in S}$ *is an* ***equilibrium collection of sets of continuation campaigns*** *if the following conditions are satisfied for all* $s \in S$*:*

(E1) $CE(s)$ *is a nonempty subset of* $\bigcup_{(k,i)\notin s} CE((s,(k,i)) \cup \{s\}$.
(E2) $s \in CE(s)$ *if and only if*

$$P(s) > P(s') \text{ for all } s' \in \bigcup_{k:(k,A)\notin s} CE((s,(k,A)))$$

and

$$1 - P(s) > 1 - P(s') \text{ for all } s' \in \bigcup_{k:(k,B)\notin s} CE((s,(k,B))).$$

(E3) *For* $s \in S$ *say that the campaign* $s' = (s,(k,i),\ldots) \in S$ *is* ***rationalizable*** *(relative to* s*) if* $s' \in CE((s,(k,i))$ *and there exists an* $s'' \in CE(s)$ *with either* $s'' = (s,(h,j),\ldots)$ *for some* $(h,j) \neq (k,i)$ *or* $s'' = s$ *such that*

$$\begin{aligned} P(s') &> P(s''), \quad \text{if } i = A, \\ \text{and} \quad 1 - P(s') &> 1 - P(s''), \quad \text{if } i = B. \end{aligned}$$

If $s' \in \bigcup_{(h,i)\notin s} CE((s,(h,i)))$ *is rationalizable, then* $s' \in CE(s)$*. Conversely, if* $s' = (s,(k,i),\ldots) \in CE(s)$ *and if either* $s \in CE(s)$ *or* $s'' = (s,(h,j),\ldots) \in CE(s)$ *for some* $(h,j) \neq (k,i)$*, then* s' *is rationalizable. If* $s' = (s,(k,i),\ldots) \in CE(s)$ *and* $\nexists j \neq i$ *such that* $s'' = (s,(h,j),\ldots) \in CE(s)$ *for some* $(h,j) \notin s$*, then*

$$\begin{aligned} P(s') &> P(s), \quad \text{if } i = A, \\ \text{and} \quad 1 - P(s') &> 1 - P(s), \quad \text{if } i = B.^2 \end{aligned}$$

Notice that in order to determine what is an equilibrium collection of continuation campaigns, one must proceed to a backward induction analysis. One must first decide whether the full campaign satisfies the conditions as a continuation of

[2] In Dutta et al. (2004) an equilibrium collection of sets of continuation campaigns is defined to satisfy (E1) and (E2). If, in addition, (E3) is satisfied, the equilibrium collection is said to be *consistent*.

each of the campaigns where only one candidate has kept silent on only one issue. Then, in view of that, the next step is to analyze whether each of the latter may be equilibrium continuations of campaigns in which either a candidate does not address two of the issues, or both candidates fail to address one issue each. After completing the backward induction, we will have one or several possible collections of equilibrium continuations. Multiplicity is possible, but we will show in section 4 that all equilibrium collections are outcome equivalent.

In what follows, after identifying those collections of continuation campaigns that satisfy our equilibrium conditions, we will focus attention on campaigns that are part of these equilibrium continuations and are continuations of themselves and of the empty set. This formalizes the notion that the disclosure of positions starts from scratch at the beginning of the campaign and follows a path leading to a campaign s, after which no further disclosures will be in the interest of anyone, given the continuations predicted from further additions.

Definition 2.2 *s^* is an **equilibrium campaign** if there exists an equilibrium collection of sets of continuation campaigns $(CE(s))_s$ with $s^* \in CE(\varnothing)$.*

The reader may wonder how decisive our choice of specific vote shares is to drive our results. In fact, since what matters for equilibrium is just the order of preference that the vote shares imply for the candidates' ranking of campaigns, we can make sure that any ordering of vote shares can be explained as being associated with some profile of voters' preferences. This follows from a result in Debord (1987) who proves that for a finite set of alternatives X and for any collection $(m(x,y))_{x \neq y}$ of integers with $m(x,y) = -m(y,x)$ for all $x \neq y$ in X such that $m(x,y)$ is either even for all $x \neq y$ or odd for all $x \neq y$ in X, there exists a set of voters V and strict preference orderings P_i for all voters $i \in V$ such that $m(x,y)$ is the majority margin for x over y for all alternatives $x \neq y$ at the given preference profile, i.e. $m(x,y) = |\{i \in V|\, xP_iy\}| - |\{i \in V|\, yP_ix\}|$.[3] Since

$$m(x,y) = 2N\frac{|\{i \in V|\, xP_iy\}|}{N} - N,$$

[3]Le Breton (2005, Remark 2.5 and Section 4) provides an alternative proof of Debord's theorem.

where $N = |V|$ it follows that the majority margin for x over y is increasing in the proportion of voters strictly preferring x over y which is the vote share for alternative x. Hence, any given ordering of vote shares can be generated by some preference profile.

3 Special Cases and Issue Ownership

As we stated in the introduction, one of our purposes is to discuss relevant features of the process of campaign formation, and to do it within a model that is stark, and yet powerful enough to generate the phenomena that have been considered most salient by previous analysts. In this section we discuss three special cases and use them with a double purpose.

One is to present the reader with examples of the workings of our general model and equilibrium notion. The other is to show that, indeed, their analysis reveals the basic phenomena that we shall later try to extend to the general case.

In our first subsection we study the case in which only one issue is at stake and provide a full characterization of its equilibria. One first conclusion from this analysis is that issue convergence or issue divergence may arise, depending on the vote shares at the different campaigns, among other configurations; in fact, all possible combinations of equilibrium campaigns may arise.

In the second subsection we present the two-issue case, and different examples confirming that, again, all possible forms of campaign may arise, including now, among others, different combinations of divergence and convergence.

In the third subsection we study an intermediate case: the situation where two issues are available for discussion, but each candidate can only address one of them. The reason to propose this case is that it nicely incorporates the idea that, because of budgetary reasons or others, the candidates may be constrained in their choices. In that case, we can again offer a full characterization of equilibrium configurations.

Finally, in the last subsection we consider the important idea of issue ownership, in a simple form.

3.1 One issue

Assume that there is only one issue, i.e. $K = 1$. In this case we shortly write $(A), (B), (A, B), (B, A)$ for the campaigns, where only candidate A, only candidate B, first candidate A and then B, first candidate B and then A have addressed the unique issue.

Recall that every campaign s defines a state $(z^A(s), z^B(s))$, where $z^i(s) \in \{0, 1\}$ for $i = A, B$, and $z^i(s) = 1$ if and only if candidate $i \in \{A, B\}$ has addressed the issue at campaign s. Campaigns then can be represented in a square (see Figure 1).

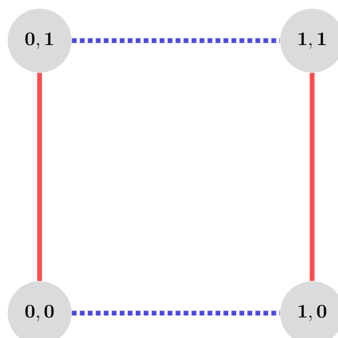

Figure 1: Representation of campaigns in a square. Edges denote feasible moves between two campaigns. Candidate A moves along the horizontal edges (dotted). Candidate B moves along the vertical edges (solid).

In the following we characterize equilibrium campaigns in terms of the vote shares $p(0, 0), p(1, 0), p(0, 1)$ and $p(1, 1)$.

For illustration, consider the case where

$$p(1, 1) < p(0, 0) < p(0, 1) \text{ and } p(1, 1) < p(1, 0).$$

In this case, (E1) and (E2) imply that

$$CE(A) = \{(A, B)\} \text{ and } CE(B) = \{(B)\},$$

which in turn implies that $CE(\varnothing) \subset \{\varnothing, (B), (A, B)\}$ by (E1). (E2) then implies that $\varnothing \in CE(\varnothing)$ and (E3) implies that

$$CE(\varnothing) = \{\varnothing\}.$$

This case is illustrated in Figure 2.

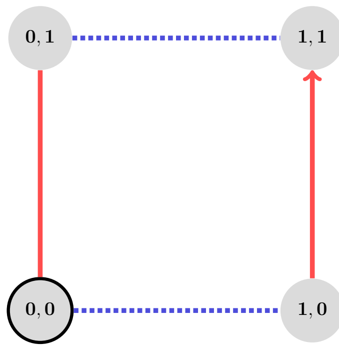

Figure 2: Equilibrium continuations for $p(1,1) < p(0,0) < p(0,1)$ and $p(1,1) < p(1,0)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Table 1 summarizes the necessary and sufficient conditions on the vote shares for all possible equilibrium campaigns. Note that for all orderings of the vote shares the equilibrium campaign is unique. The detailed analysis can be found in the appendix.

$CE(\varnothing)$		Conditions on vote shares
$\{\varnothing\}$		$p(1,1) < p(0,0) < p(0,1)$ and $p(1,1) < p(1,0)$
	or	$p(1,0) < p(0,0) < p(1,1)$ and $p(0,1) < p(1,1)$
	or	$p(1,0) < p(0,0) < p(0,1)$ and $p(1,0) < p(1,1) < p(0,1)$
$\{(A)\}$		$p(0,0) < p(1,0) < p(1,1)$
$\{(B)\}$		$p(1,1) < p(0,1) < p(0,0)$
$\{(A,B)\}$		$p(0,0) < p(1,1) < \min\{p(1,0), p(0,1)\}$
	or	$p(0,1) < p(1,1) < p(1,0)$ and $p(0,0) < p(1,1)$
$\{(B,A)\}$		$\max\{p(1,0), p(0,1)\} < p(1,1) < p(0,0)$
	or	$p(0,1) < p(1,1) < p(1,0)$ and $p(1,1) < p(0,0)$

Table 1: Vote shares and equilibrium campaigns for one issue.

3.2 Two issues

Let there be two issues, i.e. $K = 2$. Campaigns then can be represented in the 4-hypercube, also called *tesseract*, in Figure 3, where the edges denote feasible moves: There is an edge between two nodes if and only if there is exactly one candidate and one issue which is addressed by the candidate in one node, but not in the other.

Note that by assumption the candidates' vote shares at a given campaign only depend on the issues that have been addressed by the different candidates, but not on the order in which the issues have been addressed.

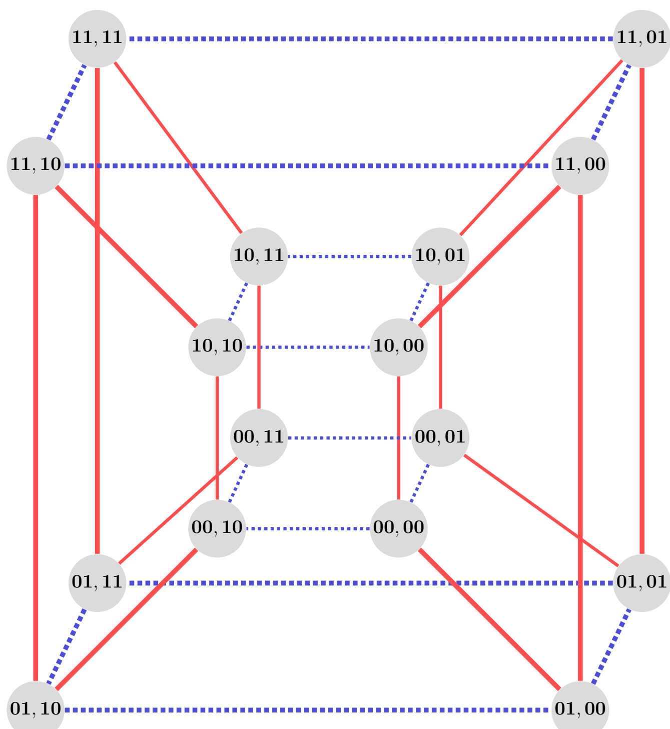

Figure 3: Representation of campaigns in a tesseract. Edges denote feasible moves between two campaigns. Candidate A moves between adjacent campaigns in the same level of the tesseract (dotted edge). Candidate B moves between adjacent campaigns in different levels of the tesseract (solid edge).

In the following we will present some examples to illustrate that any outcome can obtain in equilibrium, i.e. for any possible state we can find vote shares that the given state is the unique outcome in equilibrium. In particular, we present examples for issue convergence (both candidates address the same issue) and issue divergence (both candidates address different issues). The general characterization of equilibrium campaigns in terms of properties of the vote shares can be found in section 4 where we consider the general case with an arbitrary number of issues.

Example 3.1 This is an example for issue convergence. Let candidate A's vote share be given in the following table.

		z^B			
		00	10	01	11
z^A	00	0.5	0.3	0.7	0.65
	10	0.75	0.45	0.9	0.95
	01	0.6	0.4	0.1	0.2
	11	0.8	0.85	0.55	0.35

Figure 4 illustrates the equilibrium continuation campaigns. As we see there is a unique equilibrium campaign, where first candidate B and then A address issue 1:

$$CE(\varnothing) = \{((1, B), (1, A))\}.$$

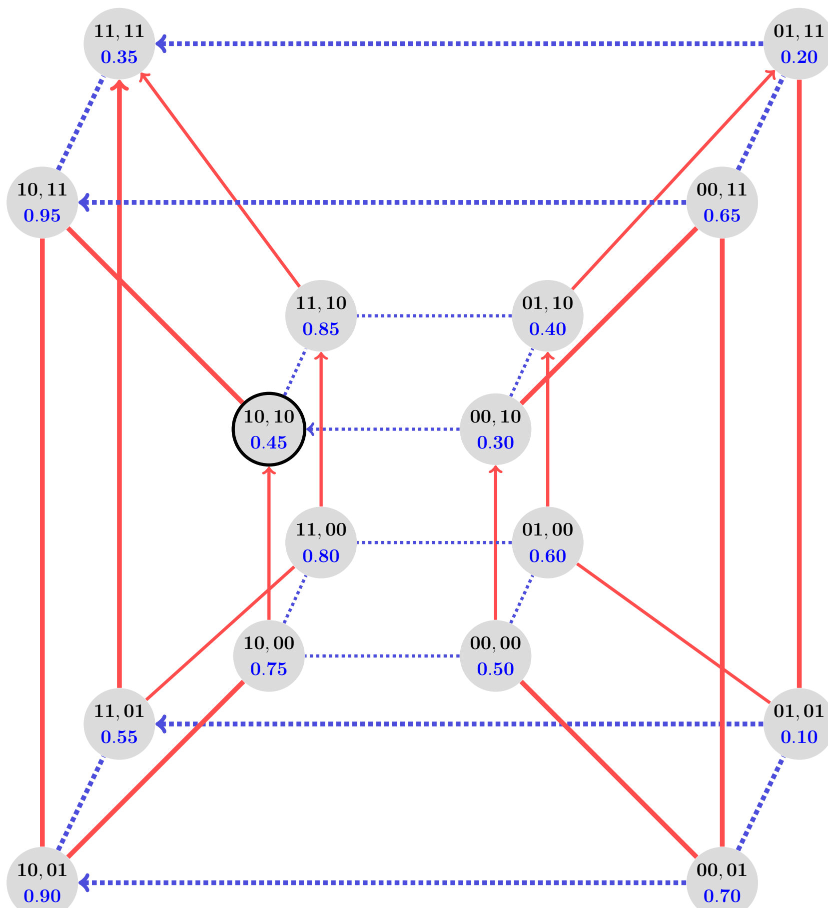

Figure 4: Equilibrium continuations in Example 3.1. The lower number in a node is candidate A's vote share at the given campaign. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Example 3.2 This is an example for issue divergence. Let candidate A's vote share be given in the following table.

z^A \ z^B	00	10	01	11
00	0.5	0.88	0.31	0.73
10	0.72	0.8	0.58	0.74
01	0.2	0.67	0.25	0.55
11	0.51	0.7	0.43	0.62

Figure 5 illustrates the equilibrium continuation campaigns. As we see there is a unique equilibrium campaign, where first candidate A addresses issue 1 and then candidate B addresses issue 2:

$$CE(\varnothing) = \{((1, A), (2, B))\}.$$

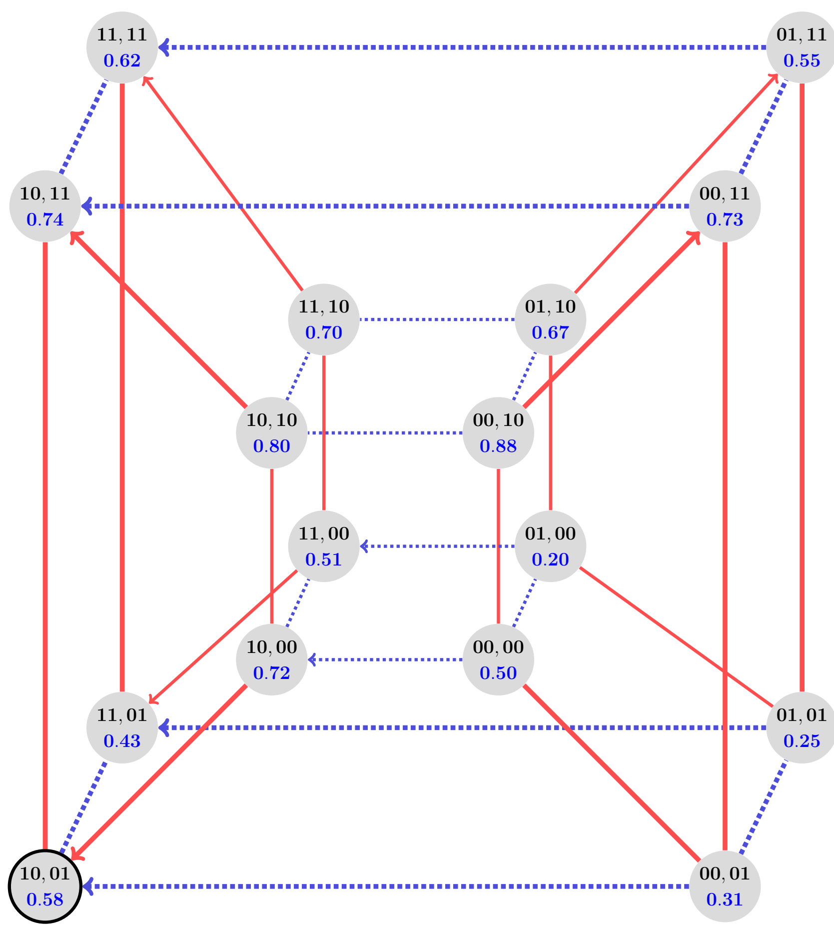

Figure 5: Equilibrium continuations in Example 3.2. The lower number in a node is candidate A's vote share at the given campaign. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Example 3.3 This is an example where no candidate addresses any issue in equilibrium. Let candidate A's vote share be given in the following table.

		z^B			
		00	10	01	11
z^A	00	0.5	0.7	0.68	0.76
	10	0.32	0.28	0.2	0.58
	01	0.3	0.21	0.29	0.59
	11	0.24	0.41	0.42	0.19

Figure 6 illustrates the equilibrium continuation campaigns. Note that there are multiple equilibrium continuations at state $(11, 00)$: If $z(s) = (11, 00)$, then $CE(s) = \{(s, (1, B), (2, B))\}$ and $CE(s) = \{(s, (2, B), (1, B))\}$ are two singleton sets of continuation campaigns which are outcome equivalent. Despite this multiplicity the equilibrium campaign is unique and given by

$$CE(\varnothing) = \{\varnothing\}.$$

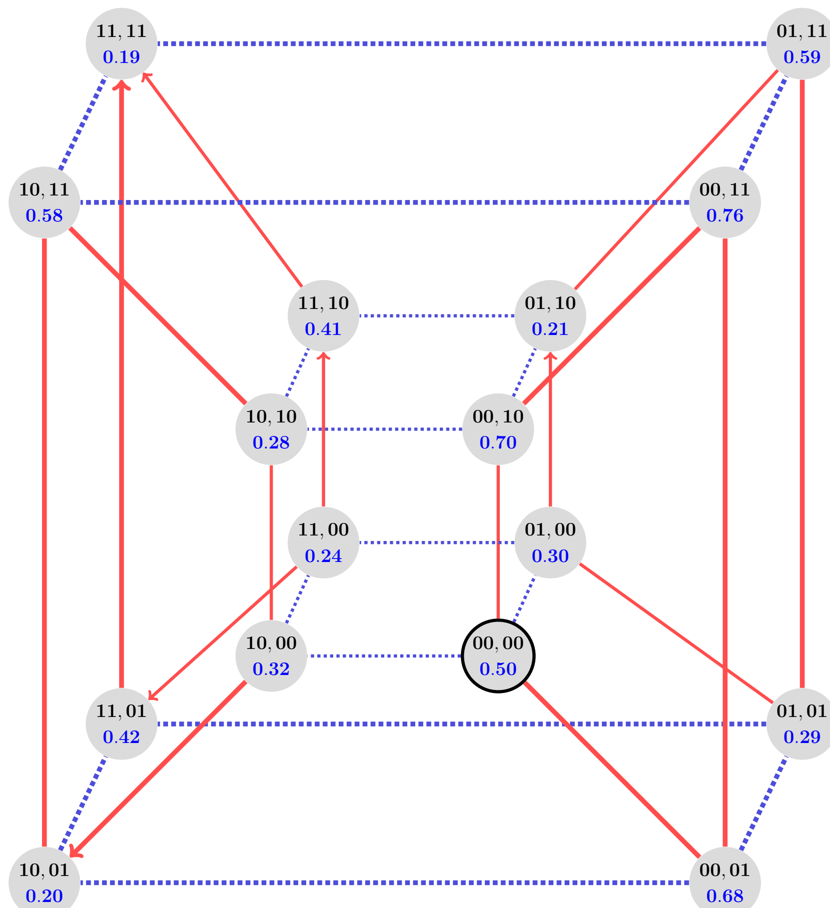

Figure 6: Equilibrium continuations in Example 3.3. The lower number in a node is candidate A's vote share at the given campaign. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Example 3.4 This is an example where both candidates address both issues in equilibrium. Let candidate A's vote share be given in the following table.

		z^B			
		00	10	01	11
z^A	00	0.5	0.2	0.19	0.09
	10	0.72	0.66	0.61	0.49
	01	0.75	0.59	0.7	0.45
	11	0.82	0.79	0.78	0.74

Figure 7 illustrates the equilibrium continuation campaigns. Note that there are several states with multiple equilibrium continuations, where all equilibrium continuations are initiated by the same candidate and lead to the same outcome. This is also true for the initial state $(00, 00)$, so we get multiple equilibrium collections and multiple equilibrium campaigns which all give the same outcome, where both candidates have addressed both issues. All equilibrium campaigns are initiated by candidate A which is the candidate who gains from moving to state $(11, 11)$ relative to the initial state $(00, 00)$:

$$\begin{aligned}
& CE(\varnothing) &=& \{((1, A), (2, A), (1, B), (2, B))\} \\
\text{or} \quad & CE(\varnothing) &=& \{((1, A), (2, A), (2, B), (1, B))\} \\
\text{or} \quad & CE(\varnothing) &=& \{((2, A), (1, B), (1, A), (2, B))\} \\
\text{or} \quad & CE(\varnothing) &=& \{((2, A), (2, B), (1, A), (1, B))\}
\end{aligned}$$

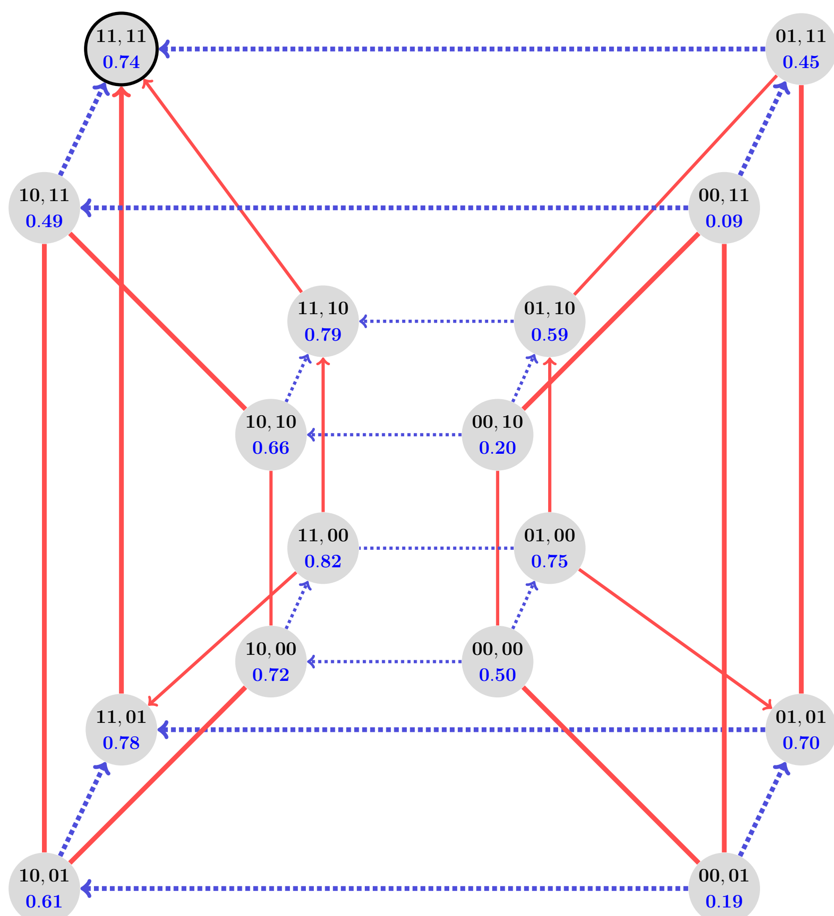

Figure 7: Equilibrium continuations in Example 3.4. The lower number in a node is candidate A's vote share at the given campaign. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

3.3 Candidates can only address one issue

Let there be two issues, i.e. $K = 2$, but now assume that candidates can address at most one issue. The reasons may be manifold. For example, candidates may not have enough time to campaign with more than one issue or because candidates do not have enough funds for a campaign with more than one issue. Let $\bar{S} \subset S$ denote the set of campaigns under the restriction that candidates can address at most one issue. The possible campaigns can then be represented in a grid, where edges denote feasible moves between two edges (see Figure 8).

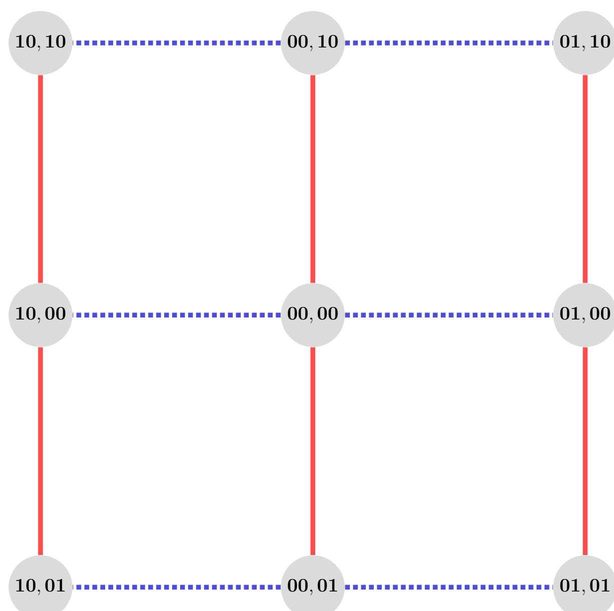

Figure 8: Representation of states in a grid. Edges denote feasible moves between two campaigns. Candidate A moves along the horizontal edges (dotted). Candidate B moves along the vertical edges (solid).

We then have the following result:

Theorem 3.1 *Let* $Z = \{(z^A, z^B) |\, z_1^i + z_2^i \leq 1$ *for* $i = A, B\}$.

(i) There exists a unique equilibrium collection of sets of continuation campaigns $(CE(s))_{s\in\bar{S}}$ *and for all* $s \in \bar{S}$ *there is a unique equilibrium continuation in* $CE(s)$*, i.e.* $CE(s) = \{s'\}$ *for some* $s' \in \bar{S}$.

(ii) For all $(z^A, z^B) \in Z$ *let* $p^*(z^A, z^B)$ *be the vote share of candidate* A *in the unique continuation equilibrium in* $CE(s)$ *where* $s \in \bar{S}$ *is such that* $z(s) = (z^A, z^B)$*. Then*

$$\begin{aligned} p^*(z^A, z^B) &= p(z^A, z^B), \ \textit{if } z_1^i + z_2^i = 1 \textit{ for } i = A, B, \\ p^*(z^A, 00) &= \min\{p(z^A, 00), p(z^A, 10), p(z^A, 01)\}, \ \textit{if } z_1^A + z_2^A = 1, \\ p^*(00, z^B) &= \max\{p(00, z^B), p(10, z^B), p(01, z^B)\} \ \textit{if } z_1^B + z_2^B = 1, \end{aligned}$$

and

$$p^*(00,00) = $$

$$\begin{cases} \min\{p^*(00,10), p^*(00,01)\}, \ \textit{if } p(00,00) > \min\{p^*(00,10), p^*(00,01)\} \\ p(00,00), \ \textit{if } \min\{p^*(00,10), p^*(00,01)\} > p(00,00) > \\ \qquad\qquad\qquad\qquad\qquad\qquad \max\{p^*(10,00), p^*(01,00)\} \\ \max\{p^*(10,00), p^*(01,00)\}, \ \textit{if } \max\{p^*(10,00), p^*(01,00)\} > p(00,00) \end{cases}.$$

In particular, it is true that

$$\min\{p^*(00,10), p^*(00,01)\} \geq p^*(00,00) \geq \max\{p^*(10,00), p^*(01,00)\}.$$

Theorem 3.1 provides a full characterization of the unique equilibrium outcome in terms of the vote shares at different campaigns. It shows that any outcome can obtain for some ordering of the vote shares at the different states. For example, candidate A addressing issue 1 and B addressing issue 2 ("issue divergence") is the unique equilibrium outcome if and only if $p^*(00,00) = p(10,01)$

which by Theorem 3.1 is equivalent to

$$\begin{aligned} p(00,00) > \min\{p^*(00,10), p^*(00,01)\} &= p(10,01) && (1)\\ \text{or} \quad p(00,00) < \max\{p^*(10,00), p^*(01,00)\} &= p(10,01). && (2) \end{aligned}$$

Note that (1) is equivalent to

$$\max\{p(00,01), p(01,01)\} < p(10,01) < \max\{p(00,10), p(10,10), p(01,10)\}$$

$$\text{and} \quad p(10,01) < p(00,00),$$

and (2) is equivalent to

$$\min\{p(01,00), p(01,10), p(01,01)\} < p(10,01) < \min\{p(10,00), p(10,10)\}$$

$$\text{and} \quad p(00,00) < p(10,01).$$

Also, both candidates addressing issue 1 ("issue convergence") is the unique equilibrium outcome if and only if $p^*(00,00) = p(10,10)$ which by Theorem 3.1 is equivalent to

$$\begin{aligned} p(00,00) > \min\{p^*(00,10), p^*(00,01)\} &= p(10,10) && (3)\\ \text{or} \quad p(00,00) < \max\{p^*(10,00), p^*(01,00)\} &= p(10,10). && (4) \end{aligned}$$

Note that (3) is equivalent to

$$\max\{p(00,10), p(01,10)\} < p(10,10) < \max\{p(00,01), p(10,01), p(01,01)\}$$

$$\text{and} \quad p(10,10) < p(00,00),$$

and (4) is equivalent to

$$\min\{p(01,00), p(01,10), p(01,01)\} < p(10,10) < \min\{p(10,00), p(10,01)\}$$

$$\text{and} \quad p(00,00) < p(10,10).$$

3.4 Issue ownership

We will now explore the effect of issue ownership on equilibrium outcomes in the limited case we discussed in the last subsection. Issue ownership captures the fact that a candidate has an a priori advantage in dealing with some issue and that this is reflected in her vote share (see Petrocik, 1996). Therefore, defining ownership first requires to introduce some notion of competence or reliability. We then illustrate how issue ownership may lead to issue convergence or divergence in more specific terms.

Assume that all voters care about one and only one issue and let α be the share of voters who only care about issue 1 and let $1-\alpha$ be the share of voters who only care about issue 2, where $0<\alpha<1$. Moreover, for $k=1,2$, let γ_k be the share of voters who consider candidate A more competent or more reliable on issue k than candidate B.

If the candidates address different issues (issue divergence) voters vote for the candidate who has addressed the issue they care about. If the candidates address the same issue (issue convergence), the voters who care about this issue vote for the candidate they consider more competent on the issue and the voters who do not care about the issue split their vote evenly between the candidates.[4] Moreover, if one candidate does not address any issue and the other candidate addresses issue k, the voters who care about issue k vote for the candidate who addresses this issue and the voters who do not care about the issue split their vote evenly. Finally, if no candidate addresses any issue the total vote share is also split at value 0.5.

Under these assumptions we get the following vote shares of candidate A at the different outcomes of a campaign:

			z^B	
		00	10	01
	00	0.5	$0.5(1-\alpha)$	0.5α
z^A	10	$\alpha+0.5(1-\alpha)$	$\gamma_1\alpha+0.5(1-\alpha)$	α
	01	$1-\alpha+0.5\alpha$	$1-\alpha$	$\gamma_2(1-\alpha)+0.5\alpha$

[4] In case of a finite and odd number of voters, just assume that an odd number of voters abstain.

Note that the vote shares differ across states if and only if $\alpha \neq 0.5$ and $\gamma_k \neq 0.5$ for $k = 1, 2$, which we will assume in the following. We will now explore under which conditions on the parameters of our voting model the candidates address the same or different issues in equilibrium. In particular, we are interested in the implications of issue ownership, which is modelled by the parameters γ_1 and γ_2. To this end we define candidate A to "own" issue k if a majority of voters considers A more competent on issue k than B, i.e. if $\gamma_k > 0.5$. Similarly, B owns issue k if $\gamma_k < 0.5$.

Consider first the case of issue divergence. W.l.o.g. let candidate A address issue 1 and let candidate B address issue 2 in equilibrium. Then $p^*(00, 00) = p(10, 01)$ which by Theorem 3.1 is the case if and only if

$$\max\{0.5\alpha, \gamma_2(1-\alpha) + 0.5\alpha\} < \alpha < \max\{0.5(1-\alpha), \gamma_1\alpha + 0.5(1-\alpha), 1-\alpha\} \quad (5)$$

$$\text{and} \quad \alpha < 0.5, \quad (6)$$

or

$$\min\{1-\alpha+0.5\alpha, 1-\alpha, \gamma_2(1-\alpha)+0.5\alpha\} < \alpha < \min\{\alpha+0.5(1-\alpha), \gamma_1\alpha+0.5(1-\alpha)\} \quad (7)$$

$$\text{and} \quad 0.5 < \alpha. \quad (8)$$

(5) and (6) hold if and only if

$$\frac{\gamma_2}{\gamma_2 + 0.5} < \alpha < 0.5 \quad (9)$$

and (7) and (8) hold if and only if

$$0.5 < \alpha < \frac{0.5}{1.5 - \gamma_1}. \quad (10)$$

From (9) and (10) we conclude that a necessary and sufficient condition for issue divergence is that one of the two candidates owns the issue she addresses and that a majority of voters cares about this issue but there is also a sufficiently large share of voters who care about the other issue.[5] The required minimum

[5]Note that (9) implies that $\gamma_2 < 0.5$, i.e. that candidate B owns issue 2, and (10) implies that $\gamma_1 > 0.5$, i.e. that candidate A owns issue 1.

share of voters is decreasing in the share of voters who consider the candidate who addresses the issue she owns more competent on this issue than the other candidate.

Consider next the case of issue convergence. W.l.o.g. let both candidates address issue 1 in equilibrium. Then $p^*(00,00) = p(10,10)$ which by Theorem 3.1 is the case if and only if

$$\max\{0.5(1-\alpha), 1-\alpha\} < \gamma_1\alpha + 0.5(1-\alpha) < \max\{0.5\alpha, \alpha, \gamma_2(1-\alpha)+0.5\alpha\} \quad (11)$$

$$\text{and} \quad \gamma_1\alpha + 0.5(1-\alpha) < 0.5, \quad (12)$$

or

$$\min\{1-\alpha+0.5\alpha, 1-\alpha, \gamma_2(1-\alpha)+0.5\alpha\} < \gamma_1\alpha + 0.5(1-\alpha) < \min\{\alpha+0.5(1-\alpha), \alpha\} \quad (13)$$

$$\text{and} \quad \gamma_1\alpha + 0.5(1-\alpha) > 0.5. \quad (14)$$

(11) and (12) hold if and only if

$$\gamma_1 < 0.5 \quad \text{and} \quad \alpha > \frac{0.5}{0.5+\gamma_1} \quad (15)$$

and (13) and (14) hold if and only if

$$\gamma_1 > 0.5 \quad \text{and} \quad \alpha > \frac{0.5}{1.5-\gamma_1}. \quad (16)$$

From (15) and (16) we conclude that a necessary and sufficient condition for issue convergence is that one candidate owns the issue addressed by both candidates and that a sufficiently large share of voters care about this issue. The required minimum share is always larger than 0.5 and increasing in the share of voters who consider the issue owner more competent than the other candidate.

4 The General Case

Consider now the general case with $K \geq 2$ issues and no restrictions on the number of issues that can be addressed by the candidates. We first provide sufficient conditions on the vote shares to obtain arbitrary equilibrium outcomes.

Let $\mathbf{1} \in \{0,1\}^K$ denote the vector with $\mathbf{1}_k = 1$ for all k. $(\hat{z}^A, \hat{z}^B)$ is a *continuation* of (z^A, z^B) if for all $k = 1 \ldots, K, z_k^i = 1$ implies that $\hat{z}_k^i = 1$.[6]

Let $G_k > 0$ and $L_k > 0$ for $k = 1, \ldots, K$, be such that the following conditions are satisfied:

$$G_k > 0 \text{ and } L_k > 0 \text{ for } k = 1, \ldots, K, \tag{17}$$

$$\sum_{k=1}^{K} G_k < 0.5, \tag{18}$$

$$\sum_{k=1}^{K} L_k < 0.5, \tag{19}$$

$$\sum_{k \in M_1} G_k \neq \sum_{k \in M_2} L_k \text{ for all } M_1, M_2 \subset K. \tag{20}$$

For all (z^A, z^B) let

$$\hat{p}(z^A, z^B) = 0.5 + \sum_{k=1}^{K} (z_k^A G_k - z_k^B L_k)$$

be the *standard vote share* for candidate A in state (z^A, z^B). Note that conditions (17)-(20) imply that $0 < \hat{p}(z^A, z^B) < 1$ for all $(z^A, z^B), \hat{p}(\mathbf{0}, \mathbf{0}) = 0.5$, and $\hat{p}(z^A, z^B) \neq \hat{p}(\hat{z}^A, \hat{z}^B)$ for all $(z^A, z^B) \neq (\hat{z}^A, \hat{z}^B)$.

If (z^A, z^B) and all its continuations have standard vote shares then in equilibrium any campaign that starts at state (z^A, z^B) will end at state $(\mathbf{1}, \mathbf{1})$ corresponding to the full campaign. We state this result in the following lemma:

Lemma 4.1 *Let (z^A, z^B) be such that $p(\hat{z}^A, \hat{z}^B) = \hat{p}(\hat{z}^A, \hat{z}^B)$ for all continuations $(\hat{z}^A, \hat{z}^B)$ of (z^A, z^B). Then any continuation equilibrium at a campaign s with $z(s) = (z^A, z^B)$ is a full campaign, i.e. $(\mathbf{1}, \mathbf{1})$ is the unique outcome at any continuation equilibrium at (z^A, z^B).*

The next theorem provides sufficient conditions for arbitrary equilibrium outcomes.

[6]Note that by this definition (z^A, z^B) is a continuation of itself.

Theorem 4.1

1. *Let* $p(z^A, z^B) = \hat{p}(\hat{z}^A, \hat{z}^B)$ *for all* $(z^A, z^B) \in \{0,1\}^{2K}$. *Then any equilibrium campaign is a full campaign.*

2. *Let* $(z^{*A}, z^{*B}) \neq (\mathbf{1}, \mathbf{1})$ *and let the parameters of the standard vote shares be such that*
$$\hat{p}(z^{*A}, z^{*B}) < \hat{p}(\mathbf{1}, \mathbf{1}).$$
If $z_k^{*A} = 0$ *for some* k *and the vote shares satisfy*
$$p(z^A, z^B) < \hat{p}(z^{*A}, z^{*B})$$
for all (z^A, z^B) *with* $z^B \leq z^{*B}$ *and* $z_k^A = 1$ *for some* k *with* $z_k^{*A} = 0$, *and*
$$p(z^A, z^B) = \hat{p}(z^A, z^B)$$
for all remaining campaigns, then (z^{*A}, z^{*B}) *is the unique outcome at any equilibrium campaign.*[7]

We now go back to the case where vote shares are arbitrary and only satisfy $p(z^A, z^B) \neq p(\hat{z}^A, \hat{z}^B)$ for $(z^A, z^B) \neq (\hat{z}^A, \hat{z}^B)$. The following theorem presents the remarkable result that all continuation equilibria are outcome equivalent. Moreover, similar to Theorem 3.1 the theorem provides a recursive procedure to determine candidate A's vote share in the unique equilibrium outcome.

The uniqueness result is anything but an obvious implication of our equilibrium notion. As we will show in the next section multiple equilibrium outcomes may obtain if we relax the assumption that the electoral campaign is a zero-sum game.

[7] Analogous conditions can be provided if $z_k^{*A} = 1$ for all k and $z_k^{*B} = 0$ for some k.

Theorem 4.2 *For $k = 1, \ldots, K$, let $e^k \in \{0,1\}^K$ be such that $e^k_k = 1$ and $e^k_l = 0$ for all $l \neq k$.*

(i) If $(CE(s))_{s\in S}$ is an equilibrium collection of sets of continuation campaigns, then for all $s \in S$, there is a unique continuation equilibrium in $CE(s)$. Moreover, all equilibrium collections of sets of continuation campaigns are outcome equivalent, i.e. if $(CE(s))_{s\in S}$ and $\left(\widehat{CE}(s)\right)_{s\in S}$ are two equilibrium collections of sets of continuation campaigns, then for all $s \in S$, $CE(s) = \{s'\}$ and $\widehat{CE}(s) = \{s''\}$ implies that $z(s') = z(s'')$.

(ii) For all (z^A, z^B) let $p^*(z^A, z^B)$ be the unique vote share of candidate A in all continuation equilibria at s where $z(s) = (z^A, z^B)$. Then

$$p^*(z^A, z^B) =$$

$$\begin{cases} \min\limits_{k:z^B_k=0} p^*(z^A, z^B + e^k), \ \text{if } p(z^A, z^B) > \min\limits_{k:z^B_k=0} p^*(z^A, z^B + e^k) \\ p(z^A, z^B), \ \text{if } \min\limits_{k:z^B_k=0} p^*(z^A, z^B + e^k) > p(z^A, z^B) > \max\limits_{k:z^A_k=0} p^*(z^A + e^k, z^B) \\ \max\limits_{k:z^A_k=0} p^*(z^A + e^k, z^B), \ \text{if } \max\limits_{k:z^A_k=0} p^*(z^A + e^k, z^B) > p(z^A, z^B) \end{cases}$$

where the minimum (maximum) over the empty set is defined to be ∞ ($-\infty$). In particular, it is true that

$$\min_{k:z^B_k=0} p^*(z^A, z^B + e^k) \geq p^*(z^A, z^B) \geq \max_{k:z^A_k=0} p^*(z^A + e^k, z^B).$$

Theorem 4.2 can also be used to determine all equilibrium collections of sets of continuation campaigns. The construction is by backwards induction:

Let s be a full campaign, i.e. $z(s) = (\mathbf{1}, \mathbf{1})$. Then it is obviously true that $CE(s) = \{s\}$. Now consider a campaign s with $z(s) < (\mathbf{1}, \mathbf{1})$ and assume that $CE(\hat{s})$ has been determined for all $\hat{s}$ with $z(\hat{s}) > z(s)$. Then from Theorem 4.2 (ii) there are three cases.

1. If $p^*(z^A, z^B) = p(z^A, z^B)$ the campaign ends at s and $CE(s) = \{s\}$.

2. If $p^*(z^A, z^B) = \min_{k:z^B_k=0} p^*(z^A, z^B + e^k)$, then $CE(s) = \{\hat{s}\}$ for some $\hat{s}$ with $\hat{s} = (s, (k, B), \ldots)$, where $CE((s, (k, B)) = \{\hat{s}\}$ and $p^*(z^A, z^B + e^k) = \min_{l:z^B_l=0} p^*(z^A, z^B + e^l)$.

3. If $p^*(z^A, z^B) = \max_{k:z^A_k=0} p^*(z^A + e^k, z^B)$, then $CE(s) = \{\hat{s}\}$ for some $\hat{s}$ with $\hat{s} = (s, (k, A), \ldots)$, where $CE((s, (k, A)) = \{\hat{s}\}$ and $p^*(z^A + e^k, z^B) = \max_{l:z^A_l=0} p^*(z^A + e^l, z^B)$.

5 An Extension

So far, we have assumed that candidates want to maximize their vote shares, and that they have the same perception regarding their chances. One consequence of this zero-sum game property is the uniqueness of the equilibrium outcome (see Theorem 4.2). Now we will explore what happens if we abandon the zero-sum game property. More specifically, we will consider an extension of our model, where candidates face costs for addressing issues. Our particular way of turning the game into a non-zero sum one, by combining vote shares with campaigning costs is only an illustrative example. Obviously, payoff functions may have more complex origins.

Let $c^i \geq 0$ be the cost of candidate $i \in \{A, B\}$ for addressing an issue. We assume that costs are additive, i.e. if $z^i \in \{0, 1\}^K$ records the issues addressed by candidate i, then i's total cost is

$$c^i \sum_{k=1}^{K} z^i_k.$$

If (z^A, z^B) is the outcome at a campaign, then A's utility is

$$U^A(z^A, z^B) = p(z^A, z^B) - c^A \sum_{k=1}^{K} z^A_k,$$

and B's utility is

$$U^B(z^A, z^B) = 1 - p(z^A, z^B) - c^B \sum_{k=1}^{K} z^B_k.$$

The adjustment of the equilibrium conditions then is straightforward. We just have to replace $P(s)$ by $U^A(z(s))$ and $1 - P(s)$ by $U^B(z(s))$.

Note that with positive costs the electoral campaign is not a zero-sum game anymore. This changes the equilibrium predictions as we can already demonstrate for the case with one issue.

Let $K = 1$. Then under the following conditions $CE(\varnothing) = \{(A), (B)\}$, i.e. different from the case with zero costs (see Theorem 4.2) there are two equilibrium campaigns which are not outcome equivalent. In one equilibrium campaign only candidate A addresses the issue and in the other only candidate B addresses the issue:

$$
\begin{aligned}
p(0,1) &> p(1,1) - c^A && (21)\\
1 - p(1,0) &> 1 - p(1,1) - c^B && (22)\\
p(1,0) - c^A &> p(0,0) && (23)\\
p(1,0) - c^A &> p(0,1) && (24)\\
1 - p(0,1) - c^B &> 1 - p(1,0) && (25)
\end{aligned}
$$

For example, conditions (21)-(25) are satisfied if

$$
\begin{aligned}
&p(0,0) = 0.5,\ p(1,0) = 0.8,\ p(0,1) = 0.6,\ p(1,1) = 0.7,\\
&c^A = c^B = 0.15.
\end{aligned}
$$

(21) implies that $CE(B) = \{(B)\}$ and (22) implies that $CE(A) = \{(A)\}$, i.e. the campaign stops when one candidate has addressed the issue (here we use equilibrium conditions (E2) and (E3)).

(23) implies that stopping is no equilibrium at the empty campaign, i.e. $\varnothing \notin CE(\varnothing)$ (here we use equilibrium condition (E2)).

(24) implies that candidate A prefers (A) over (B) and (25) implies that candidate B prefers (B) over (A). Using equilibrium condition (E3) this implies that $CE(\varnothing) = \{(A), (B)\}$.

Conditions (21)-(25) cannot be satisfied simultaneously if $c^A = c^B = 0$ because in that case (21) implies that

$$p(0,1) > p(1,1)$$

and (22) implies that

$$p(1,1) > p(1,0).$$

Hence, $p(0,1) > p(1,0)$ which violates (24).

Note that by (E3) having multiple equilibrium continuations at some campaign s that are not outcome equivalent implies that there exist at least two equilibrium continuations at s that are not initiated by the same candidate. Otherwise, one of the continuations would not be rationalizable. We may then have situations where a candidate, let's say A, continues a campaign because one of the possible equilibrium continuations is such that A moves again and this is favorable for A relative to stopping, even if there is another equilibrium continuation, where B moves that is unfavorable for A relative to stopping.[8] This suggests that we may get very different equilibrium predictions if we would assume a fixed order of moves of the candidates rather than deriving the order endogenously as part of our protocol-free equilibrium concept. The following example illustrates this point for the case with two issues.

Example 5.1 Let $K = 2$ and let candidate A's vote share be given in the following table.

		z^B			
		00	10	01	11
z^A	00	0.5	0.3	0.95	0.99
	10	0.94	0.51	0.61	0.6
	01	0.3	0.82	0.2	0.4
	11	0.1	0.8	0.83	0.7

[8]Recall equilibrium condition (E2) according to which the campaign stops if and only if the addition of one further issues makes each candidate worse off, no matter which equilibrium continuation is taken from there.

Let costs be given by $c^A = c^B = 0.15$. Figure 9 then illustrates the equilibrium continuation campaigns. The equilibrium campaign is unique and given by

$$CE(\varnothing) = \{\varnothing\}.$$

Note that there are multiple equilibrium continuations at states $(10, 10)$ and $(10, 01)$ which are not outcome equivalent and which are initiated by different candidates. Consider a campaign s with state $z(s) = (10, 00)$. (E2) then implies that stopping is no equilibrium continuation because if B continues to $(10, 10)$, then there is an equilibrium continuation leading to $(10, 11)$ which gives B a higher utility than at $(10, 00)$. (E3) then implies that $CE(s) = \{(s, (1, B), (2, B))\}$ or $CE(s) = \{(s, (2, B), (1, B))\}$.

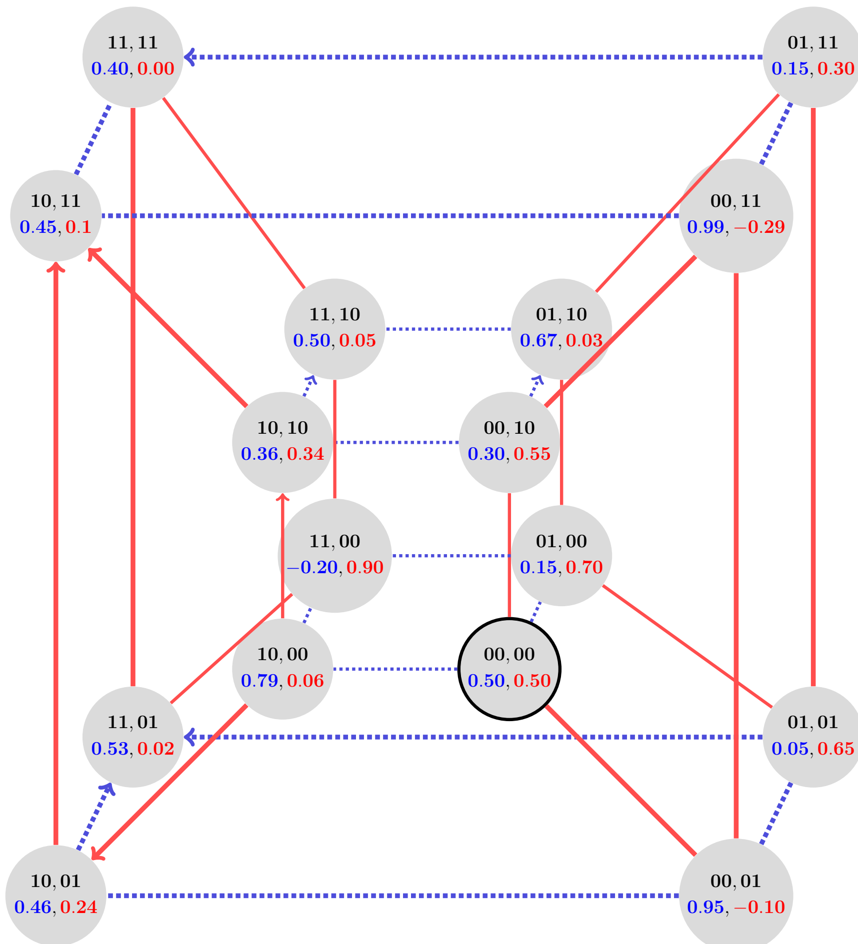

Figure 9: Equilibrium continuations in Example 5.1. The lower numbers in a node are the candidates' utilities U^A, U^B, at the given campaign. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Example 5.1 illustrates the difference between our protocol-free equilibrium notion and subgame perfect Nash equilibrium in an extensive form game where candidates take turn and each candidate can either address one single issue or pass if it is her turn. The game ends if both candidates have addressed both issues or if one candidate has addressed both issues and the other passes or if there are two passes in a row. The reader can then verify that independent of which candidate is the first mover, the unique subgame perfect equilibrium outcome for the parameters in Example 5.1 is $(10, 00)$, i.e. candidate A addresses issue 1 and candidate B addresses no issue.

6 Conclusion

We offer a stark but attractive model to study the phenomena associated with campaign formation. Solving for equilibria may be demanding if there are many issues, but the backwards induction procedure is no more complex than solving for subgame perfect Nash equilibria in an extensive form game where candidates move according to some exogenously given order. We are convinced that we add significant realism to the study of campaign formation decisions by dispensing from the aprioristic assumptions introduced by any protocol determining a fixed order of play. Our analysis highlights the effect of allowing players to decide whether or not to move at any point of the game, and identifies the paths that will lead rational players to equilibrium outcomes. Even in the case considered here, where campaign payoffs are independent of the order in which issues were included in the agenda, not any path leading to equilibrium is admissible as part of an equilibrium in our sense.

Our model and results highlight the interaction between issues, thus challenging the possibility of identifying the role of each one of them separately. We show that the characteristics of equilibrium can be very sensitive to parameter changes allowing for different combinations of silences and voice in campaigns. We provide sufficient conditions for any campaign configuration to arise in equilibrium, and present a general result (Theorem 4.2) on the uniqueness of equilibrium outcomes and on properties of the candidates vote shares in equilibrium. For some relevant special cases we have also provided explicit characterizations of equilibria.

Moreover, we show that issue ownership is a useful concept to better understand the shape of equilibrium campaigns, but not the unique determinant of their shape, even in simple contexts. We prove that multiple equilibrium outcomes may arise if payoffs are not zero-sum, but not if that condition is met, as it is the case if payoffs only value the vote shares of each candidate.

We do not deny the importance in reality of many variables that our model omits. Our concern has been to prove that, in fact, relevant insights can be obtained even before appealing to other qualifications.

Appendix

Equilibrium campaigns for one issue

There are the following cases.

Case 1: $p(1,1) < \min\{p(1,0), p(0,1)\}$

In this case, (E1) and (E2) imply that

$$CE(A) = \{(A,B)\} \text{ and } CE(B) = \{(B)\},$$

which in turn implies that $CE(\varnothing) \subset \{\varnothing, (B), (A,B)\}$ by (E1). We then obtain the following subcases:

(i) $p(0,0) < p(1,1)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{(A,B)\}.$$

This case is illustrated in Figure 10.

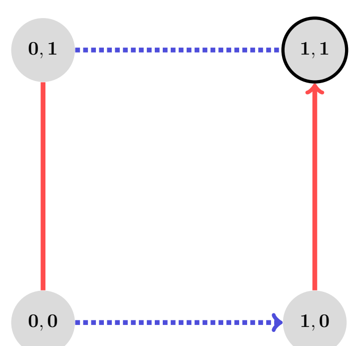

Figure 10: Equilibrium continuations for $p(0,0) < p(1,1) < \min\{p(1,0), p(0,1)\}$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

(ii) $p(1,1) < p(0,0) < p(0,1)$

This case was considered in the main text.

(iii) $p(0,1) < p(0,0)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{(B)\}.$$

This case is illustrated in Figure 11.

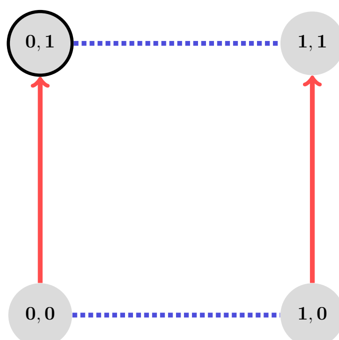

Figure 11: Equilibrium continuations for $p(1,1) < p(0,1) < p(0,0)$ and $p(1,1) < p(1,0)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Case 2: $\max\{p(1,0), p(0,1)\} < p(1,1)$

In this case, (E1) and (E2) imply that

$$CE(A) = \{(A)\} \text{ and } CE(B) = \{(B, A)\},$$

which in turn implies that $CE(\varnothing) \subset \{\varnothing, (A), (B, A)\}$ by (E1). We then obtain the following subcases:

(i) $p(0,0) < p(1,0)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{(A)\}.$$

This case is illustrated in Figure 12.

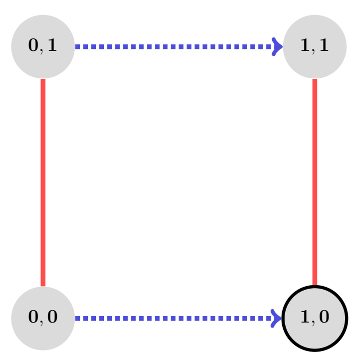

Figure 12: Equilibrium continuations for $p(0,0) < p(1,0) < p(1,1)$ and $p(0,1) < p(1,1)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

(ii) $p(1,0) < p(0,0) < p(1,1)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{\varnothing\}.$$

This case is illustrated in Figure 13.

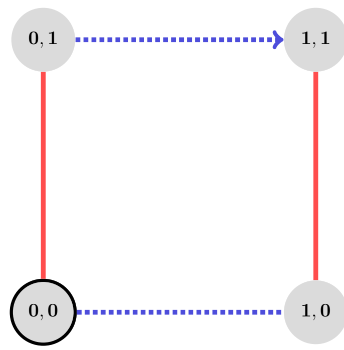

Figure 13: Equilibrium continuations for $p(1,0) < p(0,0) < p(1,1)$ and $p(0,1) < p(1,1)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

(iii) $p(1,1) < p(0,0)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{(B, A)\}.$$

This case is illustrated in Figure 14.

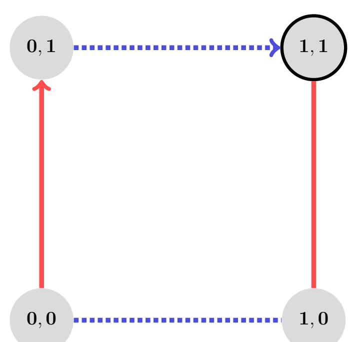

Figure 14: Equilibrium continuations for $\max\{p(1,0), p(0,1)\} < p(1,1) < p(0,0)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Case 3: $p(1,0) < p(1,1) < p(0,1)$
In this case, (E1) and (E2) imply that

$$CE(A) = \{(A)\} \text{ and } CE(B) = \{(B)\},$$

which in turn implies that $CE(\varnothing) \subset \{\varnothing, (A), (B)\}$ by (E1). We then obtain the following subcases:

(i) $p(0,0) < p(1,0) < p(0,1)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{(A)\}.$$

This case is illustrated in Figure 15.

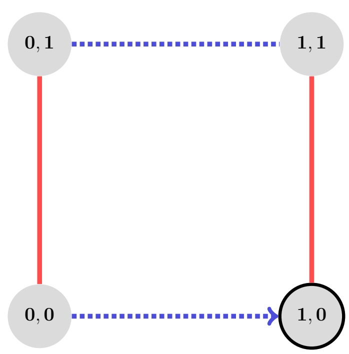

Figure 15: Equilibrium continuations for $p(0,0) < p(1,0) < p(1,1) < p(0,1)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

(ii) $p(1,0) < p(0,0) < p(0,1)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{\varnothing\}.$$

This case is illustrated in Figure 16.

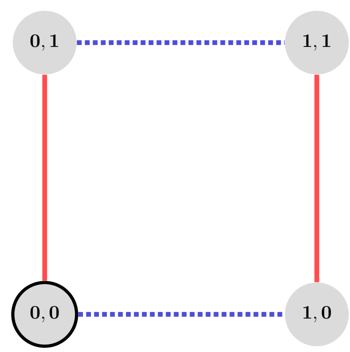

Figure 16: Equilibrium continuations for $p(1,0) < p(0,0) < p(0,1)$ and $p(1,0) < p(1,1) < p(0,1)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

(iii) $p(1,0) < p(0,1) < p(0,0)$

In this case, (E2) and (E3) imply that

$$CE(\varnothing) = \{(B)\}.$$

This case is illustrated in Figure 17.

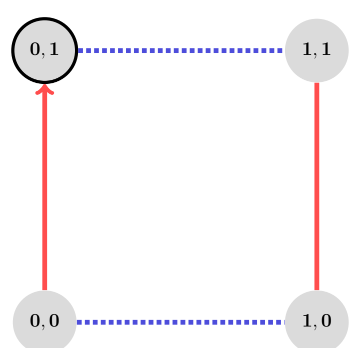

Figure 17: Equilibrium continuations for $p(1,0) < p(1,1) < p(0,1) < p(0,0)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Case 4: $p(0,1) < p(1,1) < p(1,0)$
In this case, (E1) and (E2) imply that

$$CE(A) = \{(A,B)\} \text{ and } CE(B) = \{(B,A)\},$$

which in turn implies that $CE(\varnothing) \subset \{\varnothing, (A,B), (B,A)\}$ by (E1). By (E2), $\varnothing \notin CE(\varnothing)$. If $p(0,0) < p(1,1)$, then (E3) implies that

$$CE(\varnothing) = \{(A,B)\}.$$

This case is illustrated in Figure 18.

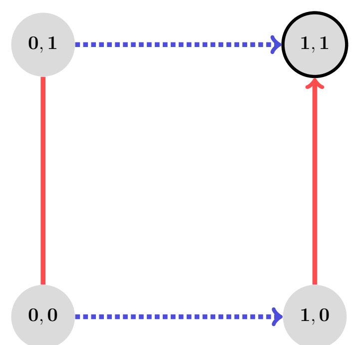

Figure 18: Equilibrium continuations for $p(0,1) < p(1,1) < p(1,0)$ and $p(0,0) < p(1,1)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

If $p(1,1) < p(0,0)$, then (E3) implies that

$$CE(\varnothing) = \{(B,A)\}.$$

This case is illustrated in Figure 19.

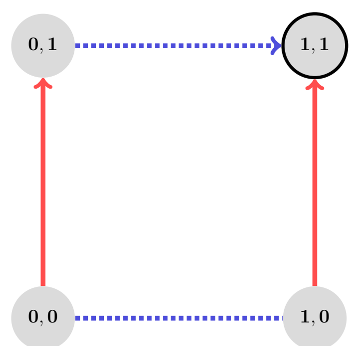

Figure 19: Equilibrium continuations for $p(0,1) < p(1,1) < p(1,0)$ and $p(1,1) < p(0,0)$. Arrows denote the equilibrium path. The bordered node corresponds to the equilibrium continuation at $\varnothing$.

Proofs

Proof of Theorem 3.1: Let $p(z^A, z^B) \neq p(\hat{z}^A, \hat{z}^B)$ for all $(z^A, z^B), (\hat{z}^A, \hat{z}^B) \in Z$ with $(z^A, z^B) \neq (\hat{z}^A, \hat{z}^B)$. Let $(CE(s))_{s \in \bar{S}}$ be an equilibrium collection of sets of continuation campaigns and let $(z^A, z^B) \in Z$. If $z_1^i + z_2^i = 1$ for $i = A, B$, then (E1) implies that $CE(s) = \{s\}$ for s with $z(s) = (z^A, z^B)$. This implies that $p^*(z^A, z^B) = p(z^A, z^B)$.

Next consider $(z^A, z^B) = (z^A, 00)$ with $z_1^A + z_2^A = 1$ and let s be such that $z(s) = (z^A, 00)$. Then (E1) implies that $CE(s) \subseteq \{s, (s, (1, B)), (s, (2, B))\}$. By (E2) $s \in CE(s)$ if and only if $p(z^A, 00) < \min\{p(z^A, 10), p(z^A, 01)\}$. Moreover, if the latter condition holds, then neither $(s, (1, B))$ nor $(s, (2, B))$ are rationalizable and hence (E3) implies that $CE(s) = \{s\}$ which in turn implies that $p^*(z^A, 00) = p(z^A, 00)$.

If $p(z^A, 00) > \min\{p(z^A, 10), p(z^A, 01)\}$, then $CE(s) \subseteq \{(s, (1, B)), (s, (2, B))\}$. If $p(z^A, 10) < p(z^A, 01)$, then suppose by way of contradiction that $(s, (2, B)) \in CE(s)$. Then $(s, (1, B))$ is rationalizable and hence (E3) implies that $(s, (1, B)) \in CE(s)$. But then $(s, (2, B))$ is not rationalizable and hence by (E3) $(s, (2, B)) \notin CE(s)$ which is a contradiction. We conclude that $p(z^A, 10) < \min\{p(z^A, 00), p(z^A, 01)\}$ implies that $CE(s) = \{(s, (1, B))\}$ and therefore $p^*(z^A, 00) = p(z^A, 10)$.

In the same way one proves that $p(z^A, 01) < \min\{p(z^A, 00), p(z^A, 10)\}$ implies that $CE(s) = \{(s, (2, B))\}$ and therefore $p^*(z^A, 00) = p(z^A, 01)$.

Summarizing, if s is such that $z(s) = (z^A, 00)$ with $z_1^A + z_2^A = 1$, then there is a unique continuation equilibrium in $CE(s)$ and

$$p^*(z^A, 00) = \min\{p(z^A, 00), p(z^A, 10), p(z^A, 01)\}. \tag{26}$$

Now consider $(z^A, z^B) = (00, z^B)$ with $z_1^B + z_2^B = 1$ and let s be such that $z(s) = (00, z^B)$. Then (E1) implies that $CE(s) \subseteq \{s, (s, (1, A)), (s, (2, A))\}$. By (E2) $s \in CE(s)$ if and only if $p(00, z^B) > \max\{p(10, z^B), p(01, z^B)\}$. Moreover, if the latter condition holds, then neither $(s, (1, A))$ nor $(s, (2, A))$ are rationalizable and hence (E3) implies that $CE(s) = \{s\}$ which in turn implies that $p^*(00, z^B) = p(00, z^B)$.

If $p(00, z^B) < \max\{p(10, z^B), p(01, z^B)\}$, then $CE(s) \subseteq \{(s, (1, A)), (s, (2, A))\}$. If $p(10, z^B) > p(01, z^B)$, then suppose by way of contradiction that $(s, (2, A)) \in$

$CE(s)$. Then $(s,(1,A))$ is rationalizable and hence (E3) implies that $(s,(1,A)) \in CE(s)$. But then $(s,(2,A))$ is not rationalizable and hence by (E3) $(s,(2,A)) \notin CE(s)$ which is a contradiction. We conclude that $p(10,z^B) > \max\{p(00,z^B), p(01,z^B)\}$ implies that $CE(s) = \{(s,(1,A))\}$ and therefore $p^*(00,z^B) = p(10,z^B)$.

In the same way one proves that $p(01,z^B) > \max\{p(00,z^B), p(10,z^B)\}$ implies that $CE(s) = \{(s,(2,A))\}$ and therefore $p^*(00,z^B) = p(01,z^B)$.

Summarizing, if s is such that $z(s) = (00,z^B)$ with $z_1^B + z_2^B = 1$, then there is a unique continuation equilibrium in $CE(s)$ and

$$p^*(00,z^B) = \max\{p(00,z^B), p(10,z^B), p(01,z^B)\}. \tag{27}$$

(26) and (27) imply that

$$\begin{aligned}
p^*(10,00) &\le p(10,10) \le p^*(00,10),\\
p^*(10,00) &\le p(10,01) \le p^*(00,01),\\
p^*(01,00) &\le p(01,10) \le p^*(00,10),\\
p^*(01,00) &\le p(01,01) \le p^*(00,01),
\end{aligned}$$

from which we conclude that

$$\max\{p^*(10,00), p^*(01,00)\} \le \min\{p^*(00,10), p^*(00,01)\}.$$

Finally, consider $(z^A, z^B) = (00,00)$ and let $s = \varnothing$. From the previous analysis we know that there is a unique continuation equilibrium at $s = (k,i)$ for $k = 1,2$, and $i = A,B$. Let $CE((k,i)) = \{s_k^i\}$ for $k = 1,2$, and $i = A,B$. (E1) then implies that $CE(\varnothing) \subseteq \{\varnothing, s_1^A, s_2^A, s_1^B, s_2^B\}$. By (E2) $\varnothing \in CE(s)$ if and only if

$$\max\{p^*(10,00), p^*(01,00)\} < p(00,00) < \min\{p^*(00,10), p^*(00,01)\}.$$

Moreover, if the latter condition holds then s_k^i is not rationalizable for $k = 1,2$, and $i = A,B$. Hence, (E3) implies that $CE(\varnothing) = \{\varnothing\}$ which in turn implies that $p^*(00,00) = p(00,00)$.

If $p(00,00) > \min\{p^*(00,10), p^*(00,01)\} \ge \max\{p^*(10,00), p^*(01,00)\}$, then $CE(\varnothing) \subseteq \{s_1^A, s_2^A, s_1^B, s_2^B\}$. W.l.o.g. let $p^*(10,00) \le p^*(01,00)$. Then s_1^A is not rationalizable and (E3) implies that $s_1^A \notin CE(\varnothing)$ which in turn implies that s_2^A is not rationalizable and hence $s_2^A \notin CE(\varnothing)$ by (E3). Therefore, $CE(\varnothing) \subseteq \{s_1^B, s_2^B\}$. Since $z(s_1^B) \neq z(s_2^B)$ it follows that $p^*(00,10) = p(z(s_1^B)) \neq p(z(s_2^B)) = p^*(00,01)$.

If $p^*(00,10) > p^*(00,01)$, then suppose by way of contradiction that $s_1^B \in CE(\varnothing)$. Then s_2^B is rationalizable and hence (E3) implies that $s_2^B \in CE(\varnothing)$. But then s_1^B is not rationalizable and therefore $s_1^B \notin CE(\varnothing)$ by (E3) which is a contradiction. Therefore, $p^*(00,10) > p^*(00,01)$ implies that $CE(\varnothing) = \{s_2^B\}$ and $p^*(00,00) = p^*(00,01)$. Similarly, $p^*(00,10) < p^*(00,01)$ implies that $CE(\varnothing) = \{s_1^B\}$ and $p^*(00,00) = p^*(00,10)$.

In any case we conclude that if

$$p(00,00) > \min\{p^*(00,10), p^*(00,01)\} \geq \max\{p^*(10,00), p^*(01,00)\},$$

then there is a unique equilibrium continuation in $CE(\varnothing)$ and

$$p^*(00,00) = \min\{p^*(00,10), p^*(00,01)\}.$$

Finally, if $p(00,00) < \max\{p^*(00,10), p^*(00,01)\} \leq \min\{p^*(00,10), p^*(00,01)\}$, then $CE(\varnothing) \subseteq \{s_1^A, s_2^A, s_1^B, s_2^B\}$. W.l.o.g. let $p^*(00,01) \leq p^*(00,10)$. Then s_1^B is not rationalizable and (E3) implies that $s_1^B \notin CE(\varnothing)$ which in turn implies that s_2^B is not rationalizable and hence $s_2^B \notin CE(\varnothing)$ by (E3). Therefore, $CE(\varnothing) \subseteq \{s_1^A, s_2^A\}$. Since $z(s_1^A) \neq z(s_2^A)$ it follows that $p^*(10,00) = p(z(s_1^A)) \neq p(z(s_2^A)) = p^*(01,00)$.

If $p^*(10,00) < p^*(01,00)$, then suppose by way of contradiction that $s_1^A \in CE(\varnothing)$. Then s_2^A is rationalizable and hence (E3) implies that $s_2^A \in CE(\varnothing)$. But then s_1^A is not rationalizable and therefore $s_1^A \notin CE(\varnothing)$ by (E3) which is a contradiction. Therefore, $p^*(10,00) < p^*(01,00)$ implies that $CE(\varnothing) = \{s_2^A\}$ and $p^*(00,00) = p^*(01,00)$. Similarly, $p^*(00,01) < p^*(10,00)$ implies that $CE(\varnothing) = \{s_1^A\}$ and $p^*(00,00) = p^*(10,00)$.

In any case we conclude that if

$$\min\{p^*(00,10), p^*(00,01)\} \geq \max\{p^*(10,00), p^*(01,00)\} > p(00,00),$$

then there is a unique equilibrium continuation in $CE(\varnothing)$ and

$$p^*(00,00) = \max\{p^*(10,00), p^*(01,00)\}.$$

This proves the theorem.

$\square$

Proof of Lemma 4.1: The proof is by backwards induction: Consider a state where one candidate i has addressed all issues and the other candidate $j \neq i$ has addressed all but one issue. Then (17) implies that j can increase her vote share by addressing the remaining issue. Hence, the unique equilibrium continuation is a full campaign.

Consider now an arbitrary state (z^A, z^B) and assume that for all $(\hat{z}^A, \hat{z}^B) > (z^A, z^B)$ it is true that any continuation equilibrium at a campaign s with $z(s) = (\hat{z}^A, \hat{z}^B)$ is a full campaign. Then (20) implies that either candidate A or candidate B can increase her vote share by continuing from (z^A, z^B) which will then lead to a full campaign by the induction hypothesis. Hence, the unique equilibrium continuation at (z^A, z^B) is a full campaign.

□

Proof of Theorem 4.1: For all k, let $e^k \in \{0,1\}^K$ be such that $e^k_k = 1$ and $e^k_l = 0$ for all $l \neq k$.

1. The claim immediately follows from Lemma 4.1.

2. If candidate A continues from (z^{*A}, z^{*B}) to $(z^{*A} + e^k, z^{*B})$ for some k with $z^{*A}_k = 0$, then A's vote share in any continuation equilibrium will be below $\hat{p}(z^{*A}, z^{*B}) = p(z^{*A}, z^{*B})$ by assumption on the vote shares. To see this note that B will not add further issues because this would lead to a full campaign by Lemma 4.1 and $p(\mathbf{1}, \mathbf{1}) = \hat{p}(\mathbf{1}, \mathbf{1}) > \hat{p}(z^{*A}, z^{*B}) > p(z^A, z^{*B})$ for all z^A with $z^A_k = 1$ for some k with $z^{*A}_k = 0$. For the same reason candidate B will not continue from (z^{*A}, z^{*B}) to $(z^{*A}, z^{*B} + e^k)$ for some k with $z^{*B}_k = 0$. Hence, the campaign will stop at (z^{*A}, z^{*B}).

   We will now prove by backwards induction that any equilibrium continuation at a campaign with state $(z^A, z^B) < (z^{*A}, z^{*B})$ is a campaign with state (z^{*A}, z^{*B}). Let $(z^A, z^B) = (z^{*A} - e^k, z^{*B})$ for some k with $z^{*A}_k = 1$. Then A can increase her vote share by continuing to (z^{*A}, z^{*B}) while any continuation to $(z^{*A} - e^k + e^l, z^{*B})$ for some l with $z^{*A}_l = 0$ leads to a vote share below $\hat{p}(z^{*A}, z^{*B}) = p(z^{*A}, z^{*B})$ for the same reason as above. Moreover, if B continues to $(z^{*A} - e^k, z^{*B} + e^l)$ for some l with $z^{*B}_l = 0$, then any equilibrium continuation will be a full campaign by Lemma 4.1 which is worse for candidate B than (z^{*A}, z^{*B}). Hence, the unique state in any equilibrium continuation at $(z^{*A} - e^k, z^{*B})$ is (z^{*A}, z^{*B}).

Let $(z^A, z^B) = (z^{*A}, z^{*B} - e^k)$ for some k with $z_k^{*B} = 1$. Then B can increase her vote share by continuing to (z^{*A}, z^{*B}) while any continuation to $(z^{*A}, z^{*B} - e^k + e^l)$ for some l with $z_l^{*B} = 0$ leads to a full campaign where A's vote share is above $p(z^{*A}, z^{*B})$. Moreover, if A continues to $(z^{*A} + e^l, z^{*B} - e^k)$ for some l with $z_l^{*A} = 0$, then A's vote share will be below $\hat{p}(z^{*A}, z^{*B}) = p(z^{*A}, z^{*B})$ in any continuation equilibrium which is worse for candidate A than (z^{*A}, z^{*B}). Hence, the unique state in any equilibrium continuation at $(z^{*A}, z^{*B} - e^k)$ is (z^{*A}, z^{*B}).

Consider now an arbitrary state $(z^A, z^B) < (z^{*A}, z^{*B})$ and assume that for all $(\hat{z}^A, \hat{z}^B)$ with $(z^A, z^B) < (\hat{z}^A, \hat{z}^B) < (z^{*A}, z^{*B})$ it is true that the unique state in any continuation equilibrium at $(\hat{z}^A, \hat{z}^B)$ is (z^{*A}, z^{*B}). By (20) either candidate A or candidate B can increase their vote share by continuing to $(z^A + e^k, z^B)$ for some k with $z_k^A = 0$ and $z_k^{*A} = 1$, or to $(z^A, z^B + e^k)$ for some k with $z_k^B = 0$ and $z_k^{*B} = 1$ as this will lead to state (z^{*A}, z^{*B}) by the induction hypothesis. Moreover, for the same reason as above neither A nor B will continue to $(z^A + e^k, z^B)$ with $z_k^{*A} = 0$ or to $(z^A, z^B + e^k)$ with $z_k^{*B} = 0$. Hence, the unique state in any equilibrium continuation at (z^A, z^B) is (z^{*A}, z^{*B}).

□

Proof of Theorem 4.2: Let $(CE(s))_{s \in S}$ be an equilibrium collection of sets of continuation campaigns and let s be a campaign with $z(s) = (z^A, z^B)$. The proof is by induction over L, where $L = \#\{(i,k) |\, z_k^i = 0\}$. Note that $0 \leq L \leq 2K$.

If s is such that $L = 0$, then $(z^A, z^B) = (1 \ldots 1, 1 \ldots 1)$ and by (E1) $CE(s) = \{s\}$ which implies $p^*(1 \ldots 1, 1 \ldots 1) = p(1 \ldots 1, 1 \ldots 1)$. This proves the claim for $L = 0$.

Let s be such that $L = 1$. First consider the case where $z_k^A = 0$ for some k. W.l.o.g. let $k = 1$ which implies $(z^A, z^B) = (01 \ldots 1, 1 \ldots 1)$. Then from the case $L = 0$ we know that $(1 \ldots 1, 1 \ldots 1)$ is the unique continuation campaign at $(z^A + e^1, z^B)$. Moreover, by (E1) $CE(s) \subseteq \{s, (s, (1, A))\}$. By (E2)

$s \in CE(s)$ if and only if $p(01\ldots1,1\ldots1) > p(1\ldots1,1\ldots1)$. If the latter condition is satisfied, then $s \in CE(s)$ and (E3) implies that $(s,(1,A)) \notin CE(s)$ because if it were true that $(s,(1,A)) \in CE(s)$, then by (E3) it would have to be true that $p(1\ldots1,1\ldots1) > p(01\ldots1,1\ldots1)$ which is not the case. Hence, if $p(01\ldots1,1\ldots1) > p(1\ldots1,1\ldots1)$, then $CE(s) = \{s\}$. If $p(01\ldots1,1\ldots1) < p(1\ldots1,1\ldots1)$, then $s \notin CE(s)$ by (E2) and nonemptiness of $CE(s)$ implies that $CE(s) = \{(s,(1,A))\}$ which also satisfies (E3) because $p(1\ldots1,1\ldots1) > p(01\ldots1,1\ldots1)$. In any case there is a unique continuation equilibrium in $CE(s)$ and A's vote share $p^*(01\ldots1,1\ldots1)$ in the unique continuation equilibrium satisfies

$$p^*(01\ldots1,1\ldots1) = \max\{p^*(1\ldots1,1\ldots1), p(01\ldots1,1\ldots1)\}.$$

Next consider the case where $z_k^B = 0$ for some k. W.l.o.g. let $k = 1$ which implies $(z^A, z^B) = (1\ldots1, 01\ldots1)$. Then from the case $L = 0$ we know that $(1\ldots1,1\ldots1)$ is the unique continuation campaign at (z^A, z^B+e^1). Moreover, by (E1) $CE(s) \subseteq \{s,(s,(1,B))\}$. By (E2) $s \in CE(s)$ if and only if $p(1\ldots1,1\ldots1) > p(1\ldots1,01\ldots1)$. If the latter condition is satisfied, then $s \in CE(s)$ and (E3) implies that $(s,(1,B)) \notin CE(s)$ because if it were true that $(s,(1,B)) \in CE(s)$, then by (E3) it would have to be true that $p(1\ldots1,1\ldots1) < p(01\ldots1,1\ldots1)$ which is not the case. Hence, if $p(1\ldots1,01\ldots1) < p(1\ldots1,1\ldots1)$, then $CE(s) = \{s\}$. If $p(1\ldots1,1\ldots1) < p(1\ldots1,01\ldots1)$, then $s \notin CE(s)$ by (E2) and nonemptiness of $CE(s)$ implies that $CE(s) = \{(s,(1,B))\}$ which also satisfies (E3) because $p(1\ldots1,1\ldots1) < p(1\ldots1,01\ldots1)$. In any case there is a unique continuation equilibrium in $CE(s)$ and A's vote share $p^*(1\ldots1,01\ldots1)$ in the unique continuation equilibrium satisfies

$$p^*(1\ldots1,01\ldots1) = \min\{p^*(1\ldots1,1\ldots1), p(1\ldots1,01\ldots1)\}.$$

This proves the claim for $L = 1$.

Let $2 \leq M \leq 2K$ and assume that the claim has been proved for all L with $0 \leq L \leq M-1$. Let s be such that $L = M$. From the induction hypothesis we then know that for all k with $z_k^i = 0$ all continuation equilibria in $CE((s,(k,i)))$ are outcome equivalent. Moreover, if l is such that $z_l^A = 0$, then by the induction hypothesis

$$\min_{l':z^B_{l'}=0} p^*(z^A+e^l, z^B+e^{l'}) \geq p^*(z^A+e^l, z^B) \geq \max_{l'\neq l:z^A_{l'}=0} p^*(z^A+e^l+e^{l'}, z^B), \quad (28)$$

and if k is such that $z^B_k=0$, then

$$\min_{k'\neq k:z^B_{k'}=0} p^*(z^A, z^B+e^k+e^{k'}) \geq p^*(z^A, z^B+e^k) \geq \max_{k':z^A_{k'}=0} p^*(z^A+e^{k'}, z^B+e^k), \quad (29)$$

where the minimum (maximum) over the empty set is defined to be ∞ $(-\infty)$. (28) and (29) imply that for all k and l such that $z^A_l=0$ and $z^B_k=0$,

$$p^*(z^A, z^B+e^k) \geq p^*(z^A+e^l, z^B+e^k) \geq p^*(z^A+e^l, z^B). \quad (30)$$

(30) implies that

$$\min_{k:z^B_k=0} p^*(z^A, z^B+e^k) \geq \max_{k:z^A_k=0} p^*(z^A+e^k, z^B). \quad (31)$$

From (E1) we know that $CE(s) \subseteq \bigcup_{(k,i):z^i_k=0} CE((s,(k,i))) \cup \{s\}$. By (E2) $s \in CE(s)$ if and only if

$$\min_{k:z^B_k=0} p^*(z^A, z^B+e^k) > p(z^A, z^B) > \max_{k:z^A_k=0} p^*(z^A+e^k, z^B). \quad (32)$$

Assume that (32) is satisfied which implies $s \in CE(s)$. Suppose by way of contradiction that there exist some (k,i) with $z^i_k=0$ such that $s' \in CE(s) \cap CE((s,(k,i)))$. W.l.o.g. let $i=A$. Let

$$K^A = \{k|\, z^A_k=0 \text{ and } \exists\, s' \in CE(s) \cap CE((s,(k,A)))\} \quad (33)$$

and let

$$\hat{k} \in \{k \in K^A|\, p^*(z^A+e^k, z^B) \leq p^*(z^A+e^l, z^B) \text{ for all } l \in K^A\}. \quad (34)$$

Then $s' \in CE(s) \cap CE((s,(\hat{k},A)))$ is not rationalizable which contradicts (E3). Hence, if (32) is satisfied, then $CE(s)=\{s\}$ and

$$p^*(z^A, z^B) = p(z^A, z^B).$$

Assume now that

$$p(z^A, z^B) > \min_{k:z^B_k=0} p^*(z^A, z^B+e^k) \geq \max_{k:z^A_k=0} p^*(z^A+e^k, z^B).$$

Then (32) is violated and (E2) implies that $s \notin CE(s)$. Suppose by way of contradiction that $\exists\, s' \in CE(s) \cap CE((s,(k,A)))$ for some k with $z_k^A = 0$. Let K^A and $\hat{k}$ be defined as in (33) and (34), respectively. Then $s' \in CE(s) \cap CE((s,(\hat{k},A)))$ is not rationalizable which together with $p^*(z^A + e^{\hat{k}}, z^B) < p(z^A, z^B)$ implies that (E3) is violated. Hence, $CE(s) \cap CE((s,(k,A))) = \emptyset$ for all k with $z_k^A = 0$. (E1) then implies that

$$CE(s) \subseteq \bigcup_{(l,B):z_k^B=0} CE((s,(l,B))).$$

Let

$$L^B = \{l|\, z_l^B = 0 \text{ and } p^*(z^A, z^B + e^l) \leq p^*(z^A, z^B + e^k) \text{ for all } k \text{ with } z_k^B = 0\}. \quad (35)$$

Suppose by way of contradiction that $CE(s) \cap CE((s,(l,B))) = \emptyset$ for all $l \in L^B$ and let $s' \in CE((s,(l,B)))$ for some $l \in L^B$. Then s' is rationalizable and hence (E3) implies that $s' \in CE(s)$ which is a contradiction. Hence, there exists some $l \in L^B$ and some $s' \in CE(s) \cap CE((s,(l,B)))$.

Suppose by way of contradiction that $CE(s) \cap CE((s,(l,B))) \neq \emptyset$ for some $l \notin L^B$ with $z_l^B = 0$. Let $\bar{l} \notin L^B$ be such that there exists some $s' \in CE(s) \cap CE((s,(\bar{l},B)))$ and $p^*(z^A, z^B + e^{\bar{l}}) \geq p^*(z^A, z^B + e^l)$ for all l with $CE(s) \cap CE((s,(l,B))) \neq \emptyset$. Then s' is not rationalizable and hence (E3) is violated which is a contradiction. Hence, $CE(s) \cap CE((s,(l,B))) = \emptyset$ for all $l \notin L^B$ with $z_l^B = 0$. This implies that all continuation equilibria in $CE(s)$ are outcome equivalent and

$$p^*(z^A, z^B) = \min_{k:z_k^B=0} p^*(z^A, z^B + e^k).$$

Finally, assume that

$$\min_{k:z_k^B=0} p^*(z^A, z^B + e^k) \geq \max_{k:z_k^A=0} p^*(z^A + e^k, z^B) > p(z^A, z^B).$$

Then analogously to the previous case ones shows that all continuation equilibria in $CE(s)$ are outcome equivalent and

$$p^*(z^A, z^B) = \max_{k:z_k^A=0} p^*(z^A + e^k, z^B).$$

This proves the claim for $L = M$ and concludes the proof of the theorem.

$\square$

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