Barcelona School of Economics

WORKING PAPERS

The Bright Side of the Doom Loop: Banks' Exposure and Default Incentives

BSE Working Paper 1143
December 2019 (Revised: December 2021)

Luis E. Rojas, Dominik Thaler

The Bright Side of the Doom Loop: Banks' Exposure and Default Incentives
latest version

Luis E. Rojas and Dominik Thaler*

December 7, 2021

Abstract

The feedback loop between sovereign and financial sector insolvency has been identified as a key driver of the European debt crisis and has motivated an array of policy proposals. We revisit this "doom-loop" focusing on governments' incentives to default. To this end, we present a simple 3-period model with strategic sovereign default, where debt is held by domestic banks and foreign investors. The government maximizes domestic welfare, and thus the temptation to default increases with externally-held debt. Importantly, the costs of default arise endogenously from the damage that default causes to domestic banks' balance sheets. Internally-held debt thus serves as a commitment device for the government. We show that two prominent policy prescriptions – lower exposure of banks to domestic sovereign debt or a commitment not to bailout banks – can backfire, since default incentives depend not only on the quantity of debt, but also on who holds it. Conversely, allowing banks to buy additional sovereign debt in times of sovereign distress can avert the doom loop.

Keywords: Sovereign Default; Bailout; Doom Loop; Self-fulfilling Crises

JEL Classification E44, E6, F34.

*Luis E. Rojas: UAB, MOVE and Barcelona School of Economics; luis.rojas@uab.es. Dominik Thaler: European Central Bank; dominik.thaler@eui.eu. Rojas acknowledges support from the Research Council Advanced Grant (APMPAL) GA 324048; and from the Spanish Ministry of Economy and Competitiveness, through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

1 Introduction

The "doom loop" or "sovereign-bank nexus" has been identified as a key driver of the European debt crisis and has come back into the spotlight in these current times, since the response to the public health crisis is causing sovereign debt to skyrocket.[1] The core of the argument is that problems with sovereign debt sustainability and with financial sector stability reinforce each other due to the mutual exposure between the two systems. If sovereign debt loses value due to deteriorating creditworthiness of the public sector, financial sector balance sheets are hurt, due to the large amounts of public debt these hold. Weakened financial institutions, in turn, force the government to bailout the financial system – banks for short. Bailouts entail expenses for the government and hence a further deterioration of its fiscal capacity.[2] This vicious circle can amplify fundamental shocks (Acharay et al., 2014, Farhi and Tirole, 2016) or even give rise to crises that are entirely generated by self-fulfilling pessimistic expectations (Brunnermeier et al., 2016, Brunnermeier et al. (2017), Cooper and Nikolov, 2013), hence explaining how sovereign crises can develop suddenly and spiral out of control easily.

However, the existing doom loop theory has focused on how financial sector exposure to the sovereign affects government incentives to bail out banks while abstracting how it affects the incentives to repay. In fact, there is a strand of the literature emphasizing that the larger the exposure, the larger the incentives to repay. For example, Bolton and Jeanne (2011) show that domestic banks' default is particularly costly when these hold sovereign debt. In this framework, domestic exposure to sovereign risk serves as a commitment device for governments to pay back their debt. Supporting this view, Gennaioli et al. (2014) show that sovereign default premia decrease in domestic banks' exposure to their sovereign.

In this paper we extend the doom-loop theory by allowing the government to default strategically and by subsequently studying how its incentives to honor its debt are shaped by the exposure of the domestic financial sector. We identify two channels through which the exposure of the financial sector to sovereign debt shapes the default incentives. The *temptation channel* establishes that the higher the exposure, the lower the foreign creditors holding of debt and consequently the less tempting it is to default. The *commitment channel* captures that the higher the exposure, the larger the disruption on the financial sector and consequently the larger the cost of a default.

We use our setup to evaluate two prominent policy proposals that have been put forward to address the doom loop. In particular, Cooper and Nikolov (2013) advocate a commitment by the sovereign not to bailout banks, while Brunnermeier et al. (2016, 2017) suggest to reduce the exposure of domestic banks to the government, by reducing their public debt holdings or by pooling and tranching debt of several sovereigns, to the same end. Our theory puts into question the suitability of the above mentioned policies to break the doom loop. Such policies may come at a cost that makes them undesirable, while policies that may appear counterproductive at

[1]E.g. Brunnermeier (2015), Benassy-Quere et al. (2018)

[2]As Brunnermeier et al. (2016) argue, weaker balance sheets also affect the public sector indirectly by causing a credit crunch, which leads to lower output and hence a reduction in the tax base.

first, such as allowing banks to stack up on domestic debt, may turn out to be beneficial under given circumstances.

We develop these arguments through a simple 3-period model of sovereign debt and banks with multiple equilibria, similar to the ones used in the papers cited above. In period 0, the government has to finance a fixed expenditure by issuing debt, which is bought by foreign investors and domestic banks. Furthermore, banks also make loans to entrepreneurs financed through deposits and equity. In period 1, a sunspot shock may hit the economy. If it does, the government bond price may drop to a lower value, causing banks to fail. Bank failure entails a cost, because a fraction of the banks' loans gets destroyed in that event, so that the government has an incentive to intervene and bail banks out. To do so, it needs to issue more debt, which, in our baseline model, is bought by foreign investors. In period 2, all debt comes to maturity. The government defaults strategically, but does not discriminate across creditors. Sovereign default is costly in two ways: first, default causes national output to drop; second, if default drives banks to bankruptcy, then entrepreneurs that were financed by banks are more likely to fail. The model features a sunspot equilibrium in which the sunspot variable triggers the Doom Loop: a drop in the bond price causes a bank bailout, which increases debt and hence makes default more likely, thus validating the initial drop in bond prices.

We then analyze several policy options under our novel theoretical framework, proceeding in three steps. In the first step, we show that if the bailout is carried out by capitalizing banks directly with the transfer of sovereign debt, then the Doom Loop ceases to exist. This result comes from the fact that the newly issued debt to implement the bailout is held locally and therefore it does not increase the default incentives. Alternatively, this outcome could also be achieved with secondary markets, as long as banks are allowed to buy sovereign debt with the bailout funds. The key to the results in this step is what we refer to as the *temptation channel:* incentives to default on foreign debt are higher than on domestic debt.

In the second step, we consider policies that rule out the doom loop but may nevertheless be undesirable. In particular, we show that increasing banks' equity ratios or rebalancing their portfolio away from domestic sovereign debt does disable the doom loop – but on the other hand eliminates the commitment device that local banks' exposure provides to the government. Therefore, such policies undermine sovereign debt sustainability and make sovereign debt more costly, potentially reducing welfare as a consequence. This argument also applies to the case in which the increase in banks' equity ratios is the result of a no-bailout commitment, as in Cooper and Nikolov (2013).[3] This analysis highlights the *commitment channel:* banks' exposure increases the costs of default thus providing additional commitment

In a third step, we consider two extensions. First, we consider a multi-country extension of the model, to show that bundling and tranching bonds of many countries belonging to a union, as suggested by Brunnermeier et al. (2017) in the context of the ESBies scheme, does not resolve the doom loop and can be detrimental for welfare. Second, while our baseline specification focuses on multiplicity of equilibria, in another extension we show that the same

[3] Cooper and Nikolov (2013) argue in favor of such a commitment not because it is effective per se, but because it incentivizes banks to self-insure against sovereign default by increasing their equity sufficiently.

model also generates amplification. We illustrate this for the example of a news shock, a case that may resemble the current health crisis. We show that all the policy conclusions from earlier on apply to this case: policies that do not succeed in ruling out multiple equilibria are also not effective in ruling out amplification; policies that rule out amplification by increasing equity or reducing domestic bond holdings come at a large cost and may hence be undesirable.

The model we propose is attractive because, despite its simplicity, it is able to illustrate how a significant exposure of the domestic financial sector to the sovereign may not be as problematic as suggested in previous theory; debt re-nationalization in adverse times may be just what is needed to prevent a market turmoil from developing into a full-blown crisis. This argument is particularly relevant today, as government debt skyrockets across Europe and beyond. We thus provide an argument against policies that restrict the financial sectors exposure to domestic debt, which was prominently advocated by a group of German and French economists (Benassy-Quere et al., 2018). This proposal was soon criticized by Messori and Micossi (2018). Indeed, our model provides a formalization of their critique.

2 Related literature

Our paper builds a bridge between two strands of literature. The first is the literature on the doom loop. Brunnermeier et al. (2016, 2017) and Cooper and Nikolov (2013) propose 3 period models that are very similar to ours. In their models multiplicity arises through the exact same mechanisms. Leonello (2017) shows that the doom loop can exist even if banks hold no explicit claims to the government on their balance sheets (bonds or debt) but enjoy government guarantees (deposit insurance, bailouts) and resolves the multiplicity of equilibria through global games. Acharay et al. (2014) and Farhi and Tirole (2016) provide a slightly different notion of the doom loop. Instead of generating multiple equilibria, the doom loop serves as an amplifier, so that small fundamental changes can lead to large changes in the equilibrium. Our paper incorporates both notions of the doom loop and the policy conclusion apply to both as well. What distinguishes our model is that in these models the default incentives increase in total sovereign debt, while in our model the default incentives increase only in foreign held debt but not in bank held debt, which rather serves as a commitment device. This leads us to arrive to contrary policy conclusions.

The second strand regards the commitment role of domestic exposure to sovereign debt. This idea has been developed both in 3 period models (Balloch 2016, Basu 2010, Bolton and Jeanne 2011, Brutti 2011, Erce 2012, Gennaioli et al. 2014 and Mayer 2011) as well as in quantitative dynamic models (Boz et al. 2014, Balke 2018, Engler and Grosse Steffen 2016, Mallucci 2014, Sosa-Padilla 2018, Perez 2015, Thaler 2019). It is furthermore backed by empirical evidence. E.g. Gennaioli et al. (2014) show that sovereign default premia decrease in domestic bank's exposure to their sovereign.[4] Relative to this literature we contribute by adding a notion of the doom loop.

[4]For similar evidence see Sturzenegger and Zettelmeyer (2007), Acharay et al. (2014), Bolton and Jeanne (2011), Reinhart and Rogoff (2011), and Balteanu and Erce (2017)

3 Model

3.1 Setup

We consider a three period economy $t = 0, 1, 2$ with five types of agents: Bankers, households, entrepreneurs, the government and international creditors. The key player is the government, which has to decide on bailouts and debt repayment.

First we present a brief overview of the timing and sequence of events and then we move on to describe in detail the players and contracts available.

At $t = 0$ the government sells bonds to international creditors and local banks in order to finance an exogenous level of expenditure. Entrepreneurs require external financing to undertake risky investment projects that payoff at $t = 2$ and households have initial resources and decide on savings. Banks intermediate resources by providing the loans to entrepreneurs and collecting deposits from households.

At $t = 1$ the realization of a sunspot variable is revealed $s \in \{n, p\}$ unexpectedly, $s = p$ constitutes an unforeseen panic state without any effect on fundamentals that can coordinate beliefs on a possibly lower expected repayment by the government.[5] Then the government reacts by announcing if it plans to bailout the banks and issue debt to finance the bailout. The depositors follow by deciding if to early withdraw their deposits. In case depositors decide to withdraw the bank has to liquidate their assets whereby liquidating the loans to entrepreneurs has the cost that it generates that a fraction of the projects fail. This is the cost of a banking disruption.

At $t = 2$ the productivity of the investment projects is revealed. The government decides if to repay or default on all of its outstanding debt. Default has a direct cost on output and an indirect cost through the banking sector by possibly generating a bank run and loans costly liquidation. Depositors decide if to early withdraw before production takes place or if to wait until production is completed. Production takes place with the surviving projects and finally agents consume.

The households and the deposit contract

There is a continuum of measure 1 of households with an initial endowment at $t = 0$ of Y_0^h. Households only derive utility from consumption at $t = 2$ and are risk neutral. Savings can be done in two ways: using an storage technology with zero net return or placing deposits at a bank.

A deposit is an asset issued by the bank at $t = 0$ that has a face value of 1 at the end of period $t = 2$ after production takes place. If the depositor decides to early withdraw in $t = 1$ or in $t = 2$, then the face value are in each case λ_1 and λ_2 correspondingly. The face values set an upper bound on the recovery rate a depositor can get in case of early withdrawal, it can be

[5] Could be thought of as having a prior in $t = 0$ of $s = n$ with probability 1. Or an MIT type of shock, that is non-fundamental and can only have effects on the allocation through beliefs.

lower if the bank is insolvent. We will limit attention to the case where $\lambda_1 < \lambda_2 < 1$ such that waiting to withdraw offers a positive interest rate at face value.

At $t = 0$ the household decides how many deposits to hold. Denote p_0^D to be the price of placing a deposit (buying the asset), then the budget constraint is given by

$$Y_0^h \geq p_0^D D_0 + S_0^h$$

where D_0 is the number of deposits placed and S_0^h are the resources in the storage technology. The household uncertainty is with respect to the payoff of the deposit.

At $t = 1$, after the sunspot is revealed and the government announced if it plans a bailout to the banks, the households decide if to early withdraw their deposits or not. If they early withdraw they obtain a payment of $\tilde{\lambda}_1$ per unit of deposits (recovery rate), otherwise they just keep the asset. Finally at $t = 2$, after the government announces the repayment decision, the households decide if to early withdraw with a recovery rate $\tilde{\lambda}_2$ or if to wait until production takes place to claim repayment from the bank.

The depositor takes as given the price of deposits p_0^D, and the recovery rates $\tilde{\lambda}_t$ in each possible scenario, these objects will de obtained in equilibrium.

Entrepreneurs and international creditors

There is a measure K_0 of entrepreneurs, each of them with an investment project that provides a random gross return ω in $t = 2$ in case the project survives. Each project requires an initial investment of 1 that is financed with loans by the banks. In case the loan is liquidated at $t-1, 2$ there is a probability θ_t the project fails. All projects that succeed repay in full the loan.

The international creditors are risk neutral investors with deep pockets. They have an opportunity cost of their funds of a zero net rate or return. Therefore they will buy sovereign debt at $t = 0$ and at $t = 1$ as long as the expected net return is greater or equal than zero, Since the face value of the sovereign debt is equal to 1, then as long as the international creditors are the marginal investors, the price of sovereign debt q is given by the repayment probability.

The bankers and the lending contract

There is a continuum of measure one of bankers with an initial endowment of Y_0^b. Bankers derive utility from consumption at $t = 2$ and are risk neutral. Just as households, bankers have access to a storage technology with zero net return. On top of this, bankers can take deposits, buy sovereign debt and provide loans to entrepreneurs.

To approximate monopoly power and give raise to intermediation margins we assume that as an industry banks collude in the deposit market to set the price p_0^D such that depositors are indifferent between the storage technology or the deposits. So in the deposit market, the banks have market power but they are competitive in the loans market.[6]

[6] As we discuss in the next sections, most of our results go through with competitive banks in both markets. What is relevant from the imperfect competition structure we assume is that the banks are more competitive in the loans market than in the deposit markets, as we characterize how the intermediation margin varies with

The loans to entrepreneurs are assets with face value 1 that mature at $t = 2$ after production. If the bank has to early liquidate this asset then a fraction of funded projects fail and default. If the liquidation is in $t = 1$, θ_1 projects fail. If the liquidation happens in $t = 2$ before production, then the failed projects is $\theta_2 \leq \theta_1$.

Denote p_0^L the price of the loan asset and L_0 the total number of projects financed. Then the balance sheet of the representative bank is characterized by

$$Y_0^b + p_0^d D_0 = S_0 + p_0^L L_0 + q_0 B_0^h \tag{1}$$

where the left hand side is the initial capital Y_0^b and the resources obtained by issuing deposits. The right hand side is the amount of resources placed in the storage technology S_0, the resources given to the loans issued $p_0^L L_0$ and the resources devoted to buy sovereign debt $q_0 B_0^h$; where q_0 is the price and B_0^h the quantity held by the local bank. Portfolio composition and leverage are the key decisions of the banker to maximize expected consumption at $t = 2$. The following two periods the bank is passive and just liquidates the necessary asset holdings to satisfy the early withdrawers and can possibly receive a bailout.

At $t = 1$ the bank might receive a transfer from the government (bailout) in safe assets ΔS_1. In case there is a bank run at $t = 1, 2$ the bank is supposed to pay to depositors $\lambda_t D_0$ by liquidating its assets. Nevertheless the value of the bank portfolio might not be enough to cover the face value of deposits, consequently the effective repayment rate $\tilde{\lambda}_t$ can be lower and given by

$$\tilde{\lambda}_t = \max \left\{ \lambda_t, \frac{S_1 + (1 - \theta_t) p_t^L L_0 + q_t B_t^h}{D_0} \right\}$$

where the safe asset holdings $S_1 = S_0 + \Delta S_1$ captures the possibility that the bank received already a bailout and the loans are multiplied by $(1 - \theta_t) < 1$ since their liquidation imply a destruction of θ_t.

In case the bank does not face a bank run, then the consumption of bankers is given by

$$C^b = (1 - d) B_t^h + S_1 + L_0 - D_0$$

where d is a dummy variable that takes the value of 1 in case of government default.

The government and sovereign debt

The government has to cover a fix level of expenses R in $t = 0$, for which the only possible source of funding is the issuance of sovereign debt with zero-coupon bonds that mature at $t = 2$. Therefore we have that the initial issuance satisfies

$$B_0 = \frac{R}{q_0} \tag{2}$$

the default probability of the sovereign.

where q_0 is the price of sovereign debt. The government takes into account that the price of debt can be a function of the issuance $q_0(B_0)$, and consequently in case there are multiple values of B_0 that satisfy equation (2), then the government selects the minimum of those. This is shown to be consistent with the final objective of the government, the maximization of consumption at $t = 2$ by locals (bankers, entrepreneurs and households). The initial issuance is acquired by local banks B_0^h and foreign creditors B_0^f.

At $t = 1$ the government can issue further debt to finance a bailout transfer to the banks, in order to avoid the inefficient liquidation and consequent failure of loans. Let ΔS_1 be the required bailout, then the amount of new debt that has to be issued is given by

$$\Delta B_1 = \frac{\Delta S_1}{q_1}$$

where q_1 is the price of debt at $t = 1$.

At $t = 2$, after the productivity draw ω is realized, the government decides if to repay in full the outstanding debt or default. The government takes this decision to maximize the local economy consumption, that is aggregated over entrepreneurs, bankers and households. The consumption at $t = 2$ is given by

$$C = Y_2 + S_1 - (1 - d)B_1^f$$

where Y_2 is output at $t = 2$, S_1 the safe assets held by locals and $(1 - d)B_1^f$ the repayment to foreign creditors; where in case of default $d = 1$ is zero. Output is given by

$$Y_2 = (1 - d\vartheta)\omega K_2 \tag{3}$$

where ϑ captures the direct output loss of default. K_2 are the surviving projects when production takes place, that is equal to L_0 if there is no bank liquidation or is given by $K_2 = (1-\theta_t)L_0$ if there is a liquidation of loans at t. This captures an indirect cost of default, as default can trigger a bank run and banks liquidation.

3.2 Bank's solvency, costs of default and the Doom Loop.

For a bank run to be an equilibrium outcome it has to be the case that a single depositor has no incentives to deviate and decide to keep his funds at the bank. This is the case if after a bank run the bank becomes insolvent, since in this situation the holdout depositors will receive no repayment at all. The condition for insolvency after a bank run is given by[7]

$$\lambda_t D_t \geq S_1 + (1 - \theta_t)p_t^L L_0 + q_t B_t^h \tag{4}$$

that states that the repayment required by withdrawals at face value is larger than the value of the banks portfolio if it is liquidated.

[7]A formal derivation and discussion is in appendix C.1

The condition (4) establishes the key link between sovereign and financial risk. If international creditors perceive a high default probability, then sovereign debt prices q_t will be low, and this opens the possibility of bank insolvency by lowering the value of the banks portfolio.

On the other hand, a financial crises generates higher sovereign risk by lowering the cost of default. From equation (3) we have that the cost of a default is given by $\vartheta\omega K_2$ where if there is no banking crisis we have that $K_2 = L_0$. Instead if there banking disruption at $t = 2$ then θ_2 projects are fail and consequently the cost of default is lower and given by

$$\vartheta(1-\theta_2)\omega L_0$$

Overall, the proportional output loss of a joint sovereign and banking crisis at $t = 2$ is given by the parameter $\Theta = 1-(1-\vartheta)(1-\theta_2) \leq 1$, this parameter will play a central role in our analysis since we will focus on the case where a sovereign defaults triggers a banking crisis.

The two way relationship between sovereign and financial risk is what gives rise to the possibility of the Doom Loop. This feedback generates multiple equilibria in our main setup and amplification of shocks in the extension considered in section (7.2).

3.3 Equilibrium and assumptions

We use sub-game perfection as our equilibrium concept and focus on symmetric equilibria, where all agents of the same type take the same decision. In our definition we exploit the fact that a bank run can be sustained only if condition 4 is satisfied.

Definition 1. The equilibrium is given by the initial issuance of debt B_0; banks balance sheet $\left\{B_0^h, L_0, D_0, S_0\right\}$; a bailout decision rule $b(s)$; a default decision rule $d(\omega, s)$; asset prices $\left\{q_0, q_1^s, p_0^L, p_1^{L,s}, p_2^{L,s}, p_0^D, p_1^{D,s}, p_2^{D,s}\right\}_{s=\{n,p\}}$ and the bank run condition 4 such that

1. The initial issuance raises enough revenue to cover for expenses R at price q_0,

2. The bank's balance sheet maximizes the banker expected consumption at $t = 2$ taking as given the asset prices; the government decision rules for bailout and default; and the bank run condition.

3. The bailout decision maximized the expected national consumption at $t = 2$ taking as given the default policy function (no commitment), asset pricing functions and the bank run condition. The default policy function maximizes consumption at $t = 2$, taking as given the pricing functions and the bank run condition.

4. The asset prices are equal to the expected repayment to be obtained by the marginal buyer.

The equilibrium definition already incorporates the optimal household behavior by using the bank run condition and requiring that the price of deposits equal the expected repayment and consequently the opportunity costs of depositors that is the safe asset.

Condition 4. of the definition is explicit about the repayment obtained by the marginal buyer. This is relevant because in case banks are liquidated, the banker receives no returns on his portfolio. Consequently when the banker values an asset he takes into account only the payoff of the asset in case of bank solvency. In the next section we discuss how this mechanism generates a spillover from sovereign risk to the interest rate on loans.

The equilibrium definition takes into account that the government cannot commit ex-ante to a specified default or bailout policy. In Section 6.2 we discuss the possibility of a no bailout commitment.

In order for the possibility of the doom loop to arise in our model we focus in the case where the banking system is exposed to sovereign debt risk and depending on the valuation of sovereign debt it can become insolvent. The following assumptions guarantee that:

Assumption 1. *Household's endowment is large enough to satisfy*

$$\xi_2 \geq \xi_1 > 0$$

where

$$\xi_1 = \lambda_1 Y_0^d - (1-\theta_1)K_0$$
$$\xi_2 = \lambda_2 Y_0^d - (1-\theta_2)K_0$$

This assumption guarantees that the bank is exposed to a bank run in case of an expected default, since the deposits the bank take are partially invested on sovereign debt. It states that the face value of early withdrawals is larger than the surviving projects in case of liquidation and consequently that there is a low enough price of sovereign debt for which the insolvency condition is satisfied. We refer to ξ_1 as exposure in period $t=1$, as it is a lower bound of the market value of sovereign debt at $t=1$ such that banks remain solvent. If this value is positive then clearly banks are exposed to sovereign debt.

Assumption 2. *Solvency condition: The bankers capital is large enough to satisfy*

$$Y_0^b > \xi_2 - \left(Y_0^d - K_0\right)$$

This condition guarantees that the banks own capital is large enough to sustain an equilibrium where the bank is solvent. A single bank would never face in equilibrium a bank run, we focus then only to runs to the banking system.

Assumption 3. *Default along the equilibrium path*

$$F\left(\frac{1}{\Theta}\frac{K_0}{R+K_0-Y^b-Y^h}\right) > 0$$

This condition guarantees that there is default along the equilibrium path. Ruling out an equilibrium with $q_0 = 1$ equilibrium.

Assumption 4. *There is a run equilibrium if there is no bailout, the resources needed by the government R are large enough such that*

$$F\left(\frac{R-Y^b-Y^h+K_0}{\vartheta(1-\theta_1)K_0}\right) > 1-\frac{\lambda_1 Y_0^h-(1-\theta_1)K_0}{\left(Y_0^h+Y_0^b-K_0\right)}$$

This assumption guarantees that there is an equilibrium where if there is no bailout then there is a bank run. As a bank run lowers the value of assets enough to increase the default probability sufficiently to sustain the bank run in the first place.

Our specification with full default (100% haircut) simplifies the analysis but is not crucial for our results, which can be extended for an arbitrary haircut, adjusting assumptions 1 and 2 accordingly so that banks are exposed to default and solvent in case of no panic. Empirically haircuts are found to be on average 37% (Cruces and Trebesch (2013)).

We rule out the possibility of selective default. In particular, the government cannot decide to discriminate across debt issued at different periods or by the identity of the creditor. These margins seem not to be empirically relevant[8] and as Broner et al. (2010) show, if creditors where to be discriminated then they would trade in secondary markets avoiding that the government can effectively target specific creditors. An alternative way to discriminate would be to default on all outstanding debt and bailout banks in period $t=2$. We also abstract from this possibility, nevertheless in our setup it is equivalent to have the cost of financial disruption θ_2 equal to zero. As the government can always avoid financial disruption if a bailout last period where available. By having $\theta_2>0$ we focus on the case where if banks are exposed to sovereign debt, then default triggers a costly financial disruption.

4 The doom loop

This model allows for several equilibria. We start by describing an equilibrium in which the bond price is unaffected by the sunspot and banks are solvent in period 1. Then we describe an equilibrium where banks are solvent in normal times ($s=n$), but insolvent in panic times ($s=p$). In the second equilibrium, the allocation in normal times coincides with the allocation of the first equilibrium.

Proposition 1. *There exists an equilibrium where the price of debt does not depend on s, the banks are not exposed to bank runs in $t=1$, and no bailout is implemented.*

The initial issuance of debt held by foreigners is the minimum level B_0^f that solves

$$B_0^f=\frac{R+K_0-\left(Y^h+Y^b\right)}{1-F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)}$$

[8] As the few instances of repayment discrimination were carried across other dimensions, such as the currency of issuance and the legal jurusdiction.

and the equilibrium price of debt is given by

$$q_0 = q_1 = 1 - F(\omega^n)$$

where $\omega^n = \frac{1}{\Theta}\frac{B_0^f}{K_0}$ *is the productivity threshold above which the government decides to repay in period* $t = 2$*.*

The proof (Appendix C.1) is simply the solution of the model by backward induction. First solving for the optimal default decision default threshold at $t = 2$, where we obtain the optimal default policy is intuitive and in line with much of the literature on sovereign default: The government defaults for TFP below a threshold $\omega^n = \frac{1}{\Theta}\frac{B_0^f}{K_0}$. The larger the foreign debt burden B_0^f, the larger the incentives to default. Conversely, the higher TFP ω and the greater the number of productive assets K_0 available at the last period, the lower the incentives of default. After all, the "punishment" for default is a proportional loss of the output produced by the productive asset.

Given this optimal default strategy by the government then any issuance of debt to foreigners is priced accordingly and the price is equal to the repayment probability $q = 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$. The resources obtained from abroad are then given by $B_0^f\left(1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)\right)$.

In equilibrium there is a spread between the price of deposits and that of loans to firms. Since in equilibrium banks are exposed to sovereign debt and a default leads to a bank run, then when pricing loans the banker only values the repayment of the loan in case of no sovereign default. This generates that the price of the loans asset equalizes that of sovereign debt, since the payments are perfectly correlated for the banker. On the other hand, deposits are partially repaid also in case of default, and consequently have a higher price (lower implied interest rate). This intermediation margin makes it optimal for the bank to take leverage to the maximum and finance all investment projects.

Consequently the level of resources the local economy devotes to cover government expenditures is given by $Y_0^b + Y_0^h - K_0$ and therefore the foreign investors have to finance the rest and we have that

$$R - \left(Y_0^b + Y_0^h - K_0\right) = B_0^f\left(1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)\right)$$

that is our equilibrium condition in Proposition (1).

A Panic in $t = 1$

The "Doom-Loop" is driven by a perceived higher sovereign default probability that lowers sovereign debt prices to the point that without a bailout there would be a bank run. The condition for a bank run to happen is[9]

$$\lambda_1 D_0 < (1 - \theta_1)L_0 + q_1^p B_0^h$$

[9]Where we have already substituted $S_0 = 0$ since banks hold no safe assets in the initial portfolio.

where q_1^p refers to the price of sovereign debt in case of panic. The term $\lambda_1 D_0$ is the nominal value of deposits if all savers withdrawal at $t = 1$; $(1-\theta_1)L_0$ is the value of the liquidated loans that is the face value of loans L_0 times the fraction of projects that do not fail because of the early liquidation. Therefore the condition establishes that the value at liquidation of the bank is lower than its liabilities with depositors.

The government has the possibility to bailout the bank and avoid the bank run, the bailout transfer to the banks is given by

$$Bailout = \lambda_1 D_0 - (1-\theta_1)L_0 - q_1^p B_0^h$$

that is financed by issuing debt to international creditors, and consequently the increase in debt held by foreigners is given by

$$\Delta B_1^f = \frac{1}{q_1^p} Bailout$$

Even after a bailout, at $t = 2$ a default generates a bank run (Assumption (1)) and consequently the cost Θ to the overall economy. Then the tradeoff faced by the government to default or repay follows the same structure as when there is no panic, only that the outstanding debt held by foreigners is larger. The default threshold is then given by

$$\begin{aligned}\tilde{\omega}^p &= \frac{B_0^f + \Delta B_1^f}{\Theta K_0} \\ &= \omega^n + \frac{1}{\Theta K_0}\frac{Bailout}{q_1^p}\end{aligned}$$

and for investors to break even in expectation when purchasing the newly issued debt, then we have that the price of debt equal the repayment probability $q_1^p = 1 - F(\tilde{\omega}^p)$. The next proposition establishes that under our assumptions such an equilibrium exists and that bailout is optimal for the government.

Proposition 2. *A sunspot equilibrium exists where in case of panic $s = p$ the price of sovereign debt falls in $t = 1$ and banks are bailed out by the new issuance of debt by the government.*

The price of debt in case of panic is given by the solution to the system

$$\begin{aligned}q_1^p &= 1 - F\left(\tilde{\omega}^p\right) \\ \tilde{\omega}^p &= \omega^n + \frac{1}{\Theta K_0}\frac{Bailout}{q_1^p} \\ Bailout &= \lambda_1 D_0 - (1-\theta_1)L_0 - q_1^p B_0^h\end{aligned}$$

where $\tilde{\omega}^p$ is the default threshold in case of panic. The equilibrium variables at $t = 0$ and in case $s = n$ coincide with Proposition 1.

The proof (Appendix C.2) shows that as long as banks are exposed $\xi_1 > 0$ and we have a bounded support for productivity, there is an equilibrium where panic can emerge.[10] The idea is that for positive exposure there are always low enough prices of sovereign debt that make a run possible, and the bounded support guarantees that repayment probability can be as low as necessary as the debt held by foreigners increase and for a finite level of foreign held debt it would be zero.

Note that this is the infamous Doom Loop at play. Just as in Brunnermeier et al. (2016) and Cooper and Nikolov (2013), pessimistic expectations become self-fulfilling. If agents happen to coordinate on the lower bond price, banks become insolvent, forcing the government to increase its debt to finance a bailout, which makes it more likely that the government defaults later on (red curve). A higher default probability in turn justifies a lower bond price (blue curve). The pessimistic expectations are hence validated.

In this discussion of the sunspot equilibrium we focused on the case where bailout is the government's optimal choice. This is guaranteed by the condition that the loans losses in period 1 (θ_1) are a large enough (see Appendix C.2). This assumption is natural – if it were not satisfied, banks would never be bailed out and hence the doom loop would not exist.

For the bailout to be optimal it also has to be feasible. If creditors at $t = 1$ would expect a default with certainty and consequently the price of sovereign debt were zero, then the government could not bailout banks by issuing further debt – a bailout would be infeasible. However, as long as there is some probability mass above $\omega^{p,0}$ (the default threshold in case of no bailout) this panic is not self-fulfilling and consequently not an equilibrium. We focus on this case here.

We can rewrite the system of equations by solving for the bailout to have

$$
\begin{aligned}
q_1^p &= 1 - F\left(\tilde{\omega}^p\right) \\
\tilde{\omega}^p &= \omega^n + \frac{1}{\Theta K_0} \frac{\lambda_1 D_0 - (1-\theta_1) L_0 - q_1^p B_0^h}{q_1^p}
\end{aligned}
$$

the equilibrium is a solution to this system. Let $q(\omega)$ represent the mapping from the default threshold to prices in the first equation and $\omega(q)$ the mapping from prices to the default decision in the second equation. Figure (1) represents these two mappings and the normal and panic equilibrium at $t = 1$, where the shaded region represents the region where sovereign debt prices are low enough to generate a bank run in case of no bailout.

Finally, note that the panic equilibrium, which we depict in the figure, is not locally stable under best response dynamics (as in Brunnermeier et al. (2016)). However, as Cooper and Nikolov (2013) show it is easy to obtain a stable panic equilibrium price by putting adequate restrictions on the c.d.f. $F(\omega)$; modifying then the mapping $q(\omega)$ to have multiple crossing in our setup. All of our analysis goes through if we were to restrict our attention to such a stable equilibrium.

[10]The bounded support is a sufficient but not a necessary condition. All we require is a sufficiently thin right tail.

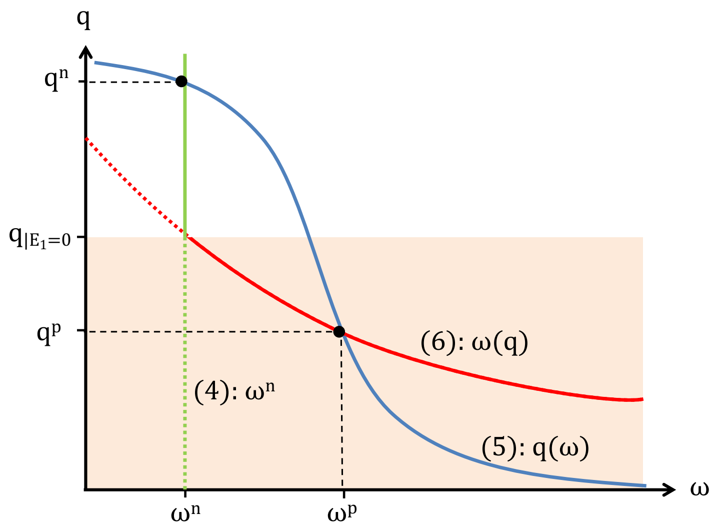

Figure 1: Equilibrium

5 Bailout financing and the Doom Loop

Consider the model as in section 3.1 but now assume that in a bailout the government directly provides the bank with additional sovereign debt, instead of borrowing abroad to finance the safe investment. In this case a bailout no longer increases the foreign debt burden, which remains at its initial value B_0^f. Hence the benefit of default no longer increases in the size of the bailout and thus the bond price $q^{p,1}$ has no reason to fall. Unlike before, the temptation channel is mute.[11] That means that the doom loop, which leads to multiplicity of equilibria in the baseline model, is no longer active and we can rule out the sunspot equilibrium.

Proposition 3. *When the bank is bailed out with domestic bonds, the sunspot equilibrium ceases to exist.*

Thus increasing the exposure of banks by issuing additional debt in times of self-fulfilling expectations driven crises is benign and, in this simple model, in fact rules out such crises altogether. This is so because such a bailout does not interact with the default incentives like a foreign debt financed bailout does. Models where only total debt determines repayment, but not the composition of bond holders, such as Brunnermeier et al. (2017), do not share this feature.

The result in Proposition 3 could also be decentralized by originally financing the bailout with international creditors and letting the newly issued debt to be traded in secondary markets, as we show in Appendix A. What is key in that case is that banks face no restrictions to expand their holdings of sovereign debt by using their bailout funds.

[11]The cost of default does not change with the bailout or the bond price either, because banks are bankrupt in case of default and solvent in case of repayment no matter how many bonds they got in the course of the bailout in period 1.

We can generalize what we have done now by noting that the bailout required by banks to avoid a run is given by

$$Bailout = \lambda_1 D_0 - (1-\theta_1)L_0 - q_1^p B_0^h$$

and the increase on the levels of debt held by foreigners are given by

$$\Delta B_1^f = \chi Bailout$$

where χ parametrizes the cost of financing the bailout for the local economy; how many bonds with face value one have to be handed to the foreign investors.

For a given cost of the bailout we have the expected repayment is given by

$$q_1^p = 1 - F\left(\omega^n + \frac{\chi}{\Theta}\frac{Bailout}{K_0}\right)$$

and consequently the system of equations that an equilibrium has to satisfy are

$$\begin{aligned} Bailout &= \lambda_1 D_0 - (1-\theta_1)L_0 - q_1^p B_0^h \\ q_1^p &= 1 - F\left(\omega^n + \frac{\chi}{\Theta}\frac{Bailout}{K_0}\right) \end{aligned}$$

the Doom Loop we have considered is the special case where $\chi = 1/q_1^p$ and the bailout with bonds is the special case where $\chi = 0$. Overall the doom loop is avoided if χ is low enough. A loan by an international institution or if the marginal investor of newly issued debt charges a low enough interest rate then the doom loop can be avoided. Note that the low χ needs not to be observed in equilibrium. Here is when the Mario Draghi speech "whatever it takes", as the willingness to provide support to sovereign at low cost if considered as a promise on a low χ it can avoid the Doom Loop crisis in our setup.

6 Reducing exposure to break the doom loop?

In the previous section we showed that allowing debt re-nationalization during crises or cheap funding can destroy the loop. The result rest on the fact that only foreign debt *increases default incentives* – the *temptation channel.* In this section we highlight a second difference between domestic and foreign debt: Domestic debt *reduces default incentives* by increasing the costs of default due to the impact on banks balance sheets in period 2 – the *commitment channel.*

Before our analysis didn't depend on the nature of the default costs – all that mattered was $\Theta > 0$ as default always triggered both the exogenous output loss ϑ and the endogenous loss θ_2 caused by bank insolvency. By contrast, now we compare situations in which default triggers bank insolvency to others when it does not. In other words, we now explore the role of bank bond holdings as a commitment device for the government, which requires $\theta_2 > 0$.

Note that this commitment device is of discrete nature. If banks are sufficiently exposed

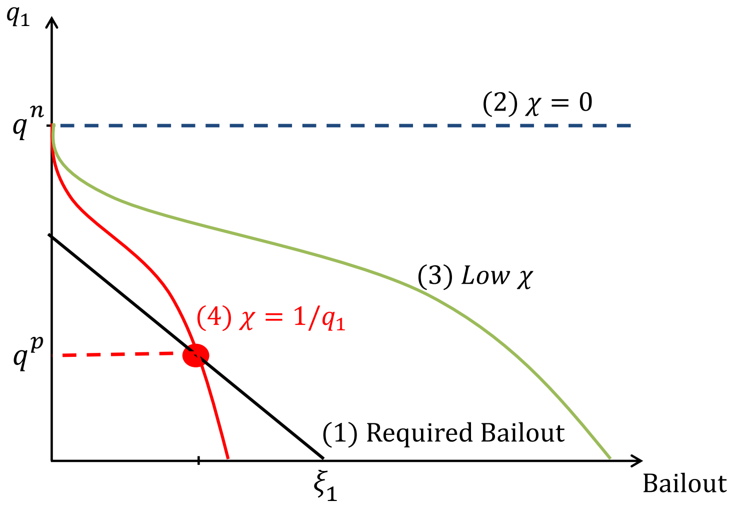

Figure 2: Multiple Equilibria.

default causes them to have negative equity, causing the additional default cost θ. Else, banks have positive equity no matter what and default does not cause the cost θ. This is a simplifying assumption that allowed us to highlight that the results in the previous section do not rely on θ. It could be relaxed by making default costs increasing in the equity shortfall.

6.1 Reducing exposure enough to kill the doom loop

Since the doom loop arose from the bank's fragility, the doom loop can be avoided by reducing the bank's exposure sufficiently to make them immune to fluctuations in the value of sovereign debt. As Brunnermeier et al. (2016) show, this can be achieved by either raising the bank's equity ratio or by reducing their sovereign bond holdings. Indeed, Cooper and Nikolov (2013) use this insight to argue for a no bailout commitment: In their model such a commitment induces banks to self-insure and hold enough equity to never be in need of a bailout.[12] We now revisit these policy proposals.

Compare our baseline economy from before, in which banks are exposed to fluctuations in the price of debt with an alternative economy where banks are not exposed because their safe assets cover their deposit liabilities

$$\lambda_t D_0^{ne} = (1 - \theta_t) L_0 + S_0^{ne} \tag{5}$$

where superscripts *ne* refers to no exposure. In this latter economy banks' equity is non-negative even if the bonds loose all their value. The doom loop thus disappears. Motivated

[12]We do not model the bank's funding choice. However, since the no bailout commitment is irrelevant if the banks has enough equity, we can mimic their policy proposal by simply assuming that the equity ratio is high enough.

by the above cited literature, we consider two variants of the no exposure economy. First, we consider the case that the banks adjusts its liabilities structure, i.e. increases its equity ratio, this is achieved by shifting resources from households to bankers but maintaining the same total endowments for the local economy. Second, we consider the case that bank adjusts its asset structure by buying less domestic debt and instead purchasing more of the safe asset, all else equal. $S_0^{ne,S} > S_0$, $q_0^{ne,S} B_0^{h,ne,S} < q_0 B_0^h$. The superscripts E and S refer to the two variants, no exposure by larger equity E or by higher holding of safe assets S.

The following proposition characterizes the equilibrium with no exposure and establishes that in that case there is no Doom Loop possibility in the economy.

Proposition 4. *In the no exposure economy, for ϑ large enough, there only exists a no-sunspot equilibrium where banks are solvent in $t = 1$ and in $t = 2$.*

If exposure is avoided by having a required level of safe assets holdings we have that $S_0^{ne,S} = \lambda_2 Y_0^h - (1 - \theta_2) K_0$ while if exposure is achieved with higher bank equity then $S_0^{ne,E} = 0$.

The initial issuance of debt held by foreigners in each case is the minimum level $B_0^{f,i}$ that solves

$$B_0^{f,i} = \frac{R + K_0 + S_0^{ne,i} - \left(Y^h + Y^b\right)}{1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right)} \quad \text{for } i \in \{S, E\}$$

and the equilibrium price of debt is given by

$$q_0^{ne,i} = q_1^{ne,i} = 1 - F(\omega^{ne,i})$$

where $\omega^{ne,i} = \frac{1}{\vartheta}\frac{B_0^{f,i}}{K_0}$ is the productivity threshold for which the government decides to repay in period $t = 2$.

6.2 The commitment value of exposure

The amount of funds that the government has to raise from international creditors is unaffected by the endowment of bankers (initial equity of the bank), as long as the total national endowment is constant. However, when the bank has enough equity to be solvent even if the government defaults, the costs of default are lower ($\vartheta < \Theta$). Banks exposure no longer serves as a commitment device. Thus, the default threshold $\omega^{ne} = \frac{1}{\vartheta}\frac{B_0^f}{K_0}$ is higher and the bond price lower $q_0^{ne,E} < q_0$ through the *commitment channel*. That in turn raises the sovereign debt necessary to finance the expenditures R $\left(B_0^{f,ne,E} > B_0^f\right)$. The latter makes default even more attractive through the *temptation channel* mentioned in the introduction, which amplifies the initial drop in the bond price.

Furthermore, if exposure is adjusted by lowering the bank's holding of sovereign debt, then the government is required to raise more funds from the international creditors. This implies that default becomes even more tempting through the *temptation channel*, over and above the loss of commitment already discussed for the previous case.

The economies with no exposure then face larger spreads in normal times, as the government bonds are sold at discount to compensate for the lower repayment probability. The next proposition establishes that spreads are higher in the no exposure economy and that reaching no exposure by accumulating the safe asset increases spreads further.

Proposition 5. *Assume $\theta_2 > 0$. Bond prices are lower in the economies with no exposure than the economy with exposure $q_0^{ne} \leq q_0$. Prices are lower if exposure is avoided by substituting local sovereign debt for the safe asset than if exposure is avoided by increasing equity $q_0^{ne,S} \leq q_0^{ne,E}$. The equalities are strict if $F'(\omega^n) > 0$..*

In sum, no matter how banks exposure is eliminated, the foreign debt burden increases and the cost of default decrease such that default becomes more likely. If there is enough probability mass that the productivity draw can fall in between the default thresholds $\omega^{ne,E}$ and $\omega^{ne,S}$ determining the bond prices in Proposition 5, then the ordering of prices translates into an ordering of welfare: the lower the price the lower expected consumption and thus welfare. The next proposition formalizes this claim:

Proposition 6. *Assume $\theta_2, \vartheta > 0$. If $f(\omega)\omega$ is non-decreasing in the interval $\left[0, \omega^{ne,S}\right]$ then welfare is lower in the economy with no exposure than in the economy with exposure in the no sunspot equilibrium (Proposition 1). If the exposure of banks is avoided by substituting local sovereign debt for the safe asset then welfare falls more than if exposure is avoided by increasing equity.*

The condition that $f(\omega)\omega$ is non-decreasing in the interval $\left[0, \omega^{ne,S}\right]$ is sufficient but not necessary. It guarantees that the probability of default in the economy with no exposure is sufficiently higher such that the expected default costs increase despite of facing a lower cost in case of default. Note that a uniform distribution satisfies this condition, as does a bell shaped distribution to the extent that default is a tail risk. Even if the density is decreasing over this interval the condition can be satisfied as long as $f'(\omega) > -f(\omega)/\omega$.

Note further that the existence of the "normal times" equilibrium is no longer guaranteed. If ϑ is too small – which we have ruled out in the proposition –, then the government simply doesn't have enough (exogenous) commitment to finance its expenditures. That is the *commitment channel* would kick in so strongly that it would be unable to finance the expenditures R. This should certainly have some significant welfare costs, but they are outside our simple model.

We have compared prices and welfare assigning the panic a probability of zero for simplicity. As the probability of the panic increases, prices and welfare in the exposure economy decrease and could fall below those of the no exposure economies. However, by continuity our results continue to hold if the probability of the panic occurring is small enough.

In sum, under certain conditions it is undesirable to kill the doom loop by ex-ante restricting banks exposure to the sovereign. This is true for both policies considered here: higher bank capital ratios and substitution towards safe assets, such as ESBies. Furthermore, the latter policy is more harmful than the first. By extension, a no-bailout commitment is also undesirable even if it causes banks to increase their capital ratios as in Cooper and Nikolov (2013).

Our findings conflict with the policy conclusions of Brunnermeier et al. (2016) and Cooper and Nikolov (2013). The reason for these different conclusions lies in the modeling of default incentives. In their models the incentives to default do not depend on the bank's exposure, only on total debt. Hence ruling out the sunspot equilibrium comes free of any cost for public debt sustainability. On the contrary, in our model policies that kill the doom loop by reducing banks exposure affect default incentives negatively, reducing the commitment to repay and thus increasing default probabilities.[13] At the same time muting the doom loop by allowing banks to act as lenders of last resort, as discussed in section 5, comes without these costs.

Lowering exposure can also open the possibility of multiplicity in $t = 0$. We have assumed so far that the government is able to issue the minimum amount of debt in case there are many equilibrium price/debt levels at $t = 0$. Avoiding exposure can be itself a source of multiplicity at $t = 0$, as by lowering the cost of default then government can be exposed to an early panic.

Recall that the debt held by foreigners is given by

$$B_0^f = \frac{R + K_0 - \left(Y^h + Y^b\right)}{q_0}$$

and the lower the price the largest the face value fo debt held abroad. This is the continuous line in figure (3). On the other hand foreign creditors price sovereign debt according to their expected repayment probability. As given by

$$q_0 = 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$$

in case of exposure, and

$$q_0 = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right)$$

if the banks are not exposed.

By starting in a situation where there is no multiplicity with exposure, then we can have that by removing exposure there could be multiplicity as the commitment value of exposure is lost. This is Illustrated in figure (3) and appendix (B) specifies an F that behaves accordingly. So eliminating the doom loop can result in exposure to multiplicity steaming from the lack of commitment of the government to repay its debts.

7 Extensions

7.1 Diversification, ESBies and the doom loop

One particular proposal to break the doom loop in response to the European debt crisis has been the creation of European Safe Bonds (ESBies) (see Brunnermeier et al. 2017). This proposal

[13] In Brunnermeier et al. and in Cooper and Nikolov's baseline model, default is non-strategic and driven directly by an exogenous "tax capacity" process. Cooper and Nikolov consider strategic default in an extension, but the default incentives are modeled as independent of the bank's balance sheet. Brunnermeier et al. (2016) do however not analyze welfare or claim desirability.

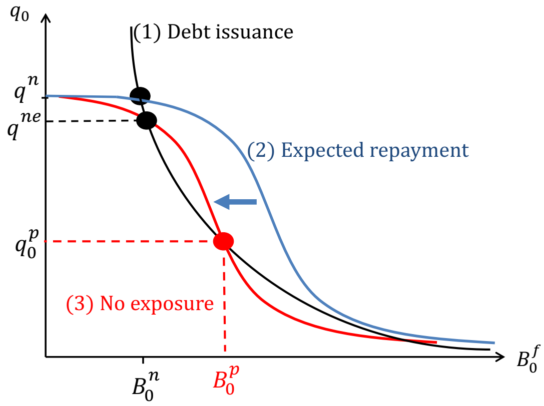

Figure 3: Equilibrium Multiplicity at $t = 0$

consists in creating a European safe asset by tranching a bundle of European sovereign debt. The senior tranche of this collateralized security would constitute a safe asset, and by restricting banks to hold only the senior tranch, as opposed to local sovereign debt, banks would become less exposed to the domestic sovereign and the doom loop would be avoided. If this policy is successful at creating a safe asset, then the doom loop would indeed disappear, as we have shown in the previous section. However, it would also lead to more debt being held by foreigners and thus the commitment value of banks' exposure would be lost. This would come at the costs of higher spreads in normal times and possibly cause welfare losses, as we have shown in the same section.

In this section we turn to another issue that can arise if this policy is implemented, and that renders it even less beneficial: If banks hold a diversified portfolio of sovereign debt, the doom loop may still be present, even if the bundle is tranched (ESBies). Just that the panic happens at the European level, and not at the level of a single country.

To make this point, we extend our single country model to a continuum of identical countries. We show that if the countries in isolation are exposed to the doom loop, then diversification and tranching do not remove that risk. The doom loop persists in the ESBies economy, and its mechanism is closely related to the original doom loop: If investors expect a surge in default rate among European sovereigns in t=1, then the value of the sovereign debt bundle falls, causing bank insolvency and the need for bailouts in all countries. Since bailouts are financed by additional debt issuance, more countries end up defaulting in $t = 2$, validating the initial beliefs. The main two differences with the single country doom loop are: (i) a larger fraction of debt is held by foreigners; (ii) the default decision of a given country has no impact on the local financial system, thus banks' sovereign debt holdings no longer serve as a commitment device and the temptation to default is larger – these are again the two channels highlighted throughout the paper.

Consider a continuum of measure one of ex-ante identical countries, which are each characterized as in the baseline model. The total amount of debt issued by the continuum of countries is split between the amount held to create the asset bundle and the amount held directly by non-European foreign investors as follows

$$\int_0^1 B_0^i di = \mathcal{B}_0 + \int_0^1 B_0^{i,f}$$

where B_0^i is the debt issued at $t = 0$ by country i, $\mathcal{B}_0$ is bundle of sovereign debt and $B_0^{i,f}$ is the bonds issued by country i held by foreign investors. The bundle $\mathcal{B}_0$ is tranched into a senior tranche $\mathcal{B}_0^s$ (the ESBies) and a junior tranch $\mathcal{B}_0^j$ where the subordination level is given by ς. If $\varsigma = 0$ then there is no tranching, only diversification. We normalize the face value of a unit of the senior and the junior tranch equal to one, consequently the total issuance of the senior and junior tranches are given by $\mathcal{B}_0^s = (1 - \varsigma)\mathcal{B}_0$ and $\mathcal{B}_0^j = \varsigma\mathcal{B}_0$. Since we assume all countries to be identical, we focus on a symmetric equilibrium and from now on omit the superscript i.

Assume that banks can only hold the senior tranch of the bundle and, for simplicity, that the volume of the senior tranch $\mathcal{B}_0^s$ is set to exactly satisfy European banks' demand for sovereign debt. The junior tranch will be held by non-European investors. Let Q_t^s be the price of the senior tranch and Q_t^j of the junior tranch. The constraint that total resources equal the value of liabilities and assets of the bank now reads:

$$R = D_0 + E_0 = Q_0^s \mathcal{B}_0^s + L_0 \tag{6}$$

where, as before, the quantities R, L_0 and E_0 are exogenously given parameters.

The timing and decisions are as in the baseline model. However, now at $t = 1$ a sunspot $S = \{N, P\}$ is revealed that affects *all* the countries, where N refers to normal and P to Paneuropean panic. After S is revealed the governments have to decide if to bailout banks in case their equity becomes negative, in order to avoid the destruction of a fraction ϕ of loans. The bailout is financed with the issuance of additional sovereign debt sold to non-European investors. At $t = 2$ countries learn about their idiosyncratic productivity, which is iid distributed. Then each government decides if to repay the outstanding debt. Note that banks' solvency does not depend on the domestic default decision, since they hold the ESBies whose return is certainby the law of large numbers. Therefore only the exogenous output cost ϑ matter.

The following proposition shows that an equilibrium with the doom loop exists.

Proposition 7. *For θ_1 sufficiently large, a subgame-perfect sunspot equilibrium exists and is characterized by the following:*

1. *The initial debt issuance is given by the minium level of debt B_0 that solves*

$$B_0 = R\left(1 - F\left(\frac{1}{\vartheta}\frac{B_0}{L_0}\right)\right)$$

   (a) *The price at issuance is given by*

$$q_0 = 1 - F(\omega^N)$$

   where $\omega^N = \frac{1}{\vartheta}\frac{B_0}{L_0}$.

   (b) *The price of the senior tranch is given by*

$$Q_0^s = \min\left\{1, \frac{1 - F(\omega^N)}{1 - \varsigma}\right\}$$

   and of the junior tranch

$$Q_0^j = \max\left\{0, \frac{\varsigma - F(\omega^N)}{1 - \varsigma}\right\}$$

2. *For $S = N$ (normal times) no bailout is necessary. The prices are the same as at issuance and the default threshold is ω^N*

3. *For $S = P$ (panic) a bailout is implemented and the default threshold and the price of debt are the solution to the system*

$$\begin{aligned}
\omega^{esbies} &= \frac{1}{\vartheta}\frac{B_0}{L_0} + \frac{1}{\vartheta}\frac{\frac{\lambda_2 Y_0^h - (1-\theta_2)L_0}{q_1^p} - B_0}{L_0} \\
q_1^p &= 1 - F\left(\omega^{esbies}\right)
\end{aligned}$$

   where ω^{esbies} is the default threshold. The price of the two tranches are given by

$$\begin{aligned}
Q^{P,j} &= 0 \\
Q^{P,s} &= \frac{1 - F(\omega^{esbies})}{1 - \varsigma}
\end{aligned}$$

The conditions for the existence of a panic equilibrium are the same as in Proposition 2. As long as banks are exposed to sovereign debt $D_0 > L_0$, bailouts are desirable and TFP ω has bounded support such an equilibrium exists. Diversification and tranching do not remove this equilibrium.

So how does the sunspot equilibrium without ESBies from Proposition 2 compare to the sunspot equilibrium with ESBies here? One evident difference is that the default threshold in normal times now depends on the ratio of total debt to exogenous default costs B_0/ϑ, as opposed to the ratio of foreign held debt and the sum of the exogenous costs of default and the financial disruption B_0^f/Θ. This outcome is essentially the same as in Proposition 5, just that here we assume that all debt is pooled and thus held by foreigners, whereas there we focused on the case where only a part is sold to foreigners. ESBies just provide a way to create the safe asset, which before we simply assumed to exist.[14] Extending Propositions 5 and 6 it is straightforward to rank the bond prices q^n and q^N from Propositions 2 and 7 and the associated levels of welfare.[15] Conditional on the normal state, the bond price is lower with ESBies than without and the default probability higher. Furthermore, welfare is lower with ESBies under the conditions on F in proposition 6.

The possibility of a panic $S = P$, and the associated higher default rates show that the introduction of ESBies do not rule out the doom loop. It nevertheless changes the nature of it, as now the panic affects all the countries at once. This result holds for any level of subordination. What a higher level of subordination does is to increase the spreads and consequently the fraction of defaults observed in the panic equilibrium. The higher the subordination, the greater the panic needs to be to make the banks insolvent and consequently the higher the cost of the bailouts. This translates into welfare (conditional on the panic) decreasing as subordination goes up.

Our results are in stark contrast with Brunnermeier et al. (2016, 2017). In those papers, diversification and tranching effectively rules out the doom loop. In particular for a high enough level of bank capitalization, the introduction of ESBies removes the risk of the doom loop. There are several differences between the two setups,[16] but the two key difference that explain why ESBies are effective in their setup are the following: First, they assume that bailouts are financed by issuing senior government debt that is paid back with certainty even if the remaining debt is partially defaulted upon and that is thus sold at face value. By contrast, in our model the government finances the bailout by issuing additional sovereign debt that has no preferential treatment with respect to previously issued debt and consequently is valued at market prices. Second, in our model default is strategic such that bond holders may end up getting nothing, while in theirs the government mechanically pays bond holders as much as it can given an exogenous tax capacity, such that they always get something. The first difference strengthens the strategic complementarities in our model, the second ensures the existence of a sunspot equilibrium with a nonzero bond price.

Since the panic is driven by a sunspot, our model of course remains silent about the probabilities of the panic in the ESBies and the baseline model. Furthermore, symmetry may not be a reasonable assumption, since ESBies may help less solvent countries to benefit from more solvent countries. Yet the results that normal times get worse (through the *temptation and*

[14] Safe in the sense that there is no uncertainty about the payoff of the asset, even if the payoff is below the face value as some countries do default.

[15] The ESBies economy in normal times (N) resembles the no exposure economy (ne, S) from Propositions **??** and **??** with the twist that $B_0^{h,ne,S} = 0$.

[16] The default is not strategic in Brunnermeier et al. (2017) and the government repayment is restricted by the primary surplus that is a random variable with a binary distribution. Therefore the composition of debt holders is irrelevant for repayment as opposed to our setup, where repayment incentives depend on how much debt is held by local banks and foreign investors. The underlying structure to generate the doom loop is basically the same: there is a sunspot variable that generates debt repricing and if banks become insolvent then a fraction of the loans are destroyed.

commitment channels) and that panics may still happen serve as warning.

7.2 Amplification

As Cooper and Ross (1998) show, strategic complementarities – such as the one between regarding the bond price which our model considers – typically lead not only to multiplicity of equilibria, but also to amplification. We illustrate this next and show that our policy results carry over, thus highlighting how our model relates to the doom loop's amplifying role highlighted by Farhi and Tirole (2016) and Acharay et al. (2014).

To this end, we remove the sunspot shock and rule out the self fulfilling sunspot equilibrium and add a fundamental shock to agents expectations about the distribution of future productivity. That is, we replace the sunspot shock by a news shock in period 1. This shock may capture diverse negative developments, including the outbreak of a global pandemic that shifts down the distribution of expected GDP. We choose this shock to illustrate how the doom loop amplifies fundamental shocks, in this case a shock to future productivity. The nature of the shock is irrelevant. A contemporaneous shock to the asset quality of banks or the world interest rate, for example, would be amplified in the same way.

Now we have a new state $s = r$ where the distribution of GDP in $t = 2$ shifts to the left. To parametrize in a simple way this shift, we put mass probability ϵ at the lower bound of the support of ω and preserve scale the pdf to satisfy the probability axions. This implies that the CDF in case $s = r$ is given by

$$F^r(\omega) = F(\omega)\,(1-\epsilon) + \epsilon \quad \text{for } \omega \in (\underline{\omega}, \overline{\omega})$$

where $\overline{\omega}^r$ is the new upper bound $F^r(\overline{\omega}^r) = 1$. This way the probability of having a productivity draw below a given threshold is higher $\epsilon\%$ in case $s = r$.

I have to write an assumption about the solvency of banks in case the shock hits. So if the shock hits and the government does nothing then the solvency condition is

$$B_0^h q_1^r + (1-\theta)(1-\phi)L_0 < \lambda_1 D_0$$

and this would happen for $q_1^r = F^r\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$

Assumption 5. *Banks are not solvent if the productivity distribution is F^r, or equivalently*

$$1 - \frac{\xi_1}{Y_0^h + Y_0^b - K_0} < \epsilon$$

That is, we assume the opposite for the normal and the recession state: By assumption **??** banks are solvent in normal times, but by assumption **??** they are insolvent in recessions times. The equilibrium now depends on the fundamental shock, but otherwise closely resembles the sunspot equilibrium in proposition **??**:

Proposition 8. *The equilibrium exists and in case of recession $s = r$ the price of sovereign debt falls in $t = 1$ and banks are bailed out by the new issuance of debt by the government.*

The price of debt in case of panic is given by the solution to the system

$$q_1^r = (1 - F(\tilde{\omega}^p))\,(1 - \epsilon) \tag{7}$$

$$\tilde{\omega}^r = \omega^n + \frac{1}{\Theta} \frac{\frac{1}{q_1^r}\,(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0) - B_0^h}{K_0} \tag{8}$$

where $\tilde{\omega}^r$ is the default threshold in case of recession. The equilibrium variables at $t = 0$ and in case $s = n$ coincide with Proposition 1.

Due to the doom loop the solution of the system ((7))-((8)) is not unique. Since we focus on amplification in this section, we restrict our attention to the equilibrium with the highest $q^{r,1}$.

To understand how the doom loop amplifies the news shock, consider an alternative version of the model where negative bank equity in period 1 is inconsequential ($\theta_1 = 0$) such that the government would never bail out banks. In that case the equilibrium default threshold for ω would be always the same, no matter whether good or bad news arrive. The bond price would however reflect the relevant distribution of future productivity.

$$q^{r*} = 1 - F^r(\omega^*) \tag{9}$$

$$q^{n*} = 1 - F(\omega^*) \tag{10}$$

$$d^*(\omega) = \begin{cases} 1 & \omega < \omega^* \\ 0 & \omega \geq \omega^* \end{cases} \tag{11}$$

$$\text{where } \omega^* = \frac{1}{\Theta}\frac{B_0^f}{L_0} \tag{12}$$

Comparing the baseline and this alternative model, it is clear that nothing changes in good times. In bad times however the doom loop matters. Even if it is absent, the bond price drops in bad times, but if it is present, the drop is larger ($q^{r,0} < q^{r*}$). That is, the doom loop amplifies the drop in bond prices caused by the fundamental shock. The same holds for the associated drop in welfare.

Figure 4 illustrates this graphically. When the bad state materializes, the bond price drops from q^n to q^{r*} in the absence of the doom loop. The doom loop then amplifies this initial drop and pushes the bond price further down to q^r.

Since it is the same strategic complementary that generates amplification and multiplicity, the results from the policy exercises in section 6 propositions **??** - **??** carry over to the case of amplification. Policies that (do not) help with multiplicity also (do not) help with amplification. Specifically: (i) reducing ex ante exposure makes normal times worse without removing amplification; (ii) domestic bailouts or (iii) secondary markets and a loose enough limit on bank bond holdings disable the doom loop and hence its amplifying effect; (iv) a no-bailout

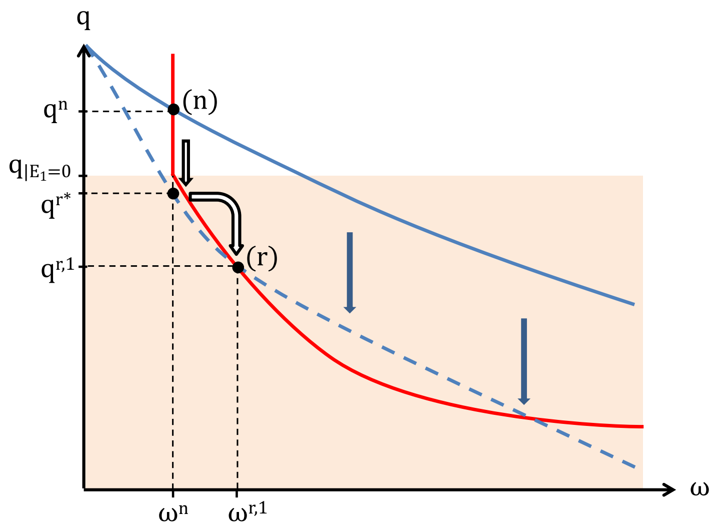

Figure 4: Amplification of a news shock

commitment does not disable the doom loop.

Furthermore, as in section 6, reducing the bank's exposure sufficiently by either increasing its equity ratio or decreasing its domestic bond holdings rules out the doom loop. This applies here too. However, propositions 5 and 6 apply as well, that is the success of these policies to rule out amplification has a cost in normal times: (v) reducing banks exposure to sovereign debt to the point that they are solvent regardless of the price of sovereign debt reduces the bond price in normal times and (vi) reduces welfare, conditional on normal times and hence if the recession state is sufficiently unlikely. We summarizes these points in the following proposition:

Proposition 9. *Define amplification as a situation where $\omega^n < \omega^r$. Then:*

(i) Lower exposure generates a lower price of sovereign debt and higher default probability in the normal state: $\frac{\partial q^n}{\partial S_0} < 0$.

(ii) Lower exposure generates a higher price of sovereign debt and lower default probability in the recession state: $\frac{\partial q^r}{\partial S_0} > 0$.

(ii) When the bank is bailed out with domestic bonds, there is no amplification.

Lower amplification at the cost of higher spread in normal times.

8 Conclusion

Banks' exposure to sovereign debt give rise to the doom loop: A fall in the price of debt can require a bailout, which raises debt and hence the default probability, justifying the fall in the price of debt. However, the same exposure also provides commitment to the government, thus sustaining sovereign debt. This paper combines these two views to challenge two conclusions that can be derived from looking at the doom loop in isolation: (i) that banks' exposure to their government should be reduced (ii) that it is desirable to commit not to bailout banks.

We show that an increase in the bank equity ratio – which may be a response to the no bailout commitment – or a reduction of bank bond holdings that is sufficiently large to mute the doom loop comes at a cost: Sovereign spreads in the no doom loop scenario rise as the commitment value of bank's exposure disappears and welfare drops. Rather, we argue that it is desirable that banks expand their exposure to public debt in times of sovereign distress, thus acting as lenders of last resort and breaking the doom loop.

These result may serve as a warning to the policy makers, which often express discomfort especially about banks high exposures to domestic sovereign debt. Maybe such exposure has more upsides than downsides after all. This is of particular relevance now that public debt is soaring due to the public health crisis.

While our model is no doubt stylized, it is straightforward to extend our analysis along several dimensions. First, parts of our analysis for simplicity assumed that the doom loop is perceived as a zero probability event. Yet by continuity our results would hold as long as it is sufficiently unlikely. Second, in our analysis banks' exposure did not affect the government's default cost *at the margin*. This allowed us to clearly separate between the effect negative and positive effects that foreign and domestically held debt have on repayment incentives. Allowing bank's exposure to also have a positive effect at the margin would strengthen our mechanism further.

Finally one caveat is in place. Our government is benevolent and maximizes national welfare. Thus there is no role for asset markets to discipline undesirable overspending by self-interested politicians, which might reduce the benefits of the additional commitment that bank's exposure provides.

Bibliography

Acharay, V., Drechsler, I., and Schnabl, P. (2014). A pyrrhic victory? bank bailouts and sovereign credit risk. *The Journal of Finance*, 69(6):2689–2739.

Andreeva, D. C. and Vlassopoulos, T. (2019). Home Bias in Bank Sovereign Bond Purchases and the Bank-Sovereign Nexus. *International Journal of Central Banking*, 15(1):157–197.

Balke, N. (2018). The Employment Cost of Sovereign Default. Chicago University Working Paper.

Balloch, C. M. (2016). Default, commitment, and domestic bank holdings of sovereign debt. Working Paper.

Balteanu, I. and Erce, A. (2017). Linking Bank Crises and Sovereign Defaults: Evidence from Emerging Markets. Working Papers 22, European Stability Mechanism.

Basu, S. (2010). Sovereign Debt and Domestic Economic Fragility.

Benassy-Quere, A., Brunnermeier, M., Enderlein, H., Farhi, E., Fratzscher, M., Fuest, C., Gourinchas, P.-O., Martin, P., Pisani-Ferry, Jeanand Rey, H., Schnabel, I., Veron, N., Weder di Mauro, B., and Zettelmeyer, J. (2018). Reconciling risk sharing with market discipline: A constructive approach to euro area reform. Working Paper.

Bolton, P. and Jeanne, O. (2011). Sovereign Default Risk and Bank Fragility in Financially Integrated Economies. *IMF Economic Review*, 59(2):162–194.

Boz, E., D'Erasmo, P., and Durdu, B. (2014). Sovereign Risk and Bank Balance Sheets: The Role of Macroprudential Policies. Working Paper.

Broner, F., Martin, A., and Ventura, J. (2010). Sovereign Risk and Secondary Markets. *American Economic Review*, 100(4):1523–1555.

Brunnermeier, M. (2015). Financial dominance. Baffi Lecture.

Brunnermeier, M. K., Garicano, L., Lane, P. R., Pagano, M., Reis, R., Santos, T., Thesmar, D., Nieuwerburgh, S. V., and Vayanos, D. (2016). The Sovereign-Bank Diabolic Loop and ESBies. *American Economic Review*, 106(5):508–512.

Brunnermeier, M. K., Langfield, S., Pagano, M., Reis, R., Nieuwerburgh, S. V., and Vayanos, D. (2017). ESBies: safety in the tranches. *Economic Policy*, 32(90):175–219.

Brutti, F. (2011). Sovereign defaults and liquidity crises. *Journal of International Economics*, 84(1):65–72.

Cooper, R. and Nikolov, K. (2013). Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty. NBER Working Papers 19278, National Bureau of Economic Research, Inc.

Cooper, R. and Ross, T. W. (1998). Bank runs: Liquidity costs and investment distortions. *Journal of Monetary Economics*, 41(1):27–38.

Cruces, J. J. and Trebesch, C. (2013). Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics*, 5(3):85–117.

Engler, P. and Grosse Steffen, C. (2016). Sovereign risk, interbank freezes, and aggregate fluctuations. *European Economic Review*, 87:34–61.

Erce, A. (2012). Selective sovereign defaults. Globalization and Monetary Policy Institute Working Paper 127, Federal Reserve Bank of Dallas.

Farhi, E. and Tirole, J. (2016). Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops. Working Paper 164191, Harvard University OpenScholar.

Gennaioli, N., Martin, A., and Rossi, S. (2014). Sovereign Default, Domestic Banks, and Financial Institutions. *Journal of Finance*, 69(2):819–866.

Leonello, A. (2017). Government guarantees and the bank-sovereign nexus. *Research Bulletin*, 35.

Mallucci, E. (2014). Sovereign Defaults, Wholesale Funding and Banking Crises. Working Paper.

Mayer, M. (2011). Sovereign Credit Risk and Banking Crises. SSRN Scholarly Paper ID 1785893, Social Science Research Network, Rochester, NY.

Messori, M. and Micossi, S. (2018). Counterproductive Proposals on Euro Area Reform by French and German Economists. Working Paper.

Perez, D. (2015). Sovereign Debt, Domestic Banks and the Provision of Public Liquidity. Working Paper, Department of Economics, Stanford.

Reinhart, C. M. and Rogoff, K. S. (2011). From Financial Crash to Debt Crisis. *American Economic Review*, 101(5):1676–1706.

Sosa-Padilla, C. (2018). Sovereign Defaults and Banking Crises. *Journal of Monetary Economics*, 99:88–105.

Sturzenegger, F. and Zettelmeyer, J. (2007). Creditors' Losses Versus Debt Relief: Results from a Decade of Sovereign Debt Crises. *Journal of the European Economic Association*, 5(2-3):343–351.

Thaler, D. (2019). Sovereign Default, Domestic Banks and Exclusion from International Capital Markets. Working Paper.

Appendix

A Portfolio re-optimization and secondary markets

So far our bank was extremely passive; it had no decision to take. Now we extend the model to allow the bank to re-optimize its portfolio in period 1. Loans and deposits are assumed to be illiquid, but the bank can now choose whether to invest the bailout funds it receives (S_1) in domestic debt, subject to a regulatory maximal exposure constraint $q^{p,1}B_1^h \leq \bar{B}$.

Just as in the basic setup, the value of the bailout S_1 is given by

$$S_1 = \max\left\{D_0 - L_0 - q^{p,1}B_0^h, 0\right\}$$

and it is financed with the issuance of new debt such that

$$\Delta B_1 = \frac{1}{q^{p,1}}S_1$$

where the new debt is allocated to foreign investors or local banks through the secondary markets satisfying $\Delta B_1 = \Delta B_1^f + \Delta B_1^h$.

The bank operates under limited liability. Its objective hence is to maximize the expected non-negative part of equity in period 2 i.e. $\mathbb{E}\left[max\left\{E_2, 0\right\}\right]$, subject to the budget constraint $S_1 + qB_0^h \leq S_2 + qB_1^h$, where S_2 denotes the bailout funds that the bank keeps in the safe asset. The bank is atomistic and thus takes all prices and the governments actions as given. Due to limited liability the bank always has an incentive to buy as much debt as possible, whenever the default probability is positive.[17,18] It thus invests all the bailout funds into sovereign debt, if the exposure limit permits, or up to the limit $\bar{B}$ otherwise, and invests the rest of the bailout funds in the safe asset. Then we have that the change in sovereign debt holdings and the safe asset holdings by the bank are given by:

$$\begin{aligned} \Delta B_1^h &= \min\left\{\bar{B}/q^{p,1} - B_0^h, \Delta B_1\right\} \\ S_2 &= S_1 - q^{p,1}\Delta B_1^h \end{aligned}$$

It is immediately evident that for $\bar{B} \to \infty$ the equilibrium of this economy coincides with that of the economy analyzed before in Proposition 3, where the government bails out the bank with sovereign debt but no trading is possible after the bailout. On the other hand, for finite $\bar{B}$ the economy could feature multiple equilibria. The following proposition characterizes for which values of $\overline{B}$ this is the case.

Proposition 10. *If $\overline{B} \geq \lambda_1 D_0 - (1-\phi)(1-\theta)L_0$ there is no sunspot equilibrium and the equilibrium corresponds to the equilibrium described in Proposition 1. If $\overline{B} < \lambda_1 D_0 - (1-\phi)(1-\theta)L_0$ there is a sunspot equilibrium.*

This result establishes that to rule out the doom loop it is sufficient that the regulatory maximal exposure in the panic state $\overline{B}$ is greater than the equity shortfall in case of default $(D_0 - L_0)$. From Assumption **??** we have that in normal times the exposure is already higher than this threshold as $q^{1,n}B_0^h > D_0 - L_0$. Consequently, the doom loop only arises if banks face constraints that force them to lower bond holdings at market value sufficiently during panics. In such scenario, the panic is self-fulfilling as the lower exposure of banks weakens the incentives of the sovereign to repay. On the contrary, if banks in panic times are allowed to hold bonds up to a value not too much lower than in normal times, the doom loop ceases to exist.

[17]By Assumption **??** a bailout is only necessary if the price is lower than 1 and hence if the default probability is positive.

[18]Andreeva and Vlassopoulos (2019) find that banks that had its risks more highly correlated with the local sovereign increased more their demand for local sovereign debt during the European sovereign debt crisis. The motive they propose to rationalize their finding is the same as here, risk-shifting. There are other reasons why banks may have a higher valuation of government bonds than foreign investors, especially in times of crisis, such as: Regulatory reasons, financial repression, or non-atomistic behavior of banks. See also Andreeva and Vlassopoulos (2019) for a discussion of this literature and the empirical support for the risk shifting hipothesis.

B Panic at $t = 0$. Avoiding the Doom Loop to get into the classic self-fulfilling debt trap at the initial issuance.

There are F for which there is no classical self-fulfillign debt crisis with exposure but there is the possibility of a classical self-fulfilling debt crisis without exposure.

Proof. Let F be such that there is a unique solution for the system

$$B_0^f = \frac{R + K_0 - \left(Y^h + Y^b\right)}{q_0}$$
$$q_0 = 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$$

but multiple solutions to the system

$$B_0^f = \frac{R + K_0 - \left(Y^h + Y^b\right)}{q_0}$$
$$q_0 = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right)$$

this can be simply achieved by a F that is constructed to be equal to 0 up to the point $B_0^{f*} = R + K_0 - \left(Y^h + Y^b\right) + T$, where $T = \left(\frac{\Theta}{\vartheta} - 1\right)\tilde{B}_0^f$ such that the crossing is at $q = 1$ and $B_0^f = R + K_0 - \left(Y^h + Y^b\right)$.

Let then F be given by

$$F(\omega) = 1 - \frac{R + K_0 - \left(Y^h + Y^b\right)}{\Theta K_0 \omega - T}$$

for $\omega \geq \frac{R+K_0-\left(Y^h+Y^b\right)+T}{K_0\Theta}$. Then at the no exposure case there is a second crossing at $\tilde{B}_0^f$. □

C Proofs of propositions

C.1 Proposition 1

Proof. First consider the decision of depositors at $t = 2$ after the government announced the repayment decision. For a given productivity draw ω and a bank balance sheet given by $(D_1, , , L_1, S_1, B_1^h)$ where there are K_1 projects active.

If the government announces full repayment then the bank assets are given by $L_1 + B_1^h$ and a bank run would require a transfer to depositors given by $\lambda_2 D_1$. The bank would remain solvent, even after satisfying the early withdrawls if

$$(1 - \theta_2)L_1 + B_1^h + S_1 > \lambda_2 D_1 \tag{13}$$

in this case then bank run is not an equilibrium. Since a single depositor can profitably deviate and wait for repayment after production takes place. This deviation given him the return of 1 per unit of deposit, since the bank is solvent and can repay the depositor in full. Instead if he withdraws the depositor gets $\lambda_1 < 1$ per unit of deposit. Note that loans L_1 are multiplied by $(1-\theta_2)$ to take into account that a fraction θ_2 of the projects fail in case of liquidation and default whle the remaining fraction repays in full.

On the other hand not running is an equilibrium after government repayment if we have that

$$L_1 + B_1^h + S_1 \geq D_1 \tag{14}$$

since in this case the bank is solvent, can repay depositors in full and consequently no depositor has incentives to deviate and early withdraw. Since $\theta_2 \in (0,1)$ then if 13 is satisifed then 14 is also satisifed.

Alternatively if the government decides to default then there is a bank run equilibrium if

$$(1-\theta)L_1 + S_1 < \lambda_2 D_1 \tag{15}$$

since in this case any depositor that does ot early withdraw in case of a bank run would get zero repayment by wating until production takes place since the bank had to liuidate all their assets to repay the early withdrawers.

For now we assume that conditions 13 and 15 are satisfied and then we will verify this is the case.

Now consider the default decision by the government in $t = 2$ for a given productivity draw of the government and a bank balance sheet given by $(D_1, , , L_1, S_1, B_1^h)$ where there are K_1 projects active and the level of foreign held debt is B_1^f. If the government decides to repay, then aggregate consumption is given by

$$C^r = \omega K_1 - B_1^f + S_1$$

on the other hand if the government defaults we have

$$C^d = \omega(1-\vartheta)(1-\theta_2)K_1 + S_1$$

where production in case of default takes into account the cost of the bank run θ and the direct default cost ϑ. In this case the government optimally decides to repay whenever $C^r \geq C^d, ,$ and that is accomplished by a threshold strategy that defaults whenever $\omega < \omega^n, , ,$ where ω^n is given by

$$\omega^n = \frac{1}{\Theta}\frac{B_1^f}{K_1}$$

where Θ is given by $(1-(1-\vartheta)(1-\theta_2))$. The correponding default probability is given by $F(\omega^n)$.

At $t = 1$ the depositors decide if to withdraw for a given bank balance sheet $(D_1, , , L_1, S_1, B_1^h)$. The bank remains solvent after a run if

$$(1 - \theta_1)L_1 + q_1 B_1^h + S_1 > \lambda_1 D_1 \tag{16}$$

where $(1 - \theta_1)$ is the fraction of surviving loans if there is a liquidation at $t = 1$. In this case running on the bank is not an equilibrium since waiting would give the depositor a repament of 1 per unit of deposit instead of $\lambda_1 < 1$.

For now we also assume that condition 16 is satisified for the balance sheet from $t = 0$, so in that case

$$(1 - \theta)(1 - \phi)L_0 + q_1 B_0^h + S_0 > \lambda_1 D_0 \tag{17}$$

and consequently there is no bank run at $t = 1$ even if the government does not intervene. There is no bailout required to avoid a bank run at $t = 1$ and consequently the levels of debt are the same as the initial issuances and the balanace sheet of the bank also remains the same. We verify that our assumptions guarantee that indeed 17 in equilibrium.

Note that $q_1 = 1 - F(\omega^n)$, where in this case since no new debt is issued or projects destroyed in $t = 1$ is given by

$$q_1 = 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{K_0}\right)$$

where note that we restrict attention here to the no sunspot equilibrium and consequently q_1 does not depend on s.

Now at $t = 0$ we have to consider the initial issuance of debt and the portfolio choice of the bank.

The bank can invest their resources on sovereign debt or providing loans. Since from 13 and 15, when the government defaults there is a bank run, and when the government repays the bank is solvent then the valuation of both assets are the same for the banker. Since whenever the government defaults, the return of loans for the banker is also zero. Therefore we have that $p_0^L = q_0$.

As long as $q_0 < 1$ the banker strictly prefers holding sovereign debt or loans, as opposed to the safe asset S_0. Since the cost of creating a unit of safe assets is higher $1 > q_0$ than buying sovereign debt and the expected repayment to the banker is the same. Since in case of sovereign default the banker becomes insolvent. Consequently it is optimal for the banker to set $S_0 = 0$, as long as 13 and 15 are satisfied.

On the other hand the price of deposits is such that depositors are indiferent between the storage technology and deposits. The net return of the storage technology is zero, consequently the net expected return of deposits has to be also equal to zero, this condition can be written as

$$E\left\{r_0^D\right\} = \frac{F(\omega^n)(1-\theta)\frac{L_0}{D_0}}{p_0^D} + \frac{(1 - F(\omega^n))}{p_0^D} - 1 = 0$$

where the first term is the gross return in case of sovereign default $\frac{(1-\theta)\frac{L_0}{D_0}}{p_0^D}$ times he probability of default and the second is the return in case of repayment times the probability of repayment. We have used the fact that in case of government repayment the depositors are fully repaid, while in case of default they get the liquidated value of the loans. The solution is a price for deposits that corresponds to

$$p_0^D = q_0 \left(1 - \frac{(1-q_0)}{q_0}(1-\theta)\frac{K_0}{Y_0^d}\right)^{-1} < q_0$$

where we have used tohe fact that total loans issued is given by K_0/p_0^L and $p_0^L = q_0$. Since $p_0^d < q_0 = p_0^L$ then the bank optimally decides to take as many deposits as possible and earn in expectations the intermediation margin. Note that since the deposit rate is not contractable on the portfolio of the bank then we have a case of risk shifting where the bank decides not to use the safe storage.

The initial balance sheet of the bank has to satisfy the constraint

$$Y_0^b + p_0^d D_0 = p_0^L L_0 + q_0 B_0^h + S_0$$

so total internal plus external funding equals the portfolio cost of the bank. Using the fact that $p_0^L = q_0$, $S_0 = 0$, $p_0^L L_0 = K_0$ and $p_0^d D_0 = Y_0^h$ then we have that

$$B_0^h = \frac{Y_0^b + Y_0^h - K_0}{q_0} \tag{18}$$

The initial issuance of debt by the government has to be the minimum ammount B_0 that satisifes

$$R = q_0 B_0$$

then by using $B_0 = \left(B_0^f + B_0^h\right)$ and 18 we have

$$\Longrightarrow B_0^f = \frac{R - Y^b - Y^h + K_0}{q_0}$$

and then the initial holdings of foreigners is the minimum level that solves the equation

$$B_0^f = \frac{R - Y^b - Y^h + K_0}{1 - F\left(\frac{1}{\Theta}\frac{K_0}{B_0^f}\right)}$$

Finally we are left to verify that conditions 13, 15 and 17 hold in equilibrium. We verify that they hold for $q_0 = 0$ and by continuity they hold for q_0 close enough to 1.

First start with 13, that replacing the equilibirum levels of L_1, S_1 and D_1 corresponds to

$$(1-\theta_2)\frac{K_0}{p_0^L} + B_0^h > \lambda_2 \frac{Y_0^h}{p_0^D}$$

$$(1-\theta_2)\frac{K_0}{q_0} + \frac{Y_0^b + Y_0^h - K_0}{q_0} > \lambda_2 \frac{Y_0^h}{p_0^D}$$

$$Y_0^b + Y_0^h - \theta_2 K_0 > \lambda_2 Y_0^h \left(1 - \frac{(1-q_0)}{q_0}(1-\theta_2)\frac{K_0}{Y_0^d}\right)$$

at $q_0 = 1$ we have

$$\begin{aligned} Y_0^b + Y_0^h - \theta_2 K_0 &> \lambda_2 Y_0^h \\ Y_0^b + (1-\lambda_2) Y_0^h &> \theta_2 K_0 \end{aligned}$$

that is implied by Assumption 2 on the solvency condition.

Second the condition 15 corresponds in equilibrium to

$$\begin{aligned} (1-\theta_2)\frac{K_0}{q_0} &< \lambda_2 \frac{Y_0^h}{q_0\left(1 - \frac{(1-q_0)}{q_0}(1-\theta_2)\frac{K_0}{Y_0^d}\right)^{-1}} \\ (1-\theta_2)K_0 &< \lambda_2 Y_0^h \left(1 - \frac{(1-q_0)}{q_0}(1-\theta_2)\frac{K_0}{Y_0^d}\right) \end{aligned}$$

that at $q_0 = 1$ corresponds to

$$(1-\theta_2)K_0 < \lambda_2 Y_0^h$$

that is implied by Assumption **??** (exposure at t=2).

Finnally condition 17 corresponds to

$$\begin{aligned} (1-\theta_1)\frac{K_0}{p_0^L} + B_0^h &> \lambda_1 \frac{Y_0^h}{p_0^D} \\ Y_0^b + Y_0^h - \Theta K_0 &> \lambda_1 Y_0^h \left(1 - \frac{(1-q_0)}{q_0}(1-\theta)\frac{K_0}{Y_0^d}\right) \end{aligned}$$

at $q_0 = 1$ we have

$$Y_0^b + (1-\lambda_1) Y_0^h > \Theta K_0$$

is also guranteed by Assumption 2. □

C.2 Proposition 2

Proof. The variables at $t = 0$ are given in the proof of Proposition 1 (no sunspot) and are the same here since we assume the sunspot is perceived to have probability zero.

We proceed by guessing that in case of panic $s = p$ the price of government debt falls and the banks can remain solvent only with a bailout. Then we verify that the increase of debt to finance the bailout sustains the initial fall in the price of debt. We show that the bailout is optimal given that in the abscence of a bailout there would be a bank run at $t = 1$ that implies a lower welfare.

Let the price of sovereign debt at $t = 1$ for $s = p$ and with an announced bailout be given by q_1^p, we conjecture that q_1^p is low enough to make the bank insolvent

$$q_1^p B_0^h + (1-\theta_1)L_0 < \lambda_1 D_0$$

such that the liquidation value of the banks assets is below the resources required to satisfy the early withdrawls. The necessary bailout transfer to make the bank solvent and rule out a bank run is given by a level of safe assets S_1 given by

$$S_1 = \lambda_1 D_0 - \left(q_1^p B_0^h + (1-\theta_1)L_0\right)$$

Since this transfer is financed with the issuance of sovereign debt, the new issuance required to fiannce this transfer is given by

$$\begin{aligned} \Delta B_1^f &= \frac{S_1}{q_1^p} \\ \implies \Delta B_1^f &= \frac{\lambda_1 D_0 - (1-\theta_1)L_0}{q_1^p} - B_0^h \end{aligned}$$

and consequently the total debt held by foreigners is

$$B_1^f = B_0^f + \frac{\lambda_1 D_0 - (1-\theta_1)L_0}{q_1^p} - B_0^h$$

and we see how it depends on the price of sovereign debt.

The price of debt q_1^p is given by the repayment probability

$$q_1^p = 1 - F(\tilde{\omega}^p)$$

where $\tilde{\omega}^p$ is the repayment threshold, that as shown in the previous proof is a function of foreign held debt as follows

$$\begin{aligned} \tilde{\omega}^p &= \frac{1}{\Theta}\frac{B_1^f}{K_0} \\ &= \omega^n + \frac{1}{\Theta}\frac{\frac{\lambda_1 D_0-(1-\theta_1)L_0}{q_1^s} - B_0^h}{K_0} \end{aligned}$$

then we have that the equilibrium price of debt q_1^p and the default threshold $\tilde{\omega}^p$ are the solution to the system of equations

$$\begin{aligned} q_1^p &= 1 - F\left(\tilde{\omega}^p\right) \\ \tilde{\omega}^p &= \omega^n + \frac{1}{\Theta}\frac{\frac{\lambda_1 D_0-(1-\theta_1)L_0}{q_1^p} - B_0^h}{K_0} \end{aligned}$$

This system has a solution, note that the system can be rewritten as

$$G_1(\omega) = q_1^p(\omega) = 1 - F\left(\omega\right)$$

$$G_2(\omega) = q_1^p(\omega) = \frac{\lambda_1 D_0 - (1-\theta_1)L_0}{B_0^h + \Theta K_0\left(\omega - \tilde{\omega}^n\right)}$$

where we have defined functions $G_1(\omega)$ and $G_2(\omega)$ for convenience. An equilibrium price q_1^p is sustained by a threshold $\tilde{\omega}^p$ if $G_1(\tilde{\omega}^p) = G_1(\tilde{\omega}^p) = q_1^p$.

First note we have that $G_1(\tilde{\omega}^n) > G_2(\tilde{\omega}^n)$ since while

$$G_2(\tilde{\omega}^n)B_0^h = \lambda_1 D_0 - (1-\phi)(1-\theta)L_0$$

we have from assumption 2 that for q_0 close enough to one it follows that

$$G_1(\tilde{\omega}^n)B_0^h = \left(1 - F(\tilde{\omega}^n)\right)B_0^h > \lambda_1 D_0 - (1-\phi)(1-\theta)L_0$$

Second note that both functions are non-increasing in ω and both have the limit of 0 as $\omega \to \infty$.

Third since we assume that the support of omega is bounded above, there is a $\bar{\omega}$ for which $F(\bar{\omega}) = 1 \implies G_1(\bar{\omega}) = 0$ while $G_2(\omega) > 0$ for all ω as given by assumption 1. So it has to be that the two curves cross at least once. We focus on the equilibrium with the highest price.

We are left to show that in case of panic and no bailout there would be a bank run and that welfare in case of a bank run is lower than if a bailout is implemented.

Let $q_1^{p,0}$ be the price of debt in case of panic and with the announcement of no bailout. For a bank ru to happen in equilibrium it has to be that the liquidatio value of the bank is below the early withdrawals as given by

$$q_1^{p,0}B_0^h + (1-\theta_1)L_0 < \lambda_1 D_0$$

where he price is equal to the default probability

$$q_1^{p,0} = 1 - F(\tilde{\omega}^{p,0})$$

where $\tilde{\omega}^{p,0}$ is the default threshold in case there is no bailout.

In this case total consumption in case of repayment is given by

$$C^r(\omega) = (1-\theta_1)\omega K_0 - B_0^f$$

and in case of default

$$C^d(\omega) = (1-\theta_1)(1-\vartheta)\omega K_0$$

and consequently the default threshold is given by

$$\tilde{\omega}^{run} = \frac{1}{\vartheta(1-\theta_1)}\frac{B_0^f}{K_0} \tag{19}$$

and consequently the run condition in case of no bailout is given by

$$\left(1 - F\left(\frac{1}{\vartheta(1-\theta_1)}\frac{B_0^f}{K_0}\right)\right) B_0^h + (1-\theta_1)L_0 < \lambda_1 D_0$$

or equivalently by

$$1 - \frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0}{B_0^h} < F\left(\frac{1}{\vartheta(1-\theta_1)}\frac{B_0^f}{K_0}\right)$$

that for q_0 close enough to 1 is guaranteed by Assumption 4. So if the government decides not to bailout there will be a bank run in case of panic.

Let ϕ be such that $(1-\theta_1) = (1-\phi)(1-\theta_2)$, so it captures how many more projects are destroyed when the liquidation happens at $T = 1$instead that at $t = 2$. We show now that for ϕ high enough a bailout is optimal. Consumption in case of a run in $t = 1$ and a subsequent default in $t = 2$ is given by

$$C^{run,d}(\omega) = (1-\theta_1)(1-\vartheta)\omega K_0$$

while consumption in case of default after a bailout is given by

$$C^{bail,d}(\omega) = (1-\theta_2)(1-\vartheta)\omega K_0 + S_1$$

and since $\phi > 0$ we have that for every ω, $C^{run,d}(\omega) < C^{bail,d}(\omega)$.

The consumption in case of a run after repayment is given by

$$C^{run,r}(\omega) = (1-\theta_1)\omega K_0 - B_0^f$$

and define $\tilde{\omega}^*$ as the value for which $C^{bail,d}(\omega) = C^{run,r}(\omega)$, then we have $C^{bail,d}(\omega) \geq C^{run,r}(\omega)$ if

$$\begin{aligned}(1-\theta_2)(1-\vartheta)\omega K_0 + S_1 &\geq (1-\theta_1)\omega K_0 - B_0^f \\ \implies \omega \geq \tilde{\omega}^* &= \frac{1}{(\vartheta-\phi)(1-\theta)}\frac{B_0^f + S_1}{K_0}\end{aligned}$$

if $\tilde{\omega}^* \leq 0$ then we have that for all ω the consumption after a bailout and a default is larger than consumption after a bank run, either after repayment or default. Consequently $(\phi \geq \vartheta)$ is a sufficient condition to have that a bailout is optimal. A bailout implies greater consumption in every possible state of nature.

Consider also the case when $\tilde{\omega}^* > 0$ in this case for $\omega > \tilde{\omega}^*$ we have that $C^{run,d}(\omega) > C^{bail,d}(\omega)$. Note that the consumption after repayment in case of a bailout is given by

$$C^{bail,r}(\omega) = \omega K_0 - B_1^f + S_1$$

and consequently $\frac{\partial C^{bail,r}(\omega)}{\omega} = K_0 > \frac{\partial C^{run,r}(\omega)}{\omega} = (1-\phi)(1-\theta_2)K_0$ so if for a $\hat{\omega}$ we have that $C^{bail,r}(\hat{\omega}) \geq C^{run,r}(\hat{\omega})$ then for every $\omega \geq \hat{\omega}$ we have that $C^{bail,r}(\hat{\omega}) \geq C^{run,r}(\hat{\omega})$.

Recall that $\tilde{\omega}^p$ is the level that guarantees that $C^{bail,r}(\omega) = C^{bail,d}(\omega)$, consequently if $\tilde{\omega}^p \leq \tilde{\omega}^*$ then we have that for $\omega \leq \tilde{\omega}^p$

$$C^{bail,d}(\omega) \geq \max\{C^{run,d}(\omega), C^{run,r}(\omega)\} \tag{20}$$

and in particular

$$C^{bail,d}(\tilde{\omega}^p) = C^{bail,r}(\tilde{\omega}^p) \geq \max\{C^{run,d}(\tilde{\omega}^p), C^{run,r}(\tilde{\omega}^p)\}$$

and since we showed that $\frac{\partial C^{bail,r}(\omega)}{\omega} > \frac{\partial C^{run,r}(\omega)}{\omega}$ we have that for $\omega \geq \tilde{\omega}^p$

$$C^{bail,r}(\tilde{\omega}^p). \geq= \max\{C^{run,d}(\tilde{\omega}^p), C^{run,r}(\tilde{\omega}^p)\} \tag{21}$$

and consequently from equations 20 and 21 we have that for every ω the consumption after a bailout is larger than after a run. The condition is that

$$\tilde{\omega}^p \leq \tilde{\omega}^*$$

where what is left to note is that $\frac{\partial \tilde{\omega}^*}{\partial \phi} > 0$ and $\frac{\partial \tilde{\omega}^p}{\partial \phi} < 0$ and consequently for ϕ large enough (and smaller than ϑ) the condition is satisifed. In the limit as $\phi \to \vartheta$ we have that $\tilde{\omega}^* \to \infty$ and the condition is trivially satisfied. So all we require is that ϕ is large enough and can be strictly smaller than ϑ. □

C.3 Proposition 3

Proof. In this case the bank receives a bailout with bonds. The transfer of bonds an ammount ΔB_1^h that satisfies:

$$q_1^p B_0^h + (1-\phi)(1-\theta)L_0 + q^p \Delta B_1^h = \lambda_1 D_0$$

The default threshold in this case ends up being the same as in the normal equilibrium

$$\tilde{\omega} = \frac{1}{\Theta}\frac{B_0^f}{L_0}$$

since the aditional debt issuance does not distort the default decision. There is still a bank run in case of default and no bank run in case of full repayment. Consequently since in the normal equilibrium the bank is solvent, then there is no bailout needed and the sunspot equilibrium dissapears. □

C.4 Proof Propositions 4 and 5

Proof. **Case i: No exposure (ne) achieved by a required level of safe assets S_0.**

In the no exposure economy banks are required to hold a level of safe assets S_0 such that they do not become insolvent in case of a sovereign default. This condition is given by

$$\lambda_2 D_0 - (1-\theta_2)L_0 \leq S_0 \tag{22}$$

and since Assumption **??** guarantees that exposure at $t=2$ is larger than at $t=1$ then no exposure at $t=2$ also implies that the doom loop is ruled out. Even in $q_1^p = 0$ banks are solvent in $t=1$ if condition 22 is satisifed.

Furthermore since banks are always solvent the return they pay to depositors is equal to the storage technology and consequently $p_0^d = 1$. Also the return charged to creditors becomes equal to 1 since now they are not discounted with the sovereign default probability, as a sovereign default does not imply insolvency. This implies that the minimum level of safe assets that guarantee no exposure is given by

$$S_0 = \lambda_2 Y_0^h - (1-\theta_2)K_0 \tag{23}$$

When banks are not exposed then a sovereign default does not trigger the liquidation of the bank and consequently the output loss after default is only given by the fraction ϑ. Then repayment is optimal if

$$\begin{aligned} C^r &\geq C^d \\ \omega L_0 - B_0^f &\geq (1-\vartheta)\omega L_0 \\ \omega &\geq \frac{1}{\vartheta}\frac{B_0^f}{L_0} \end{aligned}$$

the default threshold in this case is given by $\frac{1}{\vartheta}\frac{B_0^f}{L_0}$ that is smaller than in the baseline case $\omega^n = \frac{1}{\Theta}\frac{B_0^f}{L_0}$ since $\theta > 0$ implies that $\Theta > \vartheta$. Then the system of equation that determine the price of debt and the level of debt issuance are given by

$$\begin{aligned} B_0^f &= \frac{R + K_0 + S_0 - \left(Y^h + Y^b\right)}{q_0} \\ q_0 &= 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right) \end{aligned} \tag{24}$$

that given we have that $S_0 > 0$ and $\Theta > \vartheta$ has a solution with a lower q_0 than the system in Proposition 1. To show this rewrite the system as in the proof of Proposition ##3

$$\begin{aligned} H_1(B_0^f) &= \frac{R + K_0 + S_0 - \left(Y^h + Y^b\right)}{B_0^f} \\ H_2(B_0^f) &= 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{L_0}\right) \end{aligned}$$

where an equilibrium price is the highest q_0 such that there is a B_0^f that satisifes $q_0^{ne,S} = H_1(B_0^{f,ne,S}) = H_2(B_0^{f,ne,S})$; where we have used superscript ne, S to refer to the no exposure case achieved by holding the safe asset S. In that proof we show that H_1 crosses from above H_2 at the equilibrium. First note that even for $S_0 = 0$ the fact that this system has ϑ instead of Θ implies that the curve $H_2(B_0^f)$ is displaced to the left, or formally

$$H_2(B_0^f) = 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{L_0}\right) \leq 1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)$$

consequently the equilibrium happens at an equal or lower price than in the case with exposure. The equilibrium is at a strictly lower price if there is any mass for $\omega \in \left(\frac{1}{\Theta}\frac{B_0^f}{L_0}, \frac{1}{\vartheta}\frac{B_0^f}{L_0}\right)$ as then the previous inequality becomes a strict inequality. Furthermore by having $S_0 > 0$ lowers further the price of debt as shown in the proof of Propostion ##3.

Case ii: No exposure (ne) achieved by having larger equity in the banking sector.

In the case where the no exposure is achieved by increasing bank equity we shift a fraction of households from savers to bankers. Such that the total endowment of the economy is constant $Y_0^b + Y_0^h$ but the initial resources owned by banks is larger up to the point where deposits are low enough such that

$$\lambda_2 D_0 = (1 - \theta_2) L_0$$

so exposure ξ_2 is equal to zero. In this case a single bank is indifferent between holding sovereign debt or the safe asset, although overall the total debt holdngs of the baking system determine the risk premia of bonds. We focus on the case where banks hold $S_0 = 0$ that is the case with the minimum spread for government debt. In that case the system of equations that that determine the price of debt and the level of debt issuance is given by

$$\begin{aligned} B_0^f &= \frac{R + K_0 - \left(Y^h + Y^b\right)}{q_0} \\ q_0 &= 1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{K_0}\right) \end{aligned} \tag{25}$$

where we have used the fact that the defalt cost is only ϑ as opposed to Θ. The solution to this system is the equilibrium price of debt $q_0^{ne,E}$ and debt issuance $B_0^{f,ne,E}$; where the superscript ne, E refers to the no exposure case by having higher equity.

Following the same argument as in the previous case this implies that the price of sovereign debt is lower than in the exposure case. Also, since $S_0 = 0$ and the first curve is not shifted, the price is higher than in the case where no exposure is achieved with the safe asset..

□

C.5 Proof Proposition 6

First we show that welfare is lower in the no exposure economy where bank capital is larger (ne, E), compared to economy with exposure. Then we move to show that the welfare is even lower in the no exposure economy that has larger safe assets.

Proof. **Lower welfare in the no exposure and higher bank capital economy.**

Government revenues are the same in both economies, so we have that

$$q_0^{ne,E}\left(B_0^{f,ne,E}+B_0^{h,ne,E}\right)=q_0\left(B_0^f+B_0^h\right)$$

and since in both cases the banks invest the same amount of resources, we also have

$$K_0+q_0B_0^h=K_0+q_0^{ne}B_0^{h,ne}$$

combining these two equations we get

$$q_0^{ne,E}B_0^{f,ne,E}=q_0B_0^f \tag{26}$$

Next consider the consumption levels in the two economies. Consumption in the baseline economy with exposure - conditional on $s=n$ or the no sunspot equilibrium - is, in case of repayment r or default d, given by:

$$\begin{aligned}C^{n,r}&=\omega L_0-B_0^f\\ C^{n,d}&=\omega(1-\vartheta)(1-\theta)L_0\end{aligned}$$

Expected consumption - conditional on $s=n$ or the no sunspot equilibrium - is thus

$$\begin{aligned}E(C)=&\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}}\omega(1-\vartheta)(1-\theta)L_0\partial F(\omega)+\int_{\frac{1}{\Theta}\frac{B_0^f}{L_0}}^{\bar{\omega}}\left(\omega L_0-B_0^f\right)\partial F(\omega)\\ \implies E(C)=&E(\omega)L_0-\left[1-F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)\right]B_0^f-\Theta L_0\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}}\omega f(\omega)d\omega\end{aligned}$$

Consumption in the economy with no exposure is given by

$$\begin{aligned}C^{ne,E,r}&=\omega L_0-B_0^{ne,f}\\ C^{ne,E,d}&=(1-\vartheta)\omega L_0\end{aligned}$$

and consequently expected consumption is given by

$$E(C^{ne,E})=E(\omega)L_0-\left[1-F\left(\frac{1}{\vartheta}\frac{B_0^{ne,f}}{L_0}\right)\right]B_0^{ne,f}-\vartheta L_0\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega$$

we can use 26 to write this as

$$E(C^{ne,E})=E(\omega)L_0-\left[1-F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)\right]B_0^f-\vartheta L_0\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega$$

Welfare is lower in the no exposure economy if the following condition holds

$$E\left(C\right) > E\left(C^{ne,E}\right)$$
$$\implies -\Theta L_0 \int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega) d\omega > -\vartheta L_0 \int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega) d\omega$$

that can be rewritten as

$$\frac{\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega) d\omega}{\frac{B_0^f}{\Theta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega) d\omega}{\frac{B_0^f}{\vartheta L_0}}$$

and using again equation 26 we have

$$\frac{\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega) d\omega}{\frac{B_0^f}{\Theta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}} \omega f(\omega) d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}} \frac{1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right)}{1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}\right)}$$

and since we have from Proposition 5 that

$$1 - F\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right) \geq 1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}\right)$$

a sufficient condition that guarantees $E\left(C\right) > E\left(C^{ne,E}\right)$ is

$$\frac{\int_0^{\frac{1}{\Theta}\frac{B_0^f}{L_0}} \omega f(\omega) d\omega}{\frac{B_0^f}{\Theta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,Ef}}{L_0}} \omega f(\omega) d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}}$$

Note that each side of the inequality are the averages of $\omega f(\omega)$ within an interval. So since $\frac{1}{\Theta}\frac{B_0^f}{L_0} < \frac{1}{\vartheta}\frac{B_0^{fne,E}}{L_0}$ if we have that $\omega f(\omega)$ is not decreasing in $\left[0, \frac{1}{\vartheta}\frac{B_0^{fne,E}}{L_0}\right]$ the inequality is satisfied. This condition implies$f'(\omega) \geq \frac{-f(\omega)}{\omega}$ $\forall \omega \in \left[0, \frac{1}{\vartheta}\frac{B_0^{fne,E}}{L_0}\right]$.

Lower welfare in the no exposure with the sfa easset requirement.

Consider the second variation from the previous proposition where banks have more safe assets and less bonds. Government revenues are the same in both economies, so we have that

$$q_0^{ne,E}\left(B_0^{f,ne,E} + B_0^{h,ne,E}\right) = q_0\left(B_0^f + B_0^h\right)$$

and since in both cases the banks invest the same amount of resources, we also have

$$K_0 + q_0 B_0^h = K_0 + q_0^{ne} B_0^{h,ne,S} + S_0^{ne,S}$$

combining these two equations we get

$$q_0^{ne,S}\left(B_0^{ne,S,f}\right) - S_0^{ne,S} = q_0\left(B_0^f\right) \tag{27}$$

and using Equation 23 we get

$$q_0^{ne,E}B_0^{ne,S,f} - \left(\lambda_2 Y_0^h - (1-\theta)K_0\right) = q_0 B_0^f$$

Expected consumption in the economy with no exposure by holding safe assets is given by

$$E(C^{ne,S}) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}\right)\right]B_0^{ne,S,f} - \vartheta L_0\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega + S_0^{ne,S}$$

and using equation 27 we obtain

$$E(C^{ne,S}) = E(\omega)L_0 - \left[1 - F\left(\frac{1}{\vartheta}\frac{B_0^f}{L_0}\right)\right]B_0^f - \vartheta L_0\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega$$

and consequently welfare is lower when no exposure is achieved by a safe asset requirement if

$$E\left(C^{ne,E}\right) > E\left(C^{ne,S}\right)$$
$$\implies -\vartheta L_0\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega > -\vartheta L_0\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega$$

that we can rewrite as

$$\frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}}$$

and using Equations 26 and 27 we have

$$\frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}\frac{q_0^{ne,E}}{q_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^f}{\vartheta L_0}}$$
$$\implies \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^{ne,S,f}}{\vartheta L_0}}\frac{q_0^{ne,E}}{q_0^{ne,S}}$$

as shown in Proposition **??** the ratio $\frac{q_0^{ne,E}}{q_0^{ne,S}} > 1$ and consequently a sufficient condition is that

$$\frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^{ne,E,f}}{\vartheta L_0}} < \frac{\int_0^{\frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}}\omega f(\omega)d\omega}{\frac{B_0^{ne,S,f}}{\vartheta L_0}}$$

So since $\frac{1}{\vartheta}\frac{B_0^{ne,E,f}}{L_0} < \frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}$ if we have that if $\omega f(\omega)$ is not decreasing in $\left(0, \frac{1}{\vartheta}\frac{B_0^{ne,S,f}}{L_0}\right)$ the inequality is satisfied. □

C.6 Proof Proposition 7

Proof. We solve for the equilibrium by backward induction. First focusing on the state where $S = N$ (normal state).

Now we have a continuum of countries and the banks are allowed only to hold the senior tranch of a CDO backed by the debt issued by each country. The subordination level is given by ς. Having a continuum of countries, this implies that local banks are not exposed to local sovereign debt.

We start from the conjecture that for $S = N$ no bailout has to be implemented in $t = 1$, then we verify this is the case. For $t = 2$, and $S = N$ the condition that guarantees that there is never a bank-run in $t = 2$ is given by

$$Q_2^s \mathcal{B}_0^s + (1-\theta)L_0 \geq \lambda_2 D_0$$

that we assume holds and then also verify it. So under the two conjectures of no bailout and bank solvency in $t = 2$ even in case of sovereign default, total consumption in case of default is

$$C^d = (1-\vartheta)\omega L_0 + Q_2^s \mathcal{B}_0^s$$

and in case of repayment

$$C^{ne,r} = \omega L_0 + Q_2^s \mathcal{B}_0^s - B_0$$

and consequently the default threshold is given by $\omega^{esbies} = \frac{1}{\vartheta}\frac{B_0}{L_0}$. Note that the default cost is only captured in this case by ϑ, since there is there is no need to liquidate banks after a default.

Given this default threshold, the value of the senior tranche at $t = 2$ is given by

$$Q_2^s = \min\left\{1, \frac{1 - F\left(\frac{1}{\vartheta}\frac{B_0}{L_0}\right)}{1-\varsigma}\right\}$$

and of the junior tranche

$$Q_2^j = \max\left\{0, \max\left\{0, \frac{\varsigma - F(\omega^N)}{1-\varsigma}\right\}\right\}$$

In this case there is no uncertainty in $t = 2$ with respect to the payoff of the two tranches once the state $S = N$ is revealed in $t = 1$. Furthermore, since we assume that the possibility of panic is not anticipated by agents, the prices at $t = 1$ (for $S = N$) and at issuance $t = 0$ are the same of those in $t = 2$. Furthermore the price of the bond is given by the repayment probability

$$q_0 = 1 - F\left(\frac{1}{\vartheta}\frac{B_0}{L_0}\right)$$

and consequently the debt issuance required to obtain a revenue of R is the minimum value of B_0 that solves

$$R = B_0\left(1 - F\left(\frac{1}{\vartheta}\frac{B_0}{L_0}\right)\right)$$

We are left to verify our two conjectures hold at these prices. Since bankers are always solvent the interest rate on loans and deposits is zero. This implies that $D_0 = Y_0^h$ and $L_0 = K_0$. Furthermore, the balance sheet of the bank at $t = 0$ satisfies that

$$Y_0^h + Y_0^b - K_0 = Q_0^S B_0^S$$

and replacing this in condition (2) we have

$$\begin{aligned}
& Y_0^h + Y_0^b - K_0 + (1-\theta)(1-\phi)L_0 \geq \lambda_1 D_0 \\
\Longrightarrow\ & Y_0^h + Y_0^b - K_0 + (1-\theta)(1-\phi)K_0 \geq \lambda_1 Y_0^h \\
& \qquad \Longrightarrow Y_0^h \geq \lambda_1 Y_0^h - (1-\theta)(1-\phi)K_0 - \left(Y_0^b - K_0\right)
\end{aligned}$$

the last expression is precisely the Assumption 2. So we verify that banks are solvent in $t = 1$. Following the same steps for the solvency condition at $t = 2$ we have

$$Y_0^b + (1-\lambda_2)Y_0^h \geq \theta K_0$$

and this is also guaranteed by Assumption 2. So we have verified the two conjectures hold in equilibrium.

Now we move to the case where $S = P$ (world panic). We conjecture that banks become insolvent and require a bailout and then find the equilibrium price of debt to then verify indeed banks are insolvent at that price.

If the bank had to be bailed out in $t = 1$, then the bank got a transfer S_1 and the outstanding debt increased to $B_1 = B_0 + \frac{S_1}{q_1^p}$ where q_1^p is the price of sovereign debt at $t = 1$ in case of panic. In this case the consumption in case of repayment is

$$C^{ne,r} = \omega K_0 + Q_2^{s,p}\mathcal{B}_0^s + S_1 - B_1$$

and in case of default

$$C^{ne,d} = (1-\vartheta)\omega K_0 + Q_2^{s,p}\mathcal{B}_0^s + S_1$$

the default threshold is then given by

$$\omega^{esbies} K_0 + Q_2^s \mathcal{B}_0^s + S_1 - B_1 = (1-\vartheta)\omega^{esbies} K_0 + Q_2^s \mathcal{B}_0^s + S_1$$

$$\omega^{esbies} = \frac{1}{\vartheta}\frac{B_1}{K_0}$$
$$\Longrightarrow \omega^{esbies} = \frac{1}{\vartheta}\frac{B_0}{K_0} + \frac{1}{\vartheta}\frac{S_1/q_1^p}{K_0}$$

Implicitly we have assumed that the bailout is large enough to avoid a bank run at $t = 1$ and $t = 2$. Since by our assumption, exposure is larger at $t = 2$ than at $t = 1$, then the bailout transfer is set such that the bank is still solvent at $t = 2$ and is given by

$$S_1 = \lambda_2 D_0 - (1-\theta)L_0 - Q_1^{s,p}\mathcal{B}_0^s$$

this trivially satisfied that bank are solvent in $t = 2$ and by Assumption (1) also at $t = 1$. To finance the bailout the newly issued debt is given by

$$\Delta B_1 = \frac{S_1}{q_1^p}$$
$$= \frac{\lambda_2 D_0 - Q_1^{s,p}\mathcal{B}_0^s - (1-\theta)L_0}{q_1^p}$$

replacing this in the default threshold we have

$$\omega^{esbies} = \frac{1}{\vartheta}\frac{B_0}{L_0} + \frac{1}{\vartheta}\frac{\lambda_2 D_0 - Q_1^{s,p}\mathcal{B}_0^s - (1-\theta)L_0}{q_1^p L_0}$$

and since in the panic equilibrium it has to be the senior tranche is partially defaulted we have that

$$Q_1^{s,p} = \frac{q_1^p}{1-\varsigma}$$

and since we defined the senior and junior tranche to have face value of 1, just as the bonds, we have that their total supply is given by $\mathcal{B}_0^s = (1-\varsigma)\mathcal{B}_0$ and in the symmetric equilibrium $\mathcal{B}_0 = B_0$ and replacing that in the default threshold we get

$$\omega^{esbies} = \frac{1}{\vartheta}\frac{B_0}{L_0} + \frac{1}{\vartheta}\frac{\frac{\lambda_2 D_0 - (1-\theta)L_0}{q_1^p} - B_0}{L_0}$$

then $t = 1$ the threshold for default and the price of debt is the solution to the system of equations

$$\omega^{esbies} = \frac{1}{\vartheta}\frac{B_0}{L_0} + \frac{1}{\vartheta}\frac{\frac{\lambda_2 D_0 - (1-\theta)L_0}{q_1^p} - B_0}{L_0}$$
$$q_1^p = 1 - F\left(\omega^{esbies}\right) \tag{28}$$

we can rewrite the system using two functions of q in terms of ω as

$$H_1(\omega) = \frac{\lambda_1 D_0 - q_1^{ne,L,h} L_0}{\vartheta \omega L_0}$$

$$H_2(\omega) = 1 - F(\omega)$$

and an equilibrium threshold satisfies $q_1^p = H_1(\omega^{esbies}) = H_2(\omega^{esbies})$ and q_1^p corresponds to the equilibrium price.

We have that i) both functions are decreasing in ω; ii) evaluated at $\omega^n = \frac{1}{\vartheta}\frac{B_0}{L_0}$ we have that $H_1(\omega^n) < H_2(\omega^n)$, this since $H_2(\omega^n)$ is the price of sovereign debt in the case $S = N$ and which ensures that banks are solvent. While $H_1(\omega^n)$ is the price of debt that makes the solvency condition be satisfied with equality. iii) We assume the support of ω is bounded above and and $F(\omega)$ is continuous.

Follows from i), ii) and iii) that the system of equations **??** has at least one solution with $\omega^{esbies} > \omega^N$ and consequently with a bailout.

□

C.7 Proof Proposition 8

Proof. The variables at $t = 0$ are given in the proof of Proposition 1 (no sunspot) and are the same here since we assume the recession is perceived to have probability zero.

First we show by contradiction than in case of recession, the bank becomes insolvent at $t = 1$. Suppose that for $s = r$ the bank is solvent in $t = 1$ and no bailout is requires. Then at $t = 2$ total consumption in case of repayment is given by

$$C^R = \omega K_0 - B_0^f$$

and in case of default

$$C^D = (1-\theta)(1-\vartheta)\omega K_0$$

the default threshold would then coincide with $\omega^N = \frac{1}{\Theta}\frac{B_0^f}{K_0}$. Now evaluating the solvency condition with this default threshold we have that the bank is insolvent a faces a bank run if

$$q_1^r B_0^h < \lambda_1 D_0 - (1-\phi)(1-\theta)L_0$$

where the price q_1^r is given by the repayment probability $q_1^r = 1 - F^r\left(\omega^N\right) = \left(1 - F\left(\omega^N\right)\right)(1-\epsilon)$ so ϵ determines the proportional fall in the price. Replacing this in the solvency condition we get

$$\begin{aligned} q_1^n B_0^h (1-\epsilon) &< \lambda_1 D_0 - (1-\phi)(1-\theta)L_0 \\ \left(Y_0^h + Y_0^b - K_0\right)(1-\epsilon) &< \lambda_1 D_0 - (1-\phi)(1-\theta)L_0 \end{aligned}$$

and replacing the price of loans and deposits

$$\left(Y_0^h + Y_0^b - K_0\right)(1-\epsilon) < \lambda_1 \frac{Y_0^h}{p_0^D} - (1-\phi)(1-\theta)\frac{K_0}{p_0^L}$$

that for q_0 close enough to one is guaranteed by Assumption (5). Consequently the bank is insolvent.

Let the price of sovereign debt at $t = 1$ for $s = r$ and with an announced bailout be given by q_1^r, the necessary bailout transfer to make the bank solvent and rule out a bank run is given by a level of safe assets S_1 given by

$$S_1 = \lambda_1 D_0 - \left(q_1^r B_0^h + (1-\phi)(1-\theta)L_0\right)$$

Since this transfer is financed with the issuance of sovereign debt, the new issuance required to finance this transfer is given by

$$\begin{aligned} \Delta B_1^f &= \frac{S_1}{q_1^r} \\ \implies \Delta B_1^f &= \frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0}{q_1^r} - B_0^h \end{aligned}$$

and consequently the total debt held by foreigners is

$$B_1^f = B_0^f + \frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0}{q_1^r} - B_0^h$$

and we see how it depends on the price of sovereign debt.

The price of debt q_1^r is given by the repayment probability

$$q_1^r = 1 - F^r(\tilde{\omega}^r) = (1 - F(\tilde{\omega}^r))\,(1-\epsilon)$$

where $\tilde{\omega}^r$ is the repayment threshold, that as shown in the previous proof is a function of foreign held debt as follows

$$\begin{aligned} \tilde{\omega}^r &= \frac{1}{\Theta}\frac{B_1^f}{K_0} \\ &= \omega^n + \frac{1}{\Theta}\frac{\frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0}{q_1^s} - B_0^h}{K_0} \end{aligned}$$

then we have that the equilibrium price of debt q_1^r and the default threshold $\tilde{\omega}^p$ are the solution to the system of equations

$$\begin{aligned} q_1^p &= (1 - F(\tilde{\omega}^r))\,(1-\epsilon) \\ \tilde{\omega}^p &= \omega^n + \frac{1}{\Theta}\frac{\frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0}{q_1^p} - B_0^h}{K_0} \end{aligned}$$

Existence and optimality of the bailout follows the same steps as the proof of Proposition (2).. □

C.8 Proof Proposition 9

Proof. Claim (i)

If $s = r$ and we include S_0 we have that the system of equations that determine the price of debt is

$$q = 1 - F^r(\omega)$$
$$\omega = \frac{1}{\Theta}\frac{B_0^f}{K_0} + \frac{1}{\Theta}\frac{\left(\frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - S_0}{q} - B_0^h\right)}{K_0}$$

that can be expressed as

$$G_3(\omega) = 1 - F^r(\omega)$$
$$G_4(\omega) = \frac{\lambda_1 D_0 - (1-\phi)(1-\theta)K_0/q_0 - S_0}{\Theta K_0 (\omega - \omega^n) + B_0^h}$$

where we have used that $L_0 = K_0/q_0$. The equilibrium price is the maximum value of q^r such that there exists an ω for which $q^r = G_3(\omega) = G_4(\omega)$.

First note that $G_3(\omega^n) = 1 - F^r(\omega^n) < q_0$ and $G_4(\omega^n) = \frac{\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - S_0}{B_0^h}$ where ω^n is the default threshold without recession and is given by $\frac{1}{\Theta}\frac{B_0^f}{L_0}$. Then we have that since banks are insolvent in case of recession $(1 - F^r(\omega^n))\, B_0^h < \lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - S_0$ and consequently $G_3(\omega^n) < G_4(\omega^n)$ and both non-increasing for $\omega \geq \omega^n$.

Then we have that if the two curves cross, at the crossing with the highest price $G_4(\omega)$ crosses $G_3(\omega)$ from above.

Finally since S_0 shifts $G_4(\omega)$ to the left and has no effect on $G_3(\omega)$ then the crossing is at a lower ω and consequently a higher q^r. It is the opposite case as what we showed for the panic case.

Claim (ii)

In this case the bank receives a bailout with bonds. The transfer of bonds is an amount ΔB_1^h that satisfies:

$$q_1^r B_0^h + (1-\phi)(1-\theta)L_0 + q^p \Delta B_1^h = \lambda_1 D_0$$

The default threshold in this case ends up being the same as in the normal equilibrium

$$\tilde{\omega}^r = \frac{1}{\Theta}\frac{B_0^f}{L_0}$$

since the additional debt issuance does not distort the default decision. There is still a bank run in case of default and no bank run in case of full repayment. The price of debt is nevertheless different, since the TFP distribution is shifted, and is given by

$$q_1^r = 1 - F^r\left(\frac{1}{\Theta}\frac{B_0^f}{L_0}\right) < q_1^n$$

Claim (iii)

Equivalent to the proof of Proposition **??** we have that the system of equation with a binding upper bound is given by

$$
\begin{aligned}
q_1^r &= (1 - F(\tilde{\omega}^r))\,(1-\epsilon) \\
\tilde{\omega}^r &= \omega^n + \frac{1}{\Theta}\frac{\frac{1}{q_1^r}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \overline{B}\right)}{K_0}
\end{aligned}
$$

where the new issuance of foreign debt is given by $\frac{1}{q_1^r}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \overline{B}\right)$. This implies that $\underline{B} < \lambda_1 D_0 - (1-\phi)(1-\theta)L_0$ to be binding, and consequently that a higher bound delivers $\tilde{\omega}^r = \omega^n$.

Claim (iv)

Under a no bailout commitment, if $s = r$ then Assumption 5 implies bank are insolvent and there is a bank run. In that case the default threshold is given by

$$
\tilde{\omega}^{run} = \frac{1}{(\Theta - \theta)(1-\phi)}\frac{B_0^f}{K_0} > \omega^n
$$

□

C.9 Proposition 10

Proof. The newly debt issued to finance the bailout is given by

$$
\Delta B_1 = \frac{1}{q_1^p} S_1
$$

Starting from the case where the constraint binds, we have that the number of bonds that end up in foreign creditors hands is given by

$$
\begin{aligned}
\Delta B_1^f &= \frac{1}{q_1^p} S_1 - \left(\frac{1}{q^{p,1}}\bar{B} - B_0^h\right) \\
&= \frac{1}{q_1^p}\left(\lambda_1 D_0 - \left(q^p B_0^h + (1-\phi)(1-\theta)L_0\right)\right) - \left(\frac{1}{q^p}\bar{B} - B_0^h\right) \\
&= \frac{1}{q^p}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}\right) \\
\Longrightarrow q^p \Delta B_1^f &= \lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}
\end{aligned}
$$

with these new levels of local and foreign debt, the total consumption if the government defaults is

$$
\begin{aligned}
C^d &= (1-\vartheta)(1-\theta)\omega K_0 + S_1 - q^{p,1}\left(\frac{1}{q^{p,1}}\bar{B} - B_0^h\right) \\
&= (1-\vartheta)(1-\theta)\omega K_0 - ((1-\phi)(1-\theta)L_0 - \lambda_1 D_0) - \bar{B}
\end{aligned}
$$

and under repayment is

$$\begin{aligned}
C^r &= \omega K_0 + S_1 - q^p\left(\frac{1}{q^p}\bar{B} - B_0^h\right) - B_1^f \\
&= \omega K_0 + S_1 - q^p\left(\frac{1}{q^p}\bar{B} - B_0^h\right) - \left(B_0^f + \frac{1}{q^p}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}\right)\right) \\
&= \omega K_0 + S_1 - q^p\left(\frac{1}{q^p}\bar{B} - B_0^h\right) - \left(B_0^f + \frac{1}{q^p}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}\right)\right)
\end{aligned}$$

so the default threshold is given by the value of ω that guarantees

$$\begin{aligned}
(1-\vartheta)(1-\theta)\omega K_0 + S_1 - q^p\left(\frac{1}{q^{p,1}}\bar{B} - B_0^h\right) &= \omega K_0 + S_1 - q^p\left(\frac{1}{q^p}\bar{B} - B_0^h\right) - \left(B_0^f + \frac{1}{q^p}\left(\lambda_1 D_0 - (1-\phi)(1\right.\right. \\
(1-\vartheta)(1-\theta)\omega K_0 &= \omega K_0 - \left(B_0^f + \frac{1}{q^p}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}\right)\right) \\
\omega &= \frac{1}{\Theta}\frac{B_0^f}{K_0} + \frac{\frac{1}{q^{p,1}}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}\right)}{\Theta K_0}
\end{aligned}$$

and consequently the default decision follows a threshold strategy and the system of equation which solution correspond to q^p and ω^p corresponds to

$$\begin{aligned}
q^p =& 1 - F(\omega^p) \\
\omega^p =& \frac{1}{\Theta}\frac{B_0^f}{K_0} + \frac{\frac{1}{q^p}\left(\lambda_1 D_0 - (1-\phi)(1-\theta)L_0 - \bar{B}\right)}{\Theta K_0}
\end{aligned}$$

The level of $\overline{B}$ to be binding is given by $\lambda_1 D_0 - (1-\phi)(1-\theta)L_0$, any level above that we have the whole bailout can be implemented with domestic bonds and we are back to section **??**.

By using that $D_0 = \frac{Y^h}{p_0^d}$ and $L_0 = \frac{K_0}{p_0^L}$ and that $p_0^L = q_0$ and $p_0^d = q_0\left(1 - \frac{(1-q_0)}{q_0}(1-\theta)\frac{K_0}{Y_0^d}\right)^{-1}$ we have that the level of $\overline{B}$ to be binding is

$$\frac{1}{q_0}\left(\lambda_1 Y_0^h\left(1 - \frac{(1-q_0)}{q_0}(1-\theta)\frac{K_0}{Y_0^d}\right) - (1-\phi)(1-\theta)K_0\right)$$

in the limit when $q_0 \to 1$ we have then it is sufficient to have $\bar{B}$ greater than $\xi_1 = \left(\lambda_1 Y_0^h - (1-\phi)(1-\theta)K_0\right)$

□