Higher-Order Income Risk over the Business Cycle

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Abstract

We extend the canonical income process with persistent and transitory risk to cyclical shock distributions with left-skewness and excess kurtosis. We estimate our income process by GMM for US household data. We find countercyclical variance and procyclical skewness of persistent shocks. All shock distributions are highly leptokurtic. The tax and transfer system reduces dispersion and left-skewness. We then show that in a standard incomplete-markets life-cycle model, first, higher-order risk has sizable welfare implications, which depend on risk attitudes; second, it matters quantitatively for the welfare costs of cyclical idiosyncratic risk; third, it has non-trivial implications for self-insurance against shocks.

Keywords: Idiosyncratic Income Risk, Cyclical Income Risk, Life-Cycle Model

J.E.L. classification codes: D31, E24, E32, H31, J31

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1 Introduction

The extent of idiosyncratic income risk matters for many macroeconomic questions. The first contribution of this paper is a novel parametric approach to estimate idiosyncratic income risk and its systematic variation over the business cycle within the canonical transitory-persistent decomposition (dating back at least to Gottschalk and Moffitt 1994). In our estimation framework we transparently identify skewness and kurtosis of both transitory and persistent shocks, together with their variance. The second contribution is that we systematically evaluate economic consequences of this higher-order risk. We find that, first, higher-order idiosyncratic risk has (economically relevant) implications for welfare. Second, cyclical higher-order idiosyncratic risk matters for the welfare costs of business cycles. Third, higher-order idiosyncratic risk matters for self-insurance through savings. Our moment-based approach allows for a clean decomposition into the role of the variance, skewness, and kurtosis of the shock distributions. In our analysis, we further transparently show which properties of preferences are relevant to understand the documented economic consequences.

We provide guidance for the empirical and quantitative analysis by first investigating a simple two-period model, in which agents face risky second period income. We compare a version of the model with higher-order risk to one without higher-order risk, but with the same dispersion of risky income. We show analytically that, first, larger higher-order risk (in particular: left-skewness) can have positive welfare implications (with log-utility). Second and related, the reaction of precautionary savings to larger higher-order risk is ambiguous. The utility and behavioral implications crucially depend on risk attitudes of households—and on the magnitude of (higher-order) risk. These results hold generally for shock distributions with the given moments.

Estimation of Higher-Order Risk. We characterize both transitory and persistent shocks by their second to fourth central moments, which in the case of the persistent shocks we allow to be state-contingent. We estimate these distribution moments using the second to fourth cross-sectional central moments and co-moments of incomes—while similar estimations traditionally are based solely on the variance-covariance matrix. Importantly, we do not impose any parametric distribution functions and estimate the moments of the shocks by the Generalized Method of Moments (GMM). Identification follows from the fact that the accumulated second to fourth central moments systematically differ across cohorts if these cohorts experience different macroeconomic histories—if the moments of shocks differ systematically over the business cycle. This identification idea was introduced in Storesletten et al. (2004) for the second moment and our extended estimator nests theirs as a special case. It is important to note that we include the third and fourth central moments in a way that does not affect the identification of the second moments or the persistence of the shocks: we proceed 'step-by-step', and first identify the second moments and persistence using only the variance-covariance moment conditions. We then hold persistence and second moments fixed and use the additional moment conditions only to identify the third and fourth central moments of the shocks.

While Storesletten et al. (2004) analyze household-level income including government transfers from the Panel Study of Income Dynamics (PSID) and find *countercyclical variance*[1] of persistent shocks, more recent evidence in Guvenen et al. (2014) suggests that the focus on the variance of log income changes alone misses the main characteristics of how individual risk varies with the aggregate state of the economy. Their findings based on administrative social security data (SSA) for individual males in the United States suggest that individual downside risk is larger in a contraction, while upside risk is smaller—this is reflected in a more pronounced left-skewness of the distribution of earnings changes. Related, Busch et al. (2020) conduct a non-parametric analysis of individual and household earnings dynamics in Germany, Sweden,

[1]This terminology has been introduced in the macroeconomic asset pricing literature, see Mankiw (1986), Constantinides and Duffie (1996), and Storesletten et al. (2007). Building on the conceptual framework of Storesletten et al. (2004), Bayer and Juessen (2012) focus on residual hourly wages (at the household level) and based on PSID data estimate countercyclical dispersion of persistent shocks in the United States.

France, and the US. They also find *procyclical skewness* of individual and household-level annual earnings changes.

Our estimation approach allows us to draw a richer image of income dynamics over the business cycle within the transitory-persistent framework and to thus bridge the previous analyses. Taking into account the second moment alone might lead to wrong conclusions if the change of the distribution is asymmetric, which is captured by a change of the third moment.[2]

We apply the estimation to survey data from the PSID, which allows us to control for a rich set of household-level information and to take into account several relevant transfer components. While being a smaller sample compared to administrative SSA data, it allows us to analyze features of earnings dynamics at the *household* level. Busch et al. (2020) show that the cyclical features of earnings changes at the individual level documented in Guvenen et al. (2014) are well reflected in the PSID. Also, De Nardi et al. (2020) show that many recently documented richer features of individual earnings dynamics carry over to the PSID.[3]

We estimate two separate income processes at the household level: one for joint labor income, and one for income after taxes and transfers. Comparison of the corresponding estimates is informative about the success of the existing tax and transfer scheme to dampen risk and its cyclicality. We find that both transitory and persistent shocks to pre-government earnings feature strong left-skewness, and that persistent shocks are significantly cyclical: in contractions, their distribution is more dispersed and more left-skewed. We also find that the existing tax and transfer system insures against both types of income shocks. The distribution of both shocks to post-government income (after taxes and

[2]As discussed in Busch et al. (2020), if the lower tail of a distribution expands by more than the upper tail collapses, then the distribution is more dispersed (an increase in the second moment) and more skewed to the left (a drop of the third moment).

[3]In follow-up work to Guvenen et al. (2014), Guvenen et al. (2016) document that, in a given year, most individuals experience very small earnings changes, while some workers experience very large changes of their earnings. This is summarized by a high kurtosis—relative to what the conventional assumption of log-normality implies. De Nardi et al. (2020) present similar evidence for the Netherlands, and Druedahl and Munk-Nielsen (2018) for Denmark.

transfers) is compressed relative to the respective shocks to pre-government income, but persistent shocks remain significantly cyclical. The magnitude of cyclicality of dispersion is in line with Storesletten et al. (2004). Finally, we find strong excess kurtosis of transitory and persistent shocks. It is higher for post- than for pre-government earnings suggesting that after redistribution more mass is concentrated in the center relative to the tails of the distribution. One related recent study of cyclical risk is Angelopoulos et al. (2019), who adapt a version of the GMM estimator developed in the present paper and document procyclical skewness of persistent shocks in Great Britain using data from the British Household Panel Study.

Implications of Higher-Order Risk. We next assess whether the estimated deviations from an income process with log-Normal shocks are economically significant. To this end we set up a standard incomplete-markets life-cycle model, in which households receive stochastic income following the estimated process throughout their working life, after which they enter a retirement phase and receive income through a pay-as-you-go pension system. We focus on ex-post heterogeneity, and thus the only source of inequality in the model is the risky idiosyncratic component of household income. The only means of self-insurance against the income risk explicitly present in the model is through private savings in a risk-free asset. We calibrate the model such that households face the income process estimated on post government household income, reflecting the view that it represents the amount of risk remaining after other channels of insurance against individual level risk—namely: within-household insurance and government taxes and transfers (cf. Blundell et al. 2008). We normalize all shocks in levels, and in this sense the economy does not feature aggregate risk. This reflects our interest in the role of cyclical changes in idiosyncratic risk, and in the relevance of higher-order risk. Agents have recursive preferences over consumption a la Epstein and Zin (1989, 1991), and Weil (1989), which we choose because it allows us to separately control the intertemporal elasticity of substitution and the coefficient of risk aversion. The latter also pins down the higher-order risk attitudes, and through this is a crucial determinant of the behavioral reaction to higher-order risk. To assess the implications of higher-order risk we compare model outcomes under the calibration with the estimated income process and under an alternative calibration where the process features the same dispersion of shocks, but with skewness and kurtosis of the Gaussian distribution (zero and three, respectively).

Our analysis delivers three main findings. First, evaluated from an ex-ante perspective higher-order risk has sizable negative welfare implications for strong risk attitudes: the consumption equivalent variation (CEV) that makes agents in the economy with log-Normal shocks indifferent to the economy with higher-order risk ranges between -0.4% (for a coefficient of relative risk aversion of 2) and -12.5% (relative risk aversion of 4). The dominant economic mechanism driving this welfare result is an expected reallocation of consumption over the life-cycle: when facing riskier income, risk-sensitive agents have more precautionary savings, and thus less consumption at young ages. With weak risk attitudes (specifically, for log utility), the welfare effect can be positive (CEV of 0.4%).[4]

Second, higher-order risk matters for the welfare costs of business cycles. Since Lucas (1987, 2003) argued that the gains of smoothing cycles beyond what the existing tax and transfer system does would be small, several studies have explored the role of both ex-ante and ex-post heterogeneity, wih Imrohoroglu (1989) being the first to emphasize the importance of idiosyncratic risk and incomplete markets. In a model similar to hers, Storesletten et al. (2001) allow for cyclical variance of persistent shocks as estimated in Storesletten et al. (2004). Following a similar strategy, we provide the first systematic assessment of the welfare consequences of cyclical higher-order risk as captured in a continuous distribution function, and thus bridge this approach to papers that explore cyclical downside risk in the form of unemployment (e.g.,

[4]It turns out that the mechanical relationship between the distribution of shocks in logs and the distribution of shocks in levels is important for the results: introducing left-skewness in logs (while holding the variance in logs constant) leads to a reduction of the variance in levels. In other words, the introduction of *third-order risk* (left-skewness) mechanically reduces *second-order risk* (variance) when characterizing the distribution in levels.

Krusell and Smith 1999, Krusell et al. 2009, Krebs, 2003, 2007, and Beaudry and Pages 2001). Under higher-order risk we find welfare costs (computed as CEV making households in the non-cyclical economy indifferent to the cyclical economy) which are 0.3%p (relative risk aversion of 2) to 6.4%p (relative risk aversion of 4) larger than under log-Normal shocks.

Third, we document that higher-order risk crucially matters for the degree of self-insurance against shocks. We employ a measure of self-insurance introduced in the literature by Blundell et al. (2008), who suggest to evaluate the degree of partial insurance against income shocks by estimating the pass-through of the identified transitory and permanent shocks to consumption changes. In the context of our model based analysis, we follow Kaplan and Violante (2010), who study how much of the empirically estimated partial insurance can be generated in a standard incomplete markets model. We find that incorporating higher-order risk leads to weaker pass-through of income shocks to consumption. However, this does not actually represent *better* insurance against negative shocks. In a scenario with higher-order risk agents have more precautionary savings (relative to a scenario in which they face log-Normal shocks), which implies that the consumption reaction to *positive* transitory and persistent shocks is weaker. *Negative* shocks actually translate stronger into negative consumption changes, because the higher savings do not suffice to smooth out shocks which are more pronounced relative to Normal shocks. Therefore, we caution against using only the insurance coefficient by Blundell et al. (2008) for the analysis of the degree of partial insurance against income risk.

Our paper is part of a growing literature that explicitly analyzes the implications of new insights on cyclical skewness of persistent earnings shocks for macroeconomic questions. Golosov et al. (2016) allow for time-varying skewness of idiosyncratic risk in a study of optimal fiscal policy, Catherine (2019) analyzes the implications of procyclical skewness of idiosyncratic income risk for the equity premium, and McKay (2017) links procyclical skewness to aggregate consumption dynamics. Besides the particular economic outcomes of interest, our analysis differs by providing a transparent link between moments of the shock distribution and those outcomes, emphasizing the relevant properties of preferences. Our analysis is also related to work on the implications of rich earnings dynamics in general (without considering the cyclicality of risk). De Nardi et al. (2020) feed an income process a la Arellano et al. (2017) into an incomplete markets model and study the role of richer earnings dynamics for consumption insurance and the welfare costs of idiosyncratic risk. Their analysis focuses on non-linear features of the income process and corroborates results from Karahan and Ozkan (2013) regarding the role of age-dependent persistence and distributions of shocks. Civale et al. (2017) analyze implications of left-skewed and leptokurtic idiosyncratic shocks for the interest rate and aggregate savings in an otherwise standard Aiyagari economy.

The remainder of the paper is structured as follows. Section 2 provides guidance for the analysis by discussing the role of higher-order risk in a simple two-period model. Section 3 presents our empirical approach and discusses identification of the income process. Section 4 presents the results of applying our approach to US household earnings data from the PSID. Section 5 introduces the quantitative model to analyze the economic implications of higher-order income risk, Section 6 discusses the quantitative results, and Section 7 concludes.

2 Higher-Order Risk in a Two-Period Model

2.1 Setup

Endowments. A household lives for two periods denoted by $j \in \{0, 1\}$. At period 0 the household is endowed with an exogenous income of y_0. Period 1 income is risky, $y_1 = \exp(\varepsilon)$, for some random variable ε with distribution function $\Psi(\varepsilon)$, which features higher-order income risk. Households are born with zero assets and, in the general formulation of the model, have access to a risk-free savings technology with interest factor $R = 1$. Denoting by a_1 savings in period 1, the budget constraints in the two periods are

$$a_1 = y_0 - c_0, \qquad c_1 \leq a_1 + y_1.$$

Preferences. We consider additively separable preferences over consumption c_j in the two periods of life, $j \in \{0, 1\}$. The per period utility function takes the standard iso-elastic power utility form $u(c_j) = \frac{1}{1-\theta} c_j^{1-\theta}$, with concavity parameter θ. Thus preferences are given by

$$V = \begin{cases} \frac{1}{1-\theta}\left(c_0^{1-\theta} + \int c_1^{1-\theta} d\Psi(\varepsilon)\right) & \text{for } \theta \neq 1 \\ \ln(c_0) + \int \ln(c_0)\, d\Psi(\varepsilon) & \text{for } \theta = 1. \end{cases}$$

Notice that θ captures both risk attitudes as well as the inverse of the inter-temporal substitution elasticity. In the quantitative life-cycle model we use recursive preferences a la Epstein and Zin (1989, 1991), and Weil (1989) to distinguish the two aspects of preferences. In Appendix A.5 we show that the theoretical analysis presented in this section extends naturally to recursive preferences, and that the risk attitudes are the relevant component of preferences behind consumption reactions to higher-order income risk. In the following, we thus interpret θ as representing risk attitudes when appropriate.

Since we assume an interest rate of zero and no discounting of second-period utility, there is no life-cycle savings motive in this simple model.

2.2 Analysis

Hand-to-Mouth Consumers. We first analyze the role of higher-order risk for hand-to-mouth consumers by shutting down access to the savings technology through constraint $a_1 = 0$.

Consider a fourth-order Taylor series approximation of the objective function around the mean of second period consumption, $\mu_1^c = \mathbb{E}[c_1] = \int c_1 d\Psi(\varepsilon)$. After some transformations, cf. Appendix A.1 and in line with, e.g., Eeckhoudt and Schlesinger (2006), we find that

$$U \approx \frac{c_0^{1-\theta}}{1-\theta} + \left(\frac{1}{1-\theta} - \frac{\theta}{2}\mu_2^c + \frac{\theta(1+\theta)}{6}\mu_3^c - \frac{\theta(1+\theta)(2+\theta)}{24}\mu_4^c \right), \qquad (1)$$

where we impose the restriction $\mu_1^c = 1$ for expositional reasons (which is irrelevant for the results pertaining to second- to fourth-order risk discussed here). Note that under the assumption of the binding budget constraint,the central moments[5] of the level of consumption μ_k^c, $k = 1, \ldots, 4$ coincide with the respective moments $\mu_k^{\exp(\varepsilon)}$, $k = 1, \ldots, 4$, of second period income $\exp(\varepsilon)$.

We make the following observations using the expression in (1). First, consider changing one of the central moments of the distribution while holding the others constant. An increase of the variance, μ_2^c, or of the fourth central moment, μ_4^c, or a reduction of the third central moment, μ_3^c, leads to expected utility losses. Note that changing the third central moment while holding the variance fixed implies changing the shape of the distribution as summarized by the coefficient of *skewness*. Similarly, changing the fourth central moment while holding variance fixed implies changing the relative size of the center and tails of the distribution, as summarized by the coefficient of *kurtosis*. In the remainder of the analysis, whenever we speak of an increase of risk, we refer to a change of the distribution of shocks that entails at least one of these changes (increasing second or fourth central moments, or decreasing the third central moment). Second, the utility consequences of changes of risk are governed by relative risk attitudes,[6] which in case of the employed power utility function are all pinned down by θ. Stronger *relative risk aversion* θ implies stronger adverse effects of increasing variance; stronger *relative prudence* $1+\theta$ implies stronger adverse effects of increasing negative skewness; and stronger *relative temperance* $2+\theta$ implies stronger adverse effects of increasing kurtosis. Importantly, the role of higher-order risk increases exponentially in θ: the weight attributed to risk attitudes on the variance is θ, on the third moment

[5] The k^{th} central moment of variable x is given by $\mu_k^x = \mathbb{E}\left(x - \mu_1^x\right)^k$.

[6] The relative risk attitude of order n is given by $-\frac{u^n(c)}{u^{n-1}(c)}c$, where $u^n(c)$ denotes the n^{th} derivative of the per-period utility function $u(c)$.

is $\theta(1+\theta)$ and on the fourth moment is $\theta(1+\theta)(2+\theta)$. Third, for given θ the relative importance of risk decreases in the order of risk, which is captured by the weight terms of the Taylor approximation.

These observations play a crucial role for our quantitative evaluation. In particular, while our estimates presented in Section 4.2 imply a pronounced left-skewness and a strong excess kurtosis, which may lead to sizeable welfare losses, the overall effect depends crucially on the utility weight of this risk, and thus on the calibration of θ. Indeed, in the case of log utility ($\theta = 1$) the mean-preserving introduction of left-skewness generates welfare gains. We formally derive this implication, which at first glance may appear counterintuitive, in Appendix A.2. It turns out to be crucial that moments of shocks in *levels*, $\exp(\varepsilon)$, rather than of shocks in *logs*, ε, are relevant for utility.

Precautionary Savings. We now assume that households have access to a savings technology. Using the budget constraint in the utility function it is straightforward to derive the Euler equation of the maximization problem as (cf., e.g., Eeckhoudt and Schlesinger, 2008)

$$(y_0 - a_1)^{-\theta} = \mathbb{E}\left[(\exp(\varepsilon) + a_1)^{-\theta}\right] \approx (1+a_1)^{-\theta} + \frac{\theta(1+\theta)}{2}(1+a_1)^{-(2+\theta)}\mu_2^{\exp(\varepsilon)} - \frac{\theta(1+\theta)(2+\theta)}{6}(1+a_1)^{-(3+\theta)}\mu_3^{\exp(\varepsilon)} + \frac{\theta(1+\theta)(2+\theta)(3+\theta)}{24}(1+a_1)^{-(4+\theta)}\mu_4^{\exp(\varepsilon)}. \quad (2)$$

Notice that the LHS is increasing, and the RHS is decreasing in a_1 if $\mu_3^{\exp(\varepsilon)}$ is small enough relative to $\mu_2^{\exp(\varepsilon)}$ and $\mu_4^{\exp(\varepsilon)}$.[7] Consider the effect of an increase of risk of the income shock $\exp(\varepsilon)$ (through increasing the second or fourth central moment, or reducing the third central moment). An increase of the variance increases the RHS, scaled by the product of the measures of relative prudence and relative risk aversion $\theta \cdot (1+\theta)$. A reduction of the third central moment increases the RHS, additionally scaled by the measure of relative

[7]The RHS is deacreasing in a_1 iff $\mu_3^{\exp(\varepsilon)} \leq \frac{3}{(3+\theta)}(1+a_1)\mu_2^{\exp(\varepsilon)} + \frac{(4+\theta)}{4}(1+a_1)^{-1}\mu_4^{\exp(\varepsilon)}$.

temperance $(2+\theta)$. An increase of the fourth central moment increases the RHS, additionally scaled by the measure of *relative edginess* $(3+\theta)$.[8] Similar to what we saw in equation (1), the second to fourth moments are scaled by additional weight factors $\frac{1}{2(1+a_1)^{2+\theta}}$, $\frac{1}{6(1+a_1)^{3+\theta}}$, and $\frac{1}{24(1+a_1)^{4+\theta}}$, respectively.

Therefore, an increase of risk for a given a_1 increases the RHS, which is offset by an increase of savings a_1.[9] This result is very intuitive: ordinary and higher-order income risk increases precautionary savings, through which households reduce the adverse utility consequences of risk. The intensity of the behavioral reaction crucially depends on risk attitudes as governed by θ.

3 Income Process with Higher-Order Risk

3.1 The Income Process

Let log income of household i of age j in year t be

$$y_{ijt} = f\left(\mathbf{X}_{ijt}, Y_t\right) + \tilde{y}_{ijt}, \tag{3}$$

where $f\left(\mathbf{X}_{ijt}, Y_t\right)$ is the *deterministic* component of income, i.e., the part that can be explained by observable individual and aggregate characteristics, $\mathbf{X}_{ijt}$ and Y_t, respectively, and $\tilde{y}_{ijt}$ is the *residual* part of income, which is assumed to be orthogonal to $f\left(\mathbf{X}_{ijt}, Y_t\right)$. The deterministic component $f\left(\mathbf{X}_{ijt}, Y_t\right)$ is a linear combination of a cubic in age j, $f_{age}(j)$, the log of household size, year fixed effects, and an education premium $f_{EP}(t)$ for college education, which we allow to vary over years t:

$$f\left(\mathbf{X}_{ijt}, Y_t\right) = \beta_{0t} + f_{age}\left(j\right) + \mathbb{1}_{e_{it}=c} f_{EP}(t) + \beta^{size} \log\left(hhsize_{ijt}\right) \tag{4}$$

where $f_{age}\left(j\right) = \beta_1^{age} j + \beta_2^{age} j^2 + \beta_3^{age} j^3$, $f_{EP}(t) = \beta_0^{EP} + \beta_1^{EP} t + \beta_2^{EP} t^2$, and $\mathbb{1}_{e_{it}=c}$ is an indicator function that takes on value 1 for college-educated households.

[8] The term *edginess* was coined by Lajeri-Chaherli (2004).

[9] Formally, it is straightforward to show this by taking the total differential of (2), cf. Appendix A.4.

Residual income $\tilde{y}_{ijt}$ is the main object of interest in the analysis. We model $\tilde{y}_{ijt}$ as the sum of three components: a persistent component z_{ijt}, an i.i.d. transitory shock ε_{ijt}, and an idiosyncratic *fixed effect* χ_i. The idiosyncratic fixed effect is a shock drawn once upon entering the labor market from a distribution which is the same for every cohort.[10] The persistent component is modeled as an AR(1) process with innovation η_{ijt}:

$$\tilde{y}_{ijt} = \chi_i + z_{ijt} + \varepsilon_{ijt}, \text{ where } \varepsilon_{ijt} \underset{iid}{\sim} F_\varepsilon, \ \chi_i \underset{iid}{\sim} F_\chi \tag{5a}$$

$$z_{ijt} = \rho z_{ij-1t-1} + \eta_{ijt}, \text{ where } \eta_{ijt} \underset{id}{\sim} F_\eta \left(s\left(t\right)\right), \tag{5b}$$

where F_χ, F_ε, and $F_\eta \left(s\left(t\right)\right)$ denote the density functions of χ, ε_{ijt}, and η_{ijt}, respectively. We allow the density function of the persistent shock to depend on the aggregate state of the economy in period t, denoted by $s(t)$. This income process is exactly the canonical income process (e.g., Moffitt and Gottschalk, 2011). Unlike the canonical case, we do not (implicitly) assume that the shocks to the log income process are symmetric. Instead of only focussing on the variance of the shocks, we are interested in estimating the second to fourth central moments of the density functions, and denote those by μ_2^x, μ_3^x, and μ_4^x, for $x \in \{\chi, \varepsilon, \eta(s)\}$.[11]

As in Storesletten et al. (2004), the economy can be in one of two aggregate states, which we denote by E (expansion) and C (contraction). Thus, the central moments of the persistent shock $\mu_k^\eta \left(s\left(t\right)\right)$ are equal to $\mu_k^{\eta,E}$ if $s\left(t\right) = E$ and equal to $\mu_k^{\eta,C}$ if $s\left(t\right) = C$, for $k \in \{2, 3, 4\}$. Both empirical evidence (e.g., Blundell et al. 2008) and model-based analyses (e.g., Kaplan and Violante 2010) find that households can insure well against transitory shocks. We therefore follow Storesletten et al. (2004) and only consider the cyclicality of persistent income shocks, which have long-lasting effects in the context of a

[10] Thus, from the econometric perspective, we are estimating a random effects model.

[11] One potential disadvantage of using central moments to characterize the shocks in the income process is that they are hard to interpret by themselves. However, in the samples we use, the central moments of the cross-sectional income distribution are strongly correlated with percentile-based counterparts to those moments. We are thus confident that the estimated central moments—and the implied standardized moments *skewness* and *kurtosis*—do capture the salient features of the distribution.

life-cycle decision making problem. We still do capture skewness and kurtosis of the (acyclical) transitory component and explore its quantitative role.

We assume that upon entering the labor market, in addition to drawing the fixed effect χ_i, each worker draws the first realizations of transitory and persistent shocks, ε_{it} and η_{it}, from the distributions F_ε and $F_\eta\left(s\left(t\right)\right)$, respectively. Thus, the moments of the distribution of the persistent component for the cohort entering in year t at age $j=0$ are $\mu_k(z_{i0t})=\mu_k^\eta(s(t))$.

3.2 GMM Approach to Estimation

We follow the common approach in the literature and estimate (3) and (5) in two steps. In the first step, we estimate (3), which yields residuals $\tilde{y}_{ijt}$. In the second step, we estimate the parameters of the stochastic process (5) by fitting cross-sectional moments of the distribution of residual (log) income. As is standard, the variance terms of all components of (5) can be identified by the variance-covariance matrix. Similarly, the third and fourth central moments can be identified by third and fourth central moments and co-moments. Let $\theta=\left(\rho,\mu_2^\chi,\mu_2^\varepsilon,\mu_2^{\eta,E},\mu_2^{\eta,C},\mu_3^\chi,\mu_3^\varepsilon,\mu_3^{\eta,E},\mu_3^{\eta,C},\mu_4^\chi,\mu_4^\varepsilon,\mu_4^{\eta,E},\mu_4^{\eta,C}\right)$ be the vector of second-stage parameters, and let s^t summarize the history of aggregate states up to year t.[12] We denote central moments by $\mu_k\left(\cdot\right)$ and co-moments by $\mu_{kl}\left(\cdot\right)$, where

$$\mu_k\left(\tilde{y}_{ijt};\theta\right)=E\left[\left(\tilde{y}_{ijt}-E\left[\tilde{y}_{ijt}\right]\right)^k|s^t\right] \tag{6a}$$

$$\mu_{kl}\left(\tilde{y}_{ijt},\tilde{y}_{ij+1t+1};\theta\right)=E\left[\left(\tilde{y}_{ijt}-E\left[\tilde{y}_{ijt}\right]\right)^k\left(\tilde{y}_{ij+1t+1}-E\left[\tilde{y}_{ij+1t+1}\right]\right)^l|s^t\right]. \tag{6b}$$

[12] Note that we need to condition only on s^t, not on s^{t+1}, because period $t+1$ shocks are uncorrelated with all shocks accumulated up to period t.

The imposed process implies the following moments of the distribution of residual income at age j in year t:

$$\mu_2(\tilde{y}_{ijt};\theta) = \mu_2^\chi + \mu_2^\varepsilon + \mu_2(z_{ijt}) \tag{7a}$$

$$\mu_{11}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1};\theta\right) = \mu_2^\chi + \rho\mu_2(z_{ijt}) \tag{7b}$$

$$\mu_3\left(\tilde{y}_{ijt};\theta\right) = \mu_3^\chi + \mu_3^\varepsilon + \mu_3(z_{ijt}) \tag{7c}$$

$$\mu_{21}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1};\theta\right) = \mu_3^\chi + \rho\mu_3(z_{ijt}) \tag{7d}$$

$$\mu_4\left(\tilde{y}_{ijt};\theta\right) = \mu_4^\chi + \mu_4^\varepsilon + \mu_4(z_{ijt}) + 6\left(\mu_2^\chi\mu_2^\varepsilon + \left(\mu_2^\chi + \mu_2^\varepsilon\right)\mu_2(z_{ijt})\right) \tag{7e}$$

$$\mu_{31}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1};\theta\right) = \mu_4^\chi + \rho\mu_4(z_{ijt}) + 3\left(\mu_2^\chi\mu_2^\varepsilon + \left(\mu_2^\chi + \rho\left(\mu_2^\chi + \mu_2^\varepsilon\right)\right)\mu_2(z_{ijt})\right), \tag{7f}$$

where $\mu_k(z_{ijt})$, for $k = 2, 3, 4$ is shown in Appendix A.6.

A crucial implication of equations (7c) and (7e) is that the cross-sectional distribution of $\tilde{y}_{ijt}$ does not converge to a Normal distribution, as the third and fourth central moments of the shocks accumulate over age. This allows us to identify these higher-order moments of the shock distributions based on cross-sectional moments as outlined below. Denote the empirical counterparts of the moments by $m_2(\cdot)$, $m_3(\cdot)$, $m_4(\cdot)$, $m_{11}(\cdot)$, $m_{21}(\cdot)$, and $m_{31}(\cdot)$. This gives the following set of moment conditions employed in the GMM estimation:

$$E\left[m_2\left(\tilde{y}_{ijt}\right) - \mu_2\left(\tilde{y}_{ijt};\theta\right)|s^t\right] = 0 \tag{8a}$$

$$E\left[m_{11}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1}\right) - \mu_{11}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1};\theta\right)|s^t\right] = 0 \tag{8b}$$

$$E\left[m_3\left(\tilde{y}_{ijt}\right) - \mu_3\left(\tilde{y}_{ijt};\theta\right)|s^t\right] = 0 \tag{8c}$$

$$E\left[m_{21}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1}\right) - \mu_{21}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1};\theta\right)|s^t\right] = 0 \tag{8d}$$

$$E\left[m_4\left(\tilde{y}_{ijt}\right) - \mu_4\left(\tilde{y}_{ijt};\theta\right)|s^t\right] = 0 \tag{8e}$$

$$E\left[m_{31}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1}\right) - \mu_{31}\left(\tilde{y}_{ijt}, \tilde{y}_{ij+1t+1};\theta\right)|s^t\right] = 0. \tag{8f}$$

Huggett and Kaplan (2016) use a similar strategy based on second and third central moments and co-moments, without resorting to pre-sample aggregate information in the spirit of Storesletten et al. (2004) as we do. We use moment conditions (8a) and (8b) to estimate the variance parameters and the persistence ρ. Given an estimate for ρ, we then use moment conditions (8c) and (8d) to estimate the third central moments. Likewise, given estimates for ρ and the variance parameters, we use moment conditions (8e) and (8f) to estimate the fourth central moments.

Identification. The use of cross-sectional moments for identification allows us to exploit macroeconomic information that predates the micro panel, thereby incorporating more business cycles in the analysis than covered by the sample, as pointed out by Storesletten et al. (2004). Consider the persistent component of the income process in equation (5b): the variance of the innovations accumulate as a cohort ages, as can be seen from the theoretical moment in equation (7a). If the innovation variance is higher in contractionary years, then a cohort that lived through more contractions will have a higher income variance at a given age than a cohort at the same age that lived through fewer contractions, if the persistence is high.

Our extension of Storesletten et al. (2004) is based on the insight that other central moments accumulate in a similar fashion, as seen in equations (7c) and (7e). Consider the third central moment. If the probability of a large negative income shock was higher (or that of a large positive shock lower) during a contractionary period, then this would translate into the third central moment of the shock being smaller (more negative) than in an expansion, i.e., $\mu_3^{\eta,C} < \mu_3^{\eta,E}$. For a given dispersion this implies a reduction of skewness (a more left-skewed distribution). Comparing again two cohorts when they reach a certain age, this would imply a more negative cross-sectional third central moment for the cohort that worked through more contractions.

As seen in (7a), the sum $(\mu_2^{\chi} + \mu_2^{\varepsilon})$ is identified as the intercept of the variance profile over age. The same holds for $(\mu_3^{\chi} + \mu_3^{\varepsilon})$ in (7c), which is identified via the age profile of the third central moment. Considering the sum in (7a), we see that the magnitude of the increase of the cross-sectional variance over age identifies the variance of persistent shocks. The difference between $\mu_2^{\eta,C}$ and $\mu_2^{\eta,E}$ is identified by the difference of the cross-sectional variance of different cohorts of the same age. Likewise, the difference between

$\mu_3^{\eta,C}$ and $\mu_3^{\eta,E}$ is identified by the difference of the cross-sectional third central moment of different cohorts. Note that by restricting the transitory shocks to not vary over the business cycle we do not bias the estimated cyclicality of persistent shocks, which is identified via accumulated shock distributions.

Now consider the expressions for variance and covariance in equations (7a) and (7b). The difference between the two expressions identifies μ_2^{χ} separately from μ_2^{ε}. Likewise, the difference between the expressions for the third central moment and co-moment, equations (7c) and (7d), identifies μ_3^{χ} separately from μ_3^{ε}. Given ρ and the variance parameters μ_2^x for $x \in \{\chi, \varepsilon, \eta(s)\}$, equations (7e) and (7f) identify the fourth central moments μ_4^x for $x \in \{\chi, \varepsilon, \eta(s)\}$ in the same way as for the second and third central moments.

4 Estimation of the Income Process

4.1 Data and Sample Selection

We use data from the Panel Study of Income Dynamics (PSID), which interviews households in the United States annually from 1968 to 1997 and every other year since then. The representative core sample consists of about 2,000 households in each wave, and we use data from 1977–2012.[13] We estimate the income process at the household level for both pre- and post-government household income. De Nardi et al. (2020) show that at the individual level, the PSID sample captures well the salient features of earnings dynamics documented in administrative social security data by Guvenen et al. (2016), and Busch et al. (2020) document that the cyclical changes of the distribution of annual earnings changes in the PSID reflect the dynamics in social security data documented by Guvenen et al. (2014). Similarly, Arellano et al. (2017) estimate a rich earnings process using the PSID.

Household pre-government income is defined as labor income before taxes, which we calculate as the sum of head and spouse annual labor income. Post-

[13]We do not use earlier waves because of poor coverage of income transfers before the 1977 wave.

government income is defined as household labor income plus transfers minus taxes. As measure of labor income we use annual total labor income which includes income from wages and salaries, bonuses, and the labor part of self-employment income. We impute taxes using Taxsim, and add 50% of the estimated payroll taxes to the sum of head and spouse labor incomes to obtain pre-government income. We aggregate transfers to the household level and include measures of unemployment benefits, workers' compensation, combined old-age social security and disability insurance (OASI), supplemental security income, aid to families with dependent children (AFDC), food stamps, and other welfare.

We deflate all nominal values with the annual CPI, and select households if the household head is between 25 and 60 years of age. The minimum of household pre- and post-government income needs to be above a constant threshold, which is defined as the income from working 520 hours at half the minimum wage. Central moments (especially of higher order) are imprecisely estimated in small samples. We therefore estimate the moments for a given year and age group based on a sample from a five-year window over age within the year, which also smoothes the age profiles of these moments.

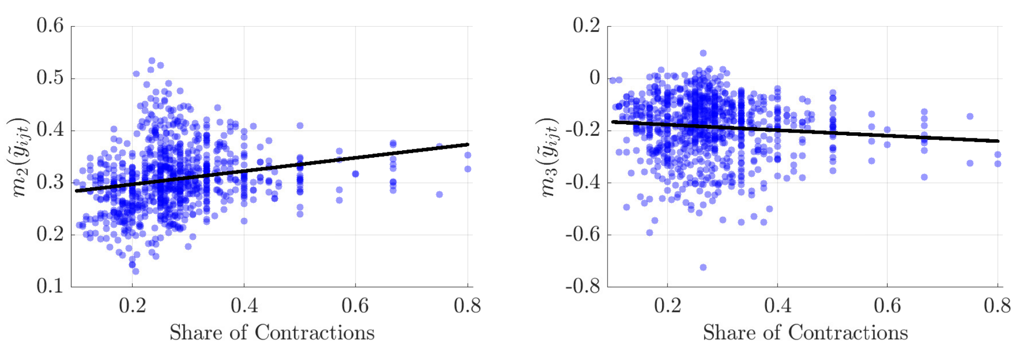
Defining Business Cycles. In order to implement the estimator we classify years as contractions or expansions. We initiate our definition on NBER peaks and trough data. The relevant time period is 1942–2012. Starting from the dating of peaks and troughs, we classify a year as a contraction if (i) it completely is in a contractionary period, which is defined as the time from peak to trough, (ii) if the peak is in the first half of the year and the contraction continues into the next year, (iii) if a contraction started before the year and the trough is in the second half of the year. Given the sluggish synchronization of labor market outcomes with the macroeconomic indicators that the NBER takes into account (cf. Guvenen et al. 2014, Huggett and Kaplan 2016), we expand the dating based on mean earnings of males in the PSID. We classify the following years as contractions: 1945, 1949, 1953, 1957, 1960, 1970, 1974, 1980–83, 1990–91, 2001–02, 2008–10. All years that are not classified as contractions are classified as expansions.

4.2 Estimation Results: Cyclical Idiosyncratic Risk

Illustration of Identification. Before turning to the estimation, in Figure 1 we plot the cross-sectional second and third central moments of residual income (for post-government income) used in the estimation, i.e., each marker denotes a moment for households of some age j in some year t, $m_k(\tilde{y}_{ijt})$ for $k = 2, 3$. The moments are plotted against the share of years classified as contractions out of all years a cohort went through since age 25 once reaching the given year. The pattern that emerges is that a higher share of contractionary years correlates positively with the cross-sectional second moments, and correlates negatively with the cross-sectional third moments. These correlations identify the cyclicality of the moments of the shocks in the esimated income process.

Figure 1: Cross-Sectional Moments by Aggregate History

(a) Second Cross-Sectional Moment (b) Third Cross-Sectional Moment



Notes: Cross-sectional moments of residual income are net of age effects. Share of contractions for a given moment is the fraction of years classified as contraction since age 25. The slopes of the fitted lines are 0.13 and –0.11 for m_2 and m_3, respectively. Moments for shares of 0 or 1 are not displayed here for visualization reasons (they are used in the GMM estimation).

Estimation. We now turn to the estimation results for household pre-government labor income (before taxes and transfers) and household post-government labor income (after taxes and transfers). We use the number of observations that contribute to an empirical moment as weights for the moment conditions, and this way assign more weight to those moments that are themselves estimated more reliably in the data. As additional moment conditions we add the averages over years of the second to fourth central moments of 1-5 year income changes. This ensures that the estimated income process is consistent both with moments of the cross-sectional distribution and with moments of income changes. We give a collective weight of 10% to the average moments of changes. In addition to the structure imposed so far, we hold the kurtosis of η fixed over the business cycle. Let α_i denote the i^{th} standardized moment: $\alpha_i = \mu_i/\mu_2^{i/2}$. Assuming $\alpha_4^\eta(s(t)) = \alpha_4^\eta$ implies $\mu_4^{\eta,C} = \alpha_4^\eta\left(\mu_2^{\eta,C}\right)^2$ and $\mu_4^{\eta,E} = \alpha_4^\eta\left(\mu_2^{\eta,E}\right)^2$. This leaves us with 12 parameters that need to be estimated. We use moment conditions (8a) and (8b) to estimate the variance parameters and the persistence ρ. Given an estimate for ρ, we then use moment conditions (8c) and (8d) to estimate the third central moments. Likewise, given estimates for ρ and the variance parameters, we use moment conditions (8e) and (8f) to estimate the fourth central moments. The third central moment of the cross-sectional distribution features a low-frequency change (see Panel (e) of Figure 2). In order to accommodate this in the estimation, and to not confound the estimated cyclicality, we add a linear trend to the third central moment of transitory shocks. We report the time average of the implied moment. For inference, we apply a block bootstrap procedure and resample households, which preserves the autocorrelation structure of the original sample. We draw 1,000 bootstrap samples. Table 1 shows the estimates, and Figure 2 illustrates the fit over age and time of the estimated process for post government income, the income variable we use in the quantitative analysis in Section 6 (see Appendix B for the implied standardized moments).

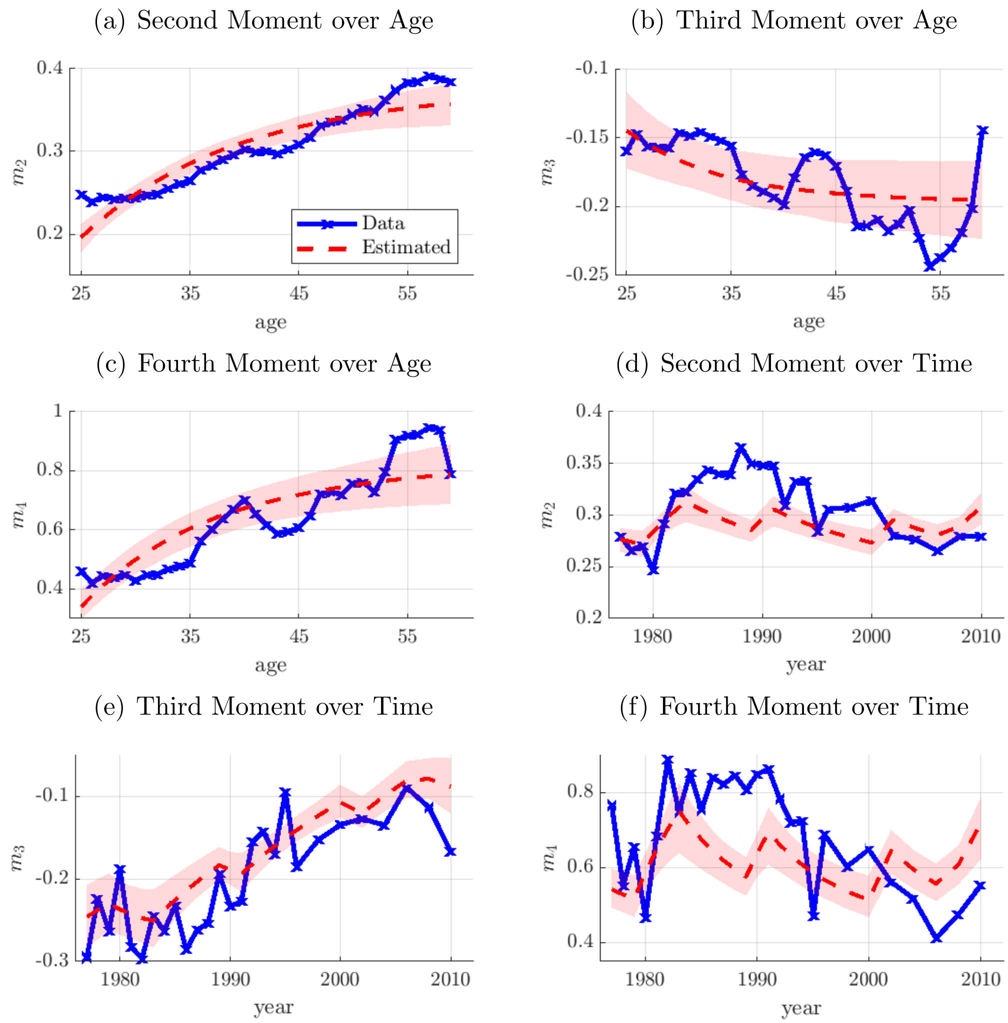
Cyclical Dispersion. The first panel of Table 1 reports the persistence of the AR(1) component of income along with the estimates of the variances of

Table 1: Estimation Results for Pre- and Post Government Income

	Estimated Central Moments		Implied Standardized Moments	
	HH Pre	HH Post	HH Pre	HH Post
ρ	0.9601	0.9683		
	[0.9412; 0.9756]	[0.9463; 0.9841]		
μ_2^{χ}	0.1591	0.1076		
	[0.1361; 0.1786]	[0.0897; 0.1237]		
μ_2^{ε}	0.1045	0.0752		
	[0.0948; 0.1133]	[0.0677; 0.0816]		
$\mu_2^{\eta,C}$	0.0375	0.0223		
	[0.0263; 0.0477]	[0.0152; 0.0291]		
$\mu_2^{\eta,E}$	0.0152	0.0085		
	[0.0099; 0.0229]	[0.0044; 0.0153]		
μ_3^{χ}	−0.1126	−0.0520	−1.77	−1.47
	[−0.1530; −0.0727]	[−0.0785; −0.0253]	[−2.47; −1.21]	[−2.32; −0.78]
μ_3^{ε}	−0.1516	−0.0866	−4.49	−4.20
	[−0.1623; −0.1374]	[−0.0935; −0.0772]	[−5.06; −3.97]	[−4.79; −3.73]
$\mu_3^{\eta,C}$	−0.0332	−0.0164	−4.59	−4.95
	[−0.0474; −0.0175]	[−0.0263; −0.0061]	[−6.73; −2.61]	[−7.80; −2.18]
$\mu_3^{\eta,E}$	−0.0047	−0.0012	−2.49	−1.54
	[−0.0128; 0.0035]	[−0.0069; 0.0040]	[−6.73; 2.26]	[−7.97; 9.09]
μ_4^{χ}	0.0607	0.0173	2.40	1.50
	[0.0000; 0.1508]	[0.0000; 0.0741]	[0.00; 5.39]	[0.00; 5.38]
μ_4^{ε}	0.4250	0.2300	38.94	40.62
	[0.3630; 0.4867]	[0.1927; 0.2664]	[34.52; 44.92]	[36.27; 47.65]
$\mu_4^{\eta,C}$	0.1359	0.0666	96.85	134.47
	[0.0856; 0.1719]	[0.0363; 0.0847]	[61.15; 141.97]	[82.02; 191.34]
$\mu_4^{\eta,E*}$	0.0225	0.0098	96.85	134.47
	[0.0089; 0.0488]	[0.0022; 0.0272]	[61.15; 141.97]	[82.02; 191.34]

Notes: Table shows estimated central moments for household earnings (HH Pre) and household income after taxes and transfers (HH Post). Brackets show 5^{th} and 95^{th} percentiles of 1,000 bootstrap estimates (in the case of post government income, 998 of the bootstrap iterations converge). $^*\mu_4^{\eta,E}$ not separately estimated.

Figure 2: Fit of Estimated Process for Post-Government Earnings



Notes: Moments are cross-sectional central moments. For each moment, age and year profiles are based on a regression of the moment on a set of age and year dummies. Blue lines: empirical moments; red dashed lines: theoretical moments implied by point estimates; shaded area denotes a 90% confidence band based on the bootstrap iterations.

the components of the income process estimated jointly. We estimate persistence parameters (ρ) of .96 and .97 for pre and post government income, respectively. The estimated variances of all components of the post-government income process are smaller than their counterparts for pre-government income. This is consistent with an interpretation that the existing tax and transfer system effectively dampens the idiosyncratic risk faced by households. Both for pre- and post-government income the estimates imply a countercyclical variance of persistent shocks: in aggregate downturns, the cross-sectional distribution of shocks is more dispersed. Our estimate of countercyclicality for post-government income is quantitatively similar to the one estimated by Storesletten et al. (2004): the estimated standard deviation of persistent shocks is 61% higher in aggregate contractions.

Cyclical Skewness. The second panel of Table 1 reports the third central moments. We find that all shock components estimated for pre-government and post-government income processes have negative third central moments, implying negative skewness of shocks. Comparing the post-government income process to the pre-government income process, the third central moments are smaller in magnitude, as expected from the reduced dispersion. For both pre and post government income, the third central moment of persistent shocks is significantly negative in contractions; point estimates of the third central moments of persistent shocks in expansions are also negative, however not statistically different from zero. The second and third central moments together translate into the third standardized moment, the coefficient of skewness, which is informative about the shape of the distribution and shown in the last two columns of Table 1. The cyclicality of the third central moment is stronger relative to the cyclicality of the second moment, which translates into the standardized moment displaying pro-cyclicality. Thus, aggregate contractions are periods in which negative persistent shocks become relatively more pronounced.

Excess Kurtosis. The third panel of Table 1 reports the fourth central moments. We restrict the kurtosis of persistent shocks to not vary with the aggregate state of the economy, i.e., $\alpha_4^\eta(s(t) = C) = \alpha_4^\eta(s(t) = E)$. Again, the last two columns of Table 1 list the implied standardized fourth moments (coefficients of kurtosis). The fixed effects are very imprecisely estimated; the point estimates imply relatively flat distributions (compared to a Normal distribtion, which has a kurtosis of 3): the implied kurtosis coefficient at the point estimates is 2.4 for pre-government income, and 1.5 for post-government income. The transitory and persistent shocks are estimated to display very pronounced excess kurtosis of about 39 and 97 for pre-government earnings, and about 41 and 134 for post-government earnings. These estimates imply that the distribution of post-government income shocks is more concentrated in the center, while some households experience shocks that are more extreme *relative* to the overall more compressed (in comparison to pre-government income) distribution. Note that while these estimates of kurtosis seem very high at first glance, they imply a good fit of the cross-sectional distribution over age and over years as shown in Figure 2. Furthermore, the estimated income process is in line with the average kurtosis of income changes.

5 A Quantitative Model

5.1 The Economy

We now set up a quantitative version of the simple two-period model of Section 2 by extending it to a standard multi-period life-cycle model with a stochastic earnings process, a zero borrowing constraint, a fixed retirement age, and an earnings-related retirement income. To calibrate higher-order risk attitudes separately from the inter-temporal elasticity of substitution, we take Epstein-Zin-Weil preferences a la Epstein and Zin (1989, 1991), and Weil (1989).

Endowments. Households earnings are exogenous and consist of a deterministic age profile and a stochastic income component with transitory and persistent shocks. The distribution of persistent shocks varies with the aggregate state $s \in \{C, E\}$, which follows a Markov process with time-invariant transition matrix Π_s. We abstract from the aggregate effects of fluctuations on wages and interest rates by holding both constant. In this sense there is *no aggregate risk*, but *cyclical idiosyncratic risk*.

Households live from age $j = 0$ to age $j = J$. They retire at the exogenously given retirement age j_r. Labor income net of taxes and transfers at age $j \in \{0, \ldots, j_r - 1\}$ in aggregate state s is given by

$$y(z, \varepsilon, j; s) = e_j \cdot \exp(z(s) + \varepsilon), \tag{9}$$

where e_j is the deterministic age profile, ε is the transitory income shock, drawn iid from distribution $\tilde{F}_\varepsilon$, and $z(s)$ is the persistent income component which obeys

$$z'(s') = \begin{cases} \rho z + \eta', & \text{where } \eta' \underset{iid}{\sim} \tilde{F}_\eta(s') \text{ for } j < j_r \\ z & \text{for } j \geq j_r, \end{cases} \tag{10}$$

where ρ is the autocorrelation coefficient and η' is the persistent income shock, drawn from distribution $\tilde{F}_\eta(s')$ that depends on aggregate state s. We assume that $\exp(\varepsilon_0) = \exp(z_0) = 1$. In retirement, $j \in \{j_r, \ldots, J\}$, households earn a fixed earnings related pension income contingent on the last income state before retirement $y_j = b(z_j)$.[14] Households have access to a risk-free savings technology with rate of return r, and face a zero borrowing constraint. Thus, the dynamic budget constraint is

$$a'(z, \varepsilon, j; s) = a(1 + r) + y(z, \varepsilon, j; s) - c \geq 0. \tag{11}$$

Preferences and Household Problem. Households born into the economy at history s^t, date t maximize recursive utility by solving a consumption-savings problem every period. They discount the future at factor $\beta > 0$. The state variables of the household's problem are age j, asset holdings a, the persistent income state z, the transitory shock ε, and the aggregate state of the

[14] With this specification we approximate the average indexed monthly earnings (AIME) of the US pension system.

economy s. The recursive problem of households is

$$V_j(a,z,\varepsilon;s) = \max_{c,a'} \begin{cases} \left((1-\tilde{\beta}) c^{1-\frac{1}{\gamma}} + \tilde{\beta} \left(v(V_{j+1}(a',z',\varepsilon';s')) \right)^{1-\frac{1}{\gamma}} \right)^{\frac{1}{1-\frac{1}{\gamma}}} & \gamma \neq 1 \\ \exp\left\{ (1-\tilde{\beta}) \ln c + \tilde{\beta} \ln \left(v(V_{j+1}(a',z',\varepsilon';s')) \right) \right\} & \text{otherwise} \end{cases}$$

$$\text{s.t. } (9), (10), \text{and } (11),$$

where $\tilde{\beta} = \frac{\beta}{1+\beta}$ denotes the relative utility weight on the certainty equivalent $v(V_{j+1})$ from next period's continuation utility $V_{j+1}(\cdot)$, which is

$$v(V_{j+1}(a',z',\varepsilon';s')) = \begin{cases} \left(\mathbb{E}_j \left[V_{j+1}(a',z',\varepsilon';s')^{1-\theta} \right] \right)^{\frac{1}{1-\theta}} & \theta \neq 1 \\ \exp\left(\mathbb{E}_j \left[\ln V_{j+1}(a',z',\varepsilon';s') \right] \right) & \text{otherwise.} \end{cases}$$

Parameter γ denotes the inter-temporal elasticity of substitution between instantaneous utility from consumption and the certainty equivalent of the continuation utility $v(V_{j+1}(\cdot))$. Given γ, parameter θ pins down the relative risk attitudes of households as discussed in Section 2, respectively in Appendix A. Conditional expectations are defined with respect to the realization of next period's aggregate state of the economy s', transitory income shock ε', and persistent income shock η'.

We solve for household policy and value functions using the method of endogenous gridpoints. We aggregate by explicit aggregation iterating forward on the cross-sectional distribution $\Phi_j(a_j, z_j, \varepsilon; s)$, which follows from the initial distribution $\Phi_0(a_0, z_0, \varepsilon_0; s)$ and the transition function $G_j(a_j, z_j, \varepsilon_j; s)$. The latter is induced by the exogenous laws of motion of z, s, the exogenous distribution of ε, and the endogenous transitions $a'_j(a_j, z_j, \varepsilon_j; s)$.

5.2 Calibration

Aggregate Shock Process. Based on our classification of time periods as contractions and expansions for the US economy, we estimate a Markov transition process on this data. We estimate $\pi(E|E) = 0.788$ and $\pi(C|C) = 0.389$, implying the stationary invariant distribution $\Pi_s = [0.257, 0.743]'$.

Age Bins and Age Productivity. Each model period corresponds to one life year. Consistent with our empirical specification, households start working at age 25 (model age $j = 0$) and retire at age 60 (model age $j = 35$).

In the economic model, we abstract from heterogeneity along the dimensions of education, labor market experience, or household size. We calibrate the age productivity process e_j by the fitted age polynomial $f_{age}(j)$ of the first stage estimation of the earnings process for household post government earnings. We take the weighted average of college and non-college age earnings profiles that display the usual hump-shaped pattern, cf. Appendix D.3, and normalize it such that average productivity is equal to one, $\frac{1}{j_r}\sum_{j=0}^{j_r-1} e_j = 1$.

Idiosyncratic Shock Processes. The most important element of the calibration is the specification of the distribution functions of the idosyncratic shocks. The goal of our approach is to directly assess the economic consequences of distributional aspects of these shocks that are summarized in the central moments—and to thus extend the illustrative analysis from Section 2, which does not need to make any (parametric) distributional assumptions, to a quantitative framework. For a given shock, our approach can be summarized in two steps. First, we use a parametric continuous distribution function, which we parameterize such that its first four central moments fit the ones estimated. Second, we discretize this distribution function. Thus, our approach allows us to translate the estimated central moments directly into the model's shock distributions without having to simulate the income process.

As distribution function we choose the Flexible Generalized Lambda Distribution (FGLD) developed by Freimer et al. (1988), which is characterized by its quantile function

$$Q(p;\lambda) = F^{-1}(p;\lambda) = x = \lambda_1 + \frac{1}{\lambda_2}\left(\frac{p^{\lambda_3}-1}{\lambda_3} - \frac{(1-p)^{\lambda_4}-1}{\lambda_4}\right), \tag{12}$$

where λ is a vector of four parameters with location parameter λ_1, scale parameter λ_2, and tail index parameters λ_3, λ_4.[15] For each shock $x \in \{\varepsilon, \eta(s)\}$,

[15] The parametric constraints are $\lambda_2 > 0$, and $\min\{\lambda_3, \lambda_4\} > -\frac{1}{4}$.

we fit these parameters such that the FGLD matches the estimated central moments $\{\hat{\mu}_i^x\}_{i=1}^4$ of distributions F_ε and $F_\eta(s)$ (as in Lakhany and Mausser 2000 and Su 2007). We numerically solve for λ_3 and λ_4 jointly to fit the third and fourth central moments.[16] Next, we determine λ_2 to match the variance and λ_1 to match the mean, both in closed form. We then discretize by spanning equidistant grids for the respective random variable $x \in \{\varepsilon, \eta(s)\}$ and by assigning to each grid point probabilities from the integrated probability density function of the distribution (details in Appendix C).

We consider two alternative parameterizations of the FGLD to which we refer as *distribution scenarios*. The first scenario features symmetric shock distributions ($\hat{\mu}_3 = 0$) with the estimated variance and a kurtosis of $\frac{\hat{\mu}_4}{\hat{\mu}_2^2} = 3$. The parameter restriction on the FGLD is that $\lambda_3 = \lambda_4$. We refer to this scenario as NORM, reflecting that it features the first four central (as well as standardized) moments of the Normal distribution. The second scenario, to which we refer as LKSW, features leptokurtic and left-skewed shock distributions with the estimated second, third, and fourth central moments; no restrictions apply to the FGLD parameters.[17]

Figure 3 shows the log density functions of the persistent shock $\eta(s)$ in contractions and expansions. Panel (a) shows the distributions in scenario LKSW which features the estimated countercyclical variance, procyclical third, and countercyclical fourth moments. For comparison, Panel (b) shows Gaussian distributions featuring the same countercyclical variance. The FGLD distribution does not nest the Gaussian distribution, and thus, while FGLD distribution NORM features the same first four central moments, it does not display the same inverse quadratic log density function. In both distributions, all odd

[16] Specifically, we solve the minimization problem $\min_{\lambda_3,\lambda_4} \sum_{i=3}^4 (\mu_i(\lambda_3,\lambda_4) - \hat{\mu}_i)^2$ s.t. $\min\{\lambda_3,\lambda_4\} > -\frac{1}{4}$, where $\hat{\mu}_i$ is the point estimate of the i^{th} moment, and $\mu_i(\cdot)$ denotes the central moment of the FGLD.

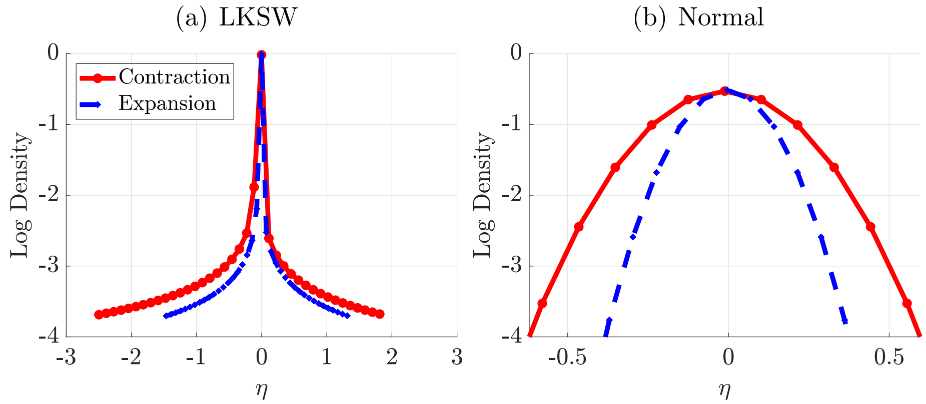
[17] We also impose a minimum post-government household income that remains unchanged across scenarios, i.e., when moving from the scenario with normally distributed shocks to the scenario with leptokurtic and left-skewed shocks, the lowest level of income that households can reach is by construction unchanged. This minimum income is expressed relative to average income. We then adjust incomes such that average income (before multiplying with the age profile) remains 1.

moments are zero, and thus the differences between FGLD distribution NORM and its Gaussian counterpart are captured by the even central moments higher than the fourth (i.e., the sixth, eighth, tenth, etc.). Following our analytical analysis in Section 2, these distributional differences mechanically become less and less important the higher the order is, and eventually their implications for utility (and consumption behavior) depend on risk attitudes. We thus also consider a scenario in which we draw shocks from the Normal distribution and discretize it using standard Gaussian quadrature methods. As documented in Appendix E.1, all quantitative results are numerically almost identical to those obtained for FGLD distribution NORM. We therefore use the latter as our benchmark to which we compare FGLD distribution LKSW. Appendix D.1 reports the estimated, fitted, and discretized moments, as well as the parameter vectors λ for all shocks under the two scenarios NORM and LKSW. In both distribution scenarios we scale down the transitory shocks because part of the estimated variance is likely due to measurement error.[18] Appendix D.3 shows central moments 2-4 in logs and levels that result from our parametrization.

Pension System. Social security benefits follow a fixed replacement schedule that approximates the current US bend point formula. We approximate average indexed monthly earnings (AIME) by the realization of the persistent income shock before entering into retirement z_{j_r-1}. We then apply the bend point formula contained in Appendix D.2 and denote the according model equivalent to the primary insurance amount (PIA) by $p(z_{j_r-1})$. To achieve budget clearing of the pension system, pension payments are further scaled by the aggregate indexation factor ϱ so that individual pension income is $b(z_{j_r-1}) = \varrho \cdot p(z_{j_r-1})$. As to contributions to the pension system, we compute the average contribution rate from the data giving $\tau^p = 11.7\%$ (which is close to the current legislation featuring a marginal contribution rate

[18] Following Huggett and Kaplan (2016) we assume that one third of the estimated variance of the transitory shock is measurement error and reduce the targeted variance accordingly. We assume that this measurement error is symmetric and accordingly adjust the third and fourth central moments such that the implied coefficients of skewness and kurtosis are unchanged.

Figure 3: Discretized Log Distribution Functions: Persistent Shock



Notes: Discretized log distribution functions for the persistent shock η. LKSW: FGLD with estimated variance, skewness, and kurtosis. Markers denote the grid points used in the discretized distribution. Normal: Normal distribution with estimated variance discretized using Gaussian quadrature method. Log density is the base 10 logarithm of the PDF.

of $\tau^p = 12.4\%$). The base for pension contributions in our model is average gross earnings. Since earnings processes in the model are based on net wages—net of all taxes and transfers—and since we normalize average net wages to one, average gross wages are $\frac{1}{1-\tau^p-\tau}$, where τ is some average labor income tax rate (including transfers). We compute τ from the data giving $\tau = 16.88\%$.

Since average labor productivity, the means of the shocks z_j, ϵ_j as well as the total population in age group j are all normalized to one, efficiency weighted aggregate labor in the economy is equal to $j_r - 1$. The number of pensioners is $J - j_r + 1$. The pension budget is therefore given by

$$\tau^p \cdot \frac{1}{1-\tau-\tau^p} \cdot (j_r - 1) = \varrho \cdot \int p(z_{j_r-1}) d\Phi(z_{j_r-1}) \cdot (J - j_r + 1) \, .$$

We calibrate ϱ in each distribution scenario so that the pension budget clears. Since contributions obey a linear tax schedule and by our normalization of income, aggregate contributions are constant across all scenarios. Recalibrating ϱ therefore implies that also average pension income is the same across all scenarios. Table D.5 in Appendix D.2 provides the accordingly calibrated values of ϱ.

Initial Assets and Interest Rate. For simplicity, we assume that all households are born with the same initial assets $a_0 = \bar{a}_0$. We compute those from the average asset to net earnings data at age 25, which we calculate from PSID data as 0.89. We set the annual interest rate of the risk-free asset to $r = 4.2\%$, based on Siegel (2002).

Table 2: Calibrated Parameters

Working period	25 ($j = 0$) to 60 ($j = j_r - 1$)
Maximum age	80
IES	$\gamma = 1$
RA	$\theta \in \{1, 2, 3, 4\}$
Discount factor (2^{nd} stage)	$\beta \in \{0.971, 0.970, 0.967, 0.965\}$
Interest rate	$r = 0.042$
Pension contribution rate	$\tau^p[\%] = 11.7\%$
Pension benefit level	See Table D.5
Average tax rate	$\tau[\%] = 16.8\%$
Aggregate shocks	$\pi(s' = c \mid s = c) = 0.39, \pi(s' = e \mid s = e) = 0.77$
Initial ass. / inc.	$\bar{a}_0 = 0.89$

Notes: Calibration parameters. IES: inter-temporal elasticity of substitution, RA: coefficient of risk aversion. The discount factor β is calibrated endogenously to match asset to income data from the PSID. The pension benefit level parameter ϱ is calibrated such that the pension budget clears.

Preferences. As we show in Section 2, risk attitudes play a crucial role for the welfare effects of higher-order income risk and for the precautionary savings motive. For each model variant we therefore consider four alternative parameterizations and vary $\theta \in \{1, 2, 3, 4\}$. Throughout, we consider risk-sensitive preferences (Tallarini 2000) and accordingly set the inter-temporal elasticity of substitution to $\gamma = 1$.[19] For each $\theta \in \{1, 2, 3, 4\}$, we determine endogenously the discount factor β to match life-cycle asset profiles scaled by

[19] Cooper and Zhu (2016) estimate a portfolio choice model where agents have Epstein-Zin-Weil preferences, and face the canonical income process with log Normal shocks. They estimate a risk aversion of 4.4 and an IES of 0.6. We choose an IES of 1 as a natural benchmark. This is also very convenient when we decompose the welfare effects as described in Appendix A.7.

net earnings, which we compute from PSID data. Since our model is not designed to match saving patterns in retirement (there is neither survival risk nor a bequest motive), we match assets for ages 25-60, the working period in our model. This calibration is done for distribution scenario LKSW, and we then hold the calibrated discount factor constant when moving to scenario NORM, for each calibration of θ.

Calibrated discount factors range from 0.971 for $\theta = 1$ to 0.965 for $\theta = 4$, see Table 2, which summarizes the calibration of the model. The reason for the decline of the calibrated discount factor in θ is that increasing θ leads to higher precautionary savings which is offset in the calibration by lowering β so that the life-cycle savings motive is less potent.

6 Quantitative Role of Higher-Order Risk

6.1 Welfare Implications of Higher-Order Income Risk

In order to assess the welfare implications of higher-order income risk, we ask which world households would prefer to be born into. Taking this ex-ante perspective, we accordingly define the Utilitarian social welfare function as the expected life-time utility function of households born with initial assets $a_0 = \bar{a}_0$, idiosyncratic persistent income state $z_0 = 0$, and transitory shock $\varepsilon = 0$. Corresponding with our notion of an ex-ante perspective we aggregate expected life-time utilities of newborns in the stationary invariant distribution of the economy. Since the transition probabilities over aggregate states are encoded in the value functions and since aggregate fluctuations in our partial equilibrium model do not affect relative prices, evaluating welfare in the stationary invariant distribution of the economy is equivalent to aggregating newborns' value functions with the stationary invariant distribution of the Markov chain process, Π_s. Accordingly, welfare is given by

$$W = \sum_s \Pi_s V_0(a_0 = \bar{a}_0, z_0 = 0, \varepsilon = 0; s).$$

We then calculate the consumption equivalent variation (CEV) that households need to receive in the world without higher-order risk (distribution scenario NORM) in order to be indifferent to a world with higher-order risk as parameterized by the distribution scenario LKSW. Given the homotheticity of the utility function, the CEV is $g_c = W^{LKSW}/W^{NORM} - 1$.

We distinguish between three different channels through which idiosyncratic risk translates into utility consequences evaluated from this ex-ante perspective, and express the CEV as the sum of three components: $g_c = g_c^{mean} + g_c^{lcd} + g_c^{csd}$ (cf. Appendix A.7 for explicit expressions). While we hold mean income constant, consumption is endogenous. When facing different (distribution) scenarios, households make different savings decisions, and thus realize different mean consumption, i.e., consumption averaged cross-sectionally and over age. We call the welfare consequence of this change of mean consumption the *mean effect*, g_c^{mean}, which is proportional to changes in mean consumption. We in turn refer to utility consequences of changes in the distribution around mean consumption as the *distribution effect*, g_c^{distr}, which we decompose into two components: the utility consequences of, first, the change of the distribution of mean consumption over the life-cycle, the *life-cycle distribution effect*, g_c^{lcd}, and, second, the change of the cross-sectional distribution of consumption around the mean life-cycle profile, the *cross-sectional distribution effect*, g_c^{csd}.

Table 3 summarizes the welfare implications of higher-order income risk by showing the CEV and its decomposition. Consistent with our analytical findings in Proposition 1 (see Appendix A.2) higher-order risk leads to welfare gains when risk attitudes are weak. With stronger risk attitudes, however, welfare losses show up, because the increasing variance and the high kurtosis dominate the welfare effects.

The main force for the welfare results is the redistribution of consumption over the life-cycle reflected in g_c^{lcd}. This is a consequence of increased precautionary savings as reflected in Panel (a) of Figure 4, which displays

Table 3: Welfare Implications of Higher-Order Income Risk: CEV in %

Risk Aversion / CEV	g_c	g_c^{mean}	g_c^{lcd}	g_c^{csd}
$\theta = 1$	0.371	-0.154	0.506	0.019
$\theta = 2$	-0.386	-0.161	-0.256	0.031
$\theta = 3$	-4.488	0.318	-4.751	-0.055
$\theta = 4$	-12.474	1.211	-13.392	-0.294

Notes: Welfare gains (positive numbers) and losses (negative numbers) of higher-order income risk, expressed as a Consumption Equivalent Variation (CEV) in scenario NORM that makes households indifferent to the higher-order income risk scenario LKSW. g_c: total CEV, g_c^{mean}: CEV from changes of mean consumption, g_c^{lcd}: CEV from changes in the distribution of consumption over the life-cycle, g_c^{csd}: CEV from changes in the cross-sectional distribution of consumption, where $g_c = g_c^{mean} + g_c^{lcd} + g_c^{csd}$.

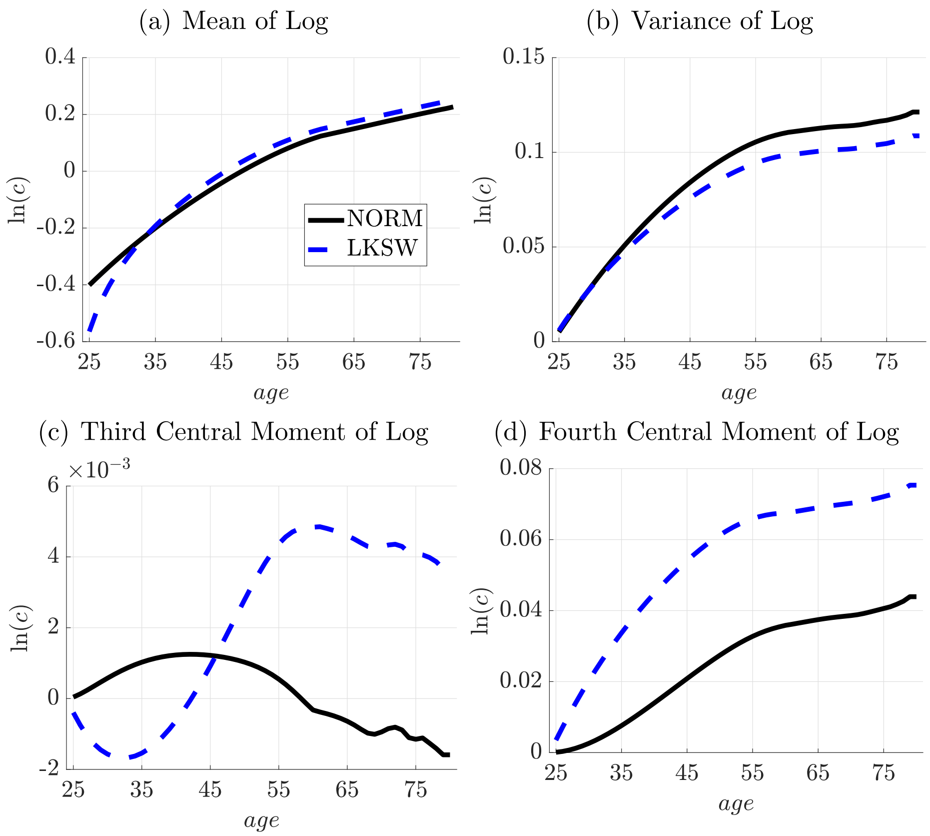
mean log consumption over the life-cycle.[20] Consumption in the higher-order income risk scenario LKSW is lower when young and higher when old compared to scenario NORM. In welfare terms lower consumption when young dominates higher consumption when old due to discounting. The mean effect g_c^{mean} instead is mostly positive because the increased consumption when old dominates (except for $\theta = 1$ and $\theta = 2$). In sum, total welfare losses for scenario LKSW range from about 0.4% (i.e., small gains) for $\theta = 1$ to -12.5% for strong risk attitudes with $\theta = 4$.

Panels (b) to (d) of Figure 4 show the second to fourth central moments of the consumption distribution over the life-cycle, which are relevant for the cross-sectional distribution effect g_c^{csd}. To interpret it observe that the variance of log consumption is lower in scenario LKSW than in scenario NORM for most ages, whereas the third central moment is initially negative and the kurtosis of the log consumption distribution is higher at all ages.[21] The lower

[20]Here we show the profile for a high risk aversion parameter of $\theta = 4$, because in this calibration the effects are most evident visually. Qualitatively, effects are the same in the other risk attitude calibrations. Note that consumption is monotonically increasing over the life-cycle and thus does not display the typical hump-shaped profile, because we do not model life-cycle consumption behavior in retirement.

[21]The Gini coefficient for assets for a risk aversion of 4 is at 0.35 in scenario NORM, and at 0.34 in scenario LKSW. Thus, introducing higher-order income risk does not increase the Gini coefficient in a quantitative model such as ours. Also, note that the Gini coefficient in our calibrated model is substantially lower than in the data and also lower than what

Figure 4: Central Moments of Log Consumption by Age ($\theta = 4$)



Notes: Moments of cross-sectional distribution of log consumption over the life-cycle. NORM: FGLD with moments of the normal distribution, LKSW: FGLD with excess kurtosis and left-skewness (in logs).

variance contributes positively to g_c^{csd}, which dominates for low risk aversion, whereas the negative skewness and the excess kurtosis contribute negatively, and dominate for strong risk attitudes.

is typically found in quantitative work; e.g., Krueger and Ludwig (2016) compute a Gini coefficient of assets of 0.55 in an overlapping generations model calibrated to the US economy. The key reason for the relatively modest asset inequality lies in our focus on ex-post heterogeneity, i.e., the only source of heterogeneity is income risk faced throughout the life cycle.

6.2 Welfare Costs of Cyclical Idiosyncratic Risk

Next, we quantify the utility consequences of *cyclical idiosyncratic risk*. To this end, for each of the two distribution scenarios NORM and LKSW we evaluate the welfare implications for households of facing the *actual* cyclical income process relative to a *counterfactual* income process in which we shut down the cyclical variation of the distribution. By holding mean wages and interest rates constant over the cycle, the welfare effects of cyclical risk we report constitute a lower bound for each scenario.[22]

As before, W^i denotes the social welfare function in the *cyclical risk scenario*, while $W^{i,ncr}$ denotes the social welfare function in the *no cyclical risk scenario*. We then compute the CEV necessary in the scenario with no cyclical risk to be indifferent to the scenario with cyclical risk, $g_c^{i,cr} = W^i / W^{i,ncr} - 1$, and decompose the total CEV from cyclical risk into its components, i.e., $g_c^{i,cr} = g_c^{i,cr,mean} + g_c^{i,cr,lcd} + g_c^{i,cr,csd}$, for $i \in \{NORM, LKSW\}$.

When computing welfare in the non-cyclical scenario $W^{i,ncr}$ we assume that households always draw from the "expansion-distribution" of the scenario rather than taking some weighted average of shock distributions for expansions and contractions. When using log-Normal distributions of shocks, one approach in the literature is to consider an *average* distribution, which features the average of expansion and contraction variances, see for example Storesletten et al. (2001). This approach is not applicable in our analysis as it is conceptually not clear what characterizes an "average" distribution once other moments than the variance are taken into account. To the extent that some average distribution represents a better non-cyclical counterfactual scenario, the pure effect of cyclical idiosyncratic risk is overstated in our analysis.[23] However, we are mainly interested in the difference of welfare costs of

[22]Note that the direct effect of business cycles is typically found to be small. For example, Storesletten et al. (2001) find the direct effect to be an order of magnitude smaller than the role of cyclical variation in idiosyncratic risk. However, there can be indirect utility "interactions" between aggregate and idiosyncratic risk, which may be large (Harenberg and Ludwig 2019), and which we abstract from here in order to focus on the role of the idiosyncratic shock distribution.

[23]Indeed, Storesletten et al. (2001) find welfare costs of cyclical risk of about 1.3%. They consider CRRA preferences with $\theta = 2$. In one of our sensitivity checks in Appendix E.2,

cyclical income risk across scenarios, i.e., the "difference in difference" comparison between $g_c^{LKSW,cr}$ and $g_c^{NORM,cr}$, i.e., $\Delta g_c^{cr} = g_c^{LKSW,cr} - g_c^{NORM,cr}$. Thus, our approach to "normalize" the economy without cyclical idiosyncratic risk is of second order importance as it is consistent across scenarios.[24]

Table 4 reports the results on the welfare costs of cyclical idiosyncratic risk in scenarios NORM and LKSW. First, note that consistent with our theoretical analysis of Section 2 in each scenario the welfare costs of business cycles increase monotonically in θ. Second, as for the welfare costs of higher-order risk, the main contributor to the welfare consequences is the redistribution of consumption over the life-cycle as quantified by g_c^{lcd}. Third, mean effects are positive. Recall that a negative g_c^{lcd} is a consequence of the counter-clockwise tilting of the consumption profile because of increased precautionary savings. Higher savings increase consumption in the middle of the life-cycle, which pushes up mean consumption. As previously, on average over the life-cycle this second effect dominates.

Consistent with the afore documented result (and with our theoretical analysis of Section 2) that with logarithmic utility the total welfare effect from higher-order income risk is positive for scenario LKSW, we now correspondingly find that welfare losses from cyclical idiosyncratic risk are about 0.28%p lower in scenario LKSW (last column in first panel of Table 4). Similarly, with moderate risk attitudes (risk aversion of 2), the welfare implications of cyclical income risk in scenario LKSW are only mildly higher than those obtained in scenario NORM. With strong risk attitudes ($\theta = 4$), the welfare losses compared to scenario NORM are significantly higher: They are about 6.4%p higher in scenario LKSW.

In Appendix E.2 we investigate the sensitivity of our results with respect to selected modeling and calibration assumptions. Specifically, we consider an

we also consider CRRA preferences with $\theta = 2$. In this case we obtain welfare costs of about 2.6% in scenario NORM. Besides other differences between our model and theirs, one reason for the higher welfare costs in our analysis lies in the different approach to characterizing the non-cyclical scenario.

[24] One alternative is to follow the "integrating out" principle (see Krusell and Smith 1999 and Krusell et al. 2009), which first isolates a true idiosyncratic component of the shock, and then integrates over the probability distribution of the aggregate state.

Table 4: Welfare Effects of Cyclical Idiosyncratic Risk

CEV	g_c	g_c^{mean}	g_c^{lcd}	g_c^{csd}	Δg_c
			Risk Aversion, $\theta = 1$		
NORM	-1.720	0.499	-2.175	-0.044	0
LKSW	-1.443	0.398	-1.806	-0.035	0.277
			Risk Aversion, $\theta = 2$		
NORM	-3.263	0.898	-4.038	-0.123	0
LKSW	-3.516	0.823	-4.228	-0.111	-0.253
			Risk Aversion, $\theta = 3$		
NORM	-4.607	1.229	-5.638	-0.198	0
LKSW	-7.177	1.379	-8.313	-0.243	-2.570
			Risk Aversion, $\theta = 4$		
NORM	-5.758	1.515	-7.009	-0.264	0
LKSW	-12.171	1.944	-13.686	-0.429	-6.413

Notes: Consumption Equivalent Variation in the non-cyclical scenario that makes households indifferent to the cyclical scenario. $g_c = g_c^{mean} + g_c^{lcd} + g_c^{csd}$, where g_c^{mean}: CEV from change of mean consumption, g_c^{lcd}: CEV from changes in the distribution of consumption over the life-cycle, g_c^{csd}: CEV from changes in the cross-sectional distribution. $\Delta g_c = g_c^{LKSW} - g_c^{NORM}$.

expected utility formulation with CRRA preferences where we restrict $\theta = \frac{1}{\gamma}$, we analyze the role of borrowing constraints in the model, and we investigate how results are affected by our choice of the interest rate. The results shown in the appendix underscore the robustness of our findings also with respect to the dominant role played by the life-cycle distribution effect g_c^{lcd}.

Furthermore, in Appendix E.3 we analyze an alternative distribution scenario, which features shocks that have excess kurtosis, but are symmetric (in logs). In the calibration with $\theta = 4$, welfare costs of cyclical risk are about 4% higher in this distribution scenario than in scenario NORM (see table E.3). Combined with the lower part of table 4 we thus find that of the differential welfare losses from higher-order risk approximately 62%($\approx 3.98/6.41 \cdot 100\%$) are due to the excess kurtosis and the remaining 38% are due to the left-skewness of shocks.

Finally, in Appendix E.4 we discuss a general equilibrium extension of our model, in which we take into account the general equilibrium effect of higher-order idiosyncratic risk on wages and interest rates, that works through the equilibrium capital allocation. We still abstract from aggregate productivity risk. We consider a solution that fully reflects higher-order risk, while being an approximation regarding its cyclicality. The equilibrium feedback effect turns out to be weak, which is explained by the life-cycle structure of the economy and the consumption profile of Figure 4. When facing scenario LKSW instead of scenario NORM, young agents have higher precautionary savings, however these savings will be dis-saved at old age. Thus, aggregate savings of the economy do not differ strongly.

Summing up, we can conclude that the welfare effects of cyclical risk are strongly underestimated in conventional approaches based on Gaussian distributions of innovations if risk attitudes are strong (levels of θ of 3 or 4) and that both features of higher-order income risk—excess kurtosis and left skewness—are quantitatively important for this finding, whereby about 60% of the effect can be attributed to the excess kurtosis.

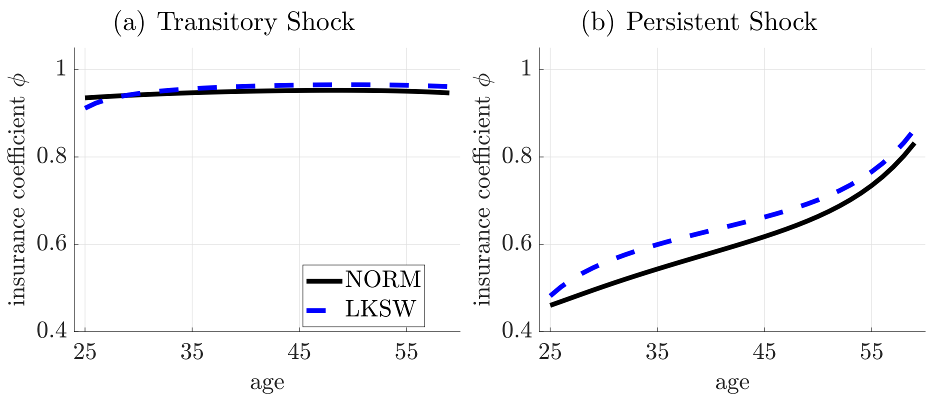
6.3 Insurance Against Idiosyncratic Risk

Finally, we adopt concepts developed in the literature on consumption insurance (Blundell et al. 2008; Kaplan and Violante 2010) to ask how households are self-insured against income shocks $x_j(s) \in \{\varepsilon_j, \eta_j(s)\}$ and how this insurance varies across scenarios. In the model, the transitory and persistent shocks are directly observed and thus we adopt the measure of Kaplan and Violante (2010) to our setting with cyclical risk. Conditional on today's aggregate state s, the insurance coefficient $\phi_j^x(s)$ is given as the share of the variance of next period's shock $x_{j+1}(s')$ that does not translate into consumption growth, and thus the pass-through coefficient $1 - \phi_j^x(s)$ is the coefficient of a linear regression of consumption growth on shock x, which captures how strongly the shock translates into consumption:

$$1 - \phi_j^x(s) = \frac{cov(\Delta \ln \left(c_{j+1}(s' \mid s)\right), x_{j+1}(s'))}{var(x_{j+1}(s'))}, \tag{13}$$

for $\Delta \ln (c_j(s' \mid s)) = \ln (c_{j+1}(s' \mid s)) - \ln (c_j(s))$.

Figure 5: Insurance Coefficients: Strong Risk Attitudes, $\theta = 4$



Notes: Figures show the degree of consumption insurance against transitory and persistent shocks separately by age.

Figure 5 reports the insurance coefficients ϕ_j^x for all ages $j \in \{0, \ldots, J\}$, as a weighted average of the coefficients in contractions and expansions[25] for the transitory shock ε in Panel (a) and for the persistent shock $\eta(s)$ in Panel (b). Results are quantitatively similar for different values of risk attitudes, so we discuss only the numbers for $\theta = 4$. For scenario LKSW, consumption insurance against both transitory and persistent shocks is improved relative to scenario NORM as measured by the ϕ-coefficients. This is a direct consequence of increased precautionary savings, which lead to shocks translating less into consumption.

Do the higher insurance coefficients in scenario LKSW really represent *better insurance*, though? Arguably, better insurance would mean that negative shocks translate less into consumption. This is not the case as can be illustrated by one simple decomposition of the pass-through of shocks to consumption changes in equation (13). Consider the aggregate (integrating over

[25] We weigh with the stationary invariant distribution Π_s.

age and averaging over states s) pass-through coefficient for shock $x \in \eta, \varepsilon$:

$$
\begin{aligned}
1-\phi^x = & \frac{E\left[\Delta \ln(c(\cdot))x\right] - E\left[\Delta \ln(c(\cdot))\right] E\left[x\right]}{var(x)} \\
= & \frac{E\left[\Delta \ln(c(\cdot))x \middle| \, x>0\right]}{var(x)} + \frac{E\left[\Delta \ln(c(\cdot))x \middle| \, x<0\right]}{var(x)} - \frac{E\left[\Delta \ln(c(\cdot))\right] E\left[x\right]}{var(x)}.
\end{aligned} \tag{14}
$$

The first two components of the sum in equation (14) give the contribution to the overall pass-through coefficient of comovements of consumption with positive and negative shocks, respectively. Table 5 shows the aggregate pass-through coefficient of the economy along with the contributions of its components. As already learned from Figure 5, the aggregate pass-through of both transitory and persistent shocks is smaller in scenario LKSW (insurance coefficient is larger). Now consider the contribution of positive and negative shocks to the aggregate pass-through coefficient. In scenario NORM, negative transitory shocks do not translate into negative consumption changes: comovements with negative realizations of ε contribute -3.4% to the pass-through coefficient. In scenario LKSW, the (negative) consumption reaction to negative shocks is important: 30.2% of the pass-through coefficient are accounted for by negative transitory shocks leading to negative consumption adjustments. At the same time, consumption reacts less strongly to positive changes. Thus, the fact that the aggregate pass-through is smaller (the insurance coefficient is larger) is indeed explained by increased precautionary savings. However, built-up savings do not suffice to smooth out the negative shocks in scenario LKSW as well as they do in scenario NORM.

For persistent shocks, the same mechanics are at work. In scenario NORM, about 40% of the pass-through coefficient is generated by consumption reductions with negative shocks, while about 59% come from consumption increases with positive shocks. In scenario LKSW, negative shocks pass-through more (51% of overall), and positive shocks pass-through less (46%). So for both transitory and persistent shocks, the reduction of the pass-through (increase of insurance coefficient) when moving from scenario NORM to scenario LKSW

Table 5: Aggregate Pass-Through and its Decomposition, $\theta = 4$

Transitory	$1-\phi^{\varepsilon}$	$E[\Delta^c \cdot \epsilon, \epsilon < 0]$	$E[\Delta^c \cdot \epsilon, \epsilon > 0]$	$-E[\Delta^c] \cdot E[\epsilon]$
NORM	0.055	-0.034	0.898	0.136
LKSW	0.047	0.302	0.525	0.173
Persistent	$1-\phi^{\eta}$	$E[\Delta^c \cdot \eta, \eta < 0]$	$E[\Delta^c \cdot \eta, \eta > 0]$	$-E[\Delta^c] \cdot E[\eta]$
NORM	0.395	0.395	0.586	0.019
LKSW	0.353	0.514	0.458	0.028

Notes: Table shows aggregate consumption pass-through coefficient (1-insurance coefficient), and its decomposition into components according to equation 14. Values are expressed as shares of total pass-through. $\Delta^c = \Delta \ln(c(\cdot))$.

is driven by an increased propensity to save, while at the same time negative shocks actually translate more into consumption.

We can thus conclude that in an economy with higher-order income risk aggregate insurance (or pass-through) coefficients are imprecise measures of insurance against risk, if one plausibly has in mind that better insurance means that negative shocks translate less into consumption.

7 Conclusion

We first develop a novel Generalized Method of Moments estimator of higher-order income risk, that starts out with the canonical income process, which captures key features of labor income risk as a combination of persistent and transitory income shocks. We show how the second to fourth central moments of the distributions of shocks can be estimated. Our estimates on PSID household-level earnings imply that the distribution of persistent income shocks exhibits strong cyclicality: the variance is countercyclical, while the third central moment is procyclical. All shock components exhibit strong excess kurtosis. The existing tax and transfer system dampens both transitory and persistent income shocks and reduces cyclicality.

In the second part of the paper we analyze the relevance of the identified *higher-order risk* from an economic perspective. Within an otherwise standard partial equilibrium life-cycle model with incomplete markets, we find that, first, higher-order risk has important welfare consequences—relative to a world with log-Normal shocks. Second, the presence of higher-order risk matters for the welfare costs of business cycles. Third, higher-order income risk affects the degree of consumption self-insurance.

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Appendix

”Higher-Order Income Risk Over the Business Cycle”

(Christopher Busch and Alexander Ludwig)

A Analytical Appendix

A.1 Derivation of Equation (1)

Take a fourth order Taylor series expansion of the age 1 subperiod utility function around $c_1 = \mu_1^c$ to get

$$U \approx \frac{c_0^{1-\theta}}{1-\theta} + \frac{1}{1-\theta}\left(\mu_1^{c^{1-\theta}} + \mathbb{E}\left[(1-\theta)\mu_1^{c^{-\theta}}(c_1 - \mu_1^c) - \frac{(1-\theta)\theta}{2}\mu_1^{c^{-(1+\theta)}}(c_1-\mu_1^c)^2 \right.\right.$$
$$\left.\left. + \frac{(1-\theta)\theta(1+\theta)}{6}\mu_1^{c^{-(2+\theta)}}(c_1-\mu_1^c)^3 - \frac{(1-\theta)\theta(1+\theta)(2+\theta)}{24}\mu_1^{c^{-(3+\theta)}}(c_1-\mu_1^c)^4\right]\right)$$

Under a binding budget constraint and the additional assumption that $\mathbb{E}[\exp(\varepsilon)] = 1$ we obtain $\mu_1^c = 1$. Also impose that $\theta = \frac{1}{\gamma}$. Using these conditions in the above we obtain (1).

A.2 Logs vs. Levels

While the transformation from logs to levels is natural, it has non-trivial implications for the welfare effects of higher-order risk: the higher-order moments of the shocks in *levels*, $\exp(\varepsilon)$, rather than of the shocks in *logs*, ε, are relevant for utility consequences. Consider a mean preserving (thus $\mathbb{E}[\exp(\epsilon)] = 1$) change of idiosyncratic risk. When introducing left-skewness in logs, probability mass is shifted to the left, which reduces the variance of the shocks in levels. Without adjustment, by Jensen's inequality for convex functions the mean of the distribution in levels would be lower, so to preserve the mean the distribution needs to be shifted up, which increases the mean in logs. Similarly, a higher variance or higher kurtosis of the distribution in logs increases the variance in levels. Without adjustment, the fanning out of the support of shocks in logs increases the mean of the distribution in levels by Jensen's inequality for convex functions. In order to preserve the mean the distribution needs to be shifted down, which reduces the mean in logs. Since with log utility and in absence of a savings technology, expected life-time utility is $U = \ln(y_0) + \mathbb{E}[\ln(y_1)]$, solely the mean of the distribution in logs matters for life-time utility and thus a mean-preserving *reduction of skewness* leads to *utility gains*. Likewise, a mean-preserving *increase of variance or kurtosis* leads to *utility losses* in expectation. This is summarized in the following

Proposition 1. *Suppose that the utility function is logarithmic* ($\theta = 1$) *and that there is no savings technology* ($a_1 = 0$)*. Then a mean-preserving* reduction of skewness *('more negative skewness') leads to* utility gains*, whereas a mean-preserving* increase of variance or kurtosis *leads to* utility losses *in expectation.*

Proof. Let $\mu_1^\varepsilon = E_\Psi[\varepsilon] = \int \varepsilon d\Psi$, $\mu_i^\varepsilon = \int (\varepsilon - \mu_1^\varepsilon)^i d\Psi$ for $i > 1$, and let $E_\Psi[\exp(\varepsilon)] = \int \exp(\varepsilon) d\Psi = 1$. Denote by $\tilde{\Psi}^{\delta_i}(\varepsilon)$ a mean preserving (constant μ_1^ε) distribution function that is obtained from $\Psi(\varepsilon)$ by changing central moment μ_i^ε holding all other moments $\mu_{\neg i}^\varepsilon$ for $i > 1$ constant. Also, define the random variable $\tilde{\varepsilon}^{\delta_i} = \varepsilon + \Delta^{\delta_i}$, which is obtained from ε by shifting all realizations by the constant Δ^{δ_i}. Let the normalization $E_{\tilde{\Psi}^{\delta_i}}[\exp(\tilde{\varepsilon}^{\delta_i})] = E_{\tilde{\Psi}^{\delta_i}}[\exp(\varepsilon + \Delta^{\delta_i})] = \int \exp(\varepsilon + \Delta^{\delta_i}) d\tilde{\Psi}^{\delta_i} = \exp(\Delta^{\delta_i}) \int \exp(\varepsilon) d\tilde{\Psi}^i = 1$ define the shift parameter $\Delta^{\delta_i} = -\ln\left(\int \exp(\varepsilon) d\tilde{\Psi}^i\right)$. Finally, observe that $E_{\tilde{\Psi}^{\delta_i}}[\varepsilon + \Delta^{\delta_i}] = E_{\tilde{\Psi}}[\varepsilon] + \Delta^{\delta_i} = E_\Psi[\varepsilon] + \Delta^{\delta_i}$ since μ_1^ε is held constant. With logarithmic utility and binding budget constraint, the expected utility difference across distributions Ψ and $\tilde{\Psi}^{\delta_i}$ is thus $\Delta U = (U \mid \Psi) - (U \mid \tilde{\Psi}) = \Delta^{\delta_i}$ and thus exclusively driven by the shift parameter. We then get the following:

- Shifting probability mass from the center to the tails, either by increasing the variance ($i = 2$) or kurtosis ($i = 4$) holding constant all $\mu_{\neg i}^\varepsilon$ for $i > 1$ increases $\int \exp(\varepsilon) d\tilde{\Psi}^i$ above one which follows from Jensen's inequality for convex functions. Thus $\Delta^{\delta_i} < 0$.

- Shifting probability mass from the right tail to the left tail decreasing the skewness ($i = 3$) (i.e., making the distribution more left-skewed), holding constant all $\mu_{\neg i}^\varepsilon$ for $i > 1$ decreases $\int \exp(\varepsilon) d\tilde{\Psi}^i$ below one which follows from Jensen's inequality for convex functions. Thus $\Delta^{\delta_i} > 0$.

□

While the finding in Proposition 1 may appear counter-intuitive at first glance, the reason is the transformation of the shocks from logs, which are typically modelled and estimated, to levels, which eventually matters for welfare.[1] In Supplementary Appendix S.B we provide a numerical illustration by considering a discrete three-point distribution. We show how

[1] Due to this re-transformation our findings are related to, but not the same, as first-order stochastic dominance, see Rothschild and Stiglitz (1970, 1971). Stochastic dominance refers to random variables in levels, in our case $\exp(\epsilon)$. Obviously, increasing the variance (or kurtosis) of $\exp(\epsilon)$, while holding the mean constant at $E[\exp(\epsilon)] = 1$, has direct negative utility consequences. In this case utility is $U = \ln(y_0) + \mathbb{E}[\ln(\exp(\epsilon))]$, which for the maintained normalization $E[\exp(\epsilon)] = 1$ we could approximate as

$$U \approx \ln(y_0) - \frac{1}{2}\mu_2^{\exp(\epsilon)} + \frac{1}{3}\mu_3^{\exp(\epsilon)} - \frac{1}{4}\mu_4^{\exp(\epsilon)}$$

from which the utility effects of increasing the variance or the kurtosis or decreasing the skewness are obviously all negative.

changing moment μ_i^ϵ by holding other moments constant can be conceptualized and how this affects the conclusions on the welfare implications of higher-order risk.

A.3 Derivation of Equation (2)

Take a fourth order Taylor series expansion of the RHS of the first-order condition around $\mathbb{E}[\exp(\varepsilon)] = 1$ to get

$$\begin{aligned}
RHS \approx \mathbb{E}\Bigg[&(1+a_1)^{-\theta} - \theta\,(1+a_1)^{-(1+\theta)}\,(\exp(\varepsilon)-1) + \frac{\theta(1+\theta)}{2}(\exp(\varepsilon)-1)^2 \\
&- \frac{\theta(1+\theta)(2+\theta)}{6}\,(1+a_1)^{-(3+\theta)}\,(\exp(\varepsilon)-1)^3 \\
&+ \frac{\theta(1+\theta)(2+\theta)(3+\theta)}{24}\,(1+a_1)^{-(4+\theta)}\,(\exp(\varepsilon)-1)^4\Bigg] \\
= (1+a_1)^{-\theta} &+ \frac{\theta(1+\theta)}{2}\,(1+a_1)^{-(2+\theta)}\,\mu_2^{\exp(\varepsilon)} \\
&- \frac{\theta(1+\theta)(2+\theta)}{6}\,(1+a_1)^{-(3+\theta)}\,\mu_3^{\exp(\varepsilon)} \\
&+ \frac{\theta(1+\theta)(2+\theta)(3+\theta)}{24}\,(1+a_1)^{-(4+\theta)}\,\mu_4^{\exp(\varepsilon)}.
\end{aligned}$$

A.4 Precautionary Savings

Rewrite the first-order condition, equation (2), as an implicit function

$$\begin{aligned}
e\left(a_1, \mu_i^{\exp(\varepsilon)}\right) = (y_0 - a_1)^{-\theta} - (1+a_1)^{-\theta} &- \frac{\theta(1+\theta)}{2}\,(1+a_1)^{-(2+\theta)}\,\mu_2^{\exp(\varepsilon)} \\
&+ \frac{\theta(1+\theta)(2+\theta)}{6}\,(1+a_1)^{-(3+\theta)}\,\mu_3^{\exp(\varepsilon)} \\
&- \frac{\theta(1+\theta)(2+\theta)(3+\theta)}{24}\,(1+a_1)^{-(4+\theta)}\,\mu_4^{\exp(\varepsilon)} = 0
\end{aligned}$$

and from the total differential of $e(\cdot)$ note that

$$\frac{da_1}{d\mu_i^{\exp(\varepsilon)}} = -\frac{\frac{\partial e(\cdot)}{\partial \mu_i^{\exp(\varepsilon)}}}{\frac{\partial e(\cdot)}{\partial a_1}}$$

Note that since $\mu_2^{\exp(\varepsilon)} > 0, \mu_3^{\exp(\varepsilon)} < 0, \mu_4^{\exp(\varepsilon)} > 0$ we have $\frac{\partial e(\cdot)}{\partial a_1} > 0$, which reflects that the marginal utility of savings is decreasing in a_1. Also note that $\frac{\partial e(\cdot)}{\partial \mu_i^{\exp(\varepsilon)}} < 0$ for $i = 2, 4$ and $\frac{\partial e(\cdot)}{\partial \mu_3^{\exp(\varepsilon)}} > 0$. Thus, $\frac{da_1}{d\mu_i^{\exp(\varepsilon)}} > 0$ for $i = 2, 4$ and $\frac{da_1}{d\mu_3^{\exp(\varepsilon)}} < 0$.

A.5 Extension of Two-Period Model to Recursive Preferences

Our results from the two-period analysis readily extend to a recursive preference specification. Of course, in the two-period model the notion of *recursive preferences* is not strictly speaking correct. We use this terminology here as we adopt Epstein-Zin-Weil preferences a la Epstein and Zin (1989, 1991), and Weil (1989) in the main analysis based on the quantitative life-cycle model. In the two-period context, with the alternative utility specification we can disentangle risk attitudes as parameterized by θ from the inter-temporal elasticity of substitution as parameterized by γ:[2]

$$U = \begin{cases} \frac{1}{1-\frac{1}{\gamma}} \left(c_0^{1-\frac{1}{\gamma}} + v\left(c_1, \theta, \Psi\right)^{1-\frac{1}{\gamma}} \right) & \text{for } \gamma \neq 1 \\ \ln\left(c_0\right) + \ln\left(v\left(c_1, \theta, \Psi\right)\right) & \text{for } \gamma = 1. \end{cases} \tag{1}$$

Thus, γ is the (inter-temporal) elasticity of substitution between c_0 and $v(\cdot)$, where $v(\cdot)$ represents the certainty equivalent from consumption in the second period, which is given by

$$v(c_1, \theta, \Psi) = \begin{cases} \left(\int c_1(\varepsilon)^{1-\theta} d\Psi(\varepsilon) \right)^{\frac{1}{1-\theta}} = \left(\mathbb{E}\left[c_1^{1-\theta} \right] \right)^{\frac{1}{1-\theta}} & \text{for } \theta \neq 1 \\ \exp\left(\int \ln(c_1(\varepsilon)) d\Psi(\varepsilon) \right) = \exp\left(\mathbb{E}\left[\ln(c_1) \right] \right) & \text{for } \theta = 1. \end{cases} \tag{2}$$

The specification of preferences gives standard CRRA preferences considered in the main text if the measure of the IES γ and the measure of risk aversion θ are reciprocals: $\theta = \frac{1}{\gamma}$.

Hand-to-Mouth Consumers. By the analogous steps to the CRRA case we can approximate the certainty equivalent (2). To this purpose write (2) as

$$v(c_1, \theta, \Psi) = \left(\int \tilde{g}(c_1(\varepsilon)) d\Psi(\varepsilon) \right)^{\frac{1}{1-\theta}}, \quad \text{where} \quad \tilde{g}(c_1(\varepsilon)) = c_1(\varepsilon)^{1-\theta}$$

[2]Notice that our representation of Epstein-Zin-Weil preferences, which goes back to Selden (1978, 1979), is a monotone transformation of the standard Epstein-Zin-Weil aggregator

$$V = \begin{cases} \left(c_0^{1-\frac{1}{\gamma}} + v(c_1, \theta, \Psi)^{1-\frac{1}{\gamma}} \right)^{\frac{1}{1-\frac{1}{\gamma}}} & \text{for } \gamma \neq 1 \\ c_0 \cdot v(c_1, \theta, \Psi) & \text{for } \gamma = 1, \end{cases}$$

where $U = \frac{1}{1-\frac{1}{\gamma}} V^{1-\frac{1}{\gamma}}$ if $\gamma \neq 1$ and $U = \ln(V)$ if $\gamma = 1$.

and take a fourth order Taylor series expansion of $\tilde{g}(c_1(\varepsilon))$ around μ_1^c, noticing that $c_1 = \exp(\varepsilon)$ and $E[\exp(\varepsilon)] = 1$ to get

$$\mathbb{E}[\tilde{g}(c_1(\varepsilon))] \approx 1 + (1-\theta)\left(-\frac{1}{2}\theta\mu_2^{\exp(\varepsilon)} + \frac{1}{6}\theta(1+\theta)\mu_3^{\exp(\varepsilon)} - \frac{1}{24}\theta(1+\theta)(2+\theta)\mu_4^{\exp(\varepsilon)}\right)$$

and thus the certainty equivalent is approximated as

$$\begin{aligned} v(c_1,\theta,\Psi) &= \left(\int c_1(\varepsilon)^{1-\theta} d\Psi(\varepsilon)\right)^{\frac{1}{1-\theta}} \\ &\approx \left(1 + (1-\theta)\left(-\frac{\theta}{2}\mu_2^c + \frac{\theta(1+\theta)}{6}\mu_3^c - \frac{\theta(1+\theta)(2+\theta)}{24}\mu_4^c\right)\right)^{\frac{1}{1-\theta}}. \end{aligned} \tag{3}$$

Since $v(g(c_1,\theta,\Psi))$, for $g(c_1,\theta,\Psi) = \int c_1(\varepsilon)^{1-\theta} d\Psi(\varepsilon)$ is decreasing in $g(\cdot)$ for $\theta > 1$ and increasing in $g(\cdot)$ for $\theta < 1$ we observe that an increase of risk of order $2-4$ reduces the certainty equivalent and thus the results for the CRRA case readily extend.

Precautionary Savings. In the general case where $\gamma \neq \frac{1}{\theta}$, we can use the resource constraint and write utility as

$$U = \frac{1}{1-\frac{1}{\gamma}}\left((y_0 - a_1)^{1-\frac{1}{\gamma}} + \left(\mathbb{E}\left[(\exp(\varepsilon) + a_1)^{1-\theta}\right]\right)^{\frac{1-\frac{1}{\gamma}}{1-\theta}}\right).$$

The first-order condition is now given by

$$(y_0 - a_1)^{-\frac{1}{\gamma}} = v(c_1,\theta,\Psi)^{\theta-\frac{1}{\gamma}}\mathbb{E}\left[(\exp(\varepsilon) + a_1)^{-\theta}\right]. \tag{4}$$

In the sequel, we follow Kimball and Weil (2009) and assume that the marginal utility of saving, the RHS of (4), is a decreasing function of a_1 (just as earlier established for CRRA utility), which establishes uniqueness of the solution. With this assumption we obtain the next proposition, as in Kimball and Weil (2009) (cf. Propositions 5 and 6):

Proposition 2. *For $\theta \neq \frac{1}{\gamma}$ an increase of (higher-order) risk leads to an increase of savings if $\gamma \leq 1$ or if $1 < \gamma \leq \frac{1}{\theta}$.*

Proof. Our proof of the proposition is adopted from Krueger and Ludwig (2019). Rewrite the RHS of the first-order condition in (4) as

$$RHS = v(c_1,\theta,\Psi)^{\theta-\frac{1}{\gamma}} f(c_1,\theta,\Psi) \tag{5}$$

$$= v(c_1,\theta,\Psi)^{1-\frac{1}{\gamma}} \frac{\mathbb{E}\left[(\exp(\varepsilon)+a_1)^{-\theta}\right]}{\mathbb{E}\left[(\exp(\varepsilon)+a_1)^{1-\theta}\right]}$$

$$= v(c_1,\theta,\Psi)^{1-\frac{1}{\gamma}} h(c_1,\theta,\Psi). \tag{6}$$

where $f(c_1,\theta,\Psi) = \mathbb{E}\left[(\exp(\varepsilon)+a_1)^{-\theta}\right]$ and $h(c_1,\theta,\Psi) = \frac{f(c_1,\theta,\Psi)}{g(c_1,\theta,\Psi)}$, where $g(c_1,\theta,\Psi) = \mathbb{E}\left[(\exp(\varepsilon)+a_1)^{1-\theta}\right]$. Consider the following case distinction:

1. $\gamma = 1$: Then the RHS is simply from (6)

$$RHS = h(c_1,\theta,\Psi)$$

giving rise to the following case distinction with respect to θ (throughout, we assume that $\theta > 0$, $\theta < \infty$):

(a) $\theta \in (0,1]$: $h(\cdot)$ is the ratio of function $f(\cdot)$ which is strictly convex in $\exp(\varepsilon)$ in the numerator and function $g(\cdot)$ which is concave in $\exp(\varepsilon)$ in the denominator (the denominator equals 1 for $\theta = 1$). Thus, an increase of (higher-order) risk increases $h(\cdot)$.

(b) $\theta > 1$: $h(\cdot)$ is the ratio of two strictly convex functions $f(\cdot), g(\cdot)$ in $\exp(\varepsilon)$, where the degree of convexity is stronger in the numerator than in the denominator (the exponent in the numerator is θ and in the denominator it is $1-\theta$). Thus, an increase of (higher-order) risk increases $h(\cdot)$.

Thus, an increase of (higher-order) risk unambiguously increases the RHS in (6), increasing precautionary savings.

2. $\gamma < 1$: For the behavior of $h(\cdot)$ the same logic as in item 1 applies. Furthermore, an increase of risk decreases $v(\cdot)$, which, for $\gamma < 1$, increases $v(\cdot)^{1-\frac{1}{\gamma}}$, since $1 - \frac{1}{\gamma} < 0$. Thus, an increase of risk unambiguously increases the RHS in (6), increasing precautionary savings.

3. $\gamma > 1$: We obtain the following case distinction from (5):

(a) $\theta \leq \frac{1}{\gamma}$: An increase of risk increases $v(\cdot)^{\theta-\frac{1}{\gamma}}$ (respectively leaves it unchanged at 1 if $\theta = \frac{1}{\gamma}$), so that an increase of risk unambiguously increases the RHS in (5), increasing precautionary savings.

(b) $\theta > \frac{1}{\gamma}$: the overall effect is ambiguous.

□

Thus, with a low IES ($\gamma \leq 1$), which since Hall (1988) most macroeconomists regard as a reasonable calibration, increasing risk leads to increasing savings. With a high IES ($\gamma > 1$), however, precautionary savings behavior *may not* arise if risk attitudes are also strong ($\gamma > \frac{1}{\theta}$). For a given degree of risk ($\mu_2^{\exp(\varepsilon)}, \mu_3^{\exp(\varepsilon)}, \mu_4^{\exp(\varepsilon)}$), the utility delivery from expected second period consumption as measured by the certainty equivalent is smaller, the stronger risk attitudes are. An increase of (higher-order) risk ($\mu_2^{\exp(\varepsilon)}, \mu_3^{\exp(\varepsilon)}, \mu_4^{\exp(\varepsilon)}$) implies a reduction of the certainty equivalent. This reduction is stronger if risk attitudes are stronger so that with a high IES the household may prefer to consume in the first period rather than to save for the second period and thus savings may decrease in response to the increase of risk.[3]

A.6 Recursive Representation of Persistent Income Component

The 2^{nd} to 4^{th} central moments of z_{ijt} are given recursively by

$$\mu_2(z_{ijt}) = \rho^2 \mu_2(z_{ij-1t-1}) + \mu_2^{\eta}(s(t)) \tag{7a}$$
$$\mu_3(z_{ijt}) = \rho^3 \mu_3(z_{ij-1t-1}) + \mu_3^{\eta}(s(t)) \tag{7b}$$
$$\mu_4(z_{ijt}) = \rho^4 \mu_4(z_{ij-1t-1}) + 6\rho^2 \mu_2(z_{ij-1t-1})\mu_2^{\eta}(s(t)) + \mu_4^{\eta}(s(t)). \tag{7c}$$

A.7 Decomposition of Consumption Equivalent Variations

We evaluate the welfare implications of higher-order risk by computing the consumption equivalent variation (CEV) that makes households that live in the world with shock distributions NORM indifferent to live with shock distributions $i \in \{LK, LKSW\}$.

A.7.1 Decomposition in the 2-Period Model

We start with the decomposition for the two-period model of Section 2, which extends to the quantitative model in a straightforward fashion, as we show in the next subsection. Under the convenient transformation[4] of utility $V = \left[\left(1 - \frac{1}{\gamma}\right) U\right]^{\frac{1}{1-\frac{1}{\gamma}}}$ we compute

$$g_c^i = \frac{V(C^i)}{V(C^{NORM})} - 1 \tag{8}$$

[3]Parts of this intuition is also discussed in Krueger and Ludwig (2019) for changes of second-order risk.
[4]I.e., we retransform to the standard EZW functional, cf. Footnote 2.

and thus the respective CEVs are defined as the percentage consumption loss in each period from the respective distribution with higher order risk relative to the distribution NORM.

We further decompose the CEV into *mean* and *distribution* effects. The mean effect is the welfare effect stemming from changes in average consumption and the distribution effect captures changes in the distribution of consumption. Formally, let $\mathbb{E}[C^i] = \frac{1}{2}\left(c_0^i + \int c_1^i(\varepsilon)d\Psi^i(\varepsilon)\right)$ for $i \in \{NORM, LK, LKSW\}$. Denote by $\delta_c^i = \frac{\mathbb{E}[C^i]}{\mathbb{E}[C^{NORM}]} - 1$ the percent change of consumption for $i \in \{LK, LKSW\}$. Then, the distribution effect corrects for the percentage change of mean consumption and is thus given by

$$g_c^{distr^i} = \frac{V\left(\frac{C^i}{1+\delta_c^i}\right)}{V(C^{NORM})} - 1 = \frac{1+g_c^i}{1+\delta_c^i} - 1. \tag{9}$$

The corresponding mean effect is accordingly

$$g_c^{mean^i} = g_c^i - g_c^{distr^i} = \frac{1+g_c^i}{1+\delta_c^i}\delta_c^i \approx \delta_c^i. \tag{10}$$

The distribution effect itself captures two changes. The first reflects the utility difference stemming from the change of the average life-cycle consumption profile, which we refer to as the *life-cycle distribution* effect. The second captures the utility change stemming from the change of the cross-sectional distribution of stochastic second period consumption, which we accordingly refer to as the *cross-sectional distribution* effect. Thus, we can rewrite $g_c^{distr^i}$ as

$$g_c^{distr^i} = g_c^{lcd^i} + g_c^{csd^i} \tag{11}$$

for the CEV stemming from the life-cycle redistribution (lcd) and cross-sectional distribution (csd) effect.

To compute the $g_c^{csd^i}$, first let $\mathbb{E}[C^i \mid j]$ denote the age j specific mean consumption, i.e., $\mathbb{E}[C^i \mid j = 0] = c_0^i$ and $\mathbb{E}[C^i \mid j = 1] = \int c_1^i(\varepsilon)d\Psi^i(\varepsilon)$. Next compute the age j specific consumption change (from $NORM$ to i) as $\delta_j^{c^i} = \frac{\mathbb{E}[C^i|j]}{\mathbb{E}[C^{NORM}|j]}$ for $i \in \{LK, LKSW\}$. Then compute the utility in distribution scenario $i \in \{LK, LKSW\}$ after correcting for mean consumption change as

$$\tilde{V}^i = \left(\left(\frac{1}{1+\delta_0^{c^i}}\right)^{1-\frac{1}{\gamma}} c_0^{i\,1-\frac{1}{\gamma}} + \left(\frac{1}{1+\delta_1^{c^i}}\right)^{1-\frac{1}{\gamma}} v(c_1^i, \theta, \Psi^i)^{1-\frac{1}{\gamma}}\right)^{\frac{1}{1-\frac{1}{\gamma}}},$$

which for $\gamma = 1$ simplifies to

$$\tilde{V}^i = \frac{1}{1+\delta_0^{c^i}} \frac{1}{1+\delta_1^{c^i}} \cdot c_0^i \cdot v(c_1^i, \theta, \Psi^i) = \frac{1}{1+\delta_0^{c^i}} \frac{1}{1+\delta_1^{c^i}} V^i.$$

Having corrected for the percent change of age-specific mean consumption, the CEV from the cross-sectional distribution effect is then

$$g_c^{csd^i} = \frac{\tilde{V}^i}{V(C^{NORM})} - 1 = \frac{1+g_c^i}{1+\delta_c^i} - 1 \tag{12}$$

and thus the life-cycle distribution effect follows as

$$g_c^{lcd^i} = g_c^{distr^i} - g_c^{csd^i}. \tag{13}$$

A.7.2 Decomposition in the Full Life Cycle Model

The decomposition into the mean and distribution effect is analogous to the two-period model, where average consumption is given by

$$\mathbb{E}[C^i] = \frac{1}{J+1} \sum_{j=0}^{J} \int c_j^i(a_j, z_j; s) d\Psi_j^i(a_j, z_j; s)$$

for $i \in \{NORM, LK, LKSW\}$, where $c_j^i(a_j, z_j; s)$ is the consumption policy function in distribution i and $\Phi_j^i(a_j, z_j; s)$ is the cross-sectional distribution.

To compute the cross-sectional distribution effect, let, as above, the age j specific consumption change be $\delta_j^{c^i} = \frac{\mathbb{E}[C^i|j]}{\mathbb{E}[C^{NORM}|j]}$ for $i \in \{LK, LKSW\}$, where now $\mathbb{E}[C^i \mid j] = \int c_j^i(a_j, z_j; s) d\Phi_j^i(a_j, z_j; s)$. Next, observe that

$$\tilde{V}_J^i = \left((1-\hat{\beta}) \left(\frac{c_J^i}{\delta_J^{c^i}} \right)^{1-\frac{1}{\gamma}} \right)^{\frac{1}{1-\frac{1}{\gamma}}} = \frac{1}{\delta_J^{c^i}} V_J^B$$

$$v\left(\tilde{V}_J^i\right) = \frac{1}{\delta_J^{c^i}} v\left(V_J^i\right).$$

and thus

$$\tilde{V}_{J-1}^i = \left((1-\tilde{\beta}) \left(\frac{1}{\delta_{J-1}^{c^i}} \right)^{1-\frac{1}{\gamma}} \left(c_{J-1}^i\right)^{1-\frac{1}{\gamma}} + \tilde{\beta} \left(v\left(\tilde{V}_J^i\right)\right)^{1-\frac{1}{\gamma}} \right)^{\frac{1}{1-\frac{1}{\gamma}}}$$

which extends to any period j as

$$\tilde{V}_j^i = \left((1-\tilde{\beta}) \left(\frac{c_j^i}{\delta_j^{c^i}} \right)^{1-\frac{1}{\gamma}} + \tilde{\beta} \left(v \left(\tilde{V}_{j+1}^i \right) \right)^{1-\frac{1}{\gamma}} \right)^{\frac{1}{1-\frac{1}{\gamma}}}.$$

With the parametric restriction $\gamma = 1$ the decomposition simplifies to

$$\tilde{V}_J^i = \exp\left((1-\tilde{\beta}) \ln \left(\frac{c_J^i}{\delta_J^{c^i}} \right) \right) = \left(\frac{1}{\delta_J^{c^i}} \right)^{1-\tilde{\beta}} V_J^i$$

and thus

$$\begin{aligned} \tilde{V}_{J-1}^i &= \exp\left((1-\tilde{\beta}) \ln \left(\frac{c_{J-1}^i}{\delta_{J-1}^{c^i}} \right) + \tilde{\beta} \ln \left(v \left(\tilde{V}_J^i \right) \right) \right) \\ &= \exp\left((1-\tilde{\beta}) \ln \left(\frac{1}{\delta_{J-1}^{c^i}} \right) + (1-\tilde{\beta}) \ln \left(c_{J-1}^i \right) + \tilde{\beta}(1-\tilde{\beta}) \ln \left(\frac{1}{\delta_J^{c^i}} \right) + \tilde{\beta} \ln \left(v \left(V_J^i \right) \right) \right) \\ &= \left(\left(\frac{1}{\delta_{J-1}^{c^i}} \right) \left(\frac{1}{\delta_J^{c^i}} \right)^{\tilde{\beta}} \right)^{1-\tilde{\beta}} V_J^i \end{aligned}$$

Continuing along these lines we get

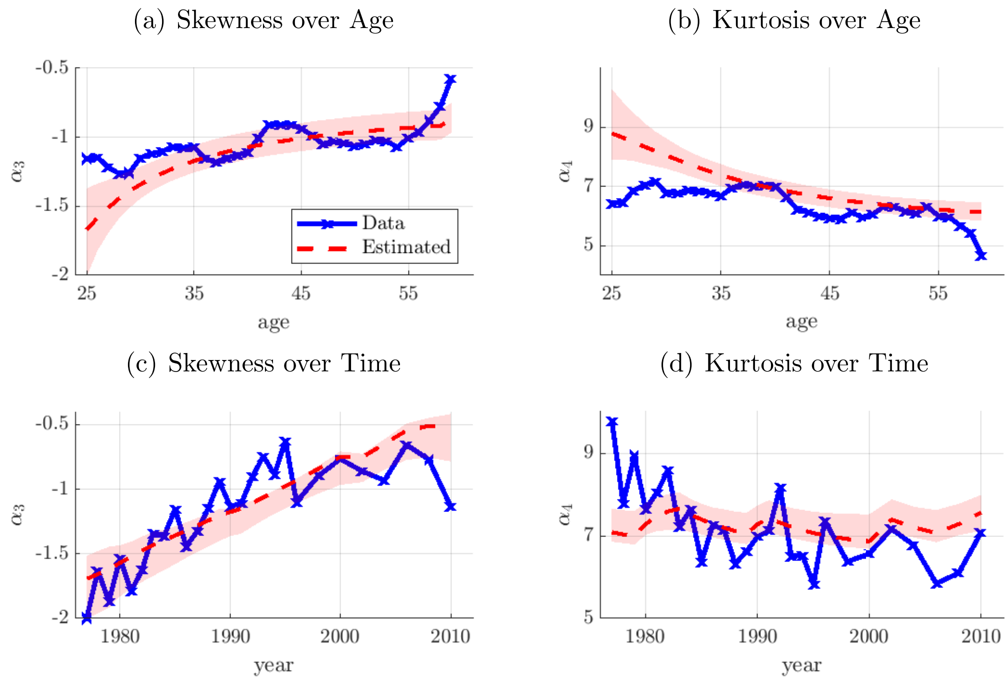
$$\tilde{V}_0^i = \left(\prod_{j=0}^{J} \left(\frac{1}{\delta_j^{c^i}} \right)^{\tilde{\beta}^j} \right)^{1-\tilde{\beta}} V_0^i.$$

With this construction we can now decompose the CEV into the cross-sectional and the life-cycle distribution effects using (12) and (13).

B Fit of Estimated Process

Figure B.1 displays age and year profiles of the standardized third and fourth moments, i.e., of the coefficients of skewness and kurtosis, implied by the estimated theoretical moments and their empirical counterparts.

Figure B.1: Fit of Estimated Process for Post-Government Earnings: Standardized Moments



Notes: Moments are cross-sectional standardized moments. For each moment, age and year profiles are based on a regression of the moment on a set of age and year dummies. Blue lines: empirical moments; red dashed lines: theoretical moments implied by point estimates; shaded area denotes a 90% confidence band based on the bootstrap iterations.

C Discretization of the FGLD

For each Flexible Generalized Lambda Distribution (FGLD) our discretization procedure is as follows:

1. Determine the endpoints of a grid $\mathcal{G}^{\tilde{x}}$ from the quantile function of the FGLD for a small probability $\tilde{\pi}_1 = \varepsilon$ such that

$$\begin{aligned} \tilde{x}_1 &= Q(\tilde{\pi}_1) \\ \tilde{x}_n &= Q(1 - \tilde{\pi}_1). \end{aligned}$$

2. Build grid $\mathcal{G}^{\tilde{x}}$ by drawing n equidistant nodes on the interval $[\tilde{x}_1, \tilde{x}_n]$.

3. For $\tilde{x}_i \in \mathcal{G}^{\tilde{x}}$, $i = 1, n-1$ compute auxiliary gridpoint $\tilde{\tilde{x}}_i = \frac{\tilde{x}_{i+1} + \tilde{x}_i}{2}$.

4. On all $\tilde{\tilde{x}}_i$ compute cumulative probability p_i from the quantile function of the FGLD. Since the quantile function of the FGLD maps $\tilde{\tilde{x}}_i = Q(p_i)$, this requires a numerical solver to compute $p_i = Q^{-1}(\tilde{\tilde{x}}_i)$.

5. Now assign to gridpoint $\tilde{x}_1$ the probability $\pi_1 = p_1$ and to all gridpoints i, $i = 2, \ldots, n-1$, the probability $\pi_i = p_i - p_{i-1}$ and to gridpoint $\tilde{x}_n$ the probability $1 - p_{n-1}$.

D Calibration Appendix

D.1 Moments of the FGLD Distribution

Tables D.1– D.3 summarize the moments for distributions NORM, LK, and LKSW, and Table D.4 contains the corresponding parameters of λ of the fitted FGLD distributions.

Table D.1: Moments: Distribution NORM

Moment	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
Transitory Shock:			
target	0.05	0	0.008
fitted	0.05	0	0.008
discrete	0.05	0	0.008
Persistent Shock—Contraction:			
target	0.022	0	0.001
fitted	0.022	0	0.001
discrete	0.022	0	0.001
Persistent Shock—Expansion:			
target	0.009	0	0
fitted	0.009	0	0
discrete	0.009	0	0

Notes: This table shows the target central moment together with the central moment of the fitted FGLD, and of the discretized FGLD for the distribution NORM, cf. Section 5.2.

Table D.2: Moments: Distribution LK

Moment	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
Transitory Shock:			
target	0.05	0	0.219
fitted	0.05	0	0.219
discrete	0.05	0	0.219
Persistent Shock—Contraction:			
target	0.022	0	0.061
fitted	0.022	0	0.061
discrete	0.022	0	0.061
Persistent Shock—Expansion:			
target	0.009	0	0.008
fitted	0.009	0	0.008
discrete	0.009	0	0.008

Notes: This table shows the target central moment together with the central moment of the fitted FGLD, and of the discretized FGLD for the distribution LK, cf. Section 5.2.

Table D.3: Moments: Distribution LKSW

Moment	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
Transitory Shock:			
target	0.05	-0.047	0.102
fitted	0.05	-0.047	0.102
discrete	0.051	-0.051	0.107
Persistent Shock—Contraction:			
target	0.022	-0.016	0.067
fitted	0.022	-0.016	0.067
discrete	0.023	-0.02	0.07
Persistent Shock—Expansion:			
target	0.009	-0.001	0.01
fitted	0.009	-0.001	0.01
discrete	0.009	-0.002	0.01

Notes: This table shows the target central moment together with the central moment of the fitted FGLD, and of the discretized FGLD for the distribution LKSW, cf. Section 5.2.

Table D.4: Fitted Parameters of FGLD

Parameter	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\hat{\lambda}_4$
		NORM		
Transitory:	1.000	0.359	5.203	5.203
Pers.—Contraction:	1.000	0.539	5.203	5.203
Pers.—Expansion:	1.000	0.871	5.203	5.203
		LK		
Transitory:	1.000	0.002	173.309	173.309
Pers.—Contraction:	1.000	0.002	244.954	244.954
Pers.—Expansion:	1.000	0.003	220.344	220.344
		LKSW		
Transitory:	0.197	0.008	92.959	57.755
Pers.—Contraction:	0.425	0.002	289.898	225.714
Pers.—Expansion:	0.894	0.003	275.612	256.735

Notes: This table shows the estimated λ-values for the fitted FGLD for distributions NORM, LK and LKSW, cf. Section 5.2.

D.2 The Bend Point Formula and the Pension Indexation Factor

Approximating the AIME with the last income state before entering into retirement z_{j_r-1} the primary insurance amount according to the bend point formula is determined as follows:

$$p(z_{j_r-1}) = \begin{cases} s_1 z_{j_r-1} & \text{for } z_{j_r-1} < b_1 \\ s_1 b_1 + s_2 \left(z_{j_r-1} - b_1\right) & \text{for } b_1 \leq z_{j_r-1} < b_2 \\ s_1 b_1 + s_2 \left(b_2 - b_1\right) + s_3 \left(z_{h_r-1} - b_2\right) & \text{for } b_2 \leq z_{j_r-1} < b_3 \\ s_1 b_1 + s_2 \left(b_2 - b_1\right) + s_3 \left(b_3 - b_2\right) & \text{for } z_{j_r-1} \geq b_3 \end{cases}$$

Table D.5 contains the calibrated values of the pension indexation factor ϱ, which is required to clear the budget of the pension system.

Table D.5: Pension Indexation Factor ϱ

	CR	NCR
NORM	0.6817	0.6692
LK	0.7007	0.6787
LKSW	0.6866	0.6758

Notes: Calibrated pension benefit level ϱ under a balanced budget. CR: cyclical risk, NCR: no cyclical risk.

D.3 Moments of the Earnings Process

Table D.6 shows cross-sectional central moments of the earnings distribution in logs and levels at labor market entry (age 25) and exit (age 60). We observe that all distributions are skewed to the right in levels and that, despite left skewness in logs, right skewness of distribution LKSW is higher in levels than of distribution NORM. Furthermore, the variance is initially lower in distribution LKSW than in distribution NORM.[5] Both features constitute a source of welfare gains from higher-order income risk, whereas the higher kurtosis in levels and the increasing variance work against it. Finally, skewness and in particular kurtosis in levels under (counterfactual) distribution LK are extremely high. Left-skewness in logs in distribution LKSW substantially reduces both moments.

[5]By construction, the variance of the log earnings distribution is the same across distribution scenarios. The difference of 0.01 showing up at age 60 is due to numerical inaccuracies of coarse grids for assets a and the persistent income state z.

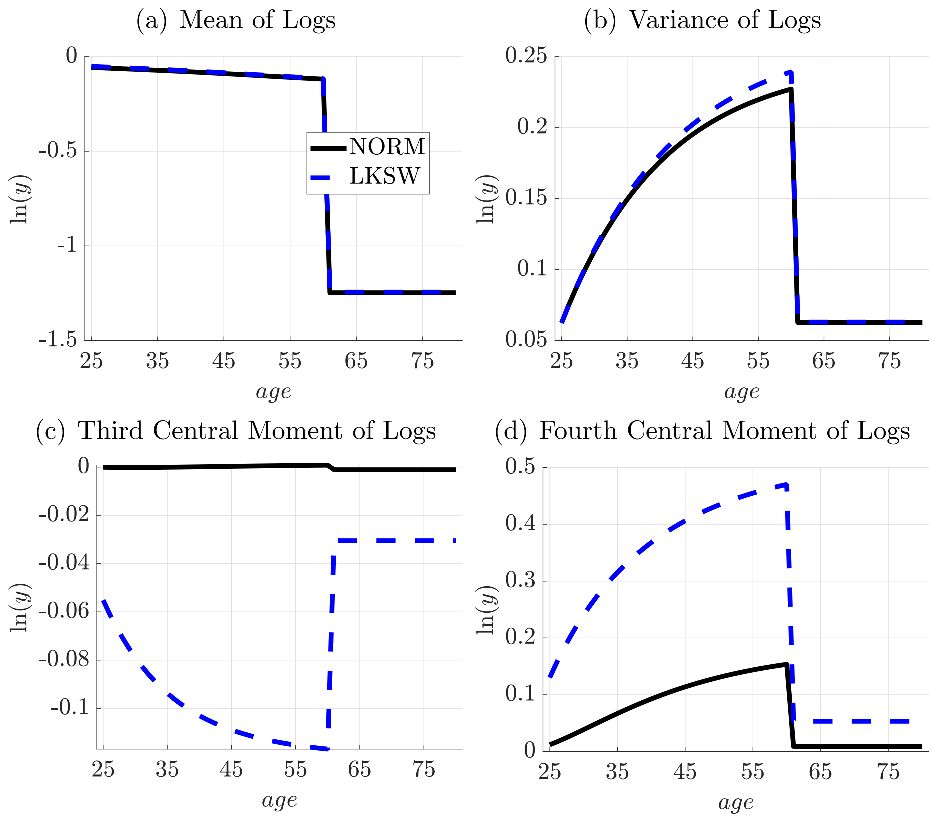
Table D.6: Moments of the Earnings Distribution in Logs and Levels

	Logs			Levels		
	Age 25 ($j = 0$)					
	NORM	LK	LKSW	NORM	LK	LKSW
μ_2	0.06	0.06	0.06	0.06	0.36	0.05
μ_3	0	0	-0.06	0.01	4.44	0.09
μ_4	0.01	0.24	0.13	0.01	129.43	0.41
	Age 60 ($j = 35$)					
μ_2	0.23	0.24	0.24	0.25	0.86	0.3
μ_3	0	0	-0.12	0.21	27.52	1.12
μ_4	0.15	0.56	0.47	0.5	27889.82	27.85

Notes: Moments of cross-sectional distribution of log earnings and earnings at ages 25 ($j = 0$) and 60 ($j = 35$) for each scenario of shock distributions. NORM: FGLD with moments of the normal distribution, LK: FGLD with excess kurtosis, LKSW: FGLD with excess kurtosis and left-skewness (in logs).

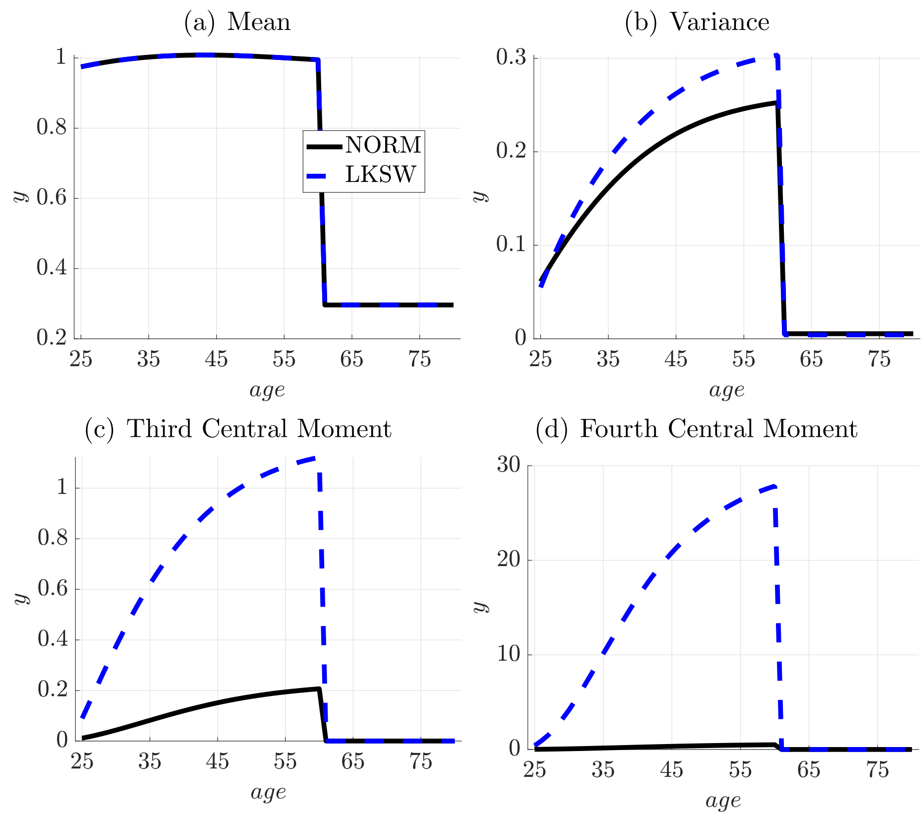
Figures D.1 and D.2 summarize the calibration of the earnings process during the working period and the pension income in retirement for central moments 1-4 of the earnings distribution in levels and logs, respectively.

Figure D.1: Moments of Life-Cycle Earnings by Age: Logs



Notes: Figures show moments of cross-sectional distribution of log earnings over the life-cycle for each scenario of shock distributions. NORM: FGLD with moments of the normal distribution, LKSW: FGLD with excess kurtosis and left-skewness (in logs).

Figure D.2: Moments of Life-Cycle Earnings by Age: Levels



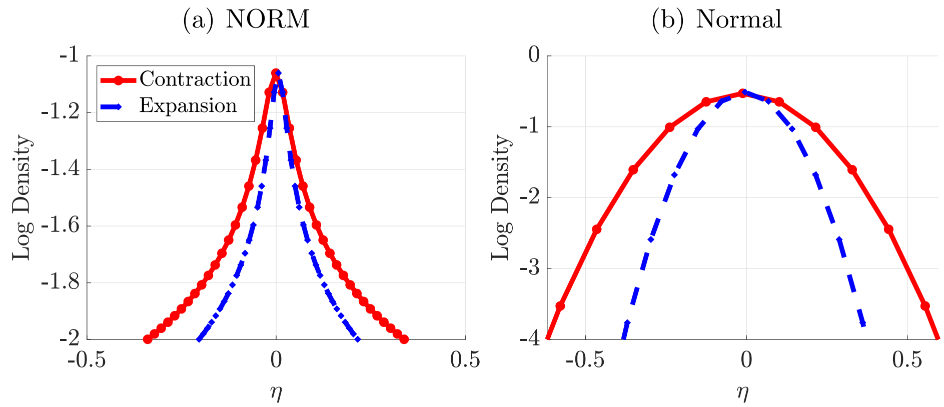
Notes: Figures show moments of cross-sectional distribution of earnings over the life-cycle for each scenario of shock distributions. NORM: FGLD with moments of the normal distribution, LKSW: FGLD with excess kurtosis and left-skewness (in logs).

E Additional Results

E.1 Comparison of FGLD with Normal Distribution

In the application in the main text, we compare the FGLD distribution with left skewness and excess kurtosis (LKSW) to the FGLD with zero skewness and kurtosis of three (NORM). Figure E.1 shows the distribution against the Normal distribution using Gaussian quadrature. The second to fourth central (and standardized) moments of the two distributions are the same—the visual differences are captured by the even moments of higher order. It turns out that these higher-order differences are quantitatively irrelevant in our application: Table E.1 documents the CEV under distribution scenario NORM in comparison to one where shocks are drawn from a Normal distribution. Thus, for the preferences used the differences of moments are not crucial in the calibrated version of the model, and therefore we choose the FGLD distribution NORM as the benchmark.

Figure E.1: Discretized Log Distribution Functions: Persistent Shock



Notes: Discretized log distribution functions for the persistent shock η. NORM: FGLD with estimated variance, zero skewness, and kurtosis of three. Markers denote the grid points used in the discretized distribution. Normal: Normal distribution with estimated variance discretized using Gaussian quadrature method. Log density is the base 10 logarithm of the PDF.

E.2 Sensitivity Analyses

In this appendix, we consider an expected utility formulation with CRRA preferences where we restrict $\theta = \frac{1}{\gamma}$, we analyze the role of borrowing constraints in the model, and we investigate how results are affected by our choice of the interest rate. Table E.2 summarizes the results.

Table E.1: Welfare Effects of Cyclical Idiosyncratic Risk: FGLD(NORM) versus Normal Distribution

CEV	g_c	g_c^{mean}	g_c^{lcd}	g_c^{csd}
Risk Aversion, $\theta = 1$				
NORM	-1.720	0.499	-2.175	-0.044
NORMAL	-1.722	0.500	-2.176	-0.045
Risk Aversion, $\theta = 2$				
NORM	-3.263	0.898	-4.038	-0.123
NORMAL	-3.268	0.898	-4.043	-0.123
Risk Aversion, $\theta = 3$				
NORM	-4.607	1.229	-5.638	-0.198
NORMAL	-4.615	1.230	-5.646	-0.199
Risk Aversion, $\theta = 4$				
NORM	-5.758	1.515	-7.009	-0.264
NORMAL	-5.767	1.516	-7.018	-0.265

Notes: Welfare gains (positive numbers) and losses (negative numbers) of cyclical idiosyncratic risk expressed as consumption equivalent variation (CEV) for FGLD distribution NORM and the normal distribution, NORMAL. g_c: total CEV, g_c^{mean}: CEV from changes of mean consumption, g_c^{lcd}: CEV from changes in the distribution of consumption over the life-cycle, g_c^{csd}: CEV from changes in the cross-sectional distribution of consumption, where $g_c = g_c^{mean} + g_c^{lcd} + g_c^{csd}$.

Table E.2: Total CEV g_c of Cyclical Idiosyncratic Risk: Sensitivity Analyses

	Baseline	CRRA	BC	IR	GE
Risk Aversion, $\theta = 1$					
NORM	-1.720	-1.720	-1.893	-1.905	-1.018
LKSW	-1.443	-1.443	-1.612	-1.611	-0.884
Risk Aversion, $\theta = 2$					
NORM	-3.263	-2.552	-3.609	-3.627	-2.000
LKSW	-3.516	-2.564	-4.293	-3.972	-2.367
Risk Aversion, $\theta = 3$					
NORM	-4.607	-3.335	-5.113	-5.123	-2.872
LKSW	-7.177	-4.456	-9.725	-8.253	-5.282
Risk Aversion, $\theta = 4$					
NORM	-5.758	-4.072	-6.404	-6.399	-3.611
LKSW	-12.171	-7.530	-17.14	-14.283	-9.619

Notes: Total welfare gains (positive numbers) and losses (negative numbers) of cyclical idiosyncratic risk expressed as Consumption Equivalent Variation (CEV) g_c in the distribution scenario NORM and the leptokurtic and left-skewed scenario LKSW. CRRA: CRRA utility, BC: "borrowing constraints", IR: interest rate, GE: general equilibrium.

CRRA Utility. Assuming CRRA preferences with $\theta = \frac{1}{\gamma}$ we conduct experiments for $\theta \in \{2, 3, 4\}$, since for $\theta = 1$ results are of course as before. As in our previous baseline analysis, we recalibrate discount factor β for each value of θ. For $\theta \in \{2, 3, 4\}$ we obtain $\beta \in \{0.982, 0.990, 0.995\}$ and thus, in contrast to our experiments with EZW utility, the calibrated discount factor is increasing in θ. With increasing risk attitudes θ the precautionary savings motive is strengthened, while the simultaneous reduction of the IES $\gamma = \frac{1}{\theta}$ reduces life-cycle savings. The second effect turns out to dominate so that calibration calls for less impatience in order to hold the average asset accumulation unchanged.

Column 2 of Table E.2 summarizes the results on the welfare effects of cyclical idiosyncratic risk for this alternative choice of preferences. In comparison to Table 4 we observe a lower increase of welfare losses from cyclical idiosyncratic risk when risk aversion is increased (the IES is decreased). Likewise, our difference in difference comparison to scenario NORM shows that higher-order income risk still substantially matters for the welfare costs of cyclical idiosyncratic risk, but less than with EZW preferences. The reason is that with a lower IES the overall consumption profile is smoother and thus reacts less to changes in risk. Thus, the simultaneous reduction of the IES when relative risk attitudes are strengthened confounds the welfare analysis.

The Role of Borrowing Constraints. In our baseline calibration households start their economic life with positive assets and calibrated impatience is relatively strong. As a consequence, very few households are borrowing constrained (numerically, the fraction is basically zero in all scenarios). We now investigate the sensitivity of our results with regard to the role of the borrowing constraint by setting initial assets to 0. In this experiment, we do not recalibrate because we aim at disentangling the role of the constraint.

As a consequence of zero initial assets, the fraction of borrowing constrained hand-to-mouth consumers increases strongly. For $\theta = 1$, roughly 6.6% of all households are constrained in scenario NORM and 4.0% in scenario LKSW. Column 3 of Table E.2 shows that this leads to higher overall welfare losses from cyclical idiosyncratic risk and an increasing importance for higher-order risk. For $\theta = 4$ the difference in the CEV between scenarios LKSW and NORM is about $-10.7\%p$, compared to $-6.4\%p$ reported in Table 4. Thus, borrowing constraints increase the role played by higher-order income risk for the welfare losses from cyclical idiosyncratic risk.

Lower Interest Rate. Next, rather than assuming an annual interest rate of 4.2% we reduce it to 2%. We recalibrate the discount factor β in all four experiments for $\theta \in \{1, 2, 3, 4\}$, which gives $\beta \in \{0.990, 0.988, 0.986, 0.983\}$ and thus the discount factors are higher because lower returns reduce life-cycle savings which is offset in calibration by stronger patience. Results are shown in column 4 of Table E.2. While the role played by higher-order income risk for the welfare losses from cyclical idiosyncratic risk is slightly increased, the difference to the baseline calibration is modest.

E.3 Separating the Role of Kurtosis

Table E.3 provides additional insights on the roles of the different components, i.e., here the excess kurtosis in isolation, of higher order risk for the high risk aversion calibration with $\theta = 4$. To this end we calibrate an additional distribution scenario, LK, that features the estimated excess kurtosis, but is symmetric (in the distribution in logs). Note that this implies for the parameters of the FGLD that $\lambda_3 = \lambda_4$. Welfare costs of cyclical risk are about 4% higher in this distribution scenario than in scenario NORM.

Table E.3: The Welfare Effects of Cyclical Idiosyncratic Risk for Distribution Scenario LK

CEV	g_c	g_c^{mean}	g_c^{lcd}	g_c^{csd}	Δg_c
			Risk Aversion, $\theta = 4$		
LK	-9.738	1.731	-11.146	-0.322	-3.980

Notes: Welfare gains (positive numbers) and losses (negative numbers) of cyclical idiosyncratic risk expressed as Consumption Equivalent Variation (CEV) in the non-cyclical scenario that makes households indifferent to the cyclical scenario. Displayed for scenario LK. g_c: total CEV, g_c^{mean}: CEV from changes of mean consumption, g_c^{lcd}: CEV from changes in the distribution of consumption over the life-cycle, g_c^{csd}: CEV from changes in the cross-sectional distribution of consumption, where $g_c = g_c^{mean} + g_c^{lcd} + g_c^{csd}$. $\Delta g_c = g_c^{LK} - g_c^{NORM}$: difference in percentage points relative to scenario NORM.

E.4 General Equilibrium

Overview. In the analyses presented in the main text we consider a partial equilibrium framework where interest rates and average wages are constant. Increased precautionary savings from higher-order risk may lead to a higher capital stock, which in general equilibrium increases wages and lowers returns on savings. To investigate the robustness of our findings with respect to this feedback, we consider a general equilibrium variant of our model, where we treat scenario LKSW with cyclical risk as a baseline for each level of risk aversion when (re)calibrating the model in general equilibrium. We focus on the role of higher-order risk itself, and deliver an approximation regarding the cyclical nature of risk: when in aggregate state $s \in \{E, C\}$ agents make choices assuming that the state of the world will remain unchanged. We then calculate ex-ante expected life-time utility by weighting the two possible states with the probabilities according to the stationary distribution of the aggregate Markov transition matrix.

As a first step, in the baseline scenario we take a normalization such that the net wage rate is one and calibrate the model to an interest rate of $r[\%] = 4.2\%$ consistent with a standard static representative firm problem in general equilibrium, and accordingly compute the implied parameters of the aggregate production function, with all calibration details provided below.

As a second step, we hold constant these parameters and compute the equilibrium interest rate and wage rate for each considered scenario, cf. Table E.4. Overall, in the economy with cyclical risk, the additional precautionary savings in general equilibrium increase the capital stock increasing wages and decreasing returns. Furthermore, this difference in net wages and returns between increases in risk aversion θ because the precautionary savings reaction is stronger with higher risk aversion. However, these changes are not large, which is a consequence of the life cycle structure of the economy. Consider moving from scenario NORM with cyclical risk to scenario LKSW with cyclical risk. While young agents have higher precautionary savings when facing higher-order risk, these savings will be dis-saved at old age. Aggregate savings of the economy will thus not change strongly.

Table E.4: Aggregate Prices in General Equilibrium Model

Variable	w^n		r	
	CR	NCR	CR	NCR
	Risk Aversion, $\theta = 1$			
NORM	1.0009	0.9979	0.0419	0.0422
LK	1.0033	1	0.0416	0.042
LKSW	1	0.9976	0.042	0.0423
	Risk Aversion, $\theta = 4$			
NORM	0.9911	0.9815	0.0431	0.0443
LK	1	0.9876	0.042	0.0435
LKSW	1	0.9856	0.042	0.0438

Notes: Net wage w^n and return r in the general equilibrium variants of the model. CR: cyclical idiosyncratic risk, NCR: no cyclical idiosyncratic risk.

Column 5 of Table E.2 shows the resulting welfare costs of cyclical risk next to the baseline results. The percentage point difference of the CEV between scenario LKSW and scenario NORM is almost identical in the general equilibrium version of the model. For instance, for $\theta = 4$ the difference now stands at $-6.00\%p$, compared to $-6.41\%p$. We therefore conclude that our main findings are robust in general equilibrium.

Calibration. We close this discussion on the GE extension by providing the details of the calibration. Assuming Cobb-Douglas production with capital elasticity α and a technology level Υ output of the representative firm is

$$Y = \Upsilon K^{\alpha} L^{1-\alpha}.$$

Denoting by $k = \frac{K}{L}$ the capital intensity and assuming a constant depreciation rate of δ the first-order conditions are given by

$$r = \Upsilon \alpha k^{\alpha-1} - \delta \tag{14a}$$

$$w = \Upsilon (1-\alpha) k^{\alpha}, \tag{14b}$$

which also implies that

$$\frac{w}{r+\delta} = \frac{1-\alpha}{\alpha} k. \tag{15}$$

Assuming capital market clearing in a closed economy so that aggregate assets are equal to the capital stock $K = A$, and knowing that aggregate efficient labor in our economy is normalized to $L = h_r - 1$, we can compute $k = \frac{A}{h_r - 1}$, and given prices r and $w = \frac{1}{1-\tau-\tau^p}$ (since net wages $w^n = 1$) the implied depreciation rate follows from using this in (15) as

$$\delta = \frac{w}{\frac{1-\alpha}{\alpha} k} - r = \frac{\frac{1}{1-\tau-\tau^p}}{\frac{1-\alpha}{\alpha} \frac{A}{h_r - 1}} - r$$

as well as the implied technology level follows from (14a) as

$$\Upsilon = \frac{r+\delta}{\alpha} k^{1-\alpha} = \frac{r+\delta}{\alpha} \left(\frac{A}{h_r - 1} \right)^{1-\alpha}.$$

Table E.5 summarizes this calibration. The calibrated depreciation rate is low, which is not surprising giving our target of a wealth to income ratio from the data and an interest rate of 4.2%.

Having determined the parameters δ, Υ in the economy CR/LKSW for each level of risk aversion as summarized in Table E.5 we then hold constant δ, Υ in all other economies and iterate on the interest rate until market clearing. In each iteration, we compute wages given the interest rate from (15) and (14b) as

$$w = \Upsilon^{\frac{1}{1-\alpha}} (1-\alpha) \left(\frac{\alpha}{r+\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

Table E.5: Technology Level and Depreciation Rate in General Equilibrium Variant

Parameter		
θ	δ	Υ
1	0.0159	0.9234
2	0.0163	0.9252
3	0.0167	0.9271
4	0.0171	0.9293

Notes: Calibrated depreciation rate δ and technology level Υ in the general equilibrium variant of the model.

and net wages as $w^n = (1 - \tau^p - \tau)w$.

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Supplementary Appendix

(Not for Publication)

"Higher-Order Income Risk Over the Business Cycle"

(Christopher Busch and Alexander Ludwig)

S.A Fitting Moments of the FGLD

This supplementary appendix describes how we fit the Flexible Generalized Lambda Distribution (FGLD). The quantile function is

$$Q(p;\lambda) = F^{-1}(p;\lambda) = x = \lambda_1 + \frac{1}{\lambda_2}\left(\frac{p^{\lambda_3}-1}{\lambda_3} - \frac{(1-p)^{\lambda_4}-1}{\lambda_4}\right) \tag{S.A.1}$$

where λ_1 is a location and λ_2 is a scale parameter, λ_3, λ_4 in turn are tail index parameters.[1]

We will need to use the relationship between the quantile function and the probability density function (PDF). Noticing that $x = F^{-1}(p) = Q(p)$ and $F(x) = p$ we can derive the PDF $f(x)$ from the quantile function $Q(p)$ by

$$f(x) = f(Q(p)) = \frac{\partial F(x)}{\partial x} = \frac{\partial p}{\partial Q(p)} = \frac{1}{\frac{\partial Q(p)}{\partial p}}. \tag{S.A.2}$$

Differentiating (S.A.1) we therefore find the PDF to be

$$f(x) = f(Q(p)) = \frac{\lambda_2}{p^{\lambda_3-1} + (1-p)^{\lambda_4-1}}. \tag{S.A.3}$$

Lakhany and Mausser (2000) and Su (2007) describe how to estimate the parameters of (S.A.1) using moments of the distribution. The kth raw moment of a random variable X is given as

$$E\left[X^k\right] = \int_{-\infty}^{\infty} x^k f(x)dx, \quad k \geq 1$$

where $f(x)$ is the distribution function. Setting $k = 1$ gives the expected value $\mu_1 = E[X]$.

[1] The parametric constraints are $\lambda_2 > 0$, and $\min\{\lambda_3, \lambda_4\} > -\frac{1}{4}$.

The kth central moment is defined as

$$E\left[(X-\mu_1)^k\right] = \int_{-\infty}^{\infty} (x-\mu_1)^k f(x)dx, \quad k \geq 1.$$

We can use binomial expansion to write central moments in terms of raw moments as

$$E\left[(X-\mu_1)^k\right] = E\left[\sum_{j=0}^{k} \binom{k}{j} (-1)^j (X)^{k-j} \mu_1^j\right] \tag{S.A.4}$$

where $\binom{k}{j}$ are binomial coefficients.

Now apply the same logic to evaluate the kth raw moment of a percentile function. Use variable substitution $p = Q^{-1}(p) = F(x)$, noticing that $Q^{-1}(-\infty) = 0$ and $Q^{-1}(\infty) = 1$ so that the integration bounds change. Furthermore, use (S.A.2) giving $f(x) = \frac{dp}{dQ(p)}$ to rewrite

$$\int_{-\infty}^{\infty} x^k f(x)dx = \int_0^1 Q(p)^k \frac{dp}{dQ(p)} dQ(p) = \int_0^1 Q(p)^k dp. \tag{S.A.5}$$

Hence the kth raw moment using quantile functions is given by

$$E\left[X^k\right] = \int_0^1 Q(p)^k dp.$$

Next, observe that (S.A.1) can be rewritten as

$$\begin{aligned} Q(p) = F^{-1}(p) = x &= \lambda_1 - \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_2\lambda_4} + \frac{1}{\lambda_2}\left(\frac{p^{\lambda_3}}{\lambda_3} - \frac{(1-p)^{\lambda_4}}{\lambda_4}\right) \\ &= a + b\tilde{Q}(p). \end{aligned}$$

Let X be the random variable with quantile function $Q(p)$ and let Y be the random variable with quantile function $\tilde{Q}(p)$. We then have

$$\begin{aligned} E[X] &= a + bE[Y], \quad k = 1 \\ E\left[(X - E[X])^k\right] &= b^k E\left[(Y - E[Y])^k\right], \quad k > 1 \end{aligned}$$

for the kth central moments. In what follows, we denote the raw moments of Y by ν, hence $\nu_k = EY^k$. Using (S.A.4) we thus get for the first four central moments (recalling that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, with $\binom{n}{n} = \binom{n}{0} = 1$):

$$\begin{aligned}\mu_1 &= E[X] = a + bE[Y] = a + b\nu_1 \\ &= \lambda_1 - \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_2\lambda_4} + \frac{1}{\lambda_2}\nu_1.\end{aligned}$$

For the remaining moments, we rewrite (S.A.4) to get

$$\begin{aligned}E\left[(Y - E[Y])^k\right] &= E\left[\sum_{j=0}^{k} \binom{k}{j} (-1)^j \, (Y)^{k-j} \, \nu(1)^j\right] \\ &= \left[\sum_{j=0}^{k} \binom{k}{j} (-1)^j \, E\left[(Y)^{k-j}\right] \nu(1)^j\right]\end{aligned}$$

We can therefore write explicitly

$$\begin{aligned}\mu_2 &= b^2 \left(E[Y^2] - (E[Y])^2\right) = \frac{1}{\lambda_2^2}(\nu_2 - \nu_1^2) \\ \mu_3 &= b^3 E\left[\sum_{j=0}^{3} \binom{3}{j} (-1)^j \, (Y)^{3-j} \, (\nu_1)^j\right] \\ &= b^3 E\left[Y^3 - 3Y^2\nu_1 + 3Y\nu_1^2 - \nu_1^3\right] \\ &= \frac{1}{\lambda_2^3}\left(\nu_3 - 3\nu_1\nu_2 + 2\nu_1^3\right] \\ \mu_4 &= b^4 E\left[\sum_{j=0}^{4} \binom{4}{j} (-1)^j \, (Y)^{4-j} \, (\nu_1)^j\right] \\ &= b^4 E\left[Y^4 - 4Y^3\nu_1 + 6Y^2\nu_1^2 - 4Y\nu_1^3 + \nu_1^4\right] \\ &= \frac{1}{\lambda_2^4}\left(\nu_4 - 4\nu_1\nu_3 + 6\nu_1^2\nu_2 - 3\nu_1^4\right).\end{aligned}$$

Finally, we need to determine expressions for the raw moments of Y. To this end, we have to evaluate

$$E\left[Y^k\right] = \nu_k = \int_0^1 \tilde{Q}(p)^k dp = \int_0^1 \left(\frac{p^{\lambda_3}}{\lambda_3} - \frac{(1-p)^{\lambda_4}}{\lambda_4}\right)^k dp$$

Again using binomial expansion, we can rewrite this integral as

$$\begin{aligned}\nu_k &= \int_0^1 \sum_{j=0}^{k} \binom{k}{j} (-1)^j \left(\frac{p^{\lambda_3}}{\lambda_3}\right)^{k-j} - \left(\frac{(1-p)^{\lambda_4}}{\lambda_4}\right)^j dp \\ &= \sum_{j=0}^{k} \binom{k}{j} \frac{(-1)^j}{\lambda_3^{k-j}\lambda_4^j} \int_0^1 \left(p^{\lambda_3(k-j)} - (1-p)^{\lambda_4 j}\right) dp \\ &= \sum_{j=0}^{k} \binom{k}{j} \frac{(-1)^j}{\lambda_3^{k-j}\lambda_4^j} \beta\left(\lambda_3(k-j)+1, \lambda_4 j+1\right),\end{aligned}$$

where $\beta(\cdot,\cdot)$ is the β-function. Observe that the β-function is only well defined if all arguments are positive. This requires that

$$\lambda_3(k-j)+1 > 0 \quad \text{and} \quad \lambda_4 j + 1 > 0$$

for all k, j. This equality can only be binding if $\lambda_3, \lambda_4 < 0$. Since $j \leq k$ we can rewrite the above inequality as

$$\min(\lambda_3, \lambda_4) > -\frac{1}{k}.$$

Observe that the RHS in the above is decreasing in k. Therefore, if we target at matching moments up to $k = 4$, the constraint reads as $\min(\lambda_3, \lambda_4) > -\frac{1}{4}$.

We can also write out ν_k, for $k = 1, \ldots, 4$ explicitly as functions of λ_3, λ_4 as:

$$\begin{aligned}
\nu_1 &= \sum_{j=0}^{1} \binom{1}{j} \frac{(-1)^j}{\lambda_3^{1-j}\lambda_4^j} \beta\left(\lambda_3(1-j)+1, \lambda_4 j+1\right) \\
&= \frac{1}{\lambda_3}\beta\left(\lambda_3+1, 1\right) - \frac{1}{\lambda_4}\beta\left(1, \lambda_4+1\right) \\
&= \frac{1}{\lambda_3(\lambda_3+1)} - \frac{1}{\lambda_4(\lambda_4+1)} \\
\nu_2 &= \sum_{j=0}^{2} \binom{2}{j} \frac{(-1)^j}{\lambda_3^{2-j}\lambda_4^j} \beta\left(\lambda_3(2-j)+1, \lambda_4 j+1\right) = \nu_1(\lambda_3, \lambda_4) \\
&= \frac{1}{\lambda_3^2}\beta\left(2\lambda_3+1, 1\right) - 2\frac{1}{\lambda_3\lambda_4}\beta\left(\lambda_3+1, \lambda_4+1\right) + \frac{1}{\lambda_4^2}\beta\left(1, 2\lambda_4+1\right) \\
&= \frac{1}{\lambda_3^2\left(2\lambda_3+1\right)} + \frac{1}{\lambda_4^2\left(2\lambda_4+1\right)} - 2\frac{1}{\lambda_3\lambda_4}\beta\left(\lambda_3+1, \lambda_4+1\right) = \nu_2(\lambda_3, \lambda_4) \\
\nu_3 &= \sum_{j=0}^{3} \binom{3}{j} \frac{(-1)^j}{\lambda_3^{3-j}\lambda_4^j} \beta\left(\lambda_3(3-j)+1, \lambda_4 j+1\right) \\
&= \frac{1}{\lambda_3^3}\beta\left(3\lambda_3+1, 1\right) - \frac{3}{\lambda_3^2\lambda_4}\beta\left(2\lambda_3+1, \lambda_4+1\right) + \frac{3}{\lambda_3\lambda_4^2}\beta\left(\lambda_3+1, 2\lambda_4+1\right) - \frac{1}{\lambda_4^3}\beta\left(1, 3\lambda_4+1\right) \\
&= \frac{1}{\lambda_3^3(3\lambda_3+1)} - \frac{1}{\lambda_4^3(3\lambda_4+1)} - \frac{3}{\lambda_3^2\lambda_4}\beta\left(2\lambda_3+1, \lambda_4+1\right) + \frac{3}{\lambda_3\lambda_4^2}\beta\left(\lambda_3+1, 2\lambda_4+1\right) = \nu_3(\lambda_3, \lambda_4) \\
\nu_4 &= \sum_{j=0}^{4} \binom{4}{j} \frac{(-1)^j}{\lambda_3^{4-j}\lambda_4^j} \beta\left(\lambda_3(4-j)+1, \lambda_4 j+1\right) \\
&= \frac{1}{\lambda_3^4}\beta\left(4\lambda_3+1, 1\right) - \frac{4}{\lambda_3^3\lambda_4}\beta\left(3\lambda_3+1, 2\lambda_4+1\right) + \frac{6}{\lambda_3^2\lambda_4^2}\beta\left(2\lambda_3+1, 2\lambda_4+1\right) - \frac{4}{\lambda_3\lambda_4^3}\beta\left(\lambda_3+1, 3\lambda_4+1\right) + \\
&\quad \frac{1}{\lambda_4^4}\beta\left(1, 4\lambda_4+1\right) \\
&= \frac{1}{\lambda_3^4(4\lambda_3+1)} + \frac{1}{\lambda_4^4(4\lambda_4+1)} - \frac{4}{\lambda_3^3\lambda_4}\beta\left(3\lambda_3+1, 2\lambda_4+1\right) - \frac{4}{\lambda_3\lambda_4^3}\beta\left(\lambda_3+1, 3\lambda_4+1\right) + \\
&\quad \frac{6}{\lambda_3^2\lambda_4^2}\beta\left(2\lambda_3+1, 2\lambda_4+1\right) = \nu_4(\lambda_3, \lambda_4).
\end{aligned}$$

From the above observe that the third and fourth central moments μ_3, μ_4 of random variable X are only functions of λ_3, λ_4. Therefore, the procedure is to determine λ_3, λ_4 jointly to target μ_3, μ_4 under the parameter restriction $\min(\lambda_3, \lambda_4) > -\frac{1}{4}$. Next, we can successively determine λ_2 from targeting μ_2 and, finally, λ_1 by targeting μ_1.

S.B A Numerical Example of the Two-Period Model

In this supplementary appendix, we present a quantitative illustration of the two-period model in order to show that higher-order income risk (in logs) may indeed lead to lower precautionary savings and utility gains. Specifically, we consider three different parameterizations of discrete PDFs $\Psi(\varepsilon)$ based on Proposition S.B.1: NORM is a symmetric distribution with a kurtosis of $\alpha_4 = 3$ as for a normal distribution. Distribution LK is also symmetric but strongly leptokurtic with a kurtosis of $\alpha_4 = 30$, and distribution LKSW additionally introduces left-skewness of $\alpha_3 = -5$. For all distributions we set the variance $\mu_2^\varepsilon = 0.5$. Throughout we normalize such that $\mathbb{E}[\exp(\varepsilon)] = 1$. To investigate the role of higher-order risk attitudes we consider two parametrizations with $\theta \in \{1, 4\}$. Throughout, we set the IES γ equal to 1, thus we focus on risk sensitive preferences.

S.B.1 Shocks

The shock ε in this two-period model is taken to be discrete. Specifically, we consider a simple lottery such that $\varepsilon \in \{\varepsilon_l, \varepsilon_0, \varepsilon_h\}$ with $\varepsilon_l < \varepsilon_0 < \varepsilon_h$ and respective probabilities $\{(1 - p) \cdot q, p, (1-p) \cdot (1-q)\}$. This simple structure enables us to derive a parametrization with a closed form representation for the variance, skewness and kurtosis of the shock process, as stated in the following proposition:[2]

Proposition S.B.0 Let $\varepsilon \in \{\varepsilon_l, \varepsilon_0, \varepsilon_h\}$, drawn with respective probabilities $\{(1 - p) \cdot q, p, (1-p) \cdot (1-q)\}$. Then, if and only if $\alpha_4 > 1$ and, for $\alpha_3 \neq 0$ in addition

1. either $\alpha_3 \in (0, \sqrt{\alpha_4 - 1})$
2. or $\alpha_3 \in (-\sqrt{\alpha_4 - 1}, 0)$,

[2] Our approach extends **?**), who analyzes skewness using a two-point distribution, to the fourth moment.

we match $\mu_2, \alpha_3, \alpha_4$, with the normalization $E[\exp(\varepsilon)] = 1$ by choosing

$$q = \frac{1}{2}\begin{cases} +\frac{1}{2}\sqrt{1 - \frac{4\frac{\alpha_4}{\alpha_3^2}-4}{4\frac{\alpha_4}{\alpha_3^2}-3}} & \text{if } \alpha_3 > 0 \\ -\frac{1}{2}\sqrt{1 - \frac{4\frac{\alpha_4}{\alpha_3^2}-4}{4\frac{\alpha_4}{\alpha_3^2}-3}} & \text{if } \alpha_3 < 0 \\ 0.5 & \text{if } \alpha_3 = 0 \end{cases}$$

$$p = \begin{cases} 1 - \frac{(2q-1)^2}{q(1-q)\alpha_3^2} & \text{if } \alpha_3 \neq 0 \\ 1 - \frac{1}{\alpha_4} & \text{if } \alpha_3 = 0 \end{cases}$$

$$\Delta_\varepsilon = \begin{cases} \frac{\sqrt{\mu_2}\alpha_3}{2q-1} & \text{if } \alpha_3 \neq 0 \\ 2\sqrt{\mu_2}\sqrt{\alpha_4} & \text{if } \alpha_3 = 0, \end{cases}$$

and

$$\begin{aligned} \varepsilon_l &= -\ln\left[p\exp\left((1-q)\Delta_\varepsilon\right) + (1-p)\left(q + (1-q)\exp(\Delta_\varepsilon)\right)\right] \\ \varepsilon_0 &= \varepsilon_l + (1-q)\Delta_\varepsilon \\ \varepsilon_h &= \varepsilon_l + \Delta_\varepsilon. \end{aligned}$$

Proof. See Section S.B.5. □

This representation of risk is useful because it enables us to transparently illustrate how higher-order income risk affects the distribution using a very simple structure with a closed-form solution from payoffs to the respective moments of higher-order income risk.

The upper part of Table S.B.1 summarizes the moments for the calibration of ε for these three distributions. The lower part shows how this translates into respective moments in level of the innovation, $\exp(\varepsilon)$. Going from distribution NORM to distribution LK we observe that not only the kurtosis increases strongly but also the variance. Simultaneously, the distribution becomes more skewed to the right. Thus, whether the higher kurtosis of the innovation ε also leads to welfare losses (or a strong increase in precautionary savings) depends on whether the effects on the variance and kurtosis dominate those on the skewness, cf. equations (1) and (2).

In turn, going from distribution NORM to distribution LKSW we observe that the distribution is now more skewed to the left and features a higher kurtosis. However, at the same time, the variance goes down quite strongly. Thus, whether the simultaneously higher kurtosis and lower skewness (or: increased left-skewness) of the innovation ε relative to distribution NORM lead to welfare losses (or a strong increase in precautionary savings) depends on whether the effects on the skewness and kurtosis dominate those on the variance, again see equations (1) and (2).

Table S.B.1: 2-Period Model: Shocks, standardized moments

Moments of Innovation in Logs, ε			
	μ_2^{ε}	α_3^{ε}	α_4^{ε}
NORM	0.5	0	3
LK	0.5	0	30
LKSW	0.5	-5	30
Moments of Innovation in Levels, $\exp(\varepsilon)$			
	$\mu_2^{\exp(\varepsilon)}$	$\alpha_3^{\exp(\varepsilon)}$	$\alpha_4^{\exp(\varepsilon)}$
NORM	0.5868	1.4885	3.7882
LK	11.6316	7.5458	57.9669
LKSW	0.1039	0.5684	4.8371

Notes: Standardized moments of the discrete shock distribution.

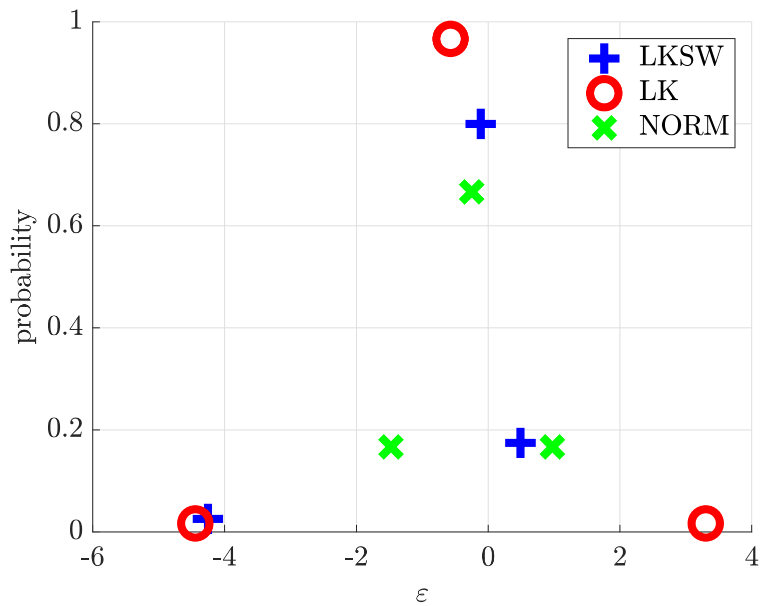
Table S.B.2: 2-Period Model: Shocks, central moments

Moments of Innovation in Logs, ε			
	μ_2^{ε}	μ_3^{ε}	μ_4^{ε}
NORM	0.5	0	0.75
LK	0.5	0	7.5
LKSW	0.5	-1.7678	7.5
Moments of Innovation in Levels, $\exp(\varepsilon)$			
	$\mu_2^{\exp(\varepsilon)}$	$\mu_3^{\exp(\varepsilon)}$	$\mu_4^{\exp(\varepsilon)}$
NORM	0.5868	0.6691	1.3045
LK	11.6316	299.3406	7842.5727
LKSW	0.1039	0.0190	0.0523

Notes: Central moments of the discrete shock distribution.

Figure S.B.1 plots the the corresponding PDFs $\Psi(\varepsilon)$. Relative to NORM, the distribution LK leads to a fanning out of the shocks. As can be seen for the realization of $\exp(\varepsilon_0)$ this induces a shift of the shock realizations to the left such that $\mathbb{E}[\varepsilon]$ is reduced from -0.24 to -0.57. Moving from distribution LK to distribution LKSW by additionally introducing skewness shifts the probability mass to the left tail such that $\mathbb{E}[\varepsilon]$ increases to -0.11. From this observation we know from Proposition 1 that with logarithmic utility ($\theta = 1$), we have welfare losses from the symmetric and leptokurtic distribution LK and welfare gains for the additionally left skewed distribution LKSW if households do not have access to a savings technology.

Figure S.B.1: Distribution of ε



Notes: Distribution function of the discrete shock with three points as in Proposition S.B.1 under the three scenarios NORM, LK, and LKSW.

S.B.2 Allocations

Table S.B.3 reports results on allocations, assuming that households have access to a savings technology. Increasing risk attitude coefficient θ leads to more precautionary savings and reduces the differences in precautionary savings across scenarios. Holding θ constant, compared to the distribution NORM we observe more precautionary savings for distribution LK and thus the effects of increased variance and kurtosis dominate the effects of higher skewness. In contrast, with θ constant we observe less precautionary savings for distribution LKSW and thus the effects of the lower variance dominate the effects of higher kurtosis and left-skewness.

Table S.B.4 displays the welfare consequence if there is no access to a savings technology under a binding budget constraint in column NST and with access in column ST. First, with $\theta = 1$, the distribution LKSW leads to utility gains. Thus, for our shock parametrization, the positive welfare effects of lower skewness dominate the losses of an increased kurtosis. This is true for both scenarios NST, cf. Proposition 1, as well as for scenario ST. Second, under NST utility consequences are strongly increasing in θ, as we learned from equation (1). Third, both gains and losses decrease in scenario ST compared to scenario NST. The rea-

Table S.B.3: Results from 2-Period Model: Allocations

	c_0	$\mathbb{E}[c_1]$	a_1
Risk Aversion, $\theta = 1$			
NORM	0.837	1.162	0.162
LK	0.773	1.226	0.226
LKSW	0.895	1.104	0.104
Risk Aversion, $\theta = 4$			
NORM	0.671	1.328	0.328
LK	0.662	1.337	0.337
LKSW	0.614	1.385	0.385

Notes: Allocations in the two-period model.

son is the precautionary savings response, which reduces utility losses from risk in both the denominator and the numerator of the CEV calculation. Fourth, as a consequence of the precautionary savings response, absolute values of the CEV are lower with higher risk aversion in scenario ST. This shows that the utility consequences of higher-order risk, expressed in terms of CEVs, may be non-monotonic in the degree of risk aversion.

Table S.B.4: Results from 2-Period Model: CEV

	NST	ST
Risk Aversion, $\theta = 1$		
LK	-14.82%	-11.75%
LKSW	7.03%	6.76%
Risk Aversion, $\theta = 4$		
LK	-66.20%	-3.22%
LKSW	-65.35%	5.66%

Notes: CEV relative to NORM. NST: no access to savings technology. ST: assess to savings technology.

S.B.3 Decomposition of Consumption Equivalent Variations

Table S.B.5 reports the results for the decomposition of the CEV, for sake of brevity only for $\theta = 1$ and with access to a savings technology (ST). With this calibration, most of the changes appear in the cross-sectional distribution effect.

Table S.B.5: Results from 2-Period Model: Decomposition of CEV for Log Utility

CEV	g_c	g_c^{mean}	g_c^{lcd}	g_c^{csd}
Baseline				
LK	-11.75%	0	-2.35%	-9.40%
LKSW	6.76%	0	2.16%	4.59%
Impatience				
LK	-11.04%	0	-9.82%	-1.22%
LKSW	-4.10%	0	-10.50%	6.40%
Positive Interest Rate				
LK	-5.56%	2.65%	-4.92%	-3.30%
LKSW	1.70%	2.63%	-4.85%	3.93%
Borrowing Constraint				
LK	-5.04%	0.34%	-0.65%	-4.73%
LKSW	2.26%	0.13%	-0.26%	2.38%

Notes: CEV relative to NORM for $\theta = 1, \rho = 1$ for scenario ST. LK: leptokurtik distribution, LKSW: leptokurtik and skewed distribution.

S.B.4 Additional Model Elements

For the remaining exercises we add step by step model elements included in the quantitative model. Throughout, we take $\theta = \frac{1}{\rho} = 1$ and only analyze the welfare consequences in terms of the consumption equivalent variation. Results are contained in the remaining rows of Table S.B.5 .

Impatience. We first add a period discount factor β of 0.96, such that the discount factor accounting for the 40-year periodicity is $0.96^{40} \approx 0.19$. This introduces a life-cycle savings motive into the model and preferences now write as (for $\rho \neq 1$)

$$U = \frac{1}{1-\rho}\left((1-\tilde{\beta})c_0^{1-\frac{1}{\rho}} + \tilde{\beta}v\left(c_1, \theta, \Psi\right)^{1-\frac{1}{\rho}}\right),$$

where $\tilde{\beta} = \frac{\beta}{1+\beta}$ and β is the raw time discount factor. As a consequence of discounting, the life-cycle distribution effect becomes more potent. Households now take on debt to finance consumption when young. Given the riskiness of second period consumption, borrowing is much lower in distributions LK and LKSW than in distribution NORM. Therefore, the life-cycle distribution effect is strongly negative.

Positive Returns. Next, we also assume a positive interest rate on savings with an annual raw interest rate of 2%. Given the length of each model period of 40 real life years, this corresponds to $R = 1.02^{40} \approx 2.2$. Thus, the budget constraints now write as

$$a_1 = y_0 - c_0, \qquad c_1 \leq a_1 \cdot R + y_1.$$

Table S.B.5 shows that now the mean effect is non-zero. The reason is that savings are inter-temporally shifted at a non-zero rate so that average consumption increases. Results also show that the aforementioned life-cycle effects are muted. Still the life-cycle distribution effects are negative.

Borrowing Constraints. Next, we add occasionally binding borrowing constraints at zero borrowing, i.e., we add the constraint

$$a_1 \geq 0.$$

For the chosen parametrization this constraint turns out to be binding only in scenario NORM. Since households are thus worse off in NORM relative to the other scenarios, welfare losses in distribution LK decrease and gains in distribution LKSW increase.

Throughout all these scenarios, we observe that the cross-sectional distribution effect is negative in scenario LK, and positive in scenario LKSW.

S.B.5 Proof of Proposition S.B.1

Proof. Take $\varepsilon_0 = \mu_1$, thus

$$\begin{aligned} \mu_1 &= p\varepsilon_0 + (1-p)\left(q\varepsilon_l + (1-q)\varepsilon_h\right) \\ &= p\mu_1 + (1-p)\left(q\varepsilon_l + (1-q)\varepsilon_h\right) \\ \Leftrightarrow \quad \mu_1 &= q\varepsilon_l + (1-q)\varepsilon_h. \end{aligned}$$

Now, let $\varepsilon_h = \varepsilon_l + \Delta_\varepsilon$ to get

$$\begin{aligned} \mu_1 &= q\varepsilon_l + (1-q)\left(\varepsilon_l + \Delta_\varepsilon\right) \\ &= \varepsilon_l + (1-q)\Delta_\varepsilon. \end{aligned}$$

For the variance we get

$$\begin{aligned}
\mu_2 &= (1-p)\left(q(\varepsilon_l-\mu_1)^2+(1-q)(\varepsilon_h-\mu_1)^2\right)\\
&= (1-p)\left(q\left(\varepsilon_l-(\varepsilon_l+(1-q)\Delta_\varepsilon)\right)^2+(1-q)\left(\varepsilon_h-(\varepsilon_l+(1-q)\Delta_\varepsilon)\right)^2\right)\\
&= (1-p)\left(q(1-q)^2+(1-q)q^2\right)\Delta_\varepsilon^2\\
&= (1-p)q(1-q)\Delta_\varepsilon^2.
\end{aligned}$$

For the third central moment μ_3 we get

$$\begin{aligned}
\mu_3 &= (1-p)\left(q(\varepsilon_l-\mu_1)^3+(1-q)(\varepsilon_h-\mu_1)^3\right)\\
&= (1-p)\left(q\left(\varepsilon_l-(\varepsilon_l+(1-q)\Delta_\varepsilon)\right)^3+(1-q)\left(\varepsilon_h-(\varepsilon_l+(1-q)\Delta_\varepsilon)\right)^3\right)\\
&= (1-p)\left(-q(1-q)^3+(1-q)q^3\right)\Delta_\varepsilon^3\\
&= (1-p)q(1-q)\left(-(1-q)^2+q^2\right)\Delta_\varepsilon^3\\
&= (1-p)q(1-q)(2q-1)\Delta_\varepsilon^3
\end{aligned}$$

and we can thus write the skewness α_3 as

$$\alpha_3 = \frac{\mu_3}{\sqrt{\mu_2}^3} = \frac{2q-1}{\sqrt{(1-p)q(1-q)}}.$$

For the fourth central moment μ_4 we get

$$\begin{aligned}
\mu_4 &= (1-p)\left(q(\varepsilon_l-\mu_1)^4+(1-q)(\varepsilon_h-\mu_1)^4\right)\\
&= (1-p)\left(q\left(\varepsilon_l-(\varepsilon_l+(1-q)\Delta_\varepsilon)\right)^4+(1-q)\left(\varepsilon_h-(\varepsilon_l+(1-q)\Delta_\varepsilon)\right)^4\right)\\
&= (1-p)\left(q(1-q)^4+(1-q)q^4\right)\Delta_\varepsilon^4\\
&= (1-p)q(1-q)\left((1-q)^3+q^3\right)\Delta_\varepsilon^4\\
&= (1-p)q(1-q)\left((1-2q+q^2)(1-q)+q^3\right)\Delta_\varepsilon^4\\
&= (1-p)q(1-q)\left(1-3q+3q^2\right)\Delta_\varepsilon^4
\end{aligned}$$

and can therefore write the kurtosis as

$$\alpha_4 = \frac{\mu_4}{\mu_2^2} = \frac{3q^2-3q+1}{(1-p)q(1-q)}.$$

To summarize, the terms we seek to match are

$$\mu_2 = (1-p)q(1-q)\Delta_\varepsilon^2, \tag{S.B.6a}$$
$$\alpha_3 = \frac{2q-1}{\sqrt{(1-p)q(1-q)}}, \tag{S.B.6b}$$
$$\alpha_4 = \frac{3q^2-3q+1}{(1-p)q(1-q)}. \tag{S.B.6c}$$

To obtain $\alpha_4 > 0$ we require $p \in (0,1)$, $q \in (0,1)$ and

$$\begin{aligned} & q^2 - q + \frac{1}{3} > 0 \\ \Leftrightarrow \qquad & \left(q - \frac{1}{2}\right)^2 > -\frac{1}{12} \end{aligned}$$

which always holds.

Let us next characterize the solution according to the following case distinction:

1. $\alpha_3 = 0$. Then we obviously have $q = 1 - q = 0.5$. We can accordingly rewrite (S.B.6a) and (S.B.6c) as

$$\mu_2 = (1-p)\frac{1}{4}\Delta_\varepsilon^2,$$
$$\alpha_4 = \frac{1}{(1-p)},$$

   and therefore

$$q = \frac{1}{2}$$
$$p = 1 - \frac{1}{\alpha_4}$$
$$\Delta_\varepsilon = 2\sqrt{\mu_2}\sqrt{\alpha_4}$$

   characterizes the solution. Notice that $\alpha_4 > 0$ and thus $p < 1$. To get $p > 0$ we require

$$1 - \frac{1}{\alpha_4} > 0 \qquad \Leftrightarrow \qquad \alpha_4 > 1.$$

2. $\alpha_3 \neq 0$. From (S.B.6a) we get

$$(1-p)q(1-q) = \frac{\mu_2}{\Delta_\varepsilon^2}$$

Using this in (S.B.6b) and (S.B.6c) we get

$$\alpha_3 = \frac{(2q-1)\Delta_\varepsilon}{\sqrt{\mu_2}}, \tag{S.B.7a}$$

$$\alpha_4 = \frac{(3q^2-3q+1)\Delta_\varepsilon^2}{\mu_2}. \tag{S.B.7b}$$

Now use (S.B.7a) in (S.B.7b) to get

$$\begin{aligned}
& \frac{(3q^2-3q+1)}{(2q-1)^2} = \frac{\alpha_4}{\alpha_3^2} \\
\Leftrightarrow \quad & (3q^2-3q+1) = \frac{\alpha_4}{\alpha_3^2}\left(4q^2-4q+1\right) \\
\Leftrightarrow \quad & q^2\left(4\frac{\alpha_4}{\alpha_3^2}-3\right) - q\left(4\frac{\alpha_4}{\alpha_3^2}-3\right) + \frac{\alpha_4}{\alpha_3^2} - 1 = 0 \\
\Leftrightarrow \quad & q^2 - q + \frac{\frac{\alpha_4}{\alpha_3^2}-1}{4\frac{\alpha_4}{\alpha_3^2}-3} = 0
\end{aligned}$$

and thus

$$q_\pm = \frac{1}{2} \pm \frac{1}{2}\sqrt{\underbrace{1 - \frac{4\frac{\alpha_4}{\alpha_3^2}-4}{4\frac{\alpha_4}{\alpha_3^2}-3}}_{=\Psi}} \tag{S.B.8}$$

Thus, the first restriction for $q_\pm \in (0,1)$ is that $\Psi > 0$. Consider the following case distinction:

(a) $4\frac{\alpha_4}{\alpha_3^2} - 3 > 0 \Leftrightarrow \frac{\alpha_4}{\alpha_3^2} > \frac{3}{4}$: Then

$$\begin{aligned}
& 1 - \frac{4\frac{\alpha_4}{\alpha_3^2}-4}{4\frac{\alpha_4}{\alpha_3^2}-3} > 0 \\
\Leftrightarrow \quad & 4\frac{\alpha_4}{\alpha_3^2} - 3 > 4\frac{\alpha_4}{\alpha_3^2} - 4 \\
\Leftrightarrow \quad & 4 > 3
\end{aligned}$$

and thus for $\frac{\alpha_4}{\alpha_3^2} > \frac{3}{4}$ we get $\Psi > 0$.

(b) $4\frac{\alpha_4}{\alpha_3^2} - 3 < 0 \Leftrightarrow \frac{\alpha_4}{\alpha_3^2} < \frac{3}{4}$ then we obviously get a contradiction.

Thus, we require $\alpha_4 > \frac{3}{4}\alpha_3^2$.

Next, for both the positive and the negative root, we further require $\Psi < 1$. Again investigate the case $\alpha_4 > \frac{3}{4}\alpha_3^2$. We get

$$\begin{aligned}
&1 - \frac{4\frac{\alpha_4}{\alpha_3^2} - 4}{4\frac{\alpha_4}{\alpha_3^2} - 3} < 1 \\
\Leftrightarrow \quad & \frac{4\frac{\alpha_4}{\alpha_3^2} - 4}{4\frac{\alpha_4}{\alpha_3^2} - 3} > 0 \\
\Leftrightarrow \quad & 4\frac{\alpha_4}{\alpha_3^2} - 4 > 0 \\
\Leftrightarrow \quad & \frac{\alpha_4}{\alpha_3^2} > 1
\end{aligned}$$

and thus a necessary and sufficient condition for $q_\pm \in (0, 1)$ is:

$$\alpha_4 > \alpha_3^2. \tag{S.B.9}$$

Since $\alpha_3 = \frac{(2q-1)\Delta_\varepsilon}{\sqrt{\mu_2}}$ and since $\Delta_\varepsilon > 0$ (by construction) and $\sqrt{\mu_2} > 0$ we choose the positive root $q^\star = q_+$ for a right-skewed distribution with $\alpha_3 > 0$ and the negative root $q^\star = q_-$ to model a left-skewed with $\alpha_3 < 0$.

We next get from (S.B.7a) that

$$\Delta_\varepsilon = \frac{\sqrt{\mu_2}\alpha_3}{2q^\star - 1}$$

and from (S.B.6a) that

$$p = 1 - \frac{\mu_2}{q^\star(1-q^\star)\Delta_\varepsilon^2} = 1 - \frac{(2q^\star - 1)^2}{q^\star(1-q^\star)\alpha_3^2}. \tag{S.B.10}$$

We have already established that under condition (S.B.9) $q^\star \in (0, 1)$. Next, we need to establish conditions such that $p \in (0, 1)$. From (S.B.10) we observe that $q^\star \in (0, 1)$ gives $p < 1$. Also observe that $p > 0$ is equivalent to

$$\alpha_3^2 > \frac{(2q^\star - 1)^2}{q^\star(1-q^\star)} \tag{S.B.11}$$

(a) Case $\alpha_3 < 0$: Recall that for this case we take the negative root $q_-^\star$, where

$$q_-^\star = \frac{1}{2} - \frac{1}{2}\sqrt{\Psi} > 0.$$

for $\Psi \in (0, 1)$ iff $\alpha_4 > \alpha_3^2$. Thus the case $\alpha_3 < 0$ implies that $\alpha_3 > -\sqrt{\alpha_4}$. Next observe that

$$(2q^\star - 1)^2 = (1 - \sqrt{\Psi} - 1)^2 = \Psi$$

and

$$\begin{aligned} q^\star(1-q^\star) &= \left(\frac{1}{2} - \frac{1}{2}\sqrt{\Psi}\right)\left(\frac{1}{2} + \frac{1}{2}\sqrt{\Psi}\right) \\ &= \frac{1}{4} - \frac{1}{4}\Psi = \frac{1}{4}\left(1-\Psi\right). \end{aligned}$$

Thus condition (S.B.11) can be rewritten as

$$\begin{aligned} & \alpha_3^2 > \frac{(2q^\star-1)^2}{q^\star(1-q^\star)} = \frac{4\Psi}{1-\Psi} \\ \Leftrightarrow \quad & \alpha_3^2(1-\Psi) > 4\Psi \\ \Leftrightarrow \quad & \alpha_3^2 \frac{\frac{4\alpha_4}{\alpha_3^2} - 4}{4\frac{\alpha_4}{\alpha_3^2} - 3} > 4\left(1 - \frac{\frac{4\alpha_4}{\alpha_3^2} - 4}{4\frac{\alpha_4}{\alpha_3^2} - 3}\right) \\ \Leftrightarrow \quad & \alpha_3^2\left(\frac{\alpha_4}{\alpha_3^2} - 1\right) > 4\frac{\alpha_4}{\alpha_3^2} - 3 - \left(4\frac{\alpha_4}{\alpha_3^2} - 4\right) \\ \Leftrightarrow \quad & \alpha_4 - \alpha_3^2 > 1 \\ \Leftrightarrow \quad & \alpha_3 > -\sqrt{\alpha_4 - 1}, \text{ since } \alpha_3 < 0 \end{aligned}$$

which also implies that we require $\alpha_4 > 1$. Since $-\sqrt{\alpha_4 - 1} > -\sqrt{\alpha_4}$ we thus obtain as a necessary and sufficient condition for the case $\alpha_3 < 0$

$$\alpha_4 > 1 \text{ and } \alpha_3 > -\sqrt{\alpha_4 - 1} \tag{S.B.12}$$

to get $q \in (0, \frac{1}{2})$, $p \in (0,1)$ and $\Delta\epsilon > 0$.

(b) Case $\alpha_3 > 0$: Recall that for this case we take the positive root $q_+^\star$ where

$$q_+^\star = \frac{1}{2} + \frac{1}{2}\sqrt{\Psi} > 0.$$

for $\Psi \in (0,1)$ iff $\alpha_4 > \alpha_3^2$ and thus $\alpha_3 < \sqrt{\alpha_4}$. Thus

$$(2q^\star - 1)^2 = \Psi$$

and

$$
\begin{aligned}
q^\star(1-q^\star) &= \left(\frac{1}{2}+\frac{1}{2}\sqrt{\Psi}\right)\left(\frac{1}{2}-\frac{1}{2}\sqrt{\Psi}\right) \\
&= \frac{1}{4}-\frac{1}{4}\Psi = \frac{1}{4}\left(1-\Psi\right).
\end{aligned}
$$

and following the steps above we thus get

$$
\begin{aligned}
& \alpha_4 - \alpha_3^2 > 1 \\
\Leftrightarrow \qquad & \alpha_3 < \sqrt{\alpha_4 - 1},
\end{aligned}
$$

Since $\sqrt{\alpha_4 - 1} < \sqrt{\alpha_4}$ we thus obtain as a necessary and sufficient condition for the case $\alpha_3 > 0$

$$
\alpha_4 > 1 \;\text{ and }\; \alpha_3 < \sqrt{\alpha_4 - 1} \tag{S.B.13}
$$

to get $q \in (\frac{1}{2}, 1)$, $p \in (0, 1)$ and $\Delta\epsilon > 0$.

Finally, for ε_l given, the mean of the exponent of the random variable x is given by

$$
\begin{aligned}
E\left[\exp(x)\right] &= p\exp\left(\varepsilon_l + (1-q)\Delta_\varepsilon\right) + (1-p)\left(q\exp\left(\varepsilon_l\right) + (1-q)\exp\left(\varepsilon_l + \Delta_\varepsilon\right)\right) \\
&= \exp(\varepsilon_l)\left[p\exp\left((1-q)\Delta_\varepsilon\right) + (1-p)\left(q + (1-q)\exp\left(\Delta_\varepsilon\right)\right)\right].
\end{aligned}
$$

Normalizing $E\left[\exp(x)\right] = 1$ we thus get

$$
\varepsilon_l = -\ln\left[p\exp\left((1-q)\Delta_\varepsilon\right) + (1-p)\left(q + (1-q)\exp\left(\Delta_\varepsilon\right)\right)\right].
$$

□