Deciding on what to Decide

Salvador Barberà
Anke Gerber

This version: February 2021

(June 2017)

Barcelona GSE Working Paper Series

Working Paper nº 973

Deciding on what to Decide*

Salvador Barberà† Anke Gerber‡

February 6, 2021

Abstract

We study two-stage collective decision-making procedures involving a voting body. In a first stage, part of the voters are in charge of deciding what issues will be debated and which ones will not be discussed and remain unsettled. In the second stage, the whole set of voters decides on the positions to be adopted regarding the issues that are in the agenda. Using a protocol-free equilibrium concept introduced by Dutta et al. (2004), we study the tension between two forces that the agenda-setters must weigh. One is the desire to include in the agenda those issues on which they could get society to adopt their preferred positions. The other is their fear that including some favorable issues might induce other agenda setters to add further issues to the agenda, on which the social position might not be favorable to their interests. Our analysis concentrates on societies that use two salient classes of voting procedures: sequential rules and voting by quota. For these rules we show that essentially any set of issues can

*The paper was previously circulated under the title "A Shut Mouth Catches No Flies: Consideration of Issues and Voting." The authors thank Jon Eguia, Matthew Jackson and three anonymous referees for valuable comments.

†MOVE, Universitat Autònoma de Barcelona and Barcelona GSE, Facultat d'Economia i Empresa, Edifici B, 08193 Bellaterra (Barcelona), Spain; e-mail: salvador.barbera@uab.cat. Salvador Barberà acknowledges financial support under grants ECO2014-53051-P, 2014SGR-515 and the Severo Ochoa Programme.

‡Department of Economics, Universität Hamburg, Von-Melle-Park 5, 20146 Hamburg, Germany; e-mail: anke.gerber@uni-hamburg.de. Anke Gerber acknowledges financial support by MOVE for a research stay at the Universitat Autònoma de Barcelona.

be obtained as an equilibrium agenda. We also discover that the power of the chair to manipulate sequential rules may go as far as to be able to avoid certain issues to get to the floor. Moreover, limited changes in the preferences of the agenda setters may result in dramatic changes in the size and composition of agendas.

Keywords: Issues, agendas, voting rules, equilibrium collections of continuation agendas
JEL-Classification: D71, D72.

1 Introduction

Legislative bodies in charge of elaborating legal texts on a broad subject typically narrow down the set of issues that will go to the floor for a vote, while leaving others out of the text, for later resolution. The decision whether to submit an issue to vote is often delegated to a subcommittee, while in other cases any legislator may have a say regarding the list. But the final stand on those issues that get to the floor is in the hands of the plenary. Similarly, the present members of clubs or learned societies vote on the admission or rejection of candidates to new membership who end up being on the ballot. Prior to that, a nominating committee may be in charge of selecting who will be in the ballot, or all members may be able to propose names. These are two examples of situations that we model and analyze in this paper.

Let us briefly point at the specific questions that we address and the reasons why we believe our study improves upon previous knowledge. We state them in terms of legislative bodies, but similar remarks apply in the case of elections in clubs. One can hardly think about any piece of legislation that treats all potential issues regarding the matter it regulates. There are may reasons for that. Even private contracts are most often incomplete, leaving room for interpretation and later resolution of aspects that are not explicitly contemplated in the initial agreement. This is also the case for laws approved in parliaments, including the most fundamental ones: the constitutions of different countries refer to a highly variable number of issues. Indeed, constitutionalists have discussed at length the question of issue deferral, whereby the writers of constitutions refrain from giving constitutional status to certain issues, thus availing future generations with different levels of flexibility regarding the treatment of those issues that are not definitely stamped as being constitutional.[1] The variability in the size of agendas can be attributed to different causes, and depend on the problem at hand. In the case of constitutions, Dixon and Ginsburg (2011) discuss several important reasons for issue deferral. One good reason is the cost of reaching agreements among parties holding extreme views, another is the possibility of making errors that may impose large costs on society at later historical stages. Each one of these general reasons may be induced by a variety of considerations regarding the nature of each issue (for example, the size of the stakes it involves) or the specific political tradition of each country (for example, regarding the degree of negotiation prior to voting that can be expected among parties).

In this paper we provide a stark model that allows us to concentrate on one of the reasons why agents, in situations like the ones we described above, may prefer to leave the social position on certain issues undecided, while fixing the social position on others. Our model abstracts from many other considerations, and the analysis is essentially focused on the balance between two possibly conflicting forces that drive the actions of each member of the nominating committee, the "agenda setters". Their purpose is to attain a satisfactory text, where the issues on which society rules against the agenda setter's preferred positions weigh less than those on which the agenda setter's position is accepted. But their tendency to propose issues on which the agenda setter's position would be accepted by the voters must be tempered by the fact that their proposal of any issue may precipitate the addition of further issues by other agenda setters, on which the vote could have a negative effect.

[1] A rough additional confirmation that issue deferral is a pervasive phenomenon can also be found in constitutional ranking statistics (http://comparativeconstitutionsproject.org/ccprankings). For example, the percentage of 70 major topics from the Comparative Constitutions Project survey that are included in any given constitution ranges from 0.21 in New Zealand to 0.81 in Zimbabwe, and the number of human rights considered in existing constitutions goes from 2 in Brunei to 99 in Ecuador. Such divergences may partly result from idiosyncratic factors, like the different quality of democracy across countries and historical periods.

Our model starts by distinguishing two separate stages in the collective decision process. In the first stage, a decision is made regarding those issues that will be voted upon, thus forming the agenda, and those that will be left out and not treated. In the second stage, a vote is taken among the different alternatives that are open after the agenda is specified. These alternatives consist in the position adopted by the voting body regarding each of the issues under consideration, along with the implicit decision to keep silent about those that were not considered. Specifically, the first stage of agenda formation describes the choice of issues to be discussed as the result of a sequence of decisions adopted by a set of agenda setters. These agenda setters may consist of the full set of members that will be involved in the second stage vote, or of a subset of the plenary.

As a result of this two-stage structure, agenda setters are assumed to engage in strategic considerations when deciding whether or not to propose the inclusion of an issue in the agenda, and when to do it. We study the process of issue inclusion in the agenda through the use of a solution concept that does not a priori impose any fixed order in the sequence of issues to be proposed or in the order of individuals who can do it. Since our model and our method of analysis are novel in several ways, let us describe where our approach differs from that of other related papers.

First of all, we introduce several elements of realism in the description of voting processes. One is to clearly differentiate between the decisions in the two periods, one of agenda setting and the second of vote. We also distinguish between the issues and the potential alternatives. The latter are the different combinations of decisions about the positions on issues in the agenda, augmented with the decision not to discuss the rest of issues. Previous models treat alternatives as the primitives of the problem, and then allow for agents to add an alternative to a previous agenda without changing the nature of those that were already in. In our model, expanding the agenda by including an additional issue completely changes the set of alternatives faced by agents, since the size of the vector of issues to be voted on is increased. An essential feature of our model is that by adding issues to an existing agenda, individuals generate irreversible changes in the sets of actions that are available to others. This facilitates the analysis, because the set of available agenda proposals shrinks along history, and the one containing all issues becomes a terminal point. We consider the distinction between issues and alternatives, and its consequences to be a realistic feature of our model, that we think was ignored in previous work. In particular, our results are not based on any cyclical pattern and differ from those obtained in the tradition of McKelvey's chaos results (McKelvey, 1979), and from recent work (see Vartiainen, 2014, and references therein) where appropriate modifications of the Banks set (Banks, 1985), and the introduction of history dependence generate predictions regarding stable agendas.

Secondly, we analyze the interaction between agenda setters by means of a solution concept that we borrow from the work of Dutta et al. (2004). The equilibrium concept that they proposed allows each agenda setter to add issues to agendas at any point, given those already introduced by her or by any other agenda setter. Its main strength is that, unlike in extensive form games, their concept does not assume any a priori established order of moves for the players, and yet is not based on simultaneous play either, as would be the case in normal form games.

Our model is certainly a simplified version of reality. We only admit two different positions on each issue. We postulate preferences of voters about alternatives as primitives, rather than dwelling on the different kinds of reasons why they would prefer each position to another, individually or in combination with others. Moreover, we assume that agenda setters have a precise knowledge of the attitudes of voters regarding each alternative, that they can introduce in the agenda any issue they wish and that once in the agenda issues cannot be taken away from it. These stylized traits allow us to discuss the reasoning of agenda setters regarding what issues to introduce into the agenda and to make some predictions about the expected consequences of their actions. The combination of our modeling decisions and our choice of equilibrium concept allows us to compute equilibrium agendas for each specification of the voters' preferences and the voting rules adopted to decide on the positions about issues in the agenda. In addition, the order in which new issues are added to those already proposed along the agenda formation process is also established as part of the equilibrium. Multiple equilibria may arise, but it is not hard to construct situations with unique equilibria which, in addition, provide interesting remarks regarding the phenomenon of issue deferral. One first and quite expected result is that one can easily construct examples where the full agenda is no equilibrium, and instead issue deferral arises in equilibrium, under several and frequently used voting methods. More interestingly, these examples can be obtained even if we assume that the preferences of voters are constrained to satisfy natural restrictions and that the voting rule is Pareto efficient. This is in contrast with the opposite prediction that obtains in Dutta et al. (2004). In their model, full agendas would arise when Pareto efficient voting rules are used.[2] This sharp contrast is due to our model's ability to separate the concept of an issue from that of an alternative, which are integrated into the same object in their formulation.

Another advantage of our model is that we can exhibit examples for an additional important fact. Not only, as it is natural to expect, will equilibrium agendas vary when voters' preferences change, but these changes can be dramatic, in terms of size and composition, even in response to apparently very local changes in these preferences. This remark fits well with the warnings of noted political scientists regarding the ever-present potential for instability and change in political situations (Riker, 1982, 1993; Cox and Shepsle, 2007). Our analysis also points at an additional form of manipulation that may be in the hands of a chair, as we already announced. Typically, chairs can manipulate sequential voting procedures by adding new alternatives to the list of possible ones (see Moser et al., 2009, and Moser, Fenn et al., 2016) and/or by changing the order of vote once the set of alternatives is given. What we find is that, in addition, the chair may induce agenda setters to change the set of issues (hence the characteristics of the set of available alternatives) under sequential voting procedures by simply announcing the order in which alternatives will be presented for a vote. This type of manipulation could be an additional instrument in the hands of a chairperson, even of one who cannot directly determine what issues or alternatives should be discussed.

Our use of a protocol-free equilibrium concept allows us to also contrast the predictions we obtain against those that could arise under models that might study the same question under a fixed order in the agenda setter' proposals.

[2]More precisely, in Dutta et al. (2004) the equilibrium agendas are such that the voting outcomes are the same as under full agendas.

Without judging the accuracy of each set of results if they were tested against data, we submit the idea that the difference in predictions justifies our challenge to those models that introduce the somewhat artificial assumptions implicit in any protocol.

Our hope is that our model, which lends itself to the computation of equilibria, might be a starting point for a more complex analysis of the issue deferral phenomenon, and eventually help researchers to add complexity to the study of the underlying causes for changes in the voters' preferences which drive our results.

The outline of the paper is the following. Section 2 gives an overview of the related literature. In Section 3 we introduce our model and equilibrium concept. Section 4 presents some examples. In Section 5 we first study sufficient conditions for equilibrium agendas to be full agendas. Then we demonstrate that an uncontroversial issue is not always on the equilibrium agenda and we provide general results about the variability of agendas for two prominent voting procedures. Section 6 concludes. All proofs are in the appendix.

2 Related literature

The literature on agenda formation is rich, and the following overview is necessarily incomplete. The literature on political agendas as reviewed in Baumgartner (2001) is mainly descriptive. Previous theoretical work mostly considers a specific protocol for the agenda formation and the subsequent voting stage. Austen-Smith (1987), Banks and Gasmi (1987), Baron and Ferejohn (1989), Miller, Grofman, and Feld (1990), Duggan (2006) and Penn (2008) all assume that voting takes place only after the agenda has been built. By contrast, Bernheim, Rangel, and Rayo (2006) and Anesi and Seidmann (2014) analyse the case of "real-time" agenda setting, where any proposal is put to an immediate vote against the current default. Chen and Eraslan (2017) study the timing at which the party in power decides to propose the subjects on which to legislate, one at a time, depending on their actual power and on the relative positions of the opposition and the status quo. In Eguia and Shepsle (2015) the bargaining protocol is endogenized and chosen by the members of a legislative assembly before the agenda is formed. While all these papers consider the case of complete information like we do, Godefroy and Perez-Richet (2013) use an incomplete information framework to study how the majority quota used to place alternatives on the agenda affects agents' behavior and hence the likelihood to change the status quo. There are only a few papers that do not rely on a specific bargaining protocol. Among the notable exceptions is Dutta et al. (2004) who study equilibrium agendas in a model with farsighted agents. Their main result is that the set of equilibrium outcomes for Pareto efficient voting rules coincides with the outcomes when all full agendas are considered. Unlike in their paper, and as we shall discuss later, we show that in our model equilibrium agendas may not contain all issues even if the voting rule is Pareto efficient.

Vartiainen (2014) completes a list of works including Anesi (2006) and Bernheim and Slavov (2009), that start from McKelvey's (1979) chaos theorem and add features to the model in order to avoid cycles and to identify stable agendas. The spirit of Vartiainen's work is similar to ours in different aspects, but there are substantial differences as well. His analysis, like ours and Dutta et al. (2004), is based on a protocol-free equilibrium concept that is implicitly history dependent. However, by clearly distinguishing between issues and alternatives, we reduce the strategic possibilities of voters in a different manner. In these other papers, including Vartiainen (2014), voters can propose any new alternative that satisfies appropriate conditions, whereas we limit their actions to proposing issues, which in addition must not have been proposed before and cannot be eliminated from the floor after they have been added to the agenda. While this makes our model specific, it also adds an important element of descriptive realism, and suggests a very different technical approach, in the line of Dutta et al. (2004), departing for good reason from the approach based on modifications of the Banks set. Another difference with Vartiainen (2014) is that we assume that individuals can, by themselves, add issues to the floor, whereas in his paper agents can only add alternatives if they reach a majority. While we think our analysis could be modified to require the consensus of more than one agent, the difference still remains that the type of changes our voters can induce on the set of alternatives in the agenda are of a different kind. Hence, although appreciative of the existing literature, we believe our paper contributes a new dimension to previous work.

Finally, let us mention that there is also a related literature that focusses on strategic candidacy (Osborne and Slivinsky, 1996; Besley and Coate, 1997; Dutta et al., 2001, 2002). The main difference with models of agenda formation like ours is that in strategic candidacy problems, the agents who take the agenda formation decision, by choosing whether to run or not to run, are different from those who will eventually vote.

3 The Model

We consider a set of voters $I = \{1, \ldots, n\}$ with $n \geq 2$ and a set of issues $\mathcal{K} = \{1, \ldots, K\}$ with $K \geq 2$. A nonempty subset of voters $J \subset I$ has the power to decide on which issues the group I will be silent, thus leaving the position on these issues undefined, and on which issues the group I will take a decision, in which case one of two positions will be voted upon and adopted for each one of those. The members of J are called *agenda setters.* A special case is the one with $J = I$ where all voters are agenda setters. We will describe the agenda setting process below.

We denote by "$-$" the decision to leave an issue out of the voting floor, and by 0 and 1 the two possible positions on issues that are voted upon. Social alternatives are then K-tuples indicating, for each issue, whether or not it was the object of a vote, and, if so, which stand was adopted on it. Accordingly, the set of alternatives then is given by $X = \{0, 1, -\}^{\mathcal{K}}$.

Voters are endowed with strict preference orderings on X.[3] Let $\mathcal{P}$ denote the set all strict preference orderings and let $\succ_i \in \mathcal{P}$ be voter i's preference ordering.

We consider a two-stage decision making process. In the first stage agents in J decide which issues to bring to the floor. The result is an agenda, which not only records the set of issues that have been proposed by the agents, but also the order in which the issues have been proposed. After that, in the second stage, agents in I cast an irreversible vote about the position on each issue of the agenda. Depending on the voting rule, agents may directly vote on the positions for all issues one after the other or they may vote simultaneously on the positions

[3] A *strict preference ordering* is a complete, asymmetric and transitive binary relation on X.

for all issues using a sequential voting procedure on the set of alternatives.[4]

Agendas

Let $1 \leq m \leq K$. An *agenda of length* m is a finite vector of issues $a = (a_1, \ldots, a_m)$ with $a_k \in \mathcal{K}$ for $k = 1, \ldots, m$, and $a_k \neq a_l$ for $k \neq l$. The empty agenda $\varnothing$ where no issue is put to vote is defined to have length 0. By A^m we denote the set of agendas of length m, where $0 \leq m \leq K$, and by $A = \bigcup_{m=0}^{K} A^m$ we denote the set of all agendas.

Let $a \in A$. If an issue k is not on the agenda a, i.e. $k \notin a$, we call k a *free issue at* a.[5] For $a \in A^m$, where $0 \leq m \leq K-1$, and $k \in \mathcal{K}, k \notin a$, (a, k) denotes the agenda $a' \in A^{m+1}$ with $a'_l = a_l$ for $l = 1, \ldots, m$, and $a'_{m+1} = k$.

Once a given set of issues constitute the agenda, the only alternatives that may be attained after voting are those where society chooses either value 0 or 1 on those issues and remains non-committed on the remaining ones. Accordingly, we define the available set of alternatives at agenda a to be the union of all those alternatives that may be potentially chosen when the agenda is a, depending on the preferences of agents. Thus, for a given agenda $a \in A$ the set of *available alternatives* at a, $X(a)$, is given by

$$X(a) = \{x \in X \,|\, \text{for all } k \in \mathcal{K}, x_k \in \{0, 1\} \text{ if and only if } k \in a\}.$$

Observe that $X(\varnothing) = \{(-, \ldots, -)\}$.

Voting

A voting procedure specifies what alternative is chosen as a function of the agenda and the preferences of agents over alternatives. Formally, a voting procedure on some domain of preference profiles $\mathcal{D} \subset \mathcal{P}^n$ is a mapping $V : A \times \mathcal{D} \to X$ with $V(a, P) \in X(a)$ for all $a \in A$ and $P \in \mathcal{D}$. Notice that a voting procedure, in our definition, associates a single outcome to each preference profile and each agenda a. Also, observe that the voting procedure need not be sensitive to the ordering

[4] In our examples and main results we will consider voting rules that only depend on the set of issues on the agenda, but not on their specific order. Yet, our model also allows for the case where the order of issues plays a role in the second stage.

[5] Here and in what follows, for a given agenda $a = (a_1, \ldots, a_m) \in A$ we write, for short, $k \in a$ ($k \notin a$) whenever $k = a_l$ for some $l \in \{1, \ldots, m\}$ ($k \neq a_l$ for all $l = 1, \ldots, m$).

of issues in a, and may only depend on the set of issues in the agenda. Below we will introduce two prominent voting procedures.

Agenda Formation

In the first stage, starting from the empty agenda each agent in J can unilaterally add issues to those already proposed by her or by others along the creation of the agenda that will eventually prevail. This process stops when either a full agenda $a \in A^K$ is reached or no agent in J wants to add further issues. Instead of modeling the agenda formation as an extensive or normal form game, where equilibrium agendas could potentially be very sensitive to the details of the game form, we follow Dutta et al. (2004) and consider an equilibrium collection of sets of continuation agendas. We first provide some motivating comments and then proceed more formally.

For $a \in A^m$, where $m \in \{0, 1, \ldots, K\}$, let $A(a)$ be the set of *continuation agendas*, i.e.

$$A(a) = \{a' \in A \,|\, a'_k = a_k \text{ for all } k = 1, \ldots, m\}.^6$$

Equilibrium collections of sets of continuation agendas express expectations about the agendas that will result starting from any given agenda a. Accordingly, our equilibrium concept is defined recursively. Since issues are assumed to be added one after the other, expectations at agenda a have to be such that they either do not involve further additions of issues, or else are equilibrium continuations with one further issue added to a (see condition (E1) below). Equilibrium continuations are not required to be unique because without a fixed protocol different agents may initiate different continuation paths. Therefore, when considering to add an issue to a given agenda an agent has to take into account all equilibrium continuations this may lead to. No further additions of issues to an agenda a are expected if and only if no agent in J would be interested in adding any additional issue after having reached a, in view of what the expected continuations would be (see condition (E2) below).

[6] Observe that $a \in A(a)$ for all $a \in A$, i.e. any agenda is a continuation agenda for itself.

Definition 3.1 *Let $P = (\succ_1, \ldots, \succ_n) \in \mathcal{D}$. An* ***equilibrium collection of sets of continuation agendas*** *is a collection $(CE(a,P))_{a \in A}$, where $CE(a,P) \subset A(a)$ for each $a \in A$, that satisfies the following two conditions for all $a \in A$:*

(E1) *$CE(a,P)$ is a nonempty subset of $\bigcup_{k \notin a} CE((a,k),P) \cup \{a\}$.*

(E2) *$a \in CE(a,P)$ if and only if $V(a,P) \succ_i V(a',P)$ for all $a' \in \bigcup_{k \notin a} CE((a,k),P)$ and for all $i \in J$.*[7]

In what follows, for a given equilibrium collection of sets of continuation agendas $(CE(a,P))_{a \in A}$, we will refer to the continuation agendas in $CE(a,P)$ for $a \in A$ as *equilibrium continuations.*

We record the following result which is a straightforward implication of condition (E1): If an agenda a^* is an equilibrium continuation at some agenda a, then it is an equilibrium continuation at every agenda along the path from a to a^*.

Lemma 3.1 *Let $V : A \times \mathcal{D} \to X$ be a voting procedure and let $(CE(a,P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas for some $P \in \mathcal{D}$. If $a^* = (a_1, \ldots, a_m) \in CE((a_1, \ldots, a_l), P)$ for some $l \le m \le K$, then*

$$a^* \in CE((a_1, \ldots, a_k), P) \text{ for all } k = l, \ldots, m.$$

In particular, a^* then is an element of $CE(a^*, P)$.

Observe that (E2) is a rather weak stopping requirement because an agent is assumed to stop adding issues to the agenda only if stopping is better than all equilibrium continuations reached when one further issue is added to the existing agenda. Nevertheless, as we will show, equilibrium continuations are not necessarily full agendas, even for restricted domains of preferences and very well behaved voting procedures.

[7]Notice that $V(a',P) \neq V(a,P)$ for all $a' \in \bigcup_{k \notin a} CE((a,k),P)$ since any such agenda a' contains at least one issue $k \notin a$ which implies that $(V(a,P))_k \neq (V(a',P))_k$.

In order to reduce the potential multiplicity of equilibrium collections of sets of continuation agendas we impose a third condition, which we call *consistency* (cf. Dutta et al., 2004). To this end, for $a \in A$ we define an agenda $a' = (a, k, \ldots) \in A$ to be *rationalizable* (relative to a) if $a' \in CE((a,k), P)$ and there exists an agent $i \in J$ and $a'' \in CE(a, P)$ with either $a'' = (a, l, \ldots)$ with $l \neq k$ or $a'' = a$ such that $V(a', P) \succ_i V(a'', P)$. Hence, the continuation agenda $(a, k, \ldots)$ is rationalizable relative to a if it is an equilibrium continuation at (a, k) and if some agent can gain from reaching it rather than sticking to some other equilibrium continuation at a.

Definition 3.2 *Let $P \in \mathcal{D}$. An equilibrium collection of sets of continuation agendas $(CE(a,P))_{a \in A}$ is* ***consistent*** *if it satisfies the following condition:*

(E3) *If $a' \in \bigcup_{l \notin a} CE((a,l), P)$ is rationalizable, then $a' \in CE(a, P)$. Conversely, if $a' = (a, k, \ldots) \in CE(a, P)$ and either $a \in CE(a, P)$ or $a'' = (a, l, \ldots) \in CE(a, P)$ for some $l \neq k$, then a' is rationalizable.*

Thus, consistency requires that an equilibrium collection of sets of continuation agendas contains all rationalizable continuation agendas. Moreover, it only contains rationalizable continuation agendas subject to the following two exceptions: The first is that the agenda a itself is an equilibrium continuation if all agents in J prefer to stop at a (condition (E2)). The second exception is when there is a unique equilibrium continuation $a' = (a, k, \ldots)$ at a which is then not required to be rationalizable. Observe, however, that the latter case only obtains if there is an agent who prefers continuing over stopping at a and if all agents in J unanimously prefer a' to adding an issue different from k to agenda a.

We will be mainly interested in the agendas that would result in equilibrium when agenda formation starts from the empty agenda where all issues are still free. Hence, it is convenient to introduce a terminology for the agendas that are elements of the set of equilibrium continuations at the empty agenda. We call these agendas "equilibrium agendas":

Definition 3.3 *Let $a^* \in A$ and $P \in \mathcal{D}$. Then a^* is a **(consistent) equilibrium agenda at P** if there exists a (consistent) equilibrium collection of sets of continuation agendas $(CE(a,P))_{a\in A}$ with $a^* \in CE(\varnothing, P)$.*

The use of our proposed equilibrium notion poses no existence problems. Moreover, the characterization of families of equilibrium continuation agendas closely follows the steps of a backward induction argument that is quite analogous to the pruning procedure suggested by Arieli and Aumann (2015) for the case of subgame perfect equilibria. Since any full agenda is its own continuation, we can start by asking whether an agenda a that contains all issues but one satisfies the equilibrium requirements. If it does, its continuation full agenda will be pruned. If it does not, then the expectation that a is its own equilibrium continuation is pruned. That leaves us with a family of potential equilibria regarding agendas where at most all issues but one are considered. Then we can continue a similar pruning process for agendas containing all but two issues, and proceed in a similar manner until we reach the case where the agenda is empty and no issue is put to vote. The family of continuation agendas that survives the pruning process is an equilibrium.

Consistent equilibrium agenda vs. subgame perfect Nash equilibrium

The order of moves of the agenda setters in our protocol-free equilibrium is endogenous and this results in equilibrium agendas that can be very different from those under a fixed order of moves. In particular, it is not true that subgame perfect equilibrium outcomes are always selections from the set of consistent equilibrium agendas. We demonstrate this by the following example where the empty agenda is a subgame perfect equilibrium outcome for some order of moves while any consistent equilibrium agenda contains all but one issue.

Example 3.1 Let there be three issues, i.e. $\mathcal{K} = \{1, 2, 3\}$, and three voters who are also agenda setters. Voters' preferences over alternatives not containing position 0 on any issue are given in Table 1. For all voters the remaining alternatives which have position 0 on at least one issue are ranked below $(1, 1, 1)$. The order of the alternatives below $(1, 1, 1)$ does not matter. So every voter has a different top alternative where exactly two issues are addressed and position 1 is adopted on those issues. This top alternative is preferred over not addressing any issue and all other alternatives are ranked below.

$\succ_1$	$\succ_2$	$\succ_3$
$(1,1,-)$	$(1,-,1)$	$(-,1,1)$
$(-,-,-)$	$(-,-,-)$	$(-,-,-)$
$(1,-,1)$	$(-,1,1)$	$(1,1,-)$
$(-,1,1,)$	$(1,1,-)$	$(1,-,1)$
$(1,-,-)$	$(1,-,-)$	$(-,1,-)$
$(-,1,-)$	$(-,-,1)$	$(-,-,1)$
$(-,-,1)$	$(-,1,-)$	$(1,-,-)$
$(1,1,1)$	$(1,1,1)$	$(1,1,1)$

Table 1: Voters' preference orderings on $\{1,-\}^3$.

Let V be a Pareto efficient voting rule, i.e. $V(a,P)$ is Pareto efficient in $X(a)$ for all $a \in A$ and all P in the domain of V, and let some preference profile P compatible with the requirements in Table 1 be in the domain of V. Let a be an arbitrary agenda. Since all voters strictly prefer position 1 on all issues $k \in a$ over any alternative with position 0 for some $k \in a$ Pareto efficiency implies that for all agendas a

$$(V(a,P))_k = 1 \text{ for all } k \in a.$$

Consider first the extensive form game with fixed order of moves $1,2,3$, which repeats until either a full agenda is reached or there were three passes in a row. Then every voter i anticipates that after she has added one issue to the agenda her successor will add one further issue and then the agenda formation process stops. But the issue that is added by the successor of i is not the one i likes to be added and thus every voter refrains from adding any issue to an empty agenda. Hence, the unique subgame perfect equilibrium agenda is empty.

By contrast, the set of consistent equilibrium agendas consists of all agendas of length 2. To see this note that at any agenda of length 1 there are two consistent equilibrium continuations where one of the remaining issues is added to the agenda. As a consequence, at the empty agenda a voter anticipates that after adding her preferred issue she may also add the other preferred issue which will lead to her top alternative. Hence, the empty agenda is no consistent equilibrium agenda but all agendas of length 2 are consistent equilibrium agendas.

A complete analysis of the example can be found in the appendix.

$\diamond$

Assumptions on Preferences

For later use we introduce specific assumptions on the domain of preferences. The first are two separability properties of preference orderings.

Definition 3.4

(1) A preference ordering $\succ$ on $X = \{0, 1, -\}^{\mathcal{K}}$ is ***separable*** *if for all $k \in \mathcal{K}$, $(x_k, x_{-k}) \succ (y_k, x_{-k})$ for some $x_{-k} \in \{0, 1, -\}^{\mathcal{K} \setminus \{k\}}$ implies that $(x_k, x'_{-k}) \succ (y_k, x'_{-k})$ for all $x'_{-k} \in \{0, 1, -\}^{\mathcal{K} \setminus \{k\}}$.*

(2) A preference ordering $\succ$ on X is ***additively separable*** *on X if there exist utility scalars $u_k(z) \in \mathbb{R}$ for all $k \in \mathcal{K}$, and for all $z \in \{0, 1, -\}$ such that for $x, y \in X$,*

$$x \succ y \iff \sum_{k=1}^{K} u_k(x_k) > \sum_{k=1}^{K} u_k(y_k).$$

Under separability a voter's preference about the positions $1, 0, -$, on any given issue is independent of the positions on the rest of issues. This is a strong regularity assumption and yet we will see that is does not impose any restrictions on the set of equilibrium agendas. Additive separability is an even stronger condition that requires the tradeoff between the positions on two issues k and l to be independent of the positions on all other issues.

Let $\mathcal{S} \subset \mathcal{P}$ be the set of all preference orderings that satisfy separability and let $\bar{\mathcal{S}} \subset \mathcal{S}$ denote the set of all preference orderings that satisfy additive separability.

Under the next condition, that we call *betweenness*, other things being equal, an agent strictly prefers the alternative that takes his preferred position on some issue k over leaving the position open, and she strictly prefers the latter to the alternative where the position is his worse.

Definition 3.5 *A preference ordering $\succ$ on X satisfies* ***betweenness*** *if for all $k \in \mathcal{K}$, and for all $x \in X$, either*

$$(1, x_{-k}) \succ (-, x_{-k}) \succ (0, x_{-k})$$

or

$$(0, x_{-k}) \succ (-, x_{-k}) \succ (1, x_{-k}).$$

Observe that betweenness is compatible with the interpretation that agents perceive the resulting indeterminacy as creating a lottery between the competing positions, to be resolved in the future. Note that betweenness will be satisfied whenever the agent's preference ordering can be represented by an expected utility function such that the utility of $(-, x_{-k})$ is the expected utility of a lottery over the set $\{(0, x_{-k}), (1, x_{-k})\}$, where the agent assigns a positive probability to both outcomes, $(0, x_{-k})$ and $(1, x_{-k})$, that is independent of the corresponding probabilities for other open positions, if any.

Voting Rules

There are many ways in which one can specify voting rules. One of them is to propose a game form that is dominance solvable for each agenda in the sense of Moulin (1979), and to associate to each preference profile the unique Nash equilibrium outcome in undominated strategies of the game induced by that profile and the game form. Another is to associate each agenda and each profile with the result of sincere voting under a sequential rule. In both cases it is well known that the same tree structure may lead to different outcomes depending on the order of vote on alternatives (see Barberà and Gerber, 2017, and references therein), and hence this order must be determined when defining the voting rule.

One prominent sequential voting rule is the *amendment procedure* (Farquharson, 1969; Miller, 1977, 1980) which assumes an exogenously given ordering of the attainable alternatives. If a is an agenda and $(x_1, x_2, \ldots, x_N)$ is the given ordering of the alternatives in $X(a)$, then under the amendment procedure the first vote is over x_1 and x_2, the second vote is over the winner of the first vote and x_3, and so on until all alternatives in $(x_1, x_2, \ldots, x_N)$ are exhausted. In every pairwise vote the winner is selected according to simple majority voting and the winning alternative is the one that survives until the end. When considering the amendment procedure below we will assume that voters are *sophisticated* (Farquharson, 1969; Brams, 1975; Moulin, 1979) and thus the voting outcome under the amendment procedure is given by the alternative chosen in an undominated Nash equilibrium, which we obtain by iterative elimination of weakly dominated strategies, where all weakly dominated strategies of all agents are simultaneously eliminated at each stage. Observe that the voting outcome under iterative elimination of weakly dominated strategies is unique if there is an odd number of agents all with strict preferences (see Moulin, 1979, and Barberà and Gerber, 2017). Moreover, it is well known that the amendment procedure is Pareto efficient (Miller, 1977, 1980; Barberà and Gerber, 2017), but it is not strategy-proof on any domain containing the set of separable preferences (and on the universal domain in particular).

Another prominent voting rule that we will consider in the following is *voting by quota*. For all $a \in A$ and all voters i let $b^i(a) \in X(a)$ be the best available alternative at a according to i's preferences, i.e.

$$b^i(a) \succ_i x \quad \text{for all } x \in X(a), x \neq b^i(a).$$

Let $q \in \{1, \dots, n\}$. Then *voting by quota* q is the voting procedure $V : A \times \mathcal{P}^n \to X$, such that for all $a \in A$, for all $k \in a$, and for all $P \in \mathcal{P}^n$,

$$(V(a,P))_k = \begin{cases} 1, & \text{if } \#\{i \,|\, b^i_k(a) = 1\} \geq q \\ 0, & \text{otherwise} \end{cases} .^{8,9} \tag{1}$$

On the domain of separable preferences $\mathcal{S}^n$ voting by quota is strategy-proof, but it is not Pareto efficient (Barberà et al., 1991). Moreover, if $P \in \mathcal{S}^n$, then the voting outcome on issue $k \in a$ under voting by quota is independent of the other issues in a and on their ordering, i.e. $(V(a,P))_k = (V(a',P))_k$ for any two agendas a, a' with $k \in a$ and $k \in a'$.

If preferences are additively separable, i.e. $P \in \bar{\mathcal{S}}^n$, then (1) is equivalent to

[9] By "#" we denote the number of elements in a set.

[9] Voting by quota is a special case of a larger class of voting procedures, called *voting by committees* (Barberà et al., 1991).

$$(V(a,P))_k = \begin{cases} 1, & \text{if } \#\{i \,|\, u_k^i(1) > u_k^i(0)\} \geq q \\ 0, & \text{otherwise} \end{cases} ,$$

where $(u_k^i(\cdot))_{k\in\mathcal{K}}$ is the collection of scalars in an additively separable utility representation of voter i's preference ordering $\succ_i$.

4 Examples

The following examples illustrate the application of our equilibrium notion for two prominent voting rules, voting by quota and the amendment procedure. The analysis is quite detailed, in order to familiarize the reader with our equilibrium concept. In addition, the examples demonstrate the tradeoff between an agent's incentives to support issues on which a vote would result in her preferred positions, and the need to take into account that adding such issues may trigger further additions to the agenda that end up being harmful to her interests. The reader will find a summary of what we learn from the examples at the end of the section.

In both examples we consider the election of new members to a society. The set of issues $\mathcal{K}$ is the set of candidates. In this case "$-$" means that the corresponding candidate is not nominated, "1" means that the candidate is nominated and elected and "0" means that the candidate is nominated and not elected.

4.1 Voting by Quota

Let there be three candidates, i.e. $\mathcal{K} = \{1, 2, 3\}$, and assume that the society currently has three members and that all members are agenda setters and voters, i.e. $I = J = \{1, 2, 3\}$. Voters have additively separable preferences with utility scalars as given in Table 2.[10]

The voting rule is voting by quota $q = 2$. For each candidate $k \in \{1, 2, 3\}$ there are two voters who prefer to elect candidate k whenever she is nominated.

[10] Note that the utility scalars are such that the resulting preference ordering on the set of alternatives is strict.

i	k	$u^i_k(1)$	$u^i_k(0)$	$u^i_k(-)$
	1	0.04	4.4	3
1	2	4.1	0.01	3
	3	4.2	0.02	3
	1	4.1	0.01	3
2	2	0.04	4.4	3
	3	4.2	0.02	3
	1	4.1	0.01	3
3	2	4.2	0.02	3
	3	0.04	4.4	3

Table 2: Utilities $u^i_k(\cdot)$ of voters $i \in \{1,2,3\}$ for candidates $k \in \{1,2,3\}$.

Hence, the voting outcome is

$$(V(a,P))_k = 1 \text{ for all agendas } a \text{ with } k \in a.$$

The remaining voter prefers not to elect the candidate and the utility for electing the candidate is so low that all voters prefer not to nominate any candidate over nominating all candidates: For all i and for all full agendas a we have that

$$V(\varnothing, P) = (-,-,-) \succ_i V(a,P) = (1,1,1) \tag{2}$$

since

$$\sum_{k=1}^{3} u^i_k(-) = 9 > \sum_{k=1}^{3} u^i_k(1) = 8.34.$$

We now solve for the equilibrium collection of sets of continuation agendas. To do that we proceed backwards starting from full agendas $a \in A^3$.[11] By (E1) we have

$$CE(a,P) = \{a\} \text{ for all } a \in A^3.$$

[11] Recall that A^m is the set of all agendas of length m, where $0 \le m \le K$, and $A = \bigcup_{m=0}^{K} A^m$.

Next consider an agenda $a \in A \setminus A^3$ with $k \in a$ for some $k \in \{1,2,3\}$. We will show that $a \notin CE(a,P)$. By definition of the agents' utility functions voter k gets his most preferred position on all candidates $l \notin a$. To see this, note that all candidates $l \in \{1,2,3\}$ are elected if nominated which is the preferred outcome for voter k if $l \neq k$. Hence,

$$V(a',P) \succ_k V(a,P) \text{ for all } a' \in \bigcup_{l \notin a} CE((a,l),P).$$

By (E2) this implies that $a \notin CE(a,P)$. Lemma 3.1 then implies that

$$CE(a,P) \subset A^3 \text{ for all } a \in A \text{ with } k \in a \text{ for some } k \in \{1,2,3\}. \tag{3}$$

We will now show that $CE(\varnothing,P) = \{\varnothing\}$. To see this, observe that any $a \in CE((k),P)$ for some $k \in \{1,2,3\}$ is a full agenda by (3). Since by (2) $V(\varnothing,P) \succ_i V(a,P)$ for all i and for all full agendas a (E2) implies that $\varnothing \in CE(\varnothing,P)$. Moreover, by (2) and the fact that any $a \in CE((k),P)$ with $k \in \{1,2,3\}$ is a full agenda by (3), we conclude that no $a \in CE((k),P)$ with $k \in \{1,2,3\}$ is rationalizable relative to $\varnothing$. (E3) then implies that $CE(\varnothing,P) = \{\varnothing\}$.

Summary

For the given preferences there is a unique consistent equilibrium agenda, where no candidate is nominated even though for each candidate two voters get their most preferred voting outcome on that candidate. The reason is that as soon as one candidate is nominated the other candidates will be nominated as well and the outcome of the election, namely that all candidates are elected, is worse for all voters than not nominating any candidate.

4.2 Amendment Procedure

Let there be two candidates, i.e. $\mathcal{K} = \{1,2\}$, and again assume that the society currently has three members and all members are agenda setters and voters, i.e. $I = J = \{1,2,3\}$. The voters' preference orderings on the set of alternatives are given in Table 3, where the alternatives in the table are listed in the order of decreasing preference.

$\succ_1$	$\succ_2$	$\succ_3$
$(0,1)$	$(1,0)$	$(1,1)$
$(0,-)$	$(-,0)$	$(-,1)$
$(-,1)$	$(1,-)$	$(0,1)$
$(-,-)$	$(-,-)$	$(1,-)$
$(0,0)$	$(0,0)$	$(-,-)$
$(-,0)$	$(0,-)$	$(0,-)$
$(1,1)$	$(1,1)$	$(1,0)$
$(1,-)$	$(-,1)$	$(-,0)$
$(1,0)$	$(0,1)$	$(0,0)$

Table 3: Preference orderings of voters $1,2,3$.

The voting rule is the amendment procedure. In order to solve for the equilibrium agendas we first determine the voting outcome for any agenda that contains at most one alternative. At the empty agenda the outcome is the unique attainable alternative $(-,-)$, i.e.

$$V(\varnothing, P) = (-,-).$$

At agenda $a = (1)$ the outcome is

$$V((1), P) = (1,-),$$

because a majority of voters prefers $(1,-)$ over $(0,-)$, and at agenda $a = (2)$ the outcome is

$$V((2), P) = (-,1),$$

since a majority of voters prefers $(-,1)$ over $(-,0)$.

It remains to determine the voting outcome at the full agendas, $(1,2)$ and $(2,1)$, with attainable sets

$$X(1,2) = X(2,1) = \{(0,0),(0,1),(1,0),(1,1)\}.$$

Figure 1 shows the dominance relation on $\{(0,0),(0,1),(1,0),(1,1)\}$ that results from pairwise simple majority voting. It follows from the characterizations in

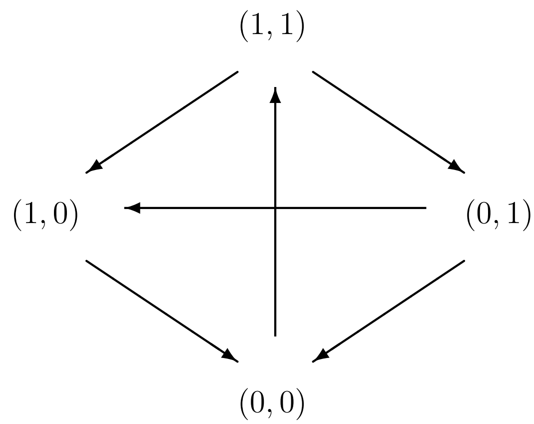

Figure 1: Dominance relation on $\{(0,0),(0,1),(1,0),(1,1)\}$ under pairwise simple majority voting. The arrows point to the alternatives that are beaten under simple majority voting.

Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) that any of the attainable alternatives except for $(1,0)$ is the outcome of sophisticated sequential voting under the amendment procedure for some ordering of the alternatives in $X(1,2) = X(2,1)$.[12] Observe that $(0,0)$ is Pareto dominated by $(-,-)$.

Case 1: $V((1,2),P) = V((2,1),P) = (0,0)$

In order to solve for the equilibrium collection of sets of continuation agendas we proceed backwards starting from the full agendas $(1,2)$ and $(2,1)$. By (E1) it must be that

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

Now consider agenda (1). By condition (E1), $CE((1),P)$ is a nonempty subset of $\{(1),(1,2)\}$. By condition (E2), $(1) \in CE((1),P)$ is ruled out since voter 1 strictly prefers the voting outcome under the equilibrium continuation $CE((1,2),P) = (1,2)$ over the outcome at agenda (1). Hence,

$$CE((1),P) = \{(1,2)\}.$$

[12] This can also be verified directly: The ordering $((0,0),(0,1),(1,0),(1,1))$ yields outcome $(0,0)$, the ordering $((1,1),(1,0),(0,0),(0,1))$ yields outcome $(1,1)$ and the ordering $((0,1),(1,0),(1,1),(0,0))$ yields outcome $(0,1)$. Finally, no ordering gives outcome $(1,0)$.

Next consider agenda (2). By condition (E1), $CE((2),P)$ is a nonempty subset of $\{(2),(2,1)\}$. By condition (E2), $(2) \in CE((2),P)$ is ruled out since voter 2 strictly prefers the voting outcome under the equilibrium continuation $CE((2,1),P) = (2,1)$ over the outcome at agenda (2). Hence,

$$CE((2),P) = \{(2,1)\}.$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing,P)$ is a nonempty subset of $\{\varnothing\} \cup CE((1),P) \cup CE((2),P) = \{\varnothing,(1,2),(2,1)\}$. Since all voters strictly prefer the voting outcome under the empty agenda over the outcome at any full agenda, (E2) implies that $\varnothing \in CE(\varnothing,P)$. We will prove that $\varnothing$ is the unique element in $CE(\varnothing,P)$. Suppose by way of contradiction that $(1,2) \in CE(\varnothing,P)$. Then, since $\varnothing \in CE(\varnothing,P)$, condition (E3) implies that $(1,2)$ is rationalizable relative to the empty agenda $\varnothing$. However, no voter prefers the voting outcome under agenda $(1,2)$ over the outcome at the empty agenda $\varnothing$ or the outcome at agenda $(2,1)$. Hence, $(1,2)$ is not rationalizable which implies that $(1,2) \notin CE(\varnothing,P)$. Similarly, one proves that $(2,1) \notin CE(\varnothing,P)$. Therefore, we conclude that

$$CE(\varnothing,P) = \{\varnothing\}.$$

Summary
If $V((1,2),P) = V((2,1),P) = (0,0)$ the unique consistent equilibrium agenda is empty and no candidate is nominated and elected.

Case 2: $V((1,2),P) = V((2,1),P) \in \{(0,1),(1,1)\}$

Observe that $(0,1)$ is the best alternative for voter 1 and $(1,1)$ is the best alternative for voter 3. Therefore, in this case only full agendas are equilibrium agendas because there is always one voter who is better off by adding an issue to the agenda that was a free issue before. Hence, for all agendas a, $CE(a,P)$ contains full agendas only.

Summary
If $V((1,2),P) = V((2,1),P) \in \{(0,1),(1,1)\}$ any consistent equilibrium agenda is a full agenda, i.e. both candidates are nominated. However, depending on the order of vote either both candidates or only candidate 2 is elected.

Effect of a small change in preferences: Assume that voter 1's preference for $(0,0)$ and $(-,-)$ is reversed, so that voter 1 now strictly prefers $(0,0)$ over $(-,-)$. The preferences of voter 1 over all other pairs of alternatives as well as the preferences of voters 2 and 3 are the same as before (see Table 3). Then it is immediate to see that all equilibrium agendas are full agendas independent of the order of the alternatives under a full agenda because even in the case where the voting outcome under a full agenda is $(0,0)$ there is now one voter, namely voter 1, who prefers to add an issue to the empty agenda such that eventually a full agenda is reached.

Summary
If the ordering of the alternatives in $X(1,2) = X(2,1)$ is such that $V((1,2),P) = V((2,1),P) = (0,0)$, then no candidate is nominated under the original preferences of voter 1 while both candidates are nominated after the small local change in voter 1's preferences.

The examples in this section illustrate the following notable points:

(1) Even if the final position on an issue is independent on the other issues on the agenda like it is the case under voting by quota with additively separable preferences, agents may refrain from adding an issue to the agenda because this may trigger further additions of issues on which they lose.

(2) The equilibrium collection of sets of continuation agendas can be very sensitive to the details of the voting rule, and in particular to the use of a fixed order of vote under sequential voting procedures. A small change in the order of vote can have a huge effect on the set of issues that are considered

in equilibrium. As a consequence, if an agent can choose the order of vote under sequential procedures, she can not only influence the outcome for a given agenda, but also the set of issues that a society may choose to leave free.

(3) The equilibrium collection of sets of continuation agendas can be very sensitive to small changes in agents' preferences. Thus, agents' preferences do not impose any structure on the set of equilibrium agendas and equilibrium agendas can be very volatile.

5 General Analysis

We start our general analysis by exploring cases where all equilibrium agendas are full agendas. It turns out that this requires rather strong assumptions on voters' preferences. After that we will show that not even an uncontroversial issue is always on the equilibrium agenda. In the last part of the section we will prove that any set of issues can be obtained as an equilibrium agenda both under voting by quota and under the amendment procedure. Apart from some minor qualifications this is even true on very restricted domains of preferences and if all voters are agenda setters.

5.1 Full Equilibrium Agendas

While Dutta et al. (2004) have shown that Pareto efficiency of the voting rule is sufficient to guarantee that the equilibrium outcomes are equivalent to those under full agendas, Example 4.2 demonstrates that this is not the case in our model. We need additional assumptions on the preferences of agenda setters and voters in order to get full agendas in equilibrium.

One obvious case where all equilibrium agendas are full agendas is when there is one agenda setter who for all issues strictly prefers any position on the issue over not taking a position on the issue. Note that this agent's preferences violate betweenness. This agent will then keep adding issues until a full agenda is reached, i.e. any equilibrium agenda is a full agenda. The following remark deals with another straightforward case. It describes a situation where some alternative that is available at a full agenda is preferred by a majority of agenda setters to all other alternatives. In this case the opportunity of adding further issues until a full agenda is reached will be seized by some agenda setter whenever the voting rule is Condorcet consistent.[13]

Remark 5.1 Let $T \subset J$ be such that $\#T \geq \frac{n+1}{2}$ and let $P = (\succ_1, \ldots, \succ_n) \in \mathcal{P}^n$ be such that there exists some $x \in \{0,1\}^{\mathcal{K}}$ with $x \succ_i y$ for all $i \in T$ and for all $y \in X, y \neq x$. Then any equilibrium agenda at P is a full agenda if the voting rule V is Condorcet consistent. To see this note that in this case $V(a,P) = x$ for all full agendas a and $x \succ y$ for all $y \in X, y \neq x$, implies that no agenda setter in T prefers to stop at an incomplete agenda. Hence, (E1) and (E2) imply that all equilibrium agendas are full agendas. $\diamond$

We now look for more general conditions on the voting procedure and on voters' and agenda setters' preferences such that only full agendas obtain in equilibrium. To this end we define the set of agenda setters J to be *representative* of the set of voters I at preference profile $P \in \mathcal{P}^n$ if any disagreement about the ranking of two alternatives among the voters implies that there is also disagreement about the ranking of the alternatives among the agenda setters:

Definition 5.1 *The set of agenda setters J is* ***representative*** *of the set of voters I at preference profile $P = (\succ_1, \ldots, \succ_n) \in \mathcal{P}^n$ if for any two alternatives $x, y \in X$,*

$$\exists\, i, i' \in I \text{ with } x \succ_i y \text{ and } y \succ_{i'} x \iff \exists\, j, j' \in J \text{ with } x \succ_j y \text{ and } y \succ_{j'} x. \quad (4)$$

Note that (4) is equivalent to

$$x \succ_i y \text{ for all } i \in I \iff x \succ_i y \text{ for all } j \in J.$$

Let $\succ_i$ be separable. Then for all $k \in \mathcal{K}$ there exists a $w_k^i \in \{0,1,-\}$ such that

$$x \succ_i (w_k^i, x_{-k}) \text{ for all } x \in \{0,1,-\}^{\mathcal{K}} \text{ with } x_k \neq w_k^i.$$

[13] A voting rule V is *Condorcet consistent* if $V(a,P) = x$ whenever $x \in X(a)$ is a Condorcet winner at the preference profile P, i.e. $\#\{i \,|\, x \succ_i y\} \geq \frac{n+1}{2}$ for all $y \in X(a), y \neq x$.

That is, w_k^i is the *worst outcome* on issue k for agent i. We then say that agent i has *almost nothing to lose on issue* k if no position on that issue, i.e. "$-$", is either the worst outcome w_k^i on issue k or is close to that in the following sense:

Definition 5.2 *Let $\succ_i \in \mathcal{S}$. Then i* ***has almost nothing to lose*** *on issue k if either $w_k^i = -$ or $(-, x_{-k})$ and (w_k^i, x_{-k}) are adjacent for all $x \in X$, i.e. there exists no $y \in X$ with $(-, x_{-k}) \succ_i y \succ_i (w_k^i, x_{-k})$.*

The following two theorems show that all equilibrium agendas are full agendas under voting by quota and under a Pareto efficient voting rule if the set of agenda setters is representative of the set of voters and in addition voters' preferences satisfy (additive) separability and voters have almost nothing to lose about any issue.

Theorem 5.1 *Let $P \in \mathcal{S}^n$ and let J be representative of I at P. If all voters $i \in I$ have almost nothing to lose on all issues and $V : A \times \mathcal{S}^n \to X$ is voting by quota q for some $q \in \{1, \ldots, n\}$, then any equilibrium agenda a^* is a full agenda.*

Theorem 5.2 *Let $P \in \mathcal{S}^n$ and let J be representative of I at P. If all voters $i \in I$ have almost nothing to lose on all issues and if $V : A \times \mathcal{S}^n \to X$ is a Pareto efficient voting procedure, i.e. $V(a, P)$ is Pareto efficient in $X(a)$ for all $a \in A$ and all $P \in \mathcal{S}^n$, then any equilibrium agenda a^* is a full agenda and $V(a^*, P)$ is Pareto efficient in X.*

We sketch the proof for Theorem 5.1 and Theorem 5.2. All details are provided in the appendix. Suppose by way of contradiction that there is an equilibrium agenda which is not a full agenda. Then using backwards induction there must exist an incomplete agenda a which is an equilibrium continuation at a and such that every equilibrium continuation at (a, k) for $k \notin a$ is a full agenda. Given (E2) this is only possible if all agenda setters prefer the voting outcome at a over the outcome at a full continuation agenda. Since the agenda setters are representative of the voters this is then also true for all voters. Because all voters have almost nothing to lose about all issues they then must prefer the alternative with their worst outcome on all free issues at a over the voting outcome at the full agenda. Yet, this is impossible if the voting rule is voting by quota or if it is Pareto efficient.

5.2 Uncontroversial Issues and Equilibrium Agendas

One may have conjectured that an uncontroversial issue, i.e. an issue for which all voters prefer the same position independent of the decisions on other issues, would always be an element of every equilibrium agenda. However, as illustrated by the following example, the conjecture is not always true even if voters' preferences satisfy separability and betweenness and even if the voting procedure is Pareto efficient. The reason is that agenda setters may refrain from adding an uncontroversial issue to the agenda because this may trigger the addition of other issues and it may also change the final positions on issues that are already on the agenda. The example also demonstrates that the alternative that is chosen in equilibrium need not be Pareto efficient even if the voting procedure is Pareto efficient.

Example 5.1 Let $K = 3$ and let there be three voters who are also agenda setters, i.e. $I = J = \{1, 2, 3\}$. The voters' preference orderings on the set of alternatives are given in Table 4, where the alternatives in the table are listed in the order of decreasing preference.

Note that all preference orderings satisfy betweenness and separability, but not additive separability. Moreover, all voters prefer position 1 over 0 on issue 1: For all $i = 1, 2, 3$, and for all $x_2, x_3 \in \{0, 1, -\}$,

$$(1, x_2, x_3) \succ_i (0, x_2, x_3).$$

Consider a voting procedure that selects the following outcomes which are Pareto efficient in $X(a)$ for all agendas a:

$$V(\varnothing, P) = (-, -, -),$$
$$V((1), P) = (1, -, -),$$
$$V((2), P) = (-, 1, -),$$
$$V((3), P) = (-, -, 1),$$

$\succ_1$	$\succ_2$	$\succ_3$
$(1,1,1)$	$(1,1,0)$	$(1,0,0)$
$(1,1,-)$	$(1,1,-)$	$(1,0,-)$
$(-,1,1)$	$(-,1,0)$	$(1,1,1)$
$(-,1,-)$	$(-,1,-)$	$(1,-,1)$
$(1,-,1)$	$(1,0,1)$	$(1,-,-)$
$(1,0,1)$	$(1,-,0)$	$(1,1,-)$
$(1,-,-)$	$(1,-,-)$	$(-,1,1)$
$(1,0,-)$	$(1,0,-)$	$(-,1,-)$
$(1,1,0)$	$(1,0,0)$	$(1,0,1)$
$(1,-,0)$	$(1,-,1)$	$(1,-,0)$
$(1,0,0)$	$(1,1,1)$	$(1,1,0)$
$(-,-,1)$	$(-,0,1)$	$(-,0,0)$
$(-,0,1)$	$(-,-,0)$	$(-,-,1)$
$(-,-,-)$	$(-,-,-)$	$(-,-,-)$
$(-,0,-)$	$(-,0,-)$	$(-,0,-)$
$(-,1,0)$	$(-,0,0)$	$(-,0,1)$
$(-,-,0)$	$(-,-,1)$	$(-,-,0)$
$(-,0,0)$	$(-,1,1)$	$(-,1,0)$
$(0,1,1)$	$(0,1,0)$	$(0,0,0)$
$(0,1,-)$	$(0,1,-)$	$(0,0,-)$
$(0,-,1)$	$(0,0,1)$	$(0,1,1)$
$(0,0,1)$	$(0,-,0)$	$(0,-,1)$
$(0,-,-)$	$(0,-,-)$	$(0,-,-)$
$(0,0,-)$	$(0,0,-)$	$(0,1,-)$
$(0,1,0)$	$(0,0,0)$	$(0,0,1)$
$(0,-,0)$	$(0,-,1)$	$(0,-,0)$
$(0,0,0)$	$(0,1,1)$	$(0,1,0)$

Table 4: Preference orderings of voters in Example 5.1.

$$
\begin{aligned}
&V((1,2),P) = V((2,1),P) = (1,0,-),\\
&V((1,3),P) = V((3,1),P) = (1,-,1),\\
&V((2,3),P) = V((3,2),P) = (-,1,1),\\
&V(a,P) = (1,0,1) \text{ for all full agendas } a.
\end{aligned}
$$

We now solve backwards for the consistent equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a,P) = \{a\}$$

for all full agendas a.

Now consider any agenda a of length 2. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,l),P) = \{a,(a,l)\}$, where $l \notin a$. By condition (E2), $a \in CE(a,P)$ is ruled out since there is always one agenda setter who strictly prefers $V((a,l),P) = (1,0,1)$ over $V(a,P)$. Hence,

$$CE((1,2),P) = \{(1,2,3)\}, CE((1,3),P) = \{(1,3,2)\}, CE((2,3),P) = \{(2,3,1)\}.$$

Next consider an agenda a of length 1. If $a = (1)$, then condition (E1) implies that $CE((1),P)$ is a nonempty subset of $\{(1)\} \cup CE((1,2),P) \cup CE((1,3),P) = \{(1),(1,2,3),(1,3,2)\}$. By condition (E2), $(1) \in CE((1),P)$ is ruled out since voter 1 prefers $V((1,2,3),P) = (1,0,1)$ over $V((1),P) = (1,-,-)$. Hence,

$$CE((1),P) \subset \{(1,2,3),(1,3,2)\}.$$

In the same way we derive that

$$CE((3),P) \subset \{(3,1,2),(3,2,1)\}.$$

If $a = (2)$, then condition (E1) implies that $CE((2),P)$ is a nonempty subset of $\{(2)\} \cup CE((2,1),P) \cup CE((2,3),P) = \{(2),(2,1,3),(2,3,1)\}$. Since all voters prefer $V((2),P) = (-,1,-)$ over $V((2,1,3),P) = V(2,3,1) = (1,0,1)$ condition (E2) implies that $(2) \in CE((2),P)$. Since neither $(2,1,3)$ nor $(2,3,1)$ is rationalizable if $(2) \in CE((2),P)$ condition (E3) implies that

$$CE((2),P) = \{(2)\}.$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing, P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{l=1}^{3} CE((l), P)$. Since $CE((2), P) = \{(2)\}$ and $CE((1), P)$ and $CE((3), P)$ contain full agendas only and since all voters prefer $V((2), P) = (-, 1, -)$ over $V(\varnothing, P) = (-, -, -)$ and over $V(a, P) = (1, 0, 1)$ for all full agendas a, conditions (E2) and (E3) imply that

$$CE(\varnothing, P) = \{(2)\}.$$

Hence, there is a unique consistent equilibrium agenda $a^* = (2)$ which does not contain issue 1. We note that $V(a^*, P) = (-, 1, -)$ is Pareto dominated by $(1, 1, -)$.

$\diamond$

The situation is different under voting by quota with separable preferences. In this case, the conjecture that any uncontroversial issue will be on every equilibrium agenda is indeed true as shown by the following theorem. What drives this result is the fact that under voting by quota with separable preferences the position taken on each issue on the agenda is independent of the positions for other issues on the agenda.

Theorem 5.3 *Let $P = (\succ_1, \ldots, \succ_n) \in \mathcal{S}^n$ and let $k \in \mathcal{K}$ be such that there exists some $z_k \in \{0, 1\}$ with*

$$(z_k, x_{-k}) \succ_i (1 - z_k, x_{-k}) \quad \textit{and} \quad (z_k, x_{-k}) \succ_i (-, x_{-k})$$

for all $x_{-k} \in \{0, 1, -\}^{\mathcal{K} \setminus \{k\}}$ and for all $i \in I$. Let $V : A \times \mathcal{S}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$ and let $J \subset I$. If a^* is a consistent equilibrium agenda at P, then $k \in a^*$.

5.3 Variable Equilibrium Agendas

We will now explore the full range of equilibrium agendas under two prominent voting procedures, the amendment procedure and voting by quota which were defined in Section 3. For both these procedures we will show that for any subset $\mathcal{F}$ of the set of issues $\mathcal{K} = \{1, \ldots, K\}$ there exists a profile of preference orderings

$P = (\succ_1, \dots, \succ_n) \in \mathcal{P}^n$, such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda at P. This is true even if $J = I$ and apart from some minor qualifications also if preferences are (additively) separable and satisfy betweenness.[14] Thus, neither of the two procedures imposes any structure on the set of equilibrium agendas.

Voting by Quota

We first consider voting by quota. Notably, on the restricted domain of additively separable preferences, for any quota q and for any set $\mathcal{F}$, there exists a preference profile such that $\mathcal{F}$ is the set of free issues at some equilibrium agenda even if $J = I$.

Theorem 5.4 *Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \dots, n\}$ and $\mathcal{F} \subset \mathcal{K}$. Then there exists a preference profile $P = (\succ_1, \dots, \succ_n) \in \bar{\mathcal{S}}^n$ such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda a^* at P if $J = I$. If $\#\mathcal{F} \neq 1$ and $\#\mathcal{F} \neq 2$ if n is odd and $q = \frac{n+1}{2}$ the preference profile $P = (\succ_1, \dots, \succ_n)$ can even be chosen such that $\succ_i$ satisfies betweenness for all i.*

The proof of the first part of Theorem 5.4 is straightforward. If preferences are not required to satisfy betweenness we can choose a profile with identical additively separable preferences for all voters such that all voters prefer to leave the issues in $\mathcal{F}$ undecided while they prefer to take either position 1 or 0 on the remaining issues over not taking a decision on the issue. The proof of the second part where voters' preferences are required to satisfy betweenness is more complex. The main idea is to partition the set of voters into nonempty subsets with identical preferences within each subset, such that (a) all voters have the same most preferred position on all issues not in $\mathcal{F}$, (b) for any agenda that

[14] Note that it is easier to obtain any set of free issues if J is a strict subset of I. For example, suppose there is only one agenda setter and there are at least two other voters whose preferences over the positions for a subset $\mathcal{F}$ of issues are opposite to the preferences of the agenda setter and whose preferences over the positions for the remaining issues are the same as the agenda setter's preferences. Then the agenda setter will add an issue to the agenda if and only if it is not in $\mathcal{F}$. Hence, $\mathcal{F}$ will be the set of free issues at any consistent equilibrium agenda.

contains at least one issue in $\mathcal{F}$ there exists a voter who is in the winning majority for all remaining issues so that any continuation equilibrium is a full agenda and (c) all voters prefer an agenda that contains all but the issues in $\mathcal{F}$ over any full agenda. With this specification of preferences one can then show that any consistent equilibrium agenda contains all issues but those in $\mathcal{F}$.

We will now argue that the conditions in Theorem 5.4 are tight in the sense that if $\mathcal{F}$ and q do not satisfy the conditions then $\mathcal{F}$ can never be the set of free issues in equilibrium if the set of agenda setters is representative of the set of voters and if voters' preferences are additively separable and satisfy a condition that is even weaker than betweenness. The following proposition deals with the case where $\#\mathcal{F} = 1$ and shows that equilibrium agendas never contain all but one issue.

Proposition 5.1 *Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota q and let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$ be such that*

$$\max\{u_k^i(1), u_k^i(0)\} > u_k^i(-) \quad \textit{for all } i \in I \textit{ and for all } k \in \mathcal{K}, \tag{5}$$

where $(u_k^i(\cdot))_{k\in\mathcal{K}}$ is the collection of utility scalars in an additively separable utility representation of $\succ_i$. If J is representative of I at P and if $(CE(a,P))_{a\in A}$ is an equilibrium collection of sets of continuation agendas and $a^* \in CE(a,P)$ for some $a \in A$, then

$$a^* \notin A^{K-1}.$$

In particular, no $a^* \in A^{K-1}$ is an equilibrium agenda at P.

Note that if preferences are additively separable and satisfy betweenness, then condition (5) is satisfied. We skip the proof of Proposition 5.1 since it is an immediate implication of Lemma 3.1 and the following result:

Lemma 5.1 *Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$ and let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$ satisfy (5). If J is representative of I at P and if $(CE(a,P))_{a\in A}$ is an equilibrium collection of sets of continuation agendas, then*

$$CE(a,P) \subset A^K \textit{ for all } a \in A^{K-1}.$$

The intuition for Lemma 5.1 is that if there is only one free issue left, then there is always one agenda setter who would get her most preferred position on that issue which by (5) is better than leaving the issue undecided. This agenda setter then is better off adding that issue to the agenda since further additions are impossible and hence nothing can deter the agenda setter from her initial move.

The following proposition deals with the other exceptional case of simple majority voting with an odd number of agents, where there can never be only two free issues at an equilibrium agenda if voters' preferences satisfy (5). The intuition is that under simple majority voting with an odd number of voters, for any two issues there exists at least one voter who is in the winning majority for both these issues. If J is representative of I there also exists an agenda setter who is in the winning majority for both these issues. Moreover, by (5) the agenda setter prefers her most preferred position on those issues over leaving the issues undecided. Hence, if a is an agenda with two free issues, then by (E2) this implies that a cannot be a continuation equilibrium at a. Hence, no continuation equilibrium can have exactly two free issues.

Proposition 5.2 *Let there be an odd number n of voters and let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q = \frac{n+1}{2}$. Let $P = (\succ_1, \ldots, \succ_n) \in \bar{\mathcal{S}}^n$ satisfy (5). If J is representative of I at P and if $(CE(a, P))_{a \in A}$ is an equilibrium collection of sets of continuation agendas, then for all $a \in A$,*

$$CE(a, P) \subset A \setminus \left(A^{K-1} \cup A^{K-2}\right).$$

In particular, no $a^* \in A^{K-2}$ is an equilibrium agenda at P, i.e. the set of free issues at an equilibrium agenda never contains two issues only.

Amendment Procedure

Suppose now that voting is according to the amendment procedure for some ordering of the alternatives and simple majority is used throughout. We then have the following result:

Theorem 5.5 *Let $n \geq 3$ be odd and let $\mathcal{F} \subset \mathcal{K} = \{1, \ldots, K\}$. Then there exists a profile of preferences $P \in \mathcal{P}^n$ and some ordering of the alternatives in $X(a)$ for all $a \in A$, such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda a^* at P if $J = I$ and if voting is according to the amendment procedure for the given orderings of the alternatives at any agenda a. If $K > 2$ or if $K = 2$ and $\#\mathcal{F} \neq 1$ the preference profile $P = (\succ_1, \ldots, \succ_n)$ can even be chosen such that $\succ_i$ satisfies betweenness and separability for all i.*

The proof of Theorem 5.5 is again constructive. In Step 1 of the proof we consider the case where all issues are free, i.e. $\mathcal{F} = \mathcal{K}$. Section 4.2 provides a preference profile for $K = 2$ where in equilibrium all issues are free. We then construct another preference profile for $K = 3$ such that all issues are free in equilibrium and show that these two preference profiles can be extended in a lexicographic way such that for any $K \geq 4$ all K issues are free in equilibrium. In Step 2 we lexicographically extend the preferences defined in Step 1 such that voters first consider the issues not in $\mathcal{F}$ and only if two alternatives have the same positions on issues not in $\mathcal{F}$ the remaining issues are considered and preferences are given by the preferences defined in Step 1. The remaining steps of the proof deal with the case of one free issue and again use a lexicographic extension of preferences for $K = 3$ to any $K \geq 4$.

The case $K = 2$ and $\#\mathcal{F} = 1$ is special. Here we can only provide a profile of preferences satisfying betweenness but not separability such that there is one free issue in equilibrium. In fact, if $K = 2$ then separability and betweenness impose such a strong structure on voters' preferences over the set of available alternatives at full agendas relative to their preferences over available alternatives at agendas with one issue only that there always exists one agenda setter who prefers to add the remaining issue to an agenda that already contains one issue. This implies that on the domain of preferences satisfying separability and betweenness the set of free issues is either empty or contains all issues if $K = 2$. Thus, Theorem 5.5 is tight as we show in the following proposition.

Proposition 5.3 *Let $n \geq 3$ be odd and let $P \in \mathcal{S}^n$ be any profile of separable preferences that satisfy betweenness. Let $J = I$ and $K = 2$ and let the voting rule be the amendment procedure for some orderings of the alternatives at any agenda a. Then the set of free issues is either empty or contains all issues.*

6 Conclusion

Issue deferral is a pervasive phenomenon in the actual works of legislative bodies and other institutions resorting to vote. We have provided a stark model to analyze a basic and general reason whereby the phenomenon can arise: members of voting bodies in charge of fixing their agenda may decide to leave some issues out of it, while choosing to fix the social position on other issues. Although our model abstracts from other, possibly idiosyncratic, considerations, it focuses on a fundamental conflict about which agenda setters will have to find a balance. On the one hand, the tendency to include in the agenda those issues on which their preferred position will prevail. On the other, the eventual need to refrain from the preceding tendency when it may generate the inclusion by others of issues on which their position would be defeated.

In spite of its simplicity, our model contains differential characteristics that add elements of realism to previous works. Distinguishing between issues and alternatives clarifies the phenomenon under study. Resorting to a new equilibrium concept eliminates any arbitrariness that a fixed protocol might introduce in the analysis, and actually makes a difference in predictions.

We think of our model as a starting point for formal work on the important and complex problem of issue deferral. Even in its present form it is able to generate a variety of clarifying results, that we have presented along the paper. These results are driven by preferences alone, and this suggest that the impact of additional variables on issue deferral could be studied by specifying how their consideration would affect the preferences of different types of voters.

Appendix

Example 3.1

We first prove that the set of consistent equilibrium agendas consists of all agendas of length 2. We proceed backwards starting from full agendas $a \in A^3$. By (E1) we have

$$CE(a, P) = \{a\} \text{ for all } a \in A^3.$$

Next consider an agenda $a = (k, l)$ for some $k \neq l$ and let $h \notin a$. (E1) implies that $CE((k, l), P)$ is a nonempty subset of $\{(k, l), (k, l, h)\}$. Since $V((k, l), P) \succ_i V((k, l, h), P) = (1, 1, 1)$ for all i (E2) implies that $(k, l) \in CE((k, l), P)$. Moreover, (k, l, h) is not rationalizable relative to a. Hence, (E3) implies that

$$CE((k, l), P) = \{(k, l)\} \text{ for all } k \neq l.$$

Next consider an agenda (k) for some $k \in \{1, 2, 3\}$. (E1) implies that $CE((k), P)$ is a nonempty subset of $\{(k), (k, l), (k, h)\}$, where $l \neq k$ and $h \neq k$. Since there exists a voter i with $V((k, l), P) \succ_i V((k), P)$ (E2) then implies that $(k) \notin CE((k), P)$. W.l.o.g. suppose $(k, l) \in CE((k), P)$. Because there exists a voter j with $V((k, h), P) \succ_j V((k, l), P)$ it follows that (k, h) is rationalizable relative to (k) and hence (E3) implies that

$$CE((k), P) = \{(k, l), (k, h)\} \text{ for all } k \in \{1, 2, 3\} \text{ and for all } l \neq l \text{ and } h \neq k.$$

Finally, consider the empty agenda $\varnothing$. (E1) implies that $CE(\varnothing, P)$ is a nonempty subset of $\{\varnothing, (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)\}$. Since $V((1, 2), P) = (1, 1, -) \succ_1 V(\varnothing, P) = (-, -, -)$ (E2) implies that $\varnothing \notin CE(\varnothing, P)$. Suppose $(k, l) \in CE(\varnothing, P)$ for some $k \neq l$ and let $h \neq k$ and $h \neq l$. Then there exists a voter i with $V((k, h), P) = V((h, k), P) \succ_i V((k, l), P)$ and a voter j with $V((l, h), P) = V((h, l), P) \succ_i V((k, l), P)$. Hence, $(k, h), (h, k), (l, h), (h, l)$ are all rationalizable relative to $\varnothing$. (E3) then implies that $(k, h), (h, k), (l, h), (h, l) \in CE(\varnothing, P)$. Repeating this argument for (k, h) instead of (k, l) we conclude that

$$CE(\varnothing, P) = \{(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)\}.$$

Consider now the extensive form game with order of moves $1, 2, 3$, which repeats until either a full agenda is reached or there were three passes in a row. We will show that in any subgame perfect perfect Nash equilibrium of the game no voter adds any issue to the agenda and hence the unique equilibrium agenda is $\varnothing$ which is not a consistent equilibrium agenda.

Since the game is finite we can solve for subgame perfect Nash equilibria by backwards induction. We start by considering any subgame such that the longest path in the subgame has length 1. This means that any action of the voter i who moves at the unique decision node in the subgame must lead to a terminal node. Let a be the given agenda at the decision node where i moves. Since all actions of i lead to a terminal node it follows that $a = (k, l)$ for some $k \neq l$ and all voters $j \neq i$ must have passed in their previous moves. Hence, if i passes the game ends with agenda (k, l) and if i adds issue h the game ends with agenda (k, l, h). Since $V((k, l), P) \succ_i V(k, l, h) = (1, 1, 1)$ i's unique best response is to pass. For a similar reason the voters who move just before voter i (these voters are first to move in a subgame of length 2 or 3), will also pass. Hence, in any subgame perfect equilibrium all voters will pass at an agenda that contains two issues.

Consider now a decision node where the given agenda contains a unique issue k and let voter i move at this node. If i adds issue $l \neq k$ to the agenda the unique subgame perfect equilibrium outcome in the subgame that is reached is (k, l). If i passes the game either ends at some agenda (k, h) or all voters pass and the outcome is (k). Since $V((k, l), P) \succ_i V((k), P)$ for all $l \neq k$ it follows that i will never pass if this leads to the final outcome (k). Hence, in any subgame perfect Nash equilibrium of the subgame starting with a move of i at agenda (k) the action taken by i is such that the final agenda is (k, l) with $V((k, l), P) \succ_i V((k, h), P)$ for $h \neq l$.[15] Table 5 shows the equilibrium agendas in a subgame perfect Nash equilibrium for a subgame that starts with a move of $i \in \{1, 2, 3\}$ at agenda (k) for all $k = 1, 2, 3$. Note that we have used the fact that the order of moves is $1, 2, 3$.

Finally, consider a decision node where the given agenda is empty and let voter i move at this node. If i adds issue k, the game ends with agenda (k, l) for

[15] There always exists a subgame perfect Nash equilibrium where i adds issue l to agenda (k), but there may also exist subgame perfect Nash equilibria where i passes and some player $j \neq i$ adds issue l to agenda (k).

	Voters		
Agenda	1	2	3
(1)	$(1,2)$	$(1,3)$	$(1,2)$
(2)	$(2,1)$	$(2,3)$	$(2,3)$
(3)	$(3,1)$	$(3,1)$	$(3,2)$

Table 5: Equilibrium agendas in a subgame perfect Nash equilibrium for a subgame that starts with a move of voter $i \in \{1,2,3\}$ at agenda (k) for all $k = 1,2,3$.

some $l \neq k$, where (k,l) is given in Table 6 which follows from Table 5.[16] Again we use the fact that the order of moves is $1,2,3$.

	Voters		
Issue	1	2	3
1	$(1,3)$	$(1,2)$	$(1,2)$
2	$(2,3)$	$(2,3)$	$(2,1)$
3	$(3,1)$	$(3,2)$	$(3,1)$

Table 6: Equilibrium agendas in a subgame perfect Nash equilibrium of the subgame that is reached if voter $i \in \{1,2,3\}$ adds issue k to the empty agenda $\varnothing$.

Suppose all voters $j \neq i$ have passed before. Then, if i passes, the game ends with the empty agenda. Note that i strictly prefers $V(\varnothing, P) = (-,-,-)$ over the voting outcome at any subgame perfect equilibrium agenda that is reached if i adds some issue k to the agenda (see Table 6).[17] Hence, the unique best response of i is to pass.

Suppose one voter $j \neq i$ has passed before and another voter $\iota \neq i$ moves after i. Then from the previous analysis we conclude that if i passes, voter ι passes as well and the game ends with the empty agenda. If instead i adds some issue k to

[16]For example, if $i = 1$ and i adds issue 1 the next mover is voter 2 at agenda (1). From Table 5 we see that the resulting equilibrium agenda is $(1,3)$.

[17]For example, if $i = 1$ then according to Table 6 the subgame perfect equilibrium agenda is either $(1,3),(2,3)$ or $(3,1)$ with outcomes $(1,-,1)$ or $(-,1,1)$ which are all strictly worse for voter 1 than the outcome $(-,-,-)$ at the empty agenda.

the agenda the game ends with an outcome that is strictly worse for i than the outcome $(-,-,-)$ at the empty agenda.

Finally, suppose all voters $j \neq i$ are moving after i. Then, if i passes, both voters $j \neq i$ will pass as well and the game ends with the empty agenda. If instead i adds some issue k to the agenda the game ends with an outcome that is strictly worse for i than the outcome $(-,-,-)$ at the empty agenda.

Therefore, we conclude that $\varnothing$ is the unique subgame perfect Nash equilibrium outcome.

$\square$

Proof of Theorem 5.1: Let $P \in \mathcal{S}^n$ and let J be representative of I at P. Moreover, let all voters $i \in I$ have almost nothing to lose on all issues and let $V : A \times \mathcal{S}^n \to X$ be voting by quota $q \in \{1, \dots n\}$. Let $(CE(a,P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas. We will prove by backwards induction that $CE(a,P) \subset A^K$ for all $a \in A$.

Obviously, the claim is true for any full agenda $a \in A^K$. Suppose the claim is true for all agendas $a \in A$ of length l, where $m+1 \leq l \leq K$ and $1 \leq m \leq K-1$. Let $a \in A^m$. By (E1), $CE(a,P)$ is a nonempty subset of $\bigcup_{k \notin a} CE((a,k),P) \cup \{a\}$. By our induction hypothesis $CE((a,k),P) \subset A^K$ for all $k \notin a$. Suppose by way of contradiction that $a \in CE(a,P)$. Then by (E2), for all agenda setters $j \in J$,

$$V(a,P) \succ_j V(a',P) \text{ for all } a' \in \bigcup_{k \notin a} CE((a,k),P).$$

Since J is representative of the set of voters I at P this implies that

$$V(a,P) \succ_i V(a',P) \text{ for all } a' \in \bigcup_{k \notin a} CE((a,k),P) \text{ for all } i \in I.$$

Let $y = V(a',P)$ for some $a' \in \bigcup_{k \notin a} CE((a,k),P)$. By the induction hypothesis a' is a full agenda, which implies that $y \in \{0,1\}^{\mathcal{K}}$. Let $x = V(a,P)$. Then $x_k = -$ for all $k \notin a$. Since every agent i has almost nothing to lose on all issues $k \notin a$ it follows that for all $k \notin a$ either $w_k^i = - = x_k$ or x and (w_k^i, x_{-k}) are adjacent to each other. Hence, for all i and for all $k \notin a$,

$$\left(w_k^i, x_{-k}\right) \succ_i y \quad \text{or} \quad \left(w_k^i, x_{-k}\right) = y.$$

In the latter case it must be that $l \in a$ for all $l \neq k$ since $y \in \{0,1\}^{\mathcal{K}}$ and $x_l = -$ for all $l \notin a$. In the former case we can iterate the previous argument for $l \notin a, l \neq k$. After a finite number of iterations we conclude that for all i,

$$\big((w_k^i)_{k\notin a}, (x_k)_{k\in a}\big) \succ_i y \quad \text{or} \quad \big((w_k^i)_{k\notin a}, (x_k)_{k\in a}\big) = y. \tag{6}$$

Since V is voting by quota $q \in \{1, \ldots, n\}$ it follows that $y_k = x_k$ for all $k \in a$ which implies that $((w_k^i)_{k\notin a}, (x_k)_{k\in a}) \succ_i y$ is impossible since w_k^i is i's worst position on issue k for all $k \notin a$. Hence, it must be true that $((w_k^i)_{k\notin a}, (x_k)_{k\in a}) = y$. But this is impossible as well since under voting by quota q there exists at least one voter i such that y_k is not i's worst position on issue k.

We conclude that $a \notin CE(a, P)$ which by the induction hypothesis implies that any agenda in $CE(a, P)$ is a full agenda. Since this holds for any $a \in A$ it follows that any equilibrium agenda $a^* \in CE(\varnothing, P)$ is a full agenda.

$\square$

Proof of Theorem 5.2: Let $P \in \mathcal{S}^n$ and let J be representative of I at P. Moreover, let all voters $i \in I$ have almost nothing to lose on all issues and let $V : A \times \mathcal{S}^n \to X$ be a Pareto efficient voting procedure. Let $(CE(a, P))_{a\in A}$ be an equilibrium collection of sets of continuation agendas. We will prove by backwards induction that $CE(a, P) \subset A^K$ for all $a \in A$.

To this end we follow the proof of Theorem 5.1 up to equation (6). Since V is Pareto efficient it follows that $y = V(a', P)$ is Pareto efficient in $\{0,1\}^{\mathcal{K}}$. If $((w_k^i)_{k\notin a}, (x_k)_{k\in a}) \succ_i y$ for all i, let $z \in \{0,1\}^{\mathcal{K}}$ be such that $z_k = x_k$ for all $k \in a$. Then, by definition of w_k^i for $k \notin a$ it follows that for all i either

$$z = \big((w_k^i)_{k\notin a}, (x_k)_{k\in a}\big) \succ_i y$$

or

$$z \succ_i \big((w_k^i)_{k\notin a}, (x_k)_{k\in a}\big) \succ_i y.$$

This contradicts our assumption that y is Pareto efficient in $\{0,1\}^{\mathcal{K}}$.

If there exists an i with $((w_k^i)_{k\notin a}, (x_k)_{k\in a}) = y$, let $z \in \{0,1\}^{\mathcal{K}}$ be such that $z_k = 1 - w_k^i$ for all $k \notin a$ and $z_k = x_k$ for all $k \in a$. Notice that $((w_k^i)_{k\notin a}, (x_k)_{k\in a}) = y = \big((w_k^j)_{k\notin a}, (x_k)_{k\in a}\big)$ for some $i \neq j$ implies that $w_k^i = w_k^j$ for all $k \notin a$. Again it follows that $z \succ_i y$ for all i which contradicts our assumption that y is Pareto efficient.

We conclude that $a \notin CE(a, P)$ which by the induction hypothesis implies that any agenda in $CE(a, P)$ is a full agenda. Since this holds for any $a \in A$ it follows that any equilibrium agenda $a^* \in CE(\varnothing, P)$ is a full agenda.

It remains to prove that $V(a^*, P)$ is Pareto efficient in X for any full agenda $a^* \in CE(\varnothing, P)$. Let $y = V(a^*, P) \in \{0,1\}^{\mathcal{K}}$ and suppose by way of contradiction that y is not Pareto efficient in X. Then there exists some $x \in X$ with $x \succ_i y$ for all $i \in I$. Since $V(a^*, P)$ is Pareto efficient in $X(a^*) = \{0,1\}^{\mathcal{K}}$ it follows that $x_k = -$ for at least one $k \in \mathcal{K}$. Since all voters have almost nothing to lose about all issues we can use the same reasoning as in the first part of the proof to show that this leads to a contradiction. Hence, $V(a^*, P)$ is Pareto efficient in X. This proves the theorem.

$\square$

Proof of Theorem 5.3: Let $P = (\succ_1, \ldots, \succ_n) \in \mathcal{S}^n$ and let $k \in \mathcal{K}$ be such that there exists some $z_k \in \{0,1\}$ with

$$(z_k, x_{-k}) \succ_i (1 - z_k, x_{-k}) \quad \text{and} \quad (z_k, x_{-k}) \succ_i (-, x_{-k}) \tag{7}$$

for all $x_{-k} \in \{0,1,-\}^{\mathcal{K}\setminus\{k\}}$ and for all $i \in I$. Let $V : A \times \mathcal{S}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$. Let $x \in \{0,1\}^{\mathcal{K}}$ be given by $(V(\hat{a}, l))_l = x_l$ for all $l \in \mathcal{K}$, where $\hat{a}$ is an arbitrary full agenda. Then by definition of voting by quota q, $(V(a, l))_l = x_l$ for all agendas a with $l \in a$ and $x_k = z_k$ for all agendas a with $k \in a$.

Let a^* be a consistent equilibrium agenda at P and suppose by way of contradiction that $k \notin a^*$. Then there exists a consistent equilibrium collection of sets of continuation agendas $(CE(a, P))_{a \in A}$ with $a^* \in CE(\varnothing, P)$. From Lemma 3.1 we know that $a^* \in CE(a^*, P)$. Let $a \in CE((a^*, k), P)$. Then (E2) implies that

$$V(a^*, P) \succ_j V(a, P) \text{ for all } j \in J. \tag{8}$$

There are two cases:

Case 1: $a = (a^*, k)$
Let $y_l = -$ for all $l \notin a^*$. Then (8) is equivalent to

$$((x_l)_{l\in a^*}, (y_l)_{l\notin a^*}) \succ_i ((x_l)_{l\in a^*}, z_k, (y_l)_{\substack{l\notin a^* \\ l\neq k}})$$

which is impossible given (7).

Case 2: $a = (a^*, k, l, \ldots)$ for some $l \notin a^*, l \neq k$.
Then (E2) and (E3) imply that there exists some $j \in J$ with $V(a, P) \succ_j V((a^*, k), P)$. Together with (8) this implies that $V(a^*, P) \succ_j V((a^*, k), P)$ which is impossible as shown in Case 1.

Hence, we conclude that $k \in a^*$ which proves the theorem.

$\square$

Proof of Theorem 5.4: Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \ldots, n\}$ and let $J = I$. Let $\mathcal{F} \subset \mathcal{K}$. Choose $(u_k(\cdot))_{k\in\mathcal{K}}$ such that the corresponding preference ordering $\succ$ is strict and

$$\begin{aligned} & u_k(-) > u_k(1) > u_k(0) \text{ for all } k \in \mathcal{F} \\ \text{and} \quad & u_k(1) > u_k(-) > u_k(0) \text{ for all } k \notin \mathcal{F}. \end{aligned}$$

Let $P = (\succ_1, \ldots, \succ_n)$ where $\succ_i = \succ$ for all i. Then $(V(a, P))_k = 1$ for all k.

Let $a \in A^m$ where $m \in \{0, 1, \ldots, K-1\}$. We will prove by induction over m that $a' \in CE(a, P)$ implies that

$$\{k \,|\, k \in a' \text{ and } k \notin a\} = \{k \,|\, k \notin \mathcal{F} \text{ and } k \notin a\}. \tag{9}$$

In particular, $CE(a, P) = \{(a)\}$ if and only if $k \in a$ for all $k \notin \mathcal{F}$. Let $a \in A^{K-1}$ and let $k \notin a$. Then for all i,

$$\begin{aligned} V(a, P) \succ_i V((a, k), P) \quad & \text{if } k \in \mathcal{F} \\ \text{and } V((a, k), P) \succ_i V(a, P) \quad & \text{if } k \notin \mathcal{F}. \end{aligned}$$

(E2) and (E3) then imply that

$$CE(a, P) = \begin{cases} \{a\} & \text{, if } k \in \mathcal{F} \\ \{(a, k)\}, & \text{if } k \notin \mathcal{F} \end{cases}.$$

This proves (9) for $m = K-1$. Suppose that the claim has been proved for all $\overline{m}$ with $1 \leq \overline{m} \leq K-1$ and let $a \in A^{\overline{m}-1}$. If $k \in a$ for all $k \notin \mathcal{F}$ then

$$V(a,P) \succ_i V(a',P) \quad \text{for all } a' \in \bigcup_{l \notin a} CE((a,l),P) \quad \text{for all } i.$$

(E2) and (E3) then imply that $CE(a,P) = \{a\}$ which proves (9) for this case. If $k \notin a$ for some $k \notin \mathcal{F}$ then by the induction hypothesis $a' \in CE((a,k),P)$ implies that

$$\{l \,|\, l \in a' \text{ and } l \notin (a,k)\} = \{l \,|\, l \notin \mathcal{F} \text{ and } l \notin (a,k)\}.$$

Hence, $V(a',P) \succ_i V(a,P)$ for all i. (E2) then implies that $a \notin CE(a,P)$. Suppose by way of contradiction that $a' \in CE(a,P)$, where $a' \in CE((a,l),P)$ for some $l \in \mathcal{F}$. Then by the induction hypothesis, for all $a'' \in CE((a,k),P)$ with $k \notin \mathcal{F}$,

$$V(a'',P) \succ_i V(a,P) \quad \text{for all } i.$$

Hence, any $a'' \in CE((a,k),P)$ with $k \notin \mathcal{F}$, is rationalizable relative to a and (E3) implies that $a'' \in CE(a,P)$. (E3) then also requires that a' is rationalizable, i.e. there must exist $a''' \in CE(a,P)$ such that

$$V(a',P) \succ_i V(a''',P) \quad \text{for some } i.$$

This implies that $a''' \in CE((a,l'),P)$ for some $l' \in \mathcal{F}$. Again (E3) requires that a''' is rationalizable. Since $\mathcal{F}$ is finite, after a finite number of steps we arrive at an agenda $\hat{a} \in CE(a,P)$ which is not rationalizable and hence must not be an element of $CE(a,P)$ by (E3). This contradiction proves that $a' \notin CE(a,P)$ for all $a' \in CE((a,l),P)$ with $l \in \mathcal{F}$. This proves (9) for $m = \overline{m}-1$.

In particular, (9) implies that $\mathcal{F}$ is the set of free issue at any consistent equilibrium agenda at P.

In the remainder of the proof we will show that for $\mathcal{F}$ with $\#\mathcal{F} \neq 1$ and $\#\mathcal{F} \neq 2$ if n is odd and $q = \frac{n+1}{2}$ the preference profile $P = (\succ_1, \ldots, \succ_n)$ can even be chosen such that $\succ_i$ satisfies betweenness for all i.

First consider the case $\mathcal{F} = \emptyset$. Take any preference ordering $\succ \in \bar{\mathcal{S}}$ that satisfies betweenness and let $P = (\succ_1, \ldots, \succ_n)$ be such that $\succ_i = \succ$ for all $i =$

$1,\dots,n$. Then, for all $a \in A \setminus A^K$ and for all $a' = (a, k, \dots)$ with $k \notin a$,

$$V(a', P) \succ_i V(a, P) \text{ for all } i \in I. \tag{10}$$

Let $(CE(a,P))_{a\in A}$ be any consistent equilibrium collection of sets of continuation agendas. (10) and (E2) then imply that $a \notin CE(a,P)$ for all $a \in A \setminus A^K$ and we conclude that

$$CE(\varnothing, P) \subset A^K$$

by Lemma 3.1. Hence, there are no free issues at any equilibrium agenda at P.

Now let $\emptyset \neq \mathcal{F} \subset \mathcal{K}$ be such that $\#\mathcal{F} > 1$ and $\#\mathcal{F} \neq 2$ if n is odd and $q = \frac{n+1}{2}$. Let $r = \#\mathcal{F}$ and w.l.o.g. let $\mathcal{F} = \{1, \dots, r\}$.

We first consider the case where $r \leq n$. In this case, for all i, choose $(u_k^i(\cdot))_{k\in\mathcal{K}}$ such that

$$u_k^i(1) > u_k^i(-) > u_k^i(0) \text{ for all } k \notin \mathcal{F}. \tag{11}$$

If $q \geq \frac{n+1}{2}$, let $\{W_1 \dots, W_r\}$ be a partition of the set of voters $\{1, \dots, n\}$ into nonempty subsets W_h, $h = 1, \dots, r$, such that $\#W_h < q$ for all $h = 1, \dots, r$. Observe that such a partition exists for any n and any $q \geq \frac{n+1}{2}$ if $r \geq 3$. It also exists for $r = 2$ if either n is even or n is odd and $q > \frac{n+1}{2}$. Then define utility scalars $(u_h^i(\cdot))_{h=1,\dots,r}$ as follows: For $h = 1, \dots, r$, and for all $i \in W_h$ let

$$\begin{aligned} u_h^i(1) &> u_h^i(-) > u_h^i(0), & (12)\\ u_{h'}^i(0) &> u_{h'}^i(-) > u_{h'}^i(1) \text{ for all } h' \in \{1, \dots, r\} \setminus \{h\}, & (13)\\ \sum_{h'=1}^{r} u_{h'}^i(-) &> \sum_{h'=1}^{r} u_{h'}^i(0). & (14) \end{aligned}$$

Observe that $(u_k^i(\cdot))_{k\in\mathcal{K}}$ can always be chosen such that conditions (11), (12), (13) and (14) are satisfied and such that the corresponding preference ordering is strict. With this specification less than q voters prefer position 1 over 0 for any $h \in \mathcal{F}$. This implies that for all $h \in \mathcal{F}$ and for all agendas $a \in A$ with $h \in a$,

$$(V(a, P))_h = 0. \tag{15}$$

If $q < \frac{n+1}{2}$, then it is straightforward to show that there are two cases: Either there exists a partition $\{W_1 \dots, W_r\}$ of the set of voters $\{1, \dots, n\}$ into nonempty subsets W_h, $h = 1, \ldots, r$, such that $\#W_h < q$ for all $h = 1, \ldots, r$, and we can use the same utility specification as in the case where $q \geq \frac{n+1}{2}$. Or there exists a partition $\{W_1 \ldots, W_r\}$ of $\{1, \ldots, n\}$ into nonempty subsets W_h, $h = 1, \ldots, r$, such that $\#W_h \geq q$ and $n - \#W_h \geq q$ for at least one $h \in \{1, \ldots, r\}$. In the latter case, choose the utility scalars $(u^i_h(\cdot))_{h=1,\ldots,r}$ as follows: For $h = 1, \ldots, r$, and for all $i \in W_h$ let

$$
\begin{aligned}
u^i_h(0) &> u^i_h(-) > u^i_h(1), && (16)\\
u^i_{h'}(1) &> u^i_{h'}(-) > u^i_{h'}(0) \text{ for all } h' \in \{1, \ldots, r\} \setminus \{h\}, && (17)\\
\sum_{h'=1}^{r} u^i_{h'}(-) &> \sum_{h'=1}^{r} u^i_{h'}(1). && (18)
\end{aligned}
$$

Again observe that $(u^i_k(\cdot))_{k\in\mathcal{K}}$ can always be chosen such that conditions (11), (16), (17) and (18) are satisfied and such that the corresponding preference ordering is strict. With this specification at least q voters prefer position 1 over 0 for any $h \in \mathcal{F}$. This implies that for all $h \in \mathcal{F}$ and for all agendas $a \in A$ with $h \in a$,

$$(V(a, P))_h = 1. \tag{19}$$

(12), (13) and (15) (resp. (16), (17)and (19)) imply that if an agenda contains at most $m \leq r-1$ issues from $\mathcal{F}$, then there exists at least one voter who gets his most preferred position on all m issues, and if an agenda contains all issues from $\mathcal{F}$, then every voter i gets his most preferred position on exactly $r - 1$ issues in $\mathcal{F}$. Moreover, given (14) (resp. (18)) every voter prefers an agenda that contains all but the issues in $\mathcal{F}$ over any full agenda: For all agendas a that contain all but the issues in $\mathcal{F}$, and for all full agendas $a' \in A^K$,

$$V(a, P) \succ_i V(a', P) \text{ for all } i. \tag{20}$$

Let $(CE(a, P))_{a\in A}$ be any consistent equilibrium collection of sets of continuation agendas and let $a \in A \setminus A^K$ with $k \in a$ for some $k \in \mathcal{F}$. We will prove that $a \notin CE(a, P)$. By definition of the voters' utility functions there exists a voter i who gets his most preferred position on all issues $l \notin a$. To see this, note that all voters agree on the position for issues not in $\mathcal{F}$ and there are at most $r-1$ issues from $\mathcal{F}$ which are not on agenda a. Hence,

$$V(a', P) \succ_i V(a, P) \text{ for all } a' \in \bigcup_{l \notin a} CE((a,l), P).$$

By (E2) this implies that $a \notin CE(a, P)$. Lemma 3.1 then implies that

$$CE(a, P) \subset A^K \text{ for all } a \in A \text{ with } k \in a \text{ for some } k \in \mathcal{F}. \tag{21}$$

Moreover, if $a \in A$ contains all issues but those in $\mathcal{F}$, then $CE(a, P) = \{a\}$. To see this, observe that any $a' \in CE((a,k), P)$ with $k \notin a$ must be a full agenda by (21). By definition of the voters' utility function it follows that

$$V(a, P) \succ_i V(a', P) \text{ for all } i. \tag{22}$$

(E2) then implies that $a \in CE(a, P)$. Moreover, by (22) and the fact that any $a'' \in CE(a, l)$ for some $l \notin a$ is a full agenda by (21), we conclude that no $a' \in CE((a,k), P)$ with $k \notin a$ is rationalizable relative to a. (E3) then implies that $CE(a, P) = \{a\}$.

Let $0 \leq m \leq K - r$ and let $a \in A^m$ with $h \notin a$ for all $h \in \mathcal{F}$. We will now show inductively over m that $a' \in CE(a, P)$ implies that a' contains all issues but those in $\mathcal{F}$. We have shown above that this is true for $m = K - r$. Suppose that the claim has been proved for all $\overline{m}$ with $\overline{m} \leq m \leq K - r$, where $1 \leq \overline{m} \leq K - r$, and let $a \in A^{\overline{m}-1}$ with $h \notin a$ for all $h \in \mathcal{F}$. By (E1), if $a' \in CE(a, P)$, then either $a' = a$ or $a' \in \bigcup_{k \notin a} CE((a,k), P)$. Let $a' \in CE((a,k), P)$ for some $k \notin a$. If $k \in \mathcal{F}$, then $a' = (a, k, \ldots) \in A^K$ by (21). If $k \notin \mathcal{F}$, then by our induction hypothesis a' contains all issues but those in $\mathcal{F}$ which implies that

$$V(a', P) \succ_i V(a, P) \text{ for all } i,$$

since all voters agree on the position for all issues not in $\mathcal{F}$. (E2) then implies that $a \notin CE(a, P)$. Hence, $CE(a, P) \subset \bigcup_{k \notin a} CE((a,k), P)$ and any $a' \in CE(a, P)$ is either a full agenda or contains all issues but those in $\mathcal{F}$.

Suppose by way of contradiction that there exists $a' \in CE(a, P)$ with $a' \in CE((a,k), P)$ for some $k \in \mathcal{F}$. Then $a' = (a, k, \ldots) \in A^K$ by (21). Let $l \notin \mathcal{F}$ and

$l \notin a$ and let $a'' \in CE((a,l),P)$. Then by the induction hypothesis $a'' = (a,l,\dots)$ contains all issues but those in $\mathcal{F}$ which implies that

$$V(a'',P) \succ_i V(a',P) \text{ for all } i.$$

Hence, a'' is rationalizable relative to a and (E3) implies that $a'' \in CE(a,P)$. Moreover, if both $a' = (a,k,\dots) \in CE(a,P)$ and $a'' = (a,l,\dots) \in CE(a,P)$ with $k \neq l$, then (E3) implies that a' is rationalizable. Therefore, there exists a voter i and some $\hat{a} \in CE(a,P)$ with $\hat{a} = (a,h,\dots)$ for some $h \neq k$ such that

$$V(a',P) \succ_i V(\hat{a},P). \tag{23}$$

However, by what we have shown above, any $\hat{a} \in CE(a,P)$ is either a full agenda or contains all issues but those in $\mathcal{F}$. Since a' is a full agenda, this contradicts (23). Hence, if $a' \in CE(a,P)$, then $a' \in CE((a,k),P)$ for some $k \notin \mathcal{F}$ and by the induction hypothesis we conclude that a' contains all issues but those in $\mathcal{F}$. This proves the claim for all m with $0 \leq m \leq K-r$. In particular, any consistent equilibrium agenda a^* at P has the property that $\mathcal{F}$ is the set of free issues at a^*.

It remains to consider the case, where $\mathcal{F} = \{1,\dots,r\}$ with $r > n$. If $1 \leq q \leq n-1$, choose $(u_k^i(\cdot))_{k\in\mathcal{K}}$ as follows. For $i = 1,\dots,n-1$, let

$$\begin{aligned}
u_i^i(0) &> u_i^i(-) > u_i^i(1), && (24)\\
u_k^i(1) &> u_k^i(-) > u_k^i(0) \quad \text{for all } k \in \mathcal{K}, k \neq i, && (25)\\
\sum_{k=1}^{r} u_k^i(-) &> \sum_{k=1}^{r} u_k^i(1), && (26)
\end{aligned}$$

and for $i = n$ let

$$\begin{aligned}
u_k^n(0) &> u_k^n(-) > u_k^n(1) \quad \text{for all } k = n,\dots,r, && (27)\\
u_k^n(1) &> u_k^n(-) > u_k^n(0) \quad \text{for all } k \in \mathcal{K}, k \notin \{n,\dots,r\}, && (28)\\
\sum_{\substack{l=1\\l\neq k}}^{r} u_l^n(-) &< \sum_{\substack{l=1\\l\neq k}}^{r} u_l^n(1) \quad \text{for all } k = n,\dots,r, && (29)\\
\sum_{k=1}^{r} u_k^n(-) &> \sum_{k=1}^{r} u_k^n(1), && (30)
\end{aligned}$$

Observe that $(u_k^i(\cdot))_{k\in\mathcal{K}}$ can always be chosen such that conditions (24)-(30) are satisfied and such that the corresponding preference ordering is strict. Moreover, note that with this specification, for every issue k there are at least $n-1$ voters who prefer position 1 over 0, which implies that

$$(V(a,P))_k = 1 \quad \text{for all } a \in A \text{ and for all } k \in a.$$

If $q = n$, choose $(u_k^i(\cdot))_{k\in\mathcal{K}}$ as follows. For $i = 1, \ldots, n-1$, let

$$\begin{aligned}
u_i^i(1) &> u_i^i(-) > u_i^i(0), && (31)\\
u_k^i(0) &> u_k^i(-) > u_k^i(1) \quad \text{for all } k \in \mathcal{K}, k \neq i, && (32)\\
\sum_{k=1}^{r} u_k^i(-) &> \sum_{k=1}^{r} u_k^i(0), && (33)
\end{aligned}$$

and for $i = n$ let

$$\begin{aligned}
u_k^n(1) &> u_k^n(-) > u_k^n(0) \quad \text{for all } k = n, \ldots, r, && (34)\\
u_k^n(0) &> u_k^n(-) > u_k^n(1) \quad \text{for all } k \in \mathcal{K}, k \notin \{n, \ldots, r\}, && (35)\\
\sum_{\substack{l=1\\ l\neq k}}^{r} u_l^n(-) &< \sum_{\substack{l=1\\ l\neq k}}^{r} u_l^n(0) \quad \text{for all } k = n, \ldots, r, && (36)\\
\sum_{k=1}^{r} u_k^n(-) &> \sum_{k=1}^{r} u_k^n(0), && (37)
\end{aligned}$$

Again observe that $(u_k^i(\cdot))_{k\in\mathcal{K}}$ can always be chosen such that conditions (31)-(37) are satisfied and such that the corresponding preference ordering is strict. Also, note that with this specification, for every issue k there is at most one voter who prefers position 1 over 0, which implies that

$$(V(a,P))_k = 0 \quad \text{for all } a \in A \text{ and for all } k \in a.$$

For all quotas q the preferences we have specified above have the following properties: Every voter $i \in \{1, \ldots, n-1\}$ gets his most preferred position on all issues but issue i, and voter n gets his most preferred position on all issues but issues $n, \ldots, r$. Moreover, all voters prefer to stop at an agenda that contains all but the issues in $\mathcal{F}$ rather than adding all issues in $\mathcal{F}$ to the given agenda.

Finally, voter n prefers to add all remaining issues in $\mathcal{F}$ to any agenda that already contains some issue in $\{n, \dots, r\}$ and all issues not in $\mathcal{F}$. We will use these properties to prove that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda.

Let $(CE(a,P))_{a \in A}$ be any consistent equilibrium collection of sets of continuation agendas and let $a \in A \setminus A^K$ with $k \in a$ for some $k \in \mathcal{F} = \{1, \dots, r\}$. We will prove that $CE(a,P) \subset A^K$. If $a \in A^K$ there is nothing to prove. Hence, let $a \in A^m$ for some $m < K$. The proof is by induction over m. Let $m = K-1$ and let $l \notin a$. If $l \notin \mathcal{F}$, then $V((a,l),P) \succ_i V(a,P)$ for all i and (E2) implies that $a \notin CE(a,P)$. It follows that $CE(a,P) = \{(a,l)\} \subset A^K$. If $l \in \mathcal{F}$, then there are $n-1$ voters who prefer $V((a,l),P)$ over $V(a,P)$ and again we conclude that $CE(a,P) = \{(a,l)\} \subset A^K$.

Now suppose that for all $\overline{m} \leq K-1$ it is true that $CE(a,P) \subset A^K$ for all $a \in A^m$ with $\overline{m} \leq m \leq K-1$ and $k \in a$ for some $k \in \mathcal{F}$. Let $a \in A^{\overline{m}-1}$. If $k \in \{1, \dots, n-1\}$, then voter k gets his most preferred position on all issues not in a, which implies that for all $l \notin a$,

$$V(a',P) \succ_k V(a,P) \text{ for all } a' \in CE((a,l),P).$$

Hence, (E2) implies that $a \notin CE(a,P)$ and

$$CE(a,P) \subset \bigcup_{l \notin a} CE((a,l),P).$$

From the induction hypothesis we conclude that $CE(a,P) \subset A^K$.

If $l \notin a$ for all $l \in \{1, \dots, n-1\}$ and $k \in a$ for some $k \in \{n, \dots, r\}$, let $a' \in CE((a,l),P)$ for some $l \in \{1, \dots, n-1\}$. Then by the induction hypothesis it follows that $a' \in A^K$ and (29), respectively (36) imply that

$$V(a',P) \succ_n V(a,P).$$

Again, (E2) implies that $a \notin CE(a,P)$ and from the induction hypothesis we conclude that $CE(a,P) \subset A^K$.

Now let $a \in A$ contain all issues but those in $\mathcal{F}$. Then using the same argument as in the first part of the proof, where we considered the case $r \leq n$, (26), (30), (33), (37) imply that $CE(a,P) = \{a\}$. Moreover, as in the first part of the proof this implies that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda. This proves the theorem.

$\square$

Proof of Lemma 5.1: Let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q \in \{1, \dots, n\}$ and let $P = (\succ_1, \dots, \succ_n) \in \bar{\mathcal{S}}^n$ be such that for all $i \in I$ and for all $k \in \mathcal{K}$,

$$\max\{u_k^i(1), u_k^i(0)\} > u_k^i(-),$$

where $(u_k^i(\cdot))_{k \in \mathcal{K}}$ is the collection of utility scalars in an additively separable utility representation of $\succ_i$. Let J be representative of I at P and let $(CE(a, P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas. Then, for $a \in A^{K-1}$ and $k \notin a$ (E1) implies that

$$CE((a, k), P) = \{(a, k)\}.$$

Moreover, $(V(a, P))_k = -$ and $(V((a, k), P))_k \in \{0, 1\}$. If $(V((a, k), P))_k = 1$ ($(V((a, k), P))_k = 0$) then there exists at least one voter i with $u_k^i(1) > u_k^i(0)$ ($u_k^i(0) > u_k^i(1)$). In either case the fact that $\max\{u_k^i(1), u_k^i(0)\} > u_k^i(-)$ implies that

$$V((a, k), P) \succ_j V(a, P)$$

for at least one agenda setter j since J is representative of I. Using (E2) we conclude that $a \notin CE(a, P)$ and hence $CE(a, P) = \{(a, k)\} \in A^K$ by (E1).

$\square$

Proof of Proposition 5.2: Let there be an odd number n of voters and let $V : A \times \bar{\mathcal{S}}^n \to X$ be voting by quota $q = \frac{n+1}{2}$. Let $P = (\succ_1, \dots, \succ_n) \in \bar{\mathcal{S}}^n$ be such that for all $i \in I$ and for all $k \in \mathcal{K}$,

$$\max\{u_k^i(1), u_k^i(0)\} > u_k^i(-), \tag{38}$$

where $(u_k^i(\cdot))_{k \in \mathcal{K}}$ is the collection of utility scalars in an additively separable utility representation of $\succ_i$. Let J be representative of I at P and let $(CE(a, P))_{a \in A}$ be an equilibrium collection of sets of continuation agendas. Obviously, for any full agenda $a \in A^K$,

$$CE(a, P) = \{a\}.$$

Now consider any agenda $a \in A^{K-1}$ and let $k \notin a$. By condition (E1) and Lemma 5.1, we conclude that

$$CE(a, P) = \{(a, k)\}.$$

Now consider any agenda $a \in A^{K-2}$ and let k and l be the free issues at a. By condition (E1) and the previous reasoning, $CE(a, P)$ is a nonempty subset of $\{a\} \cup CE(a, k) \cup CE(a, l) = \{a, (a, k, l), (a, l, k)\}$. Observe that agendas (a, k, l) and (a, l, k) are outcome equivalent since the voting procedure does not depend on the ordering of the issues in the agenda. Let $x = V(a, P)$ and $y = V((a, k, l), P) = V((a, l, k), P)$. Then $y_m = x_m$ for all $m \in a$ and $y_k, y_l \in \{0, 1\}$. By (E2) $a \in CE(a, P)$ if and only if for all agenda setters $i \in J$,

$$u_k^i(-) + u_l^i(-) > u_k^i(y_k) + u_l^i(y_l). \tag{39}$$

Since J is representative of I this implies that (39) is satisfied for all voters $i \in I$. Together with (38) this implies that no voter $i \in I$ is in the winning majority for both issues, k and l. However, if n is odd and $q = \frac{n+1}{2}$, there always exists at least one voter who belongs to the winning majority for both issues. This implies that (39) is violated for at least one voter i and hence it must be violated for at least one agenda setter $i \in J$ since J is representative of I. We conclude that

$$CE(a, P) \subset A^K \text{ for all } a \in A^{K-2}$$

and Lemma 3.1 then implies that

$$CE(a, P) \subset A \setminus \left(A^{K-1} \cup A^{K-2}\right) \text{ for all } a \in A.$$

$\square$

Proof of Theorem 5.5:

We prove the claim for $n = 3$ voters which are also agenda setters and note that the extension to an arbitrary odd number of voters $n > 3$ is straightforward: For any preference profile $(\succ_1, \succ_2, \succ_3)$ for three voterss we can define a preference profile for $n > 3$ voters, such that every voter's preference ordering is either $\succ_1, \succ_2$

or $\succ_3$ and such that the majority relation on X is preserved.[18]

From now on we assume that $n = 3$. Let $K \geq 2$ and let $\mathcal{F} = \emptyset$. Take any preference ordering $\succ \in \mathcal{S}$ that satisfies betweenness and let $P = (\succ_1, \ldots, \succ_n)$ be such that $\succ_i = \succ$ for all $i = 1, \ldots, n$. Then, for all $a \in A \setminus A^K$ and for all $a' = (a, k, \ldots)$ with $k \notin a$,

$$V(a', P) \succ_i V(a, P). \tag{40}$$

Let $(CE(a, P))_{a \in A}$ be any consistent equilibrium collection of sets of continuation agendas. (40) and (E2) then imply that $a \notin CE(a, P)$ for all $a \in A \setminus A^K$ and we conclude that

$$CE(\varnothing, P) \subset A^K$$

by Lemma 3.1. Hence, there are no free issues at any equilibrium agenda at P.

The remainder of the proof consists of four steps. Steps 1 and 2 deal with the case where there are at least two free issues. In Step 1 we show that for any $K \geq 2$ there exists a profile P of separable preferences that satisfy betweenness such that all K issues are free at any consistent equilibrium agenda at P. In Step 2 we use a lexicographic extension of the preferences defined in Step 1 to prove that for $K \geq 3$ and $\#\mathcal{F} \geq 2$ there exists a profile P of separable preferences that satisfy betweenness such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda. Steps 3 and 4 consider the case with one free issue. In Step 3 we prove that for $K = 2$ and $\#\mathcal{F} = 1$ there exists a profile P of preferences that satisfy betweenness such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda (note that there exists no profile of separable preferences with this property as we show in Proposition 5.3). In Step 4 we prove that for $K = 3$ and $\#\mathcal{F} = 1$ there exists a profile P of separable preferences that satisfy betweenness such that $\mathcal{F}$ is the set of free issues at any consistent equilibrium

[18] If $n = 5$ let the preference orderings $\succ'_i$ for voters $i = 1, \ldots, 5$, be given by $\succ'_i = \succ_1$ for $i = 1, 2$, $\succ'_i = \succ_2$ for $i = 3, 4$, and $\succ'_5 = \succ_3$. If $n \geq 7$ let $m \in \mathbb{N}$ and $k \in \{0, 1, 2\}$ be such that $n = 3m + k$, and let the preference orderings $\succ'_i$ for voters $i = 1, \ldots, n$, be given by $\succ'_i = \succ_1$ for $i = 1, \ldots, m$, $\succ'_i = \succ_2$ for $i = m + 1, \ldots, 2m$, and $\succ'_i = \succ_3$ for $i = 2m + 1, \ldots, n$. Let M be the simple majority relation at the preference profile $(\succ_1, \succ_2, \succ_3)$ and let M' be the simple majority relation at the preference profile $(\succ'_1, \ldots, \succ'_n)$. Then it is straightforward to show that for all $x, y \in X$, xMy if and only if $xM'y$.

agenda. Finally, in Step 5 we use a lexicographic extension of the preferences defined in Step 4 to extend the case with one free issue from $K = 3$ to an arbitrary number of issues $K \geq 4$.

Step 1: In the following we prove that for any set of issues $\mathcal{K} = \{1, \ldots, K\}$ with $K \geq 2$ there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$, where $\succ_i$ satisfies betweenness for all i, such that the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \mathcal{K}$. Note that this means that $\varnothing$ is the unique consistent equilibrium agenda at P.

Let $\mathcal{K} = \mathcal{F} = \{1, 2\}$. Then Section 4.2 provides an example where $\mathcal{F}$ is the set of free issues at any consistent equilibrium agenda.

Let $\mathcal{K} = \mathcal{F} = \{1, 2, 3\}$ and let the voters' preference orderings be given by Table 7. Note that the voters' preferences are separable and satisfy betweenness. It is immediate to see that there is a Condorcet winner for any agenda a which is the alternative that has position 1 for each issue on the agenda. Thus, for any agenda $a \in A$ the voting outcome under the amendment procedure satisfies

$$(V(a, P))_k = 1 \text{ for all } k \in a.$$

We now solve backwards for the equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a, P) = \{a\}$$

for all full agendas a.

Now consider an agenda a of length 2. By condition (E1), $CE(a, P)$ is a nonempty subset of $\{a\} \cup CE((a, k), P) = \{a, (a, k)\}$, where $k \notin a$. By condition (E2), $a \in CE(a, P)$ is ruled out since there is always one voter who strictly prefers $(1, 1, 1)$, which is the voting outcome at agenda (a, k), over x with $x_k = -$ and $x_l = 1$ for all $l \neq k$, which is the voting outcome at agenda a. Hence,

$$CE(a, P) = \{(a, k)\}.$$

Next consider an agenda a of length 1, i.e. $a = (k)$ for some $k \in \{1, 2, 3\}$. Let $h, l \notin a, h \neq l$. By condition (E1), $CE((k), P)$ is a nonempty subset of

$\succ_1$	$\succ_2$	$\succ_3$
$(1,1,0)$	$(1,0,1)$	$(0,1,1)$
$(1,1,-)$	$(-,0,1)$	$(-,1,1)$
$(1,-,0)$	$(1,0,-)$	$(0,-,1)$
$(1,-,-)$	$(0,0,1)$	$(0,1,-)$
$(1,0,0)$	$(-,0,-)$	$(-,1,-)$
$(-,1,0)$	$(1,-,1)$	$(0,-,-)$
$(-,1,-)$	$(1,-,-)$	$(-,-,1)$
$(0,1,0)$	$(-,-,1)$	$(-,-,-)$
$(0,1,-)$	$(0,-,1)$	$(1,1,1)$
$(-,-,0)$	$(-,-,-)$	$(1,-,1)$
$(0,-,0)$	$(1,1,1)$	$(1,1,-)$
$(-,0,0)$	$(1,1,-)$	$(1,-,-)$
$(0,0,0)$	$(-,1,1)$	$(0,0,1)$
$(-,-,-)$	$(-,1,-)$	$(0,0,-)$
$(1,1,1)$	$(0,1,1)$	$(-,0,1)$
$(1,0,-)$	$(0,0,-)$	$(-,0,-)$
$(1,-,1)$	$(1,0,0)$	$(1,0,1)$
$(-,1,1)$	$(1,-,0)$	$(1,0,-)$
$(-,-,1)$	$(-,0,0)$	$(0,1,0)$
$(1,0,1)$	$(0,-,-)$	$(-,1,0)$
$(-,0,-)$	$(0,1,-)$	$(0,-,0)$
$(-,0,1)$	$(0,0,0)$	$(-,-,0)$
$(0,1,1)$	$(-,-,0)$	$(1,1,0)$
$(0,-,-)$	$(0,-,0)$	$(1,-,0)$
$(0,0,-)$	$(1,1,0)$	$(0,0,0)$
$(0,-,1)$	$(-,1,0)$	$(-,0,0)$
$(0,0,1)$	$(0,1,0)$	$(1,0,0)$

Table 7: Preference profile for $\mathcal{K} = \mathcal{F} = \{1,2,3\}$.

$\{(k)\} \cup CE((k,h),P) \cup CE((k,l),P) = \{(k),(k,h,l),(k,l,h)\}$. By condition (E2), $(k) \in CE((k),P)$ is ruled out since there is always one voter who strictly prefers $(1,1,1)$, which is the voting outcome at agendas (k,h,l) or (k,l,h), over x with $x_k = 1$ and $x_l = x_h = -$, which is the voting outcome at agenda (k). Hence,

$$CE((k),P) \subset \{(k,h,l),(k,l,h)\}.$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing,P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{k=1}^{3} CE((k),P)$. Since all agendas in $CE((k),P)$ for $k = 1,2,3$, are full agendas with voting outcome $(1,1,1)$ and all voters strictly prefer $(-,-,-)$ over $(1,1,1)$, all voters prefer the empty agenda over any full agenda. By (E2) this implies that $\varnothing \in CE(\varnothing,P)$.

It remains to prove that $\varnothing$ is the unique consistent equilibrium agenda. Suppose by way of contradiction that $a \in CE(\varnothing,P)$ for some $a \neq \varnothing$. Then a must be a full agenda and since $\varnothing \in CE(\varnothing,P)$, condition (E3) implies that a is rationalizable relative to the empty agenda $\varnothing$. However, no voter prefers the voting outcome at a full agenda over the voting outcome at the empty agenda $\varnothing$ or any other full agenda a' which could be in $CE(\varnothing,P)$. Hence, a is not rationalizable which implies that $a \notin CE(\varnothing,P)$. Hence, we conclude that

$$CE(\varnothing,P) = \{\varnothing\}.$$

Thus, in this case the unique consistent equilibrium agenda is empty and the set of free issues is given by $\mathcal{K}$.

Let $\mathcal{K} = \mathcal{F} = \{1,2,3,4\}$. We take the preference orderings $\succ_i$ for two issues in Table 3 (Section 4) and extend them in a lexicographic way to preference orderings $\succ'_i$ on $\{0,1,-\}^{\mathcal{K}}$: For $i = 1,2,3$, let $\succ'_i$ be such that

$$(x_1,x_2,x_3,x_4) \succ'_i (y_1,y_2,y_3,y_4)$$

if and only if

$$(x_1,x_2) \succ_i (y_1,y_2)$$

or

$$(x_1,x_2) = (y_1,y_2) \text{ and } (x_3,x_4) \succ_i (y_3,y_4).$$

$\succ'_1$	$\succ'_2$	$\succ'_3$
$(0,1,0,1)$	$(1,0,1,0)$	$(1,1,1,1)$
$(0,1,0,0)$	$(1,0,0,0)$	$(1,1,0,1)$
$(0,1,1,1)$	$(1,0,1,1)$	$(1,1,1,0)$
$(0,1,1,0)$	$(1,0,0,1)$	$(1,1,0,0)$
$(0,0,0,1)$	$(0,0,1,0)$	$(0,1,1,1)$
$(0,0,0,0)$	$(0,0,0,0)$	$(0,1,0,1)$
$(0,0,1,1)$	$(0,0,1,1)$	$(0,1,1,0)$
$(0,0,1,0)$	$(0,0,0,1)$	$(0,1,0,0)$
$(1,1,0,1)$	$(1,1,1,0)$	$(1,0,1,1)$
$(1,1,0,0)$	$(1,1,0,0)$	$(1,0,0,1)$
$(1,1,1,1)$	$(1,1,1,1)$	$(1,0,1,0)$
$(1,1,1,0)$	$(1,1,0,1)$	$(1,0,0,0)$
$(1,0,0,1)$	$(0,1,1,0)$	$(0,0,1,1)$
$(1,0,0,0)$	$(0,1,0,0)$	$(0,0,0,1)$
$(1,0,1,1)$	$(0,1,1,1)$	$(0,0,1,0)$
$(1,0,1,0)$	$(0,1,0,1)$	$(0,0,0,0)$

Table 8: Lexicographic extension of the preference orderings in Table 3 to $\{0,1\}^{\mathcal{K}}$ for $\mathcal{K} = \{1,2,3,4\}$.

For illustration Table 8 gives the voters' preference orderings on $\{0,1\}^{\mathcal{K}}$.

Next we determine the voting outcome at all agendas. At the empty agenda $(-,-,-,-)$ is the unique attainable alternative which implies that

$$V(\varnothing, P) = (-,-,-,-).$$

At agenda (k) for $k \in \{1,2,3,4\}$ there are only two attainable alternatives, x and y with $x_k = 1$, $y_k = 0$ and $x_l = y_l = -$ for $l \neq k$. If $k \in \{1,3\}$, then voters 2 and 3 prefer position 1 over position 0 for issue k which implies that

$$V((1), P) = (1,-,-,-) \text{ and } V((3), P) = (-,-,1,-).$$

If $k \in \{2,4\}$, then voters 1 and 3 prefer position 1 over position 0 for issue k which implies that

$$V((2), P) = (-,1,-,-) \text{ and } V((4), P) = (-,-,-,1).$$

Next we determine the voting outcome at all agendas of length 2. The analysis of case $K = 2$ implies that there exists an ordering of the alternatives in $\{0,1\}^{\{1,2\}}$ such that

$$V((1,2), P) = V((2,1), P) = (0,0,-,-).$$

Similarly, there exists an ordering of the alternatives in $\{0,1\}^{\{3,4\}}$ such that

$$V((3,4), P) = V((3,4), P) = (-,-,0,0).$$

Consider agendas $(2,3)$ and $(3,2)$. Then $(-,1,1,-) \succ_i' (-,0,x_3,-)$ for all $x_3 \in \{0,1\}$ and $i = 1,3$, and $(-,1,1,-) \succ_i' (-,1,0,-)$ for $i = 2,3$. Hence, $(-,1,1,-)$ is the Condorcet winner in $X(2,3)$ which implies that

$$V((2,3), P) = V((3,2), P) = (-,1,1,-).$$

Similarly, we derive

$$V((1,3), P) = V((3,1), P) = (1,-,1,-),$$
$$V((1,4), P) = V((4,1), P) = (1,-,-,1),$$
$$V((2,4), P) = V((4,2), P) = (-,1,-,1).$$

Next consider all agendas of length 3. Let $a \in A^3$ with $4 \notin a$. We will now argue that there exists an ordering of the alternatives in $X(a)$ such that $(0,0,1,-)$ is the voting outcome under the amendment procedure. To see this note that by definition of the preference orderings $\succ_i'$, under simple majority voting $(0,0,1,-)$ is dominated by $(1,0,x_3,-)$ and $(0,1,x_3,-)$ for all $x_3 \in \{0,1\}$ and $(0,0,1,-)$ dominates all remaining alternatives in $X(a)$. Moreover, $(1,0,x_3,-)$ and $(0,1,x_3,-)$ are dominated by $(1,1,1,-)$ for all $x_3 \in \{0,1\}$. It follows from the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) that there exists an ordering of the alternatives in $X(a)$ such that $(0,0,1,-)$ is the voting outcome under the amendment procedure. We take this ordering and get

$$V(a,P) = (0,0,1,-) \text{ for all } \ a \in A^3 \text{ with } 4 \notin a.$$

In a similar way one shows that for all $a \in A^3$ with $4 \in a$ there exist orderings of the alternatives in $X(a)$ such that

$$V(a,P) = (-,1,0,0) \text{ for all } \ a \in A^3 \text{ with } 1 \notin a,$$
$$V(a,P) = (1,-,0,0) \text{ for all } \ a \in A^3 \text{ with } 2 \notin a,$$
$$V(a,P) = (0,0,-,1) \text{ for all } \ a \in A^3 \text{ with } 3 \notin a.$$

Finally, we determine the voting outcome at all full agendas $a \in A^4$. Note that by definition of the preference orderings $\succ_i'$, under simple majority voting $(0,0,0,0)$ is dominated by $(0,0,1,0), (0,0,0,1), (1,0,x_3,x_4)$ and $(0,1,x_3,x_4)$ for all $x_3, x_4 \in \{0,1\}$, while $(0,0,0,0)$ dominates all remaining alternatives in $X(a)$. Moreover, $(1,1,1,1)$ dominates $(0,0,1,0), (0,0,0,1), (1,0,x_3,x_4)$ and $(0,1,x_3,x_4)$ for all $x_3, x_4 \in \{0,1\}$. Again we use the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) to conclude that there exists an ordering of the alternatives in $X(a)$ such that $(0,0,0,0)$ is the voting outcome under the amendment procedure. We take this ordering and get

$$V(a,P) = (0,0,0,0) \text{ for all } \ a \in A^4.$$

We now solve backwards for the equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a,P) = \{a\} \text{ for all } a \in A^4.$$

Now consider an agenda of length 3. Let $a \in A^3$ and let $1 \notin a$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,1),P) = \{a,(a,1)\}$. By condition (E2), $a \in CE(a,P)$ is ruled out since voter 2 strictly prefers the voting outcome at agenda $(a,1)$, which is $(0,0,0,0)$ over the voting outcome at agenda a which is $(-,1,0,0)$. Hence,

$$CE(a,P) = \{(a,1)\}.$$

In the same way one proves that

$$CE(a,P) = \{(a,k)\} \text{ for all } k \notin a.$$

Next consider agendas of length 2. To begin with, let $a \in \{(1,2),(2,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,3),P) \cup CE((a,4),P) = \{a,(a,3,4),(a,4,3)\}$. By condition (E2), $a \in CE(a,P)$ since all voters prefer the outcome under (a), which is $(0,0,-,-)$, over the outcome under $(a,3,4)$ or $(a,4,3)$ which is $(0,0,0,0)$. Moreover, given $a \in CE(a,P)$ none of the agendas $(a,3,4)$ or $(a,4,3)$ is rationalizable relative to a and hence (E3) implies that

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

In a similar way it follows that

$$CE((3,4),P) = \{(3,4)\} \text{ and } CE((4,3),P) = \{(4,3)\}.$$

Let $a \in \{(1,3),(3,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,2),P) \cup CE((a,4),P) = \{a,(a,2,4),(a,4,2)\}$. By (E2), $a \in CE(a,P)$ is ruled out since voter 1 strictly prefers the voting outcome at any full agenda, which is $(0,0,0,0)$, over the voting outcome at agenda a, which is $(1,-,1,-)$. Hence,

$$CE((1,3),P), CE((3,1),P) \subset A^4.$$

In a similar way it follows that

$$CE((1,4),P), CE((4,1),P) \subset A^4,$$
$$CE((2,3),P), CE((3,2),P) \subset A^4,$$
$$CE((2,4),P), CE((4,2),P) \subset A^4.$$

Next consider agendas of length 1. To begin with, let $a = (1)$. By condition (E1), $CE((1), P)$ is a nonempty subset of $\{(1)\} \cup \bigcup_{k=2}^{4} CE((1,k), P)$. By (E2), $(1) \in CE((1), P)$ is ruled out since voter 2 strictly prefers the voting outcome at agenda $(1,2) \in CE((1,2), P)$, which is $(0, 0, -, -)$, over the voting outcome at agenda (1), which is $(1, -, -, -)$. Moreover, any agenda in $CE((1,3), P)$ or $CE((1,4), P)$ is a full agenda with voting outcome $(0, 0, 0, 0)$. If any such agenda were in $CE((1), P)$, then $(1,2)$ is rationalizable relative to (1) since all voters prefer the voting outcome under $(1,2)$, which is $(0, 0, -, -)$, over $(0, 0, 0, 0)$. (E3) then requires that $(1,2) \in CE((1), P)$ which in turn implies that no agenda in $CE((1,3), P)$ or $CE((1,4), P)$ is rationalizable. By (E3) we conclude that no agenda in $CE((1,3), P)$ or $CE((1,4), P)$ belongs to $CE((1), P)$. Hence,

$$CE((1), P) = \{(1,2)\}.$$

In a similar way it follows that

$$\begin{aligned} CE((2), P) &= \{(2,1)\}, \\ CE((3), P) &= \{(3,4)\}, \\ CE((4), P) &= \{(4,3)\}. \end{aligned}$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing, P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{k=1}^{4} CE((k), P) = \{\varnothing, (1,2), (2,1), (3,4), (4,3)\}$. Since all voters strictly prefer the voting outcome under the empty agenda, which is $(-, -, -, -)$, over the voting outcome under agendas $(1,2)$ or $(2,1)$, which is $(0, 0, -, -)$, and the voting outcome under agendas $(3,4)$ or $(4,3)$ which is $(-, -, 0, 0)$, it follows that $\varnothing \in CE(\varnothing, P)$ by (E2).

It remains to prove that $\varnothing$ is the unique consistent equilibrium agenda. To this end note that (E3) implies that if any of the agendas $(1,2), (2,1), (3,4), (4,3)$ is in $CE(\varnothing, P)$, then it must be rationalizable. However, the voting outcome under agendas $(3,4)$ and $(4,3)$ is $(-, -, 0, 0)$ which is strictly worse for all voters than the voting outcome under the empty agenda, which is $(-, -, -, -)$, and the voting outcome under agendas $(1,2)$ and $(2,1)$, which is $(0, 0, -, -)$. This implies that neither $(3,4)$ nor $(4,3)$ is rationalizable and by (E3) neither of these agendas is in $CE(\varnothing, P)$. But then neither $(1,2)$ nor $(2,1)$ are rationalizable relative to

$\varnothing$ because all voters strictly prefer $(-,-,-,-)$ over the voting outcome under agendas $(1,2)$ and $(2,1)$, which is $(0,0,-,-)$. Hence, (E3) implies that

$$CE(\varnothing, P) = \{\varnothing\}.$$

Thus, the unique consistent equilibrium agenda is empty and the set of free issues is given by $\mathcal{K} = \{1,2,3,4\}$.

Let $\mathcal{K} = \mathcal{F} = \{1,2,3,4,5\}$. We then construct a preference profile using the preference orderings in Table 3 and Table 7. For $i = 1,2,3$, let $\succ_i^2$ be voter i's preference ordering in Table 3 and let $\succ_i^3$ be voter i's preference ordering in Table 7. We extend these preferences in a lexicographic way to preference orderings $\succ_i'$ on $\{0,1,-\}^{\mathcal{K}}$: For $i = 1,2,3$, let $\succ_i'$ be such that

$$(x_1, x_2, x_3, x_4, x_5) \succ_i' (y_1, y_2, y_3, y_4, y_5)$$

if and only if

$$(x_1, x_2) \succ_i^2 (y_1, y_2)$$

or

$$(x_1, x_2) = (y_1, y_2) \text{ and } (x_3, x_4, x_5) \succ_i^3 (y_3, y_4, y_5).$$

Then, similar to case $\mathcal{K} = \mathcal{F} = \{1,2,3,4\}$ above we can show that the unique consistent equilibrium agenda is empty and hence the set of free issues is given by $\mathcal{K} = \{1,2,3,4,5\}$.

Let $\mathcal{K} = \mathcal{F} = \{1,\ldots,K\}$ with $K \geq 6$. Then either K is even or $K = 2m+3$ for some $m > 1$. In both cases we can proceed as in the previous two cases with $K = 4$ and $K = 5$ to construct an example with three voters and separable preferences such that all issues are free at the unique consistent equilibrium agenda. We simply extend the voters' preference orderings for $K = 2$ and $K = 3$ in a lexicographic way to preference orderings for the given number of issues.

Step 2: Let $\mathcal{K} = \{1,\ldots,K\}$ with $K \geq 3$ and let $\mathcal{F} = \{1,\ldots,F\} \subset \mathcal{K}$ with $F \geq 2$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$, where $\succ_i$ satisfies betweenness for all i, such that the set of free issues at the unique consistent equilibrium agenda is $\mathcal{F}$. To this end take the preference orderings $\succ_i$

used for the case $\mathcal{K} = \mathcal{F}$ in Step 1 and extend them in a lexicographic way to preference orderings $\succ'_i$ on $\{0, 1, -\}^{\mathcal{K}}$: For $i = 1, 2, 3$, let $\succ'_i$ be such that

$$(x_1, \ldots, x_K) \succ'_i (y_1, \ldots, y_K)$$

if and only if one of the following two conditions is satisfied:

(i) There exists some l with $F + 1 \leq l \leq K$, such that $x_k = y_k$ for $k = F + 1, \ldots, l - 1$, and either

$$\begin{aligned} & x_l = 1 \text{ and } y_l \in \{-, 0\} \\ \text{or} \quad & x_l = - \text{ and } y_l = 0. \end{aligned}$$

(ii) $x_k = y_k$ for $k = F + 1, \ldots, K$, and

$$(x_1, \ldots, x_F) \succ_i (y_1, \ldots, y_F).$$

Hence, all voters first consider the positions on issues $F + 1, \ldots, K$ (in that order) and all prefer position 1 over $-$ and $-$ over 0 on these issues. Only if two alternatives have the same positions on all issues $F + 1, \ldots, K$, the positions on the remaining issues are relevant. In that case voter i's preference over the alternatives is determined by the preference $\succ_i$ over the positions on issues $1, \ldots, F$.

Then, by Pareto efficiency of the amendment procedure, $(V(a, P))_k = 1$ for all agendas a with $k \in a$ and $k \in \{F + 1, \ldots, K\}$ independent of the ordering of the alternatives in $X(a)$ under the amendment procedure. Moreover, any consistent equilibrium agenda a must contain all issues in $\{F + 1, \ldots, K\}$. Suppose this were not true, i.e. there exists a consistent equilibrium agenda a with $k \notin a$ for some $k \in \{F + 1, \ldots, K\}$. Lemma 3.1 implies that $a \in CE(a, P)$. Hence, by (E2) it must be true that for all i and for all $a' \in CE((a, k), P))$,

$$V(a, P) \succ_i V(a', P).$$

However, by definition of $\succ_i$ this is impossible since $(V(a', P))_k = 1$ and $(V(a, P))_k = -$. Therefore, we conclude that any consistent equilibrium agenda contains all issues in $\{F + 1, \ldots, K\}$. We will now prove that there are no additional issues on any consistent equilibrium agenda if the order of vote under the amendment procedure is chosen in an appropriate way.

Let $(CE(a,P))_{a\in A}$ be any consistent equilibrium collection of sets of continuation agendas and let a be any agenda that is a permutation of $(F+1,\ldots,K)$. Then, given the definition of voters' preferences, Step 1 implies that there exists an order of vote under the amendment procedure, such that

$$CE(a,P)=\{a\} \tag{41}$$

Moreover, all such agendas a yield the same voting outcome x with $x_k=1$ for all $k=F+1,\ldots,K$, and $x_k=-$ for all $k=1,\ldots,K$.

Let a' be any agenda that is either empty or only contains issues in $\{F+1,\ldots,K\}$. We will prove by backwards induction over the number of issues in a', that any agenda in $CE(a',P)$ is a permutation of $(F+1,\ldots,K)$. (41) implies that the claim is true if a' is a permutation of $(F+1,\ldots,K)$. Now suppose the claim is true for all agendas that contain at least $l+1$ issues in $\{F+1,\ldots,K\}$ and no issues in $\{1,\ldots,K\}$, where $0\leq l<K-F$. Let $a'=(a_1,\ldots,a_l)$ be an agenda with $a_1,\ldots,a_l\in\{F+1,\ldots,K\}$. Since a' does not contain all issues in $\{F+1,\ldots,K\}$, (E1) implies that $CE(a',P)$ is a nonempty subset of $\bigcup_{k\notin a'}CE((a',k),P)$. By the induction hypothesis any agenda in $CE((a',k),P)$ is a permutation of $(F+1,\ldots,K)$ for all $k\notin a'$ with $k\in\{F+1,\ldots,K\}$. By definition of the voters' preferences and the proof in Step 1 it follows that there exists an order of vote under the amendment procedure, such that all voters have the same preferences over voting outcomes $V(a'',P)$ for all $a''\in CE((a',k),P)$ and for all $k\notin a'$. Moreover, all voters prefer $V(a,P)$, where a is some permutation of $(F+1,\ldots,K)$, over $V(a'',P)$ for any agenda $a''\in CE((a',k),P)$ for all $k\in\{1,\ldots,F\}, k\notin a'$. Hence, the voting outcomes $V(a'',P)$ for all $a''\in CE((a',k),P)$ and for all $k\in\{1,\ldots,K\}, k\notin a'$, are Pareto ranked with $V(a'',P)$ being preferred over $V(a''',P)$ for all $a''\in CE((a',k),P)$ with $k\notin a'$ and $k\in\{F+1,\ldots,K\}$, and for all $a'''\in CE((a',k'),P)$ for all $k'\notin a'$ and $k'\in\{1,\ldots,F\}$. Therefore, consistency (E3) implies that

$$CE(a',P)\subset\bigcup_{k\notin a',k\in\{F+1,\ldots,K\}}CE((a',k),P).$$

Hence, by the induction hypothesis any agenda in $CE(a',P)$ is a permutation of $(F+1,\ldots,K)$. This proves the claim.

We conclude that any agenda in $CE(\varnothing, P)$ is a permutation of $(F+1, \ldots, K)$, i.e. the set of free issues at any consistent equilibrium agenda is given by $\mathcal{F}$.

Step 3: Let $\mathcal{K} = \{1, 2\}$ and $\#\mathcal{F} = 1$. W.l.o.g. let $\mathcal{F} = \{2\}$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{P}^3$, where $\succ_i$ satisfies betweenness for all i, such that the set of free issues at the unique consistent equilibrium agenda is $\mathcal{F} = \{2\}$. Let preference orderings be given by Table 9. Note that the voters' preferences satisfy betweenness, but not separability.

$\succ_1$	$\succ_2$	$\succ_3$
$(1,0)$	$(0,1)$	$(1,1)$
$(1,-)$	$(1,0)$	$(1,-)$
$(-,0)$	$(1,-)$	$(0,0)$
$(0,0)$	$(-,0)$	$(-,0)$
$(-,-)$	$(-,-)$	$(-,-)$
$(0,-)$	$(0,-)$	$(0,-)$
$(0,1)$	$(0,0)$	$(-,1)$
$(-,1)$	$(-,1)$	$(0,1)$
$(1,1)$	$(1,1)$	$(1,0)$

Table 9: Preference orderings for $\mathcal{K} = \{1, 2\}$ and $\mathcal{F} = \{2\}$.

In order to solve for the equilibrium agendas we first determine the voting outcome for any agenda that contains at most one alternative. At the empty agenda the outcome is the unique attainable alternative $(-,-)$, i.e.

$$V(\varnothing, P) = (-,-).$$

At agenda $a = (1)$ the outcome is

$$V((1), P) = (1,-),$$

because a majority of voters prefers $(1,-)$ over $(0,-)$, and at agenda $a = (2)$ the outcome is

$$V((2), P) = (-,0),$$

since a majority of voters prefers $(-,0)$ over $(-,1)$. Finally, we determine the voting outcome at the full agendas, $(1,2)$ and $(2,1)$, with attainable sets

$$X(1,2) = X(2,1) = \{(0,0),(0,1),(1,0),(1,1)\}.$$

Figure 2 shows the dominance relation on $\{(0,0),(0,1),(1,0),(1,1)\}$ that results from pairwise simple majority voting.

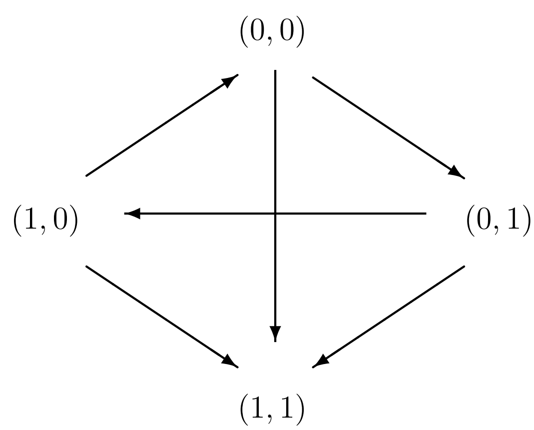

Figure 2: Dominance relation on $\{(0,0),(0,1),(1,0),(1,1)\}$ under pairwise simple majority voting for the preferences in Table 9. The arrows point to the alternatives that are beaten under simple majority voting.

Hence, there is a majority cycle among $(0,0),(0,1)$ and $(1,0)$ and all three alternatives dominate $(1,1)$. It follows from the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) that any of the attainable alternatives except for $(1,1)$ is the outcome of sophisticated sequential voting under the amendment procedure for some ordering of the alternatives in in $\{(0,0),(0,1),(1,0),(1,1)\}$. Choose an ordering of the alternatives such that

$$V((1,2),P) = V((2,1),P) = (0,0).$$

In order to solve for the equilibrium collection of sets of continuation agendas starting from the full agendas $(1,2)$ and $(2,1)$. By (E1) it must be that

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

Now consider agenda (1). By condition (E1), $CE((1), P)$ is a nonempty subset of $\{(1), (1, 2)\}$. By condition (E2), $(1) \in CE((1), P)$ since all voters strictly prefer $V((1), P) = (1, -)$ over $(0, 0)$ which is the voting outcome under the equilibrium continuation $CE((1, 2), P) = (1, 2)$. Moreover, $(1, 2)$ is not rationalizable given that $(1) \in CE((1), P)$. Hence, (E3) implies that

$$CE((1), P) = \{(1)\}.$$

Next consider agenda (2). By condition (E1), $CE((2), P)$ is a nonempty subset of $\{(2), (2, 1)\}$. By condition (E2), $(2) \in CE((2), P)$ is ruled out since voter 3 strictly prefers the voting outcome under the equilibrium continuation $CE((2, 1), P) = (2, 1)$ over $V((2), P) = (-, 0)$. Hence,

$$CE((2), P) = \{(2, 1)\}.$$

Finally, consider the empty agenda. By condition (E1), $CE(\varnothing, P)$ is a nonempty subset of $\{\varnothing\} \cup CE((1), P) \cup CE((2), P) = \{\varnothing, (1), (2, 1)\}$. Since all voters strictly prefer the voting outcome at agenda (1) over the outcome at the empty agenda, (E2) implies that $\varnothing \notin CE(\varnothing, P)$. Suppose by way of contradiction that $(2, 1) \in CE(\varnothing, P)$. Then, (1) is rationalizable and (E3) implies that $(1) \in CE(\varnothing, P)$. But then (E3) requires that $(2, 1)$ is rationalizable which is not true since all voters prefer $V((1), P) = (1, -)$ over $V((2, 1), P) = (0, 0)$. Hence, we conclude that $CE(\varnothing, P) = \{(1)\}$ which implies that the set of free issues at the unique consistent equilibrium agenda is $\mathcal{F} = \{2\}$.

Step 4: Let $\mathcal{K} = \{1, 2, 3\}$ and $\#\mathcal{F} = 1$. W.l.o.g. let $\mathcal{F} = \{3\}$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$, where $\succ_i$ satisfies betweenness for all i, such that the set of free issues at the unique consistent equilibrium agenda is $\mathcal{F} = \{3\}$. Let preference orderings be given by Table 10. Note that the voters' preferences are separable and satisfy betweenness.

In order to solve for the equilibrium agendas we first determine the voting outcome at all agendas. At the empty agenda $(-, -, -)$ is the unique attainable alternative which implies that

$$V(\varnothing, P) = (-, -, -).$$

$\succ_1$	$\succ_2$	$\succ_3$
$(0,1,1)$	$(1,0,1)$	$(1,1,0)$
$(0,-,1)$	$(-,0,1)$	$(1,-,0)$
$(0,0,1)$	$(0,0,1)$	$(1,1,-)$
$(0,1,-)$	$(1,0,-)$	$(1,-,-)$
$(0,-,-)$	$(-,0,-)$	$(-,1,0)$
$(0,0,-)$	$(1,-,1)$	$(0,1,0)$
$(-,1,1)$	$(0,0,-)$	$(-,-,0)$
$(1,1,1)$	$(1,-,-)$	$(-,1,-)$
$(-,1,-)$	$(1,1,1)$	$(0,1,-)$
$(1,1,-)$	$(-,-,1)$	$(0,-,0)$
$(-,-,1)$	$(-,1,1)$	$(1,0,0)$
$(-,0,1)$	$(0,-,1)$	$(-,-,-)$
$(1,-,1)$	$(0,1,1)$	$(1,0,-)$
$(-,-,-)$	$(-,-,-)$	$(0,-,-)$
$(-,0,-)$	$(1,1,-)$	$(-,0,0)$
$(1,0,1)$	$(0,-,-)$	$(0,0,0)$
$(1,-,-)$	$(1,0,0)$	$(-,0,-)$
$(1,0,-)$	$(1,-,0)$	$(0,0,-)$
$(0,1,0)$	$(-,0,0)$	$(1,1,1)$
$(-,1,0)$	$(-,-,0)$	$(1,-,1)$
$(0,-,0)$	$(-,1,-)$	$(-,1,1)$
$(0,0,0)$	$(0,0,0)$	$(-,-,1)$
$(-,-,0)$	$(0,-,0)$	$(0,1,1)$
$(-,0,0)$	$(0,1,-)$	$(0,-,1)$
$(1,1,0)$	$(1,1,0)$	$(1,0,1)$
$(1,-,0)$	$(-,1,0)$	$(-,0,1)$
$(1,0,0)$	$(0,1,0)$	$(0,0,1)$

Table 10: Preference orderings for $\mathcal{K} = \{1,2,3\}$ and $\mathcal{F} = \{3\}$.

At agenda (1) there are only two attainable alternatives, $(1,-,-)$ and $(0,-,-)$. Since voters 2 and 3 prefer $(1,-,-)$ over $(0,-,-)$ it follows that

$$V((1), P) = (1,-,-).$$

At agenda (2) there are only two attainable alternatives, $(-,1,-)$ and $(-,0,-)$. Since voters 1 and 3 prefer $(-,1,-)$ over $(-,0,-)$ it follows that

$$V((2), P) = (-,1,-).$$

At agenda (3) there are only two attainable alternatives, $(-,-,1)$ and $(-,-,0)$. Since voters 2 and 3 prefer $(-,-,1)$ over $(-,-,0)$ it follows that

$$V((3), P) = (-,-,1).$$

Next we consider all agendas of length 2. At agendas, $(1,2)$ and $(2,1)$ the attainable set is

$$X(1,2) = X(2,1) = \{(0,0,-),(0,1,-),(1,0,-),(1,1,-)\}.$$

Note that under simple majority voting $(0,0,-)$ is dominated by $(0,1,-)$ and $(1,0,-)$, where both of the latter agendas are dominated by $(1,1,-)$ which in turn is dominated by $(0,0,-)$. Hence, using the characterization results in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) we conclude that $(0,0,-)$ is the outcome under the amendment procedure for some ordering of the alternatives.[19] If we take this ordering of vote under the amendment procedure we obtain

$$V((1,2), P) = V((2,1), P) = (0,0,-).$$

At agendas $(1,3)$ and $(3,1)$ the attainable set is

$$X(1,3) = X(3,1) = \{(0,-,0),(0,-,1),(1,-,0),(1,-,1)\}.$$

Since $(1,-,1)$ dominates any other attainable alternative in pairwise simple majority voting it is the unique outcome under the amendment procedure for any ordering of the alternatives. Hence, we have

$$V((1,3), P) = V((3,1), P) = (1,-,1).$$

[19] The reader may also verify directly that $(0,0,-)$ is the outcome if voting takes place in the ordering $((0,0,-),(0,1,-),(1,0,-),(1,1,-))$ or $((0,0,-),(1,0,-),(0,1,-),(1,1,-))$.

Similarly, at agendas $(2,3)$ and $(3,2)$ the attainable set is

$$X(2,3) = X(3,2) = \{(-,0,0),(-0,1),(-,1,0),(-,1,1)\}$$

and $(-,1,1)$ dominates any other attainable alternative in pairwise simple majority voting. It is therefore the unique outcome under the amendment procedure for any ordering of the alternatives and we get

$$V((2,3),P) = V((3,2),P) = (-,1,1).$$

Finally, consider all full agendas. The attainable set at any full agenda $a \in A^3$ is

$$X(a) = \{(1,1,1),(1,1,0),(1,0,1),(1,0,0),(0,1,1),(0,1,0),(0,0,1),(0,0,0)\}.$$

Note that $(0,0,1)$ is the unique alternative that dominates $(1,1,1)$ under pairwise simple-majority voting. Since $(0,0,1)$ is dominated by $(1,0,1)$ which in turn is dominated by $(1,1,1)$, the characterizations in Banks (1985, Theorem 3.1) and Barberà and Gerber (2017, Theorem 3.1) imply that there exists an ordering of the alternatives in $X(a)$ such that $(1,1,1)$ is the outcome under the amendment procedure. Hence, for this ordering

$$V(a,P) = (1,1,1) \text{ for all } a \in A^3.$$

We now solve backwards for the equilibrium collection of sets of continuation agendas. By (E1) it follows that

$$CE(a,P) = \{a\} \text{ for all } a \in A^3.$$

Now consider an agenda $a \in \{(1,2),(2,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,3),P) = \{a,(a,3)\}$. Since all voters prefer $(0,0,-)$ over $(1,1,1)$ condition (E2) implies that $a \in CE(a,P)$. Moreover, in this case $(a,3)$ is not rationalizable. Hence, by (E3) we get

$$CE((1,2),P) = \{(1,2)\} \text{ and } CE((2,1),P) = \{(2,1)\}.$$

Next consider an agenda $a \in \{(1,3),(3,1)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,2),P) = \{a,(a,2)\}$. By condition (E2), $a \in$

$CE(a,P)$ is ruled out since voter 1 strictly prefers the voting outcome at agenda $(a,2)$ over the voting outcome at agenda a. Hence,

$$CE((1,3),P) = \{(1,3,2)\} \text{ and } CE((3,1),P) = \{(3,1,2)\}.$$

Next consider an agenda $a \in \{(2,3),(3,2)\}$. By condition (E1), $CE(a,P)$ is a nonempty subset of $\{a\} \cup CE((a,1),P) = \{a,(a,1)\}$. By condition (E2), $a \in CE(a,P)$ is ruled out since voter 2 strictly prefers the voting outcome at agenda $(a,1)$ over the voting outcome at agenda a. Hence,

$$CE((2,3),P) = \{(2,3,1)\} \text{ and } CE((3,2),P) = \{(3,2,1)\}.$$

We then move to agendas of length 1. By condition (E1), $CE((1),P)$ is a nonempty subset of $\{(1)\} \cup CE((1,2),P) \cup CE((1,3),P) = \{(1),(1,2),(1,3,2)\}$. By condition (E2), $(1) \in CE((1),P)$ is ruled out since voter 1 strictly prefers the voting outcome at agenda $(1,2)$ over the voting outcome at agenda (1). Suppose by way of contradiction that $(1,3,2) \in CE((1),P)$. Then $(1,2)$ is rationalizable and (E3) implies that $(1,2) \in CE((1),P)$ and that $(1,3,2)$ must be rationalizable. However, the latter is not true since all voters prefer the voting outcome at agenda $(1,2)$ over the outcome at agenda $(1,3,2)$. Contradiction. Hence, $(1,3,2) \notin CE((1),P)$ and we conclude that

$$CE((1),P) = \{(1,2)\}.$$

By condition (E1), $CE((2),P)$ is a nonempty subset of $\{(2)\} \cup CE((2,1),P) \cup CE((2,3),P) = \{(2),(2,1),(2,3,1)\}$. By condition (E2), $(2) \in CE((2),P)$ is ruled out since voter 2 strictly prefers the voting outcome at the full agenda $(2,3,1)$ over the voting outcome at agenda (2). We conclude that

$$CE((2),P) \subset \{(2,1),(2,3,1)\}.$$

Suppose by way of contradiction that $(2,3,1) \in CE((2),P)$. Since all voters prefer the voting outcome $(0,0,-)$ at agenda $(2,1)$ over the voting outcome $(1,1,1,)$ at agenda $(2,3,1)$ it follows that $(2,1)$ is rationalizable. (E3) then implies that $(2,1) \in CE((2),P)$ and that $(2,3,1)$ is rationalizable. However, the latter is not true since no voter prefers $(1,1,1)$ over $(0,0,-)$. Contradiction. Therefore, we conclude that

$$CE((2),P) = \{(2,1)\}.$$

By condition (E1), $CE((3), P)$ is a nonempty subset of $\{(3)\} \cup CE((3,1), P) \cup CE((3,2), P) = \{(3), (3,1,2), (3,2,1)\}$. By condition (E2), $(3) \in CE((3), P)$ is ruled out since voter 1 strictly prefers the voting outcome at any full agenda over the voting outcome at agenda (3). We conclude that

$$CE((3), P) \subset \{(3,1,2), (3,2,1)\},$$

where all these agendas give the same outcome $(1,1,1)$.

Finally, consider the empty agenda $\varnothing$. By condition (E1), $CE(\varnothing, P)$ is a nonempty subset of $\{\varnothing\} \cup \bigcup_{k=1}^{3} CE((k), P)$, where $CE((1), P) = \{(1,2)\}$ and $CE((2), P) = \{(2,1)\}$ both give voting outcome $(0,0,-)$, and $CE((3), P)$ contains full agendas only, which give outcome $(1,1,1)$. Since voter 1 strictly prefers $(0,0,-)$ over $(-,-,-)$, (E2) implies that $\varnothing \notin CE(\varnothing, P)$. Suppose by way of contradiction that $CE(\varnothing, P)$ contains a full agenda. Since all voters strictly prefer $(0,0,-)$ over $(1,1,1)$, agenda $(1,2)$ is rationalizable and hence is an element of $CE(\varnothing, P)$ by (E3). Moreover, in this case no full agenda is rationalizable and hence $CE(\varnothing, P)$ must not contain any full agenda. From this contradiction we conclude that

$$CE(\varnothing, P) \subset \{(1,2), (2,1)\}.$$

Thus, in this case the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \{3\}$.

Step 5: Let $\mathcal{K} = \{1, \ldots, K\}$ with $K \geq 4$ and let $\#\mathcal{F} = 1$. W.l.o.g. let $\mathcal{F} = \{3\}$. We will prove that there exists a preference profile $P = (\succ_1, \succ_2, \succ_3) \in \mathcal{S}^3$, where $\succ_i$ satisfies betweenness for all i, such that the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \{3\}$. To this end take the preference orderings $\succ_i$ in Table 10 and extend them in a lexicographic way to preference orderings $\succ_i'$ on $\{0,1,-\}^{\mathcal{K}}$: For $i = 1,2,3$, let $\succ_i'$ be such that

$$(x_1, \ldots, x_K) \succ_i' (y_1, \ldots, y_K)$$

if and only if one of the following two conditions is satisfied:

(i) There exists some l with $4 \leq l \leq K$, such that $x_k = y_k$ for $k = 4, \ldots, l-1$, and either

$$x_l = 1 \text{ and } y_l \in \{-, 0\}$$

$$\text{or} \quad x_l = - \text{ and } y_l = 0.$$

(ii) $x_k = y_k$ for $k = 4, \ldots, K$, and

$$(x_1, x_2, x_3) \succ_i (y_1, y_2, y_3).$$

Hence, all voters first consider the positions on issues $4, \ldots, K$ (in that order) and all prefer position 1 over $-$ over 0 on these issues. Only if two alternatives have the same positions on all issues $4, \ldots, K$, the positions on the remaining issues $1, 2, 3$, are relevant. In that case voter i's preference over the alternatives is determined by the preference $\succ_i$ over the positions on issues $1, 2, 3$.

Then, by Pareto efficiency of the amendment procedure, $(V(a, P))_k = 1$ for all agendas a with $k \in a$ and $k \in \{4, \ldots, K\}$ independent of the ordering of the alternatives in $X(a)$ under the amendment procedure. Then, analogously to Step 2, we can use the findings from Step 3 for $\mathcal{K} = \{1, 2, 3\}$ to prove that for any agenda $a \in A$ there exists an order of vote over the alternatives in $X(a)$ under the amendment procedure, such that the set of free issues at any consistent equilibrium agenda is $\mathcal{F} = \{3\}$.

$\square$

Proof of Proposition 5.3: Let $n \geq 3$ be odd and let $P \in \mathcal{S}^n$ be any profile of separable preferences. Let $K = 2$ and let the voting rule be the amendment procedure for some orderings of the alternatives at any agenda a. W.l.o.g. let $V((1), P) = (1, -)$ and $V((2), P) = (-, 1)$. Then there exist sets of voters M_1, M_2 with $\#M_k \geq \frac{n+1}{2}$ for $k = 1, 2$, such that

$$M_1 = \{i \,|\, (1, -) \succ_i (0, -)\} \text{ and } M_2 = \{i \,|\, (-, 1) \succ_i (-, 0)\}.$$

Separability then implies that

$$(1, 1) \succ_i (0, 1) \text{ and } (1, 0) \succ_i (0, 0) \text{ for all } i \in M_1,$$

and

$$(1, 1) \succ_i (1, 0) \text{ and } (0, 1) \succ_i (0, 0) \text{ for all } i \in M_2.$$

Suppose by way of contradiction that the set of of free issues $\mathcal{F}$ contains one issue. W.l.o.g. let $\mathcal{F} = \{2\}$. Then $CE((1), P) = \{(1)\}$ and (E2) implies that

$$(1, -) \succ_i V((1, 2), P) \text{ for all } i. \tag{42}$$

Then there are four cases.

Case 1: $V((1,2),P)=(1,1)$. Since betweenness implies that $(1,1)\succ_i(1,-)\succ_i(1,0)$ for all $i\in M_2$, this contradicts (42).

Case 2: $V((1,2),P)=(0,0)$. Since $(0,0)$ is dominated by $(1,0)$ and $(0,1)$ under pairwise simple majority voting, a necessary condition for $(0,0)$ to be the outcome under the amendment procedure for some ordering of the set of available alternatives at a full agenda is that $(0,0)$ dominates $(1,1)$ (see Barberà and Gerber (2017), Theorem 3.1). Thus, there exists a set of voters M_3 with $\#M_3\geq\frac{n+1}{2}$, such that $(0,0)\succ_i(1,1)$ for all $i\in M_3$. Since M_2 and M_3 both contain at least $\frac{n+1}{2}$ voters, there exists some $i\in M_2\cap M_3$. Betweenness implies that $(0,0)\succ_i(1,1)\succ_i(1,-)\succ_i(1,0)$ for all $i\in M_2\cap M_3$, which contradicts (42).

Case 3: $V((1,2),P)=(1,0)$. Since $(1,0)$ dominates $(0,0)$, but is dominated by $(1,1)$ which in turn dominates $(0,1)$ under pairwise simple majority voting, a necessary condition for $(1,0)$ to be the outcome under the amendment procedure for some ordering of the set of available alternatives at a full agenda is that $(0,0)$ dominates $(1,1)$ (see Barberà and Gerber (2017), Theorem 3.1). Thus, there exists a set of voters M_3 with $\#M_3\geq\frac{n+1}{2}$, such that $(0,0)\succ_i(1,1)$ for all $i\in M_3$. Since M_1 and M_3 both contain at least $\frac{n+1}{2}$ voters, there exists some $i\in M_1\cap M_3$. For all $i\in M_1\cap M_3$, $(1,0)\succ_i(0,0)\succ_i(1,1)$. Betweenness then implies that $(1,0)\succ_i(1,-)$ for all $i\in M_1\cap M_3$, which contradicts (42).

Case 4: $V((1,2),P)=(0,1)$. Since $(0,1)$ dominates $(0,0)$, but is dominated by $(1,1)$ which in turn dominates $(1,0)$ under pairwise simple majority voting, a necessary condition for $(0,1)$ to be the outcome under the amendment procedure for some ordering of the set of available alternatives at a full agenda is that $(0,0)$ dominates $(1,1)$ (see Barberà and Gerber (2017), Theorem 3.1). Thus, there exists a set of voters M_3 with $\#M_3\geq\frac{n+1}{2}$, such that $(0,0)\succ_i(1,1)$ for all $i\in M_3$. Since M_2 and M_3 both contain at least $\frac{n+1}{2}$ voters, there exists some $i\in M_2\cap M_3$. For all $i\in M_2\cap M_3$, $(0,1)\succ_i(0,0)\succ_i(1,1)\succ_i(1,0)$. Betweenness then implies that $(0,1)\succ_i(1,-)$ for all $i\in M_2\cap M_3$, which contradicts (42).

We conclude that it is impossible that there is only one free issue if $K=2$.

□

References

ANESI, V. (2006): "Committees with farsighted voters: A new interpretation of stable sets," *Social Choice and Welfare*, 27, 595–610.

ANESI, V., AND D. J. SEIDMANN (2014): "Bargaining over an endogenous agenda," *Theoretical Economics*, 9, 445–482.

ARIELI, I., AND R. J. AUMANN (2015): "The Logic of Backward Induction," *Journal of Economic Theory*, 159, 443–464.

AUSTEN-SMITH, D. (1987): "Sophisticated Sincerity: Voting over Endogenous Agendas," *The American Political Science Review*, 81, 1323–1330.

BANKS, J. S. (1985): "Sophisticated Voting Outcomes and Agenda Control," *Social Choice and Welfare*, 1, 295–306.

BANKS, J. S., AND F. GASMI (1987): "Endogenous Agenda Formation in Three-Person Committees," *Social Choice and Welfare*, 4, 133–152.

BARBERÀ, S., AND A. GERBER (2017): "Sequential Voting and Agenda Manipulation," *Theoretical Economics*, 12, 211–247.

BARBERÀ, S., H. SONNENSCHEIN, AND L. ZHOU (1991): "Voting by Committees," *Econometrica*, 59, 595–609.

BARON, D. P., AND J. A. FEREJOHN (1989): "Bargaining in Legislatures," *The American Political Science Review*, 83, 1181–1206.

BAUMGARTNER, F. R. (2001): "Agendas: Political," in *International Encyclopedia of Social and Behavioral Sciences: Political Science*, ed. by N. J. Smelser, and P. B. Baltes, pp. 288–290, New York. Elsevier Science and Oxford: Pergamon.

BERNHEIM, B. D., A. RANGEL, AND L. RAYO (2006): "The Power of the Last Word in Legislative Policy Making," *Econometrica*, 74, 1161–1190.

BERNHEIM, B. D., AND S. N. SLAVOV (2009): "A solution concept for majority rule in dynamic settings," *Review of Economic Studies*, 76, 33–62.

Besley, T., and S. Coate (1997): "An Economic Model of Representative Democracy," *The Quarterly Journal of Economics*, 112, 85–114.

Brams, S. J. (1975): *Game Theory and Politics*. Free Press, New York.

Chen, Y., and H. Eraslan (2017): "Dynamic Agenda Setting," *American Economic Journal: Microeconomics*, 9, 1–32.

Cox, G. W., and K. A. Shepsle (2007): "Majority Cycling and Agenda Manipulation: Richard McKelvey's Contributions and Legacy.," in *Positive Changes in Political Science*, ed. by J. Aldrich, J. Alt, and A. Lupia, Ann Arbor. University of Michigan Press.

Dixon, R., and T. Ginsburg (2011): "Deciding not to Decide: Deferral in Constitutional Design," *International Journal of Constitutional Law*, 9, 636–672.

Duggan, J. (2006): "Endogenous Voting Agendas," *Social Choice and Welfare*, 27, 495–530.

Dutta, B., M. O. Jackson, and M. Le Breton (2001): "Strategic Candidacy and Voting Procedures," *Econometrica*, 69, 1013–1037.

——— (2002): "Voting by Successive Elimination and Strategic Candidacy," *Journal of Economic Theory*, 103, 190–218.

——— (2004): "Equilibrium Agenda Formation," *Social Choice and Welfare*, 23, 21–57.

Eguia, J. X., and K. A. Shepsle (2015): "Legislative Bargaining with Endogenous Rules," *The Journal of Politics*, 77, 1076–1088.

Farquharson, R. (1969): *Theory of Voting*. Yale University Press, New Haven.

Godefroy, R., and E. Perez-Richet (2013): "Choosing Choices: Agenda Selection with Uncertain Issues," *Econometrica*, 81, 221–253.

McKelvey, R. D. (1979): "General Conditions for Global Intransitivities in Formal Voting Models," *Econometrica*, 47, 1085–1112.

Miller, N. R. (1977): "Graph-Theoretical Approaches to the Theory of Voting," *American Journal of Political Science*, 21, 769–803.

——— (1980): "A New Solution Set for Tournaments and Majority Voting: Further Graph-Theoretical Approaches to the Theory of Voting," *American Journal of Political Science*, 24, 68–96.

Miller, N. R., B. Grofman, and S. L. Feld (1990): "Cycle Avoiding Trajectories, Strategic Agendas and the Duality of Memory and Foresight: An Informal Exposition," *Public Choice*, 64, 265–277.

Moser, S., M. Fen, R. Ji, M. Maiden, and M. Panosian (2016): "Heresthetics and Choice from Tournaments," *Journal of Theoretical Politics*, 28, 385–407.

Moser, S., J. W. Patty, and E. M. Penn (2009): "The Structure of Heresthetical Power," *Journal of Theoretical Politics*, 21, 139–159.

Moulin, H. (1979): "Dominance Solvable Voting Schemes," *Econometrica*, 47, 1337–1351.

Osborne, M. J., and A. Slivinski (1996): "A Model of Political Competition with Citizen-Candidates," *The Quarterly Journal of Economics*, 111, 65–96.

Penn, E. M. (2008): "A Distributive N-Amendment Game with Endogenous Agenda Formation," *Public Choice*, 136, 201–213.

Riker, W. H. (1982): *Liberalism against Populism. A Confrontation Between the Theory of Democracy and the Theory of Social Choice.* Waveland Press, Inc., Prospect Heights, Illinois.

——— (ed.) (1993): *Agenda Formation.* Ann Arbor. University of Michigan Press.

Vartiainen, H. (2014): "Endogenous Agenda Formation Processes with the One-Deviation Property," *Theoretical Economics*, 9, 187–216.