Dual Decision Processes: Retrieving Preferences when some Choices are Automatic

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Dual Decision Processes:

Retrieving Preferences when some Choices are Automatic

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Abstract

Evidence from the cognitive sciences suggests that some choices are conscious and reflect individual volition while others tend to be automatic. Under these circumstances, standard economic modeling might not always be applicable because not all choices are the result of individual tastes. We propose a behavioral model that can be used in standard economic analysis that formalizes the way in which conscious and automatic choices arise. We then present a novel method capable of identifying a set of conscious choices from observed behavior and discuss its usefulness as a framework for studying asymmetric pricing and empirical puzzles in different settings.
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1 Introduction

Behavioral economics has posed a serious challenge to standard economic theory by documenting and analysing numerous behaviors inconsistent with preference maximization. Inconsistencies tend to emerge when individuals do not exert the effort to consciously analyze the problem at hand, e.g., Carroll et al. (2009). A key question then arises. When are people choosing consciously?

This paper provides the first theoretical framework that considers the coexistence of conscious and automatic behavior and provides a means to understand the nature of individual decisions in different situations once this duality is taken into account. As highlighted by Simon (1987) and Kahneman (2002), even experts such as managers, doctors, traders, or policy makers often make automatic decisions. Nevertheless, the sources of automatic choices remain underexplored in economics.[1]

Evidence from the cognitive sciences suggests that the *familiarity* of the choice environment is the key determinant of the relationship between conscious and automatic choices. Unconscious, automatic choices are made in familiar environments. Thus, if we wish to understand the role of automatic decisions in markets, we need (i) to create a model of automatic choices that takes into account the role played by the familiarity of the decision environment and (ii) to analyze whether it is possible from observed behavior to distinguish between conscious and automatic choices in order to understand the underlying preferences.

To answer the first question, we propose a simple formalization of evidence drawn from the cognitive sciences, on when and how choices should be conscious or automatic.[2] In Section 2, we present a decision maker described by this simple procedure. Whenever the decision environment is familiar, i.e., whenever its similarity with past experience, measured by a similarity function, passes a certain threshold, past behavior is replicated. This is the source of *automatic* choices. Otherwise, the best option is chosen by maximizing a

[1]Although there is a growing attention in economics to conscious and intuitive or automatic reasoning. See for example Rubinstein (2007) and Rubinstein (2016) for a distinction between conscious and intuitive strategic choices by players of a game, the distinction in Cunningham and de Quidt (2016) between implicit and explicit attitudes or the focus of Gilboa and Wang (2019) on sticking to the status quo as the result of conscious deliberation.

[2]See section 4.2 for the cognitive foundations of the model.

rational preference relation. This is the source of *conscious* choices. Consider, for example, a consumer who buys a bundle of products from the shelves of a supermarket. The first time he faces the shelves, he tries to find the bundle that best fits his preferences. Subsequently, if the prices and arrangement of products do not change *too much*, he will perceive the decision environment as familiar and will automatically stick to the bundle chosen previously. If, on the other hand, the change in prices and arrangement of the products is evident to him, his choice will again be based on preference maximization.[3]

Even in such a simple framework, there is no trivial way to distinguish which choices are made automatically and which are made consciously. Following the example, suppose our consumer is faced with a slightly different problem with more options but he sticks with the original choice. Is it because he prefers the original bundle to the new ones? Or is it because he is choosing automatically?

We show how to find conscious choices and hence restore the standard revealed preference analysis by being able to tell which environments were not familiar. Section 3.1 assumes (i) that the decision maker behaves according to our model and (ii) that the similarity function is known, while the threshold is not.[4] We then show that, for every sequence of decision problems, it is possible by means of an algorithm to identify a set of conscious observations and an interval within which the similarity threshold should lie. That is, we provide a novel method for restoring revealed preference analysis.

First notice that *new* observations, i.e., those in which the choice is an alternative never chosen before, must be conscious. No past behavior could have been replicated. Starting from these observations, the algorithm iterates the following idea. If an observation is consciously generated, any other *less familiar* observation, that is any decision problem which is less similar to those that preceded it, must be also consciously generated. Returning to our consumer, if we know that after a change in the price of the products on the shelf, the consumer chose consciously, then he must also have done so on all those occasions where the change was even more evident.

[3]This is in line with the ideas presented in Woodford (2019) where it is stated that "*. . . people should not be modeled as behaving differently in situations that they do not recognize as different. . .*"

[4]See Section 3.1 for a justification of the latter hypothesis. See the appendix for an empirical strategy for estimating the similarity function when shared in a population.

The algorithm identifies a set of automatic decisions in a similar fashion, that is, it first highlights some decisions that must be automatic and then finds *more familiar* observations to reveal other automatic decisions. Notice that knowing whether some decisions were made automatically is very important for understanding the way in which familiarity of environments is determined. Even if automatic choices do not reveal individual preferences, they tell us which problems are considered familiar, i.e., similar enough, by the decision maker, hence enabling the identification of the interval in which the similarity threshold should lie.

The algorithm assumes that the decision maker behaves according to our model, hence the falsifiability of the model becomes a central concern. In Section 3.2 we propose a testable condition that is a weakening of the Strong Axiom of Revealed Preference that characterizes our model and thus renders it falsifiable. Section 4 clarifies the connection between the model and methods introduced here and the economics and cognitive sciences literatures. Section 5 shows how the model can be an effective tool for standard economic analysis by displaying its usefulness to understand different empirical regularities. In particular, it shows that consumers described by the model can have asymmetric responses to price changes, i.e., they react more to price increases than to price decreases. Thus, firms might have incentives not to adapt prices to costs only when when costs go down. This phenomenon, known as asymmetric pricing, is an important empirical puzzle to which the model provides a possible theoretical explanation. Section 6 concludes. The appendix describes a procedure for empirically estimating the similarity function when shared by a population of otherwise heterogeneous agents.

2 Dual Decision Processes

2.1 The Model

Let X and E be two sets. The decision maker (DM) faces, at time t, a *decision problem* (A_t, e_t) with $A_t \subseteq X$ and $e_t \in E$. The set of available alternatives A_t at time t, from which the DM has to make a choice, is called the *menu*. An alternative is any choice element, such as a consumption bundle, a lottery or a consumption stream. The *environment* e_t is a description of the possible characteristics of the problem faced by the DM at time t. We denote by $a_t \in A_t$ the chosen alternative at time t. With a little abuse of the notation, we refer to the pair formed by the decision problem (A_t, e_t) and the chosen alternative a_t as *observation* t. Notice that the same menu or environment can appear more than once in the sequence.

A chosen alternative is the outcome of a two-stage choice procedure which describes the DM and formalizes the duality of automatic and conscious choices. Formally, let $\sigma : E \times E \to [0,1]$ be the *similarity function.* The value $\sigma(e, e')$ measures the degree of similarity between environment e and environment e'. The *automatic self* is endowed with a *similarity threshold* $\alpha \in [0,1]$ that delimits which pairs of environments are similar enough. Whenever $\sigma(e, e') > \alpha$ the individual considers e to be similar enough to e'. At time t, and faced with the decision problem (A_t, e_t), the automatic self executes a choice if it can replicate the choice of a previous period $s < t$ such that $\sigma(e_t, e_s) > \alpha$. The choice is the alternative a_s chosen in one such period. The *maximizing self* is endowed with a *preference relation* $\succ$ over the set of alternatives. For ease of exposition, we assume that $\succ$ is a strict order, i.e. an asymmetric, transitive and complete binary relation, defined over X. At time t, if the maximizing self is activated, it chooses the alternative a_t that maximizes $\succ$ in A_t.[5] Summarizing:

$$a_t = \begin{cases} a_s \text{ for some } s < t \text{ such that } \sigma(e_t, e_s) > \alpha \text{ and } a_s \in A_t, \\ \text{the maximal element in } A_t \text{ with respect to } \succ \text{, otherwise.} \end{cases}$$

Three remarks are useful here. First, notice that automatic and conscious decisions are separated by the behavioral parameter α. In some sense, α is summarizing the cost of using the cognitively demanding system. The higher the cost, the lower the threshold. Thus, parameter α captures individual heterogeneity as preferences do. Notice that, while the similarity function has been widely studied in cognitive sciences, e.g. Tversky (1977), Medin et al. (1993) and Hahn (2014), the cognitive costs of activating conscious decision processes remain an unknown quantity.[6]

[5] The idea that conscious behavior arises from the maximization of a preference relation is a simplification which we use to focus the analysis on the main novelties of the framework presented in this paper. For a more detailed discussion regarding this point, see section 6.

[6] There is growing attention in decision theory though to the empirical estimation of cognitive parameters

As a second remark, notice that we are describing a class of models because we do not impose any particular structure on replicating behavior. We do not specify which alternative will be chosen when more than one past choice can be replicated. This class can accommodate many different behaviors , e.g., choosing the alternative that was chosen in the most similar past environment, or choosing the alternative that maximizes the preference relation over those chosen in similar enough past environments, etc.[7] All of the following analysis is valid for the class as a whole.

As a final remark, notice that an environment can be as simple as the menu or as complex as the framing of alternatives. For illustrative purposes, we propose some examples of relevant environments for economic applications and a possible similarity function for use in such cases.

Environments as Menus: *In many economic applications it seems sensible to view the entire menu of alternatives, e.g. the budget set, as the main driver of analogies. That is, E could be the set of all possible menus and two decision problems are perceived to be as similar as their menus are. In this framework, $E = 2^X \setminus \{\emptyset\}$.*

Environments as Attributes: *The alternatives faced by decision makers are often bundles of attributes. In such contexts, it is reasonable to assume that the attributes of the available alternatives determine the decision environment. If $\mathcal{A}$ is the set containing all possible attributes, then $E = 2^{\mathcal{A}} \setminus \{\emptyset\}$.*

Environments as Frames: *We can think of the set E as the set of frames or ancillary conditions as described in Salant and Rubinstein (2008) and Bernheim and Rangel (2009). Frames are observable information that is irrelevant for the rational assessment of alternatives; for example, the way the products are displayed on a shelf. Every frame can be seen as a set of irrelevant features of the decision environment. Thus, if the set containing all possible irrelevant features is F, we have $E = 2^F \setminus \{\emptyset\}$.*

The cognitive sciences provide different ways to model the similarity function based on em-

for richer theoretical models, e.g., Cerreia-Vioglio et al. (2012), Rustichini et al. (2016), and Dardanoni et al. (2020).

[7]This last specification of the model can be seen as a theory for consideration set formation through similarity with past experiences hence relating the model to the literature interested in limited attention, e.g. Masatlioglu et al. (2012), Manzini and Mariotti (2014a) and Cattaneo et al. (2020)

pirical evidence. In all the previous examples it is natural to assume that the similarity function relates different environments according to their commonalities and differences. For example, $\sigma(e,e') = \frac{|e\cap e'|}{|e\cup e'|}$, that is, the similarity of two environments increases with the proportion of shared characteristics . This function is just a symmetric specification of the more general class considered in Tversky (1977). Henceforth, we take E and σ as given.[8]

2.2 An example

This section provides an example to illustrate the behavioral process we are modeling. Although the example is abstract, it shows very directly how the model works. For a more concrete application of the model to an interesting economic setting, please refer to section 5.1.

Let $X = \{1, 2, 3, ..., 10\}$. We assume that environments are menus, i.e. E is the set of all non-empty subsets of X and that $\sigma(A, A') = \frac{|A\cap A'|}{|A\cup A'|}$. Suppose that the automatic self is described by $\alpha = .55$ and that the preference $1 \succ 2 \succ 3 \succ \cdots \succ 10$ describes the maximizing one. We now clarify the choices our DM makes from the following ordered menus:

$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$
$3,4,5,6$	$1,2,3,4,5,6$	$1,3,4,7$	$2,4,7$	$1,3,6$	$1,2,3,4$	$2,4,8$	$2,4,8,9,10$
$a_1=3$	$a_2=3$	$a_3=1$	$a_4=2$	$a_5=1$	$a_6=1$	$a_7=2$	$a_8=2$

In the first period, the DM has no prior experiences, therefore the maximizing self is active. Thus, the choice results from the maximization of preferences, that is, $a_1 = 3$. Then, in the following period, given that $\sigma(A_2, A_1) = \frac{4}{6} > .55$, the automatic self is active and therefore we will observe a replication of past choices, that is, $a_2 = 3$. Period 1's environment makes period 2's one fluent. Now, in period 3, notice that the similarity between A_3 and A_2 or A_1 is below the similarity threshold and that this makes the maximizing self active. The preference relation is maximized and therefore $a_3 = 1$. By similar reasoning applied for the fourth and fifth periods, it can be seen that $a_4 = 2$ and $a_5 = 1$. Then, in period six, the automatic self is active given that $\sigma(A_6, A_3) = \frac{3}{5} > .55$, leading to $a_6 = 1$. In period seven,

[8] It is not always possible to obtain all the information relating to the similarity function, that is why in the online appendix we consider the possibility of running the whole analysis with just incomplete relative information regarding the comparison between environments.

given no sufficiently similar past environment, the maximizing self is active and therefore $a_7 = 2$. Finally, in the last period, the automatic self again becomes active, given that $\sigma(A_8, A_7) = \frac{3}{5} > .55$ and behavior will therefore be replicated, i.e. $a_8 = 2$.

One may wonder what an external observer would be able to infer from this choice behavior. Section 3.1 addresses this point.

3 Theoretical Analysis of Dual Decision Processes

Section 3.1 analyzes how to classify observations generated by the model. In particular, it proposes an algorithm that can identify two sets, one containing only observations generated by the maximizing self and one containing observations automatically generated. This can be seen as a way of identifying the *welfare relevant domain* as proposed in Bernheim and Rangel (2009). Section 3.2 then shows that a simple restriction on one of these sets can characterize the whole class of models.

3.1 Revealed Preferences

In this section, we discuss how to recognize which observations were automatically or consciously generated in a DD process. This information is crucial to elicit the unobservables in the model that are the sources of individual heterogeneity, that is, the preference relation and the similarity threshold. In fact, we can think of the similarity function as a cultural component common to most individuals, while the similarity threshold is the source of individual heterogeneity in automatic processes. Thus, the similarity function is taken as given, while the similarity threshold α is elicited from observed behavior.[9] Notice that the similarity function and the similarity threshold define a binary similarity function that is individual specific. Thus, by separating the similarity function from the similarity threshold we are

[9]This assumption is in line with the stance taken in the cognitive sciences. In fact, similarity comparisons are generally taken as a more interpersonal component while cognitive costs are specific to the individual. See for example Tversky (1977), Medin et al. (1993) or Hahn (2014) and subsequent related literature , which also provides a variety of methods for identifying the similarity function. The idea here is that, even if , everyone in a certain society implicitly agrees that an Italian restaurant is more similar to a French one than to a Chinese one, they might not all perceive the Italian and the French ones, for example, to be so close as to be indistinguishable, i.e., to be similar enough. The appendix uses this assumption to estimate the similarity function. In the online appendix, the function is identified with rich enough data.

able to associate individual heterogeneity to a parameter relating to individual cognitive costs without too many degrees of freedom hampering the technical analysis.

It is easy to recognize a set of observations that are undoubtedly generated by the maximizing self. Notice that all those observations in which the chosen alternative was not a previous choice must belong to this category. This is so because, as no past behavior has been replicated, the automatic self could not be active. We call these observations *new* observations.

In order to identify a first set of automatically-generated observations, notice that, being rational, the maximizing self cannot generate cycles of revealed preference. As is standard, a set of observations $t_1, t_2, \ldots, t_k$ forms a *cycle* if $a_{t_{i+1}} \in A_{t_i}$, $i = 1, \ldots, k-1$ and $a_{t_1} \in A_{t_k}$, where all chosen alternatives are different. Given the above reasoning, for every cycle there must be at least one observation that is automatically generated. Intuition suggests that the one corresponding to the most familiar environment should be a decision mimicking past behavior. The following definition helps in formalizing this idea.

The *unconditional familiarity* of observation t is

$$f(t) = \max_{s<t, a_s \in A_t} \sigma(e_t, e_s).$$

Whenever there is no $s < t$ such that $a_s \in A_t$, we say $f(t) = 0$.

That is, unconditional familiarity measures how similar observation t is to past observations from which behavior *could be* replicated, i.e., those past decision problems for which the chosen alternative is present at t. Then, we say that observation t is a *most familiar in a cycle* if it is part of a cycle of observations within which it maximizes the unconditional familiarity value. Given the above reasoning, these observations must be generated by the automatic self.

The major challenge is to relate pairs of observations in such a way as to allow the knowledge of which self generated one of them to be transferred to the other. In order to do so, we introduce a second measure of familiarity of an observation t, which we call *conditional familiarity*. Formally,

$$f(t|a_t) = \max_{s<t, a_s = a_t} \sigma(e_t, e_s).$$

Whenever there is no $s < t$ such that $a_s = a_t$, we say $f(t|a_t) = 0$.

That is, conditional familiarity measures how similar observation t is with past observations from which behavior *could have been* replicated, i.e., those past decision problems for which the choice is the same as at t. The main difference between $f(t)$ and $f(t|a_t)$ is that the former is an ex-ante concept, i.e., before considering the choice, while the latter is an ex-post concept, i.e., after considering the choice. Our key definition uses these two measures of familiarity to relate pairs of observations.

Definition 1 (Linked Observations) *We say that observation t is* linked *to observation s, and we write $t \in L(s)$, whenever $f(t|a_t) \leq f(s)$. We say that observation t is* indirectly linked *to observation s if there exists a sequence of observations $t_1, \ldots, t_k$ such that $t = t_1$, $t_k = s$ and $t_i \in L(t_{i+1})$ for every $i = 1, 2, \ldots, k-1$.*

The definition formalizes the key idea behind the algorithm. Observation t is linked to observation s if its conditional familiarity is lower than the unconditional familiarity of s. As explained below, once an observation is categorized as consciously or automatically generated, it is through its link to other observations that such knowledge can be extended.

First consider an observation t which is known to be new, and hence consciously generated. This implies that its corresponding environment is not similar enough to any other previous environment. In other words, $f(t) \leq \alpha$. That is, $f(t)$ is a lower bound for α. Then, any observation s for which the conditional familiarity is less than $f(t)$ ($s \in L(t)$) must also be generated by the maximizing self. In fact, $f(s|a_s) \leq f(t) \leq \alpha$ implies that no past behavior that could have been replicated in s comes from an environment that is similar enough to the one in s. This implies that $f(s) \leq \alpha$, potentially refining the lower bound for α. In fact, it could be that $f(t) < f(s)$. It is easy to see that this reasoning can be iterated by finding the observations linked to s. In fact, if $f(t) < f(s)$, there might exists observations that are linked to s but not to t (observations that are *indirectly* linked to t). Denote by D^N the set of all those observations that are *indirectly* linked to new observations, that is the set obtained through the iteration of the previous procedure.

Similarly, consider a most familiar observation in a cycle t, that is known to be automatically generated. Any observation s for which the unconditional familiarity is greater than the conditional familiarity of t ($t \in L(s)$) must also be generated by the automatic self. In fact, we know that $\alpha < f(t|a_t)$ because t is generated by the automatic self. That is, $f(t|a_t)$ defines an upper bound for α. Then, any observation s to which t is linked has unconditional familiarity greater than α, which implies that some past behavior *could be* replicated by the automatic self, and therefore that such an observation must also be generated by the automatic self. This implies $\alpha < f(s|a_s)$, potentially refining the upper bound for α. In fact, it could be that $f(s|a_s) < f(t|a_t)$. Again, the reasoning can be iterated by finding the observations to which s is linked. In fact, if $f(s|a_s) < f(t|a_t)$, s might be linked to observations to which t is not (observations to which t is *indirectly* linked). Let D^C be the set obtained by iterating the reasoning, that is, the set of all observations to which most familiar observations in a cycle are *indirectly* linked.

The binary relation determined by the concept of linked observations is clearly reflexive; thus, new observations and most familiar observations in a cycle are contained in D^N and D^C respectively. Thus, we can start from a small subset of observations undoubtedly generated either automatically or consciously, and thence infer which other observations are of the same type.

Now we can state the main result of this section. The previous reasoning establishes that observations in D^N are generated by the maximizing self, while observations in D^C are generated by the automatic self. As a consequence, it must be that the revealed preference of observations in D^N is informative with respect to the preferences of the individual. Moreover, we can identify an interval within which the similarity threshold must lie. In fact, notice that since observations in D^N are generated by the maximizing self, we know that $\max_{t \in D^N} f(t) \leq \alpha$ and also that, since observations in D^C are generated by the automatic self, $\alpha < \min_{t \in D^C} f(t|a_t)$.

Before stating the proposition, it is useful to highlight that x is revealed preferred to y for a set of observations O, i.e., $xR(O)y$, if there is a sequence of different alternatives $x_1, x_2, \ldots, x_k$ such that $x_1 = x, x_k = y$ and for every $i = 1, 2, \ldots, k-1 \in O$, it is $x_i = a_t$ and $x_{i+1} \in A_t$ for some t.

Proposition 1 *For every collection of observations D generated by a DD process:*

1. *all observations in D^N are generated by the maximizing self while all observations in D^C are generated by the automatic self,*

2. *if x is revealed preferred to y for the set of observations D^N, then $x \succ y$,*

3. $\max_{t \in D^N} f(t) \leq \alpha < \min_{t \in D^C} f(t|a_t)$.

Proof. In the text. ■

We here use the example in section 2.2 to show how the algorithm works.

Algorithm: Example of Section 2.2

Suppose that we observe the decisions made by the DM in the example in section 2.2, without any knowledge regarding his preferences $\succ$ or similarity threshold α.

We can easily notice that the only new observations are observations 1, 3 *and* 4*; hence, we can directly infer that the corresponding choices were conscious.*

Given no other new observations are present, we now need to go one step further. We know that the choice was conscious in period 3 *and one can easily find that* $f(5|a_5) = \frac{2}{5} \leq \frac{3}{7} = f(3)$*, which means that observation* 5 *is linked with observation* 3 *and, by Proposition 1, this reveals that it, too, was conscious.*

Now that we know that observation 5 *was conscious, a similar argument can be applied to observation* 7*. We cannot directly link observation* 7 *either to observation* 1, 3 *or* 4*, because* $f(7|a_7) = \frac{1}{2} > \max\{f(1), f(3), f(4)\}$*. However, we can indirectly link observation* 7 *to observation* 3 *through observation* 5*, because* $f(7|a_7) = \frac{1}{2} \leq \frac{1}{2} = f(5)$*, thus making* 7 *an element of* D^N*. No other observation is indirectly linked to observations* 1, 3 *or* 4 *and hence,* $D^N = \{1, 3, 4, 5, 7\}$*. The method rightfully excludes all automatic choices from* D^N*.*

This example presents inconsistencies in the revealed preference. Observations 3 *and* 6 *are both in conflict with observation* 2*. That is, observations* 2 *and* 3 *and* 2 *and* 6 *form cycles. Then, noticing that* $\max\{f(2), f(3)\} = f(2)$ *and that* $\max\{f(2), f(6)\} = f(2) = f(6)$ *we can say that observations* 2 *and* 6 *are generated by the automatic self thanks to Proposition 1, given they are most familiar in a cycle.*

Notice then, however, that observation 6 *is linked to observation* 8 *given that* $f(6|a_6) = \frac{3}{5} \leq f(8) = \frac{3}{5}$*, thus revealing that the latter must also have been automatically generated.*

Thus, we obtain that $D^C = \{2, 6, 8\}$*, which were the observations rightfully excluded from* D^N*. No decision made by the maximizing self has been cataloged as automatic.*

The modified revealed preference exercise helps us determine that alternative 1 *is better than any alternative from* 2 *to* 7*, alternative* 3 *is better than any alternative from* 4 *to* 6*, and alternative* 2 *is better than alternatives* 4, 7 *and* 8*, as is indeed the case. The value of the similarity threshold* α *can, by Proposition 1, be correctly determined to be in the interval* $[0.5, 0.6)$ *thanks to the information retrieved from observations* 7 *and* 8 *respectively.*

D^N contains new observations and those indirectly linked to them.[10] It may be the case that some consciously-generated observations are not linked to any observation in D^N, thus making D^N a proper subset of the set of all consciously-generated observations. For this reason, nothing guarantees that $D \setminus D^N$ are automatically-generated observations and hence, Proposition 1 must show how to dually construct a set of automatic decisions D^C. Nonetheless, if the observations are *rich enough*, it is possible to guarantee that D^N and D^C contain all conscious and automatic decisions, respectively.[11] More importantly, notice that Proposition 1 relies on one important assumption, which is that the collection of observations is generated by a DD process. The following section addresses this issue. As it turns out, the whole class is characterized by a simple restriction on D^N.

3.2 A Characterization of Dual Decision Processes

In Section 3.1 we showed how to elicit the preferences and the similarity threshold of an individual who follows a DD process. Building upon the results of that section, we here provide a necessary and sufficient condition for a set of observations to be characterized as a DD process with a known similarity function. In other words, we provide a condition that can be used to falsify our model.

From the construction of the set D^N, we understand that a necessary condition for a dataset to be generated by a DD process is that the indirect revealed preference obtained from observations in D^N, i.e. $R(D^N)$, must be asymmetric. It turns out that this condition

[10] D^N is never empty because it always contains the first observation.

[11] Material available in the online appendix.

is not only necessary but also sufficient to represent a sequence of decision problems as if generated by a DD process. A simple postulate of choice characterizes the whole class of DD processes.

Axiom 1 (Link-Consistency) *A sequence of observations* $\{(A_t, e_t, a_t)\}_{t=1}^T$ *satisfies* Link-Consistency *if, in every cycle, there is at least one observation that is not indirectly linked to a new observation.*

This is a weakening of the Strong Axiom of Revealed Preference. In fact, it allows for cycles but only of a particular kind. Preferential information gathered from observations in D^N cannot be inconsistent. The following theorem shows that this condition is indeed necessary and sufficient to characterize DD processes with known similarity.

Theorem 1 *A sequence of observations* $\{(A_t, e_t, a_t)\}_{t=1}^T$ *satisfies Link-Consistency if and only if there exist a preference relation* $\succ$ *and a similarity threshold* α *that characterize a DD process.*

Proof. Necessity: If $\{(A_t, e_t, a_t)\}_{t=1}^T$ is generated by a DD process, then it satisfies Link-Consistency as explained in the text.

Sufficiency: Now suppose that $\{(A_t, e_t, a_t)\}_{t=1}^T$ satisfies Link-Consistency. We need to show that it can be explained as if generated by a DD process. Notice that Link-Consistency implies that the revealed preference relation defined over D^N, i.e. $R(D^N)$, is asymmetric. In fact, asymmetry of $R(D^N)$ means that it is not possible to construct cycles comprised of observations in D^N. Using standard mathematical results, we can find a transitive completion of $R(D^N)$, call it $\succ$. By construction, all decisions in D^N can be seen as the result of maximizing $\succ$ over the corresponding menu.

Define $\alpha = \max_{t \in D^N} f(t)$. Notice that by definition of D^N, there is no observation $s \notin D^N$ such that $f(s|a_s) \leq f(t)$ for some $t \in D^N$. This implies that, for all $s \notin D^N$, $f(s|a_s) > \alpha$,; thus, it is possible to find, for each one, a preceding observation which it would appear to replicate. In particular, the one defining $f(s|a_s)$.

Thus, we can represent the choices as if generated by an individual with preference relation $\succ$ and similarity threshold α. ∎

The theorem is saying that Link-Consistency makes it possible to determine whether the DM is following a DD process or not. In particular, when the property is satisfied, we can characterize the DM's preferences with a completion of $R(D^N)$ which is asymmetric thanks to Link-Consistency and use the lower bound of α as described in Proposition 1 to characterize the similarity threshold. In fact, by construction, for any observation t outside D^N it is possible to find a preceding observation that can be replicated, i.e., the one defining $f(t|a_t)$. Notice that we do not assume any particular structure for the sequence of observations we use as data, and hence that the characterization of preferences does not need to be unique, even when the similarity is known.[12]

Finally, it is important to highlight that the whole analysis developed until now follows through even when the knowledge of the similarity is only partial and that the model is falsifiable even when there is no information regarding the similarity comparisons. To see the latter, notice that Link-Consistency implies that there must be no cycles among new observations, a concept which is independent of the particular choice of similarity function. To see the former, the online appendix contains a generalization of the identification algorithm and of the characterization, assuming that only a partial preorder, i.e., a reflexive, transitive but possibly incomplete binary relation, over pairs of environments is known. Such an extension is very general. It is potentially relevant in many contexts where it is not possible to estimate the cardinal similarity function but only the relative order it represents. It is also robust to even weaker assumptions, when, for example, only some of the binary comparisons between pairs of environments are known. Returning to the example used in the introduction, we might not know how the DM compares different prices and dispositions of products on the shelf, but we might know that, for any combination of prices, a minor change in just one price results in a more similar environment than a major change in all prices. The logic behind the algorithm extends directly by using the ordinal information contained in the preorder to define the familiarity of the environments. The only result that cannot be replicated, due to the absence of cardinal information regarding similarity comparisons, is the estimation of

[12]Material providing a full characterization of the procedure with rich enough data is available in the online appendix. Richness is obtained either by assuming that the data come from a homogeneous population, as is standard in the literature when dealing with path dependent models, or by assuming that the implicit memory of the DM is bounded and that the bound is known.

the interval in which the similarity threshold should lie.

4 The Model in Context

In this section, we show how the paper relates to the literature and what evidence from the cognitive sciences it is formalizing.

4.1 Related Literature

In our model the presence of similarity comparisons makes behavior automatic. That is, if two environments are similar enough, then behavior is replicated. This is a different approach with respect to the theory for decisions under uncertainty, which was proposed in Gilboa and Schmeidler (1995) and summarized in Gilboa and Schmeidler (2001). In case-based decision theory, as in our model, the decision maker uses a similarity function in order to assess how much alike are the problem he is facing and the ones he has in his memory. In that model the decision maker tends to choose the action that performed better in past similar cases. There are two main differences with the approach we propose here. First, from a conceptual standpoint, our model relies on the idea of two selves interacting during the decision-making process. Second, from a technical point of view, our model uses similarity in combination with a threshold to determine whether the individual replicates past behavior or maximizes preferences, while in Gilboa and Schmeidler (1995) preferences are always maximized. Thus, as Section 6 suggests, case-based decision theory may be ingrained in the more general structure proposed here. The model in Gilboa and Schmeidler (1995) can be seen as a particular way of making conscious decisions. Nevertheless, both models agree on the importance of analogies for human behavior.

We would like to stress that, although our proposed behavioral model is new, the idea that observed behavior may be the outcome of interaction between two different selves is not novel and dates back, at least, to Strotz (1955). Strotz-type models, such as Laibson (1997), Gul and Pesendorfer (2001) or Fudenberg and Levine (2006), are different from the behavioral model introduced here, since they represent the two selves as two agents with different and conflicting preferences, usually *long-run* vs *short-run* preferences.[13] In our approach, however, the two selves are *inherently* different one from the other. One uses analogies to deal with the environment in which the decision maker acts, while the other uses a preference relation to consciously choose among the available options. Furthermore, the issue of which self drives a particular decision problem depends on problems experienced in the past and their degree of similarity with the current one, and not on whether the decision affects the present or the future. Nevertheless, we do not exclude the possibility that analogy formation may be influenced to some extent by whether the decision affects the present or the future.

Finally, it is important to notice that the preference revelation strategy used in this paper agrees with the one used in Bernheim and Rangel (2009), who analyze the same problem of eliciting individual preferences from behavioral datasets, which they do in two stages. In a first stage, they take as given the *welfare relevant domain*, that is the set of observations from which individual preferences can be revealed; and then, in a second stage, they analyze the welfare-relevant observations and propose a criterion for the revelation of preferences that does not assume any particular choice procedure to make welfare judgments.[14] Albeit similar, our approach differs in two important aspects. Firstly, we model conscious and automatic choices, and, in Section 3.1, we propose a particular method for finding the welfare-relevant domain, i.e. the algorithm highlighting a set of conscious choices. Secondly, we propose a specific choice procedure, and perform standard revealed preference analysis on the relevant domain; thus our method, by being behaviorally based, is less conservative for the elicitation of individual preferences. In this sense, our stance is also similar to the one proposed in Rubinstein and Salant (2012), Masatlioglu et al. (2012) and Manzini and Mariotti (2014b), who make the case for welfare analysis based on the understanding of the behavioral process that generated the data. These differences are crucial. If we apply Bernheim and Rangel

[13] In some models, the difference between the two selves stems from the fact that they have different information. See, for example, Cunningham (2015), which proposes a model of decision making where the two selves hierarchically aggregate information before choosing an alternative. Other papers use more than two selves to rationalize individual choices, but all selves are still represented by different preference relations; see, for example, Kalai et al. (2002), Manzini and Mariotti (2007) and Ambrus and Rozen (2015).

[14] Notice that Apesteguia and Ballester (2015) also propose a choice-procedure free approach for measuring the welfare of an individual from a given dataset. They do this by providing a model-free method to measure how close actual behavior is to the preference that best reflects the choices in the dataset.

method to the example in Section 2.2, without identifying the welfare relevant domain, we would not know, for example, how the DM would compare 1 and 3 and we would find that 3 is preferred 2 which is a wrong inference that our method avoids by considering the model that generated the data. Without the tools provided by the model, it would be impossible to properly identify the relevant domain hence invalidating any inference aimed at guiding welfare enhancing policies.[15]

4.2 Cognitive Foundations of the Model

The cognitive sciences have always distinguished between conscious and unconscious processes. More recently, this duality has been incorporated into Dual Process Theory, as described in Evans and Frankish (2009) and Kahneman (2011), which divides mental processes into two categories, analytical (or conscious) and analogical (or automatic).[16] Using these findings, the objective here is twofold, (i) to propose a simple initial formalization of automatic processes with falsifiable implications and (ii) to model the interaction between conscious and automatic processes in a tractable way. We abstract from more in-depth modeling of conscious behavior, which is represented here by the rational model. Nevertheless, the framework developed in this paper is flexible enough to allow for alternative models of conscious processes, as explained in more detail in Section 6.

Unconscious or automatic processes are extremely context dependent.[17] In particular, the main determinant for the activation of non-deliberative processes is the subjective experience of ease associated with a particular situation, i.e., *fluency*, due to the characteristics of the environment. Studies surveyed by Oppenheimer (2008), one of the leading scholars in fluency

[15] Notice that this problem could be even more serious. Suppose we observe a DM described by a DD process who is first choosing between the two worse alternatives contained in X. Then, suppose she faces an increasing series of menus in which new and better alternatives are included, but the changes in the environment are not enough to activate the maximizing self. Our DM would always choose the second to last alternative. Bernheim and Rangel method, if we take the whole series of decision problems as the relevant domain, would wrongly infer that the second to last alternative is preferred to any other alternative while our method would only infer that the second to last is preferred to the worst alternative. This example makes clearer that Bernheim and Rangel method needs a model to define the relevant domain to avoid wrong inferences.

[16] Many papers helped in developing the theory, including Schneider and Shiffrin (1977), Evans (1977), McAndrews et al. (1987), Evans (1989), Reber (1989), Epstein (1994) and Evans and Over (1996).

[17] See Ouellette and Wood (1998), Bargh (2005), Wood et al. (2005), Ji and Wood (2007), Neal et al. (2011), Marteau et al. (2012), Neal et al. (2012), Lisman and Sternberg (2013), Miller et al. (2019).

research, show that in fluent, familiar, situations, individuals' decisions are less conscious, that is, more automatic. Disfluent situations, on the other hand, lead to conscious behavior.[18] Hence, it becomes crucial for the purposes of this paper to be able to formalize what fluency of an environment is. In fact, the literature does not provide a formal model to think about these problems.

Paraphrasing Oppenheimer (2008), a situation becomes more fluent as it becomes more familiar. That is, an experience is more fluent the easier is the unconscious perception of its similarity with past experiences.[19] Thus, we introduce a first formalization of familiarity here through two main channels. First, an environment is more familiar the greater its similarity with environments experienced in the past.[20] This is measured by the similarity function. Second, problems requiring new solutions due to a lack of previous solutions, cannot be considered familiar. That is, a decision problem is familiar if past behavior can be replicated. Clearly, these are simplifications; but they are in line with evidence in cognitive sciences showing that automatic processes are based on analogical reasoning, i.e., similarity judgments with the past, which activate automatic responses.[21] The main idea we are depicting is that environments that are similar with the past prompt automatic responses that are drawn from past experiences. This is also in line with the evidence in Woodford (2019) which calls for the introduction of these ideas in economics.

Then, the second important step is to model the dichotomy between conscious and automatic processes. The majority of the evidence points to the fact that analogical reasoning is parallel.[22] That is, automatic processes operate continuously; unconsciously assessing the

[18]This literature is closely related to that on limited attention in cognitive sciences, where agents consciously analyze a situation only if it is *novel enough*, see Woodford (2019).

[19]This is in line with research on *priming*, which studies the influence of unconscious or implicit memory on behavior through environmental cues, which appears to be one of the main sources of fluency. As defined in Tulving and Schacter (1990), priming is an unconscious change in the ability to identify or produce an item as a result of a specific prior encounter with that item. Priming creates a sense of familiarity and ease, which make the environment fluent and behavior automatic and coherent with the context. See Bargh (2005) for a survey on priming.

[20]Notice that , given that we are describing implicit memory, we assume perfect memory, in line with research in neurosciences as already summarized in Graf (1990). For a more recent survey, see Dew and Cabeza (2011).

[21]See Ouellette and Wood (1998), Bargh (2005), Wood et al. (2005), Ji and Wood (2007), Evans and Frankish (2009), Kahneman (2011)Neal et al. (2011), Marteau et al. (2012), Neal et al. (2012), Lisman and Sternberg (2013) and Miller et al. (2019).

[22]See Evans and Frankish (2009) for a survey of the evidence.

nature and familiarity of the environment.[23] On the other hand, conscious processes are costly and are fully activated only when the environment is disfluent or novel, i.e., unfamiliar. In the model, the parallel nature of analogical reasoning is represented by the fact that similarity comparisons are always drawn and the costly nature of the conscious system is captured by the similarity threshold. The higher the threshold, the lower the cost; that is, fewer environments will be perceived as fluent or familiar. This assumption enriches the model and aligns it more closely with the evidence. In fact, the fluency of an environment is a concept that intertwines similarity with past experiences with the cost of conscious processing.[24] Conscious processes are needed when new responses are required due to the novelty of the environment, either because it is not similar enough with past experiences or because no behavior can be replicated.[25] This is very much in line with recent research in neuroscience and other cognitive sciences which sustains that unconscious processes are context dependent and generate habits while non-habits are the outcome of conscious responses (Ouellette and Wood (1998), Wood et al. (2005), Ji and Wood (2007), Neal et al. (2011), Marteau et al. (2012), Neal et al. (2012), Lisman and Sternberg (2013), Miller et al. (2019)).

Finally, notice that, in this model, the relationship between automatic and conscious processes does not have to be completely dichotomous. As already stated, here, we present and analyze a class of models, some specifications of which make the relationship between automatic and conscious processes more complementary. For example, choosing the alternative that maximizes preferences over those chosen in similar-enough past environments. In this sense, the automatic system would be simplifying the problem by creating a *fluent* consideration set on which the conscious system would be maximizing preferences.

[23] See Sun et al. (2020) for recent evidence about this.

[24] Notice that the model abstracts from the notion that the cost of conscious processing can be a function of the environment. This is clearly a simplification to avoid having too many degrees of freedom, but it is certainly an interesting avenue for future research.

[25] We do not exclude the possibility that, in such instances, the conscious response would be to choose a *similar enough* alternative in a *similar enough* environment.

5 The Model as a Tool

The aim of this section is to show how the model can be used as a tool for standard economic analysis. It begins with an illustration of how to apply the model, highlighting its tractability and main behavioral implications. It then gives new insights into the problem by showing that DD processes are a possible explanation for the puzzling phenomenon of asymmetric pricing. Finally, the reasoning is generalized to show that the model's key implications provide a useful lens for gaining perspective on empirical regularities in a variety of fields.

5.1 An Application: Asymmetric Pricing

In what follows, the model is used to provide a simple explanation for the phenomenon of *asymmetric pricing*, that is, the asymmetric response of firms to changes in costs. Although prices are increased when costs rise, there is no tendency to reduce them when costs fall. This well-documented fact is at odds with standard economic theory, despite affecting two thirds of markets, as highlighted in the seminal paper by Peltzman (2000). The evidence in Peltzman (2000) is quite overwhelming and excludes more standard explanations, such as collusion or menu costs. In fact, the phenomenon is present in all kinds of markets, regardless of the level of competition. Various models have been put forward, as will be explained below, but, first, let us show that DD processes provide a fairly natural alternative explanation.

Consider two firms competing over a market comprised of consumers described by a DD process and whose automatic systems make analogies over price vectors, i.e., given constant wealth, over budget sets. Meanwhile, the firms face cost shocks. The key idea is that the presence of DD consumers creates a demand in the second period that is dependent on prices in the first and responds asymmetrically to price changes. In the second period, if prices rise, consumers react rationally, being unable to replicate past behavior, given that their budget sets have shrunk. However, due to heterogeneous similarity thresholds, if prices fall, not all consumers will adapt their behavior, since some will replicate past behavior. Thus, unless the price drop is big enough, some consumers will fail to perceive the change, and demand will therefore be much less elastic. Under certain circumstances, this particular demand structure gives firms incentives to maintain prices with falling costs and adjust them as costs rise. The following subsections clarify this reasoning.

5.1.1 Setting

Imagine two firms, A and B, competing à la Bertrand over two periods.[26] The game structure is common knowledge among the firms. First-period costs are symmetric, marginally constant and normalized to 1, that is, $c_A^1 = c_B^1 = 1$, where c_i^t represents the costs of firm i in period t. In period 2, costs are independently drawn for each firm $i = A, B$ as follows:

$$c_i^2 = \begin{cases} (1+\beta) \text{ with } \pi \text{ probability} \\ (1-\beta) \text{ with } 1-\pi \text{ probability} \end{cases}$$

with $\beta \in (0, 1)$.

Consumers are described by a DD process and have V units of wealth in each period. The automatic system compares price vectors. In the first period, consumers are rational, given that there is no past to replicate. In the second period, their behavior depends on firms' pricing decisions and their own similarity thresholds, which are distributed over the population with density function f. This means that the heterogeneity of the population is modeled through different costs of maximizing-self activation.

In this setting, asymmetric pricing can be defined as the asymmetric pricing response of firms to changes in costs. The following definition formalizes the concept.

Definition 2 (Asymmetric Pricing) *Firm i, with $i = A, B$, is pricing asymmetrically if, in the second period, it increases prices when costs go up while it does not decrease prices as costs go down.*

5.1.2 Analysis

In this section, we look for the subgame perfect equilibria of the game, focusing on an equilibrium in pure strategies. First of all, notice that, given the symmetry of the environment,

[26] Competition à la Bertrand assures that asymmetric pricing is not the outcome of firms' market power while also making the assumption of having just two firms less relevant. In fact, the whole analysis can be generalized to n firms.

we study firms choosing the same action in period one. For now, let $p_A^1 = p_B^1 = p^1$, where p_i^t is the price charged by firm i in period t. We now look at the second period decision. W.l.o.g., the analysis concerns firm A's decision in the second period.

The demand faced by firm A in period two depends not only on prices in period one, but also on the pricing decisions of firm B in period two. Let:

$$\mu(\alpha_n) = \int_0^{\alpha_n} f(\alpha) d\alpha$$

with

$$\alpha_n = \sigma(p_A^1, p_B^1; p_A^2, p_B^2)$$

That is, μ represents the fraction of consumers who make automatic purchase decisions in period two. Thus we have the following.
If $p_B^2 = p^1$

$$d_A^2(p_A^1, p_B^1, p_A^2, p_B^2) = \begin{cases} 0 & \text{if } p_A^2 > p_B^2, \\ \frac{1}{2}\frac{V}{p^1} & \text{if } p_A^2 = p_B^2, \\ \mu(\alpha_1)\frac{1}{2}\frac{V}{p^1} + (1-\mu(\alpha_1))1\frac{V}{p_A^2} & \text{if } p_A^2 < p_B^2. \end{cases}$$

While if $p_B^2 \neq p^1$, we get

$$d_1^2(p_A^1, p_B^1, p_A^2, p_B^2) = \begin{cases} 0 & \text{if } p_A^2 > p_B^2 \geq p^1, \\ \frac{1}{2}\frac{V}{p_A^2} & \text{if } p^1 < p_A^2 = p_B^2, \\ \frac{1}{2}\frac{V}{p^1} & \text{if } p^1 = p_A^2 = p_B^2, \\ 1\frac{V}{p^1} & \text{if } p_1^1 = p_A^2 < p_B^2, \\ \mu(\alpha_2)\frac{1}{2}\frac{V}{p^1} + (1-\mu(\alpha_2))0 & \text{if } p_B^2 < p_A^2 \leq p^1, \\ \mu(\alpha_2)\frac{1}{2}\frac{V}{p^1} + (1-\mu(\alpha_2))1\frac{V}{p_A^2} & \text{if } p_A^2 < p_B^2 \leq p^1. \end{cases}$$

Notice that $\alpha_1 \geq \alpha_2$, given that, in scenario 2, both firms might have changed prices, so fewer consumers will perceive the environment as being sufficiently similar.

First of all, to understand whether firms want to adapt prices when costs are low, let us examine what happens when costs are low for both firms, that is, $c_A^2 = c_B^2 = 1 - \beta$, let us assume that firm A believes that firm B will not change prices, that is, $p_B^2 = p^1$. Then, if firm A lowers prices it gets:

$$(p_A^2 - (1-\beta))(\mu(\alpha_1)\frac{1}{2}\frac{V}{p^1} + (1-\mu(\alpha_1))1\frac{V}{p_A^2})$$

By contrast, if it does not lower prices, it gets:

$$(p^1 - (1-\beta))\frac{1}{2}\frac{V}{p^1}$$

That is, firm A does not charge $p_A^2 = p^1 - \epsilon$ if the following condition is satisfied.

$$F(\alpha_1) \geq \frac{p^1}{p^1+\epsilon} - \frac{\epsilon}{(p^1+\epsilon)}\frac{1-\beta}{(p^1-\epsilon-(1-\beta))} \tag{1}$$

Derivation of Condition (1).

$$(p^1 - (1-\beta))\frac{1}{2}\frac{V}{p^1} \geq (p_A^2 - (1-\beta))(\mu(\alpha_1)\frac{1}{2}\frac{V}{p^1} + (1-\mu(\alpha_1))1\frac{V}{p_A^2})$$

with $p^1 > p_A^2 > (1-\beta)$. can be rewritten as:

$$\mu(\alpha_1) \geq \frac{2(p_A^2-(1-\beta))p^1 - (p^1-(1-\beta))p_A^2}{(p_A^2-(1-\beta))(2p^1-p_A^2)}$$

That is:

$$F(\alpha_1) \geq \frac{2(p_A^2-(1-\beta))p^1 - (p^1-(1-\beta))p_A^2}{(p_A^2-(1-\beta))(2p^1-p_A^2)}$$

If we let $p_A^2 = p^1 - \epsilon$, we obtain Condition (1). ■

The following subsection will show that this condition can hold under many general circumstances. For now, it is important to highlight that the condition is essential for an equilibrium with asymmetric pricing to exist. In fact, on the one hand, by standard arguments, firms always adapt prices when costs are high, as the proof of the following proposition clarifies. On the other hand, it is only if Condition (1) is satisfied that they make no price cuts when costs are low.

Proposition 2 *There is an equilibrium with asymmetric pricing only if Condition (1) holds.*

Proof. Firstly, when costs are high, i.e., $c_A^2 = 1+\beta$, firm A will adapt prices such that $p^1 \neq p_A^2 = 1+\beta$. Suppose firm A does not adapt prices, that is, $p_A^2 = p^1$. Suppose, then, that $p_A^2 = p^1 \geq 1+\beta$. In this case, firms would be making positive profits in both periods, under any circumstance, and hence would have incentives to undercut each other in the first period, thus leading to $p^1 < 1+\beta$. Clearly, it cannot be that $p_A^2 = p^1 < 1+\beta$, because firm A would have incentives to deviate in the second period in order to avoid negative profits. Thus, it must be that $p^1 \neq p_A^2 = 1+\beta$.

As a second point, notice that, when firm A has low costs in period two, i.e., $c_A^2 = 1-\beta$, while firm B has high costs, $c_B^2 = 1+\beta$, we will have that $p_A^2 = 1+\beta-\epsilon$, given that $p_B^2 = 1+\beta$, as per the previous reasoning. For simplicity, we assume that, in this case, $p_A^2 = 1+\beta$ and that firm A wins the entire market.

Now, supposing that Condition (1) is satisfied, we need to solve for p^1. Clearly, price competition among firms requires p^1 to be such that expected profits in period 1 are zero. That is:

$$(p^1-1)\frac{1}{2}\frac{V}{p^1} + \pi^2(p^1-(1-\beta))\frac{1}{2}\frac{V}{p^1} + \pi(1-\pi)2\beta\frac{V}{1+\beta} = 0$$

So we get

$$p^1 = \frac{(1+\beta)(1+(1-\beta)\pi^2)}{(1+\pi^2)(1+\beta)+\pi(1-\pi)4\beta}$$

First notice that $p^1 < 1$. In fact

$$(1+\beta)(1+\pi^2(1-\beta)) < (1+\beta)(1+\pi^2)+\pi(1-\pi)4\beta$$

given that $(1+\beta)(1+\pi^2(1-\beta)) < (1+\beta)(1+\pi^2)$ and $\pi(1-\pi)4\beta > 0$. Moreover, clearly, $p^1 > 1-\beta$. In fact:

$$p^1-(1-\beta) = \frac{\beta((1+\beta)-\pi(1-\pi)4(1-\beta))}{(1+\pi^2)(1+\beta)+\pi(1-\pi)4\beta} > 0$$

given that

$$\frac{1+\beta}{1-\beta} > 4\pi(1-\pi),$$

where the left hand side is greater than 1, while the maximum of the right hand side is 1.

To be sure that p^1 can be an equilibrium price when Condition (1) is satisfied, firms must have no incentives to deviate. In fact, they may still want to deviate unilaterally in the first period. That is, firms might be prepared to accept greater short-term losses in order to win the entire market in the second period, and thereby make greater long-term profits. Thus, for p^1 to be an equilibrium price, it must be that

$$(p^1-1)\frac{1}{2}\frac{V}{p^1} + \pi^2(p^1-(1-\beta))\frac{1}{2}\frac{V}{p^1} + \pi(1-\pi)2\beta\frac{V}{1+\beta} >$$

$$> (p^1-\epsilon-1)\frac{V}{p^1-\epsilon} + \pi^2(p^1-\epsilon-(1-\beta))\frac{V}{p^1-\epsilon} + \pi(1-\pi)2\beta\frac{V}{1+\beta}$$

Or:

$$0 > (p^1-\epsilon-1)\frac{V}{p^1-\epsilon} + \pi^2(p^1-\epsilon-(1-\beta))\frac{V}{p^1-\epsilon} + \pi(1-\pi)2\beta\frac{V}{1+\beta}.$$

which can be rewritten as

$$\frac{(1+\beta)(1+(1-\beta)\pi^2)}{(1+\pi^2)(1+\beta)+\pi(1-\pi)2\beta} > p^1-\epsilon.$$

A sufficient condition is that

$$\frac{(1+\beta)(1+(1-\beta)\pi^2)}{(1+\pi^2)(1+\beta)+\pi(1-\pi)2\beta} > p^1 = \frac{(1+\beta)(1+(1-\beta)\pi^2)}{(1+\pi^2)(1+\beta)+\pi(1-\pi)4\beta},$$

which is always true. Firms have no incentive to deviate and hence they do not change prices whenever costs decrease while they increase prices when costs inflate. This concludes the proof. ■

Notice that proposition 2 is saying two things. Firstly, if Condition (1) does not hold, then firms have incentives to deviate unilaterally and therefore, by standard arguments, prices will be equal to costs, thus there is no price asymmetry. Secondly, if Condition (1) holds, there is an equilibrium with asymmetric pricing but more equilibria are possible. In fact, the whole analysis is based on firm A's belief that firm B does not change prices. If firm A believes otherwise, then its incentives to change prices might be stronger and an additional standard equilibrium, with prices equal to costs, would be possible.[27]

Finally, it is worth highlighting the following.

Remark 1 *Whenever Condition (1) holds, there is an equilibrium at which firms enjoy a mark up in period 2 when costs are low.*

In fact, if Condition (1) is satisfied, $p^1 \in (1-\beta, 1)$ and therefore firms have negative profits in period 1 that are compensated by expected positive profits in period 2. This implies that prices align with costs only when costs are high. While not central to understanding asymmetry in pricing, this remark is interesting because it means that, in this simple setting, firms might enjoy mark-ups in pure strategies, even under Bertrand competition.

5.1.3 Analysis of Condition (1)

In this section, we analyze whether there are general settings under which Condition (1) holds.

To determine when Condition (1) is satisfied, we need to make some assumption regarding the automatic system and the distribution of the similarity threshold in the population. As a first step, let the similarity function be:

$$\sigma(p_A^1, p_B^1; p_A^2, p_B^2) = \frac{1}{1 + d((p_A^1, p_A^2), (p_B^1, p_B^2))}$$

where $d((p_A^1, p_A^2), (p_B^1, p_B^2) = ((p_A^1 - p_A^2)^2 + (p_B^1 - p_B^2)^2)^{\frac{1}{2}}$ is the Euclidean distance between price vectors in the two periods. Under the assumptions of Condition (1) we have that $((p_A^1 - p_B^1)^2 + (p_A^2 - p_B^2)^2)^{\frac{1}{2}} = \epsilon$. Thus, we have:

$$\mu(\alpha_1) = \int_0^{\frac{1}{1+\epsilon}} f(\alpha) d\alpha = F\Big(\frac{1}{1+\epsilon}\Big)$$

[27]Notice that, if Condition (1) holds, such an equilibrium would not be trembling-hand perfect, given that the belief that firm B will change prices can be sustained only by allowing mistakes by firm B.

Thus, Condition (1) can be rewritten as follows:

$$F\left(\frac{1}{1+\epsilon}\right) \geq \frac{p^1}{p^1+\epsilon} - \frac{\epsilon}{(p^1+\epsilon)} \frac{1-\beta}{(p^1-\epsilon-(1-\beta))},$$

which leads to remark 2.

Remark 2 *If* $F\left(\frac{1}{1+\epsilon}\right) \geq \frac{1}{1+\epsilon}$ *for any* ϵ*, Condition (1) is satisfied.*

Proof of Remark 2. Remark 2 stems from the fact that $p^1 < 1$ and so

$$F\left(\frac{1}{1+\epsilon}\right) \geq \frac{1}{1+\epsilon} > \frac{p^1}{p^1+\epsilon} > \frac{p^1}{p^1+\epsilon} - \frac{\epsilon}{(p^1+\epsilon)} \frac{1-\beta}{(p^1-\epsilon-(1-\beta))},$$

where the last inequality stems from the fact that the last term is always positive. ■

Notice that $F\left(\frac{1}{1+\epsilon}\right) \geq \frac{1}{1+\epsilon}$ for any ϵ for many distributions, e.g. the uniform distribution or any positively-skewed Beta distribution, such as $Beta(1,n)$ with $n > 1$. However, this is only a sufficient condition. Whenever $F\left(\frac{1}{1+\epsilon}\right) < \frac{1}{1+\epsilon}$ for some ϵ, Condition (1) must be checked and no general conclusions can be drawn without making further assumptions. However, there is an interesting implication of the model worth noting .

Remark 3 *If* $F\left(\frac{1}{1+\epsilon}\right) < \frac{p^1}{p^1+\epsilon}$ *for some* ϵ*, then there exists a* β *big enough such that Condition (1) fails.*

The remark is a direct consequence of the fact that

$$\lim_{\beta \to 1} \frac{1-\beta}{(p^1-\epsilon-(1-\beta))} = 0$$

Remark 3 is of particular interest, as it is consistent with empirical findings. In fact, Peltzman (2000) finds that asymmetric pricing is present in markets where cost shocks are not particularly big. That is, empirically, the bigger the cost shocks, the less asymmetric the behavior of firms in adjusting prices to costs. Similar results are found in Chen et al. (2008). The intuition behind Remark 3 in our setting is straightforward. *Ceteris paribus*, the higher the β, the more room there is for a firm to decrease the price to a degree that is perceived by enough consumers to make it profitable. In fact, in general, given that the second term on the right hand side of Condition (1) is decreasing in β, the greater the β, the more likely it is for Condition (1) to fail. Thus, under certain circumstances, if the cost shock is big enough, the asymmetric response of demand to price changes disappears.

This marks a crucial distinction between the theoretical explanation provided here and the models that have been presented in the literature to reconcile asymmetric pricing with economic theory. In fact, since Peltzman (2000), there have been many attempts to explain the phenomenon of asymmetric pricing by abstracting from menu costs or market power, which have been rejected by the data. All models include, as in our framework, some kind of inelasticity of demand in response to price changes.

One strand of the literature (Yang and Ye (2008), Tappata (2009), Lewis (2011), Cabral and Fishman (2012)) has used search models to explain such inelasticities. The main idea is that consumers form expectations regarding the distribution of prices based on past price or cost realizations and thus tend to search less intensely when prior price or cost realizations have been high, while searching more intensely when they have been low. This gives firms incentives to react asymmetrically to cost changes. Another strand, Levy et al. (2004), has used rational inattention to create inelastic response of demand to price changes. According to this model, consumers decide not to allocate attention to small price changes because it is costly for them to do so. This means that demand is symmetrically inelastic for small price changes, irrespective of their direction. Clearly, firms facing this kind of demand have incentives to raise prices for small changes in costs given the demand does not change but have no incentives to reduce prices for minor cost changes because they would not increase demand. Despite its simplicity, the explanation provided here has some crucial features that enable it to reconcile both strands of literature with the empirical evidence. First of all, as Remark 3 shows, it is able, like rational inattention models, to relate the presence of asymmetric pricing to the magnitude of the cost shock affecting an industry. None of the papers using search models is able to reconcile this fact, because the friction of searching would make asymmetric pricing always optimal for firms, regardless of the magnitude of the shock. Secondly, like search models, this model maintains the asymmetric response of demand to price changes, which is present in the data.[28] A third noteworthy feature of

[28]See next section for a more detailed explanation of how asymmetry of responses can arise.

the general framework presented here, which sets it apart from other settings, is that the model allows each consumer's choice to be either conscious or automatic, i.e. rational or not, depending on the features of the environment, hence making the overall proportion of agents that are rational or not in the population endogenous to the problem at hand. That is, heterogeneity of behavior is easily obtained through the distribution of α, thus making the model tractable.

Finally and most importantly, DD processes provide a new conceptual framework in which the source of inertia in behavior is due to the characteristics of the choice environment and how they affect similarity comparisons, not utility, which is a distinctive feature with implications that clearly differentiate this from other families of models. This point is made clearer in the following subsection.

5.2 Inertia, Asymmetry, Heterogeneity, and Markets

Section 5.1 showcased how the model can be easily applied to standard economic settings to obtain new insights into systematic empirical puzzles. This section shows that the key behavioral implications of Section 5.1 are, in fact, not specific to that application and can be extended to different settings to gain fresh perspectives on puzzling phenomena. The model has three general implications which serve this purpose well.

INERTIA: In Section 5.1, in the second period, consumers stick with the same firm, if the change in their budget set is not *big enough*. That is, they exhibit inertia if the environment is not *dissimilar* to what they have experienced previously. Thus, inertia is not due to consumers' preferences.

Inertia in behavior is almost ubiquitous in the data and arises in DD processes due to the influence of the decision environment characteristics on similarity comparisons. Possible orthogonality between preferences and similarity is the main difference between the model developed here and other models. In fact, habit formation and rational inattention models, which are central in the literature, also introduce inertia but they do so through the preferences of the individual.

This fundamental difference is very closely connected to the evidence from the cognitive sciences, as explained in Section 4.2. In particular, the present model reproduces two key findings from this literature. The first is that habits and inertia are highly dependent on environmental cues (Camerer et al. (2005), Wood et al. (2005), Verplanken (2006), Verplanken (2010), Marteau et al. (2012), Neal et al. (2012), Lisman and Sternberg (2013)) and the second is that this environmental dependence can be unrelated, and sometimes orthogonal, to the underlying preferences of the individual (Ouellette and Wood (1998), Ji and Wood (2007), Neal et al. (2011), Miller et al. (2019)). That is, inertia does not appear to be the outcome of individual preferences, either by comparison with a reference (habit formation), or by the optimization of scarce attentional resources (rational inattention).[29]

Thus, changes in the choice environment, which influence analogies but not preferences, should affect choices under DD processes, but not under other families of models. This is a key difference that can be tested in the data. For example, suppose that doctors are described by DD processes and make analogies between patients' symptoms. We should then expect (i) doctors bundle together patients with similar profiles in a suboptimal way, with possibly no regard for the optimality of the treatment or their preferences and, (ii) older doctors to replicate past actions by automatically responding to similar symptoms observed in the past, for instance, prescribing older (branded) drugs and treatments. Hellerstein (1998), Frank and Zeckhauser (2007), and Patel et al. (2016) find that this is indeed the case empirically. To explain the same phenomena, models that build inertia on preferences must assume that there are unreasonable costs involved in processing the information or disrupting habits and that these costs change with age.

Importantly, DD processes can also replicate empirical regularities explained by other families of models. Suppose, for example, that households are influenced in their automatic similarity comparisons by budget sets, as in the application. Then, changes in income will activate automatic responses (behavioral inertia) when they are not big enough, while activating conscious, adaptive behavior otherwise. Abundant evidence suggests that this is indeed the case. Household behavior is sticky with respect to changes in income, but the stickiness depends on the magnitude of the change (Souleles (1999), Reis (2006), Carroll et al.

[29]We do not exclude the possibility that conscious behavior may be influenced by these considerations. What we wish to underline here is that a key source of behavioral inertia are automatic processes dependent upon analogies and environmental cues and not on individual preferences.

(2011), Carroll et al. (2018), Fuster et al. (2018)).[30] Similarly traders in financial markets appear to underreact to news, but this underreaction seems to depend on the magnitude of the change (Cutler et al. (1991), Bernard (1993)). DD processes can explain those outcomes when traders' automatic systems are influenced by analogies between information sets, for example.

Asymmetric and symmetric responses to environmental changes: In Section 5.1, consumers respond asymmetrically to positive and negative changes in the environment. In fact, in the second period, the same absolute change in the price vector causes different behavioral responses depending on the sign of the change. If prices decrease, replication of past behavior is possible, as the previously chosen bundle is included in the new budget set. Hence, if the change in the environment is not enough to activate the maximizing self, past behavior should be replicated. By contrast, any increase in prices precludes replication of past behavior, hence leading to re-optimization. From an observational standpoint, consumers should show more sensitivity to adverse changes than to advantageous ones, unless the magnitude of the change is large enough to activate the maximizing self in any case. Furthermore, responses to changes that preclude replication should be more *rational* and quickly disrupt habits – as observed, for example, with *active decisions* in Carroll et al. (2009).[31] There is significant evidence that these are indeed key patterns in individual behavior.

While consumers tend to respond more strongly to negative changes of income than to positive ones, this asymmetry disappears as the magnitude of the shock increases and if consumers, e.g., savers, are able to replicate past consumption decisions, (Dargay (2001),

[30]Interestingly enough, as in the application, response of consumption to changes in income seem to be higher the greater the volatility of the income process in terms of magnitude of changes, see Reis (2006).

[31]To better understand the connection with Carroll et al. (2009), notice that they find the following: *...compelling new hires to make active decisions about 401(k) enrollment raises the initial fraction that enroll by 28 percentage points relative to a standard opt-in enrollment procedure, producing a savings distribution three months after hire that would take thirty months to achieve under standard enrollment.*
This kind of outcome would be predicted by DD processes whenever *active decisions* mean that agents have no default or compulsory choice, and hence would be expected to make a conscious decision, thus overcoming behavioral inertia. In situations in which conscious behavior can lead to optimal choice, given a DD process, this kind of regime is clearly optimal, in line with the claims made by Carroll et al. (2009). Nevertheless, even when the decision is active, the choice may still not be conscious if the decision problem is familiar to one encountered before and behavior can be replicated. This can potentially lead to suboptimal behavior, as an anonymous referee suggested. While these issues deserve further investigation, they transcend the aims of this study.

Reis (2006), Bunn et al. (2018), Fuster et al. (2018), Christelis et al. (2019), Surico and Trezzi (2019)). Consumers also react asymmetrically to changes in prices, taxes and asset returns, especially in the case of smaller changes and under higher liquidity constraints (Bidwell et al. (1995), Adeyemi and Hunt (2007), Bracha and Cooper (2014), Wadud (2014), ?).[32] If we combine inertia and asymmetry of responses, we should expect agents described by DD processes to react to negative changes in the same way as rational consumers with liquidity constraints. By contrast, we should expect *less* responsiveness to positive changes than would be rationally predicted; a pattern which indeed appears in the data.

Endogeneity of heterogeneous behavior: Section 5.1 showed that, by varying the similarity threshold in the population, it is easy to create endogenously heterogeneous aggregate behavior within the class of models presented here. At any given moment in time, the proportion of consumers choosing consciously and the proportion automatically sticking to past behavior are endogenously determined by the change in the decision environment. This key component of the models is what makes them tractable.

An important issue in macroeconomics is, in fact, how to introduce endogenously heterogeneous household behavior to match empirical data (Mankiw (2000), Sims (2003)). DD processes indicate a possible route with solid cognitive foundations. Their appeal, moreover, is threefold. Firstly, DD processes provide a simple way to reproduce many empirical facts relating to inertia and adaptation at the individual level.[33] Secondly, through the distribution of the similarity threshold, the framework can be easily calibrated to match heterogeneous aggregate behavior. Thirdly, by clearly separating automatic decisions from conscious ones, this class of models allows for an added layer of heterogeneity in behavior; households might differ in the way they make conscious decisions. Thus, standard models of heterogeneous behavior can be incorporated into a DD process. These characteristics distinguish DD processes from other frameworks.

Models of rational inattention, for example, can reproduce some of the empirical facts,

[32] The asymmetric response of demand to price changes has also received a great deal of attention in marketing (Krishnamurthi et al. (1992), Kalyanaram and Winer (1995), Mazumdar et al. (2005), Karle et al. (2015)).

[33] For further dynamic implications, see the end of this section.

but, as admitted on various occasions by Sims (Sims (2003) and Sims (2006)), they raise three key problems. First, and more importantly, at the aggregate level, informational frictions average out, thus preventing the model from reproducing the kind of aggregate behavior that is observed in the data. Next, rational inattention models become complicated once departing from the simplest specifications, thus making their application difficult. Finally, the cognitive foundations of these models are weak. In particular, people do not appear to make a rational, utilitarian choice of which information to ignore.[34]

DD processes are able to overcome these problems in a simple way. Yet, new issues arise. In fact, two key questions which need to be researched further to apply the model to different markets are: what are the relevant environments, and which similarity function best describes behavior? In this paper and in the online appendix, we provide different possible solutions to these questions. The cognitive sciences and calibration might help to push the research forward.

As a final point regarding heterogeneity, it is worth noting that DD processes also allow heterogeneous behavior to arise from experience. Since automatic processes are based on implicit memory, otherwise equal decision makers with different past experiences may behave differently. This is an important characteristic, which could enable the model to be used to analyze the impact of experience on the quality of decision making (Heckman et al. (2013)), a possibility that we leave for future research.

Finally, it is worth highlighting two dynamic implications of the model. The first is that, due to the orthogonality between preferences and similarity, it accommodates a *boiling frog* argument.[35] Any time an adverse change in the environment allows for replication of past behavior, individuals described by a DD process can be manipulated to stick to past choices,

[34] For evidence in favor of rational inattentiveness see Dewan and Neligh (2020).

[35] "*The boiling frog is a fable describing a frog being slowly boiled alive. The premise is that if a frog is put suddenly into boiling water, it will jump out, but if the frog is put in tepid water which is then brought to a boil slowly, it will not perceive the danger and will be cooked to death. The story is often used as a metaphor for the inability or unwillingness of people to react to or be aware of sinister threats that arise gradually rather than suddenly.*" Wikipedia. Although this feature of the model might appear odd, it closely maps cognitive science ideas, e.g., Chugh and Bazerman (2007) and references therein, by enabling the depiction of the *boiling frog* syndrome. See Offerman and Van Der Veen (2015) for evidence of the phenomenon in economics. Notice that this feature also differentiates the model from satisficing and rational inattention models.

simply by changing the environment as slowly as possible. For example, in the application, if consumers' wealth grows after the second period, firms can introduce very slow price increases, and thus increase their margins, as consumers continue to buy the same bundle. Combining this intuition with the one regarding firms' pricing behavior under decreasing costs, we should observe relatively frequent, small price increases and large, while infrequent, price decreases as well as seeing firms' margins increase with increasing prices. This has been well-highlighted in the data (Chen et al. (2008)), and is worth further investigation.

The second implication is that decision makers *overreact* to some environmental changes. To understand this point, suppose that a DM described by a DD process automatically sticks to past behavior at time t after a change in the environment. Now, suppose a further, larger change and that, this time, the DM behaves consciously at time $t+1$ by reacting to the change in the environment, while also taking into account previously unperceived past changes. To an observer, this appears an overreaction to the change at $t+1$ and the DM's behavior will be correlated with past changes in the environment. This is a general pattern of behavior that has been observed both in macroeconomic data (Reis (2006)) and in financial markets (De Bondt and Thaler (1985)), and can be easily accommodated by DD processes when dynamic implications are taken into account.

6 Final Remarks

The cognitive sciences have highlighted the fact that choices can be divided into two categories; conscious and automatic. This greatly hinders the use of standard economic models, which do not generally take into account the existence of automatic choices and their dependence on environmental cues.

The key contribution of this paper is to provide a novel formalization of this duality. The proposed framework allows us to restore classic revealed preference analysis and to analyze various empirical regularities, as shown in Section 5. More generally, it provides a fresh way of understanding the coexistence of sticky and adaptive behavior and generates predictions consistent with the data, but different from those made by other theories.

The model is also useful for addressing welfare issues by crucially restoring a revealed preferences approach. However, the approach proposed here highlights an issue that is usually neglected: cognitive costs. The upper and lower bounds of the similarity threshold range are defining, respectively, a lower and upper bound of the cognitive costs of activation of the maximizing self. This could be useful in welfare analysis. It also raises the question for future research of how to relate such costs to utility.[36]

The development of a new framework encompassing conscious and automatic choices required some simplifying assumptions. First, we assumed that conscious behavior is the maximization of a given preference relation. In some cases, this assumption can be too strong, since inconsistency is also possible in conscious choices. Fortunately, the framework and analysis illustrated here do not depend on the assumption of a particular type of conscious behavior . For any given decision environment, analogies determine a partition with two components, one containing problems that are similar enough to the reference environment and another containing those that are not. This partition is based on a single simple assumption; that automatic choices must stem from the replication of past behavior, which is in line with cognitive and neuro science research. Alternative conscious behaviors are possible. The only element of the formal analysis that has to be changed is the consistency requirement, which needs to be tested on those problems in D^N. Obviously, to find automatic choices, we should analyze violations of such requirements.

A second assumption was that the similarity function is known and the environments are given. Although the first of these assumptions can be relaxed, as shown in the online appendix, some knowledge of similarity comparisons is nonetheless essential.[37] Automatic behavior can be consistent with the maximization of some preference relation, thus making the observation of inconsistent behavior insufficient for the correct categorization of decisions. Analogies add a layer of complexity to the problem, thus calling for richer data. Several steps in this direction are possible. The first would be to relax the assumption of known environments. An understanding of the key elements of a decision problem for similarity comparisons is crucial for the study of individual decision making, as noted by Woodford

[36]Notice that, even when such costs can be related to utility, e.g., the similarity between two environments depends on the closeness between the respective maximum obtainable utility values , the relevance of the analysis in Section 3.1 remains intact due to the possible *boiling frog* pattern.

[37]Alternatively, one needs a large enough population of individuals sharing the same similarity function, as shown in the appendices.

(2019). The model proposed here provides a setting for the structured consideration of this issue, which is left for future research. A second possibility for a better identification of conscious and automatic decisions might be to complement the framework introduced here with richer datasets (e.g., by including response times) to categorize individual decisions. For example, Caplin and Martin (2016) and Alós-Ferrer (2018) provide formal frameworks in which automatic responses are taken as given and use drift diffusion models to predict the correlation between response times and conscious or automatic behavior. One option would be to use *unusually* long response times to identify conscious responses and use this information, together with new observations, to perform algorithmic analysis, as presented here. The models in Caplin and Martin (2016) and Alós-Ferrer (2018) can help to specify an unusually long response, thus giving structure to the exercise.[38]

Finally, an interesting avenue for future research would be to gain a deeper understanding of the impact of analogical reasoning on behavior. First, the application in Section 5 makes the simplifying assumption that the similarity function is $\sigma(e,e') = \frac{1}{1+d(e,e')}$. This, however, is not necessarily the case. In fact, the analysis throughout the paper assumes no specific functional form. Allowing for other possibilities might help to explain, for example, why a change in price from 1.99 to 1.98 triggers very different behavioral responses than a change from 1.99 to 2.00. If the similarity function is such that $\sigma(1.98, 1.99) > \sigma(2.00, 1.99)$, more people will react automatically to the first than to the second change, thus leading to different behavioral responses.[39] Second, the similarity between 1 and 2 euros, e.g., $\frac{1}{2}$, is much higher than the similarity between 100 and 200 euro cents, e.g., $\frac{1}{101}$. This might explain why people behave differently with nominal changes that do not affect real values (see for example Ert and Erev (2013)).

[38] Notice that the use of dual information, that is, the use of unusually *rapid* response times to identify automatic responses, can be somewhat more problematic, because there is no reason why responses cannot be both conscious and rapid, particularly in the case of very easy problems.

[39] I thank an anonymous referee for making this interesting connection.

A Appendix

A.1 Estimation of the Similarity Function

The similarity function is a key component of a DD process and, for the sake of exposition, in this paper, it is assumed to be at least partially known. Nonetheless, we discuss in what follows how to estimate it by studying the choice behavior of a group of individuals sharing the same similarity function.[40] In other words, we are assuming that the population follows a DD process. Notice, however, that, once the similarity is estimated, this assumption can be falsified, as explained in the main text.

Consider a continuous population of individuals sharing the similarity function σ, with a continuous and independent distribution of the similarity threshold over $[0, 1]$. Sequences of decision problems and preferences are independently distributed. Suppose, for now, that the preference distribution is known. This assumption is quite common in empirical analysis and can be relaxed in some cases, as explained below. It implies that, for every $A \subseteq X$, the proportion of agents in the population that would choose alternative $x \in A$, for any alternative $x \in A$, is known. Let $\pi(x|A)$ represent such a proportion. Before proceeding to the analysis, notice that, even in this circumstance, the revelation of individual preferences would still be relevant for policy purposes. In fact, knowing the aggregate preference distribution is not equivalent to knowing the preferences of each individual.

Now, assume that, for every pair of environments $e, e' \in E$, there exists a representative subpopulation of agents such that, for each agent:

- there exists some t' such that t' is new, $a_{t'} = x$ for some $x \in X$ and $e_{t'} = e'$.
- there exists some $t > t'$ such that $x \in A_t$, no alternative in $A_t \setminus \{x\}$ has been chosen before t and $e_t = e$.
- there is no $s \in (t', t)$ such that $a_s = x$.

The main result of this section shows that we can compare the similarity of two different pairs of environments by considering the aforementioned respective subpopulations and sampling

[40]The idea that similarity comparisons are affected by culture has been studied at least since Whorf (1941).

them. Formally, denote by $\nu(x|e,e',A_t)$ the average relative number of randomly- sampled individuals sticking to x at t, as defined in the previous richness condition. That is, for any pair of environments (e,e'), we take a sample of finite magnitude n from the aforementioned subpopulations and compute the average ratio of individuals in that sample who stick to y. This average is $\nu(x|e,e',A_t)$. Notice that the considered alternatives and sequences of decision problems that satisfy the richness condition might differ across different pairs of environments.

Proposition 3 (Eliciting the Similarity) *For every two pairs of environments (e,e') and (g,g'), $Pr(|\nu(x|e,e',A_t)-\pi(x|A_t)| \geq |\nu(x'|g,g',A_{t'})-\pi(x'|A_{t'})||\sigma(g,g')>\sigma(e,e')) \xrightarrow{p} 0$. That is, the probability of having $|\nu(x|e,e',A_t)-\pi(x|A_t)| \geq |\nu(x'|g,g',A_{t'})-\pi(x'|A_{t'})|$ when $\sigma(g,g')>\sigma(e,e')$ probabilistically converges to zero.*

Proof. First, notice that $|\nu(x|e,e',A_t)-\pi(x|A_t)| \neq 0$ for only two reasons, (i) sampling noise, (ii) automatic decisions. Given that the samples to estimate $\nu(x|e,e',A_t)$ are independent, as the dimension of the sample grows, the law of large numbers applies; therefore the first concern disappears in the limit. This leaves us with the second. Given that α is continuously and independently distributed in the population, if $|\nu(x|e,e',A_t)-\pi(x|A_t)| \geq |\nu(x'|g,g',A_{t'})-\pi(x'|A_{t'})|$ in the limit, it must be because more people are replicating behavior in sequence (e,e') than in sequence (g,g'). That is, there always exists a non-negligible part of the whole population with similarity threshold α in the interval $[\sigma(g,g'),\sigma(e,e'))$ and, since thresholds are independent of preferences, and the only behavior that can be replicated is that in which the choice of x or x' was new, the result follows. ■

The main intuition of Proposition 3 is the following. $|\nu(x|e,e',A_t)-\pi(x|A_t)|$ measures how different from the underlying primitives the behavior in the representative sample is with respect to the preferences in the whole population. There are two reasons why $|\nu(x|e,e',A_t)-\pi(x|A_t)|$ might not be zero. Either the sample has some agents making automatic choices or some noise. The law of large numbers causes the second concern to disappear, thus enabling us to reveal the similarity between different pairs of environments simply by comparing the previous differences for the different pairs.

The previous result is general and holds true for any underlying relationship between environments and menus. Nevertheless, whenever $A_t \cap e_t = \emptyset$, e.g., decision environments are frames, we can relax the assumption on the knowledge of preferences. In fact, we would only need to know $\pi(x|A)$ for all $A \subseteq X$ for just one alternative x and the analysis would follow as before, except that, this time, the same alternative can be used to estimate the similarity of different pairs of environments.[41]

[41] Even more generally, suppose there are two alternatives $x, y \in X$ such that $\{x, y\} \cap e = \emptyset$ for any $e \in E$ and that $\pi(x|\{x, y\})$ is known. Then the richness condition would be as follows. Assume that, for every pair of environments $e, e' \in E$, there exists a representative subpopulation of agents such that, for each agent:

- there exists some t' such that t' is new, $a_{t'} = x$ for some $x \in X$ and $e_{t'} = e'$.
- there exists some $t > t'$ such that $A_t = \{x, y\}$, and y has never been chosen before t and $e_t = e$.
- there is no $s \in (t', t)$ such that $a_s = x$.

then the estimation of the similarity would be exactly the same, just using this special pair of alternatives.

References

Adeyemi, O. I. and L. C. Hunt (2007). Modelling oecd industrial energy demand: Asymmetric price responses and energy-saving technical change. *Energy Economics 29*(4), 693–709.

Alós-Ferrer, C. (2018). A dual-process diffusion model. *Journal of Behavioral Decision Making 31*(2), 203–218.

Ambrus, A. and K. Rozen (2015). Rationalising choice with multi-self models. *The Economic Journal 125*(585), 1136–1156.

Apesteguia, J. and M. A. Ballester (2015). A measure of rationality and welfare. *Journal of Political Economy 123*(6), 1278–1310.

Bargh, J. A. (2005). Bypassing the will: Toward demystifying the nonconscious control of social behavior. In R. R. Hassin, J. S. Uleman, and J. A. Bargh (Eds.), *The New Unconscious*, pp. 37–58. Oxford: Oxford University Press New York.

Bernard, V. L. (1993). Stock price reactions to earnings announcements. *Advances in Behavioral Finance 1*, 303–340.

Bernheim, B. and A. Rangel (2009). Beyond revealed preference: Choice-theoretic foundations for behavioral welfare economics. *The Quarterly Journal of Economics 124*(1), 51–104.

Bidwell, M. O., B. X. Wang, and J. D. Zona (1995). An analysis of asymmetric demand response to price changes: The case of local telephone calls. *Journal of Regulatory Economics 8*(3), 285–298.

Bracha, A. and D. Cooper (2014). Asymmetric responses to income changes: The payroll tax increase versus tax refund in 2013. *Economics Letters 124*(3), 534–538.

Bunn, P., J. Le Roux, K. Reinold, and P. Surico (2018). The consumption response to positive and negative income shocks. *Journal of Monetary Economics 96*, 1–15.

Cabral, L. and A. Fishman (2012). Business as usual: A consumer search theory of sticky prices and asymmetric price adjustment. *International Journal of Industrial Organization 30*(4), 371–376.

Camerer, C., G. Loewenstein, and D. Prelec (2005). Neuroeconomics: How neuroscience can inform economics. *Journal of Economic Literature 43*(1), 9–64.

Caplin, A. and D. Martin (2016). The dual-process drift diffusion model: Evidence from response times. *Economic Inquiry 54*(2), 1274–1282.

Carroll, C. D., E. Crawley, J. Slacalek, K. Tokuoka, and M. N. White (2018). Sticky expectations and consumption dynamics. *National Bureau of Economic Research*.

Carroll, C. D., J. Slacalek, and M. Sommer (2011). International evidence on sticky consumption growth. *Review of Economics and Statistics 93*(4), 1135–1145.

Carroll, G. D., J. J. Choi, D. Laibson, B. C. Madrian, and A. Metrick (2009). Optimal defaults and active decisions. *The Quarterly Journal of Economics 124*(4), 1639–1674.

Cattaneo, M. D., X. Ma, Y. Masatlioglu, and E. Suleymanov (2020). A random attention model. *Journal of Political Economy 128*(7), 2796–2836.

Cerreia-Vioglio, S., F. Maccheroni, M. Marinacci, and A. Rustichini (2012). A theory of conscientiousness. *Working paper*.

Chen, H. A., D. Levy, S. Ray, and M. Bergen (2008). Asymmetric price adjustment in the small. *Journal of Monetary Economics 55*(4), 728–737.

Christelis, D., D. Georgarakos, T. Jappelli, L. Pistaferri, and M. Van Rooij (2019). Asymmetric consumption effects of transitory income shocks. *The Economic Journal 129*(622), 2322–2341.

Chugh, D. and M. H. Bazerman (2007). Bounded awareness: What you fail to see can hurt you. *Mind & Society 6*(1), 1–18.

Cunningham, T. (2015). Hierarchical aggregation of information and decision-making. *Manuscript*.

Cunningham, T. and J. de Quidt (2016). Implicit preferences inferred from choice. *CESifo Working Paper Series No. 5704*.

Cutler, D. M., J. M. Poterba, L. H. Summers, et al. (1991). Speculative dynamics. *Review of Economic Studies 58*(3), 529–46.

Dardanoni, V., P. Manzini, M. Mariotti, and C. J. Tyson (2020). Inferring cognitive heterogeneity from aggregate choices. *Econometrica 88*(3), 1269–1296.

Dargay, J. M. (2001). The effect of income on car ownership: Evidence of asymmetry. *Transportation Research Part A: Policy and Practice 35*(9), 807–821.

De Bondt, W. F. and R. H. Thaler (1985). Does the stock market overreact? *Journal of Finance 40*(3), 793–805.

Dew, I. T. and R. Cabeza (2011). The porous boundaries between explicit and implicit memory: Behavioral and neural evidence. *Annals of the New York Academy of Sciences 1224*(1), 174–190.

Dewan, A. and N. Neligh (2020). Estimating information cost functions in models of rational inattention. *Journal of Economic Theory 187*, 105011.

Epstein, S. (1994). Integration of the cognitive and the psychodynamic unconscious. *American Psychologist 49*(8), 709.

Ert, E. and I. Erev (2013). On the descriptive value of loss aversion in decisions under risk: Six clarifications. *Judgment and Decision Making 8*(3), 214–235.

Evans, J. S. (1977). Toward a statistical theory of reasoning. *Quarterly Journal of Experimental Psychology 29*, 297–306.

Evans, J. S. (1989). *Bias in Human Reasoning: Causes and Consequences.* Hove and London: Lawrence Erlbaum Associates, Inc.

Evans, J. S. and K. E. Frankish (2009). *In Two Minds: Dual Processes and Beyond.* Oxford: Oxford University Press.

Evans, J. S. and D. E. Over (1996). *Reasoning and Rationality.* New York: Psychology Press.

Frank, R. G. and R. J. Zeckhauser (2007). Custom-Made versus Ready-to-Wear Treatments: Behavioral Propensities in Physicians' Choices. *Journal of Health Economics 26*(6), 1101–1127.

Fudenberg, D. and D. K. Levine (2006). A dual-self model of impulse control. *American Economic Review 96*(5), 1449–1476.

Fuster, A., G. Kaplan, and B. Zafar (2018, March). What would you do with 500? spending responses to gains, losses, news and loans. Working Paper 24386, NBER.

Gilboa, I. and D. Schmeidler (1995). Case-based decision theory. *The Quarterly Journal of Economics 110*(3), 605–639.

Gilboa, I. and D. Schmeidler (2001). *A Theory of Case-Based Decisions.* Cambridge: Cambridge University Press.

Gilboa, I. and F. Wang (2019). Rational status quo. *Journal of Economic Theory 181*, 289–308.

Graf, P. (1990). Life-span changes in implicit and explicit memory. *Bulletin of the Psychonomic Society 28*(4), 353–358.

Gul, F. and W. Pesendorfer (2001). Temptation and self-control. *Econometrica 69*(6), 1403–1435.

Hahn, U. (2014). Similarity. *Wiley Interdisciplinary Reviews: Cognitive Science 5*(3), 271–280.

Heckman, J., R. Pinto, and P. Savelyev (2013). Understanding the mechanisms through which an influential early childhood program boosted adult outcomes. *American Economic Review 103*(6), 2052–86.

Hellerstein, J. K. (1998). The importance of the physician in the generic versus trade-name prescription decision. *RAND Journal of Economics 29*(1), 108–136.

Ji, M. F. and W. Wood (2007). Purchase and consumption habits: Not necessarily what you intend. *Journal of Consumer Psychology 17*(4), 261–276.

Kahneman, D. (2002). Maps of bounded rationality: A perspective on intuitive judgment and choice. *Nobel Prize Lecture 8*, 351–401.

Kahneman, D. (2011). *Thinking, Fast and Slow*. New York: Farrar, Straus and Giroux.

Kalai, G., A. Rubinstein, and R. Spiegler (2002). Rationalizing choice functions by multiple rationales. *Econometrica 70*(6), 2481–2488.

Kalyanaram, G. and R. S. Winer (1995). Empirical generalizations from reference price research. *Marketing Science 14*(3_supplement), G161–G169.

Karle, H., G. Kirchsteiger, and M. Peitz (2015). Loss aversion and consumption choice: Theory and experimental evidence. *American Economic Journal: Microeconomics 7*(2), 101–20.

Krishnamurthi, L., T. Mazumdar, and S. Raj (1992). Asymmetric response to price in consumer brand choice and purchase quantity decisions. *Journal of consumer research 19*(3), 387–400.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics 112*(2), 443–478.

Levy, D., H. A. Chen, S. Ray, and M. E. Bergen (2004). Asymmetric price adjustment in the small: An implication of rational inattention. *TC Koopmans Research Institute*, Working Paper.

Lewis, M. S. (2011). Asymmetric price adjustment and consumer search: An examination of the retail gasoline market. *Journal of Economics & Management Strategy 20*(2), 409–449.

Lisman, J. and E. J. Sternberg (2013). Habit and nonhabit systems for unconscious and conscious behavior: Implications for multitasking. *Journal of Cognitive Neuroscience 25*(2), 273–283.

Mankiw, N. G. (2000). The savers-spenders theory of fiscal policy. *American Economic Review 90*(2), 120–125.

Manzini, P. and M. Mariotti (2007). Sequentially rationalizable choice. *American Economic Review 97*(5), 1824–1839.

Manzini, P. and M. Mariotti (2014a). Stochastic choice and consideration sets. *Econometrica 82*(3), 1153–1176.

Manzini, P. and M. Mariotti (2014b). Welfare economics and bounded rationality: The case for model-based approaches. *Journal of Economic Methodology 21*(4), 343–360.

Marteau, T. M., G. J. Hollands, and P. C. Fletcher (2012). Changing human behavior to prevent disease: The importance of targeting automatic processes. *Science 337*(6101), 1492–1495.

Masatlioglu, Y., D. Nakajima, and E. Y. Ozbay (2012). Revealed attention. *American Economic Review 102*(5), 2183–2205.

Mazumdar, T., S. P. Raj, and I. Sinha (2005). Reference price research: Review and propositions. *Journal of Marketing 69*(4), 84–102.

McAndrews, M. P., E. L. Glisky, and D. L. Schacter (1987). When priming persists: Long-lasting implicit memory for a single episode in amnesic patients. *Neuropsychologia 25*(3), 497–506.

Medin, D. L., R. L. Goldstone, and D. Gentner (1993). Respects for similarity. *Psychological Review 100*(2), 254.

Miller, K. J., A. Shenhav, and E. A. Ludvig (2019). Habits without values. *Psychological Review 126*(2), 292.

Neal, D. T., W. Wood, J. S. Labrecque, and P. Lally (2012). How do habits guide behavior? perceived and actual triggers of habits in daily life. *Journal of Experimental Social Psychology 48*(2), 492–498.

Neal, D. T., W. Wood, M. Wu, and D. Kurlander (2011). The pull of the past: When do habits persist despite conflict with motives? *Personality and Social Psychology Bulletin 37*(11), 1428–1437.

Offerman, T. and A. Van Der Veen (2015). How to subsidize contributions to public goods: Does the frog jump out of the boiling water? *European Economic Review 74*, 96–108.

Oppenheimer, D. M. (2008). The secret life of fluency. *Trends in Cognitive Sciences 12*(6), 237–241.

Ouellette, J. A. and W. Wood (1998). Habit and intention in everyday life: The multiple processes by which past behavior predicts future behavior. *Psychological Bulletin 124*(1), 54.

Patel, M. S., S. C. Day, S. D. Halpern, C. W. Hanson, J. R. Martinez, S. Honeywell, and K. G. Volpp (2016). Generic medication prescription rates after health system–wide redesign of default options within the electronic health record. *JAMA Internal Medicine 176*(6), 847–848.

Peltzman, S. (2000). Prices rise faster than they fall. *Journal of Political Economy 108*(3), 466–502.

Reber, A. S. (1989). Implicit learning and tacit knowledge. *Journal of Experimental Psychology: General 118*(3), 219.

Reis, R. (2006). Inattentive consumers. *Journal of Monetary Economics 53*(8), 1761–1800.

Rubinstein, A. (2007). Instinctive and cognitive reasoning: A study of response times. *The Economic Journal 117*(523), 1243–1259.

Rubinstein, A. (2016). A typology of players: Between instinctive and contemplative. *The Quarterly Journal of Economics 131*(2), 859–890.

Rubinstein, A. and Y. Salant (2012). Eliciting welfare preferences from behavioral dataset. *Review of Economic Studies 79*, 375–387.

Rustichini, A., C. G. DeYoung, J. E. Anderson, and S. V. Burks (2016). Toward the integration of personality theory and decision theory in explaining economic behavior: An experimental investigation. *Journal of Behavioral and Experimental Economics 64*, 122–137.

Salant, Y. and A. Rubinstein (2008). (a, f): Choice with frames. *The Review of Economic Studies 75*(4), 1287–1296.

Schneider, W. and R. M. Shiffrin (1977). Controlled and automatic human information processing i: Detection, search and attention. *Psychological Review 84*(1), 1–66.

Simon, H. A. (1987). Making management decisions: The role of intuition and emotion. *The Academy of Management Executive (1987-1989)*, 57–64.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics 50*(3), 665–690.

Sims, C. A. (2006). Rational inattention: Beyond the linear-quadratic case. *American Economic Review 96*(2), 158–163.

Souleles, N. S. (1999). The response of household consumption to income tax refunds. *American Economic Review 89*(4), 947–958.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *The Review of Economic Studies 23*(3), 165–180.

Sun, C., W. Yang, J. Martin, and S. Tonegawa (2020). Hippocampal neurons represent events as transferable units of experience. *Nature Neuroscience*, 1–13.

Surico, P. and R. Trezzi (2019). Consumer spending and property taxes. *Journal of the European Economic Association 17*(2), 606–649.

Tappata, M. (2009). Rockets and feathers: Understanding asymmetric pricing. *The RAND Journal of Economics 40*(4), 673–687.

Tulving, E. and D. L. Schacter (1990). Priming and human memory systems. *Science 247*(4940), 301–306.

Tversky, A. (1977). Features of similarity. *Psychological Review 84*(4), 327.

Verplanken, B. (2006). Beyond frequency: Habit as mental construct. *British Journal of Social Psychology 45*(3), 639–656.

Verplanken, B. (2010). By force of habit. In *Handbook of Behavioral Medicine*, pp. 73–82. Springer.

Wadud, Z. (2014). The asymmetric effects of income and fuel price on air transport demand. *Transportation Research Part A: Policy and Practice 65*, 92–102.

Whorf, B. L. (1941). Languages and logic. *Foundations of Cognitive Psychology*, 707.

Wood, W., L. Tam, and M. G. Witt (2005). Changing circumstances, disrupting habits. *Journal of Personality and Social Psychology 88*(6), 918.

Woodford, M. (2019). Modeling imprecision in perception, valuation, and choice. *Annual Review of Economics 12*.

Yang, H. and L. Ye (2008). Search with learning: Understanding asymmetric price adjustments. *The RAND Journal of Economics 39*(2), 547–564.