Goal-Oriented Agents in a Market

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July 2020

Barcelona GSE Working Paper Series

Working Paper nº 1190

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Abstract

We consider a market where "standard" risk-neutral agents coexist with "goal-oriented" agents who, in addition to the expected income, seek a high-enough monetary payoff (the "trigger") to fulfill a goal. We analyze a two-sided one-to-one matching model where the matching between principals and agents and the incentive contracts are endogenous. In any equilibrium contract, goal-oriented agents are matched with the principals with best projects and receive the trigger with a positive probability. Moreover, goal and monetary incentives are complementary since goal-oriented agents receive stronger monetary incentives than standard agents. Finally, we discuss policy interventions in relevant environments.

JEL Classification numbers: D82, D86

Keywords: Goal-oriented agents, incentives, matching market

*We thank Joaquín Gómez-Miñambres for his very helpful comments. We acknowledge financial support from the GREEN-Econ research project (grant ANR-16-CE03-0005), Ministerio de Economía y Competitividad and Feder (PGC2018-094348-B-I00), Generalitat de Catalunya (2017SGR-711), and ICREA under the ICREA Academia programme.

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1 Introduction

Individuals sometimes need a certain amount of money to reach a goal. For instance, a person with an idea to start a promising business is often required to cover a percentage of the initial investment because no financier would lend him or her the total amount needed. Thus, this person needs a minimum capital level to implement the idea and become an entrepreneur. Similarly, an individual may have the goal of achieving high-school or college education for him or her, or for a descendant. However, education requires paying expensive tuition fees up front, and it also implies wage losses during schooling years. Hence, even though education may be an excellent investment opportunity for this person for both monetary and personal reasons, he or she first needs to accumulate some savings because the option to borrow for college fees is not available in many countries and may be quite limited in many others. Finally, other relevant examples where an agent's goal may be related to a monetary amount rely on purely behavioral motives. This is the case, for example, when an agent's goal is to reach an aspiration-based reference compensation level.

In this paper, we study the implications of the presence of such "goal-oriented" workers in a market. In an environment where goal-oriented agents coexist with standard ones, we analyze how the heterogeneity in the agents' population affects the equilibrium matching and contracts. We also show that the presence of goal-oriented agents creates a wedge between different markets, which results in potentially important feedback effects of policy interventions.

More precisely, we consider a labor market composed by a population of risk-neutral principals and a population of agents who are subject to limited liability. The agents' population include both "standard" risk-neutral agents who maximize expected income and goal-oriented agents who, in addition to the expected income, care about the possibility of obtaining a high-enough monetary payoff to be able to fulfill their goal. That is, such a monetary payoff constitutes the "trigger" of the goal. Each principal in the market can enter into a partnership with an agent in order to develop the principal's project, which ends up being either a success or a failure, the probability of a successful outcome depending on the agent's effort. However, any principal-agent relationship is subject to moral hazard because the agent's effort is not verifiable. Hence, the two partners sign a contract including a payment that can be contingent on the outcome. An important element of our model is that the goal of the goal-oriented agents is external to the market we study here: a principal who hires a goal-oriented agent cannot influence the characteristics of this goal, although she can influence the possibility that the agent reaches the trigger through the monetary compensation offered in the contract.

Thus, we analyze a two-sided one-to-one matching model where the matching (who partners with whom) as well as the incentive contracts implemented in the formed partnerships are endogenous.[1] In this market, we look for the equilibrium, or stable outcomes.[2] One important property of equilibrium contracts is that they are Pareto optimal.

Goal-oriented agents have stronger incentives to work hard than standard agents if the contract allows for the possibility to reach the trigger. Thus, goal-oriented agents may be more attractive to principals than standard agents. We assume an environment where the agents' population is larger than that of the principals, but where the number of goal-oriented agents is low. In this environment, all goal-oriented agents will be matched at equilibrium.

We show that, in any equilibrium contract signed in a partnership involving a goal-oriented agent, the worker receives the trigger with a positive probability. If the trigger is not too high, then the equilibrium contract includes, in addition to the base payment, a bonus that is always paid in case of success and ensures that, following a successful outcome, the compensation level is higher than the trigger. On the other hand, if the trigger is high, then the bonus (in case of success) is not always paid. The equilibrium contract specifies a bonus such that the agent receives the trigger in case of success, but the bonus is only paid with some probability. This contract structure improves on the agent's incentives and expected utility with respect to another contract specifying a lower bonus that would always be paid in case of success.

At the equilibrium, goal-oriented agents work harder than standard agents for two reasons. First, as we have seen, the equilibrium contract always opens up the possibility that the goal be achieved in case of success which, ceteris paribus, induces a goal-oriented

[1] See the seminal papers by Gale and Shapley (1962) and Shapley and Shubik (1972). The first paper introduces the marriage and college admission models (two-sided one-to-one and many-to-one matching models without transfers) whereas the second paper introduces the assignment game (two-sided one-to-one matching models with perfectly transferable utility).

[2] In our framework, utility is partially transferable among participants. Demange and Gale (1985) study the stable outcomes in this non-linear environment.

agent to exert a higher effort than a standard agent. Second, and more interestingly, at the equilibrium outcome, goal and monetary incentives are complementary: goal-oriented agents receive stronger monetary incentives than standard agents. Indeed, the competition among principals to hire the goal-oriented agents forces them to offer to those agents a higher expected utility than what the standard agents get. In the equilibrium contract, the increase in utility translates into higher incentives. Since goals are exogenous, we here stress a causality effect from goals to incentives.

We also show that the more profitable a principal's project is, the more she benefits from hiring a goal-oriented agent instead of a standard one. As a result, at equilibrium, the goal-oriented agents work for the most profitable projects. That is, the matching between principals and agents is positive assortative.

Finally, we use our comparative statics exercises to discuss the potential effects of some policy interventions. We provide this discussion in three relevant environments: when the agent's goal is to become an entrepreneur, when his goal is to buy an asset with persistent benefits, and when the goal is due to an aspiration-based reference compensation level.

Consider, for instance, the case where some agents' goal is to become entrepreneurs. When working in the market analyzed here, a goal-oriented agent's extra utility would correspond to the sum of the expected profit that the agent obtains if he could start the venture and of non-monetary benefits associated to being an entrepreneur. We first show that a policy that would increase the base salary in our market would lead to a higher effort level exerted by goal-oriented agents, but also to an increase in the probability that these agents become entrepreneurs. A second policy that would provide a subsidy to new entrepreneurs, or that would decrease the total investment needed to carry out the agent's project idea, would increase the goal-oriented agents' motivation to exert effort in their current job, which in turn would positively impact principals' profit. Moreover, it would also increase these agents' utility, as they would be more likely to become entrepreneurs. Yet, such a subsidy scheme would need to be funded by the government, and the positive effect on the goal-oriented agents' utility would need to be traded off with the cost resulting from the use of public funding. A final option would be to affect the taxes and fees paid by entrepreneurs (or self-employed people), affecting the expected income of these activities. Reducing the charges to be paid by entrepreneurs would increase the number of entrepreneurs and have an indirect positive effect on the market in which these individuals were employed as agents.

Thus, there are externalities from a policy implemented in the present labor market on the entrepreneurship activity, and policies related to the entrepreneurship activity have in turn external effects on the present labor market.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model and the equilibrium concept. Section 4 provides the results. In Section 5, we discuss the impact of some policy interventions. Finally, Section 6 concludes. All the proofs are provided in an Appendix.

2 Related literature

Our model combines three elements: a moral hazard problem with limited liability, a market where agents and principals match and sign contracts, and agents exhibiting an additional motivation.

Limited liability has been extensively used to study moral hazard problems. Given its tractability, considering that agents are risk neutral and protected by limited liability (see, e.g., Innes, 1990; and Poblete and Spulber, 2012) allows to introduce moral hazard problems in complex models. This approach has been particularly helpful to analyze principal-agent relationships in developing countries, where agents' limited wealth is an important concern, or in markets in which it is critical to have some initial capital. For example, Shetty (1988) considers the market between tenants and landlords, and analyzes the influence of tenants' wealth on tenancy contracts. Quérou et al. (2020) study a situation where the agent's effort implies not only disutility but also monetary costs, in such a way that limited resources restrict the agent's capability to exert effort.

The second element of our analysis is to introduce a moral hazard problem in a market where principals and agents meet. Several papers have considered principal-agent matching markets under moral hazard.[3] For example, Besley and Ghatak (2005) consider mission-oriented and profit-oriented principals together with agents who may or may not support the mission. Chakraborty and Citanna (2005) study an environment where individuals differ in their wealth endowment and they can either remain self-employed or engage in productive matches with another individual. Dam and Pérez-Castrillo (2006)

[3]For a recent review, see Macho-Stadler and Pérez-Castrillo (2020).

analyze the influence of the distribution of the agents' wealth on the efficiency of the matching formed. One feature of our matching model is that utility is not perfectly transferable. For these environments, Legros and Newman (2007) provide results highlighting whether the matching is positive or negative assortative depending on the characteristics of the participants on both sides of the market. By contrast, our main focus in this paper lies in understanding the characteristics of the contracts signed by the parties.

The third ingredient of our model is the interplay between monetary rewards and the existence of an additional motivation resulting from a personal goal. In our model, a sizable monetary bonus triggers an extra utility for some agents, the goal-oriented ones, because it allows these agents to reach their goal. This view is complementary to the one described in a large literature studying how monetary payments may reduce the agents' "intrinsic" motivation when they care about what they do, in addition to how much they are paid (see e.g., Deci and Ryan, 1985 and Kreps, 1997).[4] In that literature, monetary incentives crowd out the agent's intrinsic motivation. By contrast, in our setup the monetary bonus may enable the goal: hence, monetary incentives may trigger the incentives derived from the goal.

The idea that agents may have goals, or reference points, that affect their behavior has been studied in several papers. For example, Köszegi and Rabin (2006) consider that employees may have reference-dependent preferences, where the reference point corresponds to the agents' expectation about outcomes. Gómez-Miñambres (2012), Corgnet et al. (2015), and Brookins et al. (2017) study isolated principal-agent environments where a goal (in terms of output to the agent) can be set either by the principal (in the first two contributions) or by the agent himself (in the third contribution). The agent derives an intrinsic utility when fulfilling the goal and may derive an intrinsic disutility if he does not reach the goal. In contrast, the agent's goal in our setup is exogenous, idiosyncratic, and not related to the outcome of the relationship but to the wage. Moreover, we study equilibrium contracts where goal-oriented agents coexist with standard agents. Corgnet et al. (2015) and Brookins et al. (2017) find, as we do, a complementarity between the strength of the incentives and the magnitude of the agents' goal. In these contributions, an increase in the exogenous strength of incentives leads (either the principal or the agent)

[4]For more details see Köszegi (2014) and Macho-Stadler and Pérez-Castrillo (2018), as well the references included in these survey papers.

to select a higher goal. In our paper, the causality is reversed: an increase in the exogenous magnitude of the goal (which is unrelated to the output) yields stronger incentives in the goal-oriented agent's contract. Moreover, this complementarity only occurs at the market equilibrium: it would not hold in an isolated principal-agent relationship.[5]

To our knowledge, the closest contribution to ours is Ghatak et al. (2001). They consider a market similar to ours where agents experience an extra motivation from the possibility of becoming principals in the future. More precisely, Ghatak et al. (2001) study an overlapping generations model of a principal-agent problem in which contracts are determined in general equilibrium. In their model, all young workers are identical but, depending on their performance, they have different investment possibilities when being old. Old agents can choose between becoming principals (at a cost) or remaining workers. Imperfections in the credit market yield rents to principals (entrepreneurs), and these rents induce an extra motivation for all young workers. They show that the potential option to become a principal in the market induces higher effort levels and increases welfare. The authors stress the fact that policies reducing market credit imperfections (or redistributing income) may reduce welfare by dampening this effect. We model the extra motivation in a similar way as they do. In contrast to Ghatak et al. (2001), not all agents in the market exhibit the motivation to become entrepreneurs. Moreover, we consider a static model where the agents become entrepreneurs in a different market than the one they work in. Finally, we solve for a larger class of contracts. Assuming that only some agents are goal-oriented allows to discuss how the matching and the contracts depend on the scarcity or abundance of this type of agents in the economy. Considering that the goal is achieved in a different market than the one we focus on implies that the (finite) population of principals is given and, when they design the incentive schemes, they do not fear the increase in the intensity of competition in their own market. Finally, we consider random contracts, including a combination of a base salary and a bonus payment that the agent receives only with some probability in case of success.[6]

[5]Several contributions have focused on loss-averse agents (see for instance De Meza and Webb, 2007, Ditman et al., 2010, and Herweg et al., 2010). In contrast to the literature discussed above, where reaching the goal induces an extra utility to the agents (an intrinsic motivation), a loss-averse agent experiences disutility when he does not reach the goal. We discuss the consequences of the existence of a form of loss aversion in our environment in subsection 4.3.

[6]Payments that are not only tied to outcomes but also to a random device that does not depend

3 The model

We consider a labor market with n risk-neutral principals and m agents. In this market, each principal and each agent can form a partnership, that is, they can be matched. Each of these players can only participate in one partnership. Alternatively, each principal or agent can remain unmatched.

If a principal and an agent enter into a relationship, they develop a project. The outcome of the project x can be either a success or a failure. If successful, the relationship yields a return $R > 0$, while failure results in zero return.[7] The agent's effort e, $e \in [0, 1]$, determines the probability of success $p(e) = e$ of the relationship. The cost of effort e for the agent is $\frac{v}{2}e^2$. The effort is not verifiable. The principal is the residual claimant of the output and designs the contract to address this moral hazard problem.

The contract exhibits three features: a base salary w to be paid by the principal to the agent, a bonus $\Delta \geqslant 0$ that the agent may get in case of success, and the probability $p \in [0, 1]$ with which the agent gets the bonus in case of success.

The main idea to address the classical moral hazard model is the provision of a monetary motivation to the agent through an incentive scheme (w, Δ, p).[8] In our paper, we consider that some agents may also have some additional motivation related to the wage they receive. We assume that obtaining a certain level of money allows some agents to achieve a goal, for instance: undertaking an activity outside this market, buying an asset with persistent benefits, or fulfilling some aspiration. For any of these reasons, an agent may obtain an extra utility (his extra motivation) if his payoff lies above a certain threshold level, which we denote by z, $z > 0$. That is, z is the trigger of the extra motivation. We denote by k the extra utility that such an agent gets if he is able to fulfil his goal, that is, if he receives a payment that is at least equal to z.

on the agent's effort are usually not considered because they are either dominated by output-contingent contracts, if the agents are risk averse, or equivalent, if the agents are risk neutral. This property does not hold for goal-oriented agents, whether they are risk neutral or risk averse, if it allows reaching the goal with some probability.

[7] We here consider the case where the technology is homogeneous. We will extend the analysis to the case of heterogeneous technologies in Section 4.2.

[8] In order for a random scheme to be verifiable, the principal must be able to commit to the probability with which she will pay the bonus. She can delegate running the lottery to a third party or refer to the result of a third-party lottery.

Thus, in our market an agent can be of two types. First, he may be risk neutral and not have a personal goal. We will refer to this type of agent as a *standard agent.* Second, the agent may have a personal goal, which is described by the parameters (z, k). In this case, his utility function is linear in money, except at the trigger z, where there is a jump upwards. We will refer to such an agent as a *goal-oriented agent.* Hence, a standard agent corresponds to an agent with extra utility equal to 0. Formally, the utility of a standard (S) and that of a goal-oriented (G) agent who sign a contract (w, Δ, p) and exert effort e are, respectively:

$$\begin{aligned} u^S(w, \Delta, p; e) &= w + pe\Delta - \frac{v}{2}e^2, \\ u^G(w, \Delta, p; e) &= (w + I_w k) + pe\left(\Delta + I_{w+\Delta} k\right) - \frac{v}{2}e^2, \end{aligned}$$

where $I_w = 0$ if $w < z$; $I_w = 1$ if $w \geq z$; $I_{w+\Delta} = 0$ if either $w + \Delta < z$ or $w \geq z$; and $I_{w+\Delta} = 1$ if $w < z \leq w + \Delta$.

The number of goal-oriented agents is m^G and that of standard agents is m^S, with $m^G + m^S = m$. We assume that $m > n > m^G$, that is, there are less principals than agents, but there are less goal-oriented agents than principals.

The principal's expected profit under the contract (w, Δ, p) when the agent exerts effort e is:

$$\Pi(w, \Delta, p; e) = eR - pe\Delta - w.$$

The principal's profit does not depend directly on the agent's characteristics, although it may depend indirectly through the agent's choice of effort.

We assume that agents are protected by limited liability so that their final payoff cannot be lower than a minimum wage which we denote $\underline{w}$. Then, a feasible contract must satisfy the limited liability constraints (LLC):

$$w \geq \underline{w} \text{ and } w + \Delta \geq \underline{w}. \tag{1}$$

The contract must be designed also taking into account that the effort will be decided by the agent. He supplies the effort e that maximizes his expected utility, given the contract (w, Δ, p). The incentive compatibility constraint (ICC) for a standard agent is:

$$e^S(w, \Delta, p) = \frac{p\Delta}{v}.$$

For a goal-oriented agent, the ICC depends on the payments w and $w+\Delta$ and the trigger z:

$$e^G(w,\Delta,p) = \begin{cases} \frac{p(\Delta+k)}{v} \text{ if } w < z \leq w+\Delta \\ \frac{p\Delta}{v} \text{ if either } w+\Delta < z \text{ or } z \leq w. \end{cases}$$

Indeed, if $w < z \leq w+\Delta$ then a goal-oriented agent obtains the extra utility k in case of success, which provides him with extra incentives on top of the bonus Δ.

Assumption 1 specifies some restrictions on the parameters:

Assumption 1 *(i)* $\underline{w} < z$; *(ii)* $\underline{w} < \frac{1}{8v}R^2$; *and (iii)* $k+z-\underline{w} < v$ *and* $R \leq 2\left(v - \sqrt{vk}\right)$.

Part (i) ensures that the base salary is not sufficient to trigger the extra utility, whereas (ii) is necessary to ensure that the principal derives positive profits when contracting with a standard agent. Condition (iii) ensures that the effort in the equilibrium contracts lies in the interval $[0,1]$.

Finally, if an agent is not involved in a partnership, then the utility that he obtains outside the market (or when he is not matched) is $U^o = 0$. Similarly, a principal obtains zero profits if she is not matched to an agent, that is, $\Pi^o = 0$.

3.1 The equilibrium concept

In our market, each principal can contract with any agent and each agent can partner with any principal. Therefore, the (two-sided one-to-one) matching between principals and agents is endogenous. The endogeneity of the matching implies that the level of both principals' profits and agents' utilities is determined by the competition in the market. Moreover, the contract in any partnership is also endogenous. Thus, an *outcome* in the market consists of a set of partnerships, a set of principals and agents that are unmatched, and a contract for each partnership.

The equilibrium concept that we use is the classic concept of *stability* (see Shapley and Shubik, 1971, for the case of transferable utility and Demange and Gale, 1985, for the non-linear case). We will refer to a stable outcome as an equilibrium outcome.

An outcome is an *equilibrium outcome* if (i) each partner in a relationship obtains at least the same profit/utility as when she/he remains unmatched and (ii) there is no principal-agent pair and no contract such that both the principal and the agent are better-off under the new contract than in the equilibrium outcome.

Contracts at an equilibrium outcome satisfy a property that will be useful in our analysis: any contract signed in equilibrium must be Pareto optimal. Otherwise, the two members of the partnership could deviate and sign a mutually beneficial contract.

4 Characterization of equilibrium outcomes

4.1 Uniform technology

In this subsection, we consider that *all principals have the same technology* (they all have the same R). The agents only differ in whether they are standard or goal-oriented (but they are identical within each group).

Goal-oriented agents are more appealing for the principals than standard agents because of the additional utility that they can obtain, and because of the additional incentives due to the possibility of reaching the goal. This implies that, at the equilibrium, it cannot happen that a goal-oriented agent is unmatched while a standard agent is matched. Moreover, Assumption 1 (ii) ensures that a partnership between a principal and a standard agent is always profitable. Given that we consider a market where $0 < m^G < n < m$, it is necessarily the case that, at an equilibrium outcome: (a) the n principals are matched (because there is surely an agent who is unmatched and the partnerships are profitable); (b) the m^G goal-oriented agents are also matched (because some principal is certainly matched with a standard agent and she could deviate with any unmatched goal-oriented agent); and (c) some standard agents are matched and some others are not (because $m > n > m^G$ and the matching is one-to-one).

The previous properties can be used to characterize the equilibrium contract offered to the standard agents who are matched. We know that it must be Pareto optimal. Moreover, the abundance of standard agents implies that the principal can sign the best contract for her that ensures that the agent gets his outside option $U^o = 0$. The characterization of the optimal contract then follows classic arguments.

Lemma 1 *At the equilibrium outcome, in any partnership between a principal and a standard agent:*
(i) the contract is $C^{S*} \equiv \left(w = \underline{w}, \Delta = \frac{1}{2p}R, p\right)$ *with* $p \in (0, 1]$*;*
(ii) the principal obtains profit $\Pi^{S*} \equiv \Pi^S(C^{S*}) = \frac{1}{4v}R^2 - \underline{w}$*;*

(ii) the agent's effort is $e^S = \frac{R}{2v}$ *and his expected utility is* $U^{S*} \equiv U^S\left(C^{S*}\right) = \frac{1}{8v}R^2 + \underline{w}$*.*

When a principal hires a standard agent under moral hazard, and random payments are possible, several contracts are optimal. We consider without loss of generality that the optimal contract is $(w^S = \underline{w}, \Delta^S = \frac{1}{2}R, p^S = 1)$.

Lemma 1 states that standard agents who get an offer obtain U^{S*} in the market. This is higher than their outside utility U^o since, as it is common in situations characterized by limited liability, they obtain a rent. The rationale is that the principals are interested in providing incentives to induce effort supply, and the limited liability constraint is stronger than the participation constraint, which is not binding at the optimal contract.

Although hiring a goal-oriented agent is, a priori, more profitable for a principal, competition among the principals implies that those who hire them cannot benefit from the potential extra surplus. Two identical principals must obtain the same equilibrium profit if, say, principal P_1 would obtain lower profits than principal P_2, then she could deviate with the agent matched with P_2 by offering him a slightly better contract. If there is any extra surplus, it will translate into a higher utility for these agents. Lemma 2 states this property.

Lemma 2 *The principals hiring goal-oriented agents obtain equilibrium profits* $\Pi^G = \Pi^{S*} = \frac{1}{4v}R^2 - \underline{w}$*.*

We now provide some properties of the contract signed with the goal-oriented agents at the equilibrium. The equilibrium contract $C^G = (w, \Delta, p)$ designed for such an agent can in principle lie in one of three feasible sets of parameter values.

First, the contract can be such that $w + \Delta < z$ is satisfied. In this case, the goal-oriented agent is similar to a standard agent.

Second, the contract can provide extra incentives because the payment scheme satisfies $w < z \leq w + \Delta$. Here, the agent gets incentives to work because $\Delta > 0$ and he also has extra incentives because he obtains $k > 0$ with probability p in case of success. Moreover, because he obtains the extra utility k with some probability, the agent's utility level is higher than the level a standard agent would obtain under the same contract.

Finally, the contract can pay more than z in any outcome: $w \geq z$ is satisfied. In this case, the contract does not imply any additional incentive but, compared to the utility that a standard agent would obtain under the contract, the goal-oriented agent obtains an extra utility k.

Lemma 3 highlights that the equilibrium contract lies in the second region, that is, it satisfies $w < z \leq w + \Delta$. It also states the intuitive property that the base salary w is always equal to the minimum salary $\underline{w}$.

Lemma 3 *The equilibrium contract for a goal-oriented agent satisfies $w = \underline{w}$, $p \in (0,1]$, and $\Delta \geq z - \underline{w}$.*

Lemma 3 has useful implications for the characterization of the equilibrium contract signed in any partnership between a principal and a goal-oriented agent. Since the contract is Pareto optimal, we can compute it by either maximizing the principal's utility subject to the agent's participation constraint, or by maximizing the agent's expected utility subject to the constraint that the principal obtains a given level of profit. We focus on the second program because Lemma 2 provides the equilibrium level of profit obtained by the principals. Moreover, Lemma 3 ensures that, at the equilibrium contract, the goal-oriented agent can achieve his goal with some probability in case of success since $w = \underline{w} < z$ and $\Delta \geq z - \underline{w}$. Therefore, in an equilibrium contract $(\underline{w}, \Delta, p)$, the agent's effort is $e = \frac{p(\Delta+k)}{v}$.

Theorem 1 provides the equilibrium contracts, principals' profits and agents' utilities in any partnership. The equilibrium contract for the goal-oriented agents can take two different forms depending on whether the technology R is higher or lower than a threshold, which we denote $\widehat{R}$:

$$\widehat{R} \equiv 2\left(k + z - \underline{w} - \sqrt{k\left(k + z - \underline{w}\right)}\right).$$

Theorem 1 *Under Assumption 1, at an equilibrium outcome:*
(a) Standard and goal-oriented agents sign the following contracts, respectively:

$$C^S = \left(w = \underline{w}, \Delta = \frac{R}{2}, p = 1\right).$$

$$C^G = \begin{cases} \left(w = \underline{w}, \Delta = z - \underline{w}, p = \frac{R}{2(z-\underline{w})}\left(1 + \frac{\sqrt{k}}{\sqrt{k+z-\underline{w}}}\right)\right) & \text{if } R \leq \widehat{R} \\ \left(w = \underline{w}, \Delta = \frac{1}{2}\left(R + \sqrt{k\left(2R+k\right)} - k\right), p = 1\right) & \text{if } R \geq \widehat{R}. \end{cases}$$

(b) All the principals obtain profit

$$\Pi = \frac{1}{4v}R^2 - \underline{w}.$$

(c) Standard agents (if they are matched) and goal-oriented agents obtain the following utility levels, respectively:

$$U^S = \frac{1}{8v} R^2 + \underline{w}.$$

$$U^G = \begin{cases} \frac{1}{8v}\left(\frac{R}{z-\underline{w}}\left(k+z-\underline{w}+\sqrt{k\left(k+z-\underline{w}\right)}\right)\right)^2 + \underline{w} \text{ if } R \le \widehat{R} \\ \frac{1}{8v}\left(R+k+\sqrt{k\left(2R+k\right)}\right)^2 + \underline{w} \text{ if } R \ge \widehat{R}. \end{cases}$$

For values of R lower than $\widehat{R}$, the probability of getting the bonus $\Delta = z - \underline{w}$ is smaller than 1; for larger values of R, the agent gets with probability 1 a bonus larger than $z - \underline{w}$. Note that the optimal contract is continuous at $R = \widehat{R}$: the corresponding expression is $(w = \underline{w}, \Delta = z - \underline{w}, p = 1)$. The goal-oriented agent's expected utility U^G is also continuous and it is increasing in R.

Note also that the cut-off $\widehat{R}$ in Theorem 1 is decreasing in k and increasing in z. The larger the extra utility from reaching the goal k, the more valuable is the agent for the principal; hence, the larger the set of project values R that lead to contracts where the agent is rewarded more than z in case of success and where the bonus is always paid in case of success. On the other hand, since the cut-off is increasing in z, the use of this type of contract is less prevalent when the trigger is higher.

We now discuss the characteristics of the equilibrium contracts stated in Theorem 1. First, for sufficiently high values of R ($R \ge \widehat{R}$) the optimal contract exhibits an interesting and non straightforward property. The intuition suggests that, since goal-oriented agents are easier to motivate than classical ones, the optimal contract might stipulate a lower bonus for goal-oriented agents. Indeed, if one considers an isolated principal-agent contract, goal and monetary incentives are either strict substitutes (for most cases) or independent (for intermediate technology levels). Yet, where there are market interactions, it is easily checked that $\Delta > \frac{R}{2}$ if $k > 0$. That is, at the equilibrium outcome, goal and monetary incentives are complementary. The reason is that goal-oriented agents are more appealing for the principals than standard agents. Given that the principals compete for them, the goal-oriented agents obtain a higher equilibrium utility level, which translates into higher incentives rather than into a higher fixed payment, because higher incentives increase the surplus.

Second, for low values of R ($R < \widehat{R}$) the principals still benefit from the motivation derived from the existence of a goal. However, given that the trigger z is high relative to R, a principal would obtain lower profits than Π^{S*} if she would always pay z in case of success. Hence, she offers a contract where she pays z in case of success but with a probability lower than 1. Therefore, a random contract is optimal and it is offered at the equilibrium.

We discuss some potential policy implications of our analysis in Section 5, where we present three environments which are consistent with our theoretical setting. To guide this discussion, next corollary (whose proof follows easily from Theorem 1) states the effect of changes in the parameters $\underline{w}$, z, and k on the goal-oriented agent's decision at the equilibrium, as well as on the cut-off $\widehat{R}$ that characterizes the two sets of parameter values described in Theorem 1.

Corollary 1 *The comparative statics effects of $\underline{w}$, z, and k on a goal-oriented agent's equilibrium effort e^G, on his probability pe^G of triggering the additional utility k, and on the cut-off $\widehat{R}$, have the following signs:*

	$R < \widehat{R}$		$R \geq \widehat{R}$		
	e^G	pe^G	e^G	pe^G	$\widehat{R}$
$\underline{w}$	$+$	$+$	0	0	$-$
z	$-$	$-$	0	0	$+$
k	$+$	$+$	$+$	$+$	$-$
R	$+$	$+$	$+$	$+$	.

4.2 Heterogeneous principals

We now introduce the possibility that the principals are heterogeneous as they have different values associated with the success of the project: $R_1 > R_2 > ... > R_n$.[9]

Given that goal-oriented agents are more appealing than standard agents to all the principals, the first question relates to the characteristics of the principals who will hire them at equilibrium. Theorem 1 states that the goal-oriented agents' expected utility increases with the return R, which suggests that the higher the return R, the more interested is the principal in a goal-oriented agent. Lemma 4 confirms this intuition.

Lemma 4 *If a principal with R^o signs an optimal contract with a goal-oriented agent such that she obtains at least the same profits with the goal-oriented agent than with a standard*

[9] In this subsection, we amend Assumption 1 as follows: (i) $\underline{w} < z$; (ii) $\underline{w} < \frac{1}{8v}R_n^2$; and (iii) $k+z-\underline{w} < v$ and $R_1 \leq 2(v - \sqrt{vk})$.

agent, then any principal with $R > R^o$ prefers hiring the goal-oriented agent under a contract that guaranties the agent the same utility as with R^o instead of a standard agent.

An implication of Lemma 4 is that, at equilibrium, the goal-oriented agents will be hired by the principals whose value in case of success is the highest, that is, principals 1 to m^G. To attract the goal-oriented agents, these principals offer them a high-enough level of utility so that the other principals (in particular, the "marginal" principal $m^G + 1$) do not have an incentive to hire a goal-oriented instead of a standard agent. As it is common in this type of environment with heterogeneous principals and agents, there are several equilibrium outcomes in terms of levels of utility and profits, although the qualitative properties of these equilibria are similar in our framework. We are going to characterize one particular equilibrium, the one corresponding to the most profitable equilibrium outcome for the principals.

Depending on the parameter values, the specifics of the equilibrium contract can differ. For simplicity, in Theorem 2 we are going to assume that R_{m^G+1} is smaller than $\widehat{R}$, and that R_1 is not too large compared to R_{m^G+1}. We will then discuss what would happen in the other cases. The theorem uses some properties (Lemmas 1 and 3) that we have shown in the homogeneous case and that still hold when principals are heterogeneous.

Theorem 2 *Under Assumption 1, if $R_1 > R_2 > ... > R_n$, $R_{m^G+1} < \widehat{R}$ and $R_1 < \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right) R_{m^G+1}$ then:*
(a) Principal i $(i = 1, ..., m^G)$ is matched with a goal-oriented agent under the contract

$$C^G(R_i) = (w = \underline{w}, \Delta = z - \underline{w}, p = \frac{1}{2\,(z - \underline{w})} \left(1 + \sqrt{\frac{k}{k + z - \underline{w}}}\right) R_{m^G+1}),$$

and principal i $(i = m^G + 1, ..., n)$ is matched with a standard agent under the contract

$$C^S(R_i) = \left(w = \underline{w}, \Delta = \frac{R_i}{2}, p = 1\right).$$

(b) Principal i obtains profit:

$$\Pi(R_i) = \begin{cases} \frac{(z-\underline{w}+k)}{4(z-\underline{w})v} \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right) \left(2R_i - \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right) R_{m^G+1}\right) R_{m^G+1} - \underline{w} \text{ if } i \le m^G \\ \frac{1}{4v} R_i^2 - \underline{w} \text{ if } \; i > m^G. \end{cases}$$

(c) Standard and goal-oriented agents obtain the following utility levels:

$$U^S = \frac{1}{8v}R_i^2 + \underline{w} \text{ when matched with principal } i > m_G,$$
$$U^G = \frac{1}{8v}\left(\frac{R_{m^G+1}}{z-\underline{w}}\left(k+z-\underline{w}+\sqrt{k\left(k+z-\underline{w}\right)}\right)\right)^2 + \underline{w}.$$

The first implication of Theorem 2 is that there is a positive assortative matching between principals and agents: the principals with more profitable projects hire the goal-oriented agents, that is, those agents who will exert higher effort. The higher the return of a successful outcome, the more the principal benefits from hiring a goal-oriented agent compared to a standard agent.

Second, the theorem highlights that in those partnerships involving a goal-oriented agent, the contract, the principal's profits and the goal-oriented agent's utility depend on the characteristic of the (marginal) principal who does not hire a goal-oriented agent, that is, on R_{m^G+1}. Due to their scarcity in the market, goal-oriented agents have market power because they can get offers from principals who hire standard agents. The best offer comes from the marginal principal, principal m^G+1. The better the offer that they can receive from this principal, the better the contract they sign with their employers at the equilibrium, thus the higher the utility they obtain, and the lower the employers' profit. As a consequence, if the population of goal-oriented agents is smaller (that is, m^G is lower), then R_{m^G+1} is higher and they obtain better equilibrium utility.

Third, as it is usual in moral hazard problems with limited liability, the contract and the utility of standard agents depend on the profitability of the project they work on. The more profitable the principal's project, the more she is interested in a high effort and the higher the rent she leaves to the agent. Thus, similar standard agents may enjoy different levels of utility at the equilibrium.

We notice that the comparative static exercises provided in Corollary 1 (except for those with respect to R) hold in the current environment with heterogeneous principals. Under the conditions stated in Theorem 2 (that is, $R_{m^G+1} < \widehat{R}$ and $R_1 < \left(1+\sqrt{\frac{k}{k+z-\underline{w}}}\right)R_{m^G+1}$), the comparative static related to the effort and to the probability of achieving the goal are the same as those for the case where $R < \widehat{R}$. Both effort and probability of triggering k increase with $\underline{w}$ and k, and decrease with z.

Finally, we comment on the consequences of the assumptions about the population of principals in Theorem 2. The main message does not change in the other cases, although the specifics of the contract differ. For instance, if $R_{m^G+1} < \widehat{R}$ but $R_1 > \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right) R_{m^G+1}$, then the profitability of the project for some principals is so high that their optimal contract does not take into account the goal-oriented agent's participation constraint. The optimal contract only depends on the return R. Still, the comparative exercises for the effort and the probability of achieving the goal are the same as before, unless R is very high, in which case only k influences both e^G and pe^G. Finally, if $R_{m^G+1} > \widehat{R}$ then again the contract only depends on k and not on $\underline{w}$ or z. In particular, if $R_1 < \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right) R_{m^G+1}$ then the contract is the same for all the principals who hire a goal-oriented agent: it does not depend on R but on R_{m^G+1}.

4.3 When the goal is to avoid a penalty

In the previous analysis, we have considered an environment where the goal constitutes an opportunity for some agents to obtain an extra utility. Reaching the goal (that is, receiving a salary higher than the trigger) provides a goal-oriented agent with an additional utility compared to a standard agent.

We could also envision environments where obtaining a certain amount of money does not open up an opportunity to improve on the agent's utility, but it is rather a requirement for the agent to avoid a penalty. For instance, an indebted individual may experience economic or judicial consequences if he does not repay his debt. The trigger in this case corresponds to the level of the debt: if the individual does not reach the trigger z, he experiences a disutility k. We can call such an individual a "loss-avoidance agent." His utility function is identical to that of a standard agent for monetary payment levels of at least z but it is *lowered* by k if the monetary payment lies below z.[10]

As we made it clear in the previous subsections, a goal-oriented agent is more appealing for the principals than a standard agent because of two reasons: the additional incentives due to the possibility of reaching the goal, and the higher utility obtained by a goal-oriented agent. In terms of incentives, a loss-avoidance agent is similar to a goal-oriented

[10] Although the models are quite different, there are some similarities between the idea that agents lose utility if they do not reach their goal and the idea of loss-averse agents (Kahneman and Tversky, 1979, Tversky and Kahneman, 1991 and 1992). In both cases, the shape of the agent's utility function changes at some reference income. For an analysis of optimal contracts in the presence of loss-averse agents, see for instance De Meza and Webb (2007) and Herweg et al. (2010).

agent. The existence of a kick at the trigger level provides either type of agent with an extra incentive if $w < z \leq w + \Delta$ and $p > 0$. However, in terms of utility, a loss-avoidance agent may be more "expensive" not only than a goal-oriented agent but also than a standard agent. Indeed, given that they experience a lower utility than standard agents for monetary payment levels below z, loss-avoidance agents may require a higher average salary. Therefore, it is a priori not clear whether a loss-avoidance agent is more or less appealing for a principal than a standard agent.

If we consider a market where loss-avoidance agents coexist with standard agents (and the number of standard agents is large) then, as it happens in a market with goal-oriented agents, the positive effect due to the extra incentives is stronger the higher the value R associated to success. Therefore, if R is high enough, a principal prefers a loss-avoidance agent to a standard agent. What are the implications of this property for the equilibrium contracts?

First, if the principals in the market are homogeneous (as in subsection 4.1) and R is high enough then, at equilibrium, the principals will compete for the loss-avoidance agents. Given that the level of utility of these agents is determined by the degree of competition (and not by their outside option) the equilibrium contracts in this case are identical to those stated in Theorem 1. On the other hand, if R is low, then these agents are not appealing for the principals and will not be hired at equilibrium.[11]

Second, if the principals are heterogeneous in the values associated with the success of the project (as in subsection 4.2) then Lemma 4 still holds, which implies that if some loss-avoidance agents are hired in the market, they are matched to the principals with the highest R's. Depending on the value of R for the top principals, it may happen that the principals compete for the loss-avoidance agents (this is the case if the value of R for the "marginal" principal is high enough), in which case Theorem 2 still applies. But, it may also happen that only a few loss-avoidance agents (or even none of them) are hired by the principals with the highest value of R. In this case, these principals can offer a contract where a loss-avoidance agent's participation constraint is not determined by competition but by his outside option.

[11] Depending on the other model parameters, it could be the case that any principal (that is, any R) prefers a loss-avoidance agent to a standard agent or that (because of the constraint that the effort level cannot be larger than 1) any principal prefers a standard agent to a loss-avoidance agent.

5 Policy interventions

In this section, we describe three environments that explain why agents behave in a way consistent with our analysis. In the three environments, obtaining a high-enough payment allows the agent to achieve his goal. We discuss the consequences of several policy interventions making use of the comparative statics exercises provided in Corollary 1 and in the discussion following Theorem 2. To simplify the explanations, we consider situations where $R_{m^G+1} < \widehat{R}$ and $R_1 < \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right) R_{m^G+1}$ (or $R < \widehat{R}$ in case of an homogeneous technology) are satisfied. The discussion for the other cases is similar, unless the technology of the "marginal" principal (that is, R_{m^G+1}) is larger than $\widehat{R}$, a situation in which the contracts signed with the goal-oriented agents only depend on k.

5.1 Entrepreneurship

Consider an environment where the goal of some agents is to become entrepreneurs. Each of these agents has an idea that can yield profits in a different market. However, he needs some capital to develop his business idea and this is the reason why he works in the labor market that we study.

Indeed, becoming an entrepreneur requires a certain investment. The idea is risky, in the sense that it leads to a positive outcome only with some probability. To provide a simple framework we assume, following Bester and Hellwig (1987) and Tirole (2010, chap. 3), that if the idea is carried out, the entrepreneur can take two possible decisions. The venture yields positive net return only if the correct decision is taken. The alternative decision yields negative net present value but provides private benefits to the entrepreneur. In this framework, due to the entrepreneur's moral hazard problem, he only takes the correct decision if he keeps enough shares of the new venture. This puts a constraint on the shares that the entrepreneur can offer to a financier, hence, to the amount of money that he can borrow. In other words, because of the moral hazard problem concerning the entrepreneur's decision, there is a minimum amount of money that cannot come from external financiers.

The model parameters can be interpreted as follows. The trigger z corresponds to the minimum amount of money that the agent needs to invest in the venture to attract a loan and become an entrepreneur. The extra utility k models two complementary aspects.

First, it represents the expected net profit that the agent obtains if he starts the venture. Second, it may also include behavioral aspects of entrepreneurship, as studied in Gilad and Levine (1986). According to their contribution (the "pull theory" of entrepreneurship), individuals are attracted into entrepreneurial activities seeking independence and self-fulfillment, in addition to wealth.

Policy makers often have an incentive to promote the entrepreneurial activity. We now discuss interventions in the labor market that can indirectly help the entrepreneurial activity. We also show that a policy intervention in the entrepreneurial activity has indirect consequences on the labor market.

Let us first consider a change in the base salary $\underline{w}$. Increasing the base salary leads to both a higher effort exerted by goal-oriented agents and a higher probability that the bonus is awarded to these agents in case of success. This last effect translates in turn into an increase in the probability that these agents become entrepreneurs (as measured by pe) and that they enjoy k. Moreover, this change in the base salary has the indirect consequence of decreasing the threshold $\widehat{R}$, which reinforces the previous positive effect on entrepreneurship.

It is worth mentioning that a policy that increases $\underline{w}$ has also redistributive and efficiency effects. Increasing the minimum wage has ceteris paribus a positive effect for all the agents and a negative effect for all the principals. The efficiency effect exists as the agents' incentives and optimal effort increase, which brings the equilibrium effort closer to the first-best level and thus increases total welfare.

A second intervention might consist in impacting the trigger level z. In terms of the effort supplied and the likelihood to become an entrepreneur, a decrease in z is equivalent to an increase in $\underline{w}$. Such a decrease in z can be achieved, for instance, by using a policy that provides a subsidy to new entrepreneurs, or that decreases the total investment needed to carry out the project, for instance by softening the administrative and legal burden. Such a policy on the entrepreneurial activity increases the goal-oriented agents' motivation to exert effort in their current job, which in turn positively impacts principals' profit. Moreover, it also increases these agents' utility because they are more likely to become entrepreneurs.[12]

[12] A subsidy scheme that would decrease z might need to be funded by the government. Therefore, the positive effect on the goal-oriented agents' utility needs to be traded off with the cost resulting from the

Finally, the government can modify the taxes and fees paid by entrepreneurs (or self-employed people), affecting the expected income of these activities, that is, the extra utility k. As we have seen in the comparative static exercises, increasing k always has a positive effect on the goal-oriented agents' effort and the probability of becoming an entrepreneur. Therefore, reducing the charges to be paid by entrepreneurs not only increases the number of entrepreneurs but it also has an indirect positive effect on the markets where these individuals worked as agents.

5.2 Buying an asset with persistent benefits

The additional motivation of the agent can result from buying a certain good or service. Consider for simplicity that the agent has limited access to the loan market. He may have the will to get college education but lacks the monetary resources to do it. Alternatively, he may want to send his child to college (Becker, 2009). In this case, the trigger z corresponds to the minimum amount of money the agent needs to finance college education, and k corresponds to the extra salary and utility that he derives once he (or his child) obtains a college degree. The same idea can apply to the access to secondary education in many developing countries, where financing is a challenge for many families.

In another example with similar characteristics, the agent may be looking to buy some particularly relevant good characterized by long-lasting benefits or utility, like a house. In this case, the extra utility k may result from the pleasure and utility that the agent gets from owning such a house. As previous literature suggests (see, e.g., Flavin and Yamashita, 2002), many individuals have strong motivations for home ownership: getting protection against future negative shocks, using ownership as a commitment device to ensure some monetary savings, or social achievement.[13]

In this setting, a policy intervention may aim at increasing the likelihood that the

use of public funding. The effect on total welfare depends on the extra cost resulting from the use of public funding.

[13] If the agent has access to some credit market, the trigger z is the minimum amount required to obtain a loan. This minimum amount is required for reasons different from the moral hazard problem identified in Section 5.1. In the case of housing, the financial institutions only lend a share of the price of the house because there is a probability that the individual cannot repay the loan (due to unemployment or some health hazard) and there is also a probability that the market price of the house decreases, which could prevent the financier to recover the full loan in case the individual fails to make mortgage payments.

goal is realized, because the policy maker may want the agents to invest in education or housing.

The effect of a change in the base salary $\underline{w}$ or of a policy that impacts the trigger level z is similar as in the previous environment. Here, a decrease in z can be achieved, for instance, by subsidizing the people newly registered for college education, or through a policy decreasing either the frictions in the credit market or the administrative and legal burden (which may occur in the housing market). As in the previous environment, policies implemented in the labor market produce externalities on the education level or the acquisition of certain goods. Similarly, policies the affect the access to education or housing induce externalities in the labor market.

5.3 Aspiration-based trigger

Another example that is consistent with our theoretical model is based on behavioral motives. The additional motivation of the agent can result from reaching an aspiration-based reference compensation level. Specifically, some agents may exhibit extra utility based on individual aspirations. As explained in the literature (see Ray, 1998, among other references), aspirations are ambitions of achieving something which may or may not be achievable in reality. In our setting, up to a certain payment level, a goal-oriented agent might not experience extra utility as he feels like his ambition is not being fulfilled. Once his compensation is at least equal to this threshold level, the agent experiences some extra utility, as he thinks that his ambition has been satisfied.

In this case, the trigger z corresponds to the aspiration-based reference payment level, and k corresponds to the extra utility that he derives once he considers that his ambition has been fulfilled. Using the discussion provided by Génicot and Ray (2020), here aspirations are achievable, and the associate utility does not depend on the excess of outcome over the aspiration level. Our analysis shares a conclusion that is often reached in this literature, in that an increase in the level of aspiration (measured in terms of z) may prove to be detrimental to the agent, as the agent will receive the bonus in case of success with lower probability. This differs from setting where too high aspirations would be detrimental as they may create frustration (and thus disutility).[14]

[14] This setting is different from situations where the extra utility results from choosing the action that the agent considers as the right one, or from situations where the extra utility results from doing the task

In terms of public policies, changing the base salary $\underline{w}$ or impacting the trigger level z have the same consequences as above. While modifying the base salary might be implemented by using a fairly straightforward policy, impacting z might be less straightforward. Some policies aimed at changing the reference point have been discussed in the literature on aspirations and economic behaviors (we refer to La Ferrara, 2019, for an extensive discussion on this point).

6 Conclusion

In a market environment where goal-oriented agents coexist with standard ones, we analyze how the heterogeneity in the agents' population affects the equilibrium outcome in terms of matching and contracts. Compared to standard agents, goal-oriented agents derive extra-utility from achieving a high-enough monetary payoff (the "trigger") as this allows them to fulfill an individual goal. We obtain several interesting results. First, in any equilibrium contract, a goal-oriented agent reaches the trigger with a positive probability. However, a successful outcome does not ensure that goal-oriented agents achieve their ambitions: if the trigger is too high, then the bonus is only paid with some probability. We also show, at equilibrium, that goal and monetary incentives are complementary, goal-oriented agents receiving stronger monetary incentives than standard agents, and that the matching between principals and agents is positive assortative. These results have in turn interesting and important policy implications in relevant environments. Specifically, we highlight how the presence of goal-oriented agents creates a wedge between different markets, which results in important feedback effects of policy interventions. This contribution stresses the importance for contractual and policy designs of acknowledging the existence of goal-oriented agents, and calls for further research along this line.

that the agent considers as consistent with the mission of his organization (see for instance Besley and Ghatak, 2005, and Auriol and Brilon, 2014, who analyze a setting where some workers care about the mission of their organizations in the nonprofit sector).

7 References

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8 Appendix

Proof of Lemma 1: The Pareto optimal contract between a principal and a standard agent whose outside utility is $U^o = 0$ solves the following program:

$$\begin{aligned} & Max_{(w,\Delta,p)}\left\{eR - ep\Delta - w\right\} \\ & s.t. \qquad w + ep\Delta - \frac{v}{2}e^2 \geq 0 \\ & \qquad\qquad e = \frac{p\Delta}{v} \\ & \qquad w \geq \underline{w}, \quad p \geq 0, \quad p \leq 1 \end{aligned}$$

where p and Δ always appear as a product, except for the constraint that $p \in [0,1]$. Therefore, any optimal solution is equivalent to a solution where $p = 1$ and the endogenous variables are (w, Δ). Since the standard agent's "reservation utility" is zero, the problem corresponds to the classical case with limited liability and the solution is well known: given that $R \geq 2\sqrt{2v\underline{U}} = 0$ holds, one optimal contract is $(w = \underline{w}, \Delta = \frac{R}{2})$, the agent's effort is $e = \frac{R}{2v} \in [0,1]$ ($R \leq 2v$ by Assumption 1), and the principal's profit is $\frac{R^2}{4v} - \underline{w}$. The contracts described in the lemma are equivalent to $(w = \underline{w}, \Delta = \frac{R}{2}, p = 1)$. ■

Proof of Lemma 3: We prove the lemma in several steps.

Step 1. In an optimal contract, $w < z$.

Proof of step 1. We prove it by contradiction. Suppose that the equilibrium contract sets $w \geq z$. Denote U^{G*} the goal-oriented agent's utility under the contract. It is Paretoefficient, that is, it solves the following program:

$$\begin{aligned} & Max_{(w,\Delta,p)}\left\{eR - ep\Delta - w\right\} \\ s.t. \qquad & w + k + ep\Delta - \frac{v}{2}e^2 = U^{G*} \\ & e = \frac{p\Delta}{v} \\ & w \geq z \\ & \Delta \geq 0, \quad p \geq 0, \quad p \leq 1. \end{aligned}$$

The variables p and Δ always appear as a product, except for $p \in [0,1]$. Therefore, if p was smaller than 1, we could substitute p by 1 and Δ by $\Delta' = p\Delta$ and the program would be equivalent. Hence, we take $p = 1$. Moreover, using $e = \frac{1}{v}\Delta$, the program is equivalent to $w = U^{G*} - k - \frac{1}{2v}\Delta^2$ and

$$\begin{aligned} & Max_{\Delta}\left\{\frac{1}{v}R\Delta - \frac{1}{2v}\Delta^2 - EU^{G*} + k\right\} \\ s.t. \qquad & \underline{U}^{G} - k - \frac{1}{2v}\Delta^2 \geq z \qquad (\gamma) \\ & \Delta \geq 0. \qquad (\sigma) \end{aligned}$$

The FOC of the Lagrangian with respect to Δ is:

$$\frac{\partial}{\partial\Delta} = \frac{1}{v}R - \frac{1}{v}\Delta - \frac{1}{v}\Delta\gamma + \sigma = 0,$$

which implies $\gamma > 0$ because $\Delta \geq R$ would lead to negative profits, and the Kuhn-Tucker (KT) conditions imply $\sigma \geq 0$ necessarily. Hence, the candidate contract is $\Delta = \sqrt{2v\left(U^{G*} - k - z\right)}$ and $w = z$. This solution is only defined if $\sigma \geq 0$, that is, $U^{G*} \geq k + z$ holds. The principal's profit under the previous contract is

$$\Pi\left(U^{G*}\right) = R\sqrt{\frac{2}{v}\left(U^{G*} - k - z\right)} - 2U^{G*} + 2k + z.$$

The level of U^{G*} is characterized at the equilibrium. According to Lemma 2, it satisfies $\Pi\left(U^{G*}\right) = \Pi^{S*} = \frac{1}{4v}R^2 - \underline{w}$, that is:

$$R\sqrt{\frac{2}{v}\left(U^{G*} - k - z\right)} - 2U^{G*} + 2k + z = \frac{1}{4v}R^2 - \underline{w} \tag{2}$$

which, denoting $Q \equiv \frac{R}{\sqrt{v}}$ and $Y \equiv U^{G*} - k$, we write as

$$f(Y) \equiv \sqrt{2}Q\sqrt{Y - z} - 2Y + z + \underline{w} - \frac{1}{4}Q^2 = 0.$$

Note that $f'(Y) > 0$ if $Y < z + \frac{1}{8}Q^2$ and $f'(Y) < 0$ if $Y > z + \frac{1}{8}Q^2$. The minimum value of Y is $Y = z$ (because $\underline{U}^G \geq k + z$) where we have $f(z) < 0$. Then, the maximum value of $f(Y)$ is reached for $Y = z + \frac{1}{8}Q^2$. Finally, $f\left(Y = z + \frac{1}{8}Q^2\right) = \sqrt{2}Q\sqrt{\frac{1}{8}Q^2} - 2z - \frac{1}{4}Q^2 + z + \underline{w} - \frac{1}{4}Q^2 = -z + \underline{w} < 0$. Therefore, equation (2) has no solution. Hence, it is not possible that the equilibrium contract C^G includes a payment $w \geq z$.

Step 2. Any optimal contract (w, Δ, p) stipulates $p > 0$.

Proof of Step 2. If $p = 0$ the agent exerts effort $e = 0$, in which case the principal's profit is negative.

Step 3. Any contract (w, Δ, p) with $w < z$ and $\Delta < z - w$ is dominated by $(w, \Delta' = z - w, p' = p\frac{\Delta}{z-w+k})$, that is, the principal induces the extra motivation with some probability. Hence, at the equilibrium contract $\Delta \geq z - w$.

Proof of Step 3. A contract (w, Δ, p) with $w < z$ and $\Delta < z - w$ yields:

$$\begin{aligned} e(w, \Delta, p) &= \frac{p\Delta}{v} \equiv \widetilde{e}, \\ U(w, \Delta, p) &= \widetilde{e}p\Delta - \frac{v}{2}\widetilde{e}^2 + w, \\ \Pi(w, \Delta, p) &= \widetilde{e}R - \widetilde{e}p\Delta - w, \end{aligned}$$

where $\widetilde{e} \leq 1$ by Assumption 1. Now consider the contract $(w = w, \Delta' = z - w, p' = p\frac{\Delta}{z-w+k})$. Notice that $p' < p\frac{z-w}{z-w+k} < p$. Under the new contract, the agent still exerts effort $\widetilde{e}$ (because $p'(z - w + k) = p\Delta$) and he gets the same utility as under the initial contract:

$$U(w, z - w, p') = \widetilde{e}p'(k + z - w) - \frac{v}{2}\widetilde{e}^2 + w = \widetilde{e}p\Delta - \frac{v}{2}\widetilde{e}^2 + w.$$

Moreover, the principal prefers the new contract:

$$\Pi(w, z - w, p') = \widetilde{e}R - \widetilde{e}p\Delta\frac{(z - w)}{(k + z - w)} - w > \widetilde{e}R - \widetilde{e}p\Delta - w.$$

Therefore, the initial contract (w, Δ, p) is not Pareto optimal.

Step 4. Any contract (w, Δ, p) with $w \in (\underline{w}, z)$ and $\Delta \geq z - w$ is dominated by (or equivalent to, if the agent's effort is 1) the contract $(w = \underline{w}, \Delta', p)$ with $\Delta' = \Delta + \frac{w-\underline{w}}{p\widetilde{e}}$, where $\widetilde{e} = \min\left\{\frac{p(\Delta+k)}{v}, 1\right\}$. Hence, at the equilibrium contract $w = \underline{w}$.

Proof of Step 4: If $w \in (\underline{w}, z)$, $\Delta \geq z - w$ and $p > 0$ we have:

$$\begin{aligned} e(w,\Delta,p) &= \min\left\{\frac{p(\Delta+k)}{v},1\right\} \equiv \widetilde{e}, \\ U(w,\Delta,p) &= w + \widetilde{e}p\Delta + \widetilde{e}pk - \frac{v}{2}\widetilde{e}^2, \\ \Pi(w,\Delta,p) &= \widetilde{e}R - \widetilde{e}p\Delta - w. \end{aligned}$$

Consider now $(w = \underline{w}, \Delta' = \Delta + \frac{w-\underline{w}}{p\widetilde{e}}, p)$. If the agent exerts the same effort $\widetilde{e}$ he will get the same utility and the principal will obtain the same profit. If $\widetilde{e} < 1$, under the new contract the agent increases his effort level (because $\Delta' > \Delta$), and he is better off. Moreover, since profit is increasing in the agent's effort, the principal obtains higher profits than under the initial contract. ■

Proof of Theorem 1: The agent's effort is $e = \min\left\{\frac{p(\Delta+k)}{v},1\right\}$. We are going to compute the optimal contract where we substitute e by $\frac{p(\Delta+k)}{v}$. At the contract that we will find, the agent's effort is indeed lower than 1, hence it is also optimal under the true equation $e = \min\left\{\frac{p(\Delta+k)}{v},1\right\}$.

If $e = \frac{p(\Delta+k)}{v}$, the agent's expected utility is:

$$U = \underline{w} + ep(\Delta+k) - \frac{v}{2}e^2 = \underline{w} + \frac{p^2(\Delta+k)^2}{2v}.$$

Substituting the optimal agent's effort level in the principal's profit, and taking into account that her equilibrium profit must be $\frac{1}{4v}R^2 - \underline{w}$, the equilibrium contract is the solution to the following program:

$$\begin{aligned} &\underset{\Delta,p}{Max}\left\{\frac{p^2(\Delta+k)^2}{2v} + \underline{w}\right\} \\ s.t. \quad &\frac{p(\Delta+k)}{v}(R - p\Delta) - \underline{w} \geq \frac{1}{4v}R^2 - \underline{w} \\ &\Delta \geq z - \underline{w}, \ 1 \geq p, \end{aligned}$$

which is equivalent to:

$$\begin{aligned} &\underset{\Delta,p}{Max}\left\{p^2(\Delta+k)^2 + 2v\underline{w}\right\} \\ s.t. \quad &4p(\Delta+k)(R-p\Delta) - R^2 = 0 \qquad (\lambda) \\ &\Delta \geq z - \underline{w} \qquad (\alpha) \\ &1 \geq p \qquad (\gamma) \end{aligned}$$

whose FOCs are

$$\begin{aligned} \frac{\partial}{\partial \Delta} &= 2p^2(\Delta+k) - \lambda 4p(2p\Delta - R + kp) + \alpha = 0 &\text{(3a)}\\ \frac{\partial}{\partial p} &= 2p(\Delta+k)^2 + \lambda 4(\Delta+k)(R-2p\Delta) - \gamma = 0. &\text{(3b)} \end{aligned}$$

There are three possible cases.

i) If $\alpha > 0$ and $\gamma = 0$, then $\Delta = z - \underline{w}$. Constraint (λ) becomes $4p(z-\underline{w}+k)(R-p(z-\underline{w})) - R^2 = 0$. Solving the equation in p yields two solutions: $p = \frac{1}{2(z-\underline{w})}\left(1 \pm \frac{\sqrt{k}}{\sqrt{z-\underline{w}+k}}\right)R$. The smallest root is not a candidate because it would yield $R - 2p\Delta = \frac{\sqrt{k}}{\sqrt{z-\underline{w}+k}}R > 0$, in which case $(3b)$ cannot hold. Therefore,

$$p = \frac{1}{2(z-\underline{w})}\left(1 + \frac{\sqrt{k}}{\sqrt{z-\underline{w}+k}}\right)R.$$

From $(3b)$ we have $\lambda = -\frac{2p(\Delta+k)^2}{4(\Delta+k)(R-2p\Delta)} = \frac{\left(\sqrt{z-\underline{w}+k}+\sqrt{k}\right)(z-\underline{w}+k)}{4(z-\underline{w})\sqrt{k}} > 0$. Also, $(3a)$ implies $\alpha = \frac{\left(\sqrt{z-\underline{w}+k}+\sqrt{k}\right)^2(z-\underline{w}+k)}{2(z-\underline{w})^2\sqrt{z-\underline{w}+k}}\left(\frac{1}{\sqrt{k}}\left(\frac{\sqrt{k}}{\sqrt{z-\underline{w}+k}} + k\frac{1}{2(z-\underline{w})}\left(1+\frac{\sqrt{k}}{\sqrt{z-\underline{w}+k}}\right)\right) - \frac{1}{\sqrt{z-\underline{w}+k}}\right)R^2$. Therefore, $\alpha > 0$ if and only if $\frac{1}{\sqrt{k}}\left(k\frac{1}{2(z-\underline{w})}\left(1+\frac{\sqrt{k}}{\sqrt{z-\underline{w}+k}}\right)\right) > 0$, which always holds.

Finally, constraint (γ) requires

$$R \le \frac{2(z-\underline{w})\sqrt{z-\underline{w}+k}}{\sqrt{z-\underline{w}+k}+\sqrt{k}} = 2\left(z-\underline{w}+k-\sqrt{k(z-\underline{w}+k)}\right),$$

which is the condition for the contract identified in this case to be a candidate.

The agent's effort under the contract is

$$e = \frac{p(\Delta+k)}{v} = \frac{1}{2v(z-\underline{w})}\left(z-\underline{w}+k+\sqrt{k(z-\underline{w}+k)}\right)R = \frac{\sqrt{z-\underline{w}+k}}{2v\left(\sqrt{z-\underline{w}+k}-\sqrt{k}\right)}R.$$

Effort is increasing in R. In the upper limit of the interval, that is, when $R = 2\left(z-\underline{w}+k-\sqrt{k(z-\underline{w}+k)}\right)$, the effort is $e = \frac{1}{v}(z-\underline{w}+k)$, which is smaller than 1 by Assumption 1.

The agent's utility under the contract is

$$U = \frac{p^2(\Delta+k)^2}{2v} + \underline{w} = \frac{1}{8v}\left(\frac{1}{(z-\underline{w})}\left((z-\underline{w}+k)+\sqrt{k(z-\underline{w}+k)}\right)R\right)^2 + \underline{w}.$$

ii) If $\alpha = 0$ and $\gamma > 0$, then $p = 1$ holds. Also, constraint (λ) yields $4(\Delta+k)(R-\Delta) - R^2 = 0$. This equation has two solutions in Δ: $\Delta = \frac{R-k\pm\sqrt{k(2R+k)}}{2}$. The smallest root cannot be a candidate because in that case we would have $2p^2(\Delta+k) > 0$ and $4p(2p\Delta - R + kp) =$

$-4\left(\sqrt{k\left(2R+k\right)}\right)<0$, which is not compatible with equation $(3a)$. Therefore, the candidate bonus in this region is

$$\Delta=\frac{1}{2}\left(R-k+\sqrt{k\left(2R+k\right)}\right).$$

From $(3a)$ and $\alpha=0$ we deduce $\lambda=\frac{2p^2(\Delta+k)}{4p(2p\Delta-R+kp)}=\frac{R+k+\sqrt{k(2R+k)}}{4\sqrt{k(2R+k)}}$. Moreover, from $(3b)$ we have $\frac{1}{2}\left(R+k+\sqrt{k\left(2R+k\right)}\right)^2+\frac{R+k+\sqrt{k(2R+k)}}{2\sqrt{k(2R+k)}}\left(R+k+\sqrt{k\left(2R+k\right)}\right)\left(k-\sqrt{k\left(2R+k\right)}\right)-\gamma=0$, hence $\gamma=\frac{1}{2}\left(R+k+\sqrt{k\left(2R+k\right)}\right)^2\frac{k}{\sqrt{k(2R+k)}}>0$ holds.

Finally, equation (α) requires $\Delta\geq z-\underline{w}$, i.e., $\frac{1}{2}\left(R-k+\sqrt{k\left(2R+k\right)}\right)\geq z-\underline{w}$, or

$$R\geq 2\left(z-\underline{w}+k-\sqrt{k\left(z-\underline{w}+k\right)}\right).$$

The agent's effort under the contract is

$$e=\frac{p\left(\Delta+k\right)}{v}=\frac{1}{2v}\left(R+k+\sqrt{k\left(2R+k\right)}\right).$$

After some calculations, one can check that $e\leq 1$ if and only if $R\leq 2\left(v-\sqrt{vk}\right)$, which holds due to Assumption 1.

The agent's utility under the contract is

$$U=\frac{p^2\left(\Delta+k\right)^2}{2v}+\underline{w}=\frac{1}{8v}\left(R+k+\sqrt{k\left(2R+k\right)}\right)^2+\underline{w}.$$

iii) If $\alpha>0$ and $\gamma>0$, then $p=1$ and $\Delta=z-\underline{w}$. This case coincides with the limit of the regions i) and ii) and the conditions to be a candidate are identical. Condition (λ) is satisfied if $R=2\left(z-\underline{w}+k-\sqrt{k\left(z-\underline{w}+k\right)}\right)$.

iv) If $\alpha=0$ and $\gamma=0$, then easy calculations show that $(3a)$ and $(3b)$ cannot hold simultaneously if $p>0$, hence there is no candidate solution in this region. ■

Proof of Lemma 4: Denote $\underline{U}^G$ the utility obtained by the goal-oriented agent if he signs an optimal contract with a principal potentially generating value R and let (Δ,p)

be the contract. The contract (Δ, p) solves the following program (PP):

$$\begin{aligned} &\underset{\Delta,p}{Max}\left\{\frac{p\left(\Delta+k\right)}{v}\left(R-p\Delta\right)-\underline{w}\right\} \\ s.t. \quad &\frac{p^2\left(\Delta+k\right)^2}{2v}+\underline{w}\geq \underline{U}^G \qquad (\lambda) \\ &\Delta\geq z-\underline{w} \qquad (\alpha) \\ &1\geq p \qquad (\gamma) \end{aligned} \tag{PP}$$

whose FOC with respect to Δ is:

$$\frac{\partial}{\partial\Delta}=\frac{1}{v}p\left(R-2p\Delta-kp\right)+\lambda\frac{1}{v}p^2\left(\Delta+k\right)+\alpha=0. \tag{4}$$

Define $\delta(R)$ as the difference between the profit obtained by a principal whose project's value is R under contract (p,Δ) and the profit obtained with a standard agent, i.e.,

$$\delta(R)\equiv\frac{1}{v}p\left(\Delta+k\right)\left(R-p\Delta\right)-\underline{w}-\left(\frac{1}{4v}R^2-\underline{w}\right).$$

Using the envelop theorem, we have $\frac{\partial\delta}{\partial R}=\frac{1}{v}p\left(\Delta+k\right)-\frac{1}{2v}R$. Then, $\frac{\partial\delta}{\partial R}>0$ if and only if $R-2p\left(\Delta+k\right)<0$. We rewrite equation (4) as $R-2p\Delta-2kp=-kp-\lambda p\left(\Delta+k\right)-\frac{1}{p}\alpha v$; hence $\frac{\partial\delta}{\partial R}>0$. This implies that the difference in profits increases with R.

Therefore, suppose that a principal whose project generates value R^o (denote her R^o) hires a goal-oriented agent and consider a principal with $R>R^o$ (denote her R). Then, we show that at the equilibrium it is necessarily the case that principal R also hires a goal-oriented agent.

If $\underline{U}^G$ is the utility obtained by the goal-oriented agent in the contract with principal R^o, then $\underline{U}^G$ is lower than or equal to $\underline{U}$ satisfying that R^o's profits with $\underline{U}$ are equal to $\frac{1}{4v}R^2-\underline{w}$. However, if principal R would be hiring a standard agent, she would be ready to pay the goal-oriented agent a higher level of utility (as she would obtain higher profits). Therefore, this could not be an equilibrium, which proves the lemma.

Proof of Theorem 2. Denote $\underline{U}^G$ the utility obtained by the goal-oriented agent under the contract with principal R_{m^G+1} that would make this principal indifferent between hiring him and hiring a standard agent. Under this contract, the agent obtains the utility U^G characterized in Theorem 1. This is the utility level of any goal-oriented agent in the market. Since we assume that $R_{m^G+1} < \widehat{R} = 2\left(z - \underline{w} + k - \sqrt{k\left(z - \underline{w} + k\right)}\right)$, $\underline{U}^G = \frac{1}{8v}\left(\frac{R_{m^G+1}}{z-\underline{w}}\left(k + z - \underline{w} + \sqrt{k\left(k + z - \underline{w}\right)}\right)\right)^2 + \underline{w}$.

For any principal R (from R_1 to R_{m^G}), her contract (Δ, p) solves the same program (PP) as in the proof of Lemma 4. The FOCs of (PP) are:

$$\frac{\partial}{\partial\Delta} = \frac{1}{v}p\left(R - 2p\Delta - kp\right) + \lambda\frac{1}{v}p^2\left(\Delta + k\right) + \alpha = 0 \tag{5a}$$

$$\frac{\partial}{\partial p} = \frac{1}{v}\left(\Delta + k\right)\left(R - 2p\Delta\right) + \lambda\frac{1}{v}p\left(\Delta + k\right)^2 - \gamma = 0. \tag{5b}$$

We consider $\lambda > 0$, as there is no candidate with $\lambda = 0$.

i) If $\alpha > 0$ and $\gamma = 0$, then $\Delta = z - \underline{w}$. Moreover, constraint (λ) being binding yields $\frac{p^2(\Delta+k)^2}{2v} + \underline{w} = \frac{1}{8v}\left(\frac{R_{m^G+1}}{z-\underline{w}}\left(k + z - \underline{w} + \sqrt{k\left(k + z - \underline{w}\right)}\right)\right)^2 + \underline{w}$, that is,

$$p = \frac{1}{2\left(z - \underline{w}\right)}\left(1 + \sqrt{\frac{k}{k + z - \underline{w}}}\right)R_{m^G+1}.$$

We notice that $p < 1$ given that $R_{m_B+1} < \widehat{R}$. From $(5b)$, we have $\lambda = \frac{2p\Delta - R}{p(\Delta+k)}$. Therefore, $\lambda > 0$ if and only if $R - 2p\Delta < 0$, i.e., $R < \left(1 + \sqrt{\frac{k}{k+z-\underline{w}}}\right)R_{m^G+1}$, which holds according to the assumptions. Finally, from $(5a)$ we have $\alpha = \frac{1}{v}p\left(kp + 2p\Delta - R\right) - \lambda\frac{1}{v}p^2\left(\Delta + k\right)$. Then, $\alpha > 0$ iff $kp + 2p\Delta - R > 2p\Delta - R$, which always holds. Therefore, this solution is a candidate. Principal i hiring a goal-oriented agent gets the profit $\Pi(R_i) = \frac{p(\Delta+k)}{v}\left(R - p\Delta\right) - \underline{w}$, which, after substituting for the optimal contract, yields the expression provided in the theorem.

ii) If $\alpha = 0$ and $\gamma > 0$, then $p = 1$. Moreover, constraint (λ) binding yields

$$\Delta = \frac{R_{m^G+1}}{2\left(z - \underline{w}\right)}\left(k + z - \underline{w} + \sqrt{k\left(k + z - \underline{w}\right)}\right) - k.$$

Constraint (α) requires $\Delta \geq z - \underline{w}$, that is $R_{m^G+1}\left(\sqrt{k + z - \underline{w}} + \sqrt{k}\right) \geq 2\sqrt{k + z - \underline{w}}\left(z - \underline{w}\right)$. However, this inequality does not hold because $R_{m^G+1} < \widehat{R}$. ■