

# Precaution, Social Distancing and Tests in a Model of Epidemic Disease 

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#### Abstract

I develop an extension of a canonical epidemiology model in which the policy in place determines the probability of transmission of an epidemic disease. I use the model to evaluate the effects of isolating symptomatic individuals, of increasing social distancing and of tests of different quality: a poor quality test that can only discriminate between healthy and infected individuals (such as polymerase chain reaction -PCR- or Rapid Diagnostic Test), and a high quality test that is able to discriminate between immune and vulnerable healthy, and infected individuals (such as a serology test like Neutralization Assay). I find that isolating symptomatic individuals has a large effect at delaying and reducing the pick of infections. The combination of this policy with the poor quality test represents only a negligible improvement, whereas with the high


[^0]quality test there is an additional delaying and reduction in the pick of infections. Social distancing alone cannot achieve similar effects without incurring in enormous output losses. I explore the combined effect of social distancing at early stages of the epidemic with a following period of tests and find that the best outcome is obtained with a light reduction of human interaction for about three months together with a subsequent test of the population over 40 days.

Keywords: Covid-19, Social distancing, testing
JEL Classification: E1, E65, H12, I1

## 1 Introduction

As of April 2020 the rapid expansion of Covid-19 is affecting a large fraction of the population all over the world, with thousands of positive tested cases and of deaths in many countries. Precise statistics of the effects of the epidemic are scarce and display substantial variance over time in a given country and also across countries. Perhaps these differences in the available information reflect specific aspects of the local realities explain the variety of policies undertaken in different countries. The current policies mainly consist of testing the population, of social distancing to reduce human interaction and the possibility of transmission, and in isolating for some time presumably ill individuals and regions with a high density of infected population. The purpose of this paper is to propose a model suitable to study the effects of these policies on the dynamics of the the epidemic and to take into account the impact on output.

The model I propose follows a similar approach to other recent models such as Atkeson (2020), Eichenbaum, Rebelo and Tranbandt (2020), Berger, Herkenhoff and Mongey (2020) and Casares and Khan (2020). These models are based on the relatively simple SIR model of immunology (Kermack and McKendrick, 1927) and consider specific characteristics about the transmission mechanisms of the disease to obtain predictions for the number of deaths, number of infected and recovered individuals and on other economic outcomes. My approach and type of question is closer to Berger, Herkenhoff and Mongey (2020) and to Piguillem and Shi (2020): individuals can be infected, with and without symptoms, and healthy, with immunity or not, and the disease can only be transmitted from an infected individual to a healthy but vulnerable one. The policies in place determine endogenously the probability of transmission of the disease and therefore have an impact on the mass
of agents that participate in social and production activities, hence on output.
In the model in this paper I explicitly take into account the severity of social distancing, which takes the form of a reduction in the number of contacts among agents that participate in the market activity. ${ }^{1}$ I also take into account the effect of the duration of social distancing. These are important margins because in the model there are "industrial" occupations that observe severe losses with the reduction in the number of contacts among workers, and "services" occupations that can be completed from home with only a small loss of output. As an alternative to the previous policy I study a simple precautionary regime: symptomatic individuals are not tested but kept in isolation for 14 days. Finally I also consider the effects of two different test: test 1 is a poor quality test that is only able to discriminate between healthy and infected agents. This is the outcome of a polymerase chain reaction (PCR) or a Rapid Diagnostic Test (RDT) test. With this technology one needs to test repeatedly a large fraction of the population, since healthy agents participate every period in market activities and a fraction of them are likely to become infected. Then I look at test 2, a perfect test which is able to tell the type of healthy individual and the length of the infection in case the individual is infected. This second test is closer to serology testing such as Neutralization Assay that looks for antibodies in the blood of the patient. ${ }^{2}$

I evaluate the effects of the previous policies in a quantitative exercise using a calibration that is similar to the papers in this literature. I find that a simple policy such as precaution is able to delay the pick of infection under laissez-faire

[^1](or unawareness) from day 41 to day 262, and from almost $16 \%$ to about $3.6 \%{ }^{3}$ Perhaps surprisingly I find that adding the effect of test 1 to precaution is nearly irrelevant, at least at relatively low levels of coverage such as $1 \%$ of the population, but also at a level of $3 \%$. Only the combination of precaution with test 2 delays some additional 83 days the pick of infection which then affects $2.2 \%$ of the population. Furthermore, with a $3 \%$ coverage in this last combination of policies the epidemic does not take place. To obtain an infection rate below $1 \%$ using test 1 to control for the disease it would be needed to test $15 \%$ of the population over 18 months. These results suggest that there are dramatic differences in the effects of using test type 1 and test type 2. This finding is relevant because in most countries the implementation of tests type 1 is the main strategy (together with confinement) to fight against Covid-19 (as of April 19th Germany is the first European country to start large-scale coronavirus antibody testing). Finally, I find that a social distancing policy consisting in reducing interactions by $10 \%$ has positive but very limited effects under unawareness, but it is able to also prevent the epidemic from taking place under precaution and its combination with tests. In terms of the effects on output, social distancing is substantially more costly (output fall by more than $50 \%$ ) than precaution plus test 2 (less than $2 \%$ of output with a coverage of $1 \%$, and literally no loss with a coverage of $3 \%$ ). ${ }^{4}$

Not all is good about the combination of precaution and test 2: the problem is that either the population is constantly tested, or as soon as tests cease the infection is likely to start again in the near future. The reason is that with test 2 the fraction of healthy and immune individuals is too small to provide "herd immunity" (that is,

[^2]the mass of immune individuals is too small to prevent the disease from spreading again). Hence, test 2 has benefits in the short run but it may have large costs at a longer run. A severe social distancing policy will produce similar effects, to which in addition one has to add a dramatic output loss. However, a "light" social distancing may have small costs in the sort run but larger benefits in the future. In view of this I conduct a grid search to investigate the optimal combination of policies. In this final exercise I take into account the cumulative deaths, the infection rate (to control for the possible collapse of the public health network), the level of healthy and immune individuals to meet a herd immunity threshold and the cost of output. My results suggest that it is best to implement a light social distancing to reduce interactions by about $3-7 \%$ over 90 days, and then continue with test 2 covering $3 \%$ of the population for additional 40 days.

The previous policy recommendation must be taken with caution, as it rests on the fact that preferences are not lexicographic in the number of current deaths and on the fact that an effective cure or vaccine to fight Covid-19 does not seem to be available over the next 12-18 months. ${ }^{5}$ My results suggest that without a more effective medical technology the optimal policy reduces the number of deaths by about $20 \%$ in the short and in the long run, but unfortunately there is a medium run period in which the cumulative deaths are almost as large as under laissez-faire.

The rest of the paper is organized as follows. Section 2 describes the environment, Section 3 explains health distribution dynamics and the various policies, Section 4 introduces output losses, Section 5 conducts the quantitative exploration and Section 6 offers additional discussion and suggestions for further research. There is an Appendix with a few additional results from a sensitivity analysis.

[^3]
## 2 Environment

Time is discrete and there is a unit mass of agents. In any given period agents can be in one of the following four states: healthy and immune, healthy but vulnerable, infected with symptoms and infected but asymptomatic, denoted respectively $h i$, $h v, i s$ and $i a$. The distribution of health status in a period $t$ can be represented by a vector $H_{t}=\left(h i_{t}, h v_{t}, i s_{t}, i a_{t}\right)$ and it satisfies:

$$
\begin{equation*}
h i_{t}+h v_{t}+i s_{t}+i a_{t}=1 \tag{1}
\end{equation*}
$$

For future reference it is convenient to introduce $M_{t}$ as the mass of agents that participate in market and social activities. $\phi_{t}$ denotes the the fraction of healthy agents, thus $\phi_{t}=h i_{t}+h v_{t}$ and the mass of infected agents is given by $1-\phi_{t}$. It is assumed that healthy and infected but asymptomatic agents may die at an exogenous rate $1-s$, and that agents infected with symptoms die at a rate $d$ (that is, $d$ is the fatality rate). The population is constant over time because dead agents are replaced with newly born healthy but vulnerable agents.

The nature of the disease is such that the true health state may not be known to the agent. In terms of the transmission of the disease the interaction of two individuals of the same $h / i$-type is inconsequential. Only when an agent type $h v$ interacts with an agent type $i$ s or type $i a$ the disease may be transmitted.

Let $\pi_{n, m}$ denote the probability that after a meeting between an agent type $n$ and one type $m$ (with $n, m=h i, h v, i s, i a$ ) there is a change in the health status of the agent $n$. For instance, $\pi_{h i, m}=\pi_{m, h i}=0$ for all $m$, and by the same token, $\pi_{i s, m}=\pi_{i a, m}=0$ for all $m$. It is clear however that $\pi_{h v, i s}=\pi_{h v, i a}>0$ (and of course, that $\pi_{i s, h v}=\pi_{i a, h v}=0$ ). This simply reflects the fact that only the
interaction between a healthy but vulnerable agent and an infected agent is able to change the health status of the initially healthy agent. Since only $\pi_{h v, i s}=\pi_{h v, i a}$ are strictly positive I simplify notation and denote them by $\pi$.

In "normal" times (when economic activity and social interaction takes place under no constraints) each individual maintains $N$ consecutive meetings with other individuals. ${ }^{6}$ I denote by $p_{n}$ the probability of a change in the health status of an agent type $n=h i, i s, i a$ after $N$ random consecutive meetings. Given the previous assumption it follows that $p_{h i}=p_{i s}=p_{i a}=0$. Given this, let $p_{t}$ be the probability corresponding to the $h v$ type in period $t$, which satisfies

$$
\begin{equation*}
p_{t}=L\left(N ; H_{t}\right) . \tag{2}
\end{equation*}
$$

The function $L(\cdot)$ depends on the policy intervention (if any), on the number of meetings and on the distribution of health in the group that meet. The purpose of the paper is to look the effects of several configurations for $L(\cdot)$. Finally, I assume that whether the agent is infected or not is materialized at the end of the period, and in case an agent is infected the symptoms are revealed only with probability $\rho$.

## 3 Health distribution dynamics

The time-line of an infection process is as follows. Suppose that a healthy but vulnerable agent gets infected in a period $t$. I assume that the disease process is deterministic in that conditional on surviving, the infected agent faces an horizon of $t_{0}$ periods until recovery. That is, during the $t_{0}$ periods of infection the probability of dying is constant and equal to $d$. If the agent survives the $t_{0}$ periods then it is

[^4]assumed that she is totally recovered and becomes healthy and immune. In that state the probability of dying is $(1-s)$, the same as for the other healthy and for the infected but asymptomatic agents.

The dynamics of health status given an initial $H_{1}$ are described below. In particular, it holds that

$$
\begin{equation*}
h i_{t+1}=h i_{t} s+i s_{t}^{1}(1-d)+i a_{t}^{1} s \tag{3}
\end{equation*}
$$

where $i s_{t}^{1}$ and $i a_{t}^{1}$ denote the mass of infected agents that in period $t$ are one period away from recovery. The mass of $i s$ agents evolves according to

$$
\begin{equation*}
i s_{t+1}^{j}=i s_{t}^{j+1}(1-d), \tag{4}
\end{equation*}
$$

for $j=1, \ldots,\left(t_{0}-1\right)$, with

$$
\begin{equation*}
i s_{t+1}^{t_{0}}=h v_{t} p_{t} s \rho \tag{5}
\end{equation*}
$$

and where $i s_{t}=\sum_{j=1}^{t_{0}} i s_{t}^{j}$. The evolution of the as is nearly identical, with

$$
\begin{equation*}
i a_{t+1}^{j}=i a_{t}^{j+1} s \tag{6}
\end{equation*}
$$

for $j=1, \ldots,\left(t_{0}-1\right)$, with

$$
\begin{equation*}
i a_{t+1}^{t_{0}}=h v_{t} p_{t} s(1-\rho) \tag{7}
\end{equation*}
$$

and with $i a_{t}=\sum_{j=1}^{t_{0}} i a_{t}^{j}$ and such that Equation (2) holds. Finally, under the assumption that healthy but vulnerable agents participate it holds that

$$
\begin{equation*}
h v_{t+1}=1-h i_{t+1}-i s_{t+1}-i a_{t+1} . \tag{8}
\end{equation*}
$$

Notice therefore that $h v_{t+1}$ includes all $h v_{t}$ agents that survive from $t$ to $t+1$, plus all the agents in every health state that died between $t$ and $t+1$.

I now describe the configuration of $L(\cdot)$ under several canonical policies.

### 3.1 Unawareness: all agents participate

Even if the symptoms of Covid-19 are revealed, in many cases they consist of a slight increase in temperature and some coughing. I make the assumption that these symptoms are wrongly taken as symptoms of a common cold and thus all agents keep interacting and participating in the $N$ meetings. Under this assumption the mass of agents that participate in the meetings is such that $M_{t}=1$ in all periods and thus

$$
\begin{equation*}
p_{t}=\pi\left(1-\phi_{t}\right) \sum_{i=1}^{N} \phi_{t}^{i-1} . \tag{9}
\end{equation*}
$$

### 3.2 Precaution: symptomatic agents do not participate

Assume that once the symptoms are revealed, the agent is confined for $t_{0}$ periods. This assumption has dynamic effects for the size of the group that participate and on the probability $p_{t}$. In particular, the mass of agents that participate in the meetings satisfies:

$$
\begin{equation*}
M_{t}=\phi_{t}+i a_{t} \tag{10}
\end{equation*}
$$

and the probability $p_{t}$ satisfies

$$
\begin{equation*}
p_{t}=\pi \frac{i a_{t}}{M_{t}} \sum_{i=1}^{N}\left(\frac{\phi_{t}}{M_{t}}\right)^{i-1} . \tag{11}
\end{equation*}
$$

### 3.3 Testing 1: agents tested to be infected do not participate

A first policy to consider consists of randomly testing individuals among those that do not show symptoms (those with symptoms are not tested but are prevented
from participating during $t_{0}$ periods, as under precaution). This means that all non tested asymptomatic agents, plus all tested to be not infected, will participate. This policy can be implemented in two different ways: by testing a constant fraction of the asymptomatic population, and by making a fixed number of tests. These policies entail repeating the test to a fraction of the population that has been tested before, but they may be appropriate if it is difficult to verify the authenticity of tests realized in the near past. This policy may be the only one available at early stages of the epidemics when the available tests can only discriminate between infected and non infected agents.

### 3.3.1 Testing a constant fraction of the asymptomatic population

In this case the asymptomatic population in every period is $\phi_{t}+i a_{t}$, and after the test the population that participates is given by

$$
\begin{equation*}
M_{t}=\phi_{t}+i a_{t}(1-\tau) . \tag{12}
\end{equation*}
$$

The probability of infection for a healthy but vulnerable agent satisfies

$$
\begin{equation*}
p_{t}=\pi \frac{i a_{t}(1-\tau)}{M_{t}} \sum_{i=1}^{N}\left(\frac{\phi_{t}}{M_{t}}\right)^{i-1} . \tag{13}
\end{equation*}
$$

### 3.3.2 Realizing a constant number of tests

Let $\tau$ represent the number of tests which are devoted to check the asymptomatic population. If $A_{t}$ is the asymptomatic population in period $t$, then the probability of being checked is $\tau_{t}=\tau / A_{t}$ if $\tau \leq A_{t}$ (and $\tau_{t}=1$ otherwise). The mass of agents participating in the market and the probability of infection for a healthy but
vulnerable agent are given respectively by

$$
\begin{equation*}
M_{t}=\phi_{t}+i a_{t}\left(1-\tau_{t}\right) \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{t}=\pi \frac{i a_{t}\left(1-\tau_{t}\right)}{M_{t}} \sum_{i=1}^{N}\left(\frac{\phi_{t}}{M_{t}}\right)^{i-1} . \tag{15}
\end{equation*}
$$

### 3.4 Testing 2: A perfect test

Suppose now that the available test is able to discriminate among healthy and immune, healthy but vulnerable, and the periods of infection for those that are detected to be infected. In this sense the test is perfect and its results last for as long as the agent is alive. Thus some of the repeated tests that were conducted under the previous testing 1 can be more efficiently used to check the state of not previously checked agents. ${ }^{7}$ I will proceed under the assumption that there is a fixed number of tests to be done in every period. I will also assume that infected agents with symptoms are not tested but are not allowed to participate for $t_{0}$ periods as soon as the symptoms appear.

As in the case above the mass of asymptomatic agents in any period $t$ is given by $h i_{t}+h v_{t}+i a_{t}$. How many of them would need to be tested in the current period? The answer is of course all those that have not been tested before, thus one needs to keep track of the previously tested agents in each possible group. To facilitate the exposition let $\tilde{n}_{t}$ be the mass of agents type $n=h i, h v, i a$ that are alive in $t$ and that have been tested before $t$. Let also $\bar{n}_{t}$ be the mass of agents type $n=h i, h v, i a$

[^5]that are tested for the first time in $t$. With respect to the hi have that:
\[

$$
\begin{equation*}
\tilde{h} i_{t}=\left(\tilde{h} i_{t-1}+\bar{h} i_{t-1}\right) s+i s_{t-1}^{1}(1-d)+\left(\tilde{i a_{t-1}}+\overline{i a}_{t-1}^{1}\right) s, \tag{16}
\end{equation*}
$$

\]

where it is understood that $i s_{t}^{1}$ have a medical certificate which acts like the test. Agents that were tested $h v$ in the previous period would need to be tested again (since they interacted and it is not know if their type changed. The only ones that will not show up to be tested are the ones that ended the previous period with symptoms. Hence we have that it is as if

$$
\begin{equation*}
\tilde{h}_{t}=h v_{t-1} p_{t-1} s \rho \tag{17}
\end{equation*}
$$

With respect to the agents that were detected to be infected but asymptomatic it holds that

$$
\begin{equation*}
\tilde{i a_{t}^{j}}=\left(\tilde{i a}_{t-1}^{j+1}+\overline{i a}_{t-1}^{j+1}\right) s \tag{18}
\end{equation*}
$$

for $j=1, \ldots,\left(t_{0}-1\right)$, with

$$
\begin{equation*}
\tilde{i a}_{t}^{t_{0}}=\tilde{h v_{t-1}} p_{t-1} s(1-\rho) \tag{19}
\end{equation*}
$$

and with $\tilde{i a}_{t}=\sum_{j=1}^{t_{0}} \tilde{i a}_{t}^{j}$.
Hence the mass of agents that has not been tested is given by

$$
\begin{equation*}
\tilde{A}_{t}=\left(h i_{t}-\tilde{h i}_{t}\right)+\left(h v_{t}-\tilde{h}_{t}\right)+\left(i a_{t}-\tilde{i a}_{t}\right) \tag{20}
\end{equation*}
$$

and thus the probability of being tested is given by

$$
\begin{equation*}
\tau_{t}=\frac{\tau}{\tilde{A}_{t}} \tag{21}
\end{equation*}
$$

if $\tau \leq \tilde{A}_{t}$ and $\tau_{t}=1$ otherwise. The mass of newly tested agents in a period $t$ is given by $\bar{h} i_{t}=\tau_{t}\left(h i_{t}-\tilde{h} i_{t}\right), \overline{h v_{t}}=\tau_{t}\left(h v_{t}-\tilde{h} v_{t}\right)$ and $\overline{a_{t}}=\tau_{t}\left(i a_{t}-\tilde{i a}\right)$. It follows that the mass of agents that participate in period $t$ is given by

$$
\begin{equation*}
M_{t}=\phi_{t}+\left(i a_{t}-\tilde{i a}_{t}-\overline{i a}_{t}\right) \tag{22}
\end{equation*}
$$

and the corresponding probability satisfies the usual equation:

$$
\begin{equation*}
p_{t}=\pi \frac{i a_{t}-\tilde{i a}_{t}-\overline{i a_{t}}}{M_{t}} \sum_{i=1}^{N}\left(\frac{\phi_{t}}{M_{t}}\right)^{i-1} . \tag{23}
\end{equation*}
$$

### 3.5 Social distancing

For each of the previous policies the model is able to encompass social distancing by assuming that agents are able to materialize only $n<N$ interactions. Hence the value of $n$ can be seen as the severity of social distancing.

Notice that with the exception of unawareness in which $M_{t}=1$ in all $t$, in the other scenarios $M_{t}$ is the sum of the mass of healthy agents plus different fractions of infected but asymptomatic agents.

## 4 Output loss in an epidemic episode

A relevant effect of an epidemic episode is observed on the number of workers available to work. In terms of the different scenarios considered above this restriction may have no effect, in the unawareness regime, or represent a more severe limitation in the precaution regime and when tested infected agents are prevented from participating.

The model does not distinguish between social meetings, that are valuable in terms
of utility but not productive in terms of output, and productive meetings that are mainly valuable because of the output they make possible. To simplify matters it will be assumed that all interaction is related to production, thus the $N$ meetings involve commuting between home and the workplace and interacting with other peers in order to complete a number of tasks.

A policy such as social distancing introduces an additional output effect through the fact that the actual number of meetings, $n$, will be smaller than the "normal" one, $N$. That is, for a productive agent I let the value of her output be given by $y f(n)$ where $f(n)$ is non decreasing in $n$ and i) $f(0)=0$, and ii) $f(N)=1$ (this is simply a convenient normalization). With this in mind it is clear that not all occupations suffer the same loss in the face of the same reduction in $n$. For instance, in the services sector many workers are able to keep working from their homes and their output is very close the "normal" output even if $n$ is severely reduced. However, production in the industrial and in the primary sectors are nearly impossible to be moved at home, so even a small reduction in the number of interactions is able to represent a large reduction in output. I try to take into account this diversity and assume that there are two types of occupations, in the "services" sector and in the "industry" sector, such that $f_{j}=(n / N)^{\alpha_{j}}$ for $j=s, i$ and with $\alpha_{s}<1$ and $\alpha_{i}>1$. The proportion of workers in each sector is denoted respectively $S$ and $(1-S)$, and since all workers are alike in terms of their probability of infection, the same proportions prevail among healthy and among ill individuals. With these assumptions output in period $t$ at the aggregate level is given by

$$
\begin{equation*}
Y_{t}=S M_{t} y_{s}(n / N)^{\alpha_{s}}+(1-S) M_{t} y_{i}(n / N)^{\alpha_{i}} \tag{24}
\end{equation*}
$$

where $M_{t}$ is again the mass of agents that participate and where $y_{s, i}$ is the output
per worker in sector $s, i$.

## 5 A quantitative exploration

### 5.1 Calibration

In the model a period $t$ is a day. I need values for $N, \pi, s, d, t_{0}, \rho, S, y_{s}, y_{i}, \alpha_{s}$ and $\alpha_{i}$. A few of these parameters can be obtained from the literature, but unfortunately there is substantial uncertainty about their true values. The starting distribution is such that there is a fraction $0.001 \%$ of infected individuals (the rest are all healthy but vulnerable).

The value of $s$ is the survival rate in "normal" times on a daily basis. For the US economy life expectancy is about 78,54 years, so $s=0.999965$ is a reasonable approximation. There is a large variation across countries in the case fatality rate of Covid-19 (from above $13 \%$ in Algeria and Italy to as low as $0,12 \%$ in Qatar). There is evidence that age and pre-clinical condition are relevant. In the US the case fatality rate is reportes to be $5.11 \% .^{8}$ As a starting point I take $d=3.4 \%$ as a reasonable compromise. The recovery period $t_{0}$ is taken to be 14 days (but in several countries the recommendation before returning to activity is as large as 30 days). Regarding the fraction of infected individuals that show up the symptoms, Heneghan, Brassey and Jefferson (2020) suggest that based on the available evidence a fraction between $5 \%$ and $80 \%$ of tested infected individuals are asymptomatic. ${ }^{9}$ As a starting point we take $\rho=0.8$ hence the disease is manifested in $80 \%$ of the infected individuals. With respect to the transmission rate of the infection $\pi$, Berger, Herkenhoff and

[^6]Mongey (2020) assume it is 0.0091 in their model with periods of 14 hours. I take this value from that paper and in the current model with periods of 24 hours I fix $\pi=0,0156$ (this figure is similar to that in Casares and Khan (2020) in their calibration for Spain).

I use data from BLS from 2018 (the last year for which the required information is available) to calibrate $S, y_{s}$ and $y_{i} .{ }^{10}$ In 2018 there were $161.037,7$ jobs in the U.S. (in thousands), and the value of total output was $33.241,9$ Billions in chained dollars of 2012. I take employment and output in the "services" sector as including Services-providing excluding special industries minus: wholesale and retail trade, transportation and warehousing, Health care and social assistance and minus leisure and hospitality. The rest of categories are included in the "industry" sector. The fraction of jobs in the "services" sector is approximately 0.442 , so I fix $S=0.442$. The annual value of output per worker in "services" and "industry" is approximately 0.2202 and 0.1954 respectively (in Millions of chained dollars of 2012). On a daily bases this means that $y_{s}=603.28$ and $y_{i}=536.34$ Dollars. Finally, I fix the "normal" number of interactions to $N=30$ and in the examples with tests 1.1, 1.2 and 2 I evaluate several options for $\tau .{ }^{11}$

We now evaluate several counterfactual situations.

### 5.2 Counterfactual policies

I evaluate the five different scenarios under the baseline calibration discussed in the previous section and over a period of 365 days. To facilitate the exposition I introduce several figures for the fraction of infected and output effects. Figure

[^7]1 shows the evolution of the fraction of infected individuals. When the infection evolves under no control it reaches a maximum infection rate of $15.39 \%$ in 41 days, and then slowly declines: 200 additional days are needed in order to observe an infection rate below $1 \%$. Compared to this case, the other policies provide: 1) a much slower increase in infections, 2) a much smaller maximum of infections, and 3) a slower recovery. It is interesting to see that even a simple policy such as precaution (i.e., stay at home if you do not feel well or if your temperature is higher than normal), represents a substantial improvement with respect to unawareness. Tests 1.1 and 1.2 are nearly identical (hence we wont distinguish between them in the following exercises) and perform slightly better than the precaution scenario. Test 2 is clearly better: it delays substantially the pick of infections which in addition is only slightly above $2 \%$. Still, this test must be done every day and yet at a $1 \%$ level it seems unable to keep the infection rate below $1 \%$.


Figure 1: Baseline outcomes.

Figure 2 plots the effects for the infection rate of increasing the number of tests to $3 \%$ of the population. This policy has no effect on scenarios 1 and 2 (unawareness and precaution), but it delays slightly the pick of infections under test 1 (test 1.1
and 1.2 continue to be indistinguishable). Test 1 is now able to lower the infection pick to $3.1 \%$ after 297 days (before it reached $3.6 \%$ after 270 days). Increasing the number of tests to the $3 \%$ of the population has its largest effect with test 2 . In this case the fraction of infected is not zero, but it remains always below $0.0021 \%$. Thus a sufficiently large coverage with test 2 in the initial periods has a positive effect in the periods that follow. How much larger should be the coverage under test 1 to obtain this sort of results? Testing $15 \%$ of the population delivers a pick of infection of $0.92 \%$ after 551 days. This result suggests that there are dramatic differences between the two types of tests. ${ }^{12}$


Figure 2: Increasing the fraction of tests, $\tau=3 \%$.

Figure 3 plots the effects of social distancing when all agents participate: it is assumed that the number of contacts is reduced from day 1 to the end of the year to $n=20$, which represents a reduction of $33.3 \%$ of interactions. It is clear that social distancing delays a bit the pick of infections and that it is slightly smaller. This effect comes directly from the smaller $p_{t}$. In the other scenarios there is an additional effect, since not only the probability but the mass of participants will change. As an

[^8]example of these effects Figure 4 shows the probability under precaution for various $n$ 's, and Figure 5 shows the corresponding $i a_{t}$ for the first few periods. In the beginning participation declines until the first generation of healthy and immune starts participating. After this point the effect of $n$ on $p_{t}$ explains the subsequent dynamics: with a "light" social distancing participation continues to decline and the fraction of infected but asymptomatic continues to increase. However, for a sufficiently severe social distancing it happens the opposite, so after a few more periods it is as if only healthy agents participate.

Figure 6 represents the evolution of infections under precaution and the two tests (and assuming that the tests cover up to $1 \%$ of the population every day). The effect of social distancing appears to be dramatic in all cases, even when only precaution is implemented. As before the evolution of infection is better under the implementation of test 2, but the pick of infections is in all cases below $0.0018 \%$.


Figure 3: Social distancing with unawareness.

I now look at the output effects in each scenario under the previous policies. Figure 7 represents the level of output relative to the case of no epidemic taking place, so that the output loss in every period is the distance from 1. When all agents


Figure 4: Participation with Social distancing.


Figure 5: Infected asymptomatic that participate with social distancing.


Figure 6: Social distancing with precaution and tests.
participate there is of course no loss, but in the precaution and in test 1 the losses can be as large as $3 \%$ of daily output. Under test 2 the loss is delayed to the future and it is only about $1.8 \%$ (the tests cover again $1 \%$ of the population). When the fraction of tests covers $3 \%$ of the population then test 1 is able to reduce losses to $2.5 \%$, and not surprisingly, under test 2 there are essentially no losses (see Figure 8). I finally look at the effects of social distancing. It is clear in Figure 9 that reducing $n$ by $1 / 3$ has a dramatic effect on output, which falls below $50 \%$ of its "normal" level. In terms of output losses precaution is the worst scenario and both tests 1 and 2 show similar effects in the beginning. These results suggest that there is a clear trade-off between the evolution of the infection and the number of deaths, and the evolution of output.

The large negative effects on output are likely to disappear as soon as social distancing ends and $n$ returns to its normal level. If we request the infection rate be declining and no larger than $1 \%$ as the threshold level to return to normal levels of activity, then under precaution and under all tests the return to normal activity happens after $t_{0}$ periods. However, the mass of healthy and immune agents is rather
small, so without further actions the outcomes in terms of infection fall pray again of the epidemic disease. This is shown in Figure 10. It is instructive to look at the unawareness case, since in this scenario social distancing has positive but limited effects. Figure 11 shows that at the end of social distancing the fraction of infected grows again, but it rapidly declines. The reason is that in this scenario, the fraction of healthy and immune individuals is large. Figure 12 shows the dynamics of healthy and immune in each scenario. This result suggests that a successful policy to implement a gradual return to normal levels of activity should not only request a low fraction of infected, but also a large fraction of immune.


Figure 7: Output losses, baseline.

The results of the previous exercises suggest that relaying on a test like test 2 may be a very effective policy to limit the number of deaths and to preserve the economy in case of an epidemic. This policy requires to test a large fraction the population from the very beginning and over time, which may not be feasible specially at early stages of the episode. Furthermore, under this policy it is likely that a large fraction of the population remains healthy but vulnerable, which means that the population remains exposed to new episodes of the same disease in the near future. This is


Figure 8: Ouput losses with a larger number of tests.


Figure 9: Ouput losses with a larger number of tests.


Figure 10: Dynamics of infection under precaution and tests 1, 2.


Figure 11: Dynamics of infection under unawareness.


Figure 12: Dynamics of healthy and immune.
a dynamic aspect of the whole process that needs to be taken into account. On the other hand with a "light" social distancing strategy the fraction of healthy and immune individuals grows rapidly, but it has an enormous cost in terms of deaths and output losses. Social distancing therefore has costs in the short run but benefits in the long run, exactly the opposite of massive tests. It seems therefore that an optimal combination of tests and the severity of social distancing could minimize loses in the short run and maximize benefits in the long run.

I investigate this issue by combining social distancing over the first $t_{1}$ periods followed by testing the population for several additional periods. That is, I consider several lengths of social distancing (a larger $t_{1}$ period), combined with different levels of its severity (smaller $n$ ), and with several values for $\tau$ to start being applied from $t_{1}+1$ onwards. It is not obvious how to evaluate the outcomes of the different combinations. I therefore focus the attention on the cumulative deaths due to the infection and the policy in place, the fraction of the population that is healthy and immune at the time of ending social distancing, and on the average output loss per day. I simulate the economy until the mass of agents that participate is
close to 1 , the no epidemics level. This is organized as a grid search problem, in which the grid is $t_{1}=14,21,28, \ldots, 90,120$, (weeks, and months at the last two horizons), $n=N-1, \ldots, N-10$ (remember that in the baseline calibration $N=30$ ) and $\tau=0.1, \ldots, 0,3$ in increments of 0.05 . The fraction of $h i$ at the end of social distancing is a critical variable because it may prevent the disease to spread again. The threshold level of group immunity that prevents the spread of the disease is called the Herd Immunity Threshold (HIT). For instance, the HIT for influenza is between $33 \%-44 \%$, and that of SARS is between $50 \%-80 \% .{ }^{13}$ As an approximation, I will request a level of at least $40 \%$ for a policy to successfully end the confinement without suffering a new epidemic immediately afterwards.

Rather than reporting a table with all the results it is more interesting to describe three regularities that emerge from this exercise:

R1: The more sever is social distancing (the smaller is $n$ ), the larger is the number of weeks that is needed to achieve $h i\left(t_{1}\right) \geq 40 \%$. It does not seem possible to achieve the previous threshold level before 70 days, and very often requires 90 or 120 days.

R2: Cumulative deaths decrease with the severity of social distancing (the smaller is $n$ ) and given $n$ they increase in the weeks of social distancing, $t_{1}$. Furthermore, cumulative deaths decrease the larger is the fraction of the population that is tested. The positive effects of a larger coverage of tests offset the negative effects associated to "light" social distancing.

R3: Output losses are larger the more sever is social distancing (the smaller is $n$ ) and the larger is the period of social distancing (the larger is $t_{1}$ ).

These regularities are informative: R1 suggests that it is best to implement a "light" social distancing and R3 reinforces this by suggesting in addition a short period

[^9]$t_{1}$. R2 points to the same direction, provided that there is a sufficiently large coverage of tests. Given the parametrization of the model, $n=29, t_{1}=90$ and $\tau=0.03$ delivers cumulative deaths of 0.23 , a fraction of healthy and immune agents of $41.12 \%$ and an average output loss of $10.58 \%$ during the 130 days that it takes to return to normality. ${ }^{14}$ From this combination of parameters it follows that a smaller $t_{1}$ decreases cumulative deaths slightly, but the fraction of health and immune decreases very strongly. If instead $t_{1}$ is increased, then $h i\left(t_{1}\right)$ increases above $50 \%$, but cumulative deaths also go above 0.28 . Decreasing $\tau$ has no effect on $h i\left(t_{1}\right)$, but it has a negative effect on cumulative deaths. If I consider in stead $n=28$, with the same $t_{1}=90$ and $\tau=3 \%$, there is a small improvement in cumulative deaths (0.228) and a small decrease in $h_{i}\left(t_{1}\right)=40.05 \%$. However, in this case average output losses increase up to $18,40 \%$ (over the same 130 days). These seem to be the best policies, as with a more severe social distancing there is little to gain in terms of cumulative deaths, and a lot to loose in the fraction on healthy and immune and output. For completeness Figure 13 plots the outcome of the optimal policy (with a reduction of contacts of $0,33 \%, n=29$ ).


Figure 13: Combining social distancing and tests.

[^10]The series of cumulative deaths in Figure 13 measures cumulative deaths relative to the unawareness case, thus the avoided deaths with the policy are measured as the distance from 1. Figure 13 reveals that the optimal policy essentially substitutes deaths inter temporally: deaths are smaller in the short and in the long run, but in between they are nearly identical to the unawareness/no policy case.

In the calculations above the fraction of infected individuals that may attend a health center has been ignored. This is an important variable from the public health perspective because it may collapse the public health network. I computed the fraction of agents that are infected with symptoms and its maximum level is a bit above $11 \%$ in period $45 / 44$. If I assume that only individuals with severe symptoms attend a hospital and that this fraction is equal to the fraction that will die $(d=0.034)$, then I find that the fraction of individuals in the hospital is about $0.39 \%$ of the population. This is well below $1 \%$ which is some times taken as the rough threshold level at which public health may collapse. If, however, I assume that all individuals with symptoms will attend a hospital, then it is needed a reduction of about $73 \%$ in the number of contacts for 120 periods to obtain a mass of infected with symptoms of $1.1 \%$. In this case cumulative deaths decrease to 0.034 , but the fraction of healthy and infected at $t_{1}$ is only slightly above $0.8 \%$ and output losses are above $53 \%$.

The previous policy recommendation rests on the fact that preferences are not lexicographic in the number of current deaths and on the fact that an effective cure or vaccine does not seem to be available over the next 6 months. Under these premises any policy exercise conducted in this literature needs to evaluate the trade-off between deaths today versus deaths tomorrow. This trade-off is in the hands of the policy-makers and it can only improve if there are decisive medical advancements.

## 6 Final remarks

This paper proposes a model to evaluate the effect of policy on health distribution dynamics and on output in an economy that is subject to an epidemic disease. An important conclusion of this research is that a simple precautionary policy may have very positive effects to delay and reduce the pick of infections and that poor quality tests only able to discriminate healthy from infected agents do not represent a significant improvement. The precautionary policy is as easy as to check the temperature of all individuals, keeping a registration of the possibly infected and isolating them for 14 days without further tests.

It is also necessary to emphasize that the use of tests that can only discriminate between infected and non infected individuals, like the widespread PRC, does not represent a noticeable improvement upon the precautionary policy when there are no other significant social distancing measures.

A third important result in this paper is that a high quality test is able to eradicate the disease in the short run provided that there is a large enough coverage of the population. The caveat of this test is that it is likely to leave the population without herd immunity, and thus the possibility of new epidemics in the future remains positive.

I use the model to characterize the optimal policy mix. I find that the best combination of policies consists of a "light" social distancing for about three months followed with a large coverage of the population with a high quality test for additional 40 days.

The model in this paper can be improved along several dimensions, including a discrimination of the interactions between social and productive meetings and a
precise characterization of individuals along the lines in the immunology literature (age, gender, pre-clinical condition). The current representation of output losses is appropriate if the effects of the epidemic extend over a few weeks. Clearly, however, with a longer horizon not only unemployment will rise but also productive capital will be destroyed. It would be very interesting to include the health distribution dynamics of the epidemic in an Aiyagari-Huggett model and evaluate more precisely the effects of taxes and subsidies. This investigation is left for future work.

## 7 Appendix

I describe here a few additional results of a sensitivity analysis.

1. The power of test 1 . The following Table 1 summarizes the effects on the fraction of infected individuals as the coverage of test 1 is increased.

| $\tau(\%)$ | pick day | pick of infections (\%) |
| :---: | :---: | :---: |
| 4 | 301 | 3 |
| 5 | 312 | 2.8 |
| 10 | 394 | 1.8 |
| 15 | 551 | 0.91 |

Table 1: Test 1 with larger coverage.
2. The length of confinement, $t_{0}$. I check the effect of $t_{0}$ on the differences between test 1 and test 2 . The baseline value for $t_{0}$ is 14 days which is the recommendation of the WHO. I find that increasing $t_{0}$ to 18,20 or 25 days reduces the differences between the tests in the magnitude of the pick of infection and it has small effects on the pick date. Still, the type 2 test performs better than the test type 1. The results are summarized in Table 2.
3. The effects of $N$ and the power of tests 1 and 2: I described in the text that

| $t_{0}$ | TEST 1 pick day | pick of infections (\%) | TEST 2 pick day | pick of infections (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 18 | 168 | 9.24 | 192 | 7.27 |
| 20 | 160 | 11.71 | 181 | 9.61 |
| 25 | 160 | 16.83 | 179 | 14.59 |

Table 2: The effect of larger $t_{0}$.
reducing the number of contacts has a positive effect to delay the pick of infections and to reduce it. With test 2 covering $3 \%$ of the population the epidemic does not take place when $N=30$. When $N$ is increased to 35,40 and 50 I obtain that the pick of infections is delayed to day 380, day 226 and day 134 respectively, and the pick of infections increases to $1.33 \%, 3.6 \%$ and $4.5 \%$ (also respectively). The epidemic is still far from the unawareness outcome, hence test 2 is still effective at a $3 \%$ coverage of the population. I repeat the same exercise under test 1 . With 35,40 and 50 contacts the pick occurs on days 189, 146 and 105, and reaches $6 \%$, $8.15 \%$ and $10.9 \%$. The results when only precaution is in place are pick occurring at days 177,136 and 101 and reaching $6.54 \%, 8.6 \%$ and $11.24 \%$. The conclusion is that the effect of a larger $N$ goes in the expected direction, but it does not change the conclusion that test 2 is substantially more effective than test 1 to fight the epidemic. Since there are no differences in the probability of transmission of the infection across occupations changes in $N$ do not change the shape of the optimal policy.
4. The curvature in the production function. In the calibration exercise I found that the value of a job in the "services" sector is only slightly larger than that in the "industry" sector. The effect of a reduction in $n$ is different in the two sectors because of the curvature in $f(n)$. When I take the two functions simultaneously closer to the linear case output losses under the optimal policy decrease at an almost constant rate from $10.58 \%$ to $2.6 \%$, when $\alpha_{s}=\alpha_{i}=1$ and thus the loss in output is proportional
to $n / N$. Changes in the curvature do not change the optimal combination of policies.

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[^1]:    ${ }^{1}$ Casares and Khan (2020) also take this approach in a related model. Chen and Qius (2020) conduct an empirical evaluation of various non-pharmaceutical interventions for 9 countries using a dynamic-panel SIR model.
    ${ }^{2}$ For a detailed exposition of different types of test see "Serology-based tests for COVID19 " from the Center of Health Security, which is updated twice a week and can be found here https://www.centerforhealthsecurity.org/resources/COVID-19/serology/Serology-based-tests-for-COVID-19.html.

[^2]:    ${ }^{3}$ A result along this lines is also found in Berger, Herkenhoff and Mongey (2020).
    ${ }^{4}$ These results are similar to those in Piguillem and Shi (2020). Their model allows for time varying intensity of quarantine, which takes the form of a reduction in the fraction of the population taking part in the economic activity and they do not consider tests type 1 . Here in stead, social distancing does not only affect endogenously the fraction of the population that participates in economic activities but it also limits the extent of the activity, i.e., the number of contacts.

[^3]:    ${ }^{5}$ Precisely the lack of a powerful remedy makes this sort of exercise interesting. See however Eichenbaum et al. (2020) and Gonzalez-Eiras and Niepelt (2020) where the possibility of a decisive medical improvement is taken into account.

[^4]:    ${ }^{6}$ Consecutive meetings simplify a bit the exposition and description but are not fundamental for the results.

[^5]:    ${ }^{7}$ The fundamental aspect of the test is that it discriminates the type of healthy individual. The fact that the test also tells the maturity of the infection of infected individuals only simplifies to keep track of them.

[^6]:    ${ }^{8}$ See the evidence in Oke and Henghan (2020) which can be found here https://www.cebm.net/covid-19/global-covid-19-case-fatality-rates/.
    ${ }^{9}$ This can be found here https://www.cebm.net/covid-19/covid-19-what-proportion-areasymptomatic/.

[^7]:    ${ }^{10}$ The data in these calculations comes from Tables 2.1 and 2.2 from BLS, which can be found in https://www.bls.gov/emp/tables/output-by-major-industry-sector.htm and https://www.bls.gov/emp/tables/employment-by-major-industry-sector.htm.
    ${ }^{11}$ The value for $N$ is similar to 25 used in Casares and Khan (2020).

[^8]:    ${ }^{12}$ In the Appendix I report the results of a sensitivity analysis. My results suggest that the differences between test 1 and test 2 tend to decrease as $t_{0}$ is increased, but test 2 always performs better than test 1 to control the spread of the disease.

[^9]:    ${ }^{13}$ See Biggerstaff et al. (2014) and Wallinga and Teunis (2004).

[^10]:    ${ }^{14} \mathrm{I}$ checked that with $\alpha_{s}=\alpha_{i}=1$ the average output loss per day reduces to $2.6 \%$.

