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Elasticity Determinants of Inequality Reducing Income Taxation

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Abstract

The link between income inequality and progressive taxation has long been considered a fundamental normative foundation for income tax progressivity. This paper furnishes necessary and sufficient conditions on primitives, in terms of the elasticity of income with respect to ability, under which various subclasses of progressive taxes are inequality reducing. The distributional effects of progressive income taxation are decomposed into two conditions on the wage elasticity of income, the *tax rate effect* and the *subsidy effect*, each capturing different aspects of the transition between before-tax and after-tax income distributions. The results confer a degree of useful flexibility to the theory, in that they allow the analyst to expand the universe of consumer preferences by suitably restricting the set of marginal-rate progressive taxes. As an illustration of the results' practical implications, we provide a precise characterization of the subclass of (progressive) taxes that are inequality reducing for the constant elasticity of substitution (CES) and the quasi-linear utility functions.

Keywords: incentive effects of taxation, income inequality, progressive taxation, subsidy effect, tax rate effect, wage elasticity of income.
JEL classifications: D63, D71.

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1 Introduction

The link between income inequality and progressive taxation uncovered in the seminal works of Jakobsson (1976) and Fellman (1976) has long been considered a fundamental normative foundation for income tax progressivity.[1] In a recent paper, Carbonell-Nicolau and Llavador (2018) extended the classic result of Jakobsson (1976) and Fellman (1976)—according to which average-rate progressive, and only average-rate progressive income taxes, reduce income inequality—to the case of endogenous income. There it was shown that marginal-rate progressivity—in the sense of increasing marginal tax rates on income—is a necessary condition for tax structures to be inequality reducing, and necessary and sufficient conditions on preferences were identified under which progressive and only progressive taxes are inequality reducing. While this result circumvents the difficulties and the negative results emphasized by other authors in their attempts to incorporate the disincentive effects of taxation (see, e.g., Allingham (1979) and Ebert and Moyes (2003, 2007)), it confines attention to the conditions under which the set of *all* marginal-rate progressive taxes are inequality reducing. Evidently, requiring *larger* families of tax schedules to be inequality reducing results in *stronger* conditions on consumer preferences. In fact, the conditions derived in Carbonell-Nicolau and Llavador (2018) may be regarded, in some cases, as overly restrictive: while they are fulfilled by some standard classes of preferences—such as the Cobb-Douglas preferences and the so-called *GHH preferences* (see Greenwood et al., 1988)—this paper illustrates that there are important sets of preferences—such as the constant elasticity of substitution (CES) and the quasi-linear families of utility functions—that violate them. A natural question, therefore, is whether there are *subclasses* of marginal-rate progressive tax schedules that are inequality reducing for *larger* collections of preferences.

This paper identifies necessary and sufficient conditions on consumer preferences ensuring that various subclasses of progressive taxes are inequality reducing. Considering strict subclasses of progressive tax schedules allows us to work with larger families of preferences consistent with an after-tax equalization of incomes. The results obtained here are a strict generalization of those in Carbonell-Nicolau and Llavador (2018), and confer a degree of useful flexibility to the theory, in that they allow the analyst to expand the universe of consumer preferences by suitably restricting the set of marginal-rate progressive taxes.

Our analysis is formulated within the classical Mirrlees (1971) framework. We consider continuous, piecewise linear, nondecreasing tax schedules that preserve the ranking of pre-tax incomes. The allowable constraints on taxes take the form of subsidies (negative taxes)

[1] The literature on the redistributive effects of tax systems was initiated by Musgrave and Thin (1948). The contributions of Jakobsson (1976) and Fellman (1976) led to a large body of literature on the foundations of tax progressivity (see, e.g., Kakwani (1977); Hemming and Keen (1983); Eichhorn et al. (1984); Liu (1985); Formby et al. (1986); Thon (1987); Latham (1988); Thistle (1988); Moyes (1988, 1994); Le Breton et al. (1996); Ebert and Moyes (2000); Ju and Moreno-Ternero (2008)).

Other normative rationales for income tax progressivity are based on the principle of equal sacrifice (see Samuelson (1947); Young (1990); Berliant and Gouveia (1993); Ok (1995); Mitra and Ok (1996, 1997); D'Antoni (1999)) and on measures of income polarization (see Carbonell-Nicolau and Llavador (forthcoming)).

and/or subsets of $[0\%, 100\%)$ for the marginal tax rates. Each lower bound on the subsidy received by the agents in the economy, together with a subset of possible marginal tax rates for each tax bracket, gives rise to a subclass of marginal-rate progressive tax schedules. A major result of the paper (Theorem 4) characterizes, for each such subclass $\mathscr{T}$, the family of preferences that renders the members of $\mathscr{T}$ inequality reducing.

Another important contribution is the decomposition of the distributional effects of progressive income taxation into two conditions on the wage elasticity of income, the *tax rate effect* and the *subsidy effect*, each capturing different aspects of the transition between before-tax and after-tax income distributions. The subsidy effect measures how the elasticity of income with respect to ability changes when an agent receives a subsidy, while the tax rate effect measures the variation in this elasticity when an agent's income is subjected to a proportional tax rate. Our main condition characterizing inequality reducing subclasses of tax schedules is formulated in terms of the elasticity of income with respect to ability, and requires that the two effects combined reduce this elasticity. However, either effect may increase the elasticity, as long as it is offset by the other effect. Because a proportional tax rate reduces an agent's "effective ability," a negative tax rate effect implies that (before-tax) incomes are more sensitive to marginal tax rates as the ability of a worker increases; in this case, a progressive tax schedule tends to reduce income inequality. A negative subsidy effect implies that (after-tax) incomes become relatively less sensitive to ability with the introduction of a subsidy, thereby reducing income dispersion. A marginal-rate progressive tax schedule is inequality reducing if and only if the sum of the two effects is negative. For example, a sufficiently large negative subsidy effect may compensate a positive tax rate effect.

Because the wage elasticity of income can be expressed in terms of the elasticity of substitution between consumption and leisure and the wage elasticity of leisure, our main results can be reformulated directly in terms of the last two elasticities. This reformulation sheds light on the role of the elasticity of substitution between consumption and leisure in the characterization of inequality reducing tax systems.

As an illustration of the result's practical implications, we provide a precise characterization of the subclass of (progressive) taxes that are inequality reducing for the constant elasticity of substitution (CES) and the quasi-linear utility functions. These preferences are pervasive in surveys and textbooks on labor supply and fiscal policy (see, e.g., Pencavel, 1986; Killingsworth and Heckman, 1986; Auerbach and Kotlikoff, 1987; Keane, 2011; Blundell et al., 2016). In addition, the CES utility function (often in its Cobb-Douglas form) is widely used in the literature on life-cycle models (see, e.g., Heckman and MaCurdy, 1982; French, 2005; Blundell et al., 2016), while static models with fixed costs traditionally work with quasi-linear preferences (Cogan, 1981).

The remainder of the paper is organized as follows. Section 2 introduces the formal setting. It defines the set of piecewise linear tax schedules, it describes the agent's problem and introduces Lorenz dominance as the inequality criterion. The main results are presented in Section 3. Section 4 studies applications to the CES and the quasi-linear utility functions, providing a precise characterization of the inequality-reducing subclasses of progressive taxes for these preferences. Section 5 situates our assumptions on consumer preferences in the context of a broader literature that emphasizes reference dependence, loss aversion, relative consumption, inequality aversion, and tax compliance. It also compares, from a methodological perspective, our analysis with the literature on optimal income taxation, and discusses avenues for future research. All proofs are relegated to the Appendix.

2 Preliminaries

The setting is the same as that of Carbonell-Nicolau and Llavador (2018). There are n individuals. The utility function is given by a continuous utility function $u : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$ defined over consumption-labor pairs $(c,l) \in \mathbb{R}_+ \times [0,1]$ such that $u(\cdot,l)$ is strictly increasing in c for each $l \in [0,1)$, and $u(c,\cdot)$ is strictly decreasing in l for each $c>0$. The map u is assumed strictly quasiconcave on $\mathbb{R}_{++} \times [0,1)$ and twice continuously differentiable on $\mathbb{R}_{++} \times (0,1)$. For $(c,l) \in \mathbb{R}_{++} \times (0,1)$, let

$$MRS(c,l) := -\frac{u_l(c,l)}{u_c(c,l)}$$

denote the marginal rate of substitution of labor for consumption, where

$$u_c(c,l) := \frac{\partial u(c,l)}{\partial c} \quad \text{and} \quad u_l(c,l) := \frac{\partial u(c,l)}{\partial l}.$$

We assume that for each $c>0$,

$$\lim_{l \to 1^-} MRS(c,l) = +\infty \quad \text{and} \quad \lim_{l \to 0^+} MRS(c,l) < +\infty. \tag{1}$$

The set of all utility functions satisfying the above conditions is denoted by $\mathscr{U}$.

We restrict attention to nondecreasing and order-preserving piecewise linear tax schedules.

Definition 1. Let $(\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}}) = \big(\alpha_0, (t_0,\dots,t_K), (\overline{y}_0,\dots,\overline{y}_K)\big)$, where $\alpha_0 \geq 0$, $K \in \mathbb{Z}_+$, $t_k \in [0,1)$ for each $k \in \{0,\dots,K\}$, $t_k \neq t_{k+1}$ whenever $k \in \{0,\dots,K-1\}$ and $K \geq 1$, and $0 = \overline{y}_0 < \cdots < \overline{y}_K$. A $(K+1)$***-bracket piecewise linear tax schedule*** is a real-valued map T on $\mathbb{R}_+$ uniquely determined by $(\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}})$ as follows:

$$T(y) := \begin{cases} -\alpha_0 + t_0 y & \text{if } 0 = \overline{y}_0 \leq y \leq \overline{y}_1, \\ -\alpha_0 + t_0\overline{y}_1 + t_1(y - \overline{y}_1) & \text{if } \overline{y}_1 < y \leq \overline{y}_2, \\ \quad \vdots & \quad \vdots \\ -\alpha_0 + t_0\overline{y}_1 + t_1(\overline{y}_2 - \overline{y}_1) + \cdots + t_{K-1}(\overline{y}_K - \overline{y}_{K-1}) + t_K(y - \overline{y}_K) & \text{if } \overline{y}_K < y. \end{cases}$$

Here $T(y)$ is interpreted as the tax liability for gross income level y. We write $(\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}})$ and the associated map T interchangeably. Note that for $K=0$, $(\alpha_0, t_0, \overline{y}_0 = 0)$ is a linear tax with intercept a_0 and marginal tax rate t_0; for $K = 1$, $(a_0,(t_0,t_1),(\overline{y}_0,\overline{y}_1))$ is a two-bracket tax with intercept a_0, marginal tax rates t_0 and t_1, and bracket threshold $\overline{y}_1$; and so on.

The set of piecewise linear tax schedules is denoted by $\mathscr{T}$.

The following notion of tax progressivity, which requires that marginal tax rates be nondecreasing with income, plays an essential role in our results.

Definition 2. A tax schedule $T \in \mathscr{T}$ is ***marginal-rate progressive*** if it is a convex function.

The set of all marginal-rate progressive tax schedules in $\mathscr{T}$ is denoted by $\mathscr{T}_{prog}$.

Linear tax schedules play an important role in the analysis, and it is convenient to introduce their formal definition.

Definition 3. A tax schedule $T \in \mathscr{T}$ is ***linear*** if $T(y) = -b + ty$ for all $y \in \mathbb{R}_+$ and some $b \geq 0$ and $t \in [0,1)$.

Denote the set of all linear tax schedules in $\mathscr{T}$ as $\mathscr{T}_{lin}$.

Individuals differ in their abilities. An ***ability distribution*** is a vector $\boldsymbol{a} = (a_1,\dots,a_n) \in \mathbb{R}^n_{++}$ such that $a_1 \leq \cdots \leq a_n$. The set of all ability distributions is denoted by $\mathscr{A}$.

An agent of ability $a > 0$ who chooses $l \in [0,1]$ units of labor and faces a tax schedule $T \in \mathscr{T}$ consumes $c = al - T(al)$ units of the good and obtains a utility of $u(c,l)$. Thus, the agent's problem is

$$\max_{l\in[0,1]} u(al - T(al), l). \tag{2}$$

Because the members of $\mathscr{U}$ and $\mathscr{T}$ are continuous, for given $u \in \mathscr{U}$, $a > 0$, and $T \in \mathscr{T}$, the optimization problem in (2) has a solution, although it need not be unique. A ***solution function*** is a map $l^u : \mathbb{R}_{++} \times \mathscr{T} \to [0,1]$ such that $l^u(a,T)$ is a solution to (2) for each $(a,T) \in \mathbb{R}_{++} \times \mathscr{T}$. The ***pre-tax*** and ***post-tax income functions*** associated to a solution function l^u, denoted by $y^u : \mathbb{R}_{++} \times \mathscr{T} \to \mathbb{R}_+$ and $x^u : \mathbb{R}_{++} \times \mathscr{T} \to \mathbb{R}_+$ respectively, are given by

$$y^u(a,T) := al^u(a,T) \quad \text{and} \quad x^u(a,T) := al^u(a,T) - T\left(al^u(a,T)\right).$$

Given $a > 0$, let $U^a : \mathbb{R}_+ \times [0,a] \to \mathbb{R}$ be defined by $U^a(c,y) := u(c, y/a)$. For $(c,y,a) \in \mathbb{R}^3_{++}$ with $y < a$, define

$$U^a_c(c,y) := \frac{\partial U^a(c,y)}{\partial c}, \quad U^a_y(c,y) := \frac{\partial U^a(c,y)}{\partial y}, \quad \text{and } \eta^a(c,y) := -\frac{U^a_y(c,y)}{U^a_c(c,y)}.$$

The following condition was introduced by Mirrlees (1971, Assumption B, p. 182) and termed *agent monotonicity* by Seade (1982).

Definition 4. A utility function $u \in \mathscr{U}$ satisfies ***agent monotonicity*** if $\eta^a(c,y) \geq \eta^{a'}(c,y)$ for each $(c,y) \in \mathbb{R}^2_+$ and $0 < a < a'$ with $y < a$.

Agent monotonicity is a single crossing condition whereby the consumers' indifference curves in the space of pre-tax income-consumption pairs, (y,c), are flatter for more productive agents. It is equivalent to the condition that (in the absence of taxation) consumption is a nondecreasing function with respect to productivity, for any nonwage income (Mirrlees, 1971, p. 182). Any preference violating the agent monotonicity condition would necessarily treat consumption as an inferior good (Myles, 1995, p. 136).

The set of all the members of $\mathscr{U}$ satisfying agent monotonicity is denoted by $\mathscr{U}^*$.

Inequality comparisons are based on the standard relative Lorenz ordering. An ***income distribution*** is a vector $\boldsymbol{z} = (z_1, ..., z_n) \in \mathbb{R}^n_+$ of incomes arranged in increasing order, *i.e.*, $z_1 \leq \cdots \leq z_n$. Given two income distributions $\boldsymbol{z} = (z_1, ..., z_n)$ and $\boldsymbol{z}' = (z'_1, ..., z'_n)$ with $z_n, z'_n > 0$, we say that $\boldsymbol{z}$ is at least as equal as $\boldsymbol{z}'$ if $\boldsymbol{z}$ ***Lorenz dominates*** $\boldsymbol{z}'$, *i.e.*, if

$$\frac{\sum_{i=1}^k z_i}{\sum_{i=1}^n z_i} \geq \frac{\sum_{i=1}^k z'_i}{\sum_{i=1}^n z'_i}, \quad \text{for all } k \in \{1, ..., n\}. \tag{3}$$

For $u \in \mathscr{U}^*$, and given pre-tax and post-tax income functions y^u and x^u, an ability distribution $\boldsymbol{a} = (a_1, ..., a_n) \in \mathscr{A}$ and a tax schedule $T \in \mathscr{T}$ determine a ***pre-tax income distribution***

$$\boldsymbol{y}^u(\boldsymbol{a}, T) := \left(y^u(a_1, T), ..., y^u(a_n, T)\right)$$

and a ***post-tax income distribution***

$$\boldsymbol{x}^u(\boldsymbol{a}, T) := \left(x^u(a_1, T), ..., x^u(a_n, T)\right).$$ [2]

In the absence of taxation, *i.e.*, if $T \equiv 0$, one has $\boldsymbol{y}^u(\boldsymbol{a}, 0) = \boldsymbol{x}^u(\boldsymbol{a}, T)$.

The following is the central notion of inequality reducing tax schedule.

Definition 5. Let $u \in \mathscr{U}$. A tax schedule $T \in \mathscr{T}$ is ***income inequality reducing with respect to*** u, which we denote as u-***iir***, if $\boldsymbol{x}^u(\boldsymbol{a}, T)$ Lorenz dominates $\boldsymbol{y}^u(\boldsymbol{a}, 0)$ for each ability distribution $\boldsymbol{a} := (a_1, ..., a_n) \in \mathscr{A}$ and for each pre-tax and post-tax income functions y^u and x^u.

Observe that the '*iir*' relation compares post-tax income distributions with the income distribution in the absence of taxation, and requires the former to be at least as equal as the latter, according to the relative Lorenz criterion.

3 The results

To begin, we recapture a result from Carbonell-Nicolau and Llavador (2018).

Theorem 1 (Carbonell-Nicolau and Llavador (2018, Theorem 1))**.** *Given $u \in \mathscr{U}^*$, a tax schedule in $\mathscr{T}$ is u-iir only if it is marginal-rate progressive.*

Theorem 1 asserts that marginal-rate progressivity is necessary for a tax schedule to be inequality reducing. With endogenous income (and unlike in the endowment economy

[2] Under the agent monotonicity condition, in both cases the vector components are arranged in increasing order. See Carbonell-Nicolau and Llavador (2018).

framework of Jakobsson (1976) and Fellman (1976)), the effect of a tax on gross incomes, in addition to the disposition of its tax rates, determines the distributional effects of taxation. This suggests that consumer preferences, and their interaction with tax structures, are bound to play an important role in the formulation of inequality reducing properties of tax systems. The main result in Carbonell-Nicolau and Llavador (2018) demonstrates that this is indeed the case: only certain classes of preferences guarantee that the set of all marginal-rate progressive taxes are *iir*.

The contribution of this paper is threefold. First, we show that requiring *all* marginal-rate progressive tax schedules to be *iir* may be overly restrictive. Indeed, such a requirement rules out common classes of preferences, such as the constant elasticity of substitution (CES) and the quasi-linear utility functions. Second, we extend the analysis in Carbonell-Nicolau and Llavador (2018) by identifying necessary and sufficient conditions on preferences for which various *subsets* of progressive tax schedules are *iir*. This allows us to identify the utility functions within the CES and the quasi-linear families for which there exist subsets of progressive, *iir* tax schedules (Section 4). Finally, we decompose the effect of a tax schedule on income inequality into two conditions on the wage elasticity of income, called the tax rate effect and the subsidy effect, shading light on the forces that determine whether tax schedules are inequality reducing.

Consider subclasses of $\mathcal{T}_{prog}$ characterized by the number of brackets and the ranges for their intercepts with the vertical axis and their marginal tax rates. Formally, given $K \in \mathbb{Z}_+$, $B \subseteq \mathbb{R}_+$ and subsets $R_0,...,R_K$ of $[0,1)$, let $\mathcal{T}_{prog}(B,R_0,...,R_K)$ be the set of all $(K+1)$-bracket marginal-rate progressive tax schedules $(\alpha_0,(t_0,...,t_K),(\overline{y}_0,...,\overline{y}_K)) \in \mathcal{T}_{prog}$ with intercept $\alpha_0 \in B$, marginal tax rates $t_0,...,t_K$ with $t_k \in R_k$ for each $k \in \{0,...,K\}$, and bracket thresholds $\overline{y}_1,...,\overline{y}_K$, *i.e.*,

$$\mathcal{T}_{prog}(B,R_0,...,R_K) := \left\{(\alpha_0,(t_0,...,t_K),(\overline{y}_0,...,\overline{y}_K)) \in \mathcal{T}_{prog} : \alpha_0 \in B \text{ and } (t_0,...,t_K) \in R_0 \times \cdots \times R_K\right\}.$$

Let $\mathfrak{D}$ be the set of all $(B,(R_k)_{k=0}^{\infty})$ with $B \subseteq \mathbb{R}_+$ and $R_k \subseteq [0,1)$ for each k. For each $(B,(R_k)) \in \mathfrak{D}$, define

$$\mathcal{T}_{prog}(B,(R_k)) := \bigcup_{K\in\mathbb{Z}_+} \mathcal{T}_{prog}(B,R_0,...,R_K).$$

When $R_0 = R_1 = \cdots = R$, we write $\mathcal{T}_{prog}(B,R)$ for $\mathcal{T}_{prog}(B,(R_k))$. Observe that $\mathcal{T}_{prog} = \mathcal{T}_{prog}(\mathbb{R}_+,[0,1))$.

Given $B \subseteq \mathbb{R}_+$ and $R \subseteq [0,1)$, define

$$\mathcal{T}_{lin}(B,R) := \{-b + ry \in \mathcal{T}_{lin} : b \in B \text{ and } r \in R\}$$

and

$$\overline{B} := \bigcup_{b\in B} \left\{b' \in \mathbb{R}_+ : b' \geq b\right\}.$$

The next theorem provides a necessary and sufficient condition for all the members in the subclass $\mathcal{T}_{prog}(\overline{B},(R_k))$ of $\mathcal{T}_{prog}$ to be *iir*.[3]

Theorem 2. *Given $u \in \mathcal{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$,*

$$[\mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}] \quad and \quad \left[\mathcal{T}_{prog}(\overline{B},(R_k)) \subseteq \mathcal{T}_{u\text{-}iir} \Longleftrightarrow \mathcal{T}_{lin}\left(\overline{B},\bigcup_k R_k\right) \subseteq \mathcal{T}_{u\text{-}iir}\right].$$

The first containment follows immediately from Theorem 1. The bracketed equivalence asserts that the members of the set $\mathcal{T}_{prog}(\overline{B},(R_k))$ of progressive tax schedules in $\mathcal{T}_{prog}$ whose intercept α_0 is greater than or equal to the infimum of $\overline{B}$ (inf$\overline{B}$) and whose k-th marginal tax rate t_k lies in R_k are all *iir* if and only if all the linear taxes with intercept greater than or equal to inf$\overline{B}$ and marginal tax rates in $\bigcup_k R_k$ are *iir*.

For $B = \mathbb{R}_+$ and $R_k = [0,1)$ for each k, one has that $\mathcal{T}_{prog}(B,(R_k)) = \mathcal{T}_{prog}$ and $\mathcal{T}_{lin}(\overline{B},\bigcup_k R_k) = \mathcal{T}_{lin}$; in this case, Theorem 2 immediately gives Theorem 2 in Carbonell-Nicolau and Llavador (2018):

Corollary 1 (to Theorem 2)**.** *Given $u \in \mathcal{U}^*$, $\mathcal{T}_{u\text{-}iir} = \mathcal{T}_{prog}$ if and only if the members of $\mathcal{T}_{lin}$ are u-iir.*

Theorem 2 implies that in order to determine whether the members of $\mathcal{T}_{prog}(\overline{B},(R_k))$ are *iir*, one can restrict attention to the inequality reducing properties of the subclass $\mathcal{T}_{lin}(\overline{B},\bigcup_k R_k)$ of linear tax schedules.

We now provide necessary and sufficient conditions on preferences under which the subclasses $\mathcal{T}_{prog}(\overline{B},(R_k))$ of marginal-rate progressive taxes are inequality reducing. In light of Theorem 2, this is tantamount to characterizing the family of preferences for which the members of a subset $\mathcal{T}_{lin}(B,R)$ are inequality reducing (this is done in Theorem 3 below). This characterization then allows us to present a variant of Theorem 2 in terms of first principles (see Theorem 4 below).

When T is a linear tax schedule in $\mathcal{T}_{lin}$ with $T(y) = -b$, where $b \geq 0$, we write $l^u(a,b)$ for $l^u(a,T)$. For each $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$, $l^u(a,b)$ is a solution to the problem

$$\max_{l \in [0,1]} u(al + b, l). \tag{4}$$

Since u is strictly quasiconcave on $\mathbb{R}_{++} \times [0,1)$, for each $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$, there is a unique solution $l^u(a,b)$ to (4). For given $b \geq 0$, the derivative of the map $a \mapsto l^u(a,b)$ exists for all but perhaps one $a > 0$.[4]

For $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$, define the *elasticity of income with respect to ability* at ability level a and endowment b as

$$\zeta^u(a,b) := \frac{\partial(al^u(a,b)+b)}{\partial a} \cdot \frac{a}{al^u(a,b)+b}.$$

[3]The theorem generalizes Theorem 2 in Carbonell-Nicolau and Llavador (2018) and can be proven using an adaptation of the proof of that theorem. The details are provided in the Appendix.

[4]This is proved in Carbonell-Nicolau and Llavador (2018, page 45).

Given $B \subseteq \mathbb{R}_+$ and $R \subseteq [0,1)$, let $\mathscr{U}(B,R)$ be the set of all $u \in \mathscr{U}^*$ satisfying the following condition:

$$\zeta^u((1-r)a,b) \leq \zeta^u(a,0), \quad \text{for all } (a,b,r) \in \mathbb{R}_{++} \times B \times R. \tag{5}$$

The following result shows that (5) is indeed the relevant condition to characterize the familiy of preferences for which the corresponding subclass of linear tax schedules, $\mathscr{T}_{lin}(B,R)$, is inequality reducing. The proof is relegated to the Appendix.[5]

Theorem 3. *For $u \in \mathscr{U}^*$, the members of $\mathscr{T}_{lin}(B,R)$ are u-iir if and only if $u \in \mathscr{U}(B,R)$.*

Now combining Theorem 2 and Theorem 3 yields the main result of this paper.[6]

Theorem 4. *Given $u \in \mathscr{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$,*

$$\left[\mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}\right] \quad \textit{and} \quad \left[\mathscr{T}_{prog}(\overline{B},(R_k)) \subseteq \mathscr{T}_{u\text{-}iir} \Longleftrightarrow u \in \mathscr{U}\left(\overline{B}, \bigcup_k R_k\right)\right].$$

The bracketed equivalence asserts that the members of the set $\mathscr{T}_{prog}(\overline{B},(R_k))$ of progressive tax schedules in $\mathscr{T}_{prog}$ whose intercept α_0 is greater than or equal to $\inf B$ and whose k-th marginal tax rate t_k lies in R_k (for each k) are all *iir* if and only if $u \in \mathscr{U}(\overline{B}, \bigcup_k R_k)$, *i.e.*, if and only if the elasticity of income with respect to ability satisfies the following condition:

$$\zeta^u((1-r)a,b) \leq \zeta^u(a,0), \quad \text{for all } (a,b,r) \in \mathbb{R}_{++} \times \overline{B} \times \left(\bigcup_k R_k\right). \tag{6}$$

In the remainder of this section, we first obtain a decomposition of the inequality in condition (6) that is useful to develop intuition for Theorem 4 in applications, and then consider the extreme cases of perfect complementarity (resp. substitutability) between consumption and leisure, outlining some intuition. We conclude with a reformulation of our decomposition in terms of the elasticity of substitution between consumption and leisure.

As per Theorem 4, the members of the set $\mathscr{T}_{prog}(\overline{B},(R_k))$ of progressive tax schedules in $\mathscr{T}_{prog}$ are all *iir* if and only if condition (6) is fulfilled. To understand condition (6), it is useful to rewrite the inequality $\zeta^u((1-r)a,b) \leq \zeta^u(a,0)$ as

$$\zeta^u((1-r)a,b) - \zeta^u(a,0) \leq 0$$

and decompose the magnitude on the left-hand side, $\zeta^u((1-r)a,b) - \zeta^u(a,0)$, as the sum of two effects, called the *subsidy effect* (the first bracketed term below) and the *tax rate effect*

[5]Theorem 3 subsumes Theorem 3 in Carbonell-Nicolau and Llavador (2018), which states that the members of $\mathscr{T}_{lin}$ are *u-iir* if and only if $u \in \widehat{\mathscr{U}}$, where $\widehat{\mathscr{U}}$ is the class of utility functions $u \in \mathscr{U}^*$ satisfying the following two conditions: (*i*) $\zeta^u(a,b) \leq \zeta^u(a,0)$ for all $(a,b) \in \mathbb{R}_{++} \times \mathbb{R}_+$; and (*ii*) the map $a \mapsto \zeta^u(a,0)$ defined on $\mathbb{R}_{++}$ is nondecreasing. It follows from the proof of Theorem 3 in Carbonell-Nicolau and Llavador (2018) that $\widehat{\mathscr{U}} = \mathscr{U}(\mathbb{R}_+,[0,1))$.

[6]This result refines Corollary 3 in Carbonell-Nicolau and Llavador (2018).

(the second bracketed term below), respectively:

$$\zeta^u((1-r)a,b)-\zeta^u(a,0)=[\zeta^u((1-r)a,b)-\zeta^u((1-r)a,0)]+[\zeta^u((1-r)a,0)-\zeta^u(a,0)]. \quad (7)$$

The subsidy effect measures how the elasticity of income with respect to ability changes when a non-subsidized $a(1-r)$-agent receives a subsidy b, while the tax rate effect measures the change in this elasticity when ability decreases from a to $(1-r)a$.

Condition (6) requires that the two effects combined lower ζ^u, but either effect may increase this elasticity, as long as it is offset by the other effect. Note that if $\zeta^u(a,0)$ is *increasing* in a (resp., if $\zeta^u(a,b)$ is *decreasing* in b for every a), then the tax rate effect (resp. the subsidy effect) loosens the constraint in (7) (and hence condition (6)), *ceteris paribus*. Note also that, by suitably restricting the subset of progressive tax schedules, the tax rate effect can be suppressed. Indeed, if $R_k = \{0\}$ for each k, *i.e.*, if the marginal tax rates are set equal to zero, then the tax rate effect vanishes, and the total effect in (7) reduces to the subsidy effect.

A negative subsidy effect (which loosens the constraint in (6)) implies that the relative sensitivity of income with respect to ability decreases with the introduction of a subsidy, thereby reducing income dispersion. A negative tax rate effect (which loosens the constraint in (6)) implies that the elasticity of income increases with ability; in this case, any given tax rate has a higher impact on the income of higher ability individuals, thereby reducing income inequality. A positive tax rate effect works in the opposite direction, *i.e.*, it exacerbates the income differences between low ability and high ability individuals, but may be offset by a sufficiently large (and negative) subsidy effect.[7]

With this decomposition in mind, it is useful to evaluate condition (6) and its relation to the degree of substitutability of consumption and leisure. In the remainder of this section, we consider the two extreme cases of perfect complements and perfect substitutes. In Section 4, we apply our decomposition of the inequality in (6) to standard parameterized families of preferences.

Consider first the extreme case when consumption and leisure are perfect complements. Because leisure is bounded above by 1, the underlying utility function has an upper bound, with $(1,c^*)$ as the optimal bundle, where c^* is the "ideal" consumption level corresponding to the maximum leisure level (see Figure 1). Individuals of higher ability, whose consumption entails a lower opportunity cost in terms of leisure time, choose higher consumption levels. As a grows large, the optimal consumption level converges to c^*. Hence, at least for a sufficiently large ability, consumption (and hence income) increases at a declining proportion with ability. This implies that, for large enough a, $\zeta(a,0)$ *decreases* with a, implying that the tax rate effect works in the 'wrong' direction.

[7] A subsidy tends to reduce the sensitivity of income with respect to ability (rendering the subsidy effect negative) for relatively low-ability individuals.

To evaluate the subsidy effect, consider how ζ^u changes when a (non-subsidized) a-agent receives a subsidy b. Because the optimal consumption level converges to c^* as a grows large, at least for large a (and, in fact, for every a, as shown below), consumption increases at a declining proportion with the introduction of a subsidy b. Thus, $\zeta^u(a,b)$ is *decreasing* in b, and the second effect loosens condition (6).

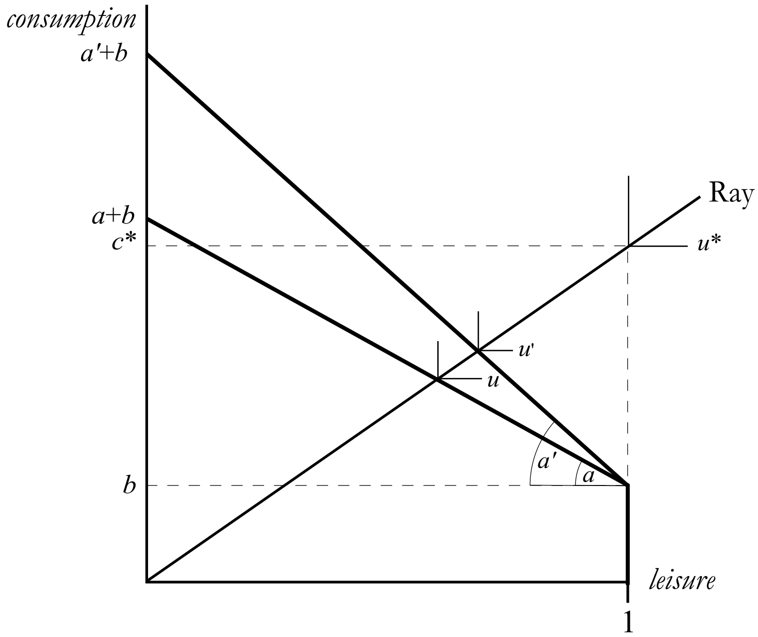

Figure 1: **Perfect complements**. Individual choice for different ability levels and exogenous income b. The ray represents the bundles with the "correct" proportions between leisure and consumption. The maximum utility level u^* is attained for the bundle $(1,c^*)$. For sufficiently high abilities, income (the chosen level of consumption) cannot increase at increasing rates. Hence, the elasticity of income to ability must decrease and not all progressive taxes are income inequality reducing (Theorem 3 and Corollary 1).

Consequently, the two effects have opposite signs, and the net effect is ambiguous. However, since the subsidy effect has the correct sign, the tax rate effect can be eliminated, as per the previous discussion, by setting the tax rates equal to zero. Therefore, any fixed subsidy is inequality reducing. Intuitively, a fixed subsidy compresses the agents' optimal leisure-consumption bundles (and hence the income/consumption levels) along the upper part of the ray in Figure 1, thereby reducing income inequality.

Next, we show that, even when the tax rate effect is positive, a sufficiently large subsidy effect offsets the tax rate effect and renders the associated tax schedule inequality reducing. Formally, if the equation $c = \alpha l$ represents the ray in Figure 1, we have

$$\zeta^u(a,b) = \begin{cases} \frac{a(\alpha-b)}{(\alpha+a)(b+a)} & \text{if } 0 \le b \le \alpha, \\ 0 & \text{if } b > \alpha. \end{cases}$$

First, note that it is always possible to reduce inequality by means of a sufficiently large subsidy; indeed, for $b \ge \alpha$, condition (6) is always satisfied. For the more interesting case

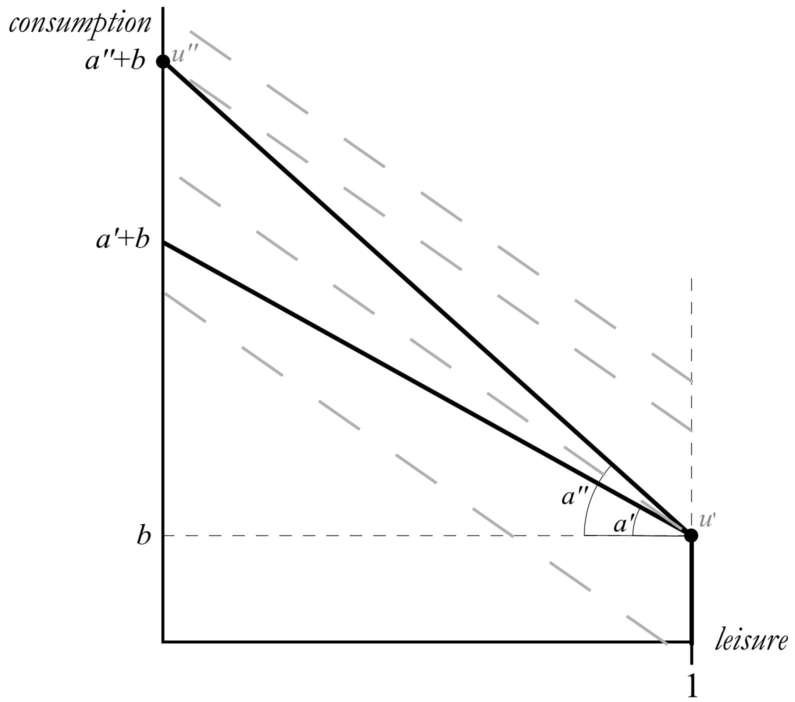

Figure 2: **Perfect substitutes**. Individual choice for different ability levels and exogenous income b. Individuals with a sufficiently high ability, like a'', choose zero leisure; while those with sufficiently low ability, like a', choose zero labor. Hence, the elasticity of income to ability is non-decreasing and all progressive taxes are income inequality reducing (Theorem 3 and Corollary 1).

when $0 \leq b < \alpha$, the tax rate effect is given by

$$\zeta^u((1-r)a,0) - \zeta^u(a,0) = \frac{\alpha r a}{(\alpha + a)(\alpha + (1-r)a)}, \tag{8}$$

while the subsidy effect is

$$\zeta^u((1-r)a,b) - \zeta^u((1-r)a,0) = \frac{-b\alpha - ab(1-r)}{(\alpha + (1-r)a)(b + (1-r)a)}.$$

Observe that the two effects vanish as a grows large. The combined effect can be expressed as

$$\zeta^u((1-r)a,b) - \zeta^u(a,0) = \frac{-b\alpha^2 - 2b\alpha a(1-r) - ba^2(1-r) + \alpha r a^2(1-r)}{(\alpha + a)(\alpha + a(1-r))(b + a(1-r))}.$$

Since condition (6) requires that the combined effect be negative for all $(a,b,r) \in \mathbb{R}_{++} \times \overline{B} \times (\bigcup_k R_k)$, a necessary and sufficient condition for a tax schedule to be inequality reducing is that the subsidy, b, satisfy $b \geq \alpha r$ for all $r \in \bigcup_k R_k$. When this condition is satisfied, the tax rate effect converges to zero (as $a \to \infty$) faster than the subsidy effect.

Next, consider the other extreme, the case when consumption and leisure are treated as perfect substitutes. This case cannot be studied using conditions (6) and (7) directly, since it does not satisfy the underlying strict quasiconcavity assumption. However, it is useful to illustrate, by means of a simple example, the trade-off between the tax rate effect and the subsidy effect in the case of linear consumer preferences.

Under perfect substitutability between consumption and leisure, and in the absence of taxation, high ability individuals choose zero leisure, while low ability individuals choose zero labor (Figure 2), and there is a threshold ability level a such that, for an a-agent, any amount of labor is optimal. A fixed subsidy b with zero marginal tax rates shifts the budget line of each agent up in a parallel fashion, leaving the slope unchanged. Therefore, it does not change the threshold ability level, and the resulting after-tax distribution is a shifted up version of the distribution with no taxes/subsidies. Using directly the definition of Lorenz domination in (3), it is easy to see that the shifted up version is inequality reducing.

Consequently, when consumption and leisure are perfect substitutes, any pure subsidy reduces inequality for any distribution of abilities. However, unlike pure subsidies, proportional tax rates need not be inequality reducing. This implies that the tax rate effect need not have the correct sign. To illustrate, consider a common marginal tax rate τ on all incomes, which reduces the slope of each agent's budget line and hence increases the threshold ability level below which agents do not work. Choose a three-level ability distribution such that, in the absence of taxes/subsidies, the lower ability agent does not work, while the other two agents consume no leisure. The associated income distribution is $(0,1,2)$. Suppose that taxing all incomes at marginal tax rate τ yields the after-tax income distribution $(0,0,2(1-\tau))$ (*i.e.*, the threshold ability level below which agents do not work rises above 1, so that, under the proportional tax, only the most productive agent with $a=2$ consumes no leisure). Using the definition of Lorenz domination in (3), it is easy to see that $(0,0,2(1-\tau))$ does not Lorenz dominate $(0,1,2)$, and so proportional taxes are not inequality reducing. However, adding a sufficiently large subsidy offsets the effect of the tax rate. Indeed, adding a subsidy b to a uniform marginal tax rate τ on all incomes does not change the threshold ability level below which agents do not work and shifts the income distribution $(0,0,2(1-\tau))$ up by the subsidy amount b. The resulting after-tax distribution, $(b,b,b+2(1-\tau))$, Lorenz dominates $(0,1,2)$ whenever $b \geq \frac{2}{3}(1-\tau)$ (recall (3)).

The analysis of the two extreme cases suggests that there is a trade-off between the tax rate effect, which may have the incorrect sign, and the subsidy effect, which reduces inequality. It also hints at forces that are likely to be relevant in intermediate cases: while, for a *given* ability distribution, subsidy levels can be found for which the subsidy effect outweighs the tax rate effect, the existence of a subsidy threshold that yields a negative net effect for *any* given ability distribution depends on the limiting behavior—and the speed of convergence—of the two effects as the ability level grows large.

Next, we turn to an alternative formulation of our main condition in (6) emphasizing the relationship between the elasticity of income with respect to ability, ζ^u, and the elasticity of substitution between consumption and leisure. This relationship is spelled out by means of the following identity:

$$\begin{aligned}\sigma^u(a,b) &:= -\frac{\partial[(1-l^u(a,b))/c^u(a,b)]}{\partial a}\cdot\frac{a}{(1-l^u(a,b))/c^u(a,b)}\\ &= -\frac{\frac{\partial(1-l^u(a,b))}{\partial a}\cdot c^u(a,b)-(1-l^u(a,b))\cdot\frac{\partial c^u(a,b)}{\partial a}}{(c^u(a,b))^2}\cdot\frac{ac^u(a,b)}{1-l^u(a,b)}\\ &= -\frac{\partial(1-l^u(a,b))}{\partial a}\cdot\frac{a}{1-l^u(a,b)}+\frac{\partial c^u(a,b)}{\partial a}\cdot\frac{a}{c^u(a,b)}\\ &= -\epsilon^u(a,b)+\zeta^u(a,b),\end{aligned}$$

where σ^u and ϵ^u denote, respectively, the *elasticity of substitution between consumption and leisure* and the *elasticity of leisure with respect to ability*.[8] In light of this identity, condition (6) can be reformulated as follows:

$$\sigma^u((1-r)a,b)+\epsilon^u((1-r)a,b)\leq\sigma^u(a,0)+\epsilon^u(a,0),\quad \text{for all } (a,b,r)\in\mathbb{R}_{++}\times\overline{B}\times\left(\bigcup_k R_k\right). \tag{9}$$

The next section evaluates the elasticity condition in (6) (via the decomposition in (7)) within two important families of preferences: the constant elasticity of substitution (CES) and the quasi-linear preferences.

4 Applications

This section characterizes the subclasses of progressive taxes that are inequality reducing for two commonly used families of income-leisure preferences: the constant elasticity of substitution (CES) and the quasi-linear preferences. The CES utility function (often in its Cobb-Douglas version) is very common in the literature on life-cycle models (Heckman and MaCurdy, 1982; French, 2005; Blundell et al., 2016), while static models with fixed costs traditionally work with quasi-linear preferences (Cogan, 1981).[9] These utilities are also dominant in surveys and textbooks on labor supply and fiscal policy (Pencavel, 1986; Killingsworth and Heckman, 1986; Auerbach and Kotlikoff, 1987; Keane, 2011; Blundell et al., 2016).

For each case, we first specify the family of utility functions and calculate their elasticities. We then characterize, as an application of Theorem 4, the utility parameters for which there

[8] Here we use the *gross* substitution definition adopted in Mas-Colell et al. (1995, p. 97), associated to the consumer problem for an a-agent whose non-wage income is b:

$$\begin{aligned}&\max_{((1-l),c)} u(1-l,c)\\ &\text{s.t.}\\ &a(1-l)+c=a+b.\end{aligned}$$

There are, however, several alternative formulations for the notion of elasticity of substitution. See, *e.g.*, Stern (2011).

[9] Static models tend to specify a labor supply function directly, which makes it difficult to identify a widely used utility function (Keane, 2011, page 966).

exist classes of *iir* tax schedules and develop intuition for our findings. Formal proofs are relegated to the Appendix.

4.1 Constant elasticity of substitution (CES) utility

Consider the well-known CES utility function

$$u(c,l) := \begin{cases} c^\gamma + \beta(1-l)^\gamma & \text{if } \gamma \in (0,1), \\ -c^\gamma - \beta(1-l)^\gamma & \text{if } \gamma < 0, \end{cases} \tag{10}$$

where $\frac{1}{1-\gamma}$ determines the elasticity of substitution between consumption and leisure, and β is a positive constant.[10] One has

$$l^{CES}(a,b) = \begin{cases} \dfrac{\left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} - b}{a + \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}}} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise,} \end{cases}$$

$$al^{CES}(a,b) + b = \begin{cases} \dfrac{\left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}}(a+b)}{a + \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}}} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ b & \text{otherwise,} \end{cases}$$

$$\zeta^{CES}(a,0) = \begin{cases} \dfrac{(1-\gamma)a^{\frac{1}{1-\gamma}} + a\beta^{\frac{1}{1-\gamma}} + b\gamma\beta^{\frac{1}{1-\gamma}}}{(1-\gamma)(a+b)\left(a^{\frac{\gamma}{1-\gamma}} + \beta^{\frac{1}{1-\gamma}}\right)} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\zeta^{CES}((1-r)a,b) = \begin{cases} \dfrac{(1-\gamma)((1-r)a)^{\frac{1}{1-\gamma}} + (1-r)a\beta^{\frac{1}{1-\gamma}} + b\gamma\beta^{\frac{1}{1-\gamma}}}{(1-\gamma)((1-r)a+b)\left(((1-r)a)^{\frac{\gamma}{1-\gamma}} + \beta^{\frac{1}{1-\gamma}}\right)} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 1. *Let u be the CES utility function given in* (10). *Suppose that $R \subseteq [0,1)$ and $\sup R < 1$. Then there exists $\underline{b} \geq 0$ such that $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir}$ if and only if $\gamma \in [\frac{1}{2},1)$, where $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$.*

As γ tends to 1, the CES utility function is approximately linear, and consumption and leisure become perfect substitutes. As γ tends to $-\infty$, the indifference curves are approximately "right angles," *i.e.*, consumption and leisure become perfect complements.

Proposition 1 states that when the elasticity of substitution is large enough, *i.e.*, when consumption and leisure substitute "sufficiently well" for each other, there are (nonempty)

[10] For the CES utility function, we have

$$\sigma^u(a,b) = \begin{cases} \frac{1}{1-\gamma} & \text{if } \left(\frac{a}{\beta}\right)^{\frac{1}{1-\gamma}} \geq b, \\ 0 & \text{otherwise.} \end{cases}$$

subclasses of progressive tax schedules whose members are inequality reducing. Specifically, in this case it suffices to choose a sufficiently large subsidy for a progressive tax schedule to be inequality reducing.

For the CES utility function, the decomposition in (7) gives a tax rate effect of

$$\zeta^{CES}((1-r)a,0)-\zeta^{CES}(a,0)=\frac{\gamma\left[a^{\frac{\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\left(1-(1-r)^{\frac{\gamma}{1-\gamma}}\right)\right]}{(1-\gamma)\left[\left(a^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\left(((1-r)a)^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\right]}\geq 0 \tag{11}$$

and a subsidy effect of

$$\zeta^{CES}((1-r)a,b)-\zeta^{CES}((1-r)a,0)=\frac{-b}{(1-r)a+b}\leq 0. \tag{12}$$

Because the tax rate effect is positive, it makes it harder for the inequality in (6) to hold, while the subsidy effect, being negative, loosens the constraint in (6). Recall that the members of $\mathcal{T}_{prog}(B^*,R)$ are inequality reducing if and only if the sum of the two effects is negative for all $(a,b,r)\in\mathbb{R}_{++}\times B^*\times R$. Consequently, we need the subsidy effect to offset the tax rate effect for all $(a,b,r)\in\mathbb{R}_{++}\times B^*\times R$. Note that while the subsidy effect is independent of the elasticity of substitution, γ, this parameter influences the tax rate effect. In particular, if the elasticity of substitution, γ, is low enough, *i.e.*, less than zero, then the tax rate effect explodes as a grows large: as a increases, consumption increases at a declining proportion, thereby reducing the elasticity of income with respect to ability, and the magnitude of this reduction increases exponentially with a.[11] Since the subsidy effect vanishes as a grows large, and, for $\gamma<0$, the tax rate effect explodes as a grows large, it is clear that, when $\gamma<0$, not all members of $\mathcal{T}_{prog}(B^*,R)$ are inequality reducing.

For relatively large values of the elasticity of substitution, *i.e.*, for $\gamma>0$, the tax rate effect vanishes as a grows large. Intuitively, for large γ, the CES utility is close to the linear case, *i.e.*, the case when consumption and leisure are perfect substitutes, and, as discussed in Section 3, in this case, high ability agents, whose consumption entails a lower opportunity cost in terms of leisure, tend to minimize their consumption of leisure, devoting most of their time to production and consumption, which leads to small changes in the elasticity of income with respect to ability, as a increases (for large enough a). While, for $\gamma>0$, the tax rate effect vanishes as a grows large, since we need the subsidy effect (which also vanishes as a grows large) to offset the tax rate effect, the subsidy effect cannot go to zero faster than the tax rate effect. In the proof of Proposition 1, we show that, for $\gamma\in(0,\frac{1}{2})$, the subsidy effect goes to zero faster than the tax rate effect, while for $\gamma\in[\frac{1}{2},1)$, the opposite is true. This explains the restrictions on γ in Proposition 1.

Alternatively, one can use the characterization of inequality reducing tax systems given in (9). For the CES family of utility functions, the elasticity of substitution, σ^u, is constant

[11]This contrasts with the case of perfect complements, discussed in Section 3, where the tax rate effect in (8) vanishes as a grows large.

(in fact, equal to $\frac{1}{1-\gamma}$ if $(a/\beta)^{\frac{1}{1-\gamma}} \geq b$, zero otherwise), and so condition (9) reduces to

$$\epsilon^u((1-r)a, b) \leq \epsilon^u(a, 0), \quad \text{for all } (a, b, r) \in \mathbb{R}_{++} \times \overline{B} \times \left(\bigcup_k R_k\right),$$

where ϵ^u is the elasticity of leisure with respect to ability, and a decomposition analogous to that in (7) can be applied directly to the above condition. Because

$$\epsilon^{CES}(a, b) = \frac{a}{a+b} - \frac{\beta^{\frac{1}{1-\gamma}} + \left(\frac{1}{1-\gamma}\right) a^{\frac{\gamma}{1-\gamma}}}{\beta^{\frac{1}{1-\gamma}} + a^{\frac{\gamma}{1-\gamma}}},$$

the tax rate effect, expressed in terms of the elasticity of leisure with respect to ability, is given by

$$\epsilon^{CES}((1-r)a, 0) - \epsilon^{CES}(a, 0) = \frac{\beta^{\frac{1}{1-\gamma}} + \left(\frac{1}{1-\gamma}\right) a^{\frac{\gamma}{1-\gamma}}}{\beta^{\frac{1}{1-\gamma}} + a^{\frac{\gamma}{1-\gamma}}} - \frac{\beta^{\frac{1}{1-\gamma}} + \left(\frac{1}{1-\gamma}\right) ((1-r)a)^{\frac{\gamma}{1-\gamma}}}{\beta^{\frac{1}{1-\gamma}} + ((1-r)a)^{\frac{\gamma}{1-\gamma}}},$$

while the subsidy effect is

$$\epsilon^{CES}((1-r)a, b) - \epsilon^{CES}((1-r)a, 0) = \frac{(1-r)a}{(1-r)a+b} - 1 \leq 0,$$

and so the latter effect has the "correct" sign. Because

$$\frac{\partial}{\partial a}\left(\frac{\beta^{\frac{1}{1-\gamma}} + \left(\frac{1}{1-\gamma}\right) a^{\frac{\gamma}{1-\gamma}}}{\beta^{\frac{1}{1-\gamma}} + a^{\frac{\gamma}{1-\gamma}}}\right) = \frac{\gamma^2 \beta^{\frac{1}{1-\gamma}} a^{\frac{2\gamma-1}{1-\gamma}}}{(1-\gamma)^2 \left(\beta^{\frac{1}{1-\gamma}} + a^{\frac{\gamma}{1-\gamma}}\right)^2} \geq 0,$$

it follows that the tax rate effect has the "wrong" sign. Both the tax rate effect and the subsidy effect vanish as a tends to infinity. Consequently, in order for the net effect to be negative for all a, the rate of convergence for the subsidy effect needs to be lower than that for the tax rate effect. As per the previous discussion, this occurs when γ ranges between $\frac{1}{2}$ and 1.

Remark 1. When $\gamma \to 0$, the CES utility function converges to the Cobb-Douglas utility function, and, in this limiting case, marginal-rate progressive and only marginal-rate progressive tax schedules are *iir*, *i.e.*, $\mathcal{T}_{u\text{-}iir} = \mathcal{T}_{prog}$.[12] The peculiarity of the Cobb-Douglas utility function is that it has no tax rate effect. Indeed, its wage elasticity of income is constant for $b = 0$: $\zeta^{CD}(a, b) = a/(a+b)$; therefore, the tax rate effect vanishes ($\zeta^{CD}((1-r)a, 0) - \zeta^{CD}(a, 0) = 0$), and the sum of the subsidy and the tax rate effects reduces to a negative subsidy effect, (12), which always satisfies condition (6).

[12]This was established in Carbonell-Nicolau and Llavador (2018, Remark 3).

Remark 2. As pointed out by several authors (see, e.g., Slemrod, 1998; Slemrod and Kopczuk, 2002; Saez et al., 2012), behavioral elasticities also play an important role in the optimal taxation literature. Roughly speaking, larger elasticities of taxable income with respect to the tax rate imply that less progressive tax systems are optimal. In our model, an income tax causes a deadweight loss as individuals substitute away from consumption to leisure, and so the deadweight loss per dollar of revenue depends on the elasticity of substitution between consumption and leisure, which is directly related to the compensated elasticity of income with respect to the tax rate. Section 5 provides a discussion on the methodological differences between our analysis and the literature on optimal taxation.

4.2 Quasi-linear utility

Consider the quasi-linear utility function

$$u^{QL}(c,l) := c + \frac{\beta(1-l)^{1-\delta}}{1-\delta}, \tag{13}$$

where $\beta > 0$ and $\delta > 0$, with $\delta \neq 1$.[13] One has

$$l^{QL}(a,b) = \begin{cases} 1 - \left(\frac{\beta}{a}\right)^{1/\delta} & \text{if } a \geq \beta \\ 0 & \text{if } a < \beta, \end{cases} \tag{14}$$

and

$$a l^{QL}(a,b) + b = \begin{cases} a + b - a\left(\frac{\beta}{a}\right)^{1/\delta} & \text{if } a \geq \beta \\ b & \text{if } a < \beta. \end{cases}$$

Define $\theta(a) := \left(\frac{\beta}{a}\right)^{1/\delta}$. Note that $\theta(a) < 1$ for $a > \beta$, and $\theta((1-r)a) - \theta(a) > 0$ for $r \in (0,1)$. Compute

$$\zeta^{QL}(a,b) = \begin{cases} \frac{(1-\delta)\theta(a)+\delta}{\delta\left(\frac{b}{a}-\theta(a)+1\right)} & \text{if } a \geq \beta, \\ 0 & \text{otherwise.} \end{cases}$$

The decomposition in (7) shows that the subsidy and tax rate effects move in opposite directions.

For $a > (1-r)a \geq \beta$, the subsidy effect is negative:[14]

$$\zeta^{QL}((1-r)a,b) - \zeta^{QL}((1-r)a,0) = \frac{-b\,(\delta + (1-\delta)\theta((1-r)a))}{\delta(1-\theta((1-r)a))((1-r)a(1-\theta((1-r)a)) + b)} \leq 0 \tag{15}$$

(recall that $\theta((1-r)a) \leq 1$).

Since the labor supply in (14) is independent of the subsidy b, introducing a subsidy b leaves the distribution of labor income intact, and so the subsidy results in a shift of all labor incomes up by the magnitude of the subsidy, which reduces inequality, and the effect of the

[13] The $MRS(c,l)$ tends to $+\infty$ as $l \to 1^-$ (recall the Inada condition in (1)) if and only if $\delta > 0$.

[14] The case when $(1-r)a < \beta$ is trivial, since the associated elasticity is zero.

income shifts on inequality tend to be smaller when the labor incomes being shifted up are large. This is consistent with a negative subsidy effect which tends to zero as a converges to infinity. In fact, it is easy to verify that the ratio in (15) vanishes as a tends to infinity.

The tax rate effect is positive:

$$\zeta^{QL}((1-r)a,0)-\zeta^{QL}(a,0)=\frac{\theta((1-r)a)-\theta(a)}{\delta(1-\theta(a))(1-\theta((1-r)a))}\geq 0. \tag{16}$$

For the quasi-linear utility function, an increase in a leads to an increase in labor (via a pure substitution effect). As a increases, the marginal rate of substitution of leisure for consumption (*i.e.*, the amount of consumption that an agent must gain in order to give up one unit of leisure) at the optimal consumption bundle increases, and the relative increase in labor and income with respect ability diminishes with a, implying a positive tax rate effect that vanishes as a grows large.

Whether the subsidy or the tax rate effect dominates depends on the elasticity of leisure with respect to ability, which is given by (minus) the inverse of the parameter δ, so that leisure is elastic for $\delta < 1$ and inelastic for $\delta > 1$.[15] Proposition 2 states that when the demand of leisure is relatively elastic (*i.e.*, $\delta \in (0,1)$), any progressive tax schedule with a sufficiently large subsidy is inequality reducing.

The parameter δ is also related to the elasticity of substitution between consumption and leisure. As $\delta \to 0$, consumption and leisure become perfect substitutes. As δ grows large, u^{QL} converges to the Leontief utility function that characterizes the case of perfect complements. The elasticity of substitution between consumption and leisure is given by

$$\sigma^{QL}(a,b)=-\epsilon^{QL}(a,b)+\zeta^{QL}(a,b)=\begin{cases}\frac{1}{\delta}\left(1+\frac{\theta(a)+\delta(1-\theta(a))}{\frac{b}{a}+1-\theta(a)}\right) & \text{if } a\geq\beta,\\ 0 & \text{otherwise,}\end{cases}$$

where, recall, $\theta(a)=(\beta/a)^{1/\delta}$. Consequently, σ^{QL} converges to zero (the elasticity of substitution for the extreme case of perfect complements) as δ tends to infinity, and σ^{QL} converges to infinity (the elasticity of substitution for the extreme case of perfect substitutes) as δ approaches zero.

Proposition 2 asserts that progressive tax schedules with sufficiently high subsidies are inequality reducing provided that the elasticity of substitution between consumption and leisure is relatively high (*i.e.*, when $\delta \in (0,1)$).

Proposition 2. *Let u be the quasi-linear utility function given in* (13)*. Suppose that $R \subseteq [0,1)$ and $\sup R < 1$. Then there exists $\underline{b} \geq 0$ such that $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir}$ if and only if $\delta \in (0,1)$, where $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$.*

[15] It is easy to see that the demand for leisure is $(\beta/a)^{1/\delta}$ (for $a \geq \beta$), and hence the elasticity of leisure with respect to ability is $-1/\delta$.

As shown in the proof of Proposition 2 (see Appendix D), the subsidy effect dominates if and only if

$$b \geq \frac{a\beta^{1/\delta}\left(1-(1-r)^{1/\delta}\right)(1-r)^{\frac{\delta-1}{\delta}}}{\delta a^{1/\delta}+(1-\delta)\beta^{1/\delta}}.$$

Because this inequality must hold for all a, the lower bound on the subsidy, $\underline{b}$, is finite if and only if $\delta \leq 1$. (Otherwise the right-hand side goes to infinity with a.)

Since both the subsidy and the tax rate effects (equations (15) and (16), respectively) vanish as the ability level tends to infinity, and since the two effects have opposite signs, if the subsidy effect goes to zero faster than the tax rate effect—as is the case here for $\delta > 1$—then, for large abilities, the tax rate effect outweighs the subsidy effect and the associated tax schedule is not inequality reducing.

5 Concluding remarks

This paper characterizes consumer preferences for which various subclasses of progressive tax schedules are inequality reducing, and provides a decomposition of the distributional effects of income tax systems into a tax rate effect and a subsidy effect, each capturing different aspects of the transition between before-tax and after-tax income distributions. The framework considered here, which subsumes that in Carbonell-Nicolau and Llavador (2018), allows one to expand the set of consumer preferences by suitably restricting the set of progressive taxes. This is illustrated in Section 4 for two standard families of utility functions: the CES and the quasi-linear utility functions.

We conclude with two comments. The first comment has to do with the methodological differences between our analysis and the literature on optimal income taxation. A first difference lies in the cardinal nature of inequality measures, which contrasts with the standard ordinal representation of preferences in consumer theory. As pointed out in Carbonell-Nicolau and Llavador (2018), alternative inequality metrics based on *welfare*, rather than *income*, pose problems in that the Lorenz ordering is not generally invariant to strictly increasing transformations of utility vectors. A second difference stems from the requirement that tax systems reduce inequality *regardless* of the distribution of abilities they are applied to. This requirement is at odds with the approach taken in the optimal taxation literature, which characterizes optimal tax structures for a *given* ability distribution. Consequently, our results are not directly comparable with those from the optimal taxation literature.

The second comment concerns our assumptions underlying consumer behavior. In adopting the framework of the standard Mirrless model, we view taxpayers as purely self-interested, tax-compliant agents. There is, however, a sizable literature on social norms, broadly defined, emphasizing aspects of preferences beyond the mere selfish pursuit of private consumption, such as relative consumption, 'social status' effects, inequality aversion, loss aversion, warm-glow and stigma effects of charitable donations, and the importance of non-pecuniary factors in voluntary tax compliance. Part of this literature assesses the empirical relevance of these behavioral assumptions (see, *e.g.*, Alesina et al., 2011; Bowles and Park, 2005; Fehr and Schmidt, 2003), while other papers study the normative and positive implications for tax systems. For example, Ireland (2001), Abel (2005), and Aronsson et al. (2016) characterize optimal income taxes in the presence of status effects and benchmark levels of consumption. In general, their results are sensitive to the distribution of abilities, as in the literature on optimal income taxation. Alesina et al. (2011) review positive redistributive theories—whereby taxes are collectively chosen via a voting mechanism—under inequality aversion. These theories are limited in that they severely restrict the set of allowable tax schedules to obtain equilibrium existence, and are generally not able to handle nonlinear taxes.

The literature on tax compliance has more direct implications for our analysis. This literature shows that concerns of fairness and reciprocity play an important role in voluntary tax compliance or *tax moral* (Luttmer and Singhal, 2014; Kleven, 2014). If tax evasion is likely affected by the perceived fairness of the tax system (Alm et al., 1995), one should expect inequality reducing tax schedules, insofar they are perceived as fairer, to induce more tax compliance.

The extension of our analysis to a broader set of social norms, along the lines of the cited literature, constitutes a natural avenue for future research.

Appendix

In this appendix, we present the proofs of Theorem 2, Theorem 3, Proposition 1, and Proposition 2. Each proof is preceded by a restatement of its corresponding theorem for the convenience of the reader.

The proofs of Theorem 2 and that of Theorem 3 adapt arguments from the proofs of Theorem 2 and Theorem 3 in Carbonell-Nicolau and Llavador (2018).

The following two lemmas, whose proofs can be found in Carbonell-Nicolau and Llavador (2018) (see their Lemma 2 and Lemma 3), are instrumental in the proofs of Theorem 2 and Theorem 3.

Lemma 1. *Given $u \in \mathscr{U}$, $(c,y) \in \mathbb{R}^2_{++}$, and $q \in (0,+\infty)$, there exists an $a > y$ such that $\eta^a(c,y) = q$.*

Lemma 2. *Given $u \in \mathscr{U}^*$, a tax schedule $T \in \mathscr{T}$ is u-iir if and only if for any ability distribution $\boldsymbol{a} \in \mathscr{A}$ and for any pre-tax and post-tax income functions y^u and x^u,*

$$\frac{x^u(a_i,T)}{y^u(a_i,0)} \geq \frac{x^u(a_{i+1},T)}{y^u(a_{i+1},0)} \qquad \forall i \in \{1,\ldots,n-1\} : y^u(a_i,0) > 0. \tag{17}$$

A Proof of Theorem 2

Theorem 2. *Given $u \in \mathscr{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$,*

$$[\mathscr{T}_{u\text{-}iir} \subseteq \mathscr{T}_{prog}] \quad \textit{and} \quad \left[\mathscr{T}_{prog}(\overline{B},(R_k)) \subseteq \mathscr{T}_{u\text{-}iir} \Longleftrightarrow \mathscr{T}_{lin}\left(\overline{B},\bigcup_k R_k\right) \subseteq \mathscr{T}_{u\text{-}iir}\right].$$

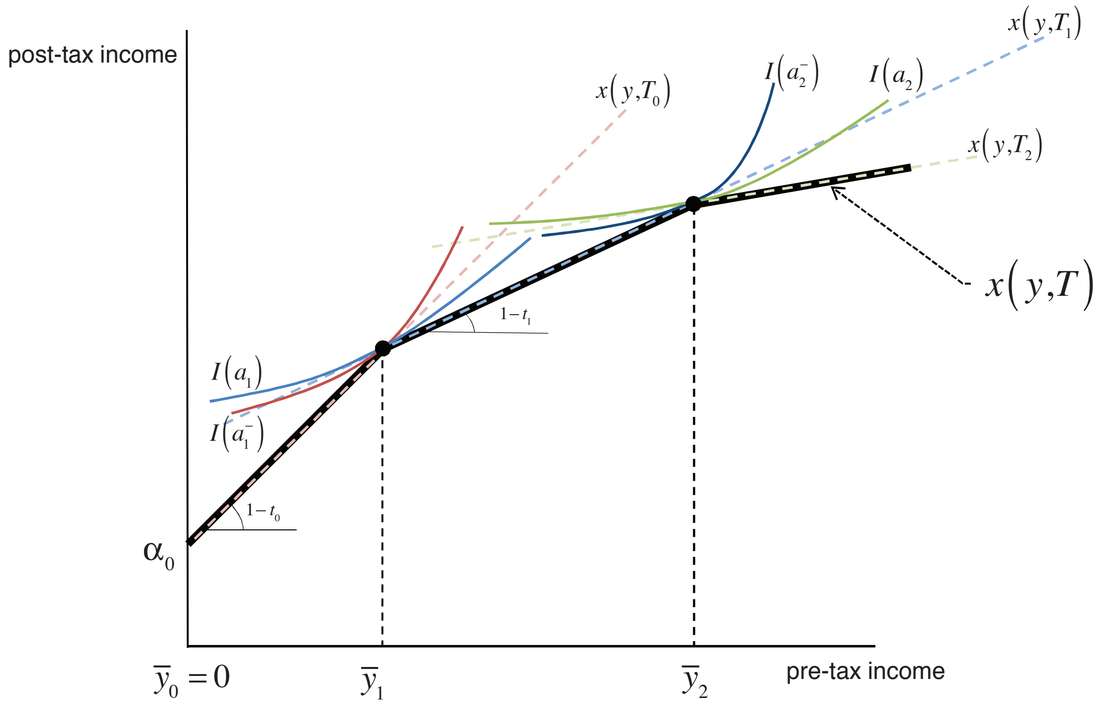

Figure 3: Figure for Theorem 2

Proof. The first containment follows immediately from Theorem 1.

Suppose that $u \in \mathscr{U}^*$ and $(B,(R_k)) \in \mathfrak{D}$.

Since $\mathscr{T}_{lin}(\overline{B}, \bigcup_k R_k) \subseteq \mathscr{T}_{prog}(\overline{B},(R_k))$, the 'only if' part of the equivalence is obvious.

Assume now that the members of $\mathscr{T}_{lin}(\overline{B}, \bigcup_k R_k) \subseteq \mathscr{T}_{u\text{-}iir}$. We need to prove that $\mathscr{T}_{prog}(\overline{B},(R_k)) \subseteq \mathscr{T}_{u\text{-}iir}$. By Lemma 2, this is equivalent to showing that that condition (17) holds for any $T \in \mathscr{T}_{prog}(\overline{B},(R_k))$, for any ability distribution $\boldsymbol{a} \in \mathscr{A}$, and for any pre-tax and post-tax income functions y^u and x^u.

Take $T = (\alpha_0, \boldsymbol{t}, \overline{\boldsymbol{y}}) \in \mathscr{T}_{prog}(\overline{B},(R_k))$ and, for each income threshold $\overline{y}_k$ of T, define the linear tax schedule $T_k(y) := t_k y - \alpha_k$ with $\alpha_0 \in \overline{B}$, $t_k \in R_k$ for $k \in \{0,...,K\}$, and $\alpha_k := \alpha_{k-1} + (t_k - t_{k-1})\overline{y}_k$ for $k \in \{1,\ldots,K\}$.

Pre-tax and post-tax income functions, y^u and x^u, are uniquely defined, since preferences are strictly quasiconcave and the tax function T is convex. For $k \in \{1,...,K\}$, define the abilities a_k^- and a_k such that

$$a_k^- := \min\{a : y^u(a, T_{k-1}) = \overline{y}_k^u\} \quad \text{and} \quad a_k := \max\{a : y(a, T_k) = \overline{y}_k\}$$

(see Figure 3). Lemma 1 guarantees that a_k^- and a_k exist and are well-defined for all $k \in \{1,\ldots,K\}$.

Furthermore, since T is marginal-rate progressive (and hence $t_{k-1} < t_k$ for all $k \in \{1,\ldots,K\}$), agent monotonicity (Definition 4) implies that $a_k^- \le a_k < a_{k+1}^-$.

Next, define the following family of sets covering $(0,+\infty)$:

$$\mathfrak{A} := \left\{(0,a_1^-], \{[a_k^-, a_k]\}_{k=1}^{K}, \{[a_k, a_{k+1}^-]\}_{k=1}^{K-1}, [a_K, +\infty)\right\}.$$

We first show that condition (17) is satisfied for ability distributions contained in each element of the family $\mathfrak{A}$.

(*i*) Consider first the interval $(0,a_1^-]$. Observe that $y^u(a,T) = y^u(a,T_0)$ for all $a \le a_1^-$. Because T_0 is a linear tax, it is *u-iir*, and so Lemma 2 gives

$$\frac{x^u(a,T)}{y^u(a,0)} = \frac{x^u(a,T_0)}{y^u(a,0)} \ge \frac{x^u(a',T_0)}{y^u(a',0)} = \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a \le a' \le a_1^-. \tag{18}$$

(*ii*) For $[a_K,+\infty)$, a symmetric argument shows that

$$\frac{x^u(a,T)}{y^u(a,0)} \ge \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a_K \le a \le a'. \tag{19}$$

(*iii*) Now consider the interval $[a_k^-, a_k]$ for $k \in \{1,\dots,K\}$. Observe that

$$y^u(a_k,T) = y^u(a_k,T_k) = \overline{y}_k = y^u(a_k^-, T_{k-1}) = y^u(a_k^-,T).$$

Because the map $a \mapsto y^u(a,T)$ is monotone (Mirrlees, 1971, Theorem 1), $y^u(a,T) = \overline{y}_k$ for all $a \in [a_k^-, a_k]$, and $y^u(a',0) \ge y^u(a,0)$ for all $a_k^- \le a \le a' \le a_k$. Therefore,

$$\frac{x^u(a,T)}{y^u(a,0)} = \frac{\overline{y}_k - T(\overline{y}_k)}{y^u(a,0)} \ge \frac{\overline{y}_k - T(\overline{y}_k)}{y^u(a',0)} = \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a,a' \in [a_k^-, a_k],\ a \le a'. \tag{20}$$

(*iv*) Finally, consider the interval $[a_k, a_{k+1}^-]$ for $k \in \{1,\dots,K-1\}$. By construction, we have $y^u(a,T) = y^u(a,T_k)$ for all $a \in [a_k, a_{k+1}^-]$. Therefore, since T_k is a linear tax in $\mathscr{T}_{lin}((\overline{B}, \bigcup_k R_k)$, and hence *u-iir*, Lemma 2 gives

$$\frac{x^u(a,T)}{y^u(a,0)} = \frac{x^u(a,T_k)}{y^u(a,0)} \ge \frac{x^u(a',T_k)}{y^u(a',0)} = \frac{x^u(a',T)}{y^u(a',0)} \qquad \forall a,a' \in [a_k, a_{k+1}^-],\ a \le a'. \tag{21}$$

Combining equations (18)-(21) we obtain (17) for every $\boldsymbol{a} \in \mathscr{A}$. ■

B Proof of Theorem 3

Theorem 3. *For $u \in \mathscr{U}^*$, the members of $\mathscr{T}_{lin}(B,R)$ are u-iir if and only if $u \in \mathscr{U}(B,R)$.*

Proof. Given $u \in \mathscr{U}^*$, $B \subseteq \mathbb{R}_+$, and $R \subseteq [0,1)$, the members $T(y) = -b + ry$ of $\mathscr{T}_{lin}(B,R)$ are *u-iir* if and only if the map

$$a \mapsto \frac{x^u(a,T)}{y^u(a,0)} = \frac{a(1-r)l^u(a,T)+b}{al^u(a,0)} = \frac{a(1-r)l^u((1-r)a,b)+b}{al^u(a,0)} \tag{22}$$

defined on $\mathbb{R}_{++}$ is nonincreasing for every $(b,r) \in B \times R$ (Lemma 2). Equivalently, the members of $\mathcal{T}_{lin}(B,R)$ are *u-iir* if and only if

$$\frac{(1-r)\left((1-r)a'\frac{\partial l^u((1-r)a',b)}{\partial a} + l^u((1-r)a',b)\right)a'l^u(a',0)}{(a'l^u(a',0))^2} - \frac{((1-r)a'l^u((1-r)a',b)+b)\left(a'\frac{\partial l^u(a',0)}{\partial a} + l^u(a',0)\right)}{(a'l^u(a',0))^2} \leq 0 \quad (23)$$

for every $(a',b,r) \in \mathbb{R}_{++} \times B \times R$.[16] Since the above inequality can be expressed as

$$\frac{(1-r)a'\left((1-r)a'\frac{\partial l^u((1-r)a',b)}{\partial a} + l^u((1-r)a',b)\right)}{(1-r)a'l^u((1-r)a',b)+b} \leq \frac{a'\left(a'\frac{\partial l^u(a',0)}{\partial a} + l^u(a',0)\right)}{a'l^u(a',0)},$$

or, equivalently, as

$$\zeta^u((1-r)a',b) \leq \zeta^u(a',0), \quad (24)$$

we see that the members of $\mathcal{T}_{lin}(B,R)$ are *u-iir* if and only if (24) holds for every $(a',b,r) \in \mathbb{R}_{++} \times B \times R$. Consequently, for $u \in \mathcal{U}^*$, the members of $\mathcal{T}_{lin}(B,R)$ are *u-iir* if and only if $u \in \mathcal{U}(B,R)$. ∎

C Proof of Proposition 1

Proposition 1. *Let u be the CES utility function given in* (10)*. Suppose that* $R \subseteq [0,1)$ *and* $\sup R < 1$*. Then there exists* $\underline{b} \geq 0$ *such that* $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ *if and only if* $\gamma \in [\frac{1}{2},1)$*, where* $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$*.*

Proof. Since $u \in \mathcal{U}^*$, given $\underline{b} \geq 0$, $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$, and $R \subseteq [0,1)$, Theorem 4 gives $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $u \in \mathcal{U}(B^*,R)$.

Given $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$, if $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} < b$, then $(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} < b$, implying $\zeta^{CES}(a,0) = 0 \geq 0 = \zeta^{CES}((1-r)a,b)$. If $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq b > (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}}$, then $\zeta^{CES}(a,0) \geq 0 = \zeta^{CES}((1-r)a,b)$. Let $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$, and split the total effect into the subsidy and tax-rate effects:

$$\zeta^{CES}((1-r)a,b) - \zeta^{CES}(a,0) = \left[\zeta^{CES}((1-r)a,b) - \zeta^{CES}((1-r)a,0)\right] + \left[\zeta^{CES}((1-r)a,0) - \zeta^{CES}(a,0)\right].$$

After some manipulation, it follows that the subsidy effect is non-positive:

$$\zeta^{CES}((1-r)a,b) - \zeta^{CES}((1-r)a,0) = \frac{-b}{(1-r)a+b} \leq 0; \quad (25)$$

[16]More precisely, the map defined in (22) is nonincreasing for every $(b,r) \in B \times R$ if and only if for every $(b,r) \in B \times R$, (23) holds for all but perhaps one $a' > 0$.

while the tax-rate effect is non-negative:

$$\zeta^{CES}((1-r)a,0)-\zeta^{CES}(a,0)=\frac{\gamma\left(a^{\frac{\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\left(1-(1-r)^{\frac{\gamma}{1-\gamma}}\right)\right)}{(1-\gamma)\left(\left(a^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\left(((1-r)a)^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\right)}\geq 0 \tag{26}$$

Therefore, the total effect is negative if and only if

$$\zeta^{CES}((1-r)a,b)-\zeta^{CES}(a,0)=\frac{\gamma\left(a^{\frac{\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\left(1-(1-r)^{\frac{\gamma}{1-\gamma}}\right)\right)}{(1-\gamma)\left(\left(a^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\left(((1-r)a)^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\right)}-\frac{b}{a+b}\leq 0$$

The previous inequality is equivalent to

$$\frac{\gamma\left(a^{\frac{1}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\left(1-(1-r)^{\frac{\gamma}{1-\gamma}}\right)\right)}{(1-\gamma)\left(\left(a^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\left(((1-r)a)^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\right)}\leq b\left(1-\frac{\gamma\left(a^{\frac{\gamma}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\left(1-(1-r)^{\frac{\gamma}{1-\gamma}}\right)\right)}{(1-\gamma)\left(\left(a^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\left(((1-r)a)^{\frac{\gamma}{1-\gamma}}+\beta^{\frac{1}{1-\gamma}}\right)\right)}\right)$$

Arranging terms gives

$$\begin{aligned}
&a^{\frac{1}{1-\gamma}}(a(1-r))^{\frac{1}{1-\gamma}}\Big[(a(1-r))^{-1}((1-\gamma)a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}})(a(1-r)\beta^{\frac{1}{1-\gamma}}+(a(1-r))^{\frac{1}{1-\gamma}})\\
&\qquad -\gamma\beta^{\frac{1}{1-\gamma}}(a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}})\Big]b\\
&\geq a^{\frac{1}{1-\gamma}}(a(1-r))^{\frac{1}{1-\gamma}}\Big[(a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}})(a(1-r)\beta^{\frac{1}{1-\gamma}}+(1-\gamma)(a(1-r))^{\frac{1}{1-\gamma}})\\
&\qquad -((1-\gamma)a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}})(a(1-r)\beta^{\frac{1}{1-\gamma}}+(a(1-r))^{\frac{1}{1-\gamma}})\Big].
\end{aligned}$$

This simplifies to

$$\begin{aligned}
&\left[\left((1-\gamma)a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}}\right)\left(\beta^{\frac{1}{1-\gamma}}+(a(1-r))^{\frac{\gamma}{1-\gamma}}\right)-\gamma\beta^{\frac{1}{1-\gamma}}\left(a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}}\right)\right]b\\
&\qquad\geq a^{1+\frac{1}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\gamma\left(1-r-(1-r)^{\frac{1}{1-\gamma}}\right).
\end{aligned} \tag{27}$$

We claim that if $\sup R<1$ there exists $\underline{b}\geq 0$ such that $\mathcal{T}_{prog}(B^*,R)\subseteq\mathcal{T}_{u\text{-}iir}\subseteq\mathcal{T}_{prog}$ if and only if $\gamma\in[\frac{1}{2},1)$. To see this, it suffices to show that (i) for $\gamma<\frac{1}{2}$ and $\underline{b}\geq 0$, there exists $(a,b,r)\in\mathbb{R}_{++}\times B^*\times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}}\geq(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}}\geq b$ such that (27) does not hold, and (ii) for $\gamma\in[\frac{1}{2},1)$, there exists $\underline{b}\geq 0$ such that for $(a,b,r)\in\mathbb{R}_{++}\times B^*\times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}}\geq(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}}\geq b$, (27) holds.

The bracketed term on the left-hand side of (27) is positive for a large enough (indeed, for any a if $\gamma<0$ or $\gamma=1/2$). Therefore, for a large enough, (27) is equivalent to

$$b\geq\frac{a^{1+\frac{1}{1-\gamma}}\beta^{\frac{1}{1-\gamma}}\gamma\left(1-r-(1-r)^{\frac{1}{1-\gamma}}\right)}{\left((1-\gamma)a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}}\right)\left(\beta^{\frac{1}{1-\gamma}}+(a(1-r))^{\frac{\gamma}{1-\gamma}}\right)-\gamma\beta^{\frac{1}{1-\gamma}}\left(a^{\frac{1}{1-\gamma}}+a\beta^{\frac{1}{1-\gamma}}\right)}=:\Lambda(a).$$

For $\gamma < 1/2, \gamma \neq 0$, $\Lambda(a)$ converges to infinity as a tends to infinity. Consequently, for any $\underline{b} \geq 0$, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{a}{\beta})^{\frac{1}{1-\gamma}} \geq (\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$ such that (27) does not hold.

If $\gamma = \frac{1}{2}$, $\Lambda(a)$ is increasing in a and converges to $r\beta^2$ as $a \to \infty$. Consequently, it suffices to set $\underline{b} = \beta^2 \sup R \leq \beta^2$.[17]

If $\gamma \in (\frac{1}{2}, 1)$, $\Lambda(a)$ tends to 0 as $a \to \infty$, and so, if $\sup R < 1$, there exists $\underline{b}$ such that for $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} \geq b$, (27) holds.[18] ■

D Proof of Proposition 2

Proposition 2. *Let u be the quasi-linear utility function given in* (13). *Suppose that $R \subseteq [0,1)$ and* $\sup R < 1$. *Then there exists $\underline{b} \geq 0$ such that $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $\delta \in (0,1)$, where $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$.*

Proof. Since $u \in \mathcal{U}^*$, given $\underline{b} \geq 0$, $B^* := \{b \in \mathbb{R}_+ : b \geq \underline{b}\}$, and $R \subseteq [0,1)$, Theorem 4 gives $\mathcal{T}_{prog}(B^*,R) \subseteq \mathcal{T}_{u\text{-}iir} \subseteq \mathcal{T}_{prog}$ if and only if $u \in \mathcal{U}(B^*,R)$.

Given $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$, if $a \leq \beta/(1-r)$, then $\zeta^u((1-r)a,b) = 0 \leq \zeta^u(a,0)$. If $(1-r)a > \beta$, then $\zeta^{QL}(a,0) \geq \zeta^{QL}((1-r)a,b)$ if and only if

$$\frac{(1-r)a(\theta((1-r)a)-\theta(a)) - b((1-\delta)\theta(a)+\delta)}{\delta(1-\theta(a))((1-r)a(1-\theta((1-r)a))+b)} \leq 0, \tag{30}$$

where $\theta(a) = \left(\frac{\beta}{a}\right)^{1/\delta}$.

Observe that $\theta(a) < 1$ since $a > \beta$; and that $\theta((1-r)a) - \theta(a) = \left(\left(\frac{1}{1-r}\right)^{1/\delta} - 1\right)\left(\frac{\beta}{a}\right)^{1/\delta} > 0$, for $r \in (0,1)$. Therefore, the denominator in (30) is positive, and the inequality holds if and only if the numerator is negative, that is,

$$(1-r)a(\theta((1-r)a)-\theta(a)) - b((1-\delta)\theta(a)+\delta) \leq 0.$$

[17] For example, if $\beta = 1$ and the maximum marginal tax rate is $\frac{1}{2}$, then it suffices to consider the set of all the marginal-rate progressive tax schedules that provide a subsidy of at least \$$\frac{1}{2}$.

[18] For instance, if $\gamma = \frac{3}{4}$, $\sup R = 0.6$, and $\beta = 1$, (27) becomes

$$\left[(0.25a^4 + a)(1 + (a(1-r))^3) - 0.75(a^4+a)\right] b \geq 0.75a^4(1-r)(1-(1-r)^3). \tag{28}$$

If $\underline{b} = 81$, then given $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(\frac{(1-r)a}{\beta})^{\frac{1}{1-\gamma}} = (1-r)^4 a^4 \geq b \geq \underline{b}$, *i.e.*, $a \geq \frac{b^{\frac{1}{4}}}{1-r} \geq \frac{\underline{b}^{\frac{1}{4}}}{1-r} = \frac{3}{1-r}$, (28) is equivalent to

$$b \geq \frac{0.75a^4(1-r)(1-(1-r)^3)}{(0.25a^4+a)(1+(a(1-r))^3) - 0.75(a^4+a)}, \tag{29}$$

and since

$$\frac{0.75a^4(1-r)(1-(1-r)^3)}{(0.25a^4+a)(1+(a(1-r))^3) - 0.75(a^4+a)} < 81$$

and $b \geq \underline{b}$, it follows that (29) holds.

After some manipulation, this is equivalent to

$$b \geq \frac{a\beta^{1/\delta}\left(1-(1-r)^{1/\delta}\right)(1-r)^{\frac{\delta-1}{\delta}}}{\delta a^{1/\delta}+(1-\delta)\beta^{1/\delta}}. \tag{31}$$

The right-hand side of the inequality is bounded above if and only if $\delta \leq 1$ (otherwise it tends to infinity as $a \to \infty$). Hence, for $\delta \in (0,1)$, there exists $\underline{b}$ such that for $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(1-r)a > \beta$, (31) holds. And, if $\delta > 1$ and $\underline{b} \geq 0$, there exists $(a,b,r) \in \mathbb{R}_{++} \times B^* \times R$ with $(1-r)a > \beta$ such that (31) does not hold. ■

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