



Optimal Crowdfunding Design

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**This version October 2016
October 2014**

Barcelona GSE Working Paper Series

Working Paper n° 871

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First draft: October 2014

This version: October 2016[‡]

Abstract

We characterize optimal reward-based crowdfunding where production is contingent on an aggregate funding threshold. Crowdfunding adapts project implementation to demand (market-testing) and its multiple prices enhance rent-extraction via pivotality, even for large crowds, indeed arbitrarily large if tastes are correlated. Adaptation raises welfare. Rent-extraction can enhance adaptation, but sometimes distorts production and lowers welfare. Threshold commitment, central to All-Or-Nothing platforms, raises profits but can lower consumer welfare. Platforms sometimes promote not-for-profits to raise success rates. When new buyers arrive ex-post, crowdfunding's market-test complements traditional finance and optimizes subsequent pricing. Crowdfunding is a general optimal mechanism in our baseline.

Keywords: Crowdfunding, mechanism design, entrepreneurial finance, market-testing, adaptation, rent-extraction.

JEL Classifications: C72, D42, L12.

*We thank Paul Belleflamme, Roberto Burguet, Ramon Caminal, Germain Gaudin, Gerard Llobet, Ángel López, Patrick Rey, Arun Sundararajan, Jean Tirole and seminar participants at IAE, Net Institute 2015, the UPF, CORE, Université Catholique de Louvain, Università di Bologna, Oxford University, Zurich University, Toulouse 2016, 9th Internet conference, Searle 2016, 7th Internet conference, Alghero 2016, 7th CRENoS workshop and GAMES 2016, Maastricht, ESEM 2016, Geneva, EARIE 2016, Lisbon, for valuable comments. Financial support from the Net Institute, <http://www.Netinst.org>, the Ministry of Economy and Competitiveness (ECO2014-59959, Ellman, and ECO2012-37065, Hurkens) and Generalitat de Catalunya (2014 SGR 510) is gratefully acknowledged.

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[‡]This supersedes *SSRN 2733537* with the same title; the longer mimeo [Ellman and Hurkens \(2015\)](#), henceforth [EH2015](#), contains supplementary results.

1 Introduction

Crowdfunding is a rapidly growing phenomenon with a major promise: to bring more socially beneficial projects to fruition. Online crowdfunding platforms have sharply reduced entrepreneurs' costs of pitching their projects to a wide range of potential funders before sinking the costs of production. We build a model of crowdfunding to investigate how the strategic interaction between entrepreneurs and funders determines consumer, producer and total welfare. Our analysis, encompassing both profit and not-for-profit motivated entrepreneurs, disentangles crowdfunding's roles in selling and funding. We locate the main social advantage of crowdfunding in its ability to adapt production to the "crowd's information" about market demand: entrepreneurs use crowdfunding to gauge the market before entering. They also use it to extract consumer rent, often further improving adaptation, but possibly lowering consumer and even total welfare relative to traditional selling mechanisms. We investigate how platform design can limit these negative effects. Introducing credit constraints, we show crowdfunding can, as its name suggests, substitute for traditional finance. But when new buyers arrive later on, crowdfunding, as a credible market signal, can actually complement finance, and optimize pricing.

We study the prominent case of reward-based crowdfunding where funders are compensated with the project's product. So the funders are buyers. Each buyer chooses a bid after the entrepreneur sets a funding threshold and a minimum price. Production occurs in the "success" event where the aggregate funds, the sum of bids, reaches the funding threshold. That is, a simple aggregate fund threshold (AFT) fully determines production. The entrepreneur then receives these funds and has to sink her production costs and deliver her product to all buyers who bid at least the minimal price. Buyers can rest assured that (1) they pay nothing in the event of funding failure and (2) they pay exactly their bids in the event of success (so bids are prices). Together with crowdfunding's defining characteristic AFT, these reassuring properties explain why so many small funders are willing to participate.¹

Crowdfunding is attractive to entrepreneurs as a tool for adaptation (market-testing) and rent extraction (price discrimination), as we now explain in our baseline model of buyers with independent valuations, high or low, for tractability. Adaptation is simplest when high types are frequent. The entrepreneur then sets the high valuation as minimum price, excluding low types, and sets her fixed cost as threshold. Welfare always rises as this threshold perfectly adapts production to actual demand; she sinks her fixed cost precisely in the demand states that are profitable. Crowdfunding effectively combines production finance with sales marketing in an ex-ante mechanism: the entrepreneur offers a sales contract *before* deciding whether to produce. Her offer, to the buyers as a

¹In 2014, 3.3 million backers pledged 529 million dollars, generating over 22,000 successfully financed projects on the major reward-based platform Kickstarter, alone. Overall, [Massolution \(2015\)](#) estimate that global crowdfunding raised 16.2 billion dollars and predict that 2015 will see this figure double.

“crowd,” is explicitly contingent on their aggregate bids reaching her threshold. The bids reveal actual demands, so this creates an incentive-compatible market test, unavailable in traditional finance and selling where the entrepreneur can pay for a market survey but must finance and implement her project before posting any price offer.²

When high types are less frequent, the entrepreneur includes low types by lowering her minimal price. In this inclusive strategy, she raises the funding threshold above her fixed cost to extract rent from high types who, being sometimes pivotal, bid extra to help reach the threshold. Caring more for project success, they pay more than low types for the exact same good. These *multiple prices*, generated by pivotality, are important. The resulting rent extraction raises welfare when it enables profitable production on otherwise invariable high cost projects. But the entrepreneur potentially sets her threshold too high. Threatening not to produce extracts rent but wastes some production opportunities. At intermediate fixed costs, welfare is lower than in traditional selling.

Multiple prices arise in our model despite our simplifying focus on a single reward. In practice, entrepreneurs often offer several rewards (*e.g.*, T-shirts, signed CDs, special editions or multiple units) at different prices. Some reward variants are merely a way of saying thanks but reward differentiation also provides a complementary tool for rent extraction. This is fully compatible with both crowdfunding’s demand adaptation and pivotality-based rent extraction. Indeed, differentiation complements inclusive pricing, by extracting more rent at a given threshold. Conversely, crowdfunding lowers the need for product differentiation compared to traditional selling.

We now illustrate adaptation and rent extraction benefits for a surprisingly large crowd, later explaining why substantial gains are representative of actual campaigns.

Illustration 1. *Esther wants to produce a CD but recording costs €2650. True fans value her music at €20, others are only willing to pay €5 and most simply have no interest. She targets 500 people who have some interest (friends, family and followers), whose values are i.i.d. draws from {5, 20} with the high value, 20, having probability $q = 0.3$ in case (i), $q = 0.2$ in case (ii). With traditional selling, Esther only produces in case (i), then selling at €20 to earn an expected profit of €350; in case (ii), her best posted price of €5 would give a loss of €150. Crowdfunding helps in both cases by adapting the production decision to actual demand:*

(i) *With $q = 0.3$, a €20 minimum price and €2650 threshold raise expected profit to €353.47 by avoiding losses when demand is low (below 133 fans).*

(ii) *With $q = 0.2$, a €5 minimum price and €2726.80 threshold motivate fans to pay €7.10, a premium of €2.10, to raise the chance of project success and getting the CD (108 fans are needed to reach the threshold). This optimal crowdfunding raises project success to 20% and expected profit to €17.30.*

²Hummel et al. (2013) show how relying on standard surveys may lead to poor production decisions.

Imposing a single crowdfunding price in case (ii) would induce a €20 minimum, decimating project success to just 0.02%. This sensitivity of success rates to rent extraction encourages platforms (with a pro-rata fee on revenues) to promote success-motivated entrepreneurs when high types are scarce. Funding successes are socially important because they generate positive externalities for ex-post consumers as well as funders, and they allow first-time entrepreneurs to demonstrate their skills and develop careers. Indeed, career motives lead most new entrepreneurs to care more about project success than project profits. So Section 3 characterizes optimal crowdfunding for each of three objectives: profits, success and welfare (relevant for some not-for-profits). The two non-profit objectives enhance crowdfunding’s adaptation role while reducing rent-extraction. Unfortunately, when high types are frequent, platforms bias towards for-profits.

Platforms also play a role in enforcing the threshold commitment. Without enforcement, entrepreneurs would self-bid, perhaps via pseudonyms, to access aggregate funds that are below threshold but above cost. So the no commitment (NC) scenario, characterized in Section 4.1, precludes above-cost thresholds. With infrequent high types, this lowers prices and raises success rates, encouraging crowdfunder participation, but otherwise NC makes all actors worse off, by provoking a switch to exclusion (higher prices). This may explain why Kickstarter enforces commitment as in our baseline, called “all-or-nothing” (AON) in the world of crowdfunding (see Mollick, 2014, p.6, and Section 4.2).

We prove reward-based crowdfunding is a general optimal mechanism in our two-type baseline in Section 6,³ and more generally, benefits entrepreneurs relative to traditional selling, but one might expect free-riding among funders in *large crowds* to trivialize pivotality-based gains, while aggregation trivializes adaptation gains by minimizing market uncertainty. Sections 4.3 and 5 show crowdfunding is still important. First, rent extraction and adaptation gains prove significant for surprisingly large crowds, as Illustration 1 demonstrated. Second, when poor targeting necessitates mass emails, adaptation gains remain significant if the expected number of active bidders stays moderate. Third, within-group preference correlations or project quality shocks imply substantial crowdfunding benefits for arbitrarily large markets. Indeed, larger crowds provide more accurate profitability signals for financiers and for ex-post pricing in Section 5.

Despite the media hype around viral projects, a typical crowdfunding project actually attracts bids from only about 50 active funders on Kickstarter. These numerous, small-to-moderate projects generate large positive externalities, via ex-post sales and entrepreneurial careers; those careers, in turn, motivate success-maximization (Section 3.2). We fit the average size and success rates of actual crowdfunding projects by recognizing that entrepreneurs self-select into crowdfunding as a function of platform fees and their ex-ante costs of pitching projects to buyers.

Our model shows that the “funding” in crowdfunding is not fundamental. An en-

³Relaxing the reassuring crowdfunding restrictions (1), (2) and AFT, does *not* permit higher profits.

trepreneur with no credit constraints uses crowdfunding purely to adapt to demand and to extract rent. Buyers do typically pay their bids in advance, instead of just committing to buy. This is a simple way to enforce payment of high bids, even if not needed for funding the fixed cost. So crowdfunding substitutes for traditional finance *if* all potential buyers participate during the 30-60 days of crowdfunding. But many people only hear about a crowdfunded project after its campaign ends. Crowdfunding can then strictly complement traditional finance (as in [Mollick and Kuppuswamy, 2014](#)) by providing a credible market test: in our Section 5 extension, the funds raised in crowdfunding ex-ante signal profitability of the ex-post market and may convince traditional financiers to step in. This signal also helps the entrepreneur to optimize pricing. The funding-contingent price dynamic that results can readily explain why prices often rise after more successful crowdfunding campaigns, unlike related models where the price can only fall.

We abstract from moral hazard. Taking funds and not fulfilling rewards is fraudulent, and empirically, very rare ([Mollick, 2014](#)). Both reputation concerns, from future careers and social networks, and ex-post selling in Section 5 plausibly motivate most entrepreneurs to sink fixed costs and deliver. Also, early backing from friends, family and local experts help screen out complete frauds with infeasible projects (see Footnote 26).

Related Literature

The field of crowdfunding has become quite crowded ([Agrawal et al. \(2014\)](#) and [Belleflamme et al. \(2015\)](#) survey early empirics and theory) but our work stands out: we characterize the *optimal* crowdfunding mechanism, showing its adaptation value as an incentive-compatible market-test and the role of multiple prices.

This harks back to the public goods literature on heterogenous private contributions towards a common goal, but (a) the general mechanism approach neglects AFT, central to crowdfunding, while (b) the simple intuitive, contribution and subscription, games that *do* impose AFT have only been solved fully with complete information or two players – hardly a crowd!⁴ We make progress on (a) and (b): with a binary type space, we fully characterize with AFT for any crowd size, proving crowdfunding is then general optimal.

All other crowdfunding models restrict to a unique crowdfunding price. The prior work of [Belleflamme et al. \(2014\)](#) assumed crowdfunders enjoy warm-glow proportional to their consumer valuations. High types pay a crowdfunding premium over a regular ex-post price. By contrast, we show warm-glow is unnecessary by moving to a finite crowd where high types pay a premium to pivot into production.

Two contemporaneous papers also model pivotality-based price discrimination. [Kumar et al. \(2015\)](#) study a continuum of consumers contradicting their pivotality claim and precluding aggregate demand uncertainty to which to adapt. [Sahm \(2015\)](#) does treat a

⁴See *e.g.*, [Cornelli \(1996\)](#); [Ledyard and Palfrey \(2007\)](#); [Schmitz \(1997\)](#) on (a) and [Alboth et al. \(2001\)](#); [Bagnoli and Lipman \(1989\)](#); [Barbieri and Malueg \(2010\)](#); [Menezes et al. \(2001\)](#) on (b).

finite crowd, but his mechanism is suboptimal (multiple crowdfunding prices improve outcomes) and he claims pivotality breaks down with any more than a few buyers which we contradict (see Illustration 1). Also he assumes traditional finance and crowdfunding are mutually exclusive, obliging at-cost thresholds despite the ex-post revenues, but we derive complementarities. More recently, [Strausz \(2015\)](#) models fraud in a simplified version of our baseline ($v_L = 0$). [Chang \(2015\)](#) treats this in a pure common value environment (see also [Hakenes and Schlegel, 2014](#)). We abstract from fraud, as motivated above.

All these papers restrict to a unique crowdfunding price, but multiple prices are salient in practice and matter in theory: they can substantially increase efficiency by enhancing demand adaptation but introduce a risk of excessive extraction, crucial to our threshold commitment results; they also reduce the need for credit and facilitate learning demand.

2 Baseline model

We present a streamlined model, deferring justification and extensions. A single entrepreneur has a project for producing a good at fixed cost $C > 0$ and constant marginal cost, normalized to zero. N buyers have unit demands for the good with private values drawn independently from the 2-type distribution: probability q on v_H and $1 - q$ on $v_L < v_H$. The number k of these N buyers with high demand v_H defines the demand state and has binomial (N, q) distribution, $f_k^N(q) = \binom{N}{k} q^k (1 - q)^{N-k}$; we follow the conventions $\binom{M}{k} = 0$ if $k < 0$ or $k > M$ and $\binom{0}{0} = 1$ and sometimes suppress q . We assume $C < Nv_H$ else production is never profitable. Finally, we define $\hat{q} = v_L/v_H$ and $S_n^M = \sum_{k=n}^M f_k^M$.

We first solve the benchmark case of traditional selling as an ex-post mechanism, where the entrepreneur decides on production before learning about the demand state k , posting a price to all buyers. Then we describe the potential for gains from an ex-ante selling mechanism, setting the stage for crowdfunding. We normalize time discounting to zero. In the baseline, the entrepreneur has unlimited wealth or unconstrained credit.

2.1 Traditional selling

In traditional selling (TS), the product is only sold after production. If the entrepreneur decides to produce, C is sunk. A single posted price p is then optimal.⁵ She gets expected revenue Np from $p \leq v_L$, qNp from $p \in (v_L, v_H]$ and 0 from higher p . So she chooses between the “exclusive” price $p = v_H$ that excludes L -types, extracting all H -type rent, and the “inclusive” price $p = v_L$ that includes L -types, leaving some rent to H -types. Exclusion is optimal if $q > \hat{q}$ and inclusion is optimal if $q \leq \hat{q}$. She indeed produces if

⁵Probabilistic offers and interpersonal bundling are useless given independent valuations. With interpersonal bundling, price offers to one buyer can depend on other buyers’ choices, but fixed costs are sunk. So fixed costs, which are crucial to all our results, are effectively zero. Nonetheless, insights on word-of-mouth advertising suggest a complementary attraction (see *e.g.*, [Chen and Zhang, 2015](#)).

$C \leq \max\{Nv_L, qNv_H\}$. So traditional selling earns her the expected profit,

$$\pi^{TS} = \begin{cases} (Nv_L - C)_+ & \text{if } q \leq \hat{q} \\ (qNv_H - C)_+ & \text{if } q > \hat{q} \end{cases} \quad (1)$$

where x_+ denotes $\max(x, 0)$ for any x .

2.2 Crowdfunding and other ex-ante selling mechanisms

As her fixed cost C is strictly positive, the entrepreneur may improve on traditional selling (posted prices) by using an “ex-ante” mechanism. First, she can adapt her fixed cost production decision to actual, instead of only expected, demand, by eliciting buyer preferences using incentive-compatible, pre-production sales contracts. Second, she can make non-production threats to induce some buyers to pay more to raise the probability of trade. For an upper bound on what the entrepreneur can earn, Section 6 characterizes the optimal ex-ante mechanism where production, consumption and monetary transfers are contingent on the full vector of buyers’ expressed demands. In general, this is too complex to be practical.

Crowdfunding is a particularly simple ex-ante mechanism: the entrepreneur chooses an **A**ggregate **F**unds **T**hreshold T and offers the good at a set of prices $\mathcal{B} \subseteq \mathbb{R}_+$ that become effective contingent on the sum of buyers’ chosen prices or bids reaching T :

AFT: *Production occurs if and only if the sum of bids exceeds threshold T .*

We call bids, prices, since buyers either pay their chosen bids and receive one unit of the good (this occurs when T is reached) or there is no production and bids are refunded. This refund property is widespread (Kickstarter makes it salient) and we know of no crowdfunding platform with non-AFT commitments.⁶

When the entrepreneur cannot restrict the set of buyer bids \mathcal{B} , she gains by setting a minimal price p that excludes buyers who bid below it. While superfluous in the baseline, we already introduce this $p > 0$ so that $b = 0$ can denote a buyer’s choice to make no bid. Requiring $\mathcal{B} \ni 0$, the timing is then:

1. The entrepreneur chooses her offer (T, \mathcal{B}, p)
2. Buyers learn their private values and simultaneously choose their bid from \mathcal{B}
3. AFT with refunds determine production, consumption and transfers

2.3 Equilibrium concept and outcomes

Given a profile of buyer strategies and realized buyer valuations $\mathbf{v} \in \{v_L, v_H\}^N$, the outcome of the simultaneous bidding game (that follows the entrepreneur’s choice of

⁶Cornelli (1996) shows refunds do not restrict the general optimum, but she never considers AFT.

mechanism) specifies: each buyer i 's probability $P_i(\mathbf{v})$ of getting the good and expected transfer $\tau_i(\mathbf{v})$ to the entrepreneur and whether the entrepreneur produces. So, treating buyers symmetrically, the outcome depends on i 's type, L or H , and the demand state, k . Since the entrepreneur sets the mechanism, standard mechanism design selects her preferred Bayesian Nash equilibrium (BNE) of the ensuing simultaneous bidding game.

We also go further by studying *full implementation* under interim efficiency, a mild equilibrium refinement (Ledyard and Palfrey, 2007). This requires *all* Pareto-undominated BNE to generate the same outcome; indeed we demonstrate that, with threshold commitment, the entrepreneur can uniquely implement her preferred outcome. This profit-maximizing outcome is itself generically unique and to simplify the exposition we tie-break non-generic cases by maximizing production.

3 Optimal crowdfunding design

We first analyze the baseline model with profit-maximizing entrepreneurs. Next, we consider not-for-profit entrepreneurs. Then Section 3.3 characterizes welfare effects.

3.1 For-profit entrepreneurs

The entrepreneur can commit to any threshold T and bid restriction $\mathcal{B} \ni 0$. She need only consider mechanisms that induce symmetric pure strategy equilibria, as we prove below. So we let L and H type buyers bid $b_L \leq b_H$, respectively, and $\mathcal{B} = \{b_L, b_H\} \cup \{0\}$. The entrepreneur's expected profit is 0 if $T > Nb_H$, $Nb_L - C$ if $b_H = b_L \geq T/N$, and

$$\pi(b_L, b_H, T, p) = \sum_{k=n}^N f_k^N (kb_H + (N - k)b_L - C)$$

if $b_H > b_L$ and $b_H \geq N/T$, where $n = \left\lceil \frac{T - Nb_L}{b_H - b_L} \right\rceil$ is the pivotal number of H -types that triggers production.⁷ Using Lemma A.1(iii) and project success rate S_n^N , we rewrite as,

$$\pi_n(b_L, b_H) = S_n^N (Nb_L - C + \mathbb{E}[k | k \geq n](b_H - b_L)) \quad (2)$$

Before trading off project success and the production-conditional expectation of revenue minus cost, we first optimize b_L and b_H , for each pivot n . That is, we maximize profit (2) subject to incentive compatibility and individual rationality constraints. In exclusive solutions, denoted by script E , $b_L = 0$ and the only constraint is $p \leq b_H \leq v_H$. In inclusive solutions, denoted by script I , $b_L > 0$, individual rationality requires $p \leq b_L \leq v_L$,

⁷We use $\lceil x \rceil$ to denote the smallest non-negative integer larger than or equal to x .

$b_H \leq v_H$ and H -type incentive compatibility requires:⁸

$$(v_H - b_H)S_{n-1}^{N-1} \geq (v_H - b_L)S_n^{N-1} \quad (\text{IC})$$

In equilibrium, each H -type believes that if he bids b_L instead of b_H , he only gets the good when at least n of the other $N - 1$ buyers are H -type, whereas if he bids b_H , only $n - 1$ other buyers need be H -type. So he trades off the higher net gain $v_H - b_L$ against lower success S_n^{N-1} . Defining hazard ratio,

$$h_n = \frac{f_{n-1}^{N-1}}{S_{n-1}^{N-1}} = 1 - \frac{S_n^{N-1}}{S_{n-1}^{N-1}}$$

we can rewrite (IC) as,

$$b_H \leq h_n v_H + (1 - h_n) b_L \quad (3)$$

or as $\delta \leq h_n(v_H - b_L)$ where $\delta = b_H - b_L$ is the H -type's voluntary additional bid.

We now characterize exclusion, then inclusion, and the overall optimal strategy.

3.1.1 Exclusion

When excluding low type buyers ($b_L = 0$), the entrepreneur can trivially extract all H -type rent by setting $b_H = v_H$ and $p \leq b_H$. So she can dedicate T to adapting implementation to demand: with $T = C$, she sinks production cost C precisely in the profitable demand states, $k : kv_H - C \geq 0$. Equivalently, she picks $n_E = \lceil \tilde{n}_E \rceil$ where $\tilde{n}_E = C/v_H$ via any $T \in ((n_E - 1)v_H, n_E v_H]$. This gives her optimized (expected) profit from exclusion,

$$\pi_{n_E}^E = \sum_{k=n_E}^N f_k^N (kv_H - C)$$

Here, crowdfunding reveals aggregate H -type demand and fully adapts production to it.

3.1.2 Inclusion

In an inclusive strategy with pivot n , $p \leq b_L \leq v_L$, the entrepreneur maximizes profit (2) subject to incentive constraint (IC), which guarantees $b_H \leq v_H$. Raising b_L relaxes (IC) and raises profits, so $b_L = v_L$, extracting all L -type rent.⁹ As b_H raises profit, (IC) binds giving $b_H = \bar{b}_n$ where,

$$\bar{b}_n = h_n v_H + (1 - h_n) v_L \quad (4)$$

⁸ L -type incentive compatibility never binds since the entrepreneur prefers the high bid.

⁹For $n = N$, these effects are weak and any b_L will do.

This inclusive n -type strategy gives profit,

$$\pi_n^I = S_n^N (Nv_L - C + \mathbb{E}[k|k \geq n]h_n(v_H - v_L)) \quad (5)$$

Any $T \in (\bar{T}_n - \delta_n, \bar{T}_n]$ suffices, where $\bar{T}_n = Nv_L + n\bar{\delta}_n$, $\bar{\delta}_n = \bar{b}_n - v_L = h_n(v_H - v_L)$. Maximum \bar{T}_n is uniquely optimal without bid restrictions (see Section 4.1).

The hazard ratio h_n determines the fraction of the H -type's rent from buying at v_L that can be extracted using the pivotality motive.¹⁰ Both h_n and $\mathbb{E}[k|k \geq n]$ strictly increase with n (see Lemma A.1(vii)), while the success rate S_n^N strictly decreases, creating a tradeoff between extracting rent with high n and ensuring production with low n . π_n^I is single-peaked in n and the optimal inclusive pivot n_I is given by:

Lemma 1. π_n^I is increasing on $n \leq n_I$ and decreasing on $n \geq n_I$, where $n_I = \lceil \tilde{n}_I \rceil$ and

$$\tilde{n}_I = \frac{C - Nv_L + q(Nv_H - C)}{v_H - v_L} \quad (6)$$

Notice that $\tilde{n}_I = \tilde{n}_E$ when $q = \hat{q}$ and $\tilde{n}_I < \tilde{n}_E$ for $q < \hat{q}$ since,

$$1 - \frac{\tilde{n}_I}{N} = \frac{1 - q}{1 - \hat{q}} \left(1 - \frac{C}{Nv_H} \right) \quad \text{and} \quad 1 - \frac{\tilde{n}_E}{N} = 1 - \frac{C}{Nv_H}$$

Also \tilde{n}_I and hence n_I are increasing in both the cost C and probability q of H -types. This is intuitive: the entrepreneur loses less from failing to produce in the event of few high types, the higher is her production cost and the less likely this event becomes.

3.1.3 Overall optimum

Comparing the optimized profits $\pi_{n_E}^E, \pi_{n_I}^I$ reveals that exclusion is optimal if and only if $q > \hat{q} = v_L/v_H$. As with posted pricing, exclusion is more attractive when the excluded L -types are less frequent; indeed, the cut-offs exactly coincide.¹¹ Summarizing,

Proposition 1. *The optimal crowdfunding outcome is characterized as follows:*

(E) For $q > \hat{q} = v_L/v_H$, L -types are excluded and H -types get the good at price v_H , when the number k of H -types weakly exceeds the pivot $n_E = \lceil C/v_H \rceil$.

(I) For $q \leq \hat{q} = v_L/v_H$, both L and H types get the good, paying v_L and \bar{b}_{n_I} respectively, when demand state k weakly exceeds the pivot n_I from Lemma 1.

*The sets of mechanisms with tight bid restrictions that uniquely implement these outcomes in Pareto-undominated BNE are.*¹²

¹⁰At $n = 0$, there is no pivotal motive, $h_0 = 0$, all buyers pay v_L and production takes place for sure. At the other extreme, with $n = N$, production requires all buyers to be H -type, so all H -types are fully pivotal, removing free-riding; $h_N = 1$. The entrepreneur then extracts all their informational rent, $\bar{b}_N = v_H$, but production only occurs with probability $S_N^N = q^N$.

¹¹At $C = 0$, interpersonal bundling has no value given independent tastes; raising C along $q = \hat{q}$ does not shift the cut-off, since $n_E = n_I$ there, cancelling out the expected cost terms under E and I .

¹²There exist Pareto-dominated bidding equilibria where some buyers do not bid or H -types bid v_L , strictly lowering project success and H -type and entrepreneur payoffs, without benefitting L -types.

$$\begin{aligned}\mathcal{M}_E &= \{(T, \{0, v_H\}, p) : 0 < p \leq v_H \text{ and } (n_E - 1)v_H < T \leq n_E v_H\} \\ \mathcal{M}_I &= \{(T, \{0, v_L, \bar{b}_{n_I}\}, p) : 0 < p \leq v_L \text{ and } \bar{T}_{n_I} - \bar{\delta}_{n_I} < T \leq \bar{T}_{n_I}\}.\end{aligned}$$

The specific mechanisms, $(T, p) = (C, v_H)$ for E and $(T, p) = (\bar{T}_{n_I}, v_L)$ for I , are natural and robust to removing bid restrictions (see Section 4.1). Note that $\bar{T}_{n_I} > C$ since: (1) $\bar{T}_N = Nv_H > C$; (2) $n_I \bar{b}_{n_I} + (N - n_I)v_L \leq C$ with $n_I < N$ would contradict n_I 's optimality since $n_I + 1$ could then avoid producing in unprofitable state n_I and strictly increase profits in higher states. So the optimum can always use $T \geq C$. This proves:

Corollary 1. *The entrepreneur's optimized profit is non-negative in every demand state.*

We illustrate this and later propositions using the representative, parameterized set,

Example 1. $N = 5$, $v_L = 5$, $v_H = 8$ (so $\hat{q} = 0.625$), $0 < q < 1$, $0 < C < 40$.

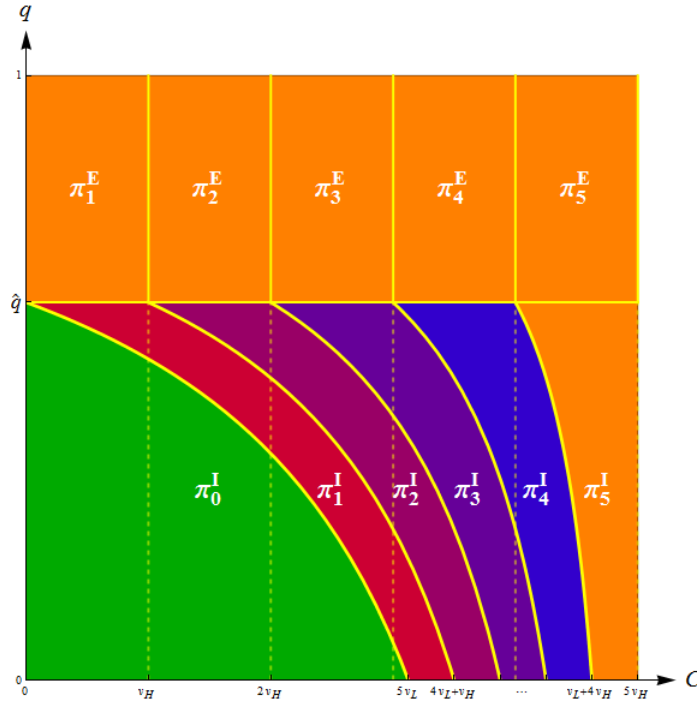


Figure 1: Profit-maximizing crowdfunding in Example 1.

Figure 1 shows the regions in (C, q) -space where each strategy type, n with E or I , is optimal for Example 1; thus, $n_I = 4$ is optimal on the (blue) region marked π_4^I , etc. We see exclusion E above line $q = \hat{q}$ and inclusion I below it. The pivots n_E, n_I increase in C , are equal on $q = \hat{q}$; n_I rises with q and has concave boundaries. Note that $\pi_5^I = \pi_5^E$ as $\bar{b}_N = v_H$ and L -types never get to buy when $n_I = N$. These properties hold generally.

The project success rate is $S_{n_E}^N(q)$ on $q > \hat{q}$ and $S_{n_I}^N(q)$ on $q \leq \hat{q}$. This decreases with C but is non-monotonic in q , as q can induce upward jumps in n_I . Nevertheless,

Corollary 2. *The entrepreneur's profit strictly decreases with C and increases with q .*

3.2 Not-for-profit entrepreneurs

We now study how crowdfunding works when entrepreneurs maximize success or welfare, instead of profits. Unconstrained success-maximizers would always produce, so we impose the canonical ex-post budget balance constraint (BB), requiring funds to always cover cost C . This means no loss in any state k and is equivalent to $T \geq C$. By Corollary 1, BB does not affect the profit-maximizing outcomes.¹³ BB is appropriate when the entrepreneur, such as an artist, is unable or unwilling to risk suffering a loss in any demand state, but also when she has no access to credit and no personal wealth.¹⁴ An ideal not-for-profit may maximize welfare, but others focus on success. Success-maximization also captures an unproven entrepreneur who maximizes long-run profits but for whom breaking even on her project is a vital career stepping stone.

The entrepreneur can guarantee maximum success and welfare, by setting $p = v_L$ and $T = C$ when $C \leq Nv_L$, so we now assume $C > Nv_L$. Given a pivot n , the not-for-profit entrepreneur still sets prices to maximize profits, to relax BB. Whether inclusive or exclusive, she also minimizes n subject to $T \geq C$. Minimal n clearly maximizes success, and also expected welfare, as production satisfying BB (no losses) always raises welfare. So $n = n_E$ is optimal among exclusive strategies and $n = n_I^{BB} = \min\{n : C \leq \bar{T}_n\}$ defines the optimal inclusive strategy. Notice $n_I^{BB} \leq n_I$. Intuitively, for-profits raise n to extract rent under inclusion. Success-maximizers only care about minimizing n so they choose inclusion unless $n_E < n_I^{BB}$. So they are more inclusive than profit-maximizers (because $n_I^{BB} \leq n_I \leq n_E$ for all $q \leq \hat{q}$), strictly so if $n_I^{BB} < n_I$. Welfare-maximizers choose inclusion even more since they strictly prefer it at a fixed success rate.

Proposition 2. *Both alternative objectives raise inclusivity compared to the profit-maximizing baseline. The inclusion region expands from $q \leq \hat{q}$ to all (C, q) with $n_I^{BB} \leq n_E$ for success-maximizers, and expands weakly further for welfare-maximizers.*

Figure 2 depicts success and welfare-maximizers' optimal design (identical in Example 1), illustrating how not-for-profit entrepreneurs are more inclusive than profit-maximizers: the (orange) exclusion region is a strict subset of that in Figure 1. Given n , prices are as before. The large rectangle (in green) with zero pivot shows not-for-profits gain nothing from crowdfunding at low costs, even at high q , but they gain more from crowdfunding at higher costs. In case (ii) of our more realistic Illustration 1, both success and welfare maximizers set inclusive pivot, $n_I^{BB} = 102 < n_E = 133$, more than doubling both the 20% success rate, from profit-maximization (pivot $n_I = 108$) to 42.9%, and welfare, from 308 to 632. Multiple prices are again key: restricted to a single crowdfunding price, the entrepreneur would set $p = 20$, yielding a mere 0.02% chance of success.

¹³See Section 8.1.2 of [EH2015](#) for similar results with ex-ante budget balance.

¹⁴Early crowdfunding platforms like SellaBand catered to creative entrepreneurs whose main objective was often to get their work out (project success) without getting into debt.

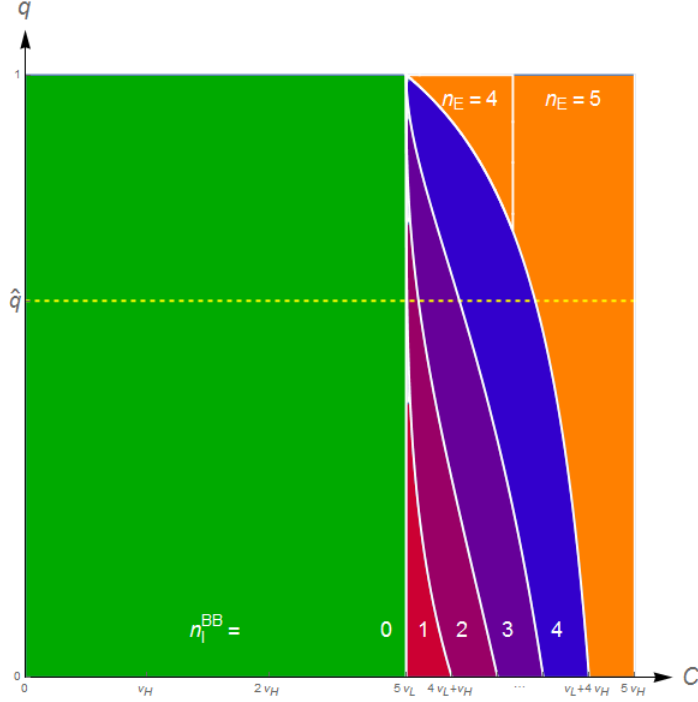


Figure 2: Success-maximizing crowdfunding in Example 1.

3.3 Welfare gains and losses from crowdfunding

In the first-best welfare-maximizing benchmark, all buyers consume whenever production occurs, namely, in all states k with $kv_H + (N - k)v_L - C \geq 0$, that is, $k \geq n^*$ where $n^* = \lceil \tilde{n}^* \rceil$ and $\tilde{n}^* = (C - Nv_L)/(v_H - v_L)$. We now analyze the welfare effects of crowdfunding compared to TS (i.e., posted pricing).

In TS, ex-post budget balance restricts to producing if and only if $C \leq Nv_L$ and has $p = v_L$ for all q . With not-for-profit entrepreneurs, crowdfunding unambiguously raises payoffs of both consumers and entrepreneurs, strictly so on $C > Nv_L$, and neutral otherwise. With profit-maximizers, the welfare effects are more involved. Figure 3 illustrates. Crowdfunding raises welfare on the orange exclusion region $q \geq \hat{q}$ by adapting production perfectly to demand, avoiding production in low demand states but producing in high ones. Consumer surplus remains zero here. Welfare is also raised on the inclusive blue rectangle with $q \leq \hat{q}$ and $C > Nv_L$ where there is no production in TS. Consumer surplus strictly increases unless $n_I = N$. However, on $q \leq \hat{q}$ and $C \leq Nv_L$, TS maximizes welfare by always producing. Here, crowdfunding can only do harm. On the convex green, low (C, q) region, crowdfunding sets $n_I = 0$ and is welfare neutral. But on the adjacent purple region, crowdfunding lowers welfare and consumer surplus by restricting production.

Proposition 3. (a) *With for-profit entrepreneurs, crowdfunding's welfare impact versus traditional selling (TS) is: (i) strictly positive if $C > Nv_L$, (ii) strictly positive for any C if $q > \hat{q}$, (iii) strictly negative if $q \leq \hat{q}$, $C \leq Nv_L$, $n_I > 0$, (iv) neutral otherwise;* (b)

With not-for-profits, its impact is: (i) strictly positive if $C > Nv_L$, (ii) neutral otherwise.

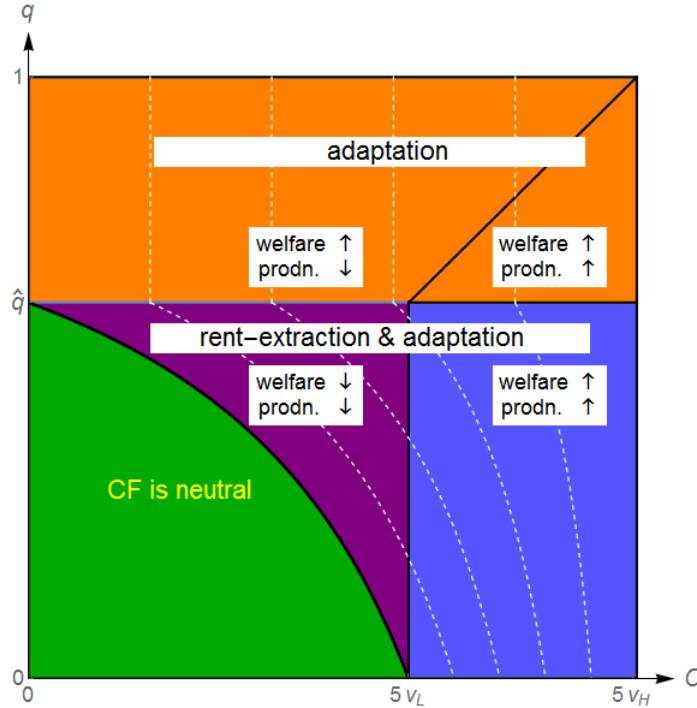


Figure 3: Welfare effects of profit-maximizing crowdfunding relative to TS, indicating adaptation to produce in higher demand states and rent extraction where thresholds induce price discrimination.

Crowdfunding has the same qualitative effect on consumer surplus as on welfare, except in having a null consumer effect when exclusive (or inclusive with $n = N$). Unsurprisingly, given crowdfunding, not-for-profits give strictly higher consumer surplus and welfare than do for-profits, whenever their strategies differ.

4 Crowdfunding in practice

This section examines the robustness of our characterization and how well it fits with observed crowdfunding practice. We revisit the entrepreneur's assumed power to restrict bids and commit to a threshold. We consider the role of platforms and policy. We also discuss crowd size and participation costs that may deter crowdfunding.

4.1 Commitment issues

Threshold commitment has a marked effect on crowdfunding outcomes. This is important because, as we explain in the next subsection, enforcing threshold commitment (and bid restrictions) is non-trivial and platforms can choose not to offer commitment. So we now characterize optimal crowdfunding when the entrepreneur lacks both types of

commitment powers, denoted NC for no commitment. We restrict attention to price-discriminating inclusivity, $q \leq \hat{q}$ and $0 < n_I < N$, since threshold commitment and bid restrictions are clearly unnecessary otherwise.¹⁵ First, we show that bid restrictions matter less than threshold commitment in that:

Lemma 2. *With threshold commitment, bid restrictions do not bind.*

Naturally, to prevent bid-shaving, the entrepreneur must now pick the highest threshold \bar{T}_{n_I} and minimum price v_L from the set of optimal mechanisms \mathcal{M}_I described in Proposition 1. (The proof rules out other bidding deviations and establishes unique implementation as before.) It follows that a crowdfunding entrepreneur essentially picks a pair (T, p) ; unlike in unique crowdfunding price models, this p is only a minimal price. In practice, bid restrictions or suggested prices may still help coordinate buyers. Consistent with this, Kickstarter encourages entrepreneurs to offer buyers a choice between multiple rewards at different prices.¹⁶ Bid restrictions also help if the entrepreneur cannot commit to a threshold or faces a richer type distribution (see [EH2015](#), Section 4.3).

No threshold commitment, nor bid restriction (NC)

With no commitment powers (NC), the entrepreneur produces precisely when the total sum of bids covers the fixed cost, effectively forcing $T = C$. This does not affect the entrepreneur in maximizing success or welfare, so we now focus on profit-maximizers.¹⁷ NC strictly lowers their profits, because an H -type can now shave his bid without lowering the success rate (recall $\bar{T}_{n_I} > C$). The entrepreneur partly counterbalances this effect by *lowering* her minimal price *below* the low type valuation, to foster high type bids.¹⁸ But if q is high enough, she instead raises p to exclude. Overall, as we derive below, NC raises production and total and consumer welfare if and only if inclusion remains optimal.

The *local* incentive compatibility condition that prevents H -types from shaving down from b_H requires equality in the n -pivotality condition (for $n > 0$): $nb_H + (N - n)b_L = C$.

Lowering p to p' substitutes for the inability to raise T above C , because L -types bidding $b_L = p' < v_L$ raises the gap $T - Nb_L = C - Nb_L$ that motivates H -type bids. Denoting the resulting b_H by b'_n and letting $\delta'_n = b'_n - p'$, we have, $nb'_n + (N - n)p' = C$, or equivalently,

$$\delta'_n = (C - Np')/n \tag{7}$$

¹⁵If only one price is ever paid in the baseline optimum, setting this as minimal price p already prevents underbidding and $T = C$ is then anyway optimal. This applies to all exclusive solutions, unique price v_H , and the corner inclusive solutions: $n_I = N$ is equivalent to exclusion; $n_I = 0$ has unique price v_L .

¹⁶Rewards typically consist of the good accompanied by small gifts like signatures, stickers and T-shirts. Often, the price differences cannot be explained by vertical differentiation.

¹⁷NC forces weakly lower pivots, but not-for-profits already choose the minimum consistent with BB.

¹⁸Surprisingly, extracting maximal rents from L -types via $p = v_L$ (so $b_L = v_L$) is no longer optimal. [Hansmann \(1981\)](#) and [Baumol and Bowen \(1968\)](#) offer evidence for such “underpricing.” They consider not-for-profit organizations with $T = C$; restricting profit distribution credibly accentuates motivations to raise welfare and success, and, in general, assures excess revenues go into production or quality.

which falls with p' . From (2), the entrepreneur's profit for $n \geq 1$ is now,

$$\pi'_n = S_n^N (\mathbb{E}[k|k \geq n] - n) \delta'_n$$

For a given $n > 0$, the entrepreneur chooses p' to maximize δ'_n , subject to L -type individual rationality, $p' \leq v_L$, and incentive compatibility of H -types not deviating to bid p' ,

$$\delta'_n \leq h_n(v_H - p') \quad (\text{IC}') \tag{IC'}$$

Solving (7) and the binding (IC') gives the optimal p' :

$$p'_n = \frac{C - nh_nv_H}{N - nh_n} \tag{8}$$

which is readily seen to decrease with n . This n -type strategy is only feasible if $p'_n \leq v_L$, or equivalently, $C \leq Nv_L + nh_n(v_H - v_L) = \bar{T}_n$. For $n = 0$, inclusion simply yields $\pi'_0 = Nv_L - C$. The optimal inclusive pivot denoted n'_I again trades off rent extraction and project success. The resulting inclusive payoff is compared to $\pi_{n_E}^E$.

Proposition 4. *Removing commitment power affects payoff outcomes in the price-discriminating case, $q \leq \hat{q}$ and $0 < n_I < N$, only. It then strictly lowers profits and:*

- (a) *if inclusion stays optimal, $n'_I \leq n_I \leq n_E$, the minimal price is strictly lower, consumer surplus is strictly higher and total surplus and the success rate also rise, strictly if $n'_I < n_I$;*
- (b) *if exclusion becomes optimal, consumer surplus is strictly lower and total surplus and the success rate also fall, strictly if $n_I < n_E$.*

Intuitively, new constraint (7) encourages the entrepreneur to lower n under inclusion. The option to lower p complicates the proof that indeed $n'_I \leq n_I$, but that still holds since reducing p directly reduces per L -type revenues and the optimal p'_n falls with n .¹⁹

This reduced minimal number of H -types reveals that, if inclusion remains optimal, non-commitment weakly raises total welfare. The entrepreneur cannot gain from lost commitment power and she strictly loses in the price-discriminating case. So consumer surplus must be strictly higher. Even L -types now get a strict positive surplus because they pay less than v_L . So total and consumer welfare rise provided that inclusion remains optimal, which holds for low values of q and C . If instead q or C is relatively high, exclusion becomes optimal and consumer surplus falls to zero.

Figure 4 illustrates these results. The dotted black curves, representing loci $C = \bar{T}_1$ through $C = \bar{T}_4$, indicate feasibility of inclusive strategies with no commitment; $n = 4$ is often feasible but exclusion always dominates it. Clearly, in the orange subregion below

¹⁹It also introduces multiple Pareto undominated equilibria: H -types prefer the equilibria with higher p and weakly lower n , while L -types prefer lower p . We now assume the entrepreneur selects her preferred equilibrium, as standard in mechanism design, but we do not need this for our key results: that $n \leq n_I$ and NC raises welfare provided the entrepreneur does not switch to exclusion.

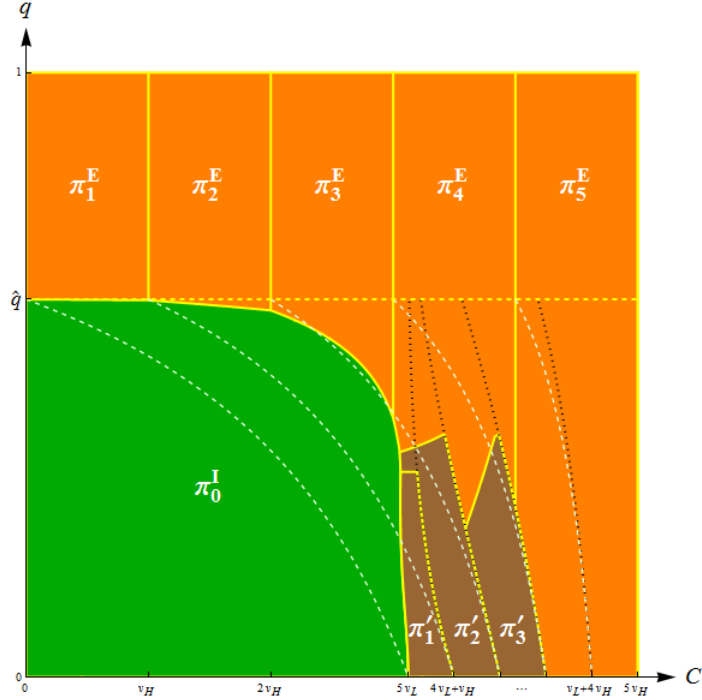


Figure 4: Profit-maximizing crowdfunding with no commitment (NC) in Example 1.

$q = \hat{q}$ where exclusion becomes optimal, consumer surplus falls to zero and profits strictly fall as $\pi_{n_I}^I$ was strictly preferred. In all other regions below $q = \hat{q}$, consumer surplus is strictly higher as both types pay less, H -types strictly, and the success rate rises.²⁰

4.2 The role of platforms

Platforms play a vital role in crowdfunding, reducing transaction costs between entrepreneurs and buyers. They support trust by leveraging social networks and defining clear obligations: buyers cannot withdraw bids once the threshold is reached and typically pay in advance, and entrepreneurs are obliged to fulfill promised rewards once funds are transferred.²¹ Not-for-profit entrepreneurs naturally do their best to fulfill these rewards. Profit-maximizers tend to do so too, thanks to potential ex-post sales and reputational concerns, enhanced by platform's feedback forums.

Platforms typically charge a share $\alpha > 0$ of revenues on successful projects, so they care about the number and size of successful projects and they value high revenues even if accompanied by high costs. Their revenue share can bias their interests towards those of entrepreneurs, but they must also attract buyers to have any successes. We apply lessons

²⁰Generically, L -types pay strictly less when $n_I' \geq 1$: L -types pay v_L only along the dotted black curve, $C = \bar{T}_{n_I'}$. In the green region marked π_0^I , all buyers pay v_L so NC does not benefit L -types strictly.

²¹Platforms warn of the inherent risks in unfinished products, but avoid direct contractual responsibility for reward delivery. This might change if fraud became at all frequent. Fraud is currently rare but delivery delays from unforeseen technical and logistical problems are quite common (Mollick, 2014).

from two-sided markets to understand platform strategies.²²

First, an important strategic choice for platforms affecting revenues is whether to provide threshold commitment or not. Kickstarter and other AON platforms exert effort to enforce this commitment by prohibiting self-bidding, inhibiting the use of pseudonyms and precluding adjustment of thresholds once set. Meanwhile, in “flexible funding” or “Keep-it-All” crowdfunding, entrepreneurs can always choose to keep all funds. Neglecting transaction costs, this corresponds to no commitment, NC, in our setting.²³ So we can apply Proposition 4 to shed light on this central platform design choice. NC always lowers entrepreneur profits, and if NC leads to more exclusion, all actors, including the platform, are better off with threshold commitment. So empirical work indicating more elastic entrepreneur than consumer participation, *or* that NC tends to provoke minimum price rises, could explain the predominance of AON and justify regulatory support in dissuading manipulation via pseudonyms.²⁴

Second, platforms can also influence outcomes by using project rankings to draw buyers’ attention to particular projects. Their revenue share biases them towards projects they expect will generate high revenues, but does this bias them towards profit-maximizing entrepreneurs to the detriment of not-for-profits? In general, platforms care more for success than profit-maximizing entrepreneurs because of the fixed cost: expected revenue equals expected net profit plus expected cost expenditure. Using Sections 3.1 and 3.2, we show that platforms sometimes prefer not-for-profits.

Proposition 5. (a) When $C \leq Nv_L$, there exist $\bar{q}(C) \geq \hat{q} \geq \underline{q}(C)$ such that the platform strictly prefers (i) not-for-profits if $\underline{q}(C) < q < \bar{q}(C)$, (ii) profit-maximizers when $q > \bar{q}(C)$ and (iii) is indifferent on $q \leq \underline{q}(C)$.

(b) When $C > Nv_L$, the platform strictly prefers (i) not-for-profits when $q \leq \hat{q}$ if $n_I^{BB} < n_I$ and (ii) profit-maximizers when $q > \hat{q}$ and $n_I^{BB} = n_E$.

In essence, the platform biases towards profit-maximizers at high q , but maximizes welfare by selecting not-for-profits at low q . The intuition is clearest in case (a): profits and success rates with not-for-profits are then independent of q , but are both increasing in q with profit-maximizers.²⁵ In an empirical study of Kickstarter, [Pitschner and Pitschner-Finn \(2014\)](#) show that not-for-profit motivated entrepreneurs have higher success rates, consistent with our finding that $n_I^{BB} \leq n_I$. Excluding the top 1%, they also find not-for-profits generate more revenues. Warm-glow (see [Belleflamme et al., 2014](#)) could explain this if such crowdfunders prefer not-for-profits, consistent with anecdotal evidence of

²²See [Agrawal et al. \(2014\)](#); [Belleflamme et al. \(2015\)](#).

²³Indiegogo’s flexible funding actually always disburses funds, but entrepreneurs with our binary investment technology would refund buyers for any below-cost aggregate funds; Indiegogo instructs them to either fulfill all rewards or fully compensate funders.

²⁴To fully investigate flexible funding would require modeling scalable projects ([Cumming et al., 2015](#)).

²⁵Welfare and success-maximizers are equivalent except on $q > \hat{q}$, $n_I^{BB} > n_E$, where welfare-maximizers may be inclusive and then platforms strictly prefer profit or success-maximizers to guarantee exclusion.

funders upset by entrepreneurs raking in profits. However, for-profit platforms may well promote not-for-profit entrepreneurs, even when crowdfunders are purely self-interested. They raise expected revenues via success rates (Proposition 5). They also raise inclusivity (Section 3.3) and reduce minimal prices (Footnote 18), which attract crowdfunders.

In practice, crowdfunding is sequential, with campaigns typically running between 30 and 60 days.²⁶ So platforms can learn about project potential during crowdfunding (see also Section 5.2) and guide buyers to particular projects accordingly. Platforms clearly gain from guiding buyers towards any project close to reaching its threshold, since converting a project into a success adds a share α on *all* the bids so far submitted. This use of project rankings is consistent with the empirical evidence that most projects either fail by a large margin or succeed by a small one (Mollick, 2014). From a welfare perspective, platforms bias attention away from projects with a smaller chance of success and positive externalities, towards projects that, having already passed their thresholds, guarantee a share α on each new bid. This raises the tendency towards viral projects. Platforms also bias towards high cost projects with a low potential to make very large revenues over smaller but safer bets with greater expected profits and social benefits. We motivate the importance of social externalities from small successes in the next subsection.

4.3 Crowd size and crowdfunding costs

In our *i.i.d.* baseline, the private and social benefits of crowdfunding through demand-adaptation and rent extraction are always positive, but could be small. In particular, they shrink away as the crowd grows, because *per capita* demand uncertainty and the relevant hazard rate both fall to zero.²⁷ Given crowdfunding’s added costs (see below) over traditional selling, this suggests that entrepreneurs will opt for the latter when crowds are very large, but we predict a vibrant future for both crowdfunding and traditional selling on two grounds. First, rent extraction and especially demand adaptation benefits may be substantial for surprisingly large crowds, as Illustration 1 shows.²⁸ We predict most crowdfunding projects to be of moderate size, and this is consistent with empirical facts. Second, massive crowds do not preclude aggregate uncertainty when valuations are correlated (see also Section 5.2).

²⁶Section 8.2 of [EH2015](#) shows: (a) substantial adaptation and rent extraction benefits under exogenous sequential bidding; (b) symmetric bidders move simultaneously if they can, as high bids are strategic substitutes. In a richer model, early funders may signal quality to later buyers (see [Agrawal et al., 2015](#)).

²⁷[Norman \(2004\)](#) proves that traditional selling is asymptotically as good as the optimal general mechanism fixing $c = C/N$; Proposition 6 in [EH2015](#) replicates in our discrete type setting.

²⁸Extraction depends on the hazard rate, not the probability of pivotality, as bids are efficient transfers.

4.3.1 Self-selection and crowd size

The media and crowdfunding platforms draw attention to projects, like PebbleWatch and Star Citizen, that attract contributions from tens of thousands of funders, but the representative crowdfunding project is far smaller. For Kickstarter projects, the average number of funders is 101.3, falling to 56.2 on excluding the top one per cent, and 41% were successfully funded, of which 75% raised less than \$10,000.²⁹ So most projects are moderate-sized and less than half are successful. We view these project failures as a sign of market-testing in action: crowdfunding filters out projects with too little demand, only sinking costs in viable projects.

To explain this predominance of moderate sizes and interior success rates, we now add two types of crowdfunding cost to the model. First, the platforms' revenue share α (typically 5–10%) on successful projects; competitive pressures lead platforms to moderate α but are limited by network effects, so traditional finance tends to be cheaper for riskless loans. Second, a (time and effort) cost ε for the entrepreneur to pitch and run her crowdfunding campaign. We let π_α^{CF} denote the re-optimized crowdfunding payoff.³⁰ The entrepreneur now opts for crowdfunding only if $\pi_\alpha^{CF} - \varepsilon \geq \pi^{TS}$.³¹ This inequality determines lower and upper bounds on the *per capita* cost of projects for which crowdfunding is optimal. High cost, low success projects are not undertaken because of the pitching cost; low cost, high success projects use traditional selling to avoid platform fees. The range of crowdfundable projects shrinks with α , ε , and N , and, for any $\alpha, \varepsilon > 0$, vanishes for N sufficiently large. This predicts that most projects self-selecting into crowdfunding will have moderate sized crowds and non-extremal average success rates.

We illustrate with $\varepsilon = 1$ and $\alpha = 0.05$. We first derive a narrow cost range for crowdfunding when the entrepreneur targets a large crowd, by expanding on Illustration 1(ii) with $N = 500$. Recall that traditional selling is viable only when $c = C/N \leq 5$ and that frictionless crowdfunding has strict positive benefits for every $0 < c < v_H = 20$. However, taking account of crowdfunding costs, the entrepreneur opts for crowdfunding only when $4.95 < c < 5.44$. For a typical crowd size of 50, the cost range giving crowdfunding, fixing other parameters, is three times larger ($4.48 < c < 6.54$), and even five times larger when pitching costs scale with N ($4.42 < c < 7.5$). Average crowdfunding success rates are about 30%. Of course, with imprecise targeting, the maximum potential crowd N may far exceed the number of observed funders. In our second example, the entrepreneur emails $N = 10,000$ people, expecting only a small number of them, $\mu = qN = 100$, to have a positive valuation, $v_H = 80$. Aggregate demand k is distributed, approximate

²⁹We exclude the 1.3% of projects with over 1,000 funders in U.C. Berkeley's Fung Institute's Kickstarter data 2009-2014, <http://rosencrantz.berkeley.edu/crowdfunding/index.php>.

³⁰Platform fees effectively reduce buyers' valuations to $(1 - \alpha)v_L$ and $(1 - \alpha)v_H$. This increases the optimal pivot but does not affect the inclusivity choice.

³¹Introducing a cost $\varepsilon^{TS} > 0$ in TS or positive interest rate or credit limit reduces π^{TS} without affecting π_α^{CF} . This would simply raise self-selection of crowdfunding over TS, qualitatively like reducing α .

normal, with mean $\mu = qN = 100$ and standard deviation $\sigma = \sqrt{q(1-q)N} \approx 10$. Fixing $qN = \mu$ as N gets large, aggregate uncertainty converges to $\sqrt{\mu} = 10$. Crowdfunding's adaptation gain remains important relative to expected surplus for a substantial range of costs: $77.8 < c < 96.9$. Its average success rate is now 18%.

These predictions are consistent with observed data. A vast majority of projects have modest goals that can be achieved when reaching a moderate sized crowd of active bidders. Mass emailing is rather ineffective, given distrust of unsolicited emails and costs of inspecting projects. Platform promotion choices can lead to a few very large projects (as discussed in Section 4.2), and correlated preferences can explain why some large projects self-select into crowdfunding (see next section). Indeed, the top 1.3% of Kickstarter projects account for nearly half of total crowdfunding revenues, but the numerous smaller projects have a bigger effect on welfare relative to their size, as each of their many successes has welfare benefits beyond the immediate realized trades between entrepreneur and funders: successes enhance entrepreneurial career prospects (see Section 3.2) and generate ongoing sales of the crowdfunded product (see Section 5).

4.3.2 Group preference model

In practice, consumer tastes can be highly correlated within demographic and other groups, like jazz-lovers.³² We now consider a crowd of size Nm , consisting of N groups with m members each. Each group's common valuation is drawn independently from a well-behaved distribution G on \mathbb{R}_+ ; so group values are independent while intra-group correlation is, for simplicity, perfect. Fixing N , aggregate demand uncertainty remains substantial for arbitrarily large m . Rent extraction via crowdfunding's price discrimination becomes ineffective as in the poor targeting example. So to simplify, we restrict to a single price p (also relevant when $m = 1$ and N is quite big if buyers need simplicity or treat small pivotality probabilities as zero). This simplification also allows us to extend beyond the two-type distributions assumed in the baseline and to deal with general well-behaved distributions G . Crowdfunding still yields substantial private and social benefits compared to TS, by adapting production to demand using $T = C$.

With price p , demand is $k \cdot m$ with probability $f_k^N(q(p))$ where $q(p) = 1 - G(p)$ and k now represents the number of groups with a valuation weakly above p . The entrepreneur's profit is then,

$$\pi(p) = \sum_{k=n(p)}^N f_k^N(kmp - C)$$

where $n(p) = \lceil C/mp \rceil$. Inside region $n(p) \equiv n$, price p satisfies the first-order condition,

$$0 = q(p) + pq'(p) + h_n [(n-1)p - cN]q'(p)$$

³²With correlated preferences, [Cremer and McLean's \(1988\)](#) general optimum extracts full rent, but breaks crowdfunding's attractive features.

where $c = C/Nm$ denotes *per capita* cost; we fix c . So inside each such region, p strictly increases with c , in contrast to the standard result of fixed cost independence. At region boundaries, n rises discretely and p jumps down to mitigate the resulting fall in success. Note that the entrepreneur sets $p^{TS} = \arg \max pq(p)$ if producing under TS, so crowdfunding's price is strictly higher for any $c > 0$. This is because crowdfunding alleviates the negative effect of raising p on $q = 1 - G(p)$; it does so by adapting production to avoid losses in the then more frequent low demand states.

Proposition 6. *With group-preferences, crowdfunding is valuable for arbitrarily large crowds: even with a single-price, the entrepreneur sets $T = C$ and $p > p^{TS}$ and makes a per-capita gain over traditional selling that is strictly positive and independent of m . This also increases consumer and total welfare when TS is in-viable.*

Given such preferences, even promoters of large concerts can have doubts about demand sufficiency (see also the arbitrarily large crowd results in the next section). Concerts and sports events usually engage in advance sales and sometimes cancel when demand proves insufficient, but the crowdfunding format is rarely salient because the typically large crowd size implies very little gain from committing to a threshold T (see Section 4.3.1). So Proposition 6 applies. It suggests that organizers of such large events may want to inflate their prices. Venue capacity constraints also push up prices.

5 Ex-post sales: credit and price-dynamics

Turning to the “funding” in crowdfunding, we now model limited crowdfunding reach and ex-post sales revenues that cannot be used to fund production. We characterize when crowdfunding substitutes for traditional finance but also when crowdfunding raises demand for, and supply of, finance: its credible demand signals reassure financiers. This mutual complementarity sheds light on evidence that venture capital and angel investors have been joining forces with entrepreneurs after successful crowdfunding campaigns (see [Mollick and Kuppuswamy, 2014](#), Table 3). The first subsection adds ex-post buyers, retaining independent demands. There, the substitution and complementarity effects are all driven by gauging the ex-ante demand. The second subsection introduces project quality differences. This generates a correlation between ex-ante and ex-post demands, so that crowdfunding's market-test signals future demand. We use this to generate a powerful complementarity result for markets where most demand arrives ex-post. We also use this project heterogeneity to study price dynamics. In particular, we explain why crowdfunding prices usually rise after a big crowdfunding success, in contrast to other crowdfunding papers that predict prices will always fall by the durable good monopoly logic. The key insight is again that the entrepreneur learns about market demand. Crowdfunding adapts

ex-post prices to expected future demand, with prices rising when crowdfunding is especially successful.³³

5.1 Crowdfunding and credit

In the preceding sections, we completely abstracted from credit constraints, proving that funding is not fundamental to crowdfunding but in practice buyers do generally pay in advance, potentially funding production. So we now ask how crowdfunding affects demand for credit from traditional finance (TF). We assume the entrepreneur has no personal funds to invest in her project and we study bank credit, normalizing the market rate of interest to zero. We denote the entrepreneur’s borrowing by B .

Corollary 1 proves that in the baseline setting, aggregate funds or revenues from buyers always cover the entrepreneur’s cost when she wants to produce. There, crowdfunding substitutes for traditional finance, reducing credit demand to $B^{CF} = 0$; this fall is strict for $C \leq \max\{Nv_L, qNv_H\}$ where traditional selling demands $B^{TS} = C$. However, in general, crowdfunding can also increase credit demand and supply. We note two minor points before focusing on the major reason. First, Corollary 1 does not apply to multi-type distributions (see Section 6); this relies on commitment to produce at a loss in some states. Second, if crowdfunding buyers commit ex-ante to buy but only pay ex-post, upon delivery say, the entrepreneur would need credit as sales revenues would arrive too late to fund production. In principle, this crowdfunding variant could eliminate any entrepreneurial moral hazard while still implementing adaptation and rent-extraction, but it is standard to force ex-ante payments to avoid needing third-party enforcement of all the buyers’ purchase commitments.

The main reason for credit demand is *limited crowdfunding reach*. Owing to costs of project inspection and advertising, many potential buyers do not participate during crowdfunding but do learn about projects after a crowdfunding success. Revenues from selling to these buyers who are reached ex-post cannot fund the fixed cost C . To model this, we distinguish two classes of buyers, all with the same independent tastes: N_1 crowdfunding participants or “funders” who can fund by buying in period 1, ex-ante, and N_2 “new buyers” who can only buy in period two, ex-post. Labels refer to ability, not choice: a funder can fund *or* buy in period 2 or not buy at all. Neither funders nor new buyers ever want to buy more than one unit. So we have:

Timing with two selling periods. (1) The entrepreneur sets crowdfunding offer (p, T) . (2) Funders choose bids. (3) If funds do not reach T , the game ends with no production and no payments.³⁴ If funds reach T , the entrepreneur receives the funds, sinks her fixed

³³Capacity constraints with demand uncertainty (*e.g.*, Dana, 2001; Gale and Holmes, 1993) or consumers learning preferences (*e.g.*, Courty and Li, 2000; Möller and Watanabe, 2010; Nocke et al., 2011) can also explain price rises but our entrepreneurial learning resonates best with crowdfunding evidence.

³⁴Platforms stop failed projects from selling to funders to prevent fee evasion and enable commitment.

cost C , delivers the goods to active funders and the game continues. (4) The entrepreneur sets her ex-post price p_2 . (5) New buyers and funders who did not buy ex-ante decide whether to purchase at p_2 .

We initially focus on $q > \hat{q}$, $N_2/N_1 \geq \hat{q}/(q-\hat{q})$ so that, though unable to commit on p_2 ex-ante, the entrepreneur always sets $p_2 = v_H$ ex-post (even if facing N_1 L -type funders who waited and N_2 unknown new buyers). In optimal crowdfunding, she sets $p = p_2 = v_H$ and $T = C - N_2qv_H < C$, reduced from C by the expected ex-post revenues. The corresponding pivotal number of H -types, $\hat{n}_E = \left\lfloor \frac{C}{v_H} - N_2q \right\rfloor \leq n_E = \left\lfloor \frac{C}{v_H} \right\rfloor$; we assume $N_2 > 1/q$ to guarantee a strict inequality: $\hat{n}_E < n_E$. Finally, assuming $C < (N_1 + qN_2)v_H$ to avoid trivial non-production, we have $\hat{n}_E \leq N_1$.

The demand for credit B^{CF} under crowdfunding is stochastic; it depends on state k . If $n_E \leq N_1$, there are three ranges: (i) for $k < \hat{n}_E$, there is no production and $B_k^{CF} = 0$; (ii) for $k \in [\hat{n}_E, n_E)$, $B_k^{CF} = C - kv_H > 0$; (iii) for $k \in [n_E, N_1]$, $B_k^{CF} = 0$. If $n_E > N_1$, range (ii) shrinks to $[\hat{n}_E, N_1]$ and range (iii) disappears.

In traditional selling (TS), only $N_1 + N_2$ matters. The optimal posted price is again $p_2 = v_H$ and the project is viable if $C \leq \hat{C}^{TS} \equiv (N_1 + N_2)qv_H$. Lacking any ex-ante revenues, the entrepreneur must borrow the full fixed cost C whenever she produces, so her credit demand $B^{TS} = C$ if $C \leq \hat{C}^{TS}$ and $B^{TS} = 0$ if $C > \hat{C}^{TS}$, both independent of k , which is only learned after finance and production under TS. So we have,

Proposition 7. (a) For high fixed costs $C > \hat{C}^{TS}$, crowdfunding raises credit demand compared to traditional selling: $B_k^{CF} \geq B_k^{TS}$ for all k , strictly on $k \in [\hat{n}_E, n_E) \neq \emptyset$. (b) For $C \leq \hat{C}^{TS}$, crowdfunding reduces credit demand, strictly on $k > 0$, all k if $\hat{n}_E > 0$.

In case (a), crowdfunding raises borrowing demand since the adaptation makes production viable and the crowd's funds do not always cover cost; it strategically complements traditional finance. Concretely, a market test revealing funders' demand $k \geq \hat{n}_E$ makes production attractive and cannot fully fund C if $k < n_E$; state k lies in this range (ii) with positive probability. In case (b), crowdfunding lowers credit demand for two reasons. First, crowdfunding is a strategic substitute since adaptation to avoid producing in unprofitable states lowers the production probability below 1. Second, crowdfunding is a direct substitute source of credit on all production states with $k > 0$.³⁵

Corollary 3. Crowdfunding and traditional finance are mutual complements for high fixed costs and crowdfunding is a substitute for credit when fixed costs are low.

³⁵The adaptation effect is trivial on $C \leq N_2qv_H$ since with $\hat{n}_E = 0$, production is optimal for all distinguishable states $k \in [0, N_1]$ (crowdfunding cannot gauge new buyers' demands). The second effect is never trivial, but we doubt crowdfunding ever purely substitutes for credit, because its costs tend to be higher: in Section 4.3.1 with $\alpha > 0$, we showed how traditional selling, and finance, instead substitute for crowdfunding when crowdfunding's adaptive and extractive benefits are small. Raising the interest rate on credit B has the opposite effect and encourages the entrepreneur to raise T to reduce B^{CF} . Also, capping credit at \bar{B} forces the entrepreneur to raise T to $C - \bar{B}$ if $\bar{B} < N_2qv_H$.

In case (a), in addition to raising credit demand by making production viable, crowdfunding credibly reveals the crowd's demand, making financiers willing to supply credit. Conversely, finance complements crowdfunding by enabling production on region (ii); without finance, crowdfunding can only produce on range (iii).

Turning to lower q , exclusion becomes less effective as funders may wait, hoping for p_2 below p . Inclusive strategies, being unaffected, become more likely. Consider $N_2 = 0$ and $q \leq \hat{q}$. Inclusion remains optimal and the entrepreneur can implement the optimum without need for credit if she sets $p = v_L$ and $T = \bar{T}_{n_I}$, since even L -types are willing to pay in advance. With free credit access, she can also set $p = \bar{b}_{n_I}$ as a unique crowdfunding price, with threshold $T = n_I \bar{b}_{n_I}$, and selling to L -types ex-post. However, if $C > n_I \bar{b}_{n_I}$, credit constraints preclude this solution. In general, the optimum requires multi-price crowdfunding.

5.2 Price dynamics and signaling future demand

So far, prices can only fall, driven by dynamic monopoly with high types buying earlier, but in practice, entrepreneurs often raise prices ex-post. We now capture the intuitive idea that prices rise when high crowdfunding sales signal high *future* demand. To do so, we let projects differ by their H -type probabilities, $q^G > q^B$. Ex-ante crowdfunding sales now signal project quality, G or B , and hence future demand. The entrepreneur uses this market-test in three ways: (1) to adapt production to expected future demand, as well as realized demand; (2) to inform traditional financiers about about future, as well as current, profitability; (3) to optimize ex-post pricing. We now show how this substantially extends the relevance of our complementarity results beyond Section 5.1, and explains why prices go up after more successful crowdfunding campaigns.

Project quality. Nature determines project quality, with G and B equiprobable, in a prior stage 0. Nobody observes project quality directly. Buyers privately learn their, conditionally independent, types; funders learn by inspecting ex-ante; new buyers learn ex-post. $\text{Prob}(v_H|G) = q^G$ and similarly for B , with $0 < q^B \leq q^G < 1$; average quality is $\bar{q} = (q^B + q^G)/2$. Buyers' private valuations provide affiliated signals of project quality.

Case A demonstrates how crowdfunding now adapts production and complements traditional finance by signalling *future* demand. Complementarity is then important even for large N_2/N_1 where adapting to the demand from the N_2 new buyers arriving ex-post is key. Recall that in Section 5.1, crowdfunding informed about total demand only by revealing funders' demand, but here it also signals the demand of the N_2 new buyers. Case B demonstrates price dynamics.

Case A: Signalling future demand $\bar{q} > \hat{q}$, $C \in (\bar{q}(N_1 + N_2)v_H, (N_1 + q^G N_2)v_H)$

Clearly, $p = v_H$. Also $p_2 = v_H$ is optimal whenever crowdfunding sales justify production given the cost lower bound (that makes TS inviable). Now crowdfunding reveals not only the number k of H -types in the crowd N_1 , but this also provides a signal of project quality, with accuracy increasing in N_1 . If N_1 is large enough, the entrepreneur adapts to this signal of current and future demand by producing when k is sufficiently high. This signal also attracts traditional finance, which is needed and valued when C exceeds ex-ante crowdfunding revenues and the ex-post crowd N_2 justifies production. In sum,

Proposition 8. *Crowdfunding and traditional finance are mutual complements in case A for sufficiently large N_1 .*

Case B: Price dynamics $\bar{q} < \hat{q} < q^G$, $N_1 v_L < C < (N_1 + N_2) v_L$

Since $q^B < \hat{q} < q^G$, if the entrepreneur learns k , she can adapt her ex-post price, even where adapting production and extracting rent help little. Setting $p = v_L$ and T just above $N_1 v_L$, so that n is positive but small, she learns k , just inducing H -types to bid above v_L and only minimally reducing the success rate. For large N_1 , the success rate is nearly one and the entrepreneur learns whether her project is good or bad with great accuracy. Optimizing her ex-post price, her overall expected profit is then approximately $N_1 v_L + \frac{1}{2}(v_L + v_H q^G) N_2 - C$, larger than $(N_1 + N_2) v_L - C$ because $q^G > \hat{q}$. This price dynamic readily explains the phenomenon of higher ex-post prices, in big successes such as PicoBrew Zymatic's automatic beer brewing appliance, sold at \$1599 or less during crowdfunding on Kickstarter and sold ex-post for \$1999 dollars.³⁶ Extending the argument of Section 4.2, platforms can also use this inference of project quality to update their ranking of projects dynamically during bidding.

To the extent entrepreneurs can influence N_1 and N_2 , they value high N_1 for raising crowdfunding's accuracy as a signal of demand and project type, but they must weigh this against advertising costs and the need to compensate buyers for their higher costs of inspecting value ex-ante than after the good has been produced and put up for sale. Also, entrepreneurs may limit the quantity sold in crowdfunding, as did Picobrew and PebbleWatch, to exploit their better adapted prices on a larger ex-post market.

6 General mechanism design analysis

The optimal general mechanism maximizes profit *without* imposing crowdfunding's characteristic restriction AFT that aggregate funds determine production. Cornelli (1996) solves this with a *continuum* of buyer types using Myerson's (1981) virtual valuation approach. EH2015 solve with a generic *discrete* type space, parallelling Bergemann and

³⁶www.kickstarter.com/projects/1708005089/picobrew-zymatic-the-automatic-beer-brewing-applia/description.

Pesendorfer's (2007) adaptation to treat discrete types in optimal auctions. Here we provide the explicit solution in our two type setting. This verifies that optimal crowdfunding is then also optimal within the class of general mechanisms.

Type L 's virtual valuation is defined by,

$$w_L = v_L - (v_H - v_L) \frac{q}{1 - q} = \frac{v_L - qv_H}{1 - q}$$

For type H , $w_H = v_H$. Production is optimal in states k where the sum of positive virtual valuations covers the fixed cost. For $q > \hat{q}$, the solution is quite trivial: $w_L < 0$ so the general optimum only includes H -types and production requires $kw_H \geq C$. Since $w_H = v_H$, this corresponds exactly to the exclusion solution under crowdfunding. For $q \leq \hat{q}$, both types are included and the condition for production is:

$$(N - k)w_L + kw_H \geq C \tag{9}$$

Substituting the virtual valuations, this condition proves to be equivalent to $k \geq n_I$ from Lemma 1. So the probability that an L -type buyer obtains the good is given by $P_L = S_{n_I}^{N-1}$ and for an H -type buyer this probability is $P_H = S_{n_I-1}^{N-1}$. Individual rationality constrains the maximal expected transfer from an L -type to $\tau_L = v_L P_L$. The maximal expected transfer τ_H from an H -type is given by the binding incentive compatibility condition that an H -type not masquerade as an L -type: $v_H P_H - \tau_H = v_H P_L - \tau_L$. Dividing by P_H yields $\tau_H/P_H = v_H(1 - P_L/P_H) + v_L(P_L/P_H) = (1 - h_{n_I})v_H + h_{n_I}v_L$. This equals b_H and similarly $\tau_L/P_L = b_L$, so the optimal crowdfunding mechanism yields the same allocation and expected payments as the general optimal mechanism.

Proposition 9. *In our baseline setting, the general optimum outcome can be implemented via crowdfunding, despite its aggregate threshold restriction (AFT).*

This result proves that mixed strategies cannot raise profits. It also shows that the two type case escapes two unattractive features of the general optimum: that the production rule generally depends on the composition, not just aggregate, of individual bids, and must sometimes commit to ex-post losses in certain states (cf., our Corollary 1). Now with three or more types of buyer: (1) crowdfunding restriction AFT can indeed be strictly costly and (2) the general mechanism and also crowdfunding can require committing to produce at a loss. We show this in a pair of three type examples. Buyer valuations are drawn independently from $V = \{v_1, v_2, v_3\}$ with probabilities $\mathbf{q} = (q_1, q_2, q_3)$. The demand state is now summarized by $\mathbf{k} = (k_1, k_2, k_3)$ where k_j is the number of buyers with valuation v_j for each $j = 1, 2, 3$.

Example 2. Crowdfunding cannot always achieve the general optimum

Let $N = 2$, $V = \{0, 1, 2\}$ and $\mathbf{q} = (1/4, 1/2, 1/4)$, with $1 < C < 2$. Then the vector

of virtual valuations is $\mathbf{w} = (-3, 1/2, 2)$ so the general optimum excludes type 1 and has production in states in $K^* = \{(0, 0, 2), (1, 0, 1), (0, 1, 1)\}$. Conditional on production, payments are $\mathbf{b}^* = (0, 1, 7/4)$ and the expected profit is $(18 - 7C)/16$. Crowdfunding cannot implement this outcome because to have production for all $k \in K^*$ would require a threshold of at least $T = \min_{\mathbf{k} \in K^*} \{\mathbf{b}^* \cdot \mathbf{k}\} = 7/4$, just reached in state $\mathbf{k} = (1, 0, 1)$, and this also generates production success in state $(0, 2, 0)$ which is not in K^* .³⁷

Notice that the general optimum requires the entrepreneur to produce in state $(1, 0, 1)$, despite making a loss if $C > 7/4$ as she then earns only $7/4$ from the single type 3 buyer. The positive commitment to produce in this state allows her to extract a high rent in other states in the production set K^* . We now show how crowdfunding can share this feature of incurring a loss in some states, using an example where crowdfunding implements the general optimum despite the three types. Example 2 still hints at room for a more complex mechanism to displace crowdfunding, but we believe crowdfunding's attractive simplicity will ensure that its current practical relevance endures.

Example 3. Crowdfunding may involve losses

Let $N = 2$, $V = \{0, 7, 10\}$ and $\mathbf{q} = (2/5, 2/5, 1/5)$, with $9.4 < C < 10$. Then virtual valuations are $\mathbf{w} = (-10.5, 5.5, 10)$ so the general optimum excludes type 1's and the production set is $K^* = \{(0, 0, 2), (1, 0, 1), (0, 1, 1), (0, 2, 0)\}$. This can be implemented using the crowdfunding mechanism, $p = 7$, $T = 9.4$ and $\mathcal{B} = \{0, 7, 9.4\}$.³⁸ In state $\mathbf{k} = (1, 0, 1)$, the entrepreneur produces at a loss, $9.4 - C < 0$.

Since traditional selling is inviable here, this also illustrates how crowdfunding and traditional finance can be mutual complements without any ex-post buyers.

7 Concluding remarks

We have characterized the optimal design of crowdfunding in a private value environment. We demonstrated the twin roles of crowdfunding's threshold mechanism in adapting production and pricing to the crowd's revealed demand and its signal of future demand, and in price discrimination that enhances adaptation except when excessive thresholds waste trade opportunities. Even in our *i.i.d.* model, both benefits can be substantial for surprisingly large crowds, and, with binary types, more general mechanisms relaxing crowdfunding's reassuring constraints cannot deliver higher profits. Sections 3 and 4 stressed the fundamental role of market-testing. Section 5 showed how reward-based crowdfunding can substitute for traditional finance *or* can be mutually complementary.

³⁷To understand why crowdfunding can fail to implement the general optimum, note that the minimum price ensures the lowest included type's bid exactly equals that type, while higher types bid less. So the AFT gives this least type the good too often compared to the general optimum, which weighs types by their virtual valuations; the latter only fully count the highest type.

³⁸Bid restrictions are not binding; type 3's bid 9.4 in any case.

Introducing investment-based elements, as in P2P lending and equity-crowdfunding, might, in principle, fully substitute for traditional finance, but we expect economies of scale from centralized monitoring (Diamond, 1984) and expertise (Gompers and Lerner, 2001) to complement the “wisdom of the crowd.” Adding financial rewards in the project-quality model of Section 5 creates a common value feature (see Hakenes and Schlegel, 2014). Our tractable framework could be extended to study *interdependent* prices, an important feature of reality in investment-based crowdfunding, missing in the literature.

Our reward-based insights are also relevant for donation-based crowdfunding. In the model, funders only cared for the good, but the results also apply exactly if funders gain recognition or “warm-glow” values v_L, v_H from contributing at least the minimum bid to a charitable cause or public good. A thorough analysis should further enrich the preference assumptions and move beyond the simple fixed-cost scenario to study scalable projects that deliver greater benefits when more is invested (see Cumming et al., 2015).

We showed how crowdfunding costs, including pro-rata revenue fees to platforms, led entrepreneurs to self-select when they expect intermediate success rates; platforms are chasing fool’s gold if they seek only project successes as that defeats crowdfunding’s purpose as a market-test. Analyzing demand aggregation led us to predict modest-sized crowds and short-run profits for the large majority of projects, consistent with empirical evidence. The pro-rata fees also explain why platforms use project rankings to guide buyers toward projects with higher revenue promise. This generated a bias towards virality and high cost projects, but platforms do sometimes favor not-for-profit entrepreneurs. We also showed why platforms often enable threshold commitment. Preventing self-bidding is then vital, particularly if crowdfunding is to serve as a signal for attracting traditional finance when there is a large ex-post market.

In sum, crowdfunding enables many entrepreneurs to bring otherwise infeasible projects to life. Its rent extraction role is particularly important at high costs. While no other paper models multiple prices within crowdfunding, Hansmann (1981) already marshals the evidence in Baumol and Bowen (1968) to argue that “voluntary price discrimination” is critical to the survival of theatres, museums and opera. He claims that vertical differentiation alone cannot explain observed contributions. Our formal derivation of rent-extraction permits a parallel conjecture that could be tested by exploiting the fact that crowdfunding is, in practice, sequential: we predict that high price rewards are chosen less often after than before the threshold is reached, since pivotality motives then disappear. Further testing of hypotheses from this analysis may require cunning techniques to estimate fixed costs and valuation distributions, but has the potential to draw more refined policy conclusions.

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Appendix A

Mostly omitting argument q , we start by stating some useful mathematical results relating $f_k^M(q)$, $S_n^M(q) = \sum_{k=n}^M f_k^M(q)$ and $h_n(q) = f_{n-1}^{N-1}(q)/S_{n-1}^{N-1}(q)$.

Lemma A.1.

- (i) $f_k^N = qf_{k-1}^{N-1} + (1-q)f_k^{N-1}$
- (ii) $S_n^N = S_{n-1}^{N-1} - (1-q)f_{n-1}^{N-1}$
- (iii) $\sum_{k=n}^N kf_k^N = qNS_{n-1}^{N-1}$ and $\mathbb{E}[k|k \geq n] = \frac{qN}{1-(1-q)h_n}$ for all $N \geq 1$ and $0 \leq n \leq N$.
- (iv) $\sum_{k=n}^N (N-k)f_k^N = (1-q)NS_n^{N-1}$, for all $N \geq 1$ and $0 \leq n \leq N$.
- (v) $\frac{\partial f_k^M(q)}{\partial q} = f_k^M \frac{k-Mq}{q(1-q)}$
- (vi) $\frac{\partial S_n^N(q)}{\partial q} = Nf_{n-1}^{N-1}$
- (vii) h_n is strictly increasing in n for $0 \leq n \leq N$, with $h_0 = 0$ and $h_N = 1$.
- (viii) For $0 < n < N$, $\frac{\partial h_n(q)}{\partial q} < 0$.
- (ix) $n(1-q)h_n \geq n - qN$ where the inequality is strict when $q > 0$ and $n < N$

Proof of Lemma A.1.

- (i) is immediate on expanding on any one draw and $N - 1$ other independent draws.
(ii) Summing (i) from $k = n$ to N and recalling that $f_N^{N-1} = 0$

$$\begin{aligned} S_n^N &= qS_{n-1}^{N-1} + (1-q)S_n^{N-1} \\ &= qS_{n-1}^{N-1} + (1-q)(S_{n-1}^{N-1} - f_{n-1}^{N-1}) \\ &= S_{n-1}^{N-1} - (1-q)f_{n-1}^{N-1} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \sum_{k=n}^N k f_k^N &= \sum_{k=n}^N k q^k (1-q)^{N-k} \frac{N!}{(N-k)!k!} \\ &= Nq \sum_{k=n}^N q^{k-1} (1-q)^{N-1-(k-1)} \frac{(N-1)!}{(N-1-(k-1))!(k-1)!} \\ &= Nq \sum_{k=n}^N f_{k-1}^{N-1} = Nq \sum_{k=n-1}^{N-1} f_k^{N-1} = Nq S_{n-1}^{N-1} \end{aligned}$$

From (ii), $S_n^N/S_{n-1}^{N-1} = 1 - (1-q)h_n$, so $\mathbb{E}[k|k \geq n] = \frac{qN S_{n-1}^{N-1}}{S_n^N} = \frac{qN}{1-(1-q)h_n}$.

- (iv) Using (ii) and (iii),

$$\begin{aligned} \sum_{k=n}^N (N-k) f_k^N(q) &= N((1-q)(S_{n-1}^{N-1} - f_{n-1}^{N-1})) - Nq S_{n-1}^{N-1} \\ &= N(1-q)(S_{n-1}^{N-1} - f_{n-1}^{N-1}) \\ &= N(1-q)S_n^{N-1} \end{aligned}$$

$$\begin{aligned} \text{(v) Differentiating,} \quad \partial f_k^M(q)/\partial q &= \binom{M}{k} q^{k-1} (1-q)^{M-k-1} \left[k(1-q) - (M-k)q \right] \\ &= \frac{\binom{M}{k} q^k (1-q)^{M-k}}{q(1-q)} (k - Mq) = f_k^M \frac{k - Mq}{q(1-q)} \end{aligned}$$

- (vi) Differentiating the summation that defines S_n^N using (v) gives,

$$\begin{aligned} \partial S_n^N(q)/\partial q &= \sum_{k=n}^N (k - Nq) f_k^N / q(1-q) \\ &= (Nq S_{n-1}^{N-1} - Nq S_n^N) / q(1-q) \text{ (from (iii))} \\ &= (S_n^N + (1-q)f_{n-1}^{N-1} - S_n^N)N / (1-q) \text{ (from (ii))} \\ &= N f_{n-1}^{N-1} \end{aligned}$$

- (vii) From the definition it is clear that $h_0 = 0$ and $h_N = 1$. We will show that h_n is

strictly increasing by induction. As a first step, note that $h_N = 1 > h_{N-1}$ since $f_{N-1}^N > 0$, for all $q \in (0, 1)$. Now suppose that $h_N > h_{N-1} > \dots > h_{n+2} > h_{n+1}$ for $N-1 \geq n+1 \geq 0$. We have to show that $h_{n+1} > h_n$ follows.

Note that

$$h_{n+2} > h_{n+1} \Leftrightarrow \frac{f_{n+1}^{N-1}}{S_{n+1}^{N-1}} > \frac{f_n^{N-1}}{S_n^{N-1}} \Leftrightarrow \frac{S_n^{N-1}}{f_n^{N-1}} > \frac{S_{n+1}^{N-1}}{f_{n+1}^{N-1}} \quad (*)$$

Next observe that for any $N-1 \geq k \geq 0$,

$$\frac{f_{k+1}^{N-1}}{f_k^{N-1}} = \frac{\binom{N-1}{k+1} q^{k+1} (1-q)^{N-k-2}}{\binom{N-1}{k} q^k (1-q)^{N-k-1}} = \frac{q}{1-q} \frac{N-k-1}{k+1}$$

This is clearly decreasing in k so that in particular,

$$\frac{f_n^{N-1}}{f_{n-1}^{N-1}} > \frac{f_{n+1}^{N-1}}{f_n^{N-1}}$$

Combined with the induction hypothesis expressed as (*), we have

$$\frac{S_n^{N-1}}{f_{n-1}^{N-1}} > \frac{S_{n+1}^{N-1}}{f_n^{N-1}}$$

Adding 1 to both sides of the inequality yields

$$\frac{S_{n-1}^{N-1}}{f_{n-1}^{N-1}} > \frac{S_n^{N-1}}{f_n^{N-1}}$$

which is precisely $1/h_n > 1/h_{n+1}$, completing the proof by induction.

$$\begin{aligned} \text{(viii)} \quad \frac{\partial h_n(q)}{\partial q} &= \left[(\partial f_{n-1}^{N-1} / \partial q) \sum_{k=n-1}^{N-1} f_k^{N-1} - f_{n-1}^{N-1} \sum_{k=n-1}^{N-1} (\partial f_k^{N-1} / \partial q) \right] / (S_{n-1}^{N-1})^2 \\ &= f_{n-1}^{N-1} \left[(n-1 - (N-1)q) \sum_{k=n}^{N-1} f_k^{N-1} - \sum_{k=n}^{N-1} f_k^{N-1} (k - (N-1)q) \right] / q(1-q)(S_{n-1}^{N-1})^2 \\ &= \frac{f_{n-1}^{N-1} \sum_{k=n}^{N-1} f_k^{N-1} (n-1-k)}{q(1-q)(S_{n-1}^{N-1})^2} < 0 \end{aligned}$$

The inequality follows from the facts that the summation is over $k > n-1$, $f_k^{N-1} > 0$ on the summation range and $f_{n-1}^{N-1} > 0$ for $n \geq 1$ and the summation range is non-trivial for $n \leq N-1$. Note that when n takes its extremal values of $n=0$ and $n=N$, the derivative equals zero since h_n is then fixed at 0 and 1, respectively.

(ix) Clearly the inequality holds when $q=0$ or $n=N$. Observe next that $\forall n < N$, $n < \mathbb{E}[k | k \geq n] = \frac{qN}{1-(1-q)h_n}$ by Lemma A.1(iii), so $qN > n(1 - (1-q)h_n)$ as claimed. ■

Proof of Lemma 1. From (5) and Lemma A.1(iii),

$$\begin{aligned}
\pi_n^I - \pi_{n+1}^I &= (Nv_L - C)f_n^N + (v_H - v_L)qN(f_{n-1}^{N-1} - f_n^{N-1}) \\
&= q(Nv_H - C)(f_{n-1}^{N-1} - f_n^{N-1}) + (Nv_L - C)f_n^{N-1} \quad \text{by Lemma A.1(i)} \\
&> 0 \Leftrightarrow \frac{n(1-q)}{(N-n)q} = \frac{f_{n-1}^{N-1}}{f_n^{N-1}} > \frac{C - Nv_L + q(Nv_H - C)}{q(Nv_H - C)}
\end{aligned}$$

The statements follow because $n(1-q)/((N-n)q)$ is increasing in n . ■

Proof of Proposition 1. We show that $\pi_{n_I}^I(q) \geq \pi_{n_E}^E(q)$ if and only if $q < \hat{q}$ with a strict inequality on $q < \hat{q}$ if $n_I < N$ and on $q > \hat{q}$ if $n_E < N$. Below we prove the stronger claim that for any N and any $0 \leq n < N$, $\pi_n^I(q) > \pi_n^E(q)$ if and only if $q < \hat{q}$. The result for optimized strategies follows quickly from this. Consider the case with $q < \hat{q}$. If $n_I < N$ and $n_E = N$ then $\pi_{n_I}^I > \pi_N^I = (Nv_H - C)q^N = \pi_N^E = \pi_{n_E}^E$.³⁹ If $n_I, n_E < N$ then $\pi_{n_I}^I \geq \max_{n < N} \{\pi_n^I\} > \max_{n < N} \{\pi_n^E\} = \pi_{n_E}^E$. The proof for $q > \hat{q}$ is an exact parallel. We now prove the stronger claim.

$$\begin{aligned}
\pi_n^I(q) - \pi_n^E(q) &= \sum_{k=n}^N f_k^N(q) \left[(N-k)v_L - k(v_H - \bar{b}_n) \right] \\
&= N \left[(1-q)S_n^{N-1}v_L - qS_{n-1}^{N-1}(1-h_n)(v_H - v_L) \right] \\
&\quad \text{(using respectively Lemma A.1(iv),(iii) and Eq. (4))} \\
&= NS_n^{N-1} \left[(1-q)v_L - q(v_H - v_L) \right] \\
&= NS_n^{N-1}(v_L - qv_H) \\
&> 0 \Leftrightarrow q < \hat{q}
\end{aligned}$$

for any $n \in \{0, 1, \dots, N-1\}$ since then $S_n^{N-1} > 0$. Proposition 9 justifies the restriction to pure strategies. ■

Proof of Corollary 2. For $q > \hat{q}$, where exclusion is optimal, the intuitive result that profits are decreasing in C and increasing in q is easily verified from the profit expression:

$$\pi_{n_E}^E = \sum_{k=n_E}^N f_k^N(kv_H - C) = \mathbb{E}_k[\max\{0, kv_H - C\}]$$

where \mathbb{E}_k denotes the expectation operator. Since an increase in q induces a first-order stochastic dominating distribution of k , and the expectation is taken over an increasing (utility) function, the expectation is increasing in q . The impact of C is more immediate: profits fall at the rate $S_{n_E}^N$.

For $q \leq \hat{q}$, note that $\pi_n^I = (Nv_L - C)S_n^N + (v_H - v_L)qNf_{n-1}^{N-1}$, which is clearly strictly

³⁹This statement holds generically, but the inequality is replaced by an equality at the knife-edge case where $n_I = N-1$ and $n = N$ deliver identical payoffs, *i.e.* where $\tilde{n}_I = N-1$. This trivial complication is just a result of the fact that profits are continuous in C, q but the integer-valued n_I is not.

decreasing in C . Taking derivatives with respect to q yields,

$$\begin{aligned}
\frac{\partial \pi_n^I}{\partial q} &= (Nv_L - C)\left(\frac{\partial S_n^N}{\partial q}\right) + N(v_H - v_L)\left(f_{n-1}^{N-1} + \frac{\partial f_{n-1}^{N-1}}{\partial q}q\right) \\
&= (Nv_L - C)Nf_{n-1}^{N-1} + N(v_H - v_L)f_{n-1}^{N-1}\left(1 + q\frac{n-1-(N-1)q}{q(1-q)}\right) \\
&\hspace{15em}(\text{by Lemmas A.1(vi) and (v)}) \\
&= (Nv_L - C)Nf_{n-1}^{N-1} + N(v_H - v_L)f_{n-1}^{N-1}\left(\frac{n-Nq}{1-q}\right) \\
&= \frac{Nf_{n-1}^{N-1}}{1-q}\left((Nv_L - C)(1-q) + (v_H - v_L)(n-Nq)\right) \\
&= \frac{N(v_H - v_L)f_{n-1}^{N-1}}{1-q}\left(n - \tilde{n}_I\right)
\end{aligned}$$

Recall that $\tilde{n}_I = \frac{C - Nv_L + q(Nv_H - C)}{v_H - v_L}$ and $n_I = \lceil \tilde{n}_I \rceil$. So $n_I > \tilde{n}_I$ except at critical values of q at which $n_I = \tilde{n}_I$. These exceptional values have measure zero; they occur on the boundary between strategy types. It follows that the maximal profit $\pi_{n_I}^I$ is strictly increasing in q . ■

Proof of Proposition 2. For the missing part of the proof, note that expected welfare is: $W^I = \sum_{k=n_I}^N f_k^N(kv_H + (N-k)v_L - C)$ under inclusivity; $W^E = \sum_{k=n_E}^N f_k^N(kv_H - C)$ under exclusivity. For all $k \geq n_I^{BB}$, $k v_H + (N - k)v_L - C \geq \max\{0, kv_H - C\}$, so clearly $W^I \geq W^E$ on $n_I^{BB} \leq n_E$ (which includes all $q \leq \hat{q}$). ■

Proof of Lemma 2. Recall that under full commitment, the profit-maximizing mechanisms restricted bids to $\mathcal{B} = \{0, v_L, \bar{b}_{n_I}\}$, had $p \leq v_L$ and $T \in (\bar{T}_{n_I} - \bar{\delta}_{n_I}, \bar{T}_{n_I}]$. Without bid restrictions, the maximal choices, $T = \bar{T}_{n_I}$ and $p = v_L$ prevent bid shaving and it is then still an equilibrium for H -types to bid \bar{b}_{n_I} and L -types to bid v_L . Clearly, L -types have no profitable deviation and H -types bid at least v_L . As before, H -types are indifferent between bidding v_L and \bar{b}_{n_I} . Bidding v_L dominates bidding in between v_L and \bar{b}_{n_I} . We now rule out bidding above \bar{b}_{n_I} .

In general, bidding above $p = v_L$ can be attractive only if it increases the probability of production. In a candidate equilibrium where L -types bid $p = v_L$, H -types bid \bar{b}_n and threshold $\bar{T}_n = n\bar{\delta}_n + Nv_L$, an individual buyer bidding $b \geq p$ generates project success rate S_ℓ^{N-1} where $\ell = \lceil \frac{b-p}{\bar{\delta}_n} \rceil$. Bidding above p reduces by ℓ the number of the other $N-1$ buyers who need to be H -type for the project to succeed. Bid increments that do not raise ℓ are weakly dominated, so we need only consider bids of the form $b = v_L + \ell\bar{\delta}_n$ for integer values of ℓ . To maintain these equilibrium choices without bid restrictions, we need to check that H -types are willing to set $\ell = 1$. Deviating to $\ell = 0$ is not a problem by incentive compatibility in the full-commitment solution. It remains to verify that deviating to a bid $b = v_L + \ell\bar{\delta}_n$ is weakly inferior for integer values of $\ell \geq 2$ in the case of $n = n_I$, but it is as simple to prove it for all n so we do.

In the putative equilibrium without bid restrictions, with $p = v_L$ and $T = Nv_L + n\bar{\delta}_n$, if the two types continue to make respective bids, $b_H = v_L + \bar{\delta}_n$ and b_L , then the production probability is S_n^N . From the perspective of a single buyer of the H -type playing the equilibrium strategy, this probability is higher at S_{n-1}^{N-1} , and falls to S_n^{N-1} if he deviates to bid v_L , but rises to $S_{n-\ell}^{N-1}$ if he deviates to the proposed bid with some $\ell \geq 2$. The first two options give this buyer the same expected utility because inequality (IC) binds as Eq. (4); this payoff is $(v_H - v_L)S_n^{N-1}$. The deviation option gives,

$$(v_H - v_L - \ell\bar{\delta}_n)S_{n-\ell}^{N-1}$$

So, substituting for $\bar{\delta}_n = h_n(v_H - v_L)$ and dividing by $(v_H - v_L)S_{n-\ell}^{N-1}$, we seek to show that,

$$(1 - \ell h_n) \leq S_n^{N-1}/S_{n-\ell}^{N-1}, \quad \forall \ell \geq 2$$

Now the right-hand side can be written as the product of $(1 - h_n)(1 - h_{n-1}) \dots (1 - h_{n-\ell})$, but h_n is increasing in n , so this expression weakly exceeds $(1 - h_n)^\ell$. Now $h_n \in [0, 1]$ so defining $a = 1 - h_n$, we have $a \in [0, 1]$, so for any $\ell \geq 1$,

$$1 - a^\ell = (1 - a)(1 + \dots + a^{\ell-1}) \leq (1 - a)\ell$$

Rearranging terms and substituting back for a , this gives $1 - \ell h_n \leq (1 - h_n)^\ell$, concluding the proof of implementation.

This optimal outcome is still uniquely implemented in pure strategy Pareto undominated equilibrium. The only candidates for alternative Pareto undominated equilibria are where $n \neq n_I$ H -types are needed who all bid $b'_n = (\bar{T}_{n_I} - (N - n)v_L)/n$. It is readily verified that this breaks H 's IC when $n < n_I$ and when $n > n_I$, it is an equilibrium but is Pareto-dominated as the entrepreneur and H -types are worse-off: an H -type buyer expects to obtain $(v_H - b'_n)S_{n-1}^{N-1} < (v_H - v_L)S_{n-1}^{N-1}$ (as $b'_n > v_L$) while in the optimal equilibrium he obtains $(v_H - \bar{b}_{n_I})S_{n_I-1}^{N-1} = (v_H - v_L)S_{n_I}^{N-1} \geq (v_H - v_L)S_{n-1}^{N-1}$. ■

Proof of Proposition 4. We prove that $n'_I \leq n_I$. The statements about profits, consumer and total welfare follow.

We define for each n ,

$$C_n(q) = \frac{N(v_L - qv_H) + n(v_H - v_L)}{1 - q} \quad (10)$$

The entrepreneur sets $n_I = n$ in the region between curves $C = C_{n-1}(q)$ and $C = C_n(q)$; Section 2.3's tie-breaker picks the more efficient, lower n_I on the boundaries. By Lemma 1, $n_I = \arg \min_n \{C \leq C_n(q)\}$. Feasibility of the n -type strategy requires $C \leq \bar{T}_n$. In particular, feasibility is guaranteed for all $n \geq 1$ when $C = Nv_L$. We show first that the

n_I -type strategy is feasible by demonstrating that $C_n(q) < \bar{T}_n$ for all $n < N$:

$$\begin{aligned} & \frac{N(v_L - qv_H) + n(v_H - v_L)}{1 - q} < Nv_L + nh_n(v_H - v_L) \\ \Leftrightarrow & N(v_L - qv_H) + n(v_H - v_L) < N(1 - q)v_L + nh_n(1 - q)(v_H - v_L) \\ \Leftrightarrow & n(v_H - v_L)(1 - h_n(1 - q)) < qN(v_H - v_L) \end{aligned}$$

The result follows from Lemma A.1(ix).

Next we show that there exist unique values $0 < q'_1 < \dots < q'_{N-1}$ so that the entrepreneur is indifferent between strategies of type n and $n + 1$ (as long as both are feasible) when $q = q'_n$, independently of C . Note that $\pi'_n = (qNS_{n-1}^{N-1} - nS_n^N)\delta'_n$ where $\delta'_n = h_n(v_H - p'_n) = h_n(Nv_H - C)/(N - nh_n)$. Hence, $[\pi'_{n+1} - \pi'_n]/(Nv_H - C)$ is independent of C . There must exist a q'_n where the entrepreneur is indifferent, because the difference is strictly negative when $q > 0$ is very small while it is strictly positive when $q < 1$ is close to one. Straightforward calculations show that a marginal increase in q above q'_n increases the difference $\pi'_{n+1} - \pi'_n$, and the uniqueness result follows.

Similar steps show that at $q_n = n/N$, $\pi'_n(q_n) > \pi'_{n+1}(q_n)$, which implies that $q'_n > q_n$. It then follows that the optimal inclusive strategy is of type n'_I where n'_I is the smallest n such that both $q \leq q'_n$ and $C \leq \bar{T}_n$. ■

Proof of Proposition 5. An inclusive strategy with pivot n gives expected revenue,

$$\begin{aligned} R_n^I(q) &= S_n^N (Nv_L + \mathbb{E}[k|k \geq n]h_n(v_H - v_L)) \\ &= S_n^N \left(\frac{(1 - h_n)Nv_L + h_n(qNv_H)}{1 - (1 - q)h_n} \right) \\ &= NS_{n-1}^{N-1} ((1 - h_n)v_L + h_n(qv_H)) \end{aligned} \quad (11)$$

using Eq. (4) and Lemma A.1(iii) then (ii). S_{n-1}^{N-1} falls with n and so does the term in parentheses for $q < \hat{q}$ since then $v_L > qv_H$; recall that h_n rises with n . $R_n^I(q)$ is then strictly decreasing in n . In particular, when $n^{BB} < n_I$ the platform makes strictly higher expected profits from not-for-profits, proving claim b(i). (This also holds on $q = \hat{q}$ except that $R_0^I(\hat{q}) = R_1^I(\hat{q})$.)

An exclusive strategy with pivot n gives expected revenue,

$$R_n^E(q) = S_n^N \mathbb{E}[k|k \geq n]v_H = NS_{n-1}^{N-1} qv_H \quad (12)$$

Clearly, $R_n^E(q)$ is strictly decreasing on $n \geq 1$ and increasing in q . Recall that entrepreneurs only use exclusive strategies on $q > \hat{q}$ and on here $qv_H > v_L$ so $R_n^E(q) > R_n^I(q)$. In particular, the platform makes strictly lower profit from success-maximizing entrepreneurs if they adopt inclusive strategies with $n_I^{BB} = n_E$, proving claim b(ii). If instead $n_I^{BB} < n_E$, the platform may prefer either not-for-profits for their higher success

probability or profit-maximizers for their higher conditional expected revenue.

When $C \leq Nv_L$, $n_I^{BB} = 0 < n_E$. For $C \leq v_H$, $n_E = 1$ and $Nv_L = R_0^I(\hat{q}) = R_1^E(\hat{q})$ and $R_0^I(q) < R_1^E(q)$ for all $q > \hat{q}$. Hence, we define $\bar{q}(C) = \hat{q}$ in this region. For $C > v_H$, $n_E > 1$ and $R_0^I(\hat{q}) > R_{n_E}^E(\hat{q})$. $R_{n_E}^E(q)$ is strictly increasing in q and $R_{n_E}^E(1) = Nv_H > Nv_L = R_0^I(q)$. Continuity implies uniqueness and existence of $\bar{q}(C)$ defined by $Nv_L = R_{n_E}^E(\bar{q}(C))$. Finally, define $\underline{q}(C) = (Nv_L - C)/(Nv_H - C)$. Then for $q \leq \underline{q}(C)$, $n_I = 0$, because $\underline{q}(C) = C_0^{-1}(q)$ where C_0 was defined in the proof of Proposition 4. This proves claims a(i),(ii),(iii). ■

Proof of Proposition 6. Using Lemma A.1 (iii) and (ii), we can rewrite the profit as,

$$\pi(p) = -S_n^N C + mpqN S_{n-1}^{N-1} = (mpqN - C)S_n^N + mpqN(1 - q)f_{n-1}^{N-1}$$

Using Lemma A.1 (vi) and (v), the optimal p must satisfy,

$$\begin{aligned} 0 = \frac{\partial \pi}{\partial p} &= (mpqN - C)N f_{n-1}^{N-1} q' + (mqN + mpq'N)S_n^N \\ &\quad + mpqN(1 - q)f_{n-1}^{N-1} \frac{n - 1 - (N - 1)q}{q(1 - q)} q' \\ &\quad + (mqN(1 - q) + mpN(1 - 2q)q')f_{n-1}^{N-1} \end{aligned}$$

Using again Lemma A.1 (ii), and defining $c = C/Nm$, this is equivalent to,

$$\begin{aligned} 0 &= (pqN - cN)f_{n-1}^{N-1} q' + (q + pq')(S_{n-1}^{N-1} - (1 - q)f_{n-1}^{N-1}) \\ &\quad + pf_{n-1}^{N-1}(n - 1 - (N - 1)q)q' \\ &\quad + (q(1 - q) + p(1 - 2q)q')f_{n-1}^{N-1} \end{aligned}$$

Taking out a factor S_{n-1}^{N-1} and rearranging yields,

$$0 = q + pq' + ((n - 1)p - cN)h_n q'. \quad \blacksquare$$

Proof of Proposition 8. Given our assumptions, if production occurs, new buyers will be charged price $p_2 = v_H$. Hence, H -type funders will bid up to v_H in crowdfunding, and, by setting $p = v_H$, the entrepreneur learns the number k of H -types among the crowd. This signals a posterior probability that a new buyer is H -type, given by

$$\eta(k) = \frac{q^G f_k^{N_1}(q^G) + q^B f_k^{N_1}(q^B)}{f_k^{N_1}(q^G) + f_k^{N_1}(q^B)} \quad (13)$$

which is increasing in k . Note that $\eta(N_1)$ tends to q^G as N_1 tends to infinity. Our assumptions then imply that we can define $\underline{N}_1 = \min\{N_1 : \eta(N_1) > (C - N_1 v_H)/(N_2 v_H)\}$. When $N_1 \geq \underline{N}_1$, signalling has some chance to convince the entrepreneur to produce and sell ex-post at v_H because that will be profitable then. We now define \hat{n}_E as the minimal $n \geq \underline{N}_1$ for which $\eta(n)N_2 v_H \geq (N_1 + N_2 - n)v_L$:

$$\hat{n}_E = \min \{k : \eta(k)N_2v_H \geq \max\{C - kv_H, (N_1 + N_2 - k)v_L\}\} \quad (14)$$

$N_1 \geq \hat{n}_E$ because $\bar{q} \geq \hat{q}$. Consider the crowdfunding mechanism with minimal price $p = v_H$ and threshold $T = \hat{n}_E v_H$. If T is reached, *i.e.*, $k \geq \hat{n}_E$, the entrepreneur produces and then sets second period price $p_2 = v_H$. This price is optimal because expected revenue in the second period is then $\eta(k)N_2v_H \geq \eta(\hat{n}_E)N_2v_H \geq (N_1 + N_2 - \hat{n}_E)v_L \geq (N_1 + N_2 - k)v_L$. Moreover, the global strategy yields positive profits because when production occurs, it delivers expected profits, $kv_H + \eta(k)N_2v_H - C \geq \hat{n}_E v_H + \eta(\hat{n}_E)N_2v_H - C > 0$. When $C > T = \hat{n}_E v_H$, aggregate funds may not suffice to cover fixed cost and additional TF is needed. ■