



Bidding for Input in Oligopoly

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Bidding for input in oligopoly*

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Abstract

We present a model where firms producing substitutes bid for inputs (especially labor) in a decentralized market. We show that downstream market power increases the intensity of competition for input through a new channel: local competitive foreclosure. In our model each unit of input (worker) is sold in a separate local market and firms try not just to get it, but also to keep it from their rivals. This externality leads to firms targeting the same units of input and the price of these is bid up. This effect mitigates the output reducing effect of downstream market power and in the limit (linear Cournot with constant returns) can even restore efficiency. As a result of coordination, there exist further equilibria, with prices above cost even with price taking suppliers – in the labor application this leads to involuntary unemployment. When, instead of targeting, firms post prices, coordination no longer plays a role and we have a unique(!) equilibrium that clears the market, still internalizing the externality. Finally, we show that targeting can also result in endogenous market segmentation and price/wage differentials. *JEL* Codes: D43, L11, L13.

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1 Introduction

Firms often compete with the same rivals in different, vertically connected, markets: upstream markets for inputs and downstream markets, where they sell their output. When these firms have significant market power, the resulting strategic interaction can become complex and closely dependent on market microstructure. It is therefore fundamental to get the latter right (c.f. Roberts, 1987). In this paper, we propose a novel model of (input) price determination that allows us to shed light on previously ignored feedback effects from the downstream market.

Our point of departure is the conventional wisdom that firms engaged in multi-market competition have an incentive to foreclose: to reduce the rivals' production by somehow starving them of input. Models often fail to capture the full ramifications of this observation.¹ To illustrate this, ignore for the moment models of vertical contracting, and instead consider the case where the – atomized – supply of input is completely elastic. It may then appear natural – though we will argue that perhaps incorrect – to model upstream competition as firms simultaneously choosing quantities (as prices are “given”). It is then clear that a result of market power in the downstream market is a lower output, resulting in a lower input demand. Note that this outcome is the aggregate consequence of two effects of opposing directions. The dominant of these is that, as a firm with market power impacts on the output price, it lowers the quantity it puts on the market. The second, mitigating, effect is that, because its rivals have market power, there is strategic substitutability: increasing its quantity leads to a lower quantity supplied by its rivals (the best response curves are downward sloping). We contend that the incentive to increase one's quantity in order to decrease the rival's is seriously underestimated if we only take into account

¹Among the few exceptions, we may cite Esó et al. (2010), Stahl (1988) and Yanelle (1997). We will comment on these below.

strategic substitutability. Consequently, the welfare loss resulting from downstream market power is significantly lower than currently believed.²

The problem we see is that this model of competition assumes away the possibility that the input choices of a firm *directly* affect the rivals' input levels. We believe that this is an unnecessary, and often unrealistic, restriction. To address this concern, we propose an alternative microstructure that is operational in any supply context, not just the elastic one assumed above.

The main feature of our model of the (upstream) market is that buyers enter into simultaneous and independent “negotiations” for exclusive deals with their potential suppliers, where the latter have fixed capacities but there are many of them. The best example of this is the labor market:³ there are many workers, but each of them works only eight hours a day and can seldom work for two different firms. As it is our focal application, henceforth, we will refer to the upstream market as the labor market, where firms hire workers.

By targeting specific workers, firms can affect whether the marginal worker they hire is at the expense of one of their rivals, even when the rivals' strategies (no longer quantity choices!) are given. In other words, by directing their demand at the same workers that the rivals expect to hire, they can potentially reduce the rivals' input. When every firm can engage in such “poaching” activity, the equilibrium strategies have to incorporate defensive tactics: competitive foreclosure ensues, magnifying the increase in competition intensity – due to strategic substitutability – described above.

The insights that we obtain are most relevant for labor markets with identifiable individuals, like top management, academics, professionals, etc., where personalized deals are common. Nonetheless, we will show that our main results continues to hold when workers are anonymous.

²In fact, in Section 8, we display an example – with linear output demand and Cournot downstream competition – where our model leads to the efficient outcome.

³Nonetheless, our model could equally apply to other markets, for example, retail space, produce, non-specialized parts, premium advertising slots etc.

We streamline bargaining by assuming that the firms make take-it-or-leave-it offers to the workers of their choice.⁴ Importantly, this mechanism is completely decentralized: whether or not a worker is hired only depends on her choice when faced with the wages set for her by the firms.

Our first result confirms that in the benchmark case, where the firms have no market power in the product market, the unique equilibrium of our targeted-offer labor market institution leads to the competitive outcome. Next, we show that, with market power, the competitive outcome cannot occur in equilibrium, but there exists another market-clearing equilibrium instead. In this equilibrium, firms target the same workers and this results in an endogenous demand for labor. This demand exceeds the competitive one as it includes a “conjectural variation” of -1 : the act of being chosen by a worker over a rival not only increases a firm’s labor force by one, but it automatically decreases the rival’s labor force by one. Due to this increased demand, both – the still common – wage and employment increase. Thus, incentives to foreclose lead to higher wages and higher employment, partially compensating for the anti-competitive effects of downstream market power.

It is important to note that this outcome is not an artifact of our assumed microstructure, as it would also result had we assumed posted prices. Even in the absence of targeting, the firms know that workers willing to accept their wage will either work for them or for the rival. For this result, the personalized offers have served only as a vehicle to make the direct effect of wage offers on the rival’s workforce more apparent.

The above is not the only equilibrium of our model. However, all other equilibria exhibit even higher wages – but lower (though supra-competitive) levels of employment than the market clearing equilibrium. Consequently, these additional equilibria are characterized by involuntary unemployment: there are workers not employed by the firms who would accept to work for less than the market wage. These equilibria

⁴We can – alternatively – construe this as each firm running a first-price auction, setting a personalized reserve wage for each worker and promising to hire them if they submit a valid bid (which, obviously, will be equal to the reserve wage). For a detailed analysis of this, “all-win”, auction, see Burguet and Sákovics (2016).

arise from the coordination-game nature of the interaction: as firms want to make offers to the same workers, if the other firms stop targeting a few workers it is a best response not to make an offer to those same workers. Attracting them would no longer have the added value of reducing the supply of the rival. It is this wedge between the value of a worker who would otherwise work for the rival and one that would not, that explains the possibility of wages above reservation wages.⁵

Our assumption that workers are identifiable is relevant for this result. Indeed, worker anonymity eliminates the coordination-game equilibria with unemployment. Unsurprisingly, anonymity retains the high-wage, high-employment, market-clearing equilibrium: as we have discussed, that outcome results when competition is in posted prices. All firms make the same wage offer to all the workers, so they know that workers willing to accept that wage will either work for them or for a rival, and thus the equivalence with posted-price competition. Moreover, this is the only outcome supported in equilibrium. Uniqueness is due to the fact that other outcomes would induce (non-posted-price)⁶ deviations, in the form of higher wage offers to fewer workers. That is, market clearing is not a consequence of downward pressure of wages, in the hypothetical case that fewer workers were hired at higher wages.

In fact, when we discourage upward wage deviations, the possibility of wages above market clearing – and so involuntary unemployment – reemerges. We show this by appending a second period to the anonymous game and supposing that, even if workers receiving an offer may not be observable, a worker being hired is, and thus in the second period firms can direct offers at workers hired by their rivals.

Most of our analysis is carried out in a symmetric framework. As a comprehensive analysis of asymmetries is beyond the scope of this article, we content ourselves with documenting the possibility of inter-industry wage differentials arising in equilibrium. To that effect, we return to our original, static model with identifiable

⁵Note that we have assumed that product demand and labor supply are independent. This is in contrast with general equilibrium (macro) models – where consumers and workers are the same – that display multiple equilibria and unemployment via different rational expectations equilibria (see Silvestre, 1993, for an excellent survey).

⁶With only posted prices, there would be a large multiplicity of equilibria (c.f. Dastidar, 1995).

workers, and suppose that – now four – firms competing in the same labor market are of two types, each selling their products in one of two identical but independent markets. As in the symmetric case, there exist a continuum of equilibria, but only two of them result in equal wages across industries. What occurs in all other equilibria is endogenous market segmentation: firms that share the same product market target the same workers, while firms from different product markets target different workers. This results in a coordination game across industries, with multiple ways in which to share labor in equilibrium.

All the above results are derived for a general, unspecified production function and downstream market interaction. To illustrate our findings, we end our essay by working out an example for the downstream market with differentiated-good price competition and constant returns to scale technology.

There is an extensive literature on foreclosure (see Rey and Tirole, 2007, for an excellent survey), mainly concerned with vertical contracting. In that literature, downstream firms with market power also have an incentive to lure upstream firms into contracts that make it harder for rivals to obtain their inputs. The focus there is on whether these incentives are stronger than upstream firms’ own incentives not to enter into these deals, or on the contractual forms that may affect competition to the contracting parties’ advantage.⁷ Instead, we have in mind an upstream market where suppliers do not possess market power, but cannot sensibly deal with more than one buyer. Foreclosure here appears through quantity “purchases”, rather than through – from a competition law point of view possibly problematic – vertical restraints.

The literature – see Bhaskar et al. (2002) and the references there – also provides abundant evidence of both wage differentials and wages above reservation values.⁸

⁷Another strand of this literature focuses on competition among upstream firms with market power, and how this may be affected by contracting tools in their deals with downstream, typically competitive retailers.

⁸Of course, efficiency wage models rank high as explanations of these phenomena. (See, for instance, Yellen, 1984.) Here, we have assumed away all traditional motives behind the rationale for efficiency wages.

Some theoretical explanations have been offered for these phenomena. Related to this paper, Shy and Stenbacka (2015), like us, allow for personalized wage offers to workers. Their motivation is poaching or anti-poaching policies when switching workers are affected by both a productivity change and a – worker specific – cost of switching. Switchers may obtain wages above the wages of stayers, so that wage differentials result from switching frictions. Our inter-firm wage segmentation, on the other hand, results from output market externalities.

Other studies have related product market imperfections with unemployment when labor (union) has bargaining power in wage negotiations.⁹ This bargaining power allows workers to capture part of firms’ rents in the output market, and so drive wages above reservation wages. In our model, workers are price takers. (In fact, our model has equilibria where workers appropriate no rents.) Wages are above reservation wages as a consequence of firms’ attempts to capture competitors’ rents.

Kaas and Madden (2004) also analyze the feedback between product and input market power, and also obtain the possibility of unemployment as an equilibrium outcome. Firms first post wages and then, after observing all choices, announce a maximum amount of labor they are willing to hire at their posted wage and, possibly, rationing follows. This two-stage competition for labor allows high wages to be equilibrium: any deviation downwards triggers a punishment by other firms in the form of large demands of labor that drives the deviator out of the market. These punishments themselves are sustained by the threat to a failing “punisher” of being also driven out of the market. Thus, in a sense unemployment is the result of collusion among firms, with collusion-type mechanisms to sustain it.¹⁰ We do not need this particular, two-stage model of the labor market or the endogenous wage rigidities that it postulates at the time employment offers are made. Rather, high wages are the consequence of firms fiercely competing with alternative employers that are also output market competitors.

⁹See Blanchard and Giavazzi (2003). Koskela and Stenbacka (2012) and Booth (2014) are recent examples of this literature.

¹⁰They require to have at least three firms competing, so that each deviant is disciplined by (at least) two punishers, each one disciplining each other.

A few papers have modeled the interaction between general input and output markets when firms have market power in both.¹¹ Stahl (1988) is perhaps among the first to analyze the effect on output market outcomes of intermediaries' competition for upstream inputs. Intermediaries compete in prices for inputs that then are sold downstream. Price competition leads to no "unemployment", by definition: all supply at the posted price is assumed to be taken by the intermediary making the offer. When ties are broken in a particular way (one winner takes all even when tying), then the output price may be larger than Walrasian, but one-price, market clearing in the input market is always guaranteed. More recently, Esó et al. (2010) discuss quantity competition in this same setting, but assume exogenous (inelastic) supply of input and efficient allocation of this input to firms. Also in the same tradition, Yanelle (1997) studies a model of bank competition that shares some interesting features with the present one. She also obtains that there is a range of equilibria, at different prices (rates) for funds. When banks and borrowers (entrepreneurs) compete with each other for funds, lenders face a coordination problem: borrowers and banks can fulfill their offers if they get sufficient other lenders on board. Thus multiplicity ensues. We also obtain a multiplicity of price equilibria for inputs as the result of a coordination problem, but in our case input buyers, not their sellers, are the ones that face this problem. Competition in the output market is the origin of the externality in the input market.

Our model could be interpreted as each worker auctioning off her services. In this sense, it is related to McAfee (1993), Peters and Severinov (1997), Burguet and Sákovics (1999), Julien et al. (2000), and De Fraja and Sákovics (2001), among others. All these papers consider similar institutions involving personalized wage offers, with the main difference that they all assume that each buyer can participate in a single auction (bid for a single worker). Thus, wage differentials (intra- or inter-firm) are a consequence of different realizations of (mixed-strategy) equilibria participation in these auctions. On the contrary, in our model, firms are allowed to make (and required to honor) offers at several auctions simultaneously.

¹¹The literature on inter-market interactions when firms act in multiple output markets is abundant. A classical reference here is Bulow et al. (1985).

Finally, note that our model is distinct from those that also model competing “retailers” bidding for input but consider the supplier as a monopolist (Marx and Shaffer, 2007, 2015, Miklos-Thal et al., 2011, Rey and Whinston, 2013). Here each retailer has a single possible contract, so foreclosure equates exclusion. Also they are concerned with reaping the industry monopoly profit in equilibrium, which is far from feasible in our model. If we considered a trade union responding to the offers of the retailers, we would not have the internalization of the external effect, which is the basis for our result.

The paper is organized as follows. Section 2 presents the model of labor market competition and the (reduced form) model of product market revenues. In Section 3, we analyze the benchmark case where firms’ labor decisions do not affect other firms’ revenues, and obtain competitive (labor market) outcomes for this model of labor market competition. Section 4 then introduces output market interaction and obtains a (non-competitive) market clearing equilibrium outcome for this case. Section 5 studies other, symmetric equilibria where the wage is even higher, and above the reservation value of non employed workers. In Section 6 we discuss how the results change when firms cannot target their wage offers. Section 7 studies the case in which all firms compete for workers, but then operate in different output markets, and shows that wage differentials may be an equilibrium outcome. Section 8 discusses micro-foundations for our revenue functions, and Section 9 concludes. All proofs are in the Appendix.

2 The base model

There are two symmetric firms competing both in the labor and product markets. Labor is measured in efficiency units, denoted by t , for talent. The measure of labor hired by Firm i is denoted by t_i , $i = 1, 2$. To avoid technical issues arising from indivisibilities, we do not model workers embodying varying measures of efficiency units, rather we assume that there are a continuum of identically productive “nano-

workers”, indexed by $z \in [0, 1]$. Their – exogenous and common knowledge¹² – reservation wages imply an aggregate supply function of labor, denoted by $S^{-1}(w)$, w.l.o.g. assumed to be weakly increasing in wage, w , so that its inverse is the inverse aggregate supply function¹³ $S(t_1 + t_2) > 0$.

The labor market operates as follows: each Firm i sets a – deterministic and Lebesgue measurable – wage schedule, $W_i(z)$, $z \in [0, 1]$, specifying a personal wage offer to each (nano-)worker.¹⁴ Let \mathbf{W} denote the full profile of wage schedules. The workers’ decisions are simple: they observe their offers, and accept (one of) the highest if it is no less than their reservation wage. When they are indifferent, the existence of an equilibrium may require that the workers’ (mixed) strategies break ties in some “appropriate” way. Unless otherwise indicated, we consider the (symmetric) strategy where, in case a worker has more than one acceptable highest offers, she randomizes among them with equal probabilities. That is, a worker, who receives a highest acceptable wage offer from both firms, accepts each with probability $\frac{1}{2}$. Note that, since $W_i(z)$ is measurable, given \mathbf{W} the set of workers that receive an offer w from Firm i above their reservation wage, and a strictly lower offer from the other firm is also measurable. Similarly, the set of workers who receive the same acceptable offer of w from both firms is measurable. Let us represent these measures by $\nu_i^1(w; \mathbf{W})$ and $\nu^2(w; \mathbf{W})$, respectively. Thus, given \mathbf{W} and the acceptance strategy of the workers, the measure of workers hired by Firm i is¹⁵

$$t_i(\mathbf{W}) = \int_0^\infty \left[\frac{\nu^2(w; \mathbf{W})}{2} + \nu_i^1(w; \mathbf{W}) \right] dw,$$

¹²Given the Law of Large Numbers, this assumption is only for simplicity. All our results would hold under the alternative assumption that each worker’s reservation wage is an independent random draw from the same distribution.

¹³In the tradition of economics, we allow for vertical segments in the inverse supply *correspondence*.

¹⁴If a firm does not want to make an offer to some workers, we simply set the wage offered at zero.

¹⁵By analogy to the Law of Large Numbers, we assume that the expected value exists and coincides with the realized value with probability 1.

and the wage bill for Firm i is

$$b_i(\mathbf{W}) = \int_0^\infty w \left[\frac{\nu^2(w; \mathbf{W})}{2} + \nu_i^1(w; \mathbf{W}) \right] dw.$$

Once this decentralized labor market has cleared, the firms produce. To maximize generality, we do not model the production process and the product market competition in detail. We simply assume that given any amount of labor hired by the other firm, t_{3-i} , Firm i 's net revenue as a function of its own workforce is $R(t_i, t_{3-i})$. This function is common knowledge and twice differentiable. That is, Firm i 's payoff, given a vector of wage schedules \mathbf{W} , is

$$\pi_i(\mathbf{W}) = R(t_i(\mathbf{W}), t_{3-i}(\mathbf{W})) - b_i(\mathbf{W}).$$

For some standard models of market competition, R would not be concave. Thus, in order to obtain sufficient conditions for equilibrium, we need to make alternative assumptions – sufficient conditions for optimality – on R . Below, the subscripts of R represent partial derivatives with respect to the corresponding variable. Also, taking advantage of symmetry, in all statements referring to $R(\cdot)$ of a firm, the first argument will refer to its own labor force, and the second to the labor employed by the rival.

Assumption 1 *i) $R_{11}(t_o, t_r) < 0$, ii) $R_{12}(t_o, t_r) \leq 0$, and iii) $R_{11}(t_o, t_r) - 2R_{12}(t_o, t_r) + R_{22}(t_o, t_r) \leq 0$.*

Assumption 1iii) implies that a firm's payoff is concave in the amount of labor that leaves the rival's firm to join the ranks of its workers.¹⁶ It is a strong assumption, but only a sufficient one, far from being necessary. It allows us to characterize the set of symmetric, pure strategy equilibria using the first-order approach. As we will see in Section 8, typical Cournot or Dixit models satisfy this assumption.

As a final preliminary, let us present a result that will help in understanding the nature of equilibrium strategies.

¹⁶Note that Assumption 1 implies, in particular, that $R(t, t)$ is concave in t , and also that both $R_1(t, t)$ and $R_1(t, t) - R_2(t, t)$ are decreasing in t .

Lemma 1 *In any equilibrium in a symmetric¹⁷ duopsony, the law of one wage holds.*

In other words, despite the flexibility afforded by the model to offer different wages to different workers, all the accepted offers in equilibrium must be at a common wage.¹⁸

3 The benchmark case: no externalities

We start our analysis by looking at the benchmark situation, where there is no interaction in the product market – for example, because the firms are price takers. Then a firm’s revenues do not depend on the amount of labor hired by its competitors, so that $R_2(t_o, t_r) \equiv 0$. In this case, the partial derivative (with respect to own labor) of net revenue, $R_1(t_o, \cdot)$, captures a firm’s marginal willingness to pay for labor when it has already hired t_o units. Therefore, the market clearing – or competitive – outcome is defined by $R_1(t^c, t^c) = S(2t^c) = w^c$.

Remark 1 *It is important to note that, if the firms were price (wage) takers, the outcome would still be (t^c, w^c) even with externalities in the product market: if a firm cannot affect the amount/price of labor its rival hires then it does not matter – for the firm’s behavior in the labor market – whether its revenues depend on the rival’s labor force.*

Remark 2 *A further relevant observation is that this competitive labor market equilibrium is only competitive in a partial equilibrium sense. Firm’s willingness to pay for labor does not equal price times marginal product of labor when the product market is not competitive. The labor market clears and the wage equals firms’ willingness to pay but less labor is hired than in the generally competitive scenario (c.f. Stahl, 1988).*

¹⁷Symmetry is only used to ensure that both firms are active.

¹⁸As we will see in Section 7, when there are more firms, this result will no longer hold.

The following result shows that our wage setting game uniquely implements the competitive outcome:¹⁹

Proposition 1 *In the absence of externalities, the unique equilibrium outcome²⁰ is market clearing: all hires are at the competitive wage, w^c , and firms hire their competitive labor demand, t^c .*

In equilibrium, each firm offers w^c to *twice* the amount of labor it wishes to hire at this wage,²¹ so that each worker willing to accept w^c receives two offers. Given this, no firm would like to attract another worker as she would require a wage above w^c , the firms' willingness to pay. Similarly, no firm would want to shed a worker, since the wage is weakly below the firms' willingness to pay. Finally, there is no way to hire any worker cheaper than for w^c either, confirming the equilibrium. Uniqueness can be established by observing that in equilibrium, practically by definition, there cannot be a positive measure of workers hired at any wage other than w^c .

This last observation explains why firms coordinate on making offers to the same set of workers, even if there are no externalities. Any worker, z , who only received a single offer would have to be hired for her reservation wage (otherwise the hiring firm would deviate and make a lower offer to her). But if this wage is lower than any wage paid by the rival, then the rival would profit from not hiring that worker and hiring z for ε more than her reservation wage instead.

¹⁹Proposition 1 generalizes to asymmetric firms but in that case equilibrium requires the workers to use a different tie-breaking rule (i.e. mixed strategy). See Theorem 1 in Burguet and Sákovic (2015).

²⁰We only consider deterministic wage schedules.

²¹Off the equilibrium path this may require the firm to hire more workers than it needs, though it still has the option not to put them to work (in case overproduction would adversely affect profits in the product market).

4 Market clearing with externalities

We now investigate the effects of (product) market power on the equilibrium outcome in the labor market by assuming that $R_2(t_o, t_r) < 0$. This externality does not only imply that the marginal value of an additional worker depends on the labor force of the rival. It has an additional – more intricate – consequence. When we totally differentiate R with respect to t_o we obtain

$$\frac{dR(t_o, t_r)}{dt_o} = R_1(t_o, t_r) + \frac{\Delta t_r}{\Delta t_o} R_2(t_o, t_r).$$

Thus, all of a sudden, $\frac{\Delta t_r}{\Delta t_o}$, by which we represent the effect an infinitesimal change in a firm's own labor force has on its rival's (holding his strategy constant, of course), comes to the fore. The crucial observation is that this effect is determined endogenously. Hiring an additional worker has a different effect on a firm's revenue depending on whether the worker would have gone to the rival firm – she is *contested* – or not. In the former case $\frac{\Delta t_r}{\Delta t_o} = -1$, while in the latter $\frac{\Delta t_r}{\Delta t_o} = 0$. Therefore, the marginal value of attracting a worker away from a rival is the sum of two (positive) effects: $R_1(t_o, t_r)$ and $-R_2(t_o, t_r)$ per efficiency unit. At the same time, the marginal value of attracting an uncontested worker is simply the increase in net revenue due to an increase in the firm's labor force, $R_1(t_o, t_r)$ – just as in the benchmark case. This increase is lower than for the case of a contested worker, since it does not include the value of reducing the labor force, and then the output, of the competitor. As we will see, in equilibrium each firm will seek to make offers to the same pool of workers as its rival – resulting in all hired workers being contested – and consequently they have the higher marginal willingness to pay for them.

Proposition 2 *In the presence of externalities, the competitive outcome (t^c, w^c) is no longer supportable in equilibrium. Nonetheless, there exists an alternative symmetric market clearing equilibrium, with each firm hiring t^* workers, paying*

them the same wage w^* , where (t^*, w^*) solve²²

$$R_1(t^*, t^*) - R_2(t^*, t^*) = S(2t^*) = w^*. \quad (1)$$

Note that t^* and w^* are the market clearing employment and wage, when the two firms make offers to the same set of workers. When each worker is contested then decreasing t_o will increase t_r by the same amount. Similarly, the best way of increasing t_o is by outbidding the rival for some of the workers it would have hired with positive probability. Thus, indeed the marginal valuation is given by $R_1(t_o, t_r) - R_2(t_o, t_r)$. Market clearing then follows from similar arguments as in the benchmark model.

Given that the market clears, and since it does not matter whether workers with higher reservation wages receive the offer or not, we have that

Remark 3 *The outcome (t^*, w^*) can be implemented by posted prices (the same offer, w^* , made to every worker) and therefore it does not rely on targeting offers.²³*

To see why the competitive outcome is not an equilibrium, simply observe that $R_1(t^c, t^c) - R_2(t^c, t^c) = w^c - R_2(t^c, t^c) > w^c$, so either firm has an incentive to outbid its rival for some of the workers it would hire with probability half if the competitive strategies were played. The following corollary is also immediate.

Corollary 1 *$w^* \geq w^c$ and, therefore, $t^* \geq t^c$. Moreover, the first inequality is strict unless labor supply is fully elastic, while the second is strict unless it is fully inelastic.*

²²Note that Assumption liii) guarantees that profits are non negative at t^* . Indeed,

$$R(t^*, t^*) - w^*t^* = \int_0^{t^*} (\Phi(x) - R_1(t^*, t^*) - R_2(t^*, t^*)) dx$$

where $\Phi(x) = R_1(x, 2t^* - x) - R_2(x, 2t^* - x)$, and the result follows noticing that $\Phi(x)$ is decreasing from Assumption liii) and the inside of the integral is zero at $x = t^*$.

²³See more on anonymity in Section 6.

In other words, the firms' market power in the product market leads to higher wages and increased employment, at least when the market clears. As we will see next, the market may not always clear, but the effects on wages and employment are qualitatively similar in all equilibria.

5 Involuntary unemployment

Proposition 2 characterizes a focal equilibrium with supra-competitive wages. We now show that there typically exist alternative equilibria, but they lead to even higher wages and consequently to the existence of a pool of workers who would be willing to work for the market wage but are not hired.

Proposition 3 *Unless either²⁴ $R(t^*, t^*) - t^*w^* = 0$ or the elasticity of supply is zero, there exists $\underline{t} < t^*$ such that for each $t \in [\underline{t}, t^*]$, there exists an equilibrium where both firms offer*

$$w = R_1(t, t) - R_2(t, t) \geq S(2t) \quad (2)$$

to (the same) $2t$ workers among those with reservation wage below w , and make no acceptable offer to the rest. Moreover, all symmetric equilibria must be characterized by such behavior for some $t^c \leq t \leq t^$.*

As we have seen in Proposition 2, when $2t^*$ measure of labor is hired the wage equals the marginal worker's reserve price, as well as the marginal willingness to pay of the firms. On the other hand, for $t < t^*$, the inequality in (2) is generically strict and thus we have an excess supply of labor. The root cause for this departure from market clearing is the presence of an externality. A worker who would be willing to accept the equilibrium wage may not receive an offer from Firm 1 for the mere fact that she does not receive an offer from Firm 2.

²⁴Non-negativity is guaranteed by Assumption 1iii. Given that we are studying firms with (downstream) market power, positive profits sound intuitive. However, as we will see in Section 8, it need not be the case: extremely simple and standard models. That is, input competition may eliminate oligopoly rents!

Finally, note that $R_1(\underline{t}, \underline{t}) \leq S(2\underline{t})$ – and, therefore, $t^c \leq \underline{t}$ – as otherwise a firm would prefer to hire additional workers (who would otherwise be unemployed). See Figure 1.

Insert Figure 1 here.

As we have mentioned, we need Assumption 1 to guarantee that a symmetric equilibrium exists, not that an equilibrium with unemployment exists. Indeed, concavity of the revenue function on the flow of workers from the rival firm has two implications that count here. Firstly, concavity guarantees second order conditions for strategies identified in a first order approach. This first order approach always identifies candidate symmetric equilibria with and without unemployment. If the revenue function is C^2 , strict local concavity at a candidate equilibrium with full employment implies strict concavity for neighboring candidate equilibria with unemployment. Secondly, concavity guarantees that a local “best response” (maximum) is also a global one. Absent concavity, a first order approach is not appropriate, whether we refer to an equilibrium with or without unemployment.

The extent of multiplicity, and thus the maximum deviation in equilibrium from the market-clearing wage (upwards) and quantity (downwards), positively depends on the elasticity of supply and on the size of the production externality – which is likely to decrease with the number of firms.

The main insight of this section is that market power in the product market creates segmented competition for otherwise homogeneous workers. Workers targeted by rival firms become more valuable than untargeted workers. Targetability separates each worker as a differentiated market and so, even if firms do not have market power in the labor market at large, they do in this more restricted, endogenously differentiated – unit – labor market. For some labor markets, targetability is not a very plausible assumption. Firms often do not observe who their rivals offer employment to, particularly when they are small in the labor market. Therefore, it is of interest to see to what extent targetability is necessary for our findings above.

6 Anonymity

Assume that firms can decide how many offers they make at what prices, but cannot address particular offers to particular workers. If we introduce *only* this modification in our model, then we obtain the competitive equilibrium as the unique outcome. Nonetheless, the mere fact of working for a rival firm constitutes a label that may be used to target a worker. Then, we can model this ability of firms to “second-hand” target in a dynamic way. If firms have a chance to react to rival offers by targeting new offers to workers hired by the rivals, then – as we will show – the possibility of endogenously segmented labor markets (and supra-competitive wages) reemerges.

Thus, suppose that a total mass of labor, T , may receive offers from two firms.²⁵ Firms do not observe the identity of the workers, so they can only decide how many offers to make at each wage. We investigate the existence of (symmetric, pure-strategy) equilibria. To simplify the discussion, we restrict attention to equilibria with a single wage.

Proposition 4 *Without targeting, in the only pure-strategy, symmetric equilibrium with a single wage, the non-competitive market clearing outcome, (t^*, w^*) , emerges.*

The intuition behind the result is simple: if both firms send offers with the same wage, a firm attracts any worker that gets no other offer plus half the workers that get the competing firm’s offer. By sending offers with slightly higher wages, the firm may increase (discrete jump) the conditional probability that an attracted worker has received an offer from the rival, unless all the workers do get one. Thus, the only possible such equilibrium must have all firms sending offers to all workers, in which case the reservation wage of any non-hired worker must be no less than the wage paid in the industry. That is, we must have market clearing.

It is worth emphasizing that any other wage cannot be an equilibrium outcome, not because this would give firms incentives to pay lower wages and still attract

²⁵Until now, there was no need to specify the total mass of workers as the “tail end” of the supply curve played no role: those workers would never be targeted.

unemployed workers, but rather the opposite: firms would have incentives to increase their wage offers and so compete more fiercely for (some) workers.

This observation indicates that the results in the static model with targeting are not artifacts of the inability of firms to react to competing firms' offers. In fact, as we show next, if we allow "some" dynamics into our model we do not even need to assume targetability of workers to get equilibrium unemployment. Indeed, assume that firms observe which workers have been hired by the rival, and may have a chance to send new offers to those workers before production takes place. Also, let us maintain the assumption that offers are binding for firms, but workers can freely walk out of a contract. Finally, suppose that, before actual production takes place and after all initial offers have been made and accepted, nature selects each firm with equal probability, and the selected firm has a new opportunity to send new offers. This time, firms can differentiate between workers that have been hired by the rival firm.

In addition to Assumption 1, let us assume that $R_{22} > 0$. Also, to simplify the analysis, let us assume that each firm makes all offers with the same wage, although different firms may set different wages. Further, we assume that in the second period only the workers hired in the first period may receive offers. Given the analysis previously displayed in this paper, the reader should convince him or herself that this would be part of an equilibrium.

Proposition 5 *In the anonymous game where each firm is equally likely to have an (exclusive) chance to target new offers at workers hired by the rival, there exist equilibria where firms hire labor t each at wage w , where $S^{-1}(w) > 2t$. That is, there are equilibria with involuntary unemployment.*

To understand the intuition behind this result, recall that unemployment equilibria with all firms offering the same wage was impossible with no second move because a firm had incentives to hire the same amount of workers with a slightly higher wage. By doing so, the firm would significantly reduce the labor hired by the rival. Hence, that deviation would also result in less talent being hired – no matter

what happened in the second period. Now, if the other firm has an opportunity to react, which occurs with 50% probability, it will basically “flip” the labor distribution, so that it would be the deviating rival who would suffer the loss. The joint profits of the two firms would drop, and (in the most favorable case), the expected profits of the deviating firm would still only be 50% of those joint profits.

7 Wage differentials and market segmentation

Qualitatively, the preceding results – that is, the non-existence of the competitive equilibrium, the existence of a market-clearing equilibrium with higher wage and employment and the existence of equilibria with involuntary unemployment – do not depend on the existence (or not) of any asymmetry between firms.²⁶ Therefore, for simplicity, we have presented them assuming symmetry. In this section, we wish to discuss an additional equilibrium phenomenon that some forms of asymmetry may induce: wage differentials.

Once we allow for heterogeneous revenue functions, the structure of externalities can be highly complex. Here we consider one of the simplest possible asymmetric settings. Thus, suppose that the T workers for hire have the common reservation wage r . The demand in this labor market comes from four firms, divided into two groups, $k = 1, 2$. Firms are symmetric within groups and the net revenue function of a firm depends only on the labor hired by the firm and its competitor in the same group: $R^k(t_{own}^k, t_{other}^k)$ for $k = 1, 2$. That is, the economy is divided into two industries (output markets), each with two firms, $i = 1, 2$.²⁷

We may now define t^{*1} and t^{*2} as the solutions to (1) for the revenue functions R^1

²⁶With asymmetry, the equilibrium strategies of workers may have to be asymmetric as well.

²⁷Note that we have introduced several types of heterogeneity: first, we have different revenues associated to the same labor composition across industries; second we have different interactions between firms in the same industry across industries; third, there is heterogeneity in the interactions between firms: the profits of a firm are affected differently by the labor market behavior of other firms, depending on whether they are interacting only in the labor market or in the product market as well.

and R^2 , respectively, with inverse labor demand fixed at $S(t) \equiv r$. Let us consider the case where $2(t^{*1} + t^{*2}) \leq T$, first. In this situation, the labor market is sufficiently thick, so that labor decisions of firms in one industry need not affect the residual labor supply for the firms in the other industry. Indeed, considering Proposition 3, it is straightforward to see that there are intervals $[\underline{t}^k, t^{*k}]$, $k = 1, 2$, such that for any $(t^1, t^2) \in [\underline{t}^1, t^{*1}] \times [\underline{t}^2, t^{*2}]$, it is an equilibrium that each firm in industry k offers the wage $R_1^k(t^k, t^k) - R_2^k(t^k, t^k)$ to $2t^k$ workers, with no worker receiving offers from different industries.

Note that this implies that all of these equilibria but the equilibrium with employment levels (t^{*1}, t^{*2}) exhibit involuntary unemployment and supra-competitive wages. Moreover, in all but a measure zero of these equilibria there will be an inter-industry wage differential: workers hired in an industry are all paid the same wage, but (identical) workers employed in different industries earn different wages.

While the abundance of labor does allow for strategic independence, one should not think that this abundance is what explains why wages can differ across industries. The possibility of a wage differential is an unavoidable consequence of the endogenous coordination of firms that was unveiled in Proposition 3. Indeed, consider the case where the labor market is thin.²⁸ That is, define \widehat{t}^k as the solution to

$$r = R_1^k(t, t),$$

for $k = 1, 2$, and assume that $2(\widehat{t}^1 + \widehat{t}^2) > T$. Note that, as $R_2^k(t, t) < 0$, this implies that $2(t^{*1} + t^{*2}) > T$. Also, define $(\widetilde{t}^1, \widetilde{t}^2)$ as the solution to the system

$$\begin{aligned} R_1^1(t^1, t^1) - R_2^1(t^1, t^1) &= R_1^2(t^2, t^2) - R_2^2(t^2, t^2), \\ 2(t^1 + t^2) &= T. \end{aligned}$$

It is a simple exercise to show that, under Assumption liii), this solution exists and is well defined. The next proposition shows that while $(\widetilde{t}^1, \widetilde{t}^2)$ is an equilibrium labor allocation, resulting in equal wages for all hired workers, it is but one of a

²⁸For simplicity, we assume that there would be excess demand even in the absence of within industry externalities. This is not necessary for establishing inter-industry wage differentials for a thin labor market.

continuum of equilibrium outcomes. In all the other equilibria, workers hired in different industries earn different wages. To visualize the next proposition it is helpful to refer to Figure 2.

Proposition 6 *Let $2(\hat{t}^1 + \hat{t}^2) > T$. For each (t^1, t^2) such that $\max_k R_1^k(t^k, t^k) \leq \min_k R_1^k(t^k, t^k) - R_2^k(t^k, t^k)$ and $2(t^1 + t^2) = T$, there exists a symmetric-inside-industries equilibrium with that labor assignment and $w^k = R_1^k(t^k, t^k) - R_2^k(t^k, t^k)$, $k = 1, 2$.*

Insert Figure 2.

One can think of these equilibria as the combination of two single-industry ones with inelastic labor supply, with the additional requirement that no firm from another industry should be willing to poach a worker who is not targeted by its direct competitor. In these equilibria the labor market is endogenously segmented: firms of the same industry choose to compete for workers (only) with each other. Indeed, each worker receives two offers, both from firms in the same industry. This, in general, leads to different wages in different industries, although within industry the wages continue to be common. Just as when $2(t^{*1} + t^{*2}) \leq T$, some workers willing to work for strictly less than the wages paid in both industries do not receive any offer.

Although Proposition 6 does not describe all possible equilibria, there exists a single alternative equilibrium outcome with a common wage, where all hired workers receive (the same) offer from all four firms.

Proposition 7 *There are only two possible symmetric-inside-industries labor assignments leading to an equilibrium outcome with a common wage: $(\tilde{t}^1, \tilde{t}^2)$ and $t^1 = t^2 = t$ with $R_1^1(t, t) - \frac{1}{3}R_2^1(t, t) = R_1^2(t, t) - \frac{1}{3}R_2^2(t, t) \geq r$.*

Note that, despite the increased competition, the second equilibrium results in a lower wage than the first one. Indeed, in this equilibrium, firms have more difficulties targeting workers that their rival would hire: each of the workers who receives an offer from the rival also receives offers from other, non-rival firms.

8 A micro-foundation of the net revenue function

In the preceding sections, we have used a reduced-form approach to modelling production and product market competition, positing general revenue functions. Of course, we needed to impose some, seemingly strong, assumptions on these functions to be able to prove our results. In this section we analyze a particular, standard output-market competition model that satisfies Assumption 1: the (linear) Dixit (1979) model of competition with differentiated goods. We use the model, which in the limit coincides with a linear Cournot model, to illustrate the results obtained. We will also discuss some welfare properties for this particular model.

Thus, assume that the representative consumer has utility function $U(m, q_1, q_2) = m + \sum_i (aq_i - \frac{b}{2}q_i^2 - cq_1q_2)$, where q_i represents the output of Firm i , and m is the rest of consumption – numeraire. The resulting inverse demand for Firm i 's product is

$$p^i(q_i, q_j) = a - bq_i - cq_j.$$

If firms operate constant (normalized to unit) returns to scale technologies, then their net revenues will be²⁹

$$R^i(t_i, t_j) = (a - bt_i - ct_j)t_i.$$

Thus, Assumption 1 is satisfied, as in this model $R_{11}(t_1, t_2) = -2b < 0$, $R_{12}(t_1, t_2) = -c < 0$, and $R_{11}(t_1, t_2) - 2R_{12}(t_1, t_2) + R_{22}(t_1, t_2) = -b + c$, which is negative as long as the price of each good is more responsive to changes in its own quantity than to changes in the quantity of the other good.

We may now compute the set of all symmetric equilibria that we have discussed in previous sections. For that purpose, assume the supply of labor is infinitely elastic, $S(t) = r < a$ for all $t > 0$. Then, the market-clearing equilibrium, discussed

²⁹Here we abstract away from the possible complications that might arise from hiring labor in advance. In that case firms may reduce output below the maximum that they could obtain using all hired labor. This may be an optimal policy when the rival firm deviates from equilibrium behavior. The analysis of these deviations would be more involved, but nothing fundamental would change.

in Proposition 2 and characterized by $R_1(t^*, t^*) - R_2(t^*, t^*) = a - 2bt^* = r$ is

$$t^* = \frac{a - r}{2b}.$$

On the other hand, the – upstream – competitive level of employment is

$$t^c = \frac{a - r}{2b + c}.$$

That is the level at which $R_1(t, t) = a - 2bt - ct = r$. Note that indeed $t^c < t^*$.

We can now obtain the set of symmetric equilibria that Proposition 3 refers to.

Corollary 2 *In the linear, Dixit model with constant returns to scale and infinitely elastic labor supply, the set of symmetric-equilibrium employment levels is $[\underline{t}, t^*]$, with the corresponding wage of $a - 2bt \geq r$ for a given t , where*

$$\underline{t} = \frac{a - r}{2b + c - \frac{c^2}{2b - c}}.$$

Note that $t^c < \underline{t}$, so all equilibria involve a higher than competitive employment. It is straightforward to see that the efficient (i.e., total surplus maximizing) labor force (per firm) would be $\bar{t} = \frac{a - r}{b + c} \geq t^*$. That is, the most efficient of all the equilibria is the market clearing, t^* . In fact, in this model total industry profits – $2(R(t, t) - wt)$ – are aligned with total surplus: they are larger the larger is t .

It is also interesting to consider the limit case, as $c \rightarrow b$. That limit case is the homogeneous-product, Cournot competition. We obtain as a limit $\underline{t} = t^* = \bar{t} = \frac{a - r}{2b} > \frac{a - r}{3b} = t^c$, with wage equal to r (and firms making zero profit). Thus, considering the feedback across markets, market power in the output market is completely bid away in the input market, and so efficiency is fully restored in this particular case. Note that the limit case itself, with linear demand and constant returns, satisfies $R(t^*, t^*) - w^*t^* = 0$; that explains the degenerate interval of symmetric equilibria.

9 Conclusion

We have explored the consequences of linkages between market power in the output market and outcomes in upstream markets, particularly labor markets. When workers are viewed as separate markets, firms have incentives to restrict rivals' access to labor, even when it is in abundant supply. This option to foreclose results in higher wages and higher employment, alleviating the anti-competitive effect of (downstream) market power.

We need not specify which subset of workers the firms choose to bid for. Obviously, a number of productivity-irrelevant characteristics (gender, race, first letter of last name etc.) could serve this purpose and in that respect our model could also be a useful vehicle for modelling discrimination (c.f. Mailath et al., 2000). Similarly, by (re)interpreting the variable t as one measuring efficiency units, our model could easily accommodate workers with heterogeneous productivities.

It is revealing to note that the targetability of offers is not necessary for the high wage/employment result. Rather, it is the explicit consideration of each worker's decision over which offer to accept that matters. Targetability does lead to additional equilibria, which exhibit even higher wages, but lower employment, leading to involuntary unemployment. These outcomes resemble those of efficiency wage models. The difference is that the wage premium paid to hired workers is not an instrument to motivate the workers, but a consequence of imperfect competition in the product market. Due to the foreclosure effect, firms are led to endogenously coordinate and make offers to the same subset of workers, while their willingness to pay for an uncontested worker is strictly lower. This same endogenous coordination may lead to inter-industry wage differentials, even in the absence of any industry specificities with respect to labor.

From an applied point of view, our analysis underscores the importance of carefully understanding the interplay of market-power rents and upstream competition for the markets involved. Competition for inputs may not only transfer rents from downstream firms to suppliers – in this case, workers –, but it may actually reduce

the size of these rents, and in so doing restore, at least partially, efficiency. It is well understood that assessing the effects and costs of market power, for instance in merger analysis, cannot be satisfactorily done without paying due attention to how the involved firms compete for their suppliers. This paper highlights one so far neglected intricacy that may characterize this competition.

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Appendix

Proof of Lemma 1:

Proof. Assume, by way of contradiction, that in equilibrium and in expectation a measure $\alpha > 0$ of workers accept offers in $[0, b]$ and a positive measure of workers accept offers in $[c, d]$ for some $0 \leq b < c \leq d$. Take a firm that offers in $[c, d]$ to a positive measure and denote by $\beta > 0$ the measure of workers it hires in expectation. Similarly, denote by $\delta \in [0, \alpha]$ the expected measure of workers this same firm hires for wages in $[0, b]$. Let $\gamma = \min(\alpha - \delta, \beta)$. Assume first that $\delta < \alpha$, and therefore $\gamma > 0$. Now if our firm deviates and outbids its rival in $[0, b]$ by $\varepsilon < c - b$, for enough workers so that it ends up in expectation with $\delta + \gamma$ workers for wages in $[0, b + \varepsilon]$, while it withdraws enough offers from $[c, d]$, so that it hires $\beta - \gamma$ from that interval, it increases its expected payoff: net revenues from the product market stay the same but the wagebill is strictly lower. If $\delta = \alpha$, and therefore $\gamma = 0$, then the offers in $[a, b]$ are not to contested workers and by the above argument the other firm cannot be making offers above b either, so it is driven out of the market. ■

Proof of Proposition 1:

Proof. First, we show that firms offering w^c to $2t^c$ workers in such a way that each worker with reservation wage below w^c receives exactly two offers – of which she chooses one with equal probabilities – is indeed an equilibrium. Suppose that Firm 2 behaves according to the hypothetical equilibrium strategy, and consider the best response of Firm 1: $W_1(\cdot)$. There are two types of worker to target: there are measure $2t^c$ workers with an offer of w^c and a reservation wage less than that, and the rest of the workers who have a reservation wage above w^c . Obviously, the firm should hire workers in increasing order of their – now perhaps determined by Firm 2's offer – reservation wage, until this wage equals marginal revenue. Thus, by the definition of t^c , the equilibrium strategy is indeed a best response.

We now show that there exists no other equilibrium outcome (with deterministic wage schedules). Assume, by way of contradiction, that there is a positive measure of workers who get hired for a wage strictly below w^c in equilibrium. Then there

must exist a firm who would be willing to change a positive measure of its offers and instead offer ε more to these workers, as the aggregate demand at infra-competitive wage strictly exceeds the supply of workers. Consequently (almost) no worker can be hired for less than w^c . Similarly, assume, by way of contradiction, that there is a positive measure of workers who get hired for a wage strictly above w^c . Then there must be workers with a reservation wage strictly below w^c who do not receive an acceptable offer, as the aggregate demand at supra-competitive wages is strictly less than the supply of workers. Consequently, (almost) all the hired workers must be paid w^c . As no firm is willing to hire more than its competitive demand at w^c , there are always workers available for hire at this wage, so each firm must hire up to its competitive quantity. ■

Proof of Proposition 2:

Proof. To see that (t^c, w^c) cannot be sustained by an equilibrium, note that as $w^c = R_1(t^c, t^c) < R_1(t^c, t^c) - R_2(t^c, t^c)$, any firm would prefer to outbid its rival by ε on some of the workers and lower its offer to zero for other ones, so that in expectation it hires the same amount of workers, but the rival hires strictly fewer. The rest of the proposition follows from Proposition 3, proven below. ■

Proof of Proposition 3:

Proof. Note that Assumption 1, ii) and iii) and $S' \geq 0$, ensure that (t^*, w^*) is well defined. We prove the first part of the proposition by contradiction. Assume there is no interval $[t, t^*]$ such that for each t in the interval the equilibrium depicted in the proposition exists. That implies that there exists a sequence $t_n \uparrow t^*$, where, for all n , Firm 1 has a better response than the proposed strategy to Firm 2 offering $w_n = R_1(t_n, t_n) - R_2(t_n, t_n)$ to the $2t_n$ workers with lowest reservation wage. Recall that workers randomize 50-50 when receiving equal offers. The response of Firm 1 amounts to finding quantities α of labor hired from the pool of $2t_n$ who receive offers from Firm 2, and β of labor hired from the pool that does not receive offers from Firm 2. Indeed, that can always be done by offering w_n to 2α of the former³⁰ and their reservation wage to workers in the interval $(2t_n, 2t_n + \beta)$. For given α and

³⁰If $\alpha > t$ then Firm 1 can offer $w + \varepsilon$ to α workers.

β , the profit of Firm 1 is then (arbitrarily close to)

$$\pi^1(\alpha, \beta) = R(\alpha + \beta, 2t_n - \alpha) - w_n\alpha - \int_{2t_n}^{2t_n + \beta} S(x)dx, \quad (3)$$

with derivatives

$$\frac{d\pi^1(\alpha, \beta)}{d\alpha} = R_1(\alpha + \beta, 2t_n - \alpha) - R_2(\alpha + \beta, 2t_n - \alpha) - w_n \quad (4)$$

and

$$\frac{d\pi^1(\alpha, \beta)}{d\beta} = R_1(\alpha + \beta, 2t_n - \alpha) - S(2t_n + \beta). \quad (5)$$

Note that $\frac{d\pi^1(\alpha, 0)}{d\alpha} = R_1(\alpha, 2t_n - \alpha) - R_2(\alpha, 2t_n - \alpha) - w_n = R_1(\alpha, 2t_n - \alpha) - R_2(\alpha, 2t_n - \alpha) - R_1(t_n, t_n) + R_2(t_n, t_n)$. Since $\frac{d\pi^1(t_n, 0)}{d\alpha} = 0$, and $R_1 - R_2$ is concave in α from Assumption 1iii), we conclude that a best response given $\beta = 0$ is $\alpha = t_n$. Thus, if there is a better response than the putative equilibrium strategy, $\alpha = t_n$, $\beta = 0$, it must be with $\beta > 0$.

Let the sequence of best responses to $\{t_n\}$ be denoted $\{\alpha_n, \beta_n\}$. This sequence is bounded, since $R(t^*, t^*) - wt^* > 0$ and $\pi^1(\alpha, \beta) < 0$ for large enough α and/or β . Thus, it has accumulation points. Suppose $(0, 0)$ is an accumulation point for this sequence. Then, for some n large enough $\pi^1(\alpha_n, \beta_n)$ is arbitrarily close to $R(0, 2t^*) = 0$. This cannot be a best response, since $\pi^1(t_n, 0)$ is arbitrarily close to $R(t^*, t^*) - wt^* > 0$.

Thus, at any accumulation point $(\hat{\alpha}, \hat{\beta})$, we have $\hat{t} = \hat{\alpha} + \hat{\beta} > 0$. That means that there exists a subsequence of $\{\alpha_n, \beta_n\}$ such that for n large $R_2(\alpha_n + \beta_n, 2t_n - \alpha_n)$ is arbitrarily close to $R_2(\hat{t}, 2t^* - \hat{\alpha}) < 0$. Since $\beta_n > 0$, (5) equals zero evaluated at (α_n, β_n) , so that $R_1(\hat{t}, 2t^* - \hat{\alpha}) = S(2t^* + \hat{\beta}) \geq S(2t^*)$. Also, since (4) is not larger than zero at these values (zero if $\alpha_n > 0$), $R_1(\hat{t}, 2t^* - \hat{\alpha}) - R_2(\hat{t}, 2t^* - \hat{\alpha}) \leq \bar{w} = R_1(t^*, t^*) - R_2(t^*, t^*) = S(2t^*)$. As $R_2(\hat{t}, 2t^* - \hat{\alpha}) < 0$, the two previous inequalities are contradictory. Therefore, there cannot exist a sequence $t_n \uparrow t^*$, where, for all n , Firm 1 has a better response than the proposed strategy when Firm 2 follows that strategy. This in turn proves that there has to exist a non-degenerate interval $[\underline{t}, t^*]$ in which the proposed strategies are indeed an equilibrium.

We now show that there is no other class of symmetric equilibria in pure wage schedules. By Lemma 1 the wage offers by both firms are identical for all the workers that are hired. Assume all workers hired receive a wage different from $R_1(t, t) - R_2(t, t)$, where t is the amount of labor hired by each firm. If the wage is larger than $R_1(t, t) - R_2(t, t)$, then a firm would profit from withdrawing some of its offers, whereas if it is lower than that amount, then a firm would profit from increasing its offers to a small mass of workers, and so “stealing” (the other) half of them from its rival. Thus, both firms must offer the same wage $w = R_1(t, t) - R_2(t, t)$ to $2t$ workers, as each hire half of them. These $2t$ workers must accept, so their reservation wage must be below w . This completes the proof. ■

Proof of Proposition 4:

Proof. Let Firm 2 play its equilibrium strategy, and so offer wage w to an amount of labor q_2 .³¹ Consider Firm 1’s strategy consisting of sending offers with the same wage w (say, b of them) or – slightly – higher wages (say, a of them), and no offers with lower wages. Any of the a offers would be accepted by the worker receiving it if her reservation wage is lower than w , whereas each of the b ones would be accepted with probability $\frac{2T - q_2}{2T}$ by these same workers. Indeed, the probability that any given offer of w made by Firm 1 is to a worker who also receives an offer from Firm 2 is $\frac{q_2}{T}$. Half of these workers will accept Firm 1’s offer if w exceeds their reservation wage. In addition, the probability that the offer is to a worker who does not receive an offer from Firm 2 is $\frac{T - q_2}{T}$, and then the offer will be accepted if w exceeds the worker’s reservation wage. Similarly, we can compute the probability that Firm 2’s offer is accepted by a worker with reservation wage below w when these strategies are used, as $\frac{2(T - a) - b}{2T}$. Thus, for $a = 0, b > 0$ to be a best response for Firm 1, these

³¹For a symmetric equilibrium with wages w , any offer below or above w must be rejected. Offers above – since offers cannot be targeted – will always have a positive probability of being accepted. Offers below too, unless the rival firm offers wage w to *all* workers. Symmetry would imply that both firms do that, eliminating the possibility of offers below w too.

values would have to maximize³²

$$R\left(\frac{S^{-1}(w)}{T}\left[a + \frac{2T - q_2}{2T}b\right], \frac{S^{-1}(w)}{T}\left[\frac{2(T - a) - b}{2T}q_2\right]\right) - w\frac{S^{-1}(w)}{T}\left(a + \frac{2T - q_2}{2T}b\right).$$

The first order conditions for $a = 0$, $b > 0$ require

$$R_1 - \frac{q_2}{2T - q_2}R_2 = w \geq R_1 - \frac{q_2}{T}R_2,$$

where we have omitted the arguments of the functions. Since $R_2 < 0$, these conditions are incompatible unless $q_2 = T$. For $b = q_2 = T$, the labor hired by each firm is $t = \frac{S^{-1}(w)}{2}$, so that we have demand equal to supply and the first order conditions imply

$$R_1(t, t) - R_2(t, t) = w = S(2t),$$

as we wanted to show. ■

Proof of Proposition 5:

Proof. We will show that there exist t and w with $S^{-1}(w) > 2t$, and a symmetric equilibrium where both firms hire t workers in the first period with offers w , and send no new offers in the second period. For that to be the case, each firm must send q offers of w , satisfying

$$t = q\frac{S^{-1}(w)}{T} \cdot \frac{2T - q}{2T}. \quad (6)$$

Suppose that Firm 2 does so, but Firm 1 deviates and sends q^1 offers with wage w^1 . Consider $w^1 \geq w$, first. It is straightforward that such a deviation can only be profitable if it leads Firm 1 to hire $t^1 = \frac{q^1}{T}S^{-1}(w^1) > t$. Firm 1's deviation also affects how much labor Firm 2 hires:

$$t^2(t^1, w^1) = q\frac{S^{-1}(w)}{T}\left[1 - \frac{t^1}{S^{-1}(w^1)}\right]. \quad (7)$$

The first term in the right hand side is measure of offers sent by Firm 2, the second is the probability that a worker that receives one has a reservation wage below w ,

³²The expression is written as if all offers a included wage offer w . Note that the profits obtained by the firm with a offers just above w can be arbitrarily close to that expression.

and the third term is the probability that one such worker has not received one of the offers sent by Firm 1.

First, it may be Firm 1 who has a chance to send new offers. Then it solves the problem

$$\max_{\Delta^1 \geq 0} \{R(t^1 + \Delta^1, t^2(t^1, w^1) - \Delta^1) - w\Delta^1\}.$$

The first order condition is

$$\Phi = R_1(t^1 + \Delta^1, t^2(t^1, w^1) - \Delta^1) - R_2(t^1 + \Delta^1, t^2(t^1, w^1) - \Delta^1) - w = 0. \quad (8)$$

The objective function is concave (in Δ^1) and continuous in $[0, t^2(t^1, w^1)]$. Let the solution be $\widehat{\Delta}^1$. Observe that

$$\frac{d\widehat{\Delta}^1}{dw^1} = -\frac{\frac{\partial t^2}{\partial w^1} [R_{12} - R_{22}]}{\partial \Phi / \partial \Delta^1} \leq 0,$$

since the denominator is negative and $\frac{\partial t^2}{\partial w^1} > 0$.

Now assume it is Firm 2 that can make new offers. It solves a similar problem, namely

$$\max_{\Delta^2 \in [0, t^1]} R(t^2(t^1, w^1) + \Delta^2, t^1 - \Delta^2) - w^1 \Delta^2.$$

This optimization problem is also concave in Δ^2 , with first order condition

$$R_1(t^2(t^1, w^1) + \Delta^2, t^1 - \Delta^2) - R_2(t^2(t^1, w^1) + \Delta^2, t^1 - \Delta^2) - w^1 = 0. \quad (9)$$

Let the solution be $\widetilde{\Delta}^2$. Similarly as before, $\frac{d\widetilde{\Delta}^2}{dw^1} \leq 0$.

Let us now consider the best deviation (with $w^1 > w$) by Firm 1 in period 1. The optimal (w^1, t^1) solves

$$\max_{w^1 \geq w, t^1 \geq t} \frac{1}{2} \{R(\widehat{t}^1, \widehat{t}^2) - w^1 t^1 - w(\widehat{t}^1 - t^1) + R(\widetilde{t}^1, \widetilde{t}^2) - w^1 \widetilde{t}^1\},$$

where $\widehat{t}^1 = t^1 + \widehat{\Delta}^1$, $\widehat{t}^2 = t^2(t^1, w^1) - \widehat{\Delta}^1$, $\widetilde{t}^1 = t^1 - \widetilde{\Delta}^2$, and $\widetilde{t}^2 = t^2(t^1, w^1) + \widetilde{\Delta}^2$. The derivative of the objective function with respect to w^1 is

$$\begin{aligned} & \frac{1}{2} \left[-t^1 + \frac{\partial \widehat{t}^1}{\partial w^1} (R_1(\widehat{t}^1, \widehat{t}^2) - w) + \frac{\partial \widehat{t}^2}{\partial w^1} R_2(\widehat{t}^1, \widehat{t}^2) \right] \\ & + \frac{1}{2} \left[-\widetilde{t}^1 + \frac{\partial \widetilde{t}^1}{\partial w^1} (R_1(\widetilde{t}^1, \widetilde{t}^2) - w^1) + \frac{\partial \widetilde{t}^2}{\partial w^1} R_2(\widetilde{t}^1, \widetilde{t}^2) \right]. \end{aligned}$$

Note that $\frac{\partial \hat{t}^1}{\partial w^1} = \frac{d\hat{\Delta}^1}{dw^1} = \frac{\partial t^2}{\partial w^1} - \frac{\partial \tilde{t}^2}{\partial w^1}$. Also, note that $\frac{\partial \tilde{t}^2}{\partial w^1} = \frac{d\tilde{\Delta}^2}{dw^1} + \frac{\partial t^2}{\partial w^1} = -\frac{\partial \tilde{t}^1}{\partial w^1} + \frac{\partial t^2}{\partial w^1}$.

Thus, the expression above can be written as

$$\begin{aligned} & \frac{1}{2} \left[-t^1 + \frac{d\hat{\Delta}^1}{dw^1} (R_1(\hat{t}^1, \hat{t}^2) - R_2(\hat{t}^1, \hat{t}^2) - w) + \frac{\partial t^2}{\partial w^1} R_2(\hat{t}^1, \hat{t}^2) \right] \\ & + \frac{1}{2} \left[-\tilde{t}^1 - \frac{d\tilde{\Delta}^2}{dw^1} (R_1(\tilde{t}^1, \tilde{t}^2) - R_2(\tilde{t}^1, \tilde{t}^2) - w^1) + \frac{\partial t^2}{\partial w^1} R_2(\tilde{t}^1, \tilde{t}^2) \right]. \end{aligned}$$

As we mentioned, $\frac{d\hat{\Delta}^1}{dw^1} \leq 0$, and $\frac{\partial t^2}{\partial w^1} > 0$ in (7). Also, note that $R_1(\hat{t}^1, \hat{t}^2) - R_2(\hat{t}^1, \hat{t}^2) = w$, from (8). Thus, this whole expression can be seen to be negative if

$$R_1(\tilde{t}^1, \tilde{t}^2) - R_2(\tilde{t}^1, \tilde{t}^2) - w^1 \leq 0.$$

Note that $\tilde{t}^1 + \tilde{t}^2 = \hat{t}^1 + \hat{t}^2 = t^1 + t^2(t^1, w^1)$, and so if $\tilde{t}^1 > \tilde{t}^2$, just as in the case of (8), the inequality follows. Thus, we must have an optimizer at w ($+\epsilon$) if $\tilde{t}^1 \geq \tilde{t}^2$. Note that, once again resorting to Assumption 1, taking into account the first- and second-order condition for the optimal choice of Firm 2 (9) and the fact that $\tilde{t}^1 + \tilde{t}^2 = t^1 + t^2(t^1, w^1) \geq 2t$, indeed $\tilde{t}^1 \geq \tilde{t}^2$.

Thus, any (optimal) deviation simply sets an amount of labor hired, $t^1 + t^2(t^1) \geq 2t$, and then results in each firm choosing how much of it to hire at wage w with probability 1/2. I.e., the best deviation results in a symmetric (expected) situation, and so same expected profits, for both firms for each t^1 . In the region with $t^1 \geq t$, and so $t^1 + t^2(t^1) \geq 2t$, the expected profits are largest when, $t^1 = t$, and so $t^2(t^1) = t$, since for any $t' > t$,

$$\begin{aligned} R(t, t) - wt &> R(t', t') - wt' \geq \\ &\geq \frac{1}{2}R(t' + \delta, t - \delta) + \frac{1}{2}R(t' - \delta, t + \delta) - t'w. \end{aligned}$$

Indeed, the first inequality follows from the fact that at given wage w , the derivative of $R(t', t') - wt'$ with respect to t' is

$$R_1(t', t') + R_2(t', t') - w < R_1(t', t') - R_2(t', t') - w,$$

and $R_1(t', t') - R_2(t', t')$ is decreasing in t' . Thus, for $w = R_1(t, t) - R_2(t, t)$, that derivative is negative. The second follows from Assumption 1.

Now, consider a deviation with $w^1 < w$. Given w^1 , t^1 would be given by

$$t^1 = S^{-1}(w^1) \frac{q^1}{T} \left(1 - \frac{q}{T}\right).$$

That is, t^1 can be chosen to be any number in $\left[0, \frac{S^{-1}(w^1)}{T}(T - q)\right]$. Observe that (6) implies that for w sufficiently close to w^* , q is arbitrarily close to T , so that this interval is arbitrarily small. That is, t^1 is arbitrarily small and so the $R(t^1, t^2) - w^1 t^1$ is arbitrarily small. That is, smaller than $R(t, t) - wt$. Also, the maximum of $R(t^1 + \Delta, t^2 - \Delta) - w^1 t^1 - w\Delta$ in Δ is attained arbitrarily close to $\Delta = t$, since $t^1 + t^2$ is arbitrarily close to $2t$. Thus, the losses in case it is Firm 2 who is allowed to send new offers more than offset any possible gains in case Firm 1 is allowed to readjust employment. This proves the result. ■

Proof of Proposition 6:

Proof. First, note that the system of inequalities

$$\begin{aligned} 2(t^1 + t^2) &= T, \\ \max_k R_1^k(t^k, t^k) &\leq \min_k (R_1^k(t^k, t^k) - R_2^k(t^k, t^k)), \end{aligned}$$

has a continuum of solutions (t^1, t^2) – the interval connecting the two endpoints of the rhombus in Figure 2. Let us consider any one of those solutions, and let both firms in Industry 2 offer $w^2 = R_1^2(t^2, t^2) - R_2^2(t^2, t^2)$ to the same $2t^2$ workers. Also, assume that Firm 2 in Industry 1 offers wage $w^1 = R_1^1(t^1, t^1) - R_2^1(t^1, t^1)$. As in the proof of Proposition 3, the best response for Firm 1 in Industry 1 may be characterized as choosing α and β so as to maximize

$$\pi_1^1(\alpha, \beta) = R^1(\alpha + \beta, 2t^1 - \alpha) - w^1 \alpha - w^2 \beta,$$

which is the same expression as (3) specialized for $S(t) = w^2$. The result then follows from Proposition 3. ■

Proof of Proposition 7:

Proof. We start by showing that in any equilibrium with a common wage and with a symmetric-inside-industries talent outcome (almost) all workers receive a highest offer from either both or none of the firms in each industry. Note that, since we

are dealing with equilibrium outcomes without price dispersion, all hired workers receive the same highest offer w . Assume that in (at least) one industry, say 1, only one firm makes this offer. Then w must equal $R_1^1(t^1, t^1)$. Then the other firm from Industry 1, who is also hiring at wage w would benefit from offering some of these workers a wage $w + \epsilon$ with $R_1^k(t^k, t^k) < w + \epsilon < R_1^k(t^k, t^k) - \frac{1}{3}R_2^k(t^k, t^k)$, where the $1/3$ is the “worst case” scenario, where the probability that the worker attracted away is from the rival is lowest (that is, when she receives three other offers).

It follows that either all firms make the same offer for all hired talent, or firms divide the market by industry. In this second case, equilibrium requires $w = R_1^k(t^k, t^k) - R_2^k(t^k, t^k)$, $k = 1, 2$, and in order for workers to accept offers, $w \geq r$. That implies that the inequality in the text of Proposition 6 is satisfied. In case all firms make the highest offer w to all hired workers, each firm hires a fourth of all talent hired. The wage must be equal to $R_1^k(t, t) - \frac{1}{3}R_2^k(t, t)$, or else a firm has incentives to either to hire less (when the wage is high), by dropping some offers (one fourth of which would have been accepted), or hire more (when the wage is low) by increasing some offers slightly (three fourths of which would have been rejected), and then attract these workers – one third of whom come from its rival in the industry – for certain. The if part is immediate: for outcomes satisfying Proposition 6 it has been proved, and for the rest of cases it is a straightforward exercise to check that the described strategies constitute an equilibrium. ■

Proof of Corollary 2

Proof. Suppose Firm 2 sends $2t$ offers with wage $w = a - 2bt$, and let x represent the total labor that Firm 1 hires, and α the labor that it hires (by sending offers $\sim w$ but larger than w) from the pool of workers that receive offers from Firm 2. Since the uncontested workers are paid r , Firm 1’s profit $\pi^1(x, \alpha; t)$ has a positive slope in α if and only if (c.f. (3) in the proof of Proposition 3)

$$x > \frac{w - r}{c} \iff t > t^c = \frac{a - r}{2b + c}.$$

Thus, given x , the optimal α , $\alpha(x)$, equals x if the inequality is satisfied and 0 if it is reversed.³³ Also, $\pi^1(x, \alpha(x); t)$, is concave in x both to the right and to the left

³³It is interesting to observe that the binding deviation from a symmetric putative equilibrium

of $\frac{w-r}{c}$, and its derivative to the right of that value – when $\alpha(x) = x$ – is zero at $x = t$. Thus, we have a symmetric equilibrium if and only if the left derivative with respect to x , evaluated at $x = \frac{w-r}{c}$ is non-negative. This is the case if

$$t \geq \underline{t} = \frac{a - r}{2b + c - \frac{c^2}{2b-c}}.$$

■

with a little lower measure of labor hired is not to hire more workers (either contested or uncontested) rather, to drop all offers to contested workers and hire some – but fewer – uncontested ones.