The Value of Useless Information

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Abstract

There are many situations in which individuals have a choice of whether or not to observe eventual outcomes. In these instances, individuals often prefer to remain ignorant. These contexts are outside the scope of analysis of the standard von Neumann-Morgenstern (vNM) expected utility model, which does not distinguish between lotteries for which the agent sees the final outcome and those for which he does not. I develop a simple model that admits preferences for making an observation or for remaining in doubt. I then use this model to analyze the connection between preferences of this nature and risk-attitude. This framework accommodates a wide array of behavioral patterns that violate the vNM model, and that may not seem related, prima facie. For instance, it admits self-handicapping, in which an agent chooses to impair his own performance. It also accommodates a status quo bias without having recourse to framing effects, or to an explicit definition of reference points. In a political economy context, voters have strict incentives to shield themselves from information. In settings with other-regarding preferences, this model predicts observed behavior that seems inconsistent with either altruism or self-interested behavior.

Keywords: Value of information, uncertainty, recursive utility, doubt, unobserved outcomes, unresolved lotteries.

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1 Introduction

Models of decision making implicitly assume that agents expect to observe the resolution of uncertainty eventually. However, there are many situations in which individuals can avoid discovering which outcome has occurred. In these cases, they often prefer to remain uninformed. For instance, people often choose not to learn about the work conditions at companies whose products they buy, and a husband may not wish to know whether he really is the biological father of the child he has raised. Consider also the classic example of Huntington's disease (HD). HD, a neurodegenerate disease with severe physical and cognitive symptoms, reduces life expectancy significantly and has no known cure. There is extensive evidence that parents prefer not to conduct a prenatal test to determine whether their unborn child will have the disease, even though there is a 50% chance of the child developing the disease if one parent is afflicted with it.[1] Observing the result is an important decision, since the prenatal test is done at a stage in which parents can choose to terminate the pregnancy. Even if parents are opposed to this choice, acquiring information could still impact the way they raise their child. The choice to remain uninformed may appear surprising, but because the average of onset of HD is high, parents may *never* observe whether their children are affected. And while there is evidence that people who have taken a predictive test for their own HD status are often unwilling to have their child tested, the two factors are distinct. A predictive test is linked to a preference for early resolution of uncertainty, as the parent will eventually find out whether he has the disease. In contrast, choosing (or refusing) to take a prenatal test mainly reveals a preference for observing an outcome over which the person could remain permanently uninformed. It is precisely this type of preferences on which this paper focuses.[2]

Preferences of this nature are pervasive, as attested by the number of well-known expressions that have arisen to describe them. Yet they might not appear rational from the standard economics perspective that information cannot have negative value. However, it is perhaps more accurate to state that these preferences are outside the scope of expected utility theory. The standard von Neumann-Morgenstern (vNM) expected utility model does not make a distinction between lotteries for which the final outcomes are observed and lotteries for which they are not, and therefore does not allow agents to

[1] See Tyler et al. (1990), Adam et al. (1993), Simpson et al. (2002). For a discussion of these choices, see Pinker (2007).

[2] In particular, this paper does not consider other factors that are present in the HD example, such as parents' concern that their child will be treated differently if it is known that he has HD, as discussed in Simpson (2002).

express preferences one way or the other. That is, the vNM framework does not provide agents with a rich enough choice set to infer their preferences over lotteries of this type. Redefining the outcome space to include whether the prize is observed does not resolve the issue.[3] For this reason, the vNM model cannot accommodate preferences for remaining ignorant. In this paper, I enrich the space over which preferences can be exhibited, allowing the agent to express his preferences over lotteries whose resolution he may not see. I then extend the basic vNM axioms to develop a model that admits strict preferences for remaining in doubt or for observing the outcome. Intuitively, these preferences arise from the difference between the agent's risk-aversion over lotteries he observes and lotteries that he does not. These attitudes need not be everywhere identical, as the notion of risk-aversion carries a different meaning when the agent does not 'consume' the final outcome. A preference for remaining ignorant does not contradict the precept that information has negative value in an expected utility setting, as information here is taken over a different domain.

While remaining close to the standard vNM model, this framework accommodates a wide array of field and experimental observations that are considered incompatible with expected utility theory. Seemingly unrelated behavioral patterns that have motivated a range of significantly different frameworks are consistent with this one. To support this claim, I use a simplified version of this model and maintain the same assumptions throughout that agents are *doubt-prone*, meaning that when given the choice between observing and not observing a lottery's resolution, they prefer *not* observing it.

I first apply this framework to self-handicapping, in which individuals choose to reduce their chances of succeeding at a task. As discussed in Benabou and Tirole (2002), people may "choose to remain ignorant about their own abilities, and [...] they sometimes deliberately impair their own performance or choose overambitious tasks in which they are sure to fail (self-handicapping)." This behavior has been studied extensively, and seems difficult to reconcile with expected utility theory. For that reason, models that study self-handicapping make a substantial departure from the standard vNM assumptions. A number of models follow Akerlof and Dickens' (1982) approach of endowing agents with manipulable beliefs or selective memory. Alternatively, Carillo and Mariotti (2000) consider a model of temporal-inconsistency, in which a game is played between the selves, and Benabou and Tirole (2002) use both manipulable beliefs and

[3] The term observation is defined as learning what the outcome is. This model does not take into account a possible disutility from the graphical nature of the observation itself. See the appendix for a discussion on the problem with redefining the outcome space to include the observation.

time-inconsistent agents.[4]

The frameworks mentioned above capture a notion of self-deception, which involves either a hard-wired form of selective memory (or perhaps a rule of thumb), or some form of conflict between distinct selves. In contrast, the model presented here simply extends the vNM framework and does not allow agents to manipulate their beliefs or to have access to any means for deceiving themselves.[5] Yet it still accommodates the decision to self-handicap, as is shown in Section 3. Intuitively, a doubt-prone agent prefers doing worse in a task if this allows him to avoid information concerning his own ability. This is essentially a formalization of the colloquial 'fear of failure'; an agent exerts less effort so as to obtain a coarser signal.

This model also accommodates a status quo bias. The status quo bias refers to the well-known tendency people have for preferring their current endowment to other alternatives. This phenomenon is often seen as a behavioral anomaly that cannot be explained using the vNM model. However, it can be accommodated using loss aversion, which refers to the agent being more averse to avoiding a loss than to making a gain (Kahneman, Knetch and Thaler (1991)). The status quo bias is therefore an immediate consequence of the agent taking the status quo to be the reference point for gains versus losses. The vNM model does not allow an agent to evaluate a bundle differently based on whether it is a gain or a loss, and hence cannot accommodate a status quo bias. Arguably, this is an important systematic violation of the vNM model and one of the main reasons cited by Kahneman, Knetch and Thaler (1991) for suggesting "a revised version of preference theory that would assign a special role to the status quo."

While the status quo bias is often explained using notions of reference points or relative gains and losses, here they are not explicitly modeled. In cases in which the choices also have an informational component on the agent's ability to perform a task well, a doubt-prone agent has an incentive to choose the bundle that is less informative.

[4]See also Compte and Postlewaite (2004), who focus on the positive welfare implications of having a degree of selective memory (assuming that such technology exists) when performance depends on emotions. Benabou (2008) and Benabou and Tirole (2006, 2011) explore further implications of belief manipulation, particularly in political economy settings, in which multiple equilibria emerge. Brunnermeier and Parker (2005) treat a general-equilbrium model in which beliefs are essentially choice variables in the first period; an agent manipulates his beliefs about the future to maximize his felicity, which depends on future utility flow. Caplin and Leahy (2001) present an axiomatic model in which agents have 'anticipatory feelings' prior to resolution of uncertainty, which may lead to time inconsistency. Koszegi (2006) considers an application of Caplin and Leahy (2001). Wu (1999) presents a model of anxiety. See Berglas and Jones (1978) for the original experiment on self-handicapping.

[5]While the theoretical framework later introduces a notion of optimism, the agents are *not* allowed to be either optimistic or pessimistic in any of the applications considered, as that can perhaps be seen as a form of belief manipulation.

This leads to a status quo bias when it is reasonable to assume that maintaining the status quo is a less informative indicator of the agent's ability than other actions. Since this model does not resort to reference points, there is no arbitrariness in defining what constitutes a gain and what constitutes a loss. The bias of a doubt-prone agent is always towards the least-informative signal of his ability. When the status quo provides the most informative signal, the bias would be *against* the status quo. For example, an individual could have an incentive to change activities frequently rather than obtaining a sharp signal of his ability in one particular field.

This framework admits other instances of seemingly paradoxical behavior. In one example, an individual pays a firm to invest for him, even though he does not expect that firm to have superior expertise. In other words, the agent's utility depends not only on the outcome, but also on who makes the decision. This result is not due to a cost of effort, but rather to the amount of information that the decision maker acquires about himself. This framework can also be applied to a political economy setting, as there are many government decisions that voters never observe. As shown in Section 3, voters may have strong incentives to remain ignorant over these issues, even if information is free. This is in line with the well-known observation that there has been a consistently high level of political ignorance among voters in the U.S. (see Bartels (1996) for details). This model suggests that if voters care more about policies that they may never observe, then they have *less* incentive to acquire information. Moreover, those who are least informed also have greater disutility from learning the truth.

Lastly, it is often the case in settings with other-regarding preferences that individuals can avoid observing the consequences of their actions. In recent experiments on this topic by Dana, Weber and Kuang (2007), an individual's generosity towards others varies significantly depending on whether he expects to see what the others receive. Specifically, the authors consider a dictator game in which the recipient's reward depends on a coin toss that the dictator can observe *before* making his choice, at no cost. Despite this, a high percentage of individuals refuse to observe the coin toss and then choose the 'selfish' option. When the option of remaining uninformed is removed, the percentage of people that choose the selfish option is much lower. These results appear difficult to reconcile with either selfish behavior or altruistic behavior, as they should not depend on the option to remain ignorant. Incorporating doubt-proneness into this context accommodates, and to some extent predicts, these choices.

Turning to the formalization of this model, I first derive a general representation that could, in principle, be empirically tested to elicit individuals' true preferences. I then analyze the interaction between the different factors that influence choice. Outside of anecdotal evidence, little is known concerning preferences to remain ignorant; therefore, a characterization of the interaction between doubt-proneness and other preferences may be of use. The agent has primitive preferences over general lotteries that lead either to outcomes that he observes or to lotteries that never resolve, from his frame of reference.[6] This is a richer domain of lotteries than in the standard vNM case. If the agent receives a lottery that never resolves, then he knows that he will not observe the outcome, and his terminal prize is the lottery itself. I apply the three standard vNM axioms to this expanded domain: weak order, continuity and independence hold. I also assume that the agent is indifferent between observing a specific outcome and receiving an unresolved lottery that places probability one on that same outcome, since he is certain of the outcome's occurrence. The observation itself has no effect on the value of the outcome in this model. This property restricts the agent's allowable preferences over unresolved lotteries, as I demonstrate in Section 4.

I obtain a representation theorem that separates the agent's risk-attitude over lotteries whose outcomes he observes from his risk attitude over unresolved lotteries. I then explore the connection between risk-aversion, doubt-proneness and a new notion of optimism over unresolved lotteries, which I formally define. Contrary to what might be expected, an agent who is both doubt-prone and risk-averse over the unresolved lotteries cannot display either optimism or pessimism. It then follows that his utility function associated with unresolved lotteries is more concave than his utility function associated with lotteries whose outcome he observes. Moreover, I show that if an agent exhibiting optimism over unresolved lotteries has the same utility function for both lotteries that resolve and lotteries that do not, then he must be doubt-prone.

Relation to the literature The approach used in this paper is related to, but distinct from, the recursive expected Utility (REU) framework introduced by Kreps and Porteus (1978), and extended by Epstein and Zin (1989), Segal (1990) and Grant, Kajii and Polak (1998, 2000).[7] These earlier contributions address the issue of temporal resolution, in which an agent has a preference for knowing now versus knowing later. While the REU framework treats the issue of the timing of the resolution, this paper treats the case of *no* resolution. Simply adding a 'never' stage, or any number of additional stages, to the REU space does not yield an equivalent representation. To demonstrate this point, I

[6] Throughout this paper, probabilities are taken to be objective.

[7] Grant, Kajii and Polak (1998) focus on preferences for early resolution of uncertainty, and Dillenberger (2011) considers preferences for one-shot resolution of uncertainty. Selden's (1978) framework is also closely related to the REU model.

place the agent in a two-stage model (Section 6), but do not allow the agent to have preferences over temporal resolution. The agent may, however, change his preferences over unresolved lotteries over time. For instance, he may prefer to avoid information in the early stage, but be curious in the later stage. Unlike REU, the dynamic extension of this model allows for commitment preferences; the agent may essentially destroy evidence to keep himself from accessing it later. This result suggests a new avenue for individuals' choices to restrict their future options. In addition to the formal differences between the two frameworks, there are also interpretational ones. The REU model captures a notion of 'anxiety' (wanting to know sooner or later), which is distinct from the notion of doubt-proneness (not wanting to know at all) addressed here.

This paper is structured as follows. Section 2 introduces a simplified version of the model, which is used in Section 3 for the applications. Section 4 presents the model, and Section 5 defines optimism and discusses the connection among doubt-proneness, optimism and risk-aversion. Section 6 extends the model to a dynamic setting, and considers preferences for commitment in the presence of both doubt preferences and 'curiosity' preferences. Section 7 concludes. All proofs are in the appendix.

2 Simplified Model

I begin with a simplified version of the model, which is sufficient for most applications of interest. The axiomatic treatment is deferred to Section 4. The objects used throughout are as follows. Let $\mathbf{Z} = [\underline{z}, \bar{z}] \subset \Re$ be the outcome space, and let $\mathfrak{L}_0$ be the set of simple probability measures on $\mathbf{Z}$. For $f = (z_1, p_1; z_2, p_2; ...; z_m, p_m) \in \mathfrak{L}_0$, z_i occurs with probability p_i. I use the notation $f(z_i)$ to mean the probability p_i (in lottery f) that z_i occurs. Let $\mathfrak{L}_1$ be the set of simple lotteries over $\mathbf{Z} \cup \mathfrak{L}_0$. For $X \in \mathfrak{L}_1$, I use the notation $X = (z_1, q_1^I; ...; z_n, q_n^I;\ f_1, q_1^N; ...; f_m, q_m^N)$. Here, z_i occurs with probability q_i^I, and lottery f_j occurs with probability q_j^N. Note that $\sum_{i=1}^{n} q_i^I + \sum_{i=1}^{m} q_i^N = 1$. The reason for using this notation, rather than the simpler enumeration $q_1, q_2, ..., q_n$, is explained shortly. Let $\succeq$ denote the agent's preferences over $\mathfrak{L}_1$, and $\succ$, $\sim$ are defined in the usual manner. Assume that the agent's preferences are monotone.

For any $X = (z_1, q_1^I; z_2, q_2^I; ...; z_n, q_n^I;\ f_1, q_1^N; f_2, q_2^N; ...; f_m, q_m^N)$, the agent expects to observe the outcome of the first-stage lottery. He knows, for instance, that with probability q_i^I, outcome z_i occurs, and furthermore he knows that he will observe it. Similarly, he knows that with probability q_i^N, lottery f_i occurs. But while he does observe that he

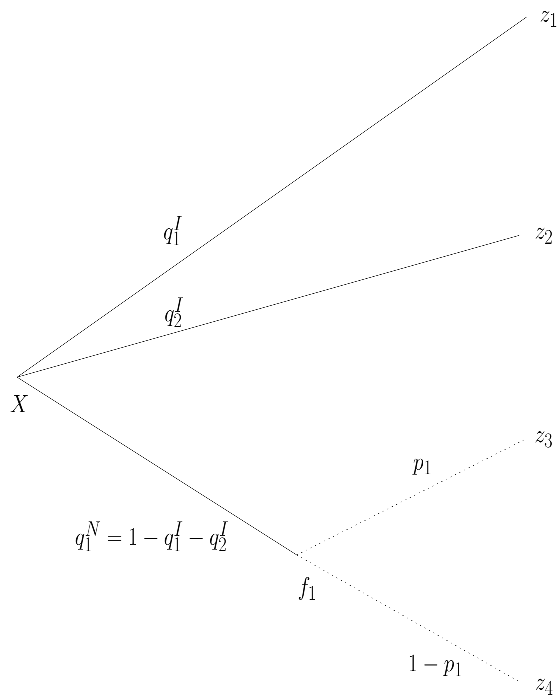

Figure 1: Lottery $X = (z_1, q_1^I; z_2, q_2^I; f_1, q_1^N)$, where $f_1 = (z_3, p_1; z_4, 1 - p_1)$

is now faced with lottery f_i, he does *not* observe the outcome of f_i. I refer to lottery f_i as an 'unresolved' lottery. I also use the notation q_i^I and q_i^N to distinguish between probabilities that lead to prizes where the agent is informed of the outcome (since he directly observes which z occurs), and probabilities that lead to prizes where he is not (since he observes only the ensuing lottery). The superscript I in q_i^I stands for 'Informed', and N in q_i^N for 'Not informed' (see Figure 1).

Denote the degenerate one-stage lottery that leads to $z_i \in \mathbf{Z}$ with certainty $\delta_{z_i} = (z_i, 1) \in \mathfrak{L}_0$. The degenerate lottery that leads to $f_i \in \mathfrak{L}_0$ with certainty is denoted $\delta_{f_i} = (f_i, 1) \in \mathfrak{L}_1$. Note that all lotteries of form $X = f$, where $f \in \mathfrak{L}_0$, are purely resolved (or 'informed') lotteries, in the sense that the agent expects to observe whatever outcome occurs. Similarly, all lotteries of form $X = \delta_f$, where $f \in \mathfrak{L}_0$, are purely unresolved lotteries. With slight abuse, the notation $f \succeq f'$ (or $\delta_f \succeq \delta_{f'}$) is used, where $f, f' \in \mathfrak{L}_0$. In addition, $f \succeq \delta_f$ (or $\delta_f \succeq f$) indicates that the agent prefers (not) observing the outcome of lottery f to remaining in doubt. Under the simplest set of axioms, the representation collapses to the following:

Simple representation. $X \succ Y$ if and only if $W(X) > W(Y)$, where for all $X = ((z_1, q_1^I; ...; z_n, q_n^I;\ f_1, q_1^N; ...; f_m, q_m^N) \in \mathfrak{L}_1$,

$$W(X) = \sum_{i=1}^{n} q_i^I u(z_i) + \sum_{j=1}^{m} q_j^N u\left(v^{-1}\left(Ev(f_j)\right)\right)$$

The functions u and v are unique up to positive affine transformation.

In this simplified setting, the representation can be shown to be essentially equivalent to a Kreps-Porteus (KP) representation, albeit with a different interpretation. Section 4 considers more general axioms, and Section 6 demonstrates that even this simplified model diverges from the KP representation if there is more than one period. It may appear that having the sequence 'today, tomorrow or never' would be equivalent to 'today, tomorrow or the day after,' but in fact they are not. For instance, the dynamic extension of this model leads to a preference for commitment that does not arise in a KP setting, as I later discuss. For the applications presented in the next section, the simplified representation suffices, as my aim is to introduce the basic mechanism at play. However, an in-depth analysis of those settings may benefit from the use of the static and the dynamic generalizations of this model.

The intuition behind the simplified representation is straightforward. The first term is of the standard expected utility form for lotteries that are eventually observed, with utility functional u. As the aim of this model is to depart as little as possible from expected utility, when there are no unobserved lotteries, the representation is indistinguishable from the standard EU form. When there are unobserved lotteries, they are treated in the same way as any other prize. Take an unobserved lottery f. The function v is used to determine how this lottery f ranks with respect to other outcomes in the outcome space $\mathbf{Z}$. In other words, v is used to obtain the certainty equivalent $v^{-1}(Ev(f))$. Then, this representation uses standard expected utility analysis with function u, using the certainty equivalent $v^{-1}(Ev(f))$ in lieu of a final outcome z. I now define doubt-proneness, which refers to a preference for remaining ignorant and not observing a lottery, in the natural way.

Definition (Doubt-proneness)

- An agent is doubt-prone *somewhere* if there exists some f such that $\delta_f \succ f$.
- An agent is doubt-prone *everywhere* if: (i) there exists no $f \in \mathfrak{L}_o$ such that $f \succ \delta_f$,

and (ii) there exists some f such that $\delta_f \succ f$.

An agent who prefers not observing the resolution of some lottery than observing it is doubt-prone somewhere. An agent who (weakly, and strictly for one lottery) prefers not to observe the outcome of *any* lottery is doubt-prone everywhere. Doubt-aversion is defined in a similar manner. Section 5 provides a thorough discussion of the relation between doubt-proneness and the functions of the more general model. The next section considers different settings in which agents are doubt-prone, so as to demonstrate that various behavioral anomalies can be accommodated naturally.

3 Applications

This section aims to illustrate the scope of this simple extension of the vNM model. Maintaining the assumption throughout that agents are doubt-prone (they would rather not observe the resolution of uncertainty), I consider three applications. In the first, an agent's utility depends on his ability, as it is linked to his self-image. While he does not directly observe this ability, his success at performing tasks provides him with an imperfect signal. In an expected utility setting, he would maximize his chances at success if effort were costless. With doubt-proneness, however, there is a tradeoff between obtaining a higher reward by exerting more effort and receiving a coarser signal of ability by exerting less. In other words, a doubt-prone agent has an incentive to self-handicap. The tension between success and unwanted signaling plays a role in a number of contexts and gives rise to other well-known behavioral patterns. For instance, it may lead to an agent exhibiting a status quo bias, and to a risk-neutral agent choosing safe bonds over riskier stocks with higher expected return. The second application considers moral preferences that appear difficult to reconcile either with selfish or other-regarding preferences alone. Often, individuals may avoid seeing the full impact they have on others, and this could influence their actions. Finally, in the third application, voters in an economy share common preferences, but they do not know which candidate is best. They can acquire this information at no cost, but there are equilibria in which the wrong candidate is as likely to win as the right candidate. Perhaps surprisingly, an increase in the intensity of their preferences diminishes their incentives to observe the truth.

3.1 Preservation of self-image

Before analyzing the implications of the results in different contexts, I first introduce a general setup in which the agent has self-image preferences. Assume that the agent places direct value on his ability, independent of the effect it has on his monetary reward. Arguably, individuals value their self-image, and believing they are of higher ability provides them with an increased sense of self-worth. But note that individuals never directly observe their intrinsic ability. Instead, their success at achieving their goals, given their effort, provides them with imperfect signals of their ability.

Suppose then that the agent is endowed with ability (or type) $t \in [\underline{t}, \overline{t}] \in \mathbb{R}$ and that he places prior probability $p(t)$ on being of type t. He chooses effort $e \in [\underline{e}, \overline{e}] \in \mathbb{R}$, to obtain a reward $m \in [\underline{m}, \overline{m}] \in \mathbb{R}$. Although the agent may never observe t, he does observe m ex-post. The reward m stochastically depends on his ability t and the effort e he exerts. Let $p(m|e,t)$ denote his probability of receiving reward m given e and t. Given his prior belief over his type t, the probability of receiving m for effort level e is $p(m|e) = \sum_{t \in [\underline{t}, \overline{t}]} p(m|e,t)p(t)$. Assume that his expected reward is larger if he exerts more effort for any given ability ($Em(e,t) > Em(e,t') \Leftrightarrow t > t'$), and that it is larger for high ability at any given effort level ($Em(e,t) > Em(e',t) \Leftrightarrow e > e'$).[8]

The agent's value function W depends on both his reward m and his type t. Assume that his utility for m is linear; more precisely, his expected utility over m is $Em(e)$. In addition, it is linearly separable from his utility over t. He is weakly risk-averse over t (for both resolved and unresolved lotteries), as well as doubt-prone.[9] Recall that u is the utility associated with resolved lotteries, and v with unresolved lotteries. In this case, these lotteries are over his ability t.

In the standard case in which the agent expects to observe both type t and reward m, he maximizes $Em(e) + Eu(t)$. Since effort is costless and $Eu(t)$ does not depend on his decision, it is immediate that he would put in the highest level of effort, $e = \overline{e}$.

But here, he does *not* necessarily observe his ability ex-post. In this case, when he receives his monetary reward, he simply updates his probability on his ability, given m and his chosen effort level e. His value function is therefore:

$$W(e) = Em(e) + \sum_m p(m|e)u\left(v^{-1}(Ev(t|m,e))\right).$$

Depending on the functional form, the agent might not put in effort $e = \overline{e}$. His effort level also depends on his incentive to obtain the least information concerning his ability, since he is doubt-prone. In other words, he takes into account that the combination

[8] All the probability distributions in this section have finite support.

[9] While the representation provided does not explicitly have a separate 'money' term, extending the model to include this term is trivial.

of his effort and the reward he obtains allows him to make inferences concerning his ability. Suppose that there is a unique effort level e_o that is entirely uninformative, i.e. $p(t|m,e_o) = p(t)$ for all $t \in [\underline{t}, \overline{t}]$ and for all $m \in [\underline{m}, \overline{m}]$. Note that e_o provides the agent with the highest expected utility over his ability. That is, define

$$C(e) \equiv u\left(v^{-1}(Ev(t))\right) - \sum_m p(m|e)u\left(v^{-1}(Ev(t|m,e))\right).$$

As shown in the appendix, it is always the case that $C(e) > 0$ (for $e \neq e_o$) for a doubt-prone agent, with $C(e_o) = 0$. Redefining the value function to be $\tilde{W}(e) = W(e) - u\left(v^{-1}(Ev(t))\right)$, the agent maximizes

$$\tilde{W}(e) = Em(e) - C(e).$$

Hence, $C(e)$ is effectively the 'shadow' cost of effort due to acquiring information that he would rather ignore. The optimal effort level depends on the importance of the expected reward $Em(e)$ relative to the agent's disutility of acquiring information concerning his ability, as is captured by $C(e)$. Suppose now that $e_0 = \underline{e}$, and that the agent obtains a more informative signal (in the Blackwell sense) for a higher effort e. Then, $C(\underline{e}) = 0$ and $C(e)$ is strictly increasing, so that the 'shadow' cost is increasing in effort level. The following simple example serves as an illustration.

Numerical Example

Let $\underline{e} = \underline{t} = 0$, $\overline{e} = \overline{t} = 1$, $p(t=0) = \frac{1}{2}$ and $p(t=1) = \frac{1}{2}$. The agent's reward m only takes value 0 and 100. The probabilities of obtaining reward $m = 100$ given e and t are $p(m = 100|t = 1, e) = e$ and $p(m = 100|t = 0, e) = 0$. The utility functions are $u = a\sqrt{t}$ for some $a > 0$, and $v = t$.

In this example, the completely uninformative effort e_o is equal to 0. At effort $e = 0$, the agent is sure to obtain 0, and his posterior on his ability is the same as his prior. As he exerts more effort, he obtains a sharper signal. If he puts in maximum effort $e = 1$, then he will fully deduce his ability ex-post: if he obtains 100 then he infers that $t = 1$, and if he obtains 0 then he infers that $t = 0$. His value function is now $\tilde{W}(e) = 50 - C(e)$, where $C(e) = \frac{a}{2}(\sqrt{2} - e - \sqrt{2 - 3e + e^2})$.

The optimal level of effort e^* is in the full range $[0,1]$, depending on a. As a increases, the monetary reward m becomes less significant, and effort level e^* decreases. As a decreases, the agent's utility over his ability becomes less significant, and effort e^* increases (see the appendix for details).

Self-handicapping

The setup presented here can be applied to several different contexts, the most immediate of which is self-handicapping. There is strong anecdotal evidence that people are sometimes restrained by a 'fear of failure' in the pursuit of their goals. Berglas and Jones (1978) find that individuals deliberately impede their own chances of success, and they attribute this behavior to people's desire to protect the image of the self.[10] In this setting, the amount of optimal self-handicapping depends on the doubt-proneness of the agent, and the precision of the signal. In maximizing $\tilde{W}(e) = Em(e) - C(e)$, choosing a higher effort level leads to a tradeoff between the improved reward $Em(e)$ and the incurred cost $C(e)$ of learning more about his actual ability. This model also confirms Berglas and Jones' intuition that those who are more likely to self-handicap are not the most successful or the least successful, but rather those who are uncertain about their own competence. Akerlof and Dickens' (1982) observation that people will remain ignorant so as to protect their ego is also consistent with the implications of this framework. But in contrast to most frameworks that analyze this behavior, here, self-handicapping stems from the agent's doubt-proneness over his ability. He updates his beliefs in a Bayesian way, and does not have access to self-deception or belief-manipulation. Notice that if an agent with these preferences were placed in a team that receives only a joint reward, his incentive to self-handicap could be mitigated, since his individual signal would be less precise.

Status quo bias

The endowment effect and the status quo bias have been analyzed extensively, and are typically explained using framing effects and loss aversion (Kahneman, Knetch and Thaler (1991)). The agent's preference for avoiding a loss is taken to be stronger than his preference for making a gain, and the reference point is usually taken to be the status quo by assumption. But in their seminal work, Samuelson and Zeckhauser (1988) do not view the status quo bias to be solely a consequence of loss aversion: "Our results show the presence of status quo bias even when there are no explicit gain/loss framing effects [...] Thus, we conclude that status quo bias is a general experimental finding – consistent with, but not solely prompted by, loss aversion." The framework introduced here can be applied to settings in which a status quo bias is present.

Suppose that e now represents a choice over different bundles rather than effort. For instance, assume that the agent only places positive probability on $\underline{e}$ and $\overline{e}$, and that

[10] See Benabou and Tirole (2002) for an explanation that uses manipulable beliefs.

$\underline{e}$ corresponds to keeping the current allocation, while $\overline{e}$ corresponds to switching to another bundle. Moreover, suppose that acquiring a bundle also carries information on the agent's decision-making ability. In this case, rather than representing a cost of effort, $C(e)$ represents the cost of deviating from the bundle that is least informative of the agent's decision-making ability. If inaction (keeping the same bundle) is uninformative ($e_0 = \underline{e}$), then the agent exhibits a status quo bias, since maintaining the status quo has information cost $C(e_0) = 0$. Intuitively, there is often more to learn from 'exploring' new options than from taking no action, since this would effectively provide the agent with more counterfactuals. That is, switching bundles may force an eventual comparison between his new acquisition and his former endowment.

While it often seems true that inaction is the least informative action, this need of course not hold in general. If, instead, keeping the status quo bundle were more informative than obtaining other bundles, then a doubt-prone agent would be biased *against* the status quo. An example would be an individual who skips from endeavor to endeavor rather than persevering with one, thereby avoiding a sharper signal of his ability in that specific field. The degree to which the signal precision of inaction compares to 'exploration' is therefore a central empirical question in determining the nature of the individual bias.

The key difference with standard expected utility theory is that this framework allows for an asymmetry between the value of acquiring and exchanging a bundle. The bundle does not change value based on whether the agent is endowed with it or not, and in that sense there is no framing effect. Instead, the acquisition of a new bundle *in itself* has different informational implications than selling it. In the case in which the unobserved prize is the agent's ability, acquiring a new bundle may provide him with more information on his ability than keeping his current allocation would.

Bonds, stocks and paternity

Consider the case in which e represents an investment decision rather than effort. A higher e represents a riskier investment, but in expectation it leads to a higher monetary reward. As before, t corresponds to a notion of ability. An individual who has greater decision-making ability makes a wiser investment choice and therefore obtains a higher expected monetary reward, given the chosen risk level. For instance, $\underline{e}$ might be a portfolio consisting solely of bonds, while $\overline{e}$ consists solely of higher-risk stocks. Maintain the assumption that $e_o = \underline{e}$. In other words, the riskless option is also least informative concerning the agent's potential as an investor.

In this setting, although the agent is risk-neutral in money, his chosen bundle e^* may still consist of more bonds than it would if the reward were purely monetary, as there is a bias towards $\underline{e}$.[11] In addition, suppose that a firm exists that offers to invest the agent's money in his place. Even if the agent does not believe that the firm has superior expertise, he will still employ its services. Since the optimal level of risk in this case is $\overline{e}$, he is willing to pay up to $Em(\overline{e}) - Em(e^*) + C(e^*)$. In fact, even if the firm were to choose the suboptimal level e^*, he would still be willing to pay up to $C(e^*)$.

In the standard EU model, the agent's choice would depend only on the monetary reward he expects to obtain. In contrast, the framework presented here allows the agent's choice to depend not only on his expected reward, but also on the decision-making process. That is, the agent bases his choice on the *manner* in which he expects to obtain the monetary reward.

3.2 Moral preferences

It is often the case with moral preferences that individuals do not observe the consequences of their actions or of those that others have taken. For instance, an individual may consume goods that have employed exploitative labor practices or that have been developed through the use of animal cruelty. He may not be sure of the effect of his actions on the environment, and he may donate money to an organization or a cause that might not use it in the way it professes to. Similarly, someone may not know whether a family member has betrayed his trust, or whether the child he has raised is, in fact, biologically his. Judges and jury members might not receive definitive proof that the judgement they have passed is fair. Preferences to remain ignorant, as described by this framework, could arise in these settings. Moreover, individuals' chosen actions could vary significantly, depending on how much information they can choose to ignore.

The notion that moral preferences depend on observing the outcome has been recently studied in experiments, notably by Dana, Weber and Kuang (2007). They consider a standard dictator game, in which the recipient does not have the possibility to refuse an offer. In addition, dictators are unsure of the amount that they are actually giving the recipient, as it depends on a hidden lottery whose outcome they do not observe. They could, however, observe this outcome at no cost before making their decision, which allows them to choose the exact quantity given to the recipient.

Their setup is as follows. The dictator has a choice between two options, A and B. In the baseline case (Table 1), there is no uncertainty; if he chooses option A, then he receives 6 and the recipient receives 1. If instead he chooses option B, then both he and

[11] A more careful study would, of course, be required to gauge the empirical significance of this effect.

Box A	Dictator: 6 Recipient: 1
Box B	Dictator: 5 Recipient: 5

Table 1: Baseline Case.

	Heads	Tails
Box A	Dictator: 6 Recipient: 1	Dictator: 6 Recipient: 5
Box B	Dictator: 5 Recipient: 5	Dictator: 5 Recipient 1

Table 2: Hidden Information Case.

the recipient receive 5. In the hidden information case (Table 2), he still receives 6 if he chooses A, and 5 if he chooses B. But now, the recipient's allocation is determined by a coin toss, prior to the dictator's choice. If it lands heads, then the recipient's allocation is 1 in option A and 5 in option B, and if it lands tails, then it is the other way around. The dictator has the choice, before making his decision, to observe the coin toss at no cost. If he chooses not to, then he never observes what the recipient receives. As for the recipient, he does not find out whether or not the dictator has seen the outcome.

A significant number of dictators choose not to observe the toss in the hidden information case and are more likely to choose option A. On average, more agents choose option A in the hidden information case than in the baseline case, and significantly fewer agents choose the altruistic (or fair) option. Notice that this behavior is inconsistent with both altruism and self-interested behavior: if the agents are indeed self-interested, then they should always choose option A, even in the case without uncertainty. If they are altruistic, then they should strictly prefer to observe the coin toss before making the decision. Incorpororating doubt-proneness accounts for these preferences.

Assume that the dictator prefers receiving more to less and that he also has other-regarding preferences. Specifically, he prefers that the recipient receives 5 to 1. His preferences are as follows: $u_S(x_S) + u_O(v_O^{-1}(p_g v_O(x_O)))$, where u_S is his utility over own-consumption; u_O and v_O are his other-regarding utility functions over resolved and unresolved lotteries, respectively; x_S and x_O are his own and the other's monetary outcomes, respectively; and $v_O(1)$ is normalized to 0.

Consider first a dictator who weakly prefers option A in the baseline case. It follows from doubt-proneness (concavity of $u_O(v_O^{-1}(\cdot))$ that he also prefers *not* to observe the outcome in the hidden information case, and that he would still prefer option A :

$$\begin{aligned}
\text{A in baseline case} \quad & \Rightarrow u_S(6) + u_O(0) \geq u_S(5) + u_O(5) \\
& \Rightarrow u_S(6) + u_O(v_O^{-1}(0.5 v_O(5))) > u_S(6) + 0.5\,(u_O(5) + u_O(0)) \\
& \Rightarrow \text{ Avoid observation, A in hidden information case.}
\end{aligned}$$

Intuitively, if his selfish preferences are stronger than his other-regarding preferences when he observes the outcome, then he has even more incentive to be selfish when he can remain ignorant about what the recipient receives. Denote this agent type **AA**.

Now consider an agent who chooses option B in the baseline case, i.e. $u_S(5)+u_O(5) \geq u_S(6)+u_O(0)$. Then, two kinds of preferences are consistent with doubt-proneness: either he avoids observing the coin toss and chooses A in the hidden information case (type **BA**), if $0.5\,(u_S(6) - u_S(5)) > u_O(5) - u_O(v_O^{-1}(0.5v_O(5)))$, or he *does* observe the coin toss and chooses whichever option provides the recipient with 5 (type **B**). Type **BA** exists *only* if the agent is doubt-prone.[12] Aside from types **AA**, **BA** and **B**, no other choices are consistent with this framework for a doubt-prone agent.

The predictions of this model appear to fit the data well: of the 32 dictators, 14 (44%) chose not to observe the outcome of the coin toss. Of those 14 individuals, 12 (86%) chose option A (consistent with type **AA** and type **BA**). Of the 18 individuals who chose to observe the coin toss, 15 (83%) chose the option for which the recipient receives 5 (consistent with type **B**). Furthermore, since types **AA** and **BA** are conflated in the hidden information case, we expect more individuals (those of type **BA**) to have chosen the fair allocation in the baseline case than in the hidden information case. This pattern emerges as well: in the baseline case, 14 out of 19 (74%) chose B, compared to 6 out of 16 (38%) who chose the fair option in the hidden information case.[13]

3.3 Political Ignorance

The high degree of voters' political ignorance has been thoroughly researched, particularly in the U.S. (see Bartels (1996)). Given the length of electoral campaigns in American politics, the amount of media coverage and the accessibility of informational sources, it seems that the cost of acquiring information should not be prohibitive for voters. Note that there are political issues whose resolution the voters may not observe. For instance, the voters may choose not to observe the amount of foreign aid given, the degree of nepotism, or the government stance on interrogation methods, or whether they were misled into going to war. For those issues, a doubt-prone agent may have a strict incentive to ignore information even if it is free. In other words, making information more accessible would not necessarily have a strong impact on how well-informed an individual is. Since voters affect the election result as a group, each individual's decision to acquire information has an externality on other voters and on *their* decision to acquire

[12] Specifically, $u_S(5) + u_O(5) \geq u_S(6) + u_O(0)$ and $0.5\,(u_S(6) - u_S(5)) > u_O(5) - u_O(v_O^{-1}(0.5v_O(5)))$ together imply that $u_O(v_O^{-1}(0.5v_O(5))) > 0.5(u_0(v_O^{-1}(v_O(5))) + u_0(v_O^{-1}(v_O(0))))$.

[13] See Table 2 in Dana, Weber and Kuang (2007).

information. To highlight the impact of doubt-proneness in this setting, I present a stylized example in which voters' information acquisition plays a dominant role in others' decisions. Although voting is sincere, there is a strategic aspect to the decision to obtain information.

Consider an economy in which N citizens care about issue $\gamma \in [0, 1]$, determined by the elected politician. They can choose not to observe what policy the politician will implement. Suppose that there are two candidates, A and B. One of them will choose policy $\gamma = 0$ if elected, and the other $\gamma = 1$. The voters cannot tell them apart ex-ante, and they place equal probability that A will choose $\gamma = 0$ and $\gamma = 1$ (and similarly for B). But they have the option of acquiring that information at no cost before voting. Let p_i be the ex-post probability that the ith agent places on the winner being the candidate who implements $\gamma = 1$, where $i \in \{1, .., N\}$. The timing is as follows. Each voter decides whether or not to observe where candidates A and B stand. He then votes sincerely, i.e. he votes for the candidate on whom he places a higher probability of implementing the policy that he prefers. If he is indifferent or if he places equal probability on either candidate implementing his preferred policy, then he tosses a fair coin and votes accordingly. The candidate who obtains the majority wins the election. In case of a tie, a coin toss determines the winner. The winner then implements the policy he prefers, and there is no possibility of reelection.

Now suppose that every voter prefers γ to be higher. In addition, every voter is strictly doubt-prone. Let his value function be W_i^I if he acquires information and W_i^N if he does not. Even though every voter prefers the candidate who implements $\gamma = 1$, and even though information is free, there is still an equilibrium in which no one acquires information, and the candidate who implements $\gamma = 0$ wins with probability $\frac{1}{2}$. This equilibrium is Pareto-dominated (in expectation) by the other equilibria, in which at least a strict majority of agents acquires information, and the candidate who implements $\gamma = 1$ wins with probability 1. This is briefly shown below.

Equilibrium in which no voter is informed. If no other voter is informed, then voter i does not acquire information either. Since $p_i \in (0, 1)$ if no one else is informed, it follows that $W_i^I < W_i^N$ (on his own, he cannot force $p_i \in \{0, 1\}$). Unless agent i is certain that either the right candidate or the wrong candidate always wins the election—i.e., that $p_i = 1$ or that $p_i = 0$—he does not acquire information. Note that the difference between W_i^I and W_i^N for a given $p_i \in (0, 1)$ is higher if the difference between the agent's utility of $\gamma = 1$ and $\gamma = 0$ is larger.

Equilibrium in which at least a strict majority is informed. If at least a strict majority is informed, then the right candidate wins with probability 1. Hence, $p_i = 1$ for each agent i, and so he is indifferent, since $W_i^I = W_i^N$. Note, however, that this equilibrium does not survive if each voter i places an arbitrarily small probability $\delta > 0$ that each of the other voters does not acquire information.

The externality of information plays an excessive role in this simple example, but the mechanism presented here would have an impact in a more realistic model. In particular, this example suggests that it is precisely those who are most affected who may end up least informed: as the difference between the doubt-prone agent's utility of the good policy and his utility of the bad policy increases, he has *less* incentive to acquire information. Moreover, a Pareto gain would be achieved if enough voters were to become informed.

4 Model

I now present the general (static) model. Recall from the previous section that the outcome space is $\mathbf{Z} = [\underline{z}, \bar{z}] \subset \Re$, that $\mathfrak{L}_0$ be the set of simple probability measures on $\mathbf{Z}$, and that $\mathfrak{L}_1$ is the set of simple lotteries over $\mathbf{Z} \cup \mathfrak{L}_0$, with typical element $X \in \mathfrak{L}_1$. Depending on the lottery, the agent may or may not observes outcomes in $\mathbf{Z}$. For instance, the agent observes all the outcomes for lottery $X = f$, and does not observe any outcome for lottery $X' = \delta_{f_i}$.

4.1 General axioms

The following certainty axiom **A.1** is assumed throughout:

AXIOM A.1 (Certainty): Take any $z_i \in \mathbf{Z}$, and let $X = \delta_{z_i} = (z_i, 1)$ and $X' = (\delta_{z_i}, 1)$. Then $X \sim X'$.

The certainty axiom **A.1** concerns the case in which an agent is certain that an outcome z_i occurs. In that case, it makes no difference whether he is presented with a resolved lottery that leads to z_i with certainty or an unresolved lottery that leads to z_i with certainty. He is indifferent between the two lotteries. Hence, axiom **A.1** does not allow the agent to have a preference for being informed of something that he already knows. This simple axiom provides a formal link between the agent's preferences over resolved lotteries and his preferences over unresolved lotteries. The following three axioms are standard.

AXIOM A.2 (Weak Order): $\succeq$ is complete and transitive.

AXIOM A.3 (Continuity): $\succeq$ is continuous in the weak convergence topology. That is, for each $X \in \mathfrak{L}_1$, the sets $\{X' \in \mathfrak{L}_1 : X' \succeq X\}$ and $\{X' \in \mathfrak{L}_1 : X \succeq X'\}$ are both closed in the weak convergence topology.

AXIOM A.4 (Independence): For all $X, Y, Z \in \mathfrak{L}_1$ and $\alpha \in (0, 1]$, $X \succ Y$ implies $\alpha X + (1 - \alpha)Z \succ \alpha Y + (1 - \alpha)Z$.

Focusing on axiom **A.4**, it is noteworthy that the agent's preferences $\succeq$ are on a richer space than in the standard framework. The independence axiom in the standard vNM model is taken on preferences over lotteries over outcomes, since all lotteries lead to outcomes that are eventually observed. In this paper, the agent's prize is not always an outcome z, and can instead be an unresolved lottery f. By assumption **A.4**, however, there is no axiomatic difference between receiving an outcome z as a prize and obtaining an unresolved lottery f as a prize. Under this approach, the rationale for using the independence axiom in the standard model holds in this case as well. Since the aim is to depart as little as possible from the vNM Expected Utility model, I assume the independence axiom **A.4** throughout.

Axioms **A.1** through **A.4** suffice for this model to subsume the standard vNM representation for preferences over outcomes that the agent eventually observes. That is, suppose that we focus on lotteries of form $X = f$, i.e. lotteries that lead to outcomes. Then, all the standard vNM axioms over these lotteries hold, and the EU representation follows directly. These axioms are not sufficient, however, to characterize the agent's preferences over lotteries that do not resolve. If, for instance, the agent receives a lottery $X = \delta_f$, it is unclear what his 'perception' of unresolved lottery f is. The next step, therefore, is to consider axioms that allow us to characterize the agent's preferences over these 'purely' unresolved lotteries of form $X = \delta_f$. As there is a natural isomorphism between lotteries of form $X = \delta_f \in \mathfrak{L}_1$ and one-stage lotteries in $\mathfrak{L}_0$, define the preference relation $\succeq_N$ in the following way:

Definition of $\succeq_N$ For any $f, f' \in \mathfrak{L}_0$, $f \succeq_N f'$ if $\delta_f \succeq \delta_{f'}$.

Define $\succ_N$ and $\sim_N$ in the usual way. I first make a continuity assumption:

AXIOM N.1 (Continuity): $\succeq_N$ is continuous in the weak convergence topology.

Assuming independence over the preference relation $\succeq_N$ would lead to the simplified representation introduced in Section 2 and used in Section 3 for all the applications considered in this paper. In this section, I consider a weaker axiom over the preference relation $\succeq_N$, to allow for more general preferences. Specifically, a weaker axiom would be required to accommodate preferences of the following type. Suppose that there are three lotteries, $f = (1000, 1/3; z, 1/3; 0, 1/3)$, $f' = (1000, 1/2; 0, 1/2)$ and $f'' = (400, 1) = \delta_{400}$. One could experimentally elicit an individual's preference among three lottery tickets with these values, to be given to a charity of his choice. Alternatively, these could be stocks that an individual leaves in a trust fund.[14] Given these alternatives, it may be that $f \succ_N f' \succ_N \delta_{400}$. If he does not observe the outcome, then he may prefer that his charity receives risky lottery f' to δ_{400}, which has a lower expected value. These preferences may be flipped if he were instead to see the final outcome. But lottery f may then be preferable to f' because it seems less risky, and information is similarly concealed in both cases. These preferences violate independence; in fact, they violate the stronger axiom of betweenness, and so do not fall in the Dekel (1986) class of preferences.[15]

In this example, two distinct notions may play a role in the agent's preferences over unresolved lotteries. The agent may be risk-averse over unresolved lotteries, and this risk-aversion manifests itself in his comparison between lottery f and the riskier lottery f'. At the same time, he may be 'optimistic' that the good outcome has occurred if he does not observe the lottery, which affects his assessment of lottery f', compared to the safe lottery δ_{400}. A single utility function v cannot capture both these notions because risk-aversion and optimism over the unobserved do not necessarily coincide. Since both risk-aversion and optimism may be contributing factors to the agent's preference to remain in doubt, this framework aims to allow the agent to express both dimensions of preference and to characterize the implications of their interaction. But observe that optimism is *not* required for the applications presented here; the aim of introducing this notion is to present a more complete framework, which could then be empirically tested.

I now assume the Rank-Dependent Utility (RDU) axioms, which are general enough to allow the previous example. The RDU representation allows for two functions, v

[14] Note that this framework can also be extended to encompass a stochastic overlapping generation model with altruism.

[15] This is a violation of independence (and betweenness) because $f' \succ_N \delta_{400}$ but the following does not hold: $f' \succ_N \frac{2}{3}f' + \frac{1}{3}\delta_{400} \succ_N \delta_{400}$.

and w—the first reweighs the outcomes (identically to the vNM model), and the second reweighs the probabilities. I show, in the following section, that an RDU representation captures notions of risk and optimism that are suitable to this model, even though my formal definition of optimism will be different from the accepted RDU definition. I later consider conditions that force the function w to be linear, essentially reducing the representation of $\succeq_N$ to a vNM representation.[16]

4.2 RDU representation for $\succeq_N$.

The following notation is convenient for the RDU representation. For lottery $f = (z_1, p_1; ; ...; z_m, p_m) \in \mathfrak{L}_o$, the $z_i's$ are ordered from smallest to highest, i.e. $z_m > ... > z_1$. Recall that the agent's preferences are monotone, which implies that $\delta_{z_m} \succ_N ... \succ_N \delta_{z_1}$. In addition, p_i^* denotes the probability of reaching outcome z_i or an outcome that is weakly preferred to z_i. That is, $p_i^* = \sum_{j=i}^{m} p_j$. Note that for the least-preferred outcome z_1, $p_1^* = 1$. Probabilities p_i^* are referred to here as 'decumulative' probabilities. The RDU form, introduced by Quiggin (1982), is defined in the following manner:[17]

Definition (RDU) Rank-dependent utility (RDU) holds if there exists a strictly increasing continuous probability weighting function $w : [0,1] \to [0,1]$ with $w(0) = 0$ and $w(1) = 1$ and a strictly increasing utility function $v : \mathbf{Z} \to \Re$ such that for all $f, f' \in \mathfrak{L}_o$,

$$f \succ_N f' \text{ if and only if } V_{RDU}(f) > V_{RDU}(f'),$$

where V_{RDU} is defined to be: for all $f = (z_1, p_1; z_2, p_2; ...; z_m, p_m)$,

$$V_{RDU}(f) = v(z_1) + \sum_{i=2}^{m} [v(z_i) - v(z_{i-1})] w(p_i^*).$$

Moreover, v is unique up to positive affine transformation.

Note that if the weighting function w is linear, then V_{RDU} reduces to the standard EU form.[18] I now briefly discuss the axiomatic foundation of the RDU representation,

[16] The notion of 'optimism' may seem at odds with the previous claim that an agent who is not allowed to manipulate his beliefs may still choose to self-handicap. That is, one interpretation of a rank-dependent utility representation is that the agent distorts the actual probability. But note that in the analysis of self-handicapping (Section 3), I do *not* allow the agent to be either optimistic or pessimistic.

[17] See also Yaari (1987) and Diecidue and Wakker (2001) for a thorough discussion of RDU.

[18] This is not the most common form of RDU; this notation is taken from Abdellaoui (2002). Given the rank-ordering above, the typical form would be $V_{RDU} = \sum_{i=1}^{n-1} [w(p_i^*) - w(p_{i+1}^*)] v(z_i) + w(p_n^*) v(z_n)$. It is easy to check that the two representations are identical.

in the context of this model. Suppose that

$$f_\alpha = (z_1, p_1; ...; \alpha, p_i; ...; z_m, p_m) \succeq_N (z'_1, p_1; ...; \beta, p_i; ...; z'_m, p_m) = f'_\beta$$
$$f'_\kappa = (z'_1, p_1; ...; \kappa, p_i; ...; z'_m, p_m) \succeq_N (z_1, p_1; ...; \gamma, p_i; ...; z_m, p_m) = f_\gamma,$$

where $\alpha, \beta, \gamma, \kappa \in \mathbf{Z}$. Comparing lotteries f_α and f_γ, the only difference is in whether α or γ is reached with probability p_i. Since all the other outcomes are the same in both lotteries and are reached with the same probabilities, the difference is in the value of outcome α compared to the value of outcome γ (and similarly for f'_β,f'_κ and β, κ). In the comparison of $f_\alpha \succeq_N f'_\beta$ and $f'_\kappa \succeq_N f_\gamma$, all the probabilities of reaching the (rank-preserved) outcomes are the same. For that reason, this model assumes that the switch in preference is due to a difference in the value of outcomes α and β relative to γ and κ, and not in the way the probabilities are aggregated. It is precisely this property that RDU provides: if $f_\alpha \succeq_N f'_\beta$ and $f'_\kappa \succeq_N, f_\gamma$, and if $\succeq_N$ is of the RDU form, then $v(\alpha) - v(\beta) \geq v(\gamma) - v(\kappa)$. Note that this does not depend on the choice of $z's$ and $p's$, and so the following axiom, adapted from Wakker (1994), must hold:

AXIOM N.RDU (Wakker tradeoff consistency for $\succeq_N$):
Let $f_\alpha = (z_1, p_1; ...; \alpha, p_i; ...; z_m, p_m)$, $f_\gamma = (z_1, p_1; ...; \gamma, p_i; ...; z_m, p_m)$,
$f'_\beta = (z'_1, p_1; ...; \beta, p_i; ...; z'_m, p_m)$ and $f'_\kappa = (z'_1, p_1; ...; \kappa, p_i; ...; z'_m, p_m)$. If:

$$f_\alpha \succeq_N f'_\beta$$
$$f'_\kappa \succeq_N f_\gamma,$$

then for any lotteries $g_\alpha = (\hat{z}_1, \hat{p}_1; ...; \alpha, \hat{p}_i; ...; \hat{z}_{\hat{m}}, \hat{p}_{\hat{m}})$, $g_\gamma = (\hat{z}_1, \hat{p}_1; ...; \gamma, \hat{p}_i; ...; \hat{z}_{\hat{m}}, \hat{p}_{\hat{m}})$, $g'_\beta = (\hat{z}'_1, \hat{p}_1; ...; \beta, \hat{p}_i; ...; \hat{z}'_{\hat{m}}, \hat{p}_{\hat{m}})$, $g'_\kappa = (\hat{z}'_1, \hat{p}_1; ...; \kappa, p_i; ...; \hat{z}'_{\hat{m}}, \hat{p}_{\hat{m}})$ such that $g_\gamma \succeq_N g'_\kappa$,
it must be that $g_\alpha \succeq_N g'_\beta$.

Under this axiom, only the values of α,β,γ and κ are relevant to the ordering of the agent's preferences when all the probabilities of reaching all other outcomes are the same across the four lotteries. In fact, as Wakker (1994) shows, this axiom is sufficient, along with stochastic dominance and continuity, for the RDU representation to hold. Using this result, the general representation theorem for $\succeq$ is as follows:

Representation Theorem. *Suppose that axioms **A.1** through **A.4** and axioms **N.1** and **N.RDU** hold. In addition, suppose that stochastic dominance holds for $\succeq_N$. Then there exist strictly increasing, continuous and bounded functions $u : \mathbf{Z} \to \mathbb{R}$, $v : \mathbf{Z} \to \mathbb{R}$, $w : [0,1] \to [0,1]$ with $w(0) = 0$ and $w(1) = 1$, such that for all $X, Y \in \mathfrak{L}_1$,*

$$X \succ Y \text{ if and only if } W(X) > W(Y),$$

where W is defined to be: for all $X = ((z_1, q_1^I; ...; z_n, q_n^I;\ f_1, q_1^N; ...; f_m, q_m^N) \in \mathfrak{L}_1$,

$$W(X) = \sum_{i=1}^{n} q_i^I u(z_i) + \sum_{j=1}^{m} q_j^N u\left(v^{-1}\left(V_{RDU}(f_j^N)\right)\right)$$

and

$$V_{RDU}(f) = v(z_1) + \textstyle\sum_{h=2}^{m} [v(z_h) - v(z_{h-1})] w(p_i^*).$$

Moreover, u and v are unique up to positive affine transformation.

Note that u remains the utility function associated with the general lotteries (and final outcomes). In addition, v is the utility function associated with unresolved lotteries, and w is the probability weighting function associated with unresolved lotteries. It is not immediately clear from this representation what doubt-proneness implies, in terms of the shapes of the functions. The next section defines optimism, and formally relates it to the accepted notion of optimism in an RDU setting. I then connect doubt-proneness, risk-aversion, and this new notion of optimism.

5 Risk-aversion, doubt-proneness and optimism

In this section, I focus on the relationship between doubt-proneness and the shapes of the functions u, v and w. I first define formally what optimism means in this context. Returning to the charity example from the previous section, recall that lottery $f = (1000, 1/3; 400, 1/3; 0, 1/3)$, lottery $f' = (1000, 1/2; 0, 1/2)$ and lottery $\delta_{400} = (400, 1)$. While $f' \succ_N \delta_{400}$, it is not the case that $f' \succ_N af' + (1+a)\delta_{400} \succ_N \delta_{400}$ for all a, which the independence axiom would imply. In this example, $f = \frac{2}{3}f' + \frac{1}{3}\delta_{400} \succ_N f'$.

I aim to capture a notion of optimism over unresolved lotteries that allows the agent to prefer more 'scrambled' information, since it essentially allows him to form a better assessment of these unresolved lotteries. Consider lottery δ_{400}, in which the agent is certain that the outcome is 400. Now suppose that it is mixed with a lottery $\tilde{f}' = (400+$

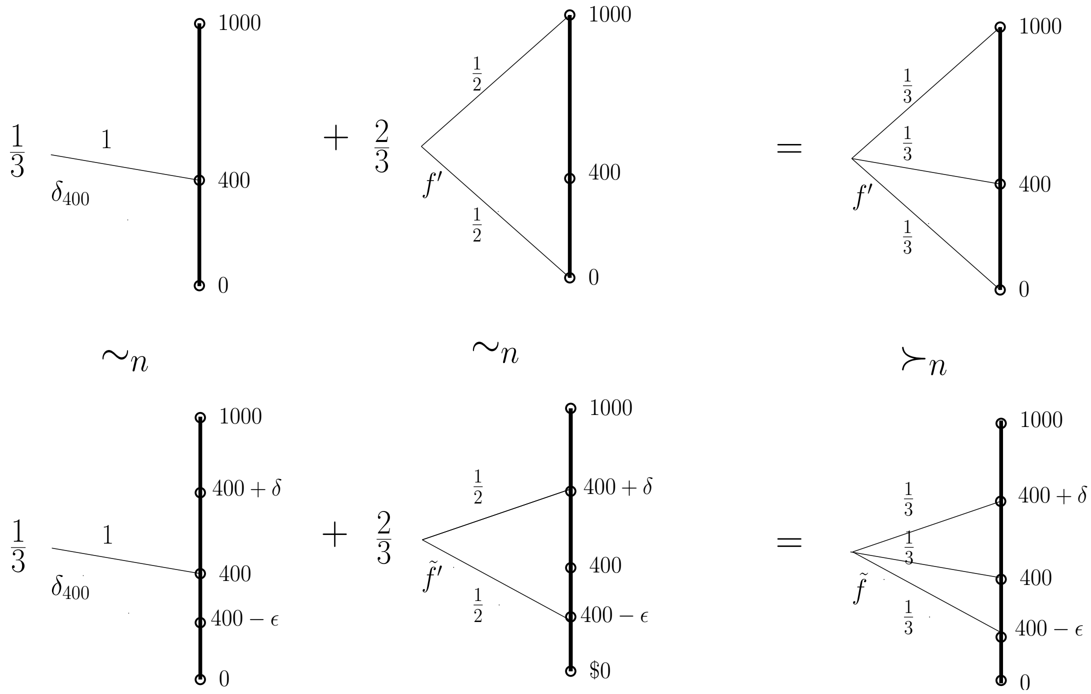

Figure 2: Optimism.

$\delta, 1/2; 400-\epsilon, 1/2)$, where $\tilde{f}'$ is chosen such that $\tilde{f}' \sim_N f'$, and ϵ is close to 0.[19] Specifically, consider the mixture $\tilde{f} = 2/3\ f' + 1/3\ \delta_{400} = (400+\delta, 1/3; 400, 1/3; 400-\epsilon, 1/3)$ (see Figure 2). If the independence axiom over unresolved lotteries were to hold, then $f \sim \tilde{f}$. But I also allow $f \succ_N \tilde{f}$, with the reasoning that the optimistic agent prefers knowing as little as as possible about the unresolved lottery. With lottery f, the optimist can form a more reassuring perception of the outcome, as it could be much higher (1000). With lottery $\tilde{f}$, however, as ϵ becomes smaller, it becomes less attractive to the optimistic agent, as he is more certain of the vicinity of the outcome. In brief, an optimist has a preference for more 'scrambled' information. A pessimistic agent, on the other hand, prefers less scrambled information, since knowing less would lead him to form a more negative perception. I allow the agent to be optimistic, pessimistic or neutral (i.e. independence may hold), but I assume that his preferences are preserved, given a specific mixture a and specific probabilities. That is, if the agent prefers unresolved lottery f to $\tilde{f}$, as in the example above, then this preference is preserved as ϵ becomes smaller. I refer to this property, which I now generalize, as 'information scrambling consistency' (ISC).

[19]For δ to also be close to 0, 400 would have to be close to the certainty equivalent of the unresolved lottery $f' = (1000, 1/2; 0, 1/2)$.

Definition (ISC)
Let $f = (z_1, p_1; ...; z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...; z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and *case 1*: $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$ (*case 2*: $(z_i, z_{i+1}) \subset (z'_i, z'_{i+1})$). If, for some $a \in (0,1)$ and some $z \in (z'_i, z'_{i+1})$:

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z,$$

then $\succeq_N$ satisfies ISC if:

$$a\tilde{f} + (1-a)\delta_{\tilde{z}} \succeq_N a\tilde{f}' + (1-a)\delta_{\tilde{z}}$$

for any $\tilde{f} = (\tilde{z}_1, p_1; ...\tilde{z}_i; p_i; \tilde{z}_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$, $\tilde{f}' = (\tilde{z}_1, p_1; ...\tilde{z}'_i; p_i; \tilde{z}'_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$ and $\tilde{z}$ such that $\tilde{z} \in (\tilde{z}'_i, \tilde{z}'_{i+1}) \subset (\tilde{z}_i, \tilde{z}_{i+1})$ (*case 2*: $\tilde{z} \in (\tilde{z}_i, \tilde{z}_{i+1}) \subset (\tilde{z}'_i, \tilde{z}'_{i+1})$).

A preference for more scrambled information (optimism) corresponds to case 1, i.e. preferring $af + (1-a)\delta_z \succ af' + (1-a)\delta_z$ when $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$. Similarly, a preference for less scrambled information (pessimism) corresponds to case 2. The appeal of the RDU representation is that it satisfies the ISC property:

Theorem 2. *Suppose that RDU holds for $\succeq_N$. Then $\succeq_N$ satisfies ISC.*

A local preference for more scrambled information, which I refer to as local optimism, does *not* correspond to the accepted RDU notion of optimism, analyzed by Wakker (1994). I focus, instead, on a global preference for more scrambled information, which is denoted (global) optimism:

Definition (Optimism) The preference relation $\succeq_N$ exhibits optimism if and only if $\succeq_N$ always exhibits a preference for more scrambled information. That is, for any $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$, and for all $a \in (0,1)$ and $z \in (z_i, z_{i+1})$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z.$$

The next theorem states that an agent who exhibits *global* optimism according to this definition also exhibits optimism according to the standard RDU definition. In other words, an agent has a global preference for more scrambled information if and only if the weighting function w is concave, which corresponds to the accepted (Wakker) RDU definition of optimism.

Theorem 3. *Suppose that* $\succeq_N$ *satisfies RDU, and let* w *be the associated weighting function. Then* w *is concave (convex) if and only if* $\succeq_N$ *exhibits optimism.*

The following result connects doubt-proneness, the properties of the utility functions, and the properties of the probability weighting function $w(p)$. A similar result holds for doubt-aversion, and is deferred to the appendix.

Theorem 4. *Suppose that axioms* ***A.1*** *through* ***A.4*** *and the RDU axioms hold, and let* u *and* v *be the utility functions associated with the resolved and unresolved lotteries, respectively, and* w *be the decision weight associated with the unresolved lotteries. In addition, suppose that* u *and* v *are both differentiable. Then:*
(i) If there exists a $p \in (0,1)$ *such that* $p < w(p)$*, then the agent is doubt-prone somewhere. Similarly, if there exists* $p' \in (0,1)$ *such that* $p' > w(p')$*, then the agent is doubt-averse somewhere.*
(ii) If the agent is doubt-prone everywhere, then $p \leq w(p)$ *for all* $p \in (0,1)$*. Moreover, if* v *exhibits stronger diminishing marginal utility than* u*, then* $\succeq_N$ *violates quasi-convexity (that is, there exists some* $f', f'' \in \mathfrak{L}_o$*, and* $\alpha \in (0,1)$ *such that* $f' \succ f''$ *and* $\alpha f' + (1-\alpha) f'' \succ_N f'$*).*

The differentiability assumption, though common, may seem bothersome as it is not taken over the primitives. Alternatively, we could make an assumption over the primitives that guarantees (for instance) strict concavity of u and v, which would in fact be sufficient for the result.[20] Given the results above, an assumption or deduction over the agent's doubt-attitude has testable implications concerning his aggregation of probabilities (w) for unresolved lottery, and vice-versa. In addition, these implications can be disentangled from the agent's diminishing marginal utility. Since it is not necessary that w satisfies the same empirical properties as the typical case considered under rank-dependent utility, an experimental study would be useful for a better understanding of the shape of w. If, in addition to doubt-proneness, mean-preserving risk-aversion (in the standard sense) of $\succeq_N$ is assumed, then the RDU representation collapses to the recursive EU representation:

Corollary 4.1. *Suppose that the conditions of Theorem 4 all hold. Then, the following two statements are equivalent:*

(i) Preference $\succeq$ *displays doubt-proneness everywhere and* $\succeq_N$ *displays mean-preserving risk-aversion.*

[20] For a discussion of the differentiability assumption, see Chew, Karni and Safra (1987).

(ii) *Function* V_{RDU} *is of the EU form (i.e.* $w(p) = p$ *for all* $p \in [0, 1]$*); both* u *and* v *are concave; and* $u = \lambda \circ v$ *for some continuous, concave, and increasing* λ*.*

This result further shows that attitude toward risk and attitude towards doubt constrain the probability weighting function and can in fact completely characterize it.[21] But note that in an RDU setting, mean-preserving risk-aversion is not identical to diminishing marginal utility. That is, the previous result does not imply that a doubt-prone agent who obeys risk-aversion cannot have a concave utility function v. I now focus on a counterexample for which doubt-proneness is entirely due to the weighting factor w, and *not* to the difference in concavity between u and v.

Consider an agent for whom functions u and v are identical. It is already immediate from Theorem 4 that for a doubt-prone agent, it is necessary that $p \leq w(p)$ for all p. In fact, this condition is sufficient.[22] The following result does not require differentiability.

Theorem 5. *Suppose that the conditions of theorem 4 all hold. Furthermore, suppose that* $u(z) = v(z)$ *for all* $z \in \mathbf{Z}$ *(or, more generally,* $u = \lambda \circ v$ *for some continuous, weakly concave, and increasing* λ*). Then, the agent is doubt-prone everywhere if and only if* $p \leq w(p)$ *(with* $p < w(p)$ *for some* $p \in (0, 1)$ *if* $u(z) = v(z)$ *for all* $z \in \mathbf{Z}$*).*

It follows that an optimistic agent for whom u is identical to v (or for whom u is more concave than v) must be doubt-prone. Therefore, these results connect optimism, doubt-proneness, and risk-aversion (in the standard sense).

Lastly, note that extensive research has been conducted on the shape of w in the usual RDU setting in which uncertainty eventually resolves.[23] As this is a different setting, I have not made similar assumptions over the shape of w. Instead, I have shown that the induced preferences to remain in doubt have strong implications for the weighting function w. Consider, for example, the common assumption that w is S-shaped (concave on the initial interval and convex beyond). In that case, it must be that the agent is doubt-prone for some lotteries and doubt-averse for others. But an empirical discussion of whether w is S-shaped in this setting is outside the scope of this paper.

[21] This last corollary is similar to a result in Grant, Kajii and Polak (2000), but with a notion of doubt-proneness that is weaker than the preference for late-resolution that would be required in the framework they use; the difference in assumptions is due to the difference in settings. It is also of note that under Grant, Kajii and Polak (2000)'s restriction, there is no need to assume differentiability, as it is implied.

[22] It is clear that if $p = w(p)$ for all $p \in (0, 1)$ and if $u(z) = v(z)$ for all $z \in \mathbf{Z}$, then the agent is doubt-neutral.

[23] See, for instance, Karni and Safra (1990), and Prelec (1998) for an axiomatic treatment of w.

6 Curiosity and Commitment

In this section, I present a dynamic version of the model. This setup corresponds to the case in which an agent may not know now whether he will ever make an observation, but he may find out later. I do this in part because this type of scenario occurs frequently; an individual is often not sure whether he can successfully avoid information in the next period. In the self-image application, for instance, the agent may take into consideration whether he might obtain information farther in the future. In this dynamic version, an agent could have preferences for committing to remaining ignorant, as he may later be 'curious' and choose to observe the resolution of uncertainty. For instance, an individual may be willing to pay to destroy a letter that contains the paternity test result for his child. Another reason for conducting this analysis is to illustrate the difference between this model and the KP representation (and, more generally, REU). I show that the models are formally distinct, even if independence axioms hold at every stage. This result may seem counterintuitive, since it may appear that a 'never' stage is formally equivalent to a 'much later' stage, but with a different interpretation. I discuss the reasons for the distinction between the two frameworks.

Suppose, for simplicity, that there are two stages of resolution (early and late) in a KP setup.[24] Assume, however, that the agent is indifferent between early and late resolution of uncertainty, so that there is a single utility function u associated with lotteries that resolve. It is clear that in this case, the KP representation is identical to an expected utility representation. But now, suppose that we include preferences over unresolved lotteries. That is, let $\mathfrak{L}_2$ be the set of simple lotteries over $\mathfrak{L}_1 \cup \mathfrak{L}_0$. For $\mathbf{X} \in \mathfrak{L}_2$, the notation $\mathbf{X} = (X_1, q^I_{1,e}; ...; X_{n_e}, q^I_{n_e,e};\ f_{1,e}, q^N_{1,e}; ...; f_{m_e,e}, q^N_{m_e,e}) \in \mathfrak{L}_2$, where $X_{i,e} \in \mathfrak{L}_1$, and $f_{j,e} \in \mathfrak{L}_0$. The subscript 'e' denotes the early stage. The agent's preferences $\succeq$ are now over $\mathfrak{L}_2$, rather than over $\mathfrak{L}_1$ (see Figure 3).

The timing is as follows. The agent first observes the outcome of the first stage lottery (the early stage). For instance, with probability $q^I_{i,e}$, he receives a second lottery $X_i \in \mathfrak{L}_1$. The superscript I ('Informed') denotes that the agent expects to observe the outcome of lottery X_i. With probability $q^N_{j,e}$, the agent receives a lottery $f^N_{j,e} \in \mathfrak{L}_0$, which does *not* resolve. Here, the superscript N ('Not informed') denotes that the agent never observes the resolution of $f^N_{j,e}$. A lottery $f^N_{j,e}$ (henceforth 'early unresolved lottery') is a terminal node, in the sense that the agent does not expect it to lead to a second stage. Now suppose that the first (early) stage lottery leads to a second (late) stage lottery $X_i = (z_1, q^I_{1,l}; z_2, q^I_{2,l}; ...; z_n, q^I_{n_l,l};\ f_{1,l}, q^N_{1,l}; f_{2,l}, q^N_{2,l}; ...; f_{m,l}, q^N_{m,l})$. This second

[24]More stages of resolution can be added in the usual way.

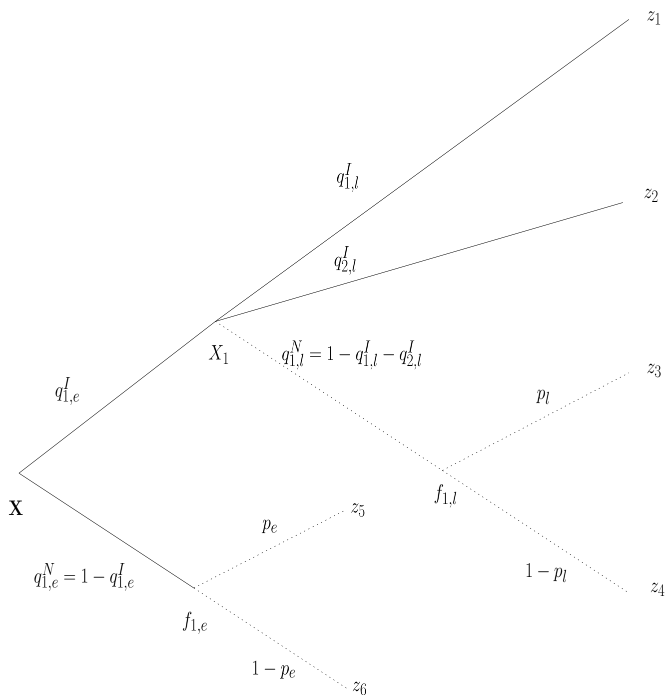

Figure 3: Lottery $\mathbf{X} = (X_1, q^I_{1,e}; f_{1,e}, q^N_{1,e} = 1 - q^I_{1,e})$, where $X_1 = (z_1, q^I_{1,l}; z_2, q^I_{2,l}; f_{1,l}, q^N_{1,l} = 1 - q^I_{1,l} - q^I_{2,l})$, $f_{1,e} = (z_5, p_e; z_6, 1 - p_e)$ and $f_{1,l} = (z_3, p_l; z_4, 1 - p_l)$.

lottery always resolves. With probability $q^I_{h,l}$, the agent receives a final outcome $z^I_{h,l}$, which he observes. With probability $q^N_{k,l}$, he receives a lottery $f_{k,l} \in \mathfrak{L}_o$ which never resolves (henceforth 'late unresolved lottery'). The difference between a lottery f_e and a lottery f_l is that the agent knows after the early stage that he receives lottery f_e which does not resolve, while he doesn't find out until the *late* stage that he receives lottery f_l which does not resolve. As before, the q^I's and q^N's are used to distinguish between the probabilities that lead to prizes where he is fully informed of the outcome (since he directly observes which z occurs), and the probabilities that lead to prizes where he is *not* informed (since he observes only the ensuing lottery).

Suppose now that continuity and independence axioms for unresolved lotteries hold at every stage. That is, define $\succeq_{N,e}$ and $\succeq_{N,l}$ in the natural way, and let continuity and independence axioms hold for each of these preferences. In this case, there are unresolved utility functions v_e, v_l associated with $\succeq_{N,e}$ and $\succeq_{N,l}$, respectively:

$$\mathbf{W}(\mathbf{X}) = \sum q^I(z)u(z) + \sum q^I_{i,e}(z) \left(\sum q^N_{i,l} u \left(v_l^{-1} \left(E v_l(z|f_{i,l}) \right) \right) \right) + \sum q^N_{i,e} u \left(v_e^{-1} \left(E v_e(z|f_{i,e}) \right) \right).$$

Note that v_e and v_l need not be the same, since $\succeq_{N,e}$ and $\succeq_{N,l}$ are separate. Hence, there are three utility functions in this setting: utility u is associated with lotteries that eventually resolve, while functions v_e and v_l are associated with early and late unresolved lotteries. Notice that under this representation, the agent may have preferences for commitment, as he may be doubt-prone in the early stage but doubt-averse in the late stage. It is immediate, therefore, that having a KP model that accommodates unresolved lotteries is formally distinct from simply adding a 'never' stage (or any number of stages) to the standard KP framework, as this can only account for additional nested utility functions.[25] The reason for this distinction is that the agent's perception of the unresolved lotteries need not be the same in the early stage as it is in the second stage.

There is another, and perhaps more fundamental, difference between temporal resolution and lack of resolution. While the early stage leads to the eventual occurrence of the late stage, there is no notion of sequence for unresolved lotteries. That is, the first unresolved lottery cannot lead to a second lottery; each unresolved lottery is a final prize, and hence a terminal node. For that reason, while the KP representation will have terms such as $u_e(u_l^{-1}(\cdot))$, there cannot be an equivalent unresolved term, $v_e(v_l^{-1}(\cdot))$. In this representation, both utility functions v_e and v_l are terminal, in the sense that the expectations are over outcomes, and not over any further lotteries. While the notation is cumbersome, this representation demonstrates that each unresolved lottery is essentially a final prize, and its value depends on whether it is obtained early or late. The agent's preferences over unresolved lotteries are allowed to vary in time, even when he has neutral preferences over the timing of resolution of uncertainty. The distinction between the KP representation and a representation that takes into account preferences for unresolved lotteries holds if the independence axioms over $\succeq_{N,e}$ and $\succeq_{N,l}$ are relaxed. In other words, this distinction carries through to more general REU representations.

7 Closing remarks

This paper provides a representation theorem for preferences over lotteries whose outcomes may never be observed. The agent's perception of the unobserved outcome, relative to his risk-aversion, induces his attitude towards doubt. This relation is captured by his resolved utility function u, his unresolved utility function v and his unresolved decision weighting function w. The model presented here is an extension of the vNM framework, and it does not entail a significant axiomatic departure. However, it can accommodate behavioral patterns that are inconsistent with expected utility, and that

[25]The KP representation would instead be of 'nested' form $Eu_1(u_2^{-1}(Eu_2(u_3^{-1}(Eu_3(...)))))$, see Kreps and Porteus (1978) for details.

have motivated a wide array of different frameworks. For instance, doubt-prone individuals have an incentive to self-handicap, and this incentive is higher if they are less certain about their competence.[26] Doubt-prone individuals are also more likely to choose the status quo bundle, if making a decision is more informative than inaction. In addition, an agent who is risk-neutral may still favor less risky investments, and would pay a firm to invest for him, even if it does not have superior expertise. The agent's attempt to preserve his self-image implies that his utility depends not only on the outcome that results, but also on the action taken. In a setting with moral or other-regarding preferences, agents may change their behavior depending on how much they expect to observe ex-post. In a political economy context, doubt-proneness encourages political ignorance. When individuals derive more utility from the policies that they are not required to observe, they have *less* incentive to acquire information. Moreover, agents have a greater disutility from acquiring information if they are more ignorant ex-ante.

Note that experiments that address the impact of anticipated regret frequently allow for foregone outcomes that individuals do not observe (see Zeelenberg (1999)). These findings would be useful in determining plausible degrees of doubt-proneness. Moreover, this model is applicable to norms of 'social diplomacy' in which individuals deliberately employ uninformative language.[27] Finally, doubt-proneness may be a factor in choices over goods that draw value from their perceived, but possibly unknown, authenticity. The analysis of these last examples is outside the scope of this paper.

[26]Recall that this model does not allow agents to be delusional, since they are unable to mislead themselves into having false beliefs.

[27]I thank Rachel Kranton for suggesting this term.

Appendix

The appendix is structured as follows. Section A.1 explains why the standard EU model is inappropriate when the agent does not expect to observe the resolution of uncertainty. Section A.2 provides an example of the 'preservation of self-image' application. All the proofs are in Section A.3.

A.1 Limitations of the standard EU model

This example illustrates the problem with using the standard vNM EU model when there are outcomes that the agent never expects to observe. Consider the simple case of an agent who has performed a task and does not fully observe its outcome. There are no future decisions that depend on his performance. For instance, as a simple adaptation of Savage's omelet example, suppose that the agent does not know whether he has fed his guests a good omelet or a bad one. With probability p_t, he has done well ($\bar{t}$), and with probability $(1-p_t)$ he has done badly ($\underline{t}$). He prefers having done well to having done badly, although this will have no future repercussions. Given the choice between remaining forever in doubt (D) and perfectly resolving the uncertainty, (ND), it might appear that he compares:

$$U_D = p_t u(\bar{t}) + (1-p_t)u(\underline{t})$$

to

$$U_{ND} = p_t u(\bar{t}) + (1-p_t)u(\underline{t})$$

and that since $U_D = U_{ND}$, he is indifferent. But U_D is *not* necessarily the right function to use if he chooses to remain in doubt because, from his frame of reference, the final outcome will not be $\bar{t}$ or $\underline{t}$. That is, he does not expect to 'obtain' ex-post utility $u(\bar{t})$ or $u(\underline{t})$ because he does not expect to observe either $\bar{t}$ or $\underline{t}$. As it is not clear what his perception of the consequence is if he does not expect the uncertainty to be resolved (from his viewpoint), his expected utility is undetermined. In its current form, the standard EU model does not offer a method for evaluating this choice. Using U_D effectively ignores that the relevant frame of reference is the agent's, not the modeler's.[28]

Redefining the outcome space to include the observation itself does not eliminate the problem. Suppose that the outcome space is taken to be $Z = \{\bar{t}_D, \underline{t}_D, \bar{t}_{ND}, \underline{t}_{ND}\}$, where $\bar{t}_D$ represents the outcome that he did well but doubts it, $\bar{t}_{ND}$ that he did well and does not doubt

[28]This issue is not resolved by starting with preferences over lotteries as primitives. In the standard framework, the agent has primitive preferences over lotteries over outcomes, and he is not allowed to choose between lotteries whose resolution he observes and lotteries whose resolution he does not observe. He is therefore not given the option to express those preferences.

it, and so forth. Therefore, he compares the following:

$$U_D = p_t u(\bar{t}_D) + (1 - p_t)u(\underline{t}_D)$$

to

$$U_{ND} = p_t u(\bar{t}_{ND}) + (1 - p_t)u(\underline{t}_{ND}).$$

It is difficult to interpret the meaning of the consequence 'did well, but doubts it' from his frame of reference, since it is not clear what it means to be in doubt if he knows that he has done well. In addition, his preferences over $\bar{t}_D$ and $\underline{t}_D$ are completely pinned down. Consider the two extremes, $p_t = 1$ and $p_t = 0$. When $p_t = 1$, there is no intrinsic difference between U_D and U_{ND}, since he knows that he has done well. Hence, $u(\bar{t}_D) = u(\bar{t}_{ND})$. Similarly, when $p_t = 0$, he knows he has done badly, and so $u(\underline{t}_D) = u(\underline{t}_{ND})$. It then follows that $U_D = U_{ND}$ for any $p_t \in [0, 1]$. This definition of the outcome space is essentially the same as simply $Z = \{\bar{t}, \underline{t}\}$. His indifference between remaining in doubt and not remaining in doubt is a consequence of following this approach; it is not implicit from the standard EU model.
Redefining the outcome space so that his utility is constant if he remains in doubt is even more problematic. Suppose that $Z = \{\bar{t}_{ND}, \underline{t}_{ND}, D\}$, letting $\bar{t}_{ND}$ be the outcome 'talented and he does not remain in doubt (he observes the outcome)', letting $\underline{T}_{ND}$ be the outcome 'untalented and he observes it' and letting D mean that he does not observe the outcome, thus remaining in doubt. He now compares:

$$U_D = u(D)$$

to

$$U_{ND} = p_t u(\bar{t}_{ND}) + (1 - p_t)u(\underline{t}_{ND})$$

However, in the limit $p_t \to 1$, U_D should approach U_{ND}, which occurs only if $u(D) = u(\bar{t}_{ND})$. But in that case, as $p_t \to 0$, U_D does not approach U_{ND}, and so there is an unavoidable discontinuity.

A.2 Applications

Numerical Example (Preservation of Self-image)

The following is a more general version of the numerical example provided in the main body of the paper. Suppose he exerts effort $e \in [0, 1]$, and obtains reward $m \in [0, 100]$. He also has an unobserved talent (or type) $t \in [0, 1]$. The agent is doubt-prone and risk-averse for both resolved and unresolved lotteries on talent. Specifically, $u = at^{1/2}$ for some $a > 0$, and v = t. His expected utility of money is linearly separable from his utility of talent, and is equal to his expected reward Em. He therefore maximizes:

$$\tilde{W}(e) = Em(e) - C(e)$$

where $C(e) \equiv u\left(v^{-1}(Ev(t))\right) - \sum_m p(m|e) u \circ v^{-1}(Ev(t|m,e))$

The agent's prior is q that talent $t = 0$, and $1-q$ that talent $t = 1$. He can put in level $e \in [\underline{e}, \overline{e}]$. Given that he has talent $t = 1$ or $t = 0$ and puts in effort e, his respective probabilities of obtaining monetary reward $m = 100$ are $p(100|t = 1, e) = e$ and $p(100|t = 0, e) = be$, for $b \in [0, 1)$.

Note that the uninformative effort e_0 in this example is $e = 0$, since he is certain to obtain $m = 0$, independently of his talent. It follows from the probabilities given above that:

$$p(0|1, e) = 1 - e$$

$$p(0|0, e) = 1 - be$$

$$p(100|e) = e(q + b(1 - q)$$

$$p(0|e) = 1 - e(q + b(1 - q))$$

$$p(1|100, e) = \frac{q}{q + b(1 - q)}$$

Solving:

$W(e) = 100 * p(100|e) + a\left(p(0|e)p(\bar{t})p(0|\bar{t}, e)\right)^{1/2} + a\left(p(100|e)p(\bar{t})p(100|\bar{t}, e)\right)^{1/2}$

$= e(100\beta + a(\beta q)^{1/2}) + aq^{1/2}\left(1 - e(1 + \beta) + \beta e^2\right)^{1/2}$

where $\beta = q + b(1 - q)$. Let $\gamma = 100\beta + a(\beta q)^{1/2}$, and $D = \frac{4\gamma^2}{a^2 q}$. Then, from the first order conditions, we obtain:

$e^2(\beta C - 4\beta^2) + e(4\beta - C)(1 + \beta) + C - (1 + \beta)^2 = 0$

The example in the text corresponds to the case $b = 0$, $q = 1/2$, and so $\beta = 1/2$, $\gamma = 50 + \frac{a}{2}$, and $d = 2D = \left(\frac{200}{a} + 2\right)^2$.

A.3 Proofs

Representation Theorem. *Proof.* Let $X = (z_1, q_1^I; z_2, q_2^I; ...; z_n, q_n^I;\ f_1, q_1^N; f_2, q_2^N; ...; f_m, q_m^N)$. By continuity, there exists a function $H : \mathfrak{L}_o \to \mathbf{Z}$ such that $\delta_{H(f)} \sim_N f$ (i.e. $\delta_{\delta_{H(f)}} \sim \delta_f$) for all $f \in \mathfrak{L}_o$. By the certainty axiom **A.3**, it follows that $\delta_{H(f)} \sim \delta_{\delta_{H(f)}}$. Hence $\delta_{H(f)} \sim \delta_f$ for any $f \in \mathfrak{L}_o$. By a well-known implication of the independence axiom **A.4**, $X \sim \tilde{X}$, where $\tilde{X} = (z_1, q_1^I; z_2, q_2^I; ...; z_n, q_n^I;\ H(f_1), q_1^N; H(f_2), q_2^N; ...; H(f_m), q_m^N)$. Defining $\tilde{Y}$ similarly, $Y \sim \tilde{Y}$. By transitivity, $X \succ Y \Rightarrow \tilde{X} \succ \tilde{Y}$. Note that all lotteries $\tilde{X}$ and $\tilde{Y}$ are one-stage lotteries, with final outcomes as prizes. Define the preference relation $\succ_I$ in the following way: $X \succ Y \Rightarrow \tilde{X} \succ_I \tilde{Y}$. All the EU axioms hold on $\succ_I$, and so $\tilde{X} \succ \tilde{Y}$ if and only if $W(\tilde{X}) > W(\tilde{Y})$, where

$$W(\tilde{X}) = \sum_{i=1}^{n} q_i^I u(z_i) + \sum_{i=1}^{m} q_i^N u\left(H(f_{z_i})\right)$$

and W is unique up to positive affine transformation. But since $X \succ Y \Rightarrow \tilde{X} \succ \tilde{Y}$, it follows that $X \succ Y$ if and only if $W(\tilde{X}) > W(\tilde{Y})$.

To obtain the representation of H: axioms **A.1-A.4** and axiom **N.1** imply that $\succeq_N$ is a weak order and that Jensen-continuity holds. The proof for the RDU representation of $\succeq_N$ then follows from Wakker (1994). Then, for any $f \in \mathfrak{L}_{\mathfrak{o}}$, we have shown that $\delta_{H(f)} \sim_N f$. Since $w(1) = 1$, it follows that $v(H(f)) = v^{-1}\left(V_{RDU}(f)\right)$, and hence $H(f) = v^{-1}\left(V_{RDU}(f)\right)$, which completes the proof.

Theorem 2. *Proof.* Case 1 is shown below, and case 2 can be proven in a similar way (by changing all the signs). Suppose RDU holds for $\succeq_N$.
There are two cases two consider:

(a) f, f' have more than 2 elements:
Let $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z_i'; p_i; z_{i+1}', p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_{\mathfrak{o}}$ such that $f \sim_N f'$, and $(z_i', z_{i+1}') \subset (z_i, z_{i+1})$. Suppose that, for some $a \in (0, 1)$ and some $z \in (z_i', z_{i+1}')$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

Since RDU holds:

$$f \sim_N f' \Rightarrow V_{RDU}(f) = V_{RDU}(f')$$

$$\begin{aligned}
&\Rightarrow v(z_1) + \sum_{j=2}^{i-1} w(p_j^*)[v(z_j) - v(z_{j-1})] + w(p_i^*)[v(z_i) - v(z_{i-1})] + w(p_{i+1}^*)[v(z_{i+1}) - v(z_i)] \\
&\qquad + w(p_{i+2}^*)[v(z_{i+2}) - v(z_{i+1})] + \sum_{j=i+3}^{n} w(p_j^*)[v(z_j) - v(z_{j-1})] \quad = \\
&v(z_1) + \sum_{j=2}^{i-1} w(p_j^*)[v(z_j) - v(z_{j-1})] + w(p_i^*)[v(z_i') - v(z_{i-1})] + w(p_{i+1}^*)[v(z_{i+1}') - v(z_i')] \\
&\qquad + w(p_{i+2}^*)[v(z_{i+2}) - v(z_{i+1}')] + \sum_{j=i+3}^{n} w(p_j^*)[v(z_j) - v(z_{j-1})]
\end{aligned}$$

$$\Rightarrow \frac{w(p_{i+1}^*) - w(p_{i+2}^*)}{w(p_i^*) - w(p_{i+1}^*)} = \frac{v(z_i') - v(z_i)}{v(z_{i+1}) - v(z_{i+1}')} \tag{1}$$

Note that $af + (1-a)\delta_z = (z_1, ap_1; ...z_i; ap_i; z, 1-a; z_{i+1}, ap_{i+1}; ...; z_n, ap_n)$, where the ranking of z is due to $z \in (z_i', z_{i+1}') \subset (z_i, z_{i+1})$. Similarly, $af' + (1-a)\delta_z = (z_1, ap_1; ...z_i'; ap_i; z, 1-$

$a; z'_{i+1}, ap_{i+1}; ...; z_n, ap_n)$. Using the condition

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

it follows that

$$\begin{aligned}\Rightarrow v(z_1) + \sum_{j=2}^{i-1} w(ap_j^* + 1 - a)[v(z_j) - v(z_{j-1})] + w(ap_i^* + 1 - a)[v(z_i) - v(z_{i-1})] \\ + w(ap_{i+1}^* + 1 - a)[v(z) - v(z_i)] + w(ap_{i+1}^*)[v(z_{i+1}) - v(z)] \\ + w(ap_{i+2}^*)[v(z_{i+2}) - v(z_{i+1})] + \sum_{j=i+3}^{n} w(ap_j^*)[v(z_j) - v(z_{j-1})] \quad \geq \\ v(z_1) + \sum_{j=2}^{i-1} w(ap_j^* + 1 - a)[v(z_j) - v(z_{j-1})] + w(ap_i^* + 1 - a)[v(z'_i) - v(z_{i-1})] \\ + w(ap_{i+1}^* + 1 - a)[v(z) - v(z'_i)] + w(ap_{i+1}^*)[v(z'_{i+1}) - v(z)] \\ + w(ap_{i+2}^*)[v(z_{i+2}) - v(z'_{i+1})] + \sum_{j=i+3}^{n} w(ap_j^*)[v(z_j) - v(z_{j-1})]\end{aligned}$$

$$\Rightarrow \frac{w(ap_{i+1}^*) - w(ap_{i+2}^*)}{w(ap_i^* + 1 - a) - w(ap_{i+1}^* + 1 - a)} \geq \frac{v(z'_i) - v(z_i)}{v(z_{i+1}) - v(z'_{i+1})} \tag{2}$$

Combining (1) and (2), we obtain:

$$\frac{w(ap_{i+1}^*) - w(ap_{i+2}^*)}{w(ap_i^* + 1 - a) - w(ap_{i+1}^* + 1 - a)} \geq \frac{w(p_{i+1}^*) - w(p_{i+2}^*)}{w(p_i^*) - w(p_{i+1}^*)} \tag{3}$$

Note that this does not depend on the utility function v, but only on the weighting function w. Take any $\tilde{f} = (\tilde{z}_1, p_1; ...\tilde{z}_i; p_i; \tilde{z}_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$, $\tilde{f}' = (\tilde{z}_1, p_1; ...\tilde{z}'_i; p_i; \tilde{z}'_{i+1}, p_{i+1}; ...; \tilde{z}_n, p_n)$ and $\tilde{z}$ such that $\tilde{z} \in (\tilde{z}'_i, \tilde{z}'_{i+1}) \subset (\tilde{z}_i, \tilde{z}_{i+1})$. It must be that $a\tilde{f} + (1-a)\delta_{\tilde{z}} \succeq_N a\tilde{f}' + (1-a)\delta_{\tilde{z}}$. Suppose not, i.e. suppose that $a\tilde{f}' + (1-a)\delta_{\tilde{z}} \succ_N a\tilde{f} + (1-a)\delta_{\tilde{z}}$. Then, redoing a similar calculation to the one above, we obtain:

$$\frac{w(ap_{i+1}^*) - w(ap_{i+2}^*)}{w(ap_i^* + 1 - a) - w(ap_{i+1}^* + 1 - a)} < \frac{w(p_{i+1}^*) - w(p_{i+2}^*)}{w(p_i^*) - w(p_{i+1}^*)} \tag{4}$$

which contradicts (3). Hence ISC holds for this case.

(b) f, f' have exactly 2 elements:
Let $f = (z_1, 1-p; z_2, p)$, $f' = (z'_1, 1-p; z'_2, p) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_1, z'_2) \subset (z_1, z_2)$. Suppose that, for some $a \in (0,1)$ and some $z \in (z'_1, z'_2)$. If $\succeq_N$ satisfies RDU, then:

$$f \sim_N f' \Rightarrow v(z_1) + w(p)[v(z_2) - v(z_1)] = v(z'_1) + w(p)[v(z'_2) - v(z'_1)]$$

$$\Rightarrow w(p) = \frac{v(z_1') - v(z_1)}{[v(z_1') - v(z_1)] + [v(z_2) - v(z_2')]}$$

$$\Rightarrow \frac{w(p)}{1 - w(p)} = \frac{v(z_1') - v(z_1)}{v(z_2) - v(z_2')} \tag{5}$$

Since $af + (1-a)\delta_z = ((z_1, a(1-p); z, 1-a; z_2, ap)$ and $af' + (1-a)\delta_z = ((z_1', a(1-p); z, 1-a; z_2', ap)$, the condition $af + (1-a)\delta_z \succ_N af' + (1-a)\delta_z$ implies (using a similar calculation to the one used for obtaining (3)) that

$$\Rightarrow \frac{w(ap)}{1 - w(ap + 1 - a)} \geq \frac{v(z_1') - v(z_1)}{v(z_2) - v(z_2')} \tag{6}$$

and combining (4) and (5), it follows that

$$\Rightarrow \frac{w(ap)}{1 - w(ap + 1 - a)} \geq \frac{w(p)}{1 - w(p)} \tag{7}$$

As before, this does not depend on the $v's$, but only on the weighting function w. Take any $\tilde{f} = (\tilde{z}_1, 1-p; \tilde{z}_2, p)$, $\tilde{f}' = (\tilde{z}_1', p_1; \tilde{z}_2', p_2)$ and $\tilde{z}$ such that $\tilde{z} \in (\tilde{z}_1', \tilde{z}_2') \subset (\tilde{z}_1, \tilde{z}_2)$. It must be that $a\tilde{f} + (1-a)\delta_{\tilde{z}} \succeq_N a\tilde{f}' + (1-a)\delta_{\tilde{z}}$. Suppose not, i.e. suppose that $a\tilde{f}' + (1-a)\delta_{\tilde{z}} \succ_N a\tilde{f} + (1-a)\delta_{\tilde{z}}$. Then, redoing a similar calculation to the one above, we obtain:

$$\Rightarrow \frac{w(ap)}{1 - w(ap + 1 - a)} < \frac{w(p)}{1 - w(p)} \tag{8}$$

which contradicts (7). Hence ISC holds for this case as well, which completes the proof.

The following lemma is used in the proof of theorem 3:

Lemma 1. *Let $w : [0,1] \to [0,1]$. Take any $p, q, p', p' \in [\underline{p}, \overline{p}] \subseteq [0,1]$ such that $p > p' > q'$, $q > q'$. Then if w is concave on $[\underline{p}, \overline{p}]$:*

$$\frac{w(p) - w(q)}{p - q} \leq \frac{w(p') - w(q')}{p' - q'}$$

if w is convex on $[\underline{p}, \overline{p}]$:

$$\frac{w(p) - w(q)}{p - q} \geq \frac{w(p') - w(q')}{p' - q'}$$

Proof. The proof is only shown for a concave function w. We make use of the following well-known result that a function f is concave if and only if for any $\tilde{p} > \tilde{q} > \tilde{r}$,

$$\frac{f(\tilde{p}) - f(\tilde{q})}{\tilde{p} - \tilde{q}} \leq \frac{f(\tilde{p}) - f(\tilde{r})}{\tilde{p} - \tilde{r}} \leq \frac{f(\tilde{q}) - f(\tilde{r})}{\tilde{q} - \tilde{r}} \tag{9}$$

We now directly prove the claim for each of the three possible cases:

(i) $p > q > p' > q'$
Using (9) twice,

$$\frac{w(p)-w(q)}{p-q} \leq \frac{w(q)-w(p')}{q-p'} \leq \frac{w(p')-w(q')}{p'-q'}$$

(ii) $p > p' > q > q'$
Using (9) twice,

$$\frac{w(p)-w(q)}{p-q} \leq \frac{w(p')-w(q)}{p'-q} \leq \frac{w(p')-w(q')}{p'-q'}$$

(iii) $p > p' = q > q'$
In this case, the result follows immediately from (9):

$$\frac{w(p)-w(q)}{p-q} \leq \frac{w(q)-w(q')}{q-q'} = \frac{w(p')-w(q')}{p'-q'}$$

which completes the proof.

Theorem 3. *Proof.* Suppose that $\succeq_N$ satisfies RDU. We first show (A) that the weighting function w is concave implies that for any $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$, and for all $a \in (0,1)$ and $z \in (z_i, z_{i+1})$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

We then prove the converse (B).
Proof of (A) Suppose that the weighting function w is concave. We proceed by contradiction. There are two cases to consider:

(a) f, f' have more than two elements: Let $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$. Suppose there exists some $a \in (0,1)$ and some $z \in (z_i, z_{i+1})$ such that $af' + (1-a)\delta_z \succ_N af + (1-a)\delta_z$. Using the derivation of theorem 3, it follows that

$$\frac{w(ap^*_{i+1}) - w(ap^*_{i+2})}{w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a)} < \frac{w(p^*_{i+1}) - w(p^*_{i+2})}{w(p^*_i) - w(p^*_{i+1})} \tag{10}$$

We now show:

(I) $w(ap^*_{i+1}) - w(ap^*_{i+2}) \geq a\left(w(p^*_{i+1}) - w(p^*_{i+2})\right)$

Note that $p^*_{i+1} > p^*_{i+2} > ap^*_{i+2}$, since $a \in (0,1)$, and using the definition of p^*. It is immediate that $ap^*_{i+1} > ap^*_{i+2}$. It follows, therefore, from lemma 1, that:

$$\frac{w(p^*_{i+1}) - w(p^*_{i+2})}{p^*_{i+1} - p^*_{i+2}} \leq \frac{w(ap^*_{i+1}) - w(ap^*_{i+2})}{ap^*_{i+1} - ap^*_{i+2}}$$

Rearranging, we obtain $w(ap^*_{i+1}) - w(ap^*_{i+2}) \geq a\left(w(p^*_{i+1}) - w(p^*_{i+2})\right)$.

(II) $w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a) \leq a\left(w(p^*_i) - w(p^*_{i+1})\right)$

Note that $ap^*_i + 1 - a > p^*_i$, since $a, p^*_i \in (0,1)$ implies that $1 - a > p^*_i(1-a)$. Similarly, $ap^*_{i+1} + 1 - a > p^*_{i+1}$, and we know that $p^*_i > p^*_{i+1}$. Using lemma 1, it follows that:

$$\frac{w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a)}{(ap^*_i + 1 - a) - \left(ap^*_{i+1} + 1 - a\right)} \leq \frac{w(p^*_i) - w(p^*_{i+1})}{p^*_i - p^*_{i+1}}$$

Rearranging, we obtain $w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a) \leq a\left(w(p^*_i) - w(p^*_{i+1})\right)$

Combining (I) and (II) (noting that both sides of (II) are greater than zero), it follows that

$$\frac{w(ap^*_{i+1}) - w(ap^*_{i+2})}{w(ap^*_i + 1 - a) - w(ap^*_{i+1} + 1 - a)} \geq \frac{w(p^*_{i+1}) - w(p^*_{i+2})}{w(p^*_i) - w(p^*_{i+1})} \tag{11}$$

which is a contradiction of (10).

(b) f, f' have exactly 2 elements:

Let $f = (z_1, 1-p; z_2, p)$, $f' = (z'_1, 1-p; z'_2, p) \in \mathfrak{L}_o$ such that $f \sim_N f'$, and $(z'_1, z'_2) \subset (z_1, z_2)$. Suppose there exists some $a \in (0,1)$ and some $z \in (z_1, z_2)$ such that $af' + (1-a)\delta_z \succ_N af + (1-a)\delta_z$. Using the derivation of theorem 3, it follows that

$$\frac{w(ap)}{1 - w(ap + 1 - a)} < \frac{w(p)}{1 - w(p)} \tag{12}$$

We now show:

(I) $w(ap) \geq aw(p)$

$a \in (0,1)$ and so $p > ap > 0$. It follows from the well-known result (9) used in proving lemma 1 that:

$$\frac{w(p) - w(0)}{p} \leq \frac{w(ap) - w(0)}{ap - 0}$$

Using $w(0) = 0$ and rearranging, we obtain $w(ap) \geq aw(p)$

(II) $1 - w(ap + 1 - a) \leq a\,(1 - w(p))$

Note that $1 > ap + 1 - a > p$, since it is immediate from $a, p \in (0,1)$ that $a > ap$ and $1 - a > p(1-a)$.

Using (9) again,
$$\frac{w(1) - w(ap+1-a)}{1-(ap+1-a)} \leq \frac{w(1) - w(p)}{1-p}$$

Using $w(1) = 1$ and rearranging, we obtain that $1 - w(ap + 1 - a) \leq a\,(1 - w(p))$.

Combining (I) and (II), we obtain

$$\frac{w(ap)}{1 - w(ap+1-a)} \geq \frac{w(p)}{1-w(p)} \tag{13}$$

which contradicts (12).

Proof of (B) Suppose that for any $f = (z_1, p_1; ...z_i; p_i; z_{i+1}, p_{i+1}; ...; z_n, p_n)$, $f' = (z_1, p_1; ...z'_i; p_i; z'_{i+1}, p_{i+1}; ...; z_n, p_n) \in \mathfrak{L}_0$ such that $f \sim_N f'$, and $(z'_i, z'_{i+1}) \subset (z_i, z_{i+1})$, and for all $a \in (0,1)$ and $z \in (z_i, z_{i+1})$,

$$af + (1-a)\delta_z \succeq_N af' + (1-a)\delta_z$$

We proceed as follows: (a) we first show that there is no interval $[\underline{p}, \overline{p}] \subseteq [0,1]$ on which w is strictly convex; (b) we then show that there is no interval $[\underline{p}, \overline{p}] \subseteq [0,1]$ such that for all $p \in [\underline{p}, \overline{p}]$, $w(p)$ is 'under the diagonal', i.e. $\frac{w(\overline{p})-w(p)}{\overline{p}-p} > \frac{w(\overline{p})-w(\underline{p})}{\overline{p}-\underline{p}} > \frac{w(p)-w(\underline{p})}{p-\underline{p}}$ (note that with stronger smoothness assumptions this would be sufficient for concavity); (c) we use results (a) and (b) to prove that w must be concave. We first note that it follows from the claim and from the derivation of theorem 3 that:

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} \geq \frac{w(p_1) - w(p_2)}{w(p_0) - w(p_1)} \tag{14}$$

for all $0 \leq p_2 < p_1 < p_0 \leq 1$ and $a \in (0,1)$.

(a) We proceed by contradiction: suppose there does exist an interval $[\underline{p}, \overline{p}] \subseteq [0,1]$ on which w is strictly convex. Let $\underline{p} < p_2 < p_1 < p_0 < \overline{p}$, and let $\{\frac{\underline{p}}{p_2}, \frac{1-\overline{p}}{1-p_0}\} < a < 1$. It follows that $\underline{p} < ap_2 < ap_1 < ap_1 + 1 - a < ap_0 + 1 - a)\overline{p}$. Using lemma 1, it follows that:

$$\frac{w(p_1) - w(p_2)}{p_1 - p_2} > \frac{w(ap_1) - w(ap_2)}{ap_1 - ap_2} \tag{15}$$

$$\frac{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)}{(ap_0 + 1 - a) - (ap_1 + 1 - a)} > \frac{w(p_0) - w(p_1)}{p_0 - p_1} \tag{16}$$

Rearranging and combining (15) and (16), it follows that

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{w(p_1) - w(p_2)}{w(p_0) - w(p_1)}$$

which contradicts (14).

(b) We proceed again by contradiction: suppose that there does exist an interval $[\underline{p}, \overline{p}] \subseteq [0, 1]$ such that $\frac{w(\overline{p})-w(p)}{\overline{p}-p} > \frac{w(\overline{p})-w(\underline{p})}{\overline{p}-\underline{p}} > \frac{w(p)-w(\underline{p})}{p-\underline{p}}$ for all $p \in [\underline{p}, \overline{p}]$.
Let $a = 1 - (\overline{p} - \underline{p}) + \epsilon$, for an arbitrarily small ϵ. Let $\tilde{p} = \underline{p}/a$. Using result (a), $[\tilde{p}, \tilde{p} + \delta]$ cannot be strictly convex, for any $\delta \in (0, 1-\tilde{p}]$. We can therefore find $\{p_0, p_1, p_2\} \in [\tilde{p}, \tilde{p}+\delta]$ such that $p_2 < p_1 < p_0$ and

$$\frac{w(p_1) - w(p_2)}{p_1 - p_2} \geq \frac{w(p_0) - w(p_1)}{p_0 - p_1} \tag{17}$$

As δ, ϵ become arbitrarily small (and $a\delta \leq \epsilon$), $ap_2 \to \underline{p}$, $ap_0 + 1 - a \to \overline{p}$ and $\{ap_2, ap_1, ap_1 + 1 - a, ap_0 + 1 - a\} \in [\underline{p}, \overline{p}]$. We therefore have that for small enough δ, ϵ,

$$\frac{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)}{(ap_0 + 1 - a) - (ap_1 + 1 - a)} > \frac{w(\overline{p}) - w(\underline{p})}{\overline{p} - \underline{p}} \tag{18}$$

and

$$\frac{w(\overline{p}) - w(\underline{p})}{\overline{p} - \underline{p}} > \frac{w(ap_1) - w(ap_2)}{a(p_1 - p_2)} \tag{19}$$

Combining (18) and (19):

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{p_1 - p_2}{p_0 - p_1} \tag{20}$$

Combining (17) and (20), we obtain:

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{w(p_1) - w(p_2)}{w(p_0) - w(p_1)}$$

which contradicts (14).

(c) We now prove that w is concave. Suppose not, i.e. suppose there exist $0 \leq p < q < r < 1$ such that

$$\frac{w(r) - w(q)}{r - q} > \frac{w(q) - w(p)}{q - p} \tag{21}$$

Let $a = 1 - (r - q) + \epsilon$, for an arbitrarily small ϵ. Let $\tilde{p} = q/a$. Using result (a), $[\tilde{p} - \delta, \tilde{p}]$ cannot be strictly convex, for any $\delta \in (0, \tilde{p}]$. We can therefore find $\{p_0, p_1, p_2\} \in [\tilde{p} - \delta, \tilde{p}]$

such that $p_2 < p_1 < p_0$ and

$$\frac{w(p_1) - w(p_2)}{p_1 - p_2} \geq \frac{w(p_0) - w(p_1)}{p_0 - p_1} \tag{22}$$

As δ, ϵ become arbitrarily small (and $a\delta \leq \epsilon$), $ap_1 \to q$, $ap_0 + 1 - a \to r$, $\{ap_2, ap_1\} \in (p, q]$ and $\{ap_1 + 1 - a, ap_0 + 1 - a\} \in [q, r]$.
Using result (b), we have can find some (small enough) δ, ϵ such that

$$\frac{w(ap_1) - w(ap_2)}{a(p_1 - p_2)} \leq \frac{w(q) - w(p)}{q - p} \tag{23}$$

$$\frac{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)}{(ap_0 + 1 - a) - (ap_1 + 1 - a)} \geq \frac{w(r) - w(q)}{r - q} \tag{24}$$

Combining (21, (23) and (24) we have

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{p_1 - p_2}{p_0 - p_1} \tag{25}$$

Combining (22) and (25), we have

$$\frac{w(ap_1) - w(ap_2)}{w(ap_0 + 1 - a) - w(ap_1 + 1 - a)} < \frac{w(p_1) - w(p_2)}{w(p_0) - w(p_1)}$$

which contradicts (14), and completes the proof.

Theorem 4. *Suppose that axioms **A.1** through **A.4** and the RDU axioms hold, and let u and v be the utility functions associated with the resolved and unresolved lotteries, respectively, and w be the decision weight associated with the unresolved lotteries. In addition, suppose that u, v are both differentiable. Then:*
(i) If there exists a $p \in (0,1)$ such that $p < w(p)$, then the agent is doubt-prone somewhere. Similarly, if there exists $p' \in (0,1)$ such that $p' > w(p')$, then the agent is doubt-averse somewhere.
(ii) If the agent is doubt-averse everywhere, then $p \geq w(p)$ for all $p \in (0,1)$. Moreover, if u exhibits stronger diminishing marginal utility than v (i.e. $u = \lambda \circ v$ for some continuous, weakly concave, and increasing λ on $v([\underline{z}, \bar{z}])$), then $\succeq_N$ violates quasi-concavity. (that is, there exists some $f', f'' \in \mathfrak{L}_o$, and $\alpha \in (0,1)$ such that $f' \succ f''$ and $f'' \succ_N \alpha f' + (1-\alpha) f''$).
If the agent is doubt-prone everywhere, then $p \leq w(p)$ for all $p \in (0,1)$. Moreover, if v exhibits stronger diminishing marginal utility than u , then $\succeq_N$ violates quasi-convexity. (that is, there exists some $f', f'' \in \mathfrak{L}_o$, and $\alpha \in (0,1)$ such that $f' \succ f''$ and $\alpha f' + (1-\alpha) f'' \succ_N f'$).

Proof. (i) Suppose not, i.e. suppose that there exists $p \in (0,1)$ such that $p < w(p)$, and that $f \succeq \delta_f$ for all $f \in \mathfrak{L}_o$. Let $f_\epsilon = (z; 1-p; z+\epsilon, p)$ for some $z \in \mathbf{Z}$, $p \in \mathfrak{L}_o$, $0 < \epsilon < \bar{z} - z$. Since $f \succeq \delta_f$, by continuity (and using the certainty axiom), there exists a $\tilde{z}_\epsilon \in (z, z+\epsilon)$ such that $f \succeq \left[\delta_{\tilde{z}_\epsilon} \sim \delta_{\delta_{\tilde{z}_\epsilon}}\right] \succeq \delta_f$. Hence:

$$(1-p)u(z) + pu(z+\epsilon) \geq u(\tilde{z}_\epsilon)$$

$$w(p)\left(v(z+\epsilon) - v(z)\right) + v(z) \leq v(\tilde{z}_\epsilon)$$

Rearranging:

$$p \geq \frac{u(\tilde{z}_\epsilon) - u(z)}{u(z+\epsilon) - u(z)}$$

$$w(p) \leq \frac{v(\tilde{z}_\epsilon) - v(z)}{v(z+\epsilon) - v(z)}$$

Hence:

$$\frac{u(\tilde{z}_\epsilon) - u(z)}{u(z+\epsilon) - u(z)} - \frac{v(\tilde{z}_\epsilon) - v(z)}{v(z+\epsilon) - v(z)} \leq p - w(p)$$

But as $\epsilon \to 0$, $\frac{u(\tilde{z}_\epsilon)-u(z)}{u(z+\epsilon)-u(z)} \to \frac{u'(z)}{u'(z)}$, and $\frac{v(\tilde{z}_\epsilon)-v(z)}{v(z+\epsilon)-v(z)} \to \frac{v'(z)}{v'(z)}$, by differentiability. Since the left-hand-side goes to $1-1=0$ in the limit, while the right-hand-side does not change, it must be that $0 \leq p - w(p)$. But this is a contradiction, since $p < w(p)$.
The second part of the result can be proved in a similar manner, for the case $p' > w(p')$.
(ii) The result is only shown for doubt-aversion; a similar reasoning holds for doubt-proneness. By the contrapositive of (i), it is immediate that if $f \succeq \delta_f$ for all $f \in \mathfrak{L}_o$, then $w(p) \leq p$ for all $p \in (0,1)$. Now suppose that $f \succ \delta_f$ for some f, and that u is a (weakly) concave transformation of v. If w is not concave, then $\succeq_N$ cannot be quasi-concave, by Wakker (1994) theorem 25. Since $w(0) = 0$, $w(1) = 1$, $w(p) \geq p$ for a concave function. We have that $w(p) \leq p$, and so it suffices to show that $w(p) < p$ for some p. Suppose not. That is, $w(p) = p$ for all p. Since u is more concave than v, it must be that $u^{-1}(EU(f)) \leq v^{-1}(EV(f))$(that is, the certainty equivalent of f for the informed lotteries is not bigger than the certainty equivalent of f for the unresolved lotteries, by a well known result). However, since $f \succ \delta_f$, it must also be that $u^{-1}(EU(f)) > v^{-1}(EV(f))$, which is a contradiction.
Note that if $f \sim \delta_f$ for all $f \in \mathfrak{L}_o$, than trivially, u is a linear transformation of v, and $w(p) = p$.

Corollary 4.1. *Proof.* To prove (i) $\Rightarrow$ (ii):If $\succeq_N$ displays mean-preserving risk-aversion, then $w(p)$ is convex, by Chew, Epstein and Safra (1986) or Grant, Kajii and Polak (2000). Since $w(0) = 0$, $w(1) = 1$, it must be that $p \geq w(p)$. Since $\delta_f \succeq f$, it follows from result (ii) that $p \leq w(p)$. Hence $w(p) = p$, implying that $\succeq_N$ satisfies expected utility.
Since $\delta_f \succeq f$ for all $f \in \mathfrak{L}_o$, and both u and v are of EU form, u must be a concave transfor-

mation of v. This is well-known, see for instance Kreps-Porteus (1978).
The other direction, (ii) $\Rightarrow$ (i), is trivial: if u and v are concave then they both display mean-preserving risk aversion by well known results, and if u is a concave transformation of v then $\delta_f \succeq f$ for all $f \in \mathfrak{L}_\circ$.

Theorem 5.

Proof. If $u(z) = v(z)$ for all $z \in \mathbf{Z}$, then $\delta_f \succeq f$ if and only if

$$u(z_1) + \sum_{i=2}^{m}[u(z_i) - u(z_{i-1})]w(p_i^*) \geq \sum_{i=1}^{m} u(z_i)p(z_i) \tag{26}$$

$$\Leftrightarrow u(z_1) + \sum_{i=2}^{m}[u(z_i) - u(z_{i-1})]w(p_i^*) \geq u(z_1) + \sum_{i=2}^{m}[u(z_i) - u(z_{i-1})]p_i^* \tag{27}$$

$$\Leftrightarrow \sum_{i=2}^{m}[u(z_i) - u(z_{i-1})](w(p_i^*) - p_i^*) \geq 0. \tag{28}$$

This expression is always true if and only if $w(p) \geq p$ for all $p \in [0, 1]$. For the agent to be doubt-prone, the inequality in (28) must be strict somewhere, hence $w(p) > p$ for some $p \in (0, 1)$. Now suppose $u = \lambda \circ v$ for some continuous, weakly concave and increasing λ. By theorem 4, the agent is doubt-prone everywhere only if $p \leq w(p)$. Now suppose that $w(p) > p$. Then using the same argument as above, we have:

$$v(z_1) + \sum_{i=2}^{m}[v(z_i) - v(z_{i-1})]w(p_i^*) \geq \sum_{i=1}^{m} v(z_i)p(z_i). \tag{29}$$

Hence:

$$u\left(v^{-1}\left(v(z_1) + \sum_{i=2}^{m}[v(z_i) - v(z_{i-1})]w(p_i^*)\right)\right) \geq u\left(v^{-1}\left(\sum_{i=1}^{m} v(z_i)p(z_i)\right)\right). \tag{30}$$

But by concavity of $u\left(v^{-1}(\cdot)\right)$, we know that

$$u\left(v^{-1}\left(\sum_{i=1}^{m} v(z_i)p(z_i)\right)\right) \geq \sum_{i=1}^{m} u(z_i)p(z_i), \tag{31}$$

with strict inequality somewhere, hence the agent is doubt-prone everywhere. This completes the proof.

Preservation of self-image. For an agent who is doubt-prone and risk-averse for both resolved and unresolved lotteries, the following holds:

$$C(e) \equiv u \circ v^{-1}(Ev(t)) - \sum_m p(m|e) u \circ v^{-1}(Ev(t|m,e)) \geq 0$$

Proof. Note that $u \circ v^{-1}$ is concave. Hence

$$\begin{aligned}
&\sum_m p(m|e) u \circ v^{-1}(Ev(t|m,e)) \leq u \circ v^{-1}\left(\sum_m p(m|e)(Ev(t|m,e))\right) \\
&\leq u \circ v^{-1}\left(\sum_m p(m|e) \sum_t \frac{p(m|t,e)p(t)}{p(m|e)} v(t)\right) \leq u \circ v^{-1}\left(\sum_m \sum_t p(m|t,e)p(t)v(t)\right) \\
&\leq u \circ v^{-1}\left(\sum_t \sum_m p(m|t,e)p(t)v(t)\right) \leq u \circ v^{-1}\left(\sum_t p(t)v(t)\right) = u \circ v^{-1}(Ev(t))
\end{aligned}$$

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