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Abstract

This paper examines the frequency of new product introductions in monopoly markets where demand is subject to temporary satiation. Consumers' taste for diversity is satisfied over time as new varieties are introduced to the market. If two varieties are introduced in consecutive periods then they become imperfect substitutes and the firm has an incentive to raise prices and sell each one to consumers with higher average valuations (better preference matching). Higher frequency can also generate market expansion. However, under strong temporary satiation, better preference matching may dominate and the frequency of new product introductions may become socially excessive.

JEL Classification numbers: L12, L13

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1 Introduction

The purpose of product diversity is to better match the preferences of heterogenous consumers at a point in time, as well as to satisfy individual consumers' taste for variety over time. In markets for leisure goods -such as books, music recordings, movies, computer games, concerts, etc.-, consumers tend to purchase only one unit of a particular variety, but engage in repeat purchases in the same product category as new varieties become available. Indeed, in most of these markets, commercialization and consumption are highly synchronized: most purchases are typically made immediately following the release of a new variety. For example, approximately 40 per cent of US cinema box-office revenues are taken during a movie's first week and very few movies generate significant revenue beyond the sixth week.[1,2] Another important characteristic of these markets is that demand is subject to temporary satiation. That is, consumption of the current variety reduces demand for the subsequent variety.[3] This begs the question as to whether suppliers have an incentive to introduce new varieties too quickly or too slowly.

Most studies of product diversity have analyzed static models, where different varieties appeal to different consumer groups.[4] This paper does not consider *static* product diversity but instead studies the market provision of *dynamic* product diversity, which is an area that, to the best or my knowledge, remains largely unexplored. It might be argued that, in examining a narrow segment of the market (for example, romantic comedies produced by big Hollywood studios, or historical novels released by major publishing houses), the static dimension becomes less relevant, whilst the dynamic aspect emerges more clearly.

This paper presents a dynamic model in which a single producer sequentially supplies different varieties of a non-durable good. Both the monopolist and consumers are infinitely-lived and form rational expectations. Consumers are ex-ante identical, but heterogeneous ex-post: they have random, variety-specific preferences. Each consumer purchases a maximum of one unit of each variety. The valuation of a new variety by a consumer who has not consumed the

[1] In this particular example the synchronization between commercialization and consumption is also influenced by two other factors. First, the heavy advertising campaigns leading up to the release date. Second, consumption externalities play a big role: consumers prefer to purchase a particular variety when aggregate consumption hits its highest level. The current model ignores these two factors but some of the assumptions generate the same effects.

[2] For a useful discussion of common practices and stylized facts in the motion picture industry, see Corts (2001), Krider and Weinberg (1998), and Einav (2007).

[3] Einav (2009) has shown that box office revenues would increase if film distributors did not cluster their releases so much.

[4] Prominent examples include Salop (1979)'s circular city model or Chen and Riordan (2007)'s spokes model. Another workhorse model of product differentiation, the Spence-Dixit-Stiglitz model (Spence, 1976; and Dixit and Stiglitz, 1977), describes consumer decisions as emanating from a 'representative consumer' with a preference for diversity. But this is only meant as a modeling short-cut rather than as a literal representation of individual consumer behavior.

previous variety in the last period is distributed uniformly over the unit interval. If the consumer did consume in the last period, their valuation is zero with some exogenous probability; whilst with the complementary probability, it is distributed uniformly over the unit interval. Hence, satiation occurs with some probability and lasts for one period.

By assuming that consumers are ex-ante identical we avoid Coasian price dynamics, analogous to those studied in durable goods markets. However, it is crucial that consumers are heterogeneous ex-post, since this implies that monopoly power generates the standard static price distortion. The cost of introducing a new variety is independent of the time that has elapsed since the last variety was introduced. As a result, the timing of product introductions is entirely demand-driven: it depends only on the speed at which consumers can absorb new varieties. The degree of product differentiation between two consecutive varieties is exogenous, which is the main reason for restricting the analysis to the monopoly case. A full analysis of competition would require an explicit examination of how product characteristics are sequentially selected by different firms, in which case we would necessarily have to consider static diversity. The analysis of competition is obviously very relevant, but is beyond the scope of this paper.

The principal research question is therefore the following: How does the monopoly's equilibrium frequency of introduction of new products compare with the socially-optimal frequency? Given that satiation lasts for only one period, the question can be reduced to analyzing whether new varieties are introduced each period (high frequency) or every other period (low frequency).

In a low-frequency equilibrium, any two varieties are independent; consumption of a particular variety does not affect the expected utility of the next available variety. As a result, prices and consumer behavior coincide with those of the 'one-shot' equilibrium. In contrast, in a high-frequency equilibrium, two consecutive varieties are imperfect substitutes. Consider the extreme version of the model in which the monopolist does not discount the future and consumers are satiated, with probability of one, if they consumed the previous variety. In this case, one sale today exactly crowds out one sale tomorrow. Also, if a consumer declines to make a purchase today they may still make a purchase tomorrow. In other words, the monopolist has two independent opportunities to make a sale. Since consumer preferences are variety-specific, then the monopolist faces a more homogeneous demand: the highest of two independent draws is less dispersed than a single draw.[5] As a result, the monopolist finds it optimal to set a higher price than when in a low-frequency equilibrium and sell each variety to consumers with higher average valuations. Clearly, the same intuition applies to the general model, although discounting and

[5] I am grateful to an anonymous referee for suggesting several intuitions, including this one.

the probability of no satiation reduces the degree of substitutability between two consecutive varieties.

In static models it has been shown (see, for instance, De Meza and Von Ungern-Stenberg, 1982) that a monopolist may provide excessive or insufficient variety with respect to the social optimum. The reason for this is that a new variety generates market expansion as well as better preference matching. If the first effect dominates, then a non-discriminating monopolist is unable to capture all the surplus generated by the new variety and thus has less incentives than the social planner to introduce a new variety. If the second effect dominates then the monopolist can sell each variety to a more homogeneous consumer group, and hence charge a higher price. Thus, the additional variety enables the monopolist to capture a larger share of the surplus from the infra-marginal varieties. As a result, the extra profit may exceed the incremental social value, generating excessive variety.

Similar effects are present in the model adopted in this paper. As a result, the equilibrium can also be characterized by insufficient, or excessive, dynamic product variety.[6] The probability of temporary satiation is the crucial determinant of the balance between market expansion and better preference matching. If temporary satiation is weak, then two varieties introduced in consecutive periods are poor substitutes. Thus, shifting from low- to high-frequency raises consumer surplus, since consumers can enjoy new varieties more often at similar prices (market expansion dominates). In this case, the monopolist has incentives to introduce new varieties more slowly than the social planner. However, as temporary satiation becomes stronger then, along a high-frequency path, the monopolist charges higher prices and is able to appropriate a larger share of the surplus in all periods. As a result, the incentives of the monopolist and the social planner are better aligned. If temporary satiation is sufficiently strong, then scenarios will exist in which the better preference matching effect dominates and equilibrium frequency of introduction of new products is socially excessive.

Temporary satiation in non-durable goods is somewhat analogous to depreciation or quality improvements in durable goods, in the sense that they both induce repeat purchases and generate a negative link between past purchases and current demand. If sellers introduce quality improvements over time then we can also consider the optimality of its frequency.[7] The litera-

[6] In the baseline model the equilibrium frequency of introduction of new products is unambiguously too slow with respect to the second-best benchmark. However, for other welfare benchmarks, as well as for various extensions of the baseline model, the equilibrium frequency may be too slow or too fast, depending on parameter values.

[7] The literature on depreciating durable goods has focused on very different issues; for instance, on the role of replacement sales in preventing the Coase conjecture (Bond and Samuelson, 1984; Driskill, 1997), or the effect of scrapping subsidies (Adda and Cooper, 2000).

ture on product innovation (quality upgrading) in durable goods, in line with the results of this paper, has shown that a monopolist may introduce more upgrades than is socially optimal (see, for instance, Waldman, 1993; Choi, 1994; Ellison and Fudenberg, 1998). These papers present two-period models and focus on network externalities and compatibility between old and new models. Their results are hardly comparable with those of the present model.

The study most closely related to this one is Fishman and Rob (2000). They examine the frequency of innovations generated by a durable-good monopolist in an infinite-horizon framework, with homogeneous consumers. They show that a monopolist introduces new products too slowly with respect to the social optimum (at least in the case of no planned obsolescence). The reason for this is that innovations are cumulative. Hence, current innovation efforts have a positive effect on all subsequent models, but consumers are willing to pay only for the incremental flow of services that the current model provides. The two models differ in two important respects. First, the current model allows for (ex-post) heterogeneous consumers; hence the static price distortion becomes a crucial ingredient of the analysis. Second, it examines the provision of different varieties of a non-durable good, instead of quality upgrades of a durable good. Consequently, it can be safely assumed that innovation efforts are independent over time (in contrast to the notion of cumulative innovations). Finally, independent, variety-specific preferences look more plausible in the context of horizontal rather than vertical differentiation.

The plan of the paper is as follows. Section 2 presents the baseline model, in which consumers and the firm discount the future at the same rate. Some preliminary results are derived in section 3. Sections 4, 5 and 6 analyze the full equilibrium for the baseline model. Section 7 examines the case of myopic consumers, and section 8 offers some concluding remarks and discusses further extensions.

2 The baseline model

An infinitely-lived monopolist sequentially supplies different varieties of a non-durable good. In each period the monopolist can introduce a new variety by incurring a fixed cost, γ. Any amount of the variety can then be produced at a constant marginal cost, which is normalized to 0. Each time a new variety is introduced, previous varieties cease to be available. The monopolist chooses both the timing of introduction of new varieties and their prices in order to maximize the expected discounted value of profits. Time periods are indexed by t, $t = 0, 1, 2...$ and the discount factor is denoted by δ, $\delta \in [0, 1)$.

There is a mass one of infinitely-lived consumers with history-dependent preferences. Each consumer purchases, at most, one unit of each variety and the utility derived from each consumption episode is variety-specific. In particular, if consumer i did not consume in period $t-1$, then their valuation of the variety introduced in period t, r_{it}, is a random variable, uniformly distributed in the interval $[0,1]$ (distribution NC). Alternatively, if consumer i did consume in period $t-1$, then r_{it} is a realization of distribution C, such that:

$$r_{it} = \begin{cases} 0, & \text{with probability } 1-\mu \\ \sim \text{uniform on } [0,1], & \text{with probability } \mu \end{cases}$$

where $\mu \in [0,1)$ is a fixed parameter. Valuations of those consumers who do not purchase the new variety immediately remain constant over time, until either they purchase one unit of this variety or a different variety is introduced.

Conditional on no consumption in $t-1$, a consumer's expected valuation of a new variety in period t is $\frac{1}{2}$. However, if a consumer did consume in $t-1$ then their expected valuation is $\frac{\mu}{2} < \frac{1}{2}$. Thus, $1-\mu$ measures the intensity of consumers' temporary satiation. Note that, for simplicity, temporary satiation lasts only for one period.

Distributions of valuations and past purchases are common knowledge. Thus, at the beginning of each period, based exclusively on this information, the monopolist decides whether or not to introduce a new variety. If it does, then it also announces the (constant) price prevailing throughout the variety's life span (until a new variety is introduced). Immediately following the launch of a new variety, consumers learn their individual valuations of the current variety, as well as its price, and make their purchasing decisions. If there is no product innovation during that period then consumers can still access the last variety at the price announced at the moment of its launching.[8]

Consumers are ex-ante identical but heterogenous ex-post. They all obtain the same long-run expected utility but they disagree about the value of each specific variety. The assumption of independent draws over time (except for the satiation effect) encapsulates the idea that consumers cannot foresee the order in which future varieties will be introduced and learn their characteristics only when they become available. Such ex-post heterogeneity implies that, at any point in time, the monopolist faces a continuous, downward sloping demand function.

Let us denote by x_t the fraction of consumers that are not satiated in period t; i.e., those who obtain a positive realization of r_{it}. Clearly, x_t will depend on x_{t-1} and consumption in $t-1$, in a way that will be specified below. Thus, x_0 is one of the exogenous parameters of the model,

[8] More generally, if the firm can set, when the new variety is introduced, the sequence of prices prevailing during its life span, then a constant price is an optimal commitment policy, and hence all results would be identical. Moreover, in section 8 I argue that, in the absence of any short-run commitment power, the main qualitative results of the paper would still remain largely unaffected. If anything, the main message would be reinforced.

but x_t, for all $t > 0$, are endogenous variables. For simplicity, attention is restricted to the case of $x_0 = 1$. It will become apparent later that considering an arbitrary value of $x_0 \in [0, 1]$ would complicate the analysis without generating any significant insight.

Consumer i obtains a net surplus of $r_{it} - p_t$ if she chooses to consume in period t at a price p_t. Otherwise she obtains 0. Consumers take their consumption decisions in order to maximize the expected discounted value of their net surpluses and, unless explicitly indicated, they discount the future using the same discount factor, δ.

In summary, this is a model with three independent parameters, δ, μ, γ. The fourth parameter, x_0, has been fixed equal to 1.[9] For future reference we will define $\omega \equiv \delta (1 - \mu)$. Such a combination of parameters will be an important determinant of equilibrium variables, indicating that strong temporary satiation (low values of μ) will have a substantial effect on equilibrium variables only if agents are forward-looking and value the future sufficiently (high δ).

3 Preliminary analysis

The role of some key assumptions. Some of the specific assumptions presented in the previous section have important implications that facilitate the analysis considerably. Firstly, the assumption of constant prices for each variety avoids intra-variety Coasian dynamics and implies that, in equilibrium, there is perfect synchronization between commercialization and consumption.[10] Secondly, the assumption of ex-ante identical consumers prevents the emergence of inter-variety Coasian dynamics. That is, if a new variety is introduced every period, since all consumers share the same perception of the future, then the monopolist is concerned only with the fraction of unsatiated consumers, but not with the composition of current demand in terms of (ex-ante) consumer types. As a result, it is possible to construct an equilibrium in which all varieties are sold at the same price. Third, the assumption of a uniform distribution allows us to compute closed-form solutions.

Interpretation of the model. Given the above, the current model aims at understanding how the frequency of introduction of new varieties is determined in markets in which consumers typically purchase a single unit of each variety, but in which the recent history of purchases affect the valuation of new varieties -consider, for example, the case of films and books. Most of these markets exhibit a high correlation between commercialization and consumption.[11] The model

[9] Note that I use Greek letters to denote parameters and Latin letters to denote endogenous variables.

[10] Section 8 examines this issue in more detail.

[11] In the case of music recordings and computer games consumers typically make a repeated use of each variety. To the extent that most of these consumption episodes occur right after the acquisition of the good, then the model can be thought of as fitting these markets as well.

is sufficiently abstract to ensure that the characteristics of subsequent varieties (and therefore the 'distance' between them) need not be made explicit. Consequently, alternative interpretations are also possible. For example, the model can also capture the case of highly perishable goods (live performances) that are available exclusively at particular points in time. Whether or not the goods supplied at various moments are physically different is not important under this interpretation. Finally, those markets where essentially the same variety is continuously available, but consumers experience temporary satiation (for example, amusement parks, restaurants, tourist destinations), also match the preference structure relatively well, whilst other aspects of the model considerably less so.

The finite-horizon game. The (subgame perfect) equilibrium properties of the finite horizon games will be used to construct the candidate equilibria of the infinite horizon game.

After an inactive period, that is, a period with no product innovation (and no consumption), all consumers will have recovered from their previous consumption episodes. Hence, history will not matter any more and the economy will revert to its initial condition, $x = 1$. Also, in the period before the inactive period, consumers and the firm care only about their short-run payoffs, since their current decisions will not affect their continuation utility. Hence, the main building block of our analysis will be a finite horizon game in which some consecutive innovative periods (a new variety is introduced in each of them) are followed by one inactive period (no innovation). In particular, a variable with superscript n (in contrast to subscripts that refer to calendar dates) will denote the equilibrium value of such variable if agents expect the current innovative period to be followed by n consecutive innovative periods, $n \geq 0$. Hence, according to this convention, period n is followed by periods $n-1, n-2, ..., 0$.

Period 0. In the last period all agents anticipate that their current decisions cannot affect the future, and hence they will simply maximize their short-run payoffs. More specifically, consumers will purchase the good if, and only if, their valuations are higher than the current price, $r_i^0 \geq p$. That is, consumers' threshold value is $\overline{r}^0(p) = p$. Given such consumer behavior, the firm will choose p in order to maximize short-run profits, $x^0(1-p)p$, and hence will optimally choose $p^0 = \frac{1}{2}$. Thus, the firm's continuation value at the beginning of period 0 is $\Pi^0(x^0) = \frac{x^0}{4} - \gamma$. A consumer's continuation value at period 0 will depend on whether or not she has consumed during the previous period. In particular, if a consumer has not purchased the good in period 1 then her continuation value in period 0 is $U_{NC}^0 = \int_{\frac{1}{2}}^{1}(r_i^0 - \frac{1}{2})dr_i^0 = \frac{1}{8}$. In contrast, if she did consume in the previous period then her continuation value is smaller, $U_C^0 = \frac{\mu}{8}$, since with probability $1-\mu$ the consumer will be satiated and hence stay out of the market. It is important to note that neither the price nor consumer behavior depends on the value of the state variable, x^0.

Consumer behavior in period n, $n > 0$. The notation introduced above can no be generalized. Let U_C^{n-1} and U_{NC}^{n-1} denote the consumers' continuation value at the beginning of period $n-1$, conditional on having and not having consumed, respectively, in period n. Consumer i's payoff, if she chooses to purchase the good in period n, is $r_i^n - p + \delta U_C^{n-1}$. Alternatively, if consumer i does not purchase the good, then her payoff is δU_{NC}^{n-1}. Notice that both U_C^{n-1} and U_{NC}^{n-1} are independent of r_i^n. Hence, all consumers adopt the same optimal decision rule: purchase the good if, and only if, $r_i^n \geq \overline{r}^n(p) = p + g^n$, where g^n (the gap) is given by $g^n = \delta\left(U_{NC}^{n-1} - U_C^{n-1}\right)$. That is, g^n represents consumers' option value of waiting; by declining to purchase the current variety a consumer increases her future expected utility by an amount g^n. If we let p^n be the equilibrium price and $\overline{r}^n$ the value of the threshold in equilibrium, $\overline{r}^n = \overline{r}^n(p^n)$, then:

$$g^n = \overline{r}^n - p^n. \tag{1}$$

Continuation values can be written in a recursive form:

$$U_{NC}^{n-1} = CS(p^{n-1}, \overline{r}^{n-1}) + \overline{r}^{n-1}\delta U_{NC}^{n-2} + \left(1 - \overline{r}^{n-1}\right)\delta U_C^{n-2}$$

$$U_C^{n-1} = \mu U_{NC}^{n-1} + (1-\mu)\,\delta U_{NC}^{n-2}$$

where $CS(p^{n-1}, \overline{r}^{n-1}) = \int_{\overline{r}^{n-1}}^{1}\left(r_i^{n-1} - p^{n-1}\right) dr_i^{n-1}$. That is, if a consumer did not purchase the good in period n, then with probability one they will obtain a positive valuation in period $n-1$, and hence their short-run expected utility is given by $CS(p^{n-1}, \overline{r}^{n-1})$. Also, with probability $\overline{r}^{n-1}$ they do not purchase and their discounted continuation value is δU_{NC}^{n-2}. With the complementary probability, $1 - \overline{r}^{n-1}$, they purchase the good and their discounted continuation value is δU_C^{n-2}. Alternatively, if the consumer did purchase the good in period n, with probability μ, their expected payoff will be exactly the payoff of those consumers who abstained in period n and, with probability $1-\mu$, their valuation will be zero and their discounted continuation value will be δU_C^{n-2}.

If we subtract these two equations and use equation (1) we obtain:

$$g^n = \omega\left[CS(p^{n-1}, \overline{r}^{n-1}) - \left(1 - \overline{r}^{n-1}\right)g^{n-1}\right] = \frac{\omega}{2}\left(1 - \overline{r}^{n-1}\right)^2 \tag{2}$$

If consumers expect the next period to be innovative then they become more selective than in the case when the next period is inactive, and only purchase the good if their current valuations are sufficiently higher than the price. In other words, because of temporary satiation, current and future varieties become imperfect substitutes and hence consumers are willing to give up some current surplus in exchange for a higher future surplus.

The firm's optimization problem. Suppose that in period n the firm expects consumers to behave according to $\overline{r}^n(p) = p + g^n$, where g^n is independent of p. In this case, the firm's objective function is $\left\{x^n(1-g^n-p)p - F + \delta\Pi^{n-1}\left(x^{n-1}\right)\right\}$, where $\Pi^{n-1}\left(x^{n-1}\right)$ is the firm's continuation value at the beginning of period $n-1$. Crucially, x^{n-1} depends on the current price, p. More specifically, if a mass $x^n(1-g^n-p)$ of consumers purchase in period n then a fraction $(1-\mu)$ of them will be out of the market during the next period and hence the transition function will be:

$$x^{n-1} = 1 - (1-\mu)(1-g^n-p)x^n \tag{3}$$

The optimal price is characterized by the first-order condition of the firm's optimization problem:

$$1 - g^n - 2p^n + \omega k^{n-1} = 0 \tag{4}$$

where $k^{n-1} = \frac{d\Pi^{n-1}}{dx_{n-1}}$. By raising the price above the short-run profit maximizing level, $\frac{1-g^n}{2}$, the firm incurs short-run losses, but enhances future profits by raising the fraction of non-satiated consumers.

The current optimal price will thus depend on k^{n-1}. By the envelop theorem:

$$k^n = (1-\overline{r}^n)\left(p^n - \omega k^{n-1}\right) \tag{5}$$

and by taking into account the first-order condition (4):

$$k^n = (1-\overline{r}^n)^2 \tag{6}$$

That is, if g^n is independent of the current price, then p^n only depends on $\overline{r}^{n-1}$ and g^n, which in turn also depends on $\overline{r}^{n-1}$. Once again, backward induction will be crucial in order to complete the argument. Since $\overline{r}^0$ is independent of x^0, then g^1 is also independent of x^0. Also, we have shown above that $k^0 = \frac{1}{4}$. Hence, from equation (4), p^1 is independent of both x^1 and x^0, and from equation (1) $\overline{r}^1$ is also independent of x^1 and x^0. We can apply this argument iteratively and conclude that in equilibrium g^n is independent of $\left(x^{n-1}, x^{n-2}, ..., x^0\right)$ and the current price, and hence equilibrium is fully characterized by equations (1), (2), (4) and (6), which can be conveniently rewritten as:

$$\overline{r}^n = \frac{1}{2} + \frac{3\omega}{4}\left(1-\overline{r}^{n-1}\right)^2 \tag{7}$$

$$p^n = \frac{1}{2} + \frac{\omega}{4}\left(1 - \overline{r}^{n-1}\right)^2 \qquad (8)$$

Summarizing this discussion:

Lemma 1 *In the unique subgame perfect equilibrium prices and consumer behavior are given by $\overline{r}^0 = p^0 = \frac{1}{2}$, and for $n > 1$ by equations (7) and (8).*

Equilibrium in period 1. For future reference it we will be useful to point out the equilibrium values in period $1: p^1 = \frac{1}{2} + \frac{\omega}{16}, g^1 = \frac{\omega}{8}, k^1 = \left(\frac{1}{2} - \frac{3\omega}{16}\right)^2$.

The limit of the finite-horizon game as n goes to infinity. Also, it is crucial to consider the limiting values of these variables as n goes to infinity, since they will be relevant to construct, for the infinite-horizon game, equilibria where the monopolist introduces a new variety every period. Notice that the difference equation (7) defines an oscillatory trajectory that converges to $\overline{r}^{\infty}$, which is given by:

$$\overline{r}^{\infty} = \frac{2 + 3\omega - \sqrt{4 + 6\omega}}{3\omega}$$

Equation (8) indicates that p^n also follows an oscillatory trajectory that converges to:

$$p^{\infty} = \frac{2 + 6\omega - \sqrt{4 + 6\omega}}{9\omega}$$

Obviously, $\overline{r}^{\infty} > p^{\infty}$, for $\omega > 0$. Also, if $\omega = 0$ then $p^{\infty} = \overline{r}^{\infty} = \frac{1}{2}$ and both increase with ω.

Finally, it is clear from equation (6), that k^n also converges to k^{∞}. If $\omega = 0$ then $k^{\infty} = \frac{1}{4}$ and it decreases with ω. Instead of providing the exact expression for k^{∞} as a function of ω, it will be more helpful to offer an alternative characterization. Evaluating equation (5) at $k^n = k^{n-1} = k^{\infty}$ we obtain:

$$k^{\infty} = \frac{\left(1 - \overline{r}^{\infty}\right) p^{\infty}}{1 + \omega\left(1 - \overline{r}^{\infty}\right)}$$

That is, the marginal effect of the current state variable on the firm's value can be expressed as the adjusted value of the short-run profits generated. The adjustment is due to the fact that a higher value of the current state variable will involve lower sales in the future.

The firm's value can be written as a function of the current state variable:[12]

$$\Pi^{\infty}(x) = xk^{\infty} + \frac{\delta k^{\infty} - \gamma}{1 - \delta}$$

[12]The continuation value in period t is $\Pi(x_t) = \sum_{s=0}^{\infty} \delta^s x_{t+s}\left[\left(1 - \overline{r}^{\infty}\right) p^{\infty} - \gamma\right]$, where $x_{t+s} = 1 - (1 - \mu)\left(1 - \overline{r}^{\infty}\right) x_{t+s-1}$.Operating and dropping the subscript of the initial period we obtain the expression in the main text.

Low-frequency equilibria. For the infinite-horizon game I will characterize equilibria in which a new variety is introduced every other period (low frequency, L). In this case, in all innovative periods $p = p^0 = \overline{r} = \overline{r}^0 = \frac{1}{2}$. The firm's continuation value, $\Pi^L(x)$, is:

$$\Pi^L(x) = \Pi^0(x) + \frac{\delta^2 \Pi^0(1)}{1-\delta^2} = \frac{x}{4} - \gamma + \frac{\delta^2\left(\frac{1}{4} - \gamma\right)}{1-\delta^2}.$$

Finally, total consumer surplus in a low-frequency equilibrium, $U^L(x)$, is:

$$U^L(x) = xCS\left(\overline{r}^0, p^0\right) + \frac{\delta^2 CS\left(\overline{r}^0, p^0\right)}{1-\delta^2} = \frac{x}{8} + \frac{\delta^2 \frac{1}{8}}{1-\delta^2}$$

High-frequency equilibria. I will also study high-frequency equilibria (H) in which a new variety is introduced every period, and where consumer and firm behavior are the limit of the finite-horizon game as n goes to infinity. Notice that in a high-frequency equilibrium the monopolists sells each variety to relatively more homogeneous consumers than in a low-frequency equilibrium; their valuations are uniformly distributed over $[\overline{r}^\infty, 1]$, where $\overline{r}^\infty > \frac{1}{2}$. As a result, it charges a higher price: $p^\infty > \frac{1}{2}$. The firm's continuation value along such equilibrium, $\Pi^H(x)$, is:

$$\Pi^H(x) = \Pi^\infty(x)$$

Total expected consumer surplus can also be written in a similar fashion:

$$U^H(x) = xu^\infty + \frac{\delta u^\infty}{1-\delta}$$

where:

$$u^\infty = \frac{CS\left(\overline{r}^\infty, p^\infty\right)}{1 + \omega\left(1 - \overline{r}^\infty\right)}$$

4 The infinite-horizon game: high-frequency equilibria

In this section a Markov perfect equilibrium is constructed, where a new variety is introduced every period. While the existence of multiple (high-frequency) equilibria cannot be ruled out, the proposed equilibrium can be understood as an equilibrium selection based on the finite-horizon model. In fact, it can be shown that this is the only equilibrium in which all varieties are equally priced.

Let us denote by q_t the variable that indicates whether or not a new variety is introduced in period t; i.e.,$q_t \in \{0, 1\}$, $q_t = 1$ indicates that t is an innovative period and $q_t = 0$ that t is an inactive period.

Since x is the fraction of unsatiated consumers in the current period, we can write the monopolist's strategy as follows:

$$\begin{cases} \text{If } x \in [\overline{x}, 1], q(x) = 1 \text{ and } p(x) = p^{\infty} \\ \text{If } x \in [0, \overline{x}), \qquad\qquad q(x) = 0 \end{cases} \tag{9}$$

where $\overline{x} = \frac{\gamma}{k^{\infty}}$. Similarly, suppose consumers adopt the following decision rule:

$$\overline{r}(p, x) = \begin{cases} p + g^{\infty} \text{ if } p \geq \widetilde{p}(x) \\ p, \qquad \text{otherwise} \end{cases} \tag{10}$$

where $\widetilde{p}(x) = 1 - g^{\infty} - \frac{1-\overline{x}}{(1-\mu)x}$. The value of $\overline{x}$ is given implicitly by the equation $\Pi^{H}(\overline{x}) = \delta\Pi^{H}(1)$. Hence, since $\Pi^{H}(x)$ increases with x then, conditional on the price p^{∞} and consumer behavior given by $\overline{r}(p, x) = p + g^{\infty}$, the firm finds it optimal to introduce a new product if, and only if, $x \geq \overline{x}$. Along the equilibrium path it must be the case that the state variable is always higher than $\overline{x}$. If the firm sets the price p^{∞} and consumers behave according to $\overline{r}(p, x) = p + g^{\infty}$, and since the transition function is (analogous to (3)):

$$x_{t+1} = 1 - (1-\mu)(1 - g^{\infty} - p)\, x_t \tag{11}$$

then the lowest value of x_t is $x_1 = 1 - (1-\mu)(1 - g^{\infty} - p^{\infty})$. Condition $x_1 \geq \overline{x}$ is equivalent to $p^{\infty} \geq \widetilde{p}(1)$. Since $\widetilde{p}(x)$ increases with x, then p^{∞} is higher than $\widetilde{p}(x)$ for all x.

Hence, if the firm restricts itself to $p \geq \widetilde{p}(x_t)$ then, for all t, $x_t \geq \overline{x}$, and according to the results of section 3, consumers' optimal behavior is given by $\overline{r}(p, x) = p + g^{\infty}$. Also, the optimal price conditional on $p \geq \widetilde{p}(x)$ and $\overline{r}(p, x) = p + g^{\infty}$ is p^{∞}.

Consumer behavior must be optimal not only along the equilibrium path, but also for any value of the state variable and at any price. Given the transition function (11), then $x_{t+1} \geq \overline{x}$ if, and only if, $p \geq \widetilde{p}(x)$. Hence, if $p \geq \widetilde{p}(x)$ consumers' optimal response is $\overline{r}(p, x) = p + g^{\infty}$. Finally, if $p < \widetilde{p}(x_t)$ then consumers should expect that the next period is inactive, $x_{t+1} < \overline{x}$. Hence, in this case, the optimal response is $\overline{r}(p, x) = p$.[13]

The proposed strategies will form a Markov perfect equilibrium only if some additional requirements are fulfilled. In particular, γ must be such that x_1 is higher than $\overline{x}$. Moreover, we need to ensure that the monopolist does not have incentives to deviate by setting a price below $\widetilde{p}(x)$, in order to try to exploit consumers' higher propensity to purchase. The Appendix shows how to compute the threshold value, γ^{H}, that defines the range of values of γ for which a high-frequency equilibrium exists.[14]

[13] It is important to notice that in the current formulation all consumers face the same price. This is why they can use the current price to make inferences about future product development plans. As discussed in Section 8, if consumers were exposed to a distribution of prices, then the inference problem could dramatically change.

[14] The next section will discuss how different parameters affect the threshold value γ^{H}.

Proposition 2 *There exists a threshold value $\gamma^H > 0$, such that strategies (9) and (10) support high frequency as a Markov perfect equilibrium if, and only if, $\gamma \leq \gamma^H$.*

5 The infinite-horizon game: low-frequency equilibria

In this section, a Markov perfect equilibrium is constructed in which the monopolist introduces a new variety every other period.[15] Consider the following strategy for the monopolist:

$$\begin{cases} \text{If } x \in [\overline{x}, 1], q(x) = 1 \text{ and } p(x) = \frac{1}{2} \\ \text{If } x \in [0, \overline{x}), \qquad q(x) = 0 \end{cases} \tag{12}$$

where $\overline{x} = \max\left\{\frac{2}{3-\mu}, \frac{\delta+4\gamma}{1+\delta}\right\}$. Similarly, suppose consumers adopt the following decision rule:

$$\overline{r}(p,x) = \begin{cases} p, \text{ if } x \in [\overline{x}, 1], \text{ and } p < \overline{p}(x) \\ p + g^1, \text{ if } x \in [\overline{x}, 1], \text{ and } p \geq \overline{p}(x) \\ p, \text{ if } x \in [0, \overline{x}), \text{ and } p \leq \overline{\overline{p}}(x) \\ p + g^1, \text{ if } x \in [0, \overline{x}), \text{ and } p > \overline{\overline{p}}(x) \end{cases} \tag{13}$$

where $\overline{p}(x) = 1 - \frac{1-\overline{x}}{(1-\mu)x}$, $\overline{\overline{p}}(x) = 1 - g^1 - \frac{1-\overline{x}}{(1-\mu)x}$. If γ is relatively high, then the threshold value of x to introduce a new product, $\overline{x}$, is given by $\overline{x} = \frac{\delta+4\gamma}{1+\delta}$; which is the value of $\overline{x}$ that satisfies $\Pi^L(\overline{x}) = \delta\Pi^L(1)$. Hence, conditional on price $\frac{1}{2}$ and consumer behavior, given by $\overline{r}(p,x) = p$, then the firm prefers to introduce a new variety if, and only if, $x \geq \overline{x}$. Along the equilibrium path prices and consumer behavior are $p^0 = \overline{r}^0 = \frac{1}{2}$. Also, it must be the case that, along the equilibrium path, for all $x_t \geq \overline{x}$, $x_{t+1} = 1 - (1-\mu)\frac{1}{2}x_t < \overline{x}$. That is, an inactive period must follow every innovation period. Such a condition is $\overline{x} \geq \frac{2}{3-\mu}$. If $\frac{\delta+4\gamma}{1+\delta} < \frac{2}{3-\mu}$, then we need to fix $\overline{x}$ equal to $\frac{2}{3-\mu}$. Notice that in this case, conditional on price $\frac{1}{2}$ and consumer behavior given by $\overline{r}(p,x) = p$, if $x \geq \overline{x}$ the firm prefers to introduce a new variety, since $\Pi^L\left(\frac{2}{3-\mu}\right) > \delta\Pi^L(1)$. From results in section 3, if the firm restricts to $p < \overline{p}(x)$ then consumers' optimal behavior is $\overline{r}(p,x) = p$, and the firm's optimal price is $\frac{1}{2}$.

Optimal consumer behavior is somewhat more complicated that in a high-frequency equilibrium. If $x \geq \overline{x}$, consumers will expect no innovation in the next period, and hence will behave according to $\overline{r}(p,x) = p$, only if the price is sufficiently low for such expectation to be self-fulfilling: $p < \overline{p}(x)$. Otherwise, consumers will expect one more innovative period, followed by an inactive period. Hence, from results in Section 3, they will optimally behave according to $\overline{r}(p,x) = p + g^1$. If $x < \overline{x}$, and in order to minimize the firm's incentives to deviate, such a threshold has been set at a lower value; that is, consumers expect no innovation in the next

[15] Whereas price and consumer behavior along the equilibrium path are uniquely determined by the results obtained in section 3, there is still room for multiple equilibria, which can be the result of different behavior out of the equilibrium path.

period only if $p \leq \overline{\overline{p}}(x) < \overline{p}(x)$. Again, if $p > \overline{\overline{p}}(x)$, consumers will expect one more innovative period followed by an inactive period.

For this type of equilibria, as suggested by the more complicated consumer strategy, the firm's optimal deviation strategy may potentially involve different prices. In particular, we need to check that if $x < \overline{x}$ the firm does not have incentives to innovate, and set a price above or below $\overline{\overline{p}}(x)$. The Appendix shows that, depending on parameter values, the optimal deviation strategy may actually involve a price higher or lower than $\overline{\overline{p}}(x)$, and hence either one can determine the minimum value of γ, γ^L, compatible with a low-frequency equilibrium. Also, note that if γ is too high the monopolist never wants to introduce a new product. Hence, the existence of a low-frequency equilibrium requires $\gamma \leq 0.25$.

Proposition 3 *There exists a threshold value* $\gamma^L > 0$, *such that strategies (12) and (13) support low frequency as a Markov perfect equilibrium if, and only if,* $\gamma \geq \left[\gamma^L, 0.25\right]$.

Multiplicity of equilibria. Table 1 reports the values of various thresholds for different values of (μ, δ). In particular, it shows that γ^H can be higher or lower than γ^L. More specifically, if the discount factor is sufficiently high and temporary satiation is sufficiently strong (low μ), then $\gamma^H \geq \gamma^L$. Consequently, if $\gamma \in \left[\gamma^L, \gamma^H\right]$ there exist multiple equilibria. That is, alternative consumer beliefs can be self-fulfilling. The intuition for this is as follows: if consumers expect at $x_0 = 1$ that new products will be introduced every period then they find it optimal to purchase only if the their current net surplus is sufficiently high (higher than g^∞). Given such consumer behavior, the monopolist prefers to set a relatively high price, p^∞, leading to a low level of sales, and a high level of the state variable in the next period, x_1. In period 1, a high level of x_1 will provide the right incentives to continue innovating. Alternatively, if consumers expect that the next period will be inactive, then they find it optimal to purchase for any positive current surplus ($g^0 = 0$). In this case, the monopolist sets a relatively low price, $\frac{1}{2}$, which results in a high level of sales, and a low level of x_1. If x_1 is low then the monopolist prefers to wait for one period to introduce a new variety.[16]

Non-existence. If δ is not so high, then $\gamma^H < \gamma^L$ for all values of μ, and hence if $\gamma \in \left[\gamma^H, \gamma^L\right]$ none of these two types of equilibria exist.[17]

[16]Whenever two pure strategy equilibria exists then it is very likely that a mixed strategy equilibrium also exists. In such equilibrium, after one inactive period the monopolists introduces a new product with a certain probability. Explicit consideration of this type of equilibria would ad very little.

[17]The search for other Markov perfect equilibria is very hard because of two features of the model. First, consumers hold expectations about a discrete variable (the number of consecutive innovative periods), and hence optimal behavior is discontinuous in (x, p). Second, the state variable, x, in a high frequency equilibrium oscillates: high values tend to be followed by low values and viceversa.

Comparative statics. Table 1 also provides some comparative static results. In particular, both γ^H and γ^L increase with μ. Table 1 only reports these thresholds for two values of δ, which suggest that both γ^H and γ^L decrease with δ. Further (unreported) numerical simulations indicate that this is indeed the general pattern. Summarizing, these thresholds decrease with the intensity of temporary satiation, ω.

6 Welfare analysis

This section examines the efficiency of equilibria, as well as the distributional implications. I first study the preferences of the two types of agents over the frequency of introduction of new products, and next the preferences of a social planner that maximizes total surplus.

The firm's ex-ante optimal frequency. The firm's payoff from low- and high-frequency paths are given by $\Pi^L = \frac{\frac{1}{4}-\gamma}{1-\delta^2}$ and $\Pi^H = \frac{k^\infty-\gamma}{1-\delta}$, respectively. Hence, the firm prefers high frequency if, and only if, $\gamma \leq \gamma^m$, where such threshold value is:

$$\gamma^m = \frac{1+\delta}{\delta}k^\infty - \frac{1}{4\delta}$$

The proof of Proposition 3 includes the following result:

Remark 4 $\gamma^H > \gamma^m$.

That is, there exist a range of values of γ for which the monopolist is 'trapped' in a high frequency equilibrium, whereas it would like to commit to a low-frequency path.

Consumers' optimal frequency. Total consumer surplus obtained in a low- and high-frequency equilibria are given by $U^L = \frac{\frac{1}{8}}{1-\delta^2}$ and $U^H = \frac{u^\infty}{1-\delta}$, respectively. In this case the comparison is unambiguous:

Remark 5 $U^H > U^L$.

As discussed above, u^∞ increases with μ; however, even if $\mu = 0$, $u^\infty > \frac{1}{8(1+\delta)}$ and hence consumers are better off in a high-frequency equilibrium. In spite of the higher prices, consumers benefit from the greater product variety.

The social optimum (second best). Consider a social planner whose goal is to maximize total surplus, $TS^j = \Pi^j + U^j$, $j = L, H$. Suppose the social planner can choose the frequency of introduction of new products but cannot affect monopoly pricing. Then, the social planner prefers high frequency if, and only if, $\gamma \leq \gamma^{sb}$, where γ^{sb} is given by:

$$\gamma^{sb} = \frac{1+\delta}{\delta}\left(k^\infty + u^\infty\right) - \frac{3}{8\delta}$$

Let us proceed in two steps. First, compare the incentives of the monopolist under full commitment with those of the social planner. Since consumers always prefer high frequency and the monopolist does not internalize the effect of frequency on consumer surplus, then a social planner would choose high frequency more often than the monopolist. That is:

Remark 6 $\gamma^{sb} > \gamma^m$.

Hence, under commitment the monopolist would tend to introduce products too slowly (too little dynamic variety). However, in the absence of commitment, we learned above that high frequency can result in equilibrium for values of γ higher than γ^m (Remark 5). Thus, whether or not excessive product variety may occur in equilibrium will depend on the distance between γ^H and γ^m. In particular, if $\gamma^H - \gamma^m$ was sufficiently high then the possibility of excessive product variety would arise, in spite of the firm's ex-ante bias in favor of low frequency. However, numerical simulations (See Table 1 for an illustration) indicates that this is not the case:[18]

Proposition 7 *When consumers discount the future at the same rate as the monopolist, the frequency of introduction of new products is either socially optimal or insufficient; that is,* $\gamma^{sb} > \gamma^H$.

Thus, in the baseline model, and from a second-best perspective, inefficient equilibria are always characterized by insufficient product variety (low frequency emerges in equilibrium, while high frequency would deliver higher total surplus).

The discussion below will reveal that this result is not very robust. At this point, it will be sufficient to note that $\gamma^{sb} - \gamma^H$ is the result of two countervailing effects. In the extreme case of no temporary satiation, $\mu = 1$, there is no intertemporal link, since $x_t = 1$ independently of past behavior. In this case, the monopolist either innovates every period (if $\gamma < 0.25$) or it never does (if $\gamma > 0.25$).[19] The monopolist has lower incentives than the social planner to introduce new products (to be active in the market) simply because higher frequency only generates market expansion, which raises consumer welfare ($\gamma^{sb} = 0.375$). However, as μ falls, the effect of which is magnified by a high discount factor, a countervailing effect becomes stronger. In this case, the monopolist considers it to be relatively more attractive to introduce new varieties every period because it can sell each variety to a more homogeneous consumer group and hence charge a higher price. Thus, shifting from a low- to a high-frequency path also generates better preference matching (which benefits the firm and hurts consumers).

[18] Like the other thresholds, γ^m and γ^{sb} also increase with μ and decrease with δ.

[19] In other words, as μ goes to 1 both γ^H and γ^L converge to 0.25.

It turns out that, in the baseline model, the first effect dominates. That is, the additional profit obtained from shifting from a low- to a high-frequency path is lower than the social benefit (that is, the equilibrium frequency is insufficient). But there is no substantial reason for such unambiguous result. In fact, relatively minor modifications of the baseline model (see sections 7 and 8) can generate equilibria with excessive frequency, if ω is sufficiently high. The specification of the welfare benchmark also matters, as discussed below.

The first-best frequency. Until now, the focus has been on a second-best scenario. Alternatively, however, we could consider the case where the social planner can control the frequency of introduction of new products as well as the price. The working paper version (Caminal, 2011) shows that in this case the social planner can implement the first-best allocation by setting the price equal to marginal cost (zero) and choosing the optimal frequency conditional on such pricing rule. In this case, it can be shown that if μ is lower than a certain threshold, $\overline{\mu}(\delta) > 0$, then the social planner's threshold, γ^{fb}, is lower than γ^m and γ^H. That is, the equilibrium frequency may be higher than in the first best, $\gamma^{fb} < \gamma^H$. Moreover, the possibility of excessive product variety survives even if we let the monopolist commit to its optimal ex-ante frequency, $\gamma^{fb} < \gamma^m$.[20]

7 The frequency of new product introductions with myopic consumers

The previous analysis has been conducted under the assumption that consumers and the firm discount the future at the same rate. Such an assumption is likely to be extreme. One could argue that consumers tend to use a lower discount factor than firms. This section considers the case of completely myopic consumers (where consumers' discount factor is set to zero). This can be interpreted as an extreme version of an extension that allows consumers to discount the future more than the monopolist.

If consumers' discount factor is zero, some of the preliminary results derived in section 3 need to be adjusted. If we denote the equilibrium variables for the case of myopic consumers with a hat, then it is clear that $\widehat{g}^n = 0$ for all n. Of course, it is still the case that $\widehat{p}^0 = \widehat{r}^0 = \frac{1}{2}$. However, for $n > 0$ we need to evaluate equations (4) and (6) at $g^n = 0$. Then, by taking the limit as n goes to infinity, we obtain:

[20] Presumably, in a first best scenario, the social planner can compensate the firm for fixed costs using revenue from non-distortionary taxation. Another more realistic welfare benchmark would allow the planner to control price and frequency but impose a break-even constraint (I am grateful to an anonymous referee for suggesting this benchmark). In this case, I would expect results somewhat in between those obtained using the two other welfare benchmarks.

$$\widehat{p}^{\infty} = \widehat{\overline{r}}^{\infty} = \frac{1 + \omega - \sqrt{1 + \omega}}{\omega}$$

Note that $\widehat{p}^{\infty}$ is equal to $\frac{1}{2}$ if $\omega = 0$, and increases with ω. Moreover, $\widehat{p}^{\infty} > p^{\infty}$. The reason is simple: myopic consumers exhibit a lower demand elasticity (demand functions shift upwards by a constant g^{∞}), and hence the optimal price is higher than in the case of forward-looking consumers. Naturally, such a demand boost leads to higher profits and lower consumer surplus. In particular:

$$\widehat{k}^{\infty} = \frac{(1 - \widehat{p}^{\infty})\,\widehat{p}^{\infty}}{1 + \omega\,(1 - \widehat{p}^{\infty})} > k^{\infty}$$

$$\widehat{u}^{\infty} = \frac{\frac{1}{2}\,(1 - \widehat{p}^{\infty})^2}{1 + \omega\,(1 - \widehat{p}^{\infty})} < u^{\infty}$$

The firm's optimal frequency. Given that consumers behave myopically, then the firm's optimization problem is time consistent. Hence, even in the absence of commitment, the monopolist can implement the ex-ante optimal frequency of introduction of new products. The firm's payoff from introducing new varieties every other period (low frequency) is $\widehat{\Pi}^L = \frac{\frac{1}{4} - \gamma}{1 - \delta^2}$, and every period (high frequency) is $\widehat{\Pi}^H = \frac{\widehat{k}^{\infty} - \gamma}{1 - \delta}$. Therefore, a high-frequency equilibrium is preferred by the monopolist, $\widehat{\Pi}^H \geq \widehat{\Pi}^L$, if and only if $\gamma \leq \widehat{\gamma}^m$, where $\widehat{\gamma}^m$ is given by:

$$\widehat{\gamma}^m = \frac{1 + \delta}{\delta}\widehat{k}^{\infty} - \frac{1}{4\delta}$$

The social optimum (second best). Suppose the social planner cannot affect monopoly pricing but can choose the frequency of introduction of new products in order to maximize the expected discounted value of total surplus. Since $\widehat{U}^L = \frac{\frac{1}{8}}{1 - \delta^2}$ and $\widehat{U}^H = \frac{\widehat{u}^{\infty}}{1 - \delta}$, and total surplus is $\widehat{TS}^j = \widehat{\Pi}^j + \widehat{U}^j$, $j = L, H$, then the social planner prefers high frequency if, and only if, $\gamma \leq \widehat{\gamma}^{sb}$, where $\widehat{\gamma}^{sb}$ is given by:

$$\widehat{\gamma}^{sb} = \frac{1 + \delta}{\delta}\left(\widehat{k}^{\infty} + \widehat{u}^{\infty}\right) - \frac{3}{8\delta} \tag{14}$$

Hence,

$$\widehat{\gamma}^m - \widehat{\gamma}^{sb} = \frac{1}{\delta}\left[\frac{1}{8} - (1 + \delta)\,\widehat{u}^{\infty}\right]$$

The next proposition shows that $\widehat{\gamma}^m$ may be higher or lower than $\widehat{\gamma}^{sb}$, depending on the intensity of temporary satiation. That is, the frequency of introduction of new products chosen by the monopolist may be insufficient or excessive from the point of view of total surplus maximization.

Proposition 8 *When consumers are myopic, the frequency of introduction of new products can be excessive if temporary satiation is sufficiently strong. More specifically, for all* $\delta > 0$, *there exists a threshold value,* $\widehat{\mu}(\delta) \in (0,1)$ *such that* $\widehat{\gamma}^m > \widehat{\gamma}^{sb}$ *if, and only if,* $\mu < \widehat{\mu}(\delta)$.

The proof of this proposition is straightforward. The likelihood of excessive diversity depends on μ, but also on δ. Indeed, the threshold $\widehat{\mu}(\delta)$ increases with δ. That is, excessive diversity is more likely as μ falls and δ increases. Thus, these two parameters are imperfect substitutes in the generation of excessive diversity. The reason is that both, a lower μ and a higher δ, make two consecutive varieties better substitutes.

As in the baseline model, obtaining excessive or insufficient product variety depends on the net balance of two countervailing effects: (i) market expansion (which works in favor of insufficient product variety); and (ii) better preference matching under high frequency (which works in favor of excessive variety). Under myopic consumers, the difference is that the second effect dominates, provided temporary satiation is sufficiently strong. The reason is that myopic consumers do not anticipate the consequences of current consumption on future utility and engage in 'excessive' consumption. As a result, along a high-frequency equilibrium, consumers become less demanding (they purchase every time they obtain a positive surplus, in contrast to the minimum surplus, $g > 0$, required by forward-looking consumers). Such an upward shift in demand induces higher equilibrium prices, and a transfer of rents from consumers to the firm: $u^\infty > \widehat{u}^\infty$ and $k^\infty < \widehat{k}^\infty$. As a result, the extra profit obtained by shifting from low- to high-frequency may exceed the incremental social value.

8 Concluding remarks

Absence of short-run commitment power. The previous analysis was conducted under the assumption that the monopolist sets a constant price for each variety. If, on the contrary, the monopolist cannot commit to keeping the price of each variety constant over its life span (until a new variety is introduced), then prices and consumer behavior along a low-frequency path would be different. In particular, in this case the monopolist would have an incentive to reduce the price in the second period of the variety's life span and sell it to consumers with lower valuations (who did not make a purchase in the first period at the 'regular' price). Clearly, the expectation of such a price reduction would decrease consumers' willingness to pay in the first period. In this scenario, as in literature on durable-goods monopoly, the inability to commit to a constant price reduces monopoly profits along any low-frequency equilibrium. Since the firm's short-run incentives to deviate from the prescribed strategy remain constant, the range of values of γ that can support a low-frequency equilibrium would shrink (higher γ^L). Analogously, the range of values of γ that support a high frequency equilibrium would expand (higher γ^H), since profits along a high-frequency path remain constant, but incentives to deviate are reduced. In summary, in the absence of short-run commitment, a high-frequency equilibrium would be more likely. Moreover, it could even be the case that $\gamma^H > \gamma^{sb}$, and hence, if $\gamma \in \left[\gamma^{sb}, \gamma^H\right]$, there would exist an equilibrium in which the frequency of introduction of new products is socially excessive.[21]

Welfare analysis (summary and additional remarks). The equilibrium frequency of introduction of new products does not always coincide with the socially optimal. Shifting from low to high frequency generates market expansion but also better preference matching. The relative strength of these two effects depends on the intensity of temporary satiation, but also on other aspects. If consumers discount the future at the same rate as the monopolist and can use the current price to make predictions about future product development plans, then as the intensity of temporary satiation increases, social and private incentives become better aligned. However, in this case, inefficiency always takes the form of insufficient dynamic diversity, $\gamma^{sb} > \gamma^H$.

Alternative scenarios would give rise to the possibility of obtaining excessive diversity. As discussed in the previous section, if the welfare benchmark is the first best (the social planner controls the frequency of introduction of new products as well as prices), and if temporary satiation is sufficiently strong, then equilibrium frequency may be socially excessive, $\gamma^{fb} < \gamma^H$. Even if we adhere to the second-best benchmark, the sign of $\gamma^{sb} - \gamma^H$ is very sensitive to some changes in the specification of the model. In particular, if consumers are myopic (section 7), and the intensity of temporary satiation is sufficiently strong, then γ^{sb} is lower than γ^H. One would expect that, by continuity, the same result could be obtained if consumers are forward-looking but their discount factor is sufficiently lower than the firm's.

Finally, I will argue that excessive dynamic variety, $\gamma^{sb} < \gamma^H$, is also possible in this model even when consumers' discount factor is the same as the firm's. In particular, we could consider an scenario where prices faced by individual consumers involve some idiosyncratic perturbation (retail prices do experience some geographic variation in the real world). In particular, we could assume that the price faced by consumer i in period t, p_{it}, is made of two components,

[21] Even in the absence of commitment power, if consumers strongly prefer to purchase the new variety during the innovative period, perhaps because of positive consumption externalities, then commercialization and consumption would again be perfectly synchronized. Krieder and Weinberg (1998) argue that externalities are indeed present in the case of the motion picture industry, where viewers prefer to watch the films immediately following their release, when total consumption is at their peak.

$p_{it} = p_t + \epsilon_{it}$, where p_t is the (wholesale) price set by the monopolist and ϵ_{it} is an $i.i.d$ random variable with zero mean (normalized retailer's mark-up). Consumers observe their individual prices, p_{it}, but not the average price, p_t. In this case, along a high-frequency equilibrium, p_{it} will affect consumer i's decision in period t, but not her beliefs about future product development plans.[22] The reason for this is that in equilibrium, consumers expect the price, p^{∞}, to be set with probability one, and any $p_{it} \neq p^{\infty}$ must be attributed to the idiosyncratic shock.[23] That is, in this case consumer behavior is given by $\overline{r}(p_{it}, x_t) = p_{it} + g^{\infty}$ for all x_t, p_{it}. In this scenario it is easy to show that a high-frequency equilibrium can be sustained for higher values of γ. The intuition is that consumer expectations are now more rigid, and the monopolist is no longer tempted to deviate by setting a sufficiently low price that induces a change in consumer expectations. In fact, γ^{sb} is in this case lower than γ^{H}, if μ is sufficiently low.[24]

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[22] This is analogous to the point made by Bagwell (1995).

[23] Adding aggregate uncertainty into the model (so that consumers cannot predict equilibrium price with infinite precision) would change again the inference problem. In particular, Bayesian updating would involve a positive weight on p_{it}. However, for small aggregate uncertainty the weight on p_{it} would also be small, and the argument in the main text would go through.

[24] If we follow the proof of Proposition 4 (Appendix) then it is immediate to realize that if consumers' beliefs are independent of prices, then equation (19) is not binding any more, and the inequality that defines the upper bound on γ is (15). That is, γ^{H} would be replaced by γ^{b}. In this case if $\mu = 0$ and $\delta = 1$ then $\gamma^{H} = 0.0919 > \gamma^{sb} = 0.0753$.

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10 Appendix

10.1 Proof of Proposition 3 (high-frequency equilibria)

Since $\overline{x} = \frac{\gamma}{k^{\infty}}$ then $x_1 \geq \overline{x}$, is equivalent to:

$$\gamma \leq \gamma^b = k^{\infty} \left[1 - (1-\mu)(1 - \overline{r}^{\infty})\right] \tag{15}$$

Also, we need to check that the monopolist does not have any incentives to deviate and set a price below $\widetilde{p}(x)$. The threshold $\widetilde{p}(1)$ could be higher than $\frac{1}{2}$. In particular, $\widetilde{p}(1) \geq \frac{1}{2}$ if, and only if, $\overline{x} \leq 1 - (1-\mu)\left(\frac{1}{2} - g^{\infty}\right)$. Since $\overline{x} = \frac{\gamma}{k^{\infty}}$ then this condition is equivalent to:

$$\gamma \geq \gamma^a = k^{\infty} \left[1 - (1-\mu)\left(\frac{1}{2} - g^{\infty}\right)\right] \tag{16}$$

Clearly, $\gamma^a < \gamma^b$. If $\widetilde{p}(1) \geq \frac{1}{2}$, since $\widetilde{p}(x)$ is an increasing function of x, then there is a range of values of x, including $x = 1$, for which the optimal deviation is to set a price equal to $\frac{1}{2}$. That is, profits from the deviation would be $x\frac{1}{4} - \gamma + \delta^2 \Pi^{\infty}(1)$, which have to be lower than $xk^{\infty} - \gamma + \delta \Pi^{\infty}(1)$. Clearly, incentives to deviate are maximized at $x = 1$. Notice that at

$x=1$ the no deviation condition means that the firm ex-ante prefers high frequency over low frequency. That is:

$$\gamma \leq \gamma^m = \frac{1}{\delta}\left[(1+\delta)\, k^{\infty} - \frac{1}{4}\right] \tag{17}$$

It can be checked that $\gamma^m < \gamma^a$; i.e., conditions (16) and (17) are incompatible and hence there is no equilibrium with $\widetilde{p}\,(1) \geq \frac{1}{2}$.

Since $\widetilde{p}\,(1) < \frac{1}{2}$ then the optimal price in case of a deviation is $\widetilde{p}\,(x)$. The expected payoff from deviating is $x\widetilde{p}\,(x)\,[1-\widetilde{p}\,(x)] - \gamma + \delta^2\Pi^{\infty}\,(1)$ which has to be lower than $xk^{\infty} - \gamma + \delta\Pi^{\infty}\,(1)$. Once again, the highest deviation incentives occur at $x=1$ and hence the no deviation condition is:

$$\gamma \leq \frac{1}{\delta}\left\{(1+\delta)\, k^{\infty} - \widetilde{p}\,(1)\,[1-\widetilde{p}\,(1)]\right\} \tag{18}$$

The left hand side of (18) is a convex function of γ that reaches a minimum at $\gamma = \gamma^a$ and takes the value γ^m $(< \gamma^a)$. Since the right hand side of (18) is the 45^o line, then the lowest value of γ that solves (18) with equality, γ^H, lies in the interval (γ^m, γ^a). Plugging the definition of $\widetilde{p}\,(x)$ and $\overline{x} = \frac{\gamma}{k^{\infty}}$ into (18), then γ^H is the lowest solution to the following quadratic function:

$$\gamma = \frac{1}{\delta}\left[(1+\delta)\, k^{\infty} - \left(1 - g^{\infty} - \frac{1-\frac{\gamma}{k^{\infty}}}{1-\mu}\right)\left(g^{\infty} + \frac{1-\frac{\gamma}{k^{\infty}}}{1-\mu}\right)\right] \tag{19}$$

Consequently, if $\gamma \leq \gamma^H$, then a high-frequency equilibrium exists. It is important to emphasize that $\gamma^H > \gamma^m$. This means that that there is a non-empty interval $\left[\gamma^m, \gamma^H\right]$ for which a high-frequency equilibrium exists, whereas the monopolist would would like to commit to low frequency. QED

10.2 Proof of Proposition 4 (low-frequency equilibria)

1) Suppose $x_t \in [0, \overline{x})$. If $p < \overline{\overline{p}}\,(x_t)$ and consumers behave according to $\overline{r}\,(p, x_t) = p$ then $x_{t+1} = 1 - (1-\mu)\,(1-p)\, x_t < \overline{x}$. Alternatively, if $p \geq \overline{\overline{p}}\,(x_t)$ and consumers behave according to $\overline{r}\,(p, x_t) = p + g^1$ then $x_{t+1} = 1 - (1-\mu)\left(1 - p - g^1\right) x_t \geq \overline{x}$. In both cases, consumers behave optimally. Let us consider a possible deviation from the firm's equilibrium strategy: suppose $q\,(x_t) = 1$.

1a) Consider the case $\overline{x} = \frac{2}{3-\mu}$. Since $\overline{\overline{p}}\left(\frac{2}{3-\mu}\right) = \frac{1}{2} - g^1$, and $\overline{\overline{p}}\,(x)$ increases with x, then $\overline{\overline{p}}\,(x_t) \leq \frac{1}{2} - g^1 < \frac{1}{2}$. Hence, provided $p \leq \overline{\overline{p}}\,(x_t)$, and since consumers behave according to $\overline{r}\,(p, x_t) = p$, the optimal price is $\overline{\overline{p}}\,(x_t)$. In this case, profits are equal to $x_t\overline{\overline{p}}\,(x_t)\left[1 - \overline{\overline{p}}\,(x_t)\right] - \gamma + \delta^2\Pi^L\,(1)$, which need to be lower than $\delta\Pi^L\,(1)$. Clearly, the highest incentives to introduce a new variety are at $x_t = \overline{x}$. That is, a low-frequency equilibrium requires that:

$$\gamma \geq \gamma_1^L = (1+\delta)\,\frac{2}{3-\mu}\left[\frac{1}{4} - \left(g^1\right)^2\right] - \frac{\delta}{4} \tag{20}$$

Alternatively, if the monopolists sets a price above $\overline{\overline{p}}\,(x_t)$, since consumers behave according to $\overline{r}\,(p, x_t) = p + g^1$, then the optimal price is p^1 and profits are equal to $x_t k^1 - \gamma + \delta\Pi^L\,(1)$. Once again, incentives to introduce a new variety increase with x, and hence a low-frequency equilibrium requires that:

$$\gamma \geq \gamma_0^L = \frac{2}{3-\mu}k^1 \tag{21}$$

Note that γ_1^L can be higher or lower than γ_0^L depending on parameter values.

1b) Consider the case $\overline{x} = \frac{\delta+4\gamma}{1+\delta} > \frac{2}{3-\mu}$. Now the optimal price, conditional on $p_t \leq \overline{\overline{p}}\,(x_t)$ is $min\left\{\frac{1}{2}, \overline{\overline{p}}\,(x_t)\right\}$. Profits from introducing a new variety increase with x, and are bounded above by $\frac{\overline{x}}{4} - \gamma + \delta^2 \Pi^L(1)$. Since $\overline{x} = \frac{\delta+4\gamma}{1+\delta}$ this upper bound is equal to $\delta\Pi^L(1)$. Therefore, it is not profitable to introduce a new variety. Finally, conditional on $p_t \geq \overline{\overline{p}}(x_t)$, and since the highest incentives to innovate are at $x = \overline{x}$, profits from innovation are equal to $\frac{\delta+4\gamma}{1+\delta}k^1 - \gamma + \delta\Pi^L(1)$. That is, deviation is not profitable if:

$$\gamma \geq \frac{\delta k^1}{1+\delta-4k^1}$$

Such a condition is implied by (21).

2) Suppose now $x_t \in [\overline{x}, 1]$. If $p < \overline{p}(x_t)$ and consumers behave according to $\overline{r}(p, x_t) = p$ then $x_{t+1} = 1-(1-\mu)(1-p)x_t < \overline{x}$. Alternatively, if $p \geq \overline{p}(x_t)$ and consumers behave according to $\overline{r}(p, x_t) = p + g^1$ then $x_{t+1} = 1-(1-\mu)\left(1-p-g^1\right)x_t \geq \overline{x}$. In both cases, consumers behave optimally.

Also note that $\overline{p}\left(\frac{2}{3-\mu}\right) = \frac{1}{2}$, In the main text it has already been shown that, conditional on $p < \overline{p}(x)$ since consumers behave according to $\overline{r}(p, x_t) = p$, the optimal price is $p = \frac{1}{2}$ and $q(x) = 1$ is optimal. Suppose now the monopolist deviates and sets a price above $\overline{p}(x)$. In this case consumers behave according to $\overline{r}(p_t, x_t) = p_t + g^1$. From Section 3, the optimal price is p^1 and profits are equal to $xk^1 - \gamma + \delta\Pi^L(1)$. Profits from the deviation increase with x at the rate k^1, but profits from $p = \frac{1}{2}$ increase at the rate $\frac{1}{4} > k^1$. As a result, incentives to deviate are the highest at $x = \overline{x}$.If $\overline{x} = \frac{2}{3-\mu}$ then $\overline{x}k^1 - \gamma + \delta\Pi(1) \leq \delta\Pi(1) \leq \Pi(\overline{x})$. The first inequality is implied by (21). Hence, (21) is also a sufficient condition for a non-profitable deviation. If instead $\overline{x} > \frac{2}{3-\mu}$ then profits from the deviation are equal to $\frac{\delta+4\gamma}{1+\delta}k^1 - \gamma + \delta\Pi(1)$ which is lower than $\Pi(\overline{x}) = \delta\Pi(1)$ if, and only if:

$$\gamma \geq \frac{\delta k^1}{1+\delta-4k^1}$$

which is again implied by $\gamma \geq \gamma_0^L$. Summarizing, a low-frequency equilibrium exists if, and only if, $\gamma \geq \gamma^L = \max\left\{\gamma_0^L, \gamma_1^L\right\}$, which are given by equations (20) and (21). QED

TABLE 1

	δ = 1.00				δ = 0.50			
μ	γ^H	γ^L	γ^m	γ^{sb}	γ^H	γ^L	γ^m	γ^{sb}
0.00	0,0669	0,0651	0.0502	0,0753	0,0807	0,1211	0.0560	0,0840
0.05	0,0722	0,0702	0.0558	0,0838	0,0869	0,1257	0.0629	0,0943
0.10	0,0778	0,0774	0.0617	0,0926	0,0933	0,1303	0.0700	0,1049
0.15	0,0836	0,0850	0.0679	0,1018	0,0999	0,1352	0.0773	0,1159
0.20	0,0897	0,0929	0.0743	0,1115	0,1067	0,1402	0.0848	0,1272
0.25	0,0960	0,1009	0.0811	0,1216	0,1137	0,1453	0.0926	0,1389
0.30	0,1027	0,1090	0.0882	0,1323	0,1209	0,1507	0.1006	0,1509
0.35	0,1097	0,1174	0.0957	0,1435	0,1284	0,1562	0.1089	0,1634
0.40	0,1170	0,1260	0.1036	0,1553	0,1360	0,1618	0.1175	0,1762
0.45	0,1248	0,1347	0.1119	0,1678	0,1439	0,1677	0.1264	0,1896
0.50	0,1330	0,1438	0.1207	0,1810	0,1521	0,1738	0.1356	0,2033
0.55	0,1417	0,1530	0.1300	0,1950	0,1605	0,1802	0.1451	0,2176
0.60	0,1508	0,1625	0.1399	0,2098	0,1692	0,1867	0.1550	0,2325
0.65	0,1606	0,1723	0.1504	0,2256	0,1782	0,1935	0.1652	0,2478
0.70	0,1709	0,1823	0.1617	0,2425	0,1875	0,2006	0.1759	0,2638
0.75	0,1820	0,1927	0.1737	0,2606	0,1971	0,2080	0.1870	0,2805
0.80	0,1938	0,2034	0.1866	0,2800	0,2071	0,2157	0.1985	0,2978
0.85	0,2064	0,2145	0.2006	0,3009	0,2173	0,2237	0.2106	0,3158
0.90	0,2199	0,2259	0.2157	0,3235	0,2280	0,2321	0.2231	0,3347
0.95	0,2345	0,2377	0.2321	0,3481	0,2389	0,2408	0.2363	0,3544
1.00	0,2500	0,2500	0.2500	0,3750	0,2500	0,2500	0.2500	0,3750