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Abstract

We consider a market where firms hire workers to run their projects and such projects differ in profitability. At any period, each firm needs two workers to successfully run its project: a junior agent, with no specific skills, and a senior worker, whose effort is not verifiable. Senior workers differ in ability and their competence is revealed after they have worked as juniors in the market. We study the length of the contractual relationships between firms and workers in an environment where the matching between firms and workers is the result of market interaction. We show that, despite in a one-firm-one-worker set-up long-term contracts are the optimal choice for firms, market forces often induce firms to use short-term contracts. Unless the market only consists of firms with very profitable projects, firms operating highly profitable projects offer short-term contracts to ensure the service of high-ability workers and those with less lucrative projects also use short-term con-

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tracts to save on the junior workers' wage. Intermediate firms may (or may not) hire workers through long-term contracts.

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1 Introduction

Partners may establish relationships that last for several periods. In job contracts, for instance, some firms hire the same worker for several years with a long-term contract, while others may prefer to sign contracts period by period, sometimes with the same worker, sometimes with different workers over time. One question that arises is when is it better for the employer to write a long-term contract that covers the whole length of the relationship and when is a short-term contract signed for a certain period of time superior, knowing that once that period is over the firm has to offer another short-term contract.

The contributions by Lambert (1983), Rogerson (1985), Malcomson and Spinnewyn (1988), and Chiappori *et al.* (1994), among others, address the previous question in settings characterized by moral hazard where one firm (the principal) enters into a long-lasting relationship with a worker (the agent). In these settings, if the firm and the agent can commit to a long-term contract, the firm can design a long-term agreement that dominates the sequence of optimal short-term contracts. In fact, long-term contracts can always replicate the sequence of the optimal short-term contracts while the reverse is, in general, not possible. This result is robust to a number of different specifications and implies that, if commitment on the part of the participants is possible, we should expect firms to use mainly long-term contracts in practice.[1] However, this is not the case and the fact that different firms in similar markets follow different time-duration contracts suggests that there may be other explanations beyond lack of commitment.

In this paper we argue that not all the characteristics of the optimal contract between an employer and a worker can be deduced from the analysis of this relationship in a one-worker, one-firm setting. As the recent contributions by Dam and Pérez-Castrillo (2006), Serfes (2008), Terviö (2008) and Alonso-Paulí and Pérez-Castrillo (forthcoming) have shown, when the analysis is enlarged to take into consideration market interaction, then the form of the optimal contracts may substantially differ from the agreements that one obtains for a given relationship studied in isolation. When heterogenous principals

[1] When commitment on the agent side is difficult, contracts may include, for example, non-compete clauses under which the agent agrees not to pursue a similar profession or trade in a firm in the same industry if he breaks the contract. Contracts may also include other clauses that will reduce mobility by increasing the cost of hiring the worker by another firm.

compete for heterogenous agents, the identity of the partners in each relationship (in addition to the contract signed) is endogenous, as it is the level of utility obtained by the agents. Hence, even if short-term contracts are not optimal when we consider an isolated relationship, they may arise as part of the market equilibrium where some firms may choose long-term contracts while other hire workers only on short-term agreements.

We consider a market where heterogenous firms hire workers to run their projects. Firms differ in the profitability of their project. At any period, each firm needs two types of workers: a junior agent, with no specific skills, and a senior experienced, worker whose expertise is crucial for the good development of the project. Every worker starts as a junior agent in the first period he works in a firm. At this point in their lives, all workers are identical. After the period as an apprentice, the worker becomes senior for the second, and final, period of his job career. In our model, the first period has a training component: workers acquire the knowledge and experience needed to run a project when seniors. We assume that this period gives human capital specific to the industry. This allows any worker that was hired (and trained) by a firm as a junior to run a project as a senior in any firm of the market.[2]

The model is dynamic not only because the relationships (may) involve several periods, but also because information about workers' characteristics changes over time: after the agent has worked for a firm as a junior, the firms and the worker himself learn his competence as senior, which was ex-ante unknown for all market participants. Therefore, while all junior workers are indistinguishable, this is not the case for senior workers as not only their responsibilities (the project they work on), but also their abilities may be quite different. Consequently, we model a very simple technology of training that combines two dimensions of learning. First, there is learning because the innate ability of the worker is revealed through his training as a junior and this information becomes common knowledge for the industry. Second, there is a learning-by-doing component since working as junior is a prerequisite to later running a project as a senior. In this respect, the paper is related, though different, to the literature on on-the-job talent discovery. Terviö (2009) also presents a situation where workers' innate ability is unknown for the market (and

[2]In other words, we do not deal with firm-specific training in the model. If there is some firm-specific ability, workers who change jobs in the second period of their lives lose their firm-specific human capital and just keep their general training in the industry. This will tend to decrease the profitability of short-term contracts.

the workers themselves) until they actually work for a firm. However, in Terviö's paper the objective is different as he is concerned by the possiblity that market imperfections hamper the process of discovering talent.

In the model, a moral hazard problem is present as senior workers' effort or decision is not contractible. On the contrary, and just for simplicity, we consider juniors' effort to be contractible. All the participants are risk neutral and they all have the capacity to commit to a long-term contract.[3] However, workers are protected by limited liability: their salary when junior and their salary when senior cannot be lower than certain thresholds.

We characterize equilibria in this market, which accounts for the type of contract offered by each firm and the characteristics of these contracts. Our equilibrium concept is close to the idea of "stability" used in the matching literature that has analyzed contracts in environments where the matching between firms and workers is endogenous.[4] To be an equilibrium, an outcome (that is, a matching and a set of contracts) must be immune to deviations. In our environment, at equilibrium, it must be the case that a firm cannot make more profit by changing its strategy, that is, by offering contracts to workers that make both the firm and the workers better-off than before.

We first show that firms signing equilibrium long-term contracts offer low salaries to junior workers together with the promise of high reward when senior. This allows the firms to alleviate the incentive problem they face with senior agents, improving the efficiency of the relationship and also their profits. Since they commit to do so, these firms will keep the agents when senior, irrespective of their ability. Firms that sign short-term contracts hire junior agents with no promise of continuation. They also sign short-term contracts with senior workers (who may or may not be the same they hired the previous period as juniors); the terms of the agreement may depend on the workers' ability level.

We have already argued that the optimal long-term contract always (at least weakly)

[3] If no participant can commit to a long-term contract, then all must be short-term contracts. If the participants in one of the sides of the market, say the firms, can commit while the others cannot, then there can still be room for long-term contracts, but they are typically less efficient than in the environment with full commitment. In terms of the commitment possibilities, we place ourselves in the best scenario for the prevalence of long-term contracts.

[4] Stability and competitive equilibrium are very close concepts. Any stable outcome is also a competitive equilibrium and vice-versa. For (early) matching models where the parties decide on money instead of contracts see, for instance, the original contribution by Shapley and Shubik (1972), and the excellent review of the literature by Roth and Sotomayor (1990).

dominates any sequence of short-term contracts when the identity of the parties matched in a job contract is predetermined. However, short-term contracts can be beneficial for firms when the firm-worker matching is endogenous. Short-term contracts allow the firms to screen workers before putting them in charge of leading a project. This non-commitment strategy gives firms the freedom to focus on the particular type of senior worker that fits their needs. Therefore, firms face a trade-off between choosing the optimal contract for a given match (long-term are superior to short-term agreements) and selecting a contract that allows a better selection (hiring high-ability senior workers is more important for some firms than for others).

The market equilibrium depends on the characteristics of the set of firms and the set of workers. We solve the model for markets where there is a large proportion of low-productivity (normal) senior workers and a small proportion of high-productivity senior workers (stars) and where these highly-talented workers really make a difference in the firms they work for.

When only firms with very profitable projects exist in the market, all of them sign long-term contracts at equilibrium. Each firm offers the same agreement that it would offer if no market would have existed. More interestingly, we show that, except in this case where the market only consists of firms with very profitable projects, there is always a set of firms that sign short-term contracts with their junior workers and specialize in a particular type of seniors. Depending on the value they attach to their projects, some firms always look for high-ability while others hire low-ability senior workers. Firms with highly profitable projects give a great deal of relevance to hiring high-ability senior agents to run their projects and, hence, they are willing to offer high wages to attract them. As a result, the expected utility of junior workers when they accept short-term contracts becomes higher because, if they turn out to be of high ability, they will obtain a high reward when senior. The expectation of this potential reward leads workers to accept, when junior, a low wage. Firms with relatively poor projects take advantage of this reduction in the wage of junior workers. These firms put more weight on the savings on juniors' wages than to the fact that they end up contracting with a low-ability senior worker. Therefore, at equilibrium, the firms with the most profitable projects use short-term contracts to ensure the services of high-ability workers while the firms with the least profitable projects use short-term contracts to save in the cost of hiring junior workers.

In this sense, the matching between firms and senior agents is positive assortative, for the sets that choose short-term contracts.[5]

The trade-off between the advantages of long-term and short-term contracts is often solved in favor of the use of long-term contracts for firms with intermediate projects. The likelihood of the coexistence of the two types of contracts is higher as the distribution of firms is more biased toward good projects, the discount rate is lower, the difference between the reservation utility of junior workers and the minimum salary is lower, the difference in performance between high and low-ability senior workers is higher, and the cost of the workers' effort is lower.

Workers receive part of the increased surplus created by the optimal sorting of senior workers promoted by the short-term agreements. Indeed, although all junior workers are identical and they perform identical tasks, those who sign short-term contracts expect a higher utility than those signing long-term contracts. Long-term agreements allow the firms to avoid the competition for the best workers, who obtain high salaries under short-term equilibrium contracts.

In our analysis, we focus on markets with a small proportion of very talented senior workers that make a difference for the firms they work for and whose level of ability is public for all the firms inside the industry. Moreover, the human capital acquired by seniors is industry-specific and not just firm-specific. This model can provide a schematic version of the university job market. The performance of researchers during the first years after the completion of their Ph.D., that we can associate to their "ability", is public information since it can be measured, for instance, by their publication record. In this job-market some universities offer Ph.D. graduates a tenure track position that guarantees tenure if, after the probationary period, the candidate satisfies some predetermined performance criteria (in terms of publications and other measures). The tenure-track system corresponds in our model to short-term contracts. Other universities sign tenure contracts from the very beginning and take the commitment of keeping the researcher independent of the outcome of further evaluation (even if contract conditions may indeed depend on performance). This corresponds to a long-term contract.

Arts and sports are also examples of markets where the ability of seniors is well-known,

[5] See Legros and Newman (2007) for conditions under which monotone matchings emerge in environments where utility is not fully transferable.

as it is subject to public scrutiny through their performance and where this human capital is mainly industry-specific. Singers or soccer players, for instance, may sign exclusive contracts (with a studio, a record company, or a club) for a long period in which they are prevented from recording an album for another company or playing with another club. Other companies, however, choose to offer shorter contracts, particularly to young singers or players. The stars of these markets, a few individuals, attain prominence and success and their value and earnings are significantly greater than the earnings of the standard worker in the markets. The same can be said about surgeons or creatives in advertising. Finally, the market for upper executives also shares some similar features: these high executives are well-known within their industry and their contracts may (or may not) include special clauses aimed at preventing them from moving to another firm.

To the best of our knowledge, ours is the first paper to study how the choice of the contractual length may be determined by market interaction. There are other papers that have studied the implications of different contractual arrangements but in widely different set-ups. Rice and Sen (2008) show how a reduction in the length of a contract can help to alleviate the moral hazard problem when explicit incentives cannot be included in the terms of the contract. In their paper, the optimal choice for the principal depends on the balance between more incentives for effort (short-term contracts) and lower wages (long-term contracts).

The contribution by Ghosh and Waldman (2010) compares two contractual arrangements: up-or-stay vs. up-or-out contracts in a setting with multiple firms competing for a worker. The paper does not address the issue of endogenous matching since it studies the firms' Bertrand competition in wages to attract the single worker available in the market. They show that up-or-out prevails when firm-specific human capital is low and when high- and low-level jobs are similar. Otherwise, standard (up-or-stay) practices are optimal. Similar to our paper, Ghatak et al. (2001) study an overlapping generations version of a principal-agent problem where contracts are determined in general equilibrium. In their model, all young workers are identical but have different investment possibilities when senior, depending on their performance. They do not allow for long term contracts because their the authors' concern is to explain the seniors' decision between becoming entrepreneurs or remaining workers.

In our paper, short-term contracts act as a form of probationary period that allows firms and workers to achieve a better matching. It is, therefore, not a way in which firms try to test if the worker is good enough for the job, but rather it allows senior workers to be matched with those firms where they are more productive. In this sense, the short-term contract serves as a sorting device. A related, but different, argument can be found in Loh (1994) where it is argued that introducing an employment probation can serve as a sorting device as it will induce self-selection by workers. Firms offering probationary employment will tend to attract workers who are more confident about their capabilities.

Finally, the coexistence of fixed payment schemes and incentive-based payment schemes related to the characteristic of the workers is also present in a static adverse selection framework where firms compete for agents. Matutes et al. (1994) study the choice of compensation schemes by two firms that compete in a labor market where agents are heterogenous and they have private information about their type. They show that, in equilibrium, if firms are not too different in the eyes of workers, one firm offers a wage rate and the other offers a piece rate. By proposing different compensation schemes, firms induce self-selection among workers, which thereby decreases the intensity of competition in the labor market.

The remainder of the paper is organized as follows. Section 2 presents the model. Sections 3, 4 and 5 analyze the candidate long-term and short-term contracts for equilibrium. Section 6 characterizes the identity of the firms and workers that enter into the relationship, the equilibrium salaries, as well as the contracts that emerge as a result of the market interaction. Finally, Section 7 concludes.

2 Model

We model the economy as an overlapping generation model where at each period t, with $t = 1, 2, ...$, firms contract with workers to develop projects. Firms are infinite-lived players and the set of firms is constant for all periods. On the other hand, workers (agents) live for two periods. Both, firms and workers discount the future according to the discount factor δ, where $\delta \in (0, 1)$.

All participants are assumed to be risk neutral. We also assume that a worker, at any age, enjoys limited liability over income. This constraint implies that his wage in any period and contingency cannot be lower than a certain threshold $\underline{w}$.

At any period t, each firm is endowed with a project. The revenue for the firm from the project is $F+R$ if it is developed successfully, whereas it is $F>0$ in case of failure. F can be interpreted as a fixed component of the revenue that is not subject to uncertainty. The additional value of the project in case of success, R, is public information, it is the same across periods for a given firm, but it varies across firms. It is distributed in the interval $\left[\underline{R},\overline{R}\right]$, with $\underline{R}>0$, according to the distribution function $G(R)$. Hence, the set of firms can also be identified as the interval $\left[\underline{R},\overline{R}\right]$.[6] We consider that the set of active firms $\left[\underline{R},\overline{R}\right]$ and the distribution function $G(R)$ are given; we will discuss at the end of the paper on these assumptions.

Each period, a generation of workers is born. We assume that the measure of the set of workers born at any period is larger than the measure of the set of firms. Therefore, at period t the market is composed by the set of firms, the set of workers that enter the market this period and the set of old workers that entered the market at period $t-1$. At period 1, there is a set of workers who are already old.

To run its project, a firm needs to hire a non-specialized worker and a specialized worker. Any agent is a non-specialized worker the first period he works in this market. After this first job, a young agent that has worked for a firm becomes a specialized worker; that is, working for a firm gives the agent the necessary skills to be in charge of a project. We will also refer to non-specialized and specialized workers as *junior* and *senior* agents, respectively.[7] According to our assumptions, there are more junior workers than non-specialized positions to fill in the market.

[6] We assume that each firm's revenue is composed of a part (R) that depends on the success of the venture and a fixed part (F) that is independent of its success. In our model, all firms have the same F while they are heterogeneous with respect to R so that a "good" firm is a firm with a high R. However, workers have no influence on F and decisions do not depend on F. Therefore, the analysis that we develop is independent of whether F is the same for all the firms or if it varies from firm to firm. We can assume an arbitrary function $F(R)$ that associates a fixed revenue F to each variable revenue R and all the results go through for this, more general, scenario. It could be the case, for example, that $F(R)$ is decreasing and that firms with a high R have lower expected profits than firms with low R.

[7] Those agents who did not work for any firm when young could also be hired as non-specialized workers when old. In other words, the time when working as a junior is both a probational and a forming period. However, we will assume that they are no longer in the market. As it will become clear once we will develop our analysis, this is just a simplification since no firm would prefer to hire an old-non-specialized instead of a junior worker.

A senior agent only enters a relationship if his expected utility is at least equal to some "outside utility" that we denote by $\underline{U}_S$. That is, $\underline{U}_S$ is the level of utility that a senior agent can secure outside our economy, at any period. It can be understood as his utility outside the labor market. Similarly, a junior agent only accepts a contract if his expected intertemporal utility is at least $\underline{U}_J + \delta \underline{U}_S$.

All workers are identical when junior. However, when senior, agents may have different abilities in this market. An agent's ability to run a project as a senior is ex-ante *unknown to all players,* including the agent himself. The ability becomes *publicly known to all players* once the agent has worked for any firm, that is, when he is senior.

A junior agent working for a firm does a routine job and exerts a predetermined and contractible level of effort.[8] We normalize the cost for the agent of exerting this effort to zero. If we denote by w_J the payment to the junior agent, then his utility at this age is equal to w_J.

A senior agent working for a firm runs the project and his effort (decision) is crucial to its good development. This specialized effort is not contractible (not verifiable) and it influences the probability of success of the project. We assume that the probability of success takes the form pe, where e is the senior agent's effort and the parameter p summarizes his ability.[9] The ability p can take two values: p_H or p_L, with $p_H > p_L$; that is, for the same effort $e > 0$, a high-ability senior agent has a higher probability of success ($p_H e$) than a low-ability senior agent ($p_L e$). As said above, it is only after being employed as a junior worker that an agent's ability is public knowledge. Ex-ante, there is a proportion q of high-ability agents in the population.

The result of the project (success or failure) is verifiable and the agents' payment can depend on it. Given that the junior agent's effort is verifiable, there is no reason to offer him a contingent payment and a fixed salary w_J is optimal. On the other hand, a contingent payment scheme should be offered to the senior agent to give him the right incentives. We denote by $C_S = (w_S, \Delta_S)$ such an incentive scheme, where the first component of the contract, w_S, is the base payment, i.e., the transfer in case of failure while

[8]The assumption that the junior agent's effort is contractible provides a simple set-up. The main conclusions of our analysis carry over a more complex model where the junior agent's effort is subject to moral hazard as long as the effort does not affect the learning of the worker's ability (this possiblity is considered for example in Holmstrom, 1999).

[9]We assume that the parameters of the model make sure that pe is always smaller than 1.

the second part, Δ_S, is the bonus in case revenue R is obtained. The contract offered may be different depending on the (publicly known) ability of the senior worker, that is, we may have C_H different from C_L. Given the contract (w_S, Δ_S), the expected utility of a senior agent of ability p is

$$w_S + pe\Delta_S - (ke)^2$$

where $(ke)^2$ represents the cost of supplying effort e. Under contract (w_S, Δ_S), the senior agent will select the effort that maximizes his expected utility, i.e.,

$$e = \arg\max_{\widehat{e}}\{w_S + p\widehat{e}\Delta_S - (k\widehat{e})^2\}.$$

Therefore, the level of effort is

$$e = \frac{1}{2k^2}p\Delta_S.$$

The previous equation represents the Incentive Compatibility Constraint (ICC) of a senior agent. The agent tends to exert a higher level of effort the lower his cost of supplying effort (k), the larger the bonus (Δ_S) and the higher his skills (p).

At any period t, the expected profits of an active firm that runs its project with a junior agent, to whom it pays the salary w_J, and with a senior agent of ability p through a payment scheme (w_S, Δ_S) are

$$F - w_J + pe(R - \Delta_S) - w_S.$$

Firms and workers in this market can sign either *Short-term* (ST) or *Long-term* (LT) *contracts*. A ST contract between a firm and a junior agent consists of a salary w_J. An ST contract with a senior agent is an incentive scheme (w_S, Δ_S), that may be different for agents with different abilities (and it can also depend on the value of the project in case of success). This senior agent was working for some firm in period $t-1$, but not necessarily for the same firm with which he is signing the ST contract at period t. An LT contract between a firm and a junior agent at period t specifies the salary that the worker will receive during the first period of the relationship and the incentive scheme that will govern their relationship in period $t+1$. The incentive scheme can be a function of the revealed ability of the agent. That is, an LT contract is a vector $(w_J, w_H, \Delta_H, w_L, \Delta_L)$ and it implies a commitment by the firm to keep the worker when senior, and a commitment by the agent to work for the same firm at period $t+1$ independent of his ability.

Each firm decides on the contracts it offers at each period and also on the type of workers it hires. Therefore, an *Outcome* in the economy specifies, for each period t, the assignment of (some) junior and senior workers to firms and the contracts that govern their relationship. Remember that a contract between a firm and a junior agent at period t is signed at this period; it can be either an ST or an LT contract. On the other hand, the contract governing the relationship between a senior worker and a firm at period t can be either an ST contract signed at this period or an LT contract that was signed at period $t-1$.

We look for equilibrium outcomes, that is, for outcomes that are immune to individual deviations. The contract for an active worker (that is, any worker who signs a contract) must be acceptable for him: he should be better off under the proposed outcome than if he did not enter the relationship. That is, the contract must be individually rational for the agent at the time he signs it. Once an agent has accepted a contract, he has to honor it. Also, a firm should not have incentives to deviate from the proposed outcome. A firm can deviate by changing the contract with its assigned workers or by contracting with other agents.[10] We assume that a firm can secure the services of a worker at period t if he did not commit to an LT contract at period $t-1$ and it offers him a contract under which he obtains the same level of utility (or a slightly higher level of utility) than in his current situation. That is, the expected level of utility of a worker who is not committed at period t is the "price" that a firm has to pay to attract him.

Hence, an *Equilibrium* in the economy is an outcome where:

a) all active workers obtain, at least, their outside utility (i.e., junior workers achieve an expected total utility of at least $\underline{U}_J + \delta \underline{U}_S$ and senior workers obtain at least $\underline{U}_S$ if they sign a contract at this age);

b) no firm would obtain higher expected intertemporal profits by changing the set of proposed contracts by another set of contracts that guarantee to each worker at least the same level of expected utility that he obtains under the current outcome.

We concentrate the analysis on *stationary equilibria*, that is, on equilibria where firms follow the same strategy every period. For simplicity, we refer to stationary equilibria simply as equilibria.

[10] For simplicity, we assume that all firms in $[\underline{R}, \overline{R}]$ are active in this market. We return to this issue at the end of Section 6.

At equilibrium, a firm may

- offer LT contracts to junior workers (so that it keeps them when senior whatever their type), or
- offer ST contracts to junior workers and to senior workers of high, or low, ability.

Therefore, at equilibrium the set of firms $\left[\underline{R}, \overline{R}\right]$ is partitioned in three subsets, some of which may be empty: the set $\mathcal{R}^{LT}$ of firms that offer LT contracts, the set $\mathcal{R}^{H}$ of firms that offer ST contracts to juniors and hire high-ability seniors and the set $\mathcal{R}^{L}$ of firms that offer ST contracts to juniors and hire low-ability seniors. Obviously, the measures of the sets $\mathcal{R}^{H}$ and $\mathcal{R}^{L}$ cannot be arbitrary as they must satisfy the feasibility constraint that the ratio of the measures of $\mathcal{R}^{H}$ and $\mathcal{R}^{L}$ must be equal to the ratio of high- and low-ability workers ($\frac{q}{1-q}$).

We first note that when one considers the analysis of one isolated firm's optimal contract, ST contracts are always (at least weakly) dominated by LT contracts. We do not prove this result since it is well-established in the literature that LT contracts typically improve the efficiency of the relationships by allowing both parties to commit on the sequence of events.[11] The intuition is that the firm can always replicate in the LT contract the optimal sequence of contingent ST contracts. Moreover, LT contracts are typically superior because the firm, when it signs ST contracts, cannot commit to paying the senior worker a utility level higher than his reservation utility. Therefore, if the firm wants to keep the senior agent independently of his type, it would obtain higher profits by signing an LT contract. In other words, we do not need to consider a fourth set of firms -those that sign ST contracts and re-hire the same worker when senior independently of his type- since this set is always empty at equilibrium.

Proposition 1 uses this idea to show a stronger result: in our environment, a situation where all firms sign (optimal) LT contracts with their workers is an equilibrium.

[11] On the optimality of LT contracts versus ST contracts in a single principal-agent model with repeated moral hazard, see, for instance, Lambert (1983), Rogerson (1985), Malcomson and Spinnewyn (1988), and Chiappori *et al.* (1994). This literature analyzes the role of commitment, reputation, memory and renegotiation. In our set-up the key element is the commitment. There is no role for memory since there is no past outcome, and there is no role for reputation since all relevant parameters are public information at any time.

Proposition 1 *There is always an equilibrium where all active firms sign LT contracts with their workers.*

If all the firms in the economy are signing LT contracts, then no single firm has incentives to deviate and offer a sequence of ST contracts. What is the advantage of committing to an LT relationship? As discussed above, the commitment allows the firm to relax the limited liability constraint of senior workers by delaying part of their payments as juniors. ST contracts typically cannot replicate this strategy. Indeed, when the firm designs the ST agreement addressed to a senior agent, the contract that governed their relationship the previous period, while he was young, is already sunk. Therefore, the firm will only give the agent the rents that maximize its second-period profits.

The next sections explore equilibria where ST and LT contracts may coexist. We first discuss how to analyze equilibrium contracts by using a firm's one-period profits.

3 Equilibria and one-period profits

In an equilibrium, no firm can obtain higher expected intertemporal profits by changing the set of proposed contracts. In this section, we relate this condition to the equilibrium profits that one firm obtains (or it may obtain by changing the contract) in one period.

To illustrate the discussion, we take a firm offering an LT contract $C^{LT} = (w_J, w_S, \Delta_S)$, with $(w_S, \Delta_S) = (w_H, \Delta_H, w_L, \Delta_L)$, at each period that considers switching to a different sequence of LT contracts $C^{LT\prime} = (w'_J, w'_S, \Delta'_S)$. If the firm decides at time t to change from C^{LT} to $C^{LT\prime}$, then it still has to keep its commitment with the current senior worker who it hired as junior at $t-1$. Therefore, the cost of the senior agent as well as the revenue it receives at time t are the same under $C^{LT'}$ and under C^{LT}, because they are determined by the realized quality of the senior agent. The only change at t concerns the payoff it offers to the junior agent (w'_J). The new contract $C^{LT\prime}$ will be fully implemented from $t+1$ on (see Figure 1). Consequently, switching to $C^{LT\prime}$ is not profitable for the firm if its profits from $t+1$ on are not higher than under C^{LT}, also taking into account the change in the cost of the junior worker at t (i.e., the change from w_J to w'_J). This is equivalent to comparing one-period profits under C^{LT} and under $C^{LT\prime}$ from the perspective of $t+1$, but considering the present value of the cost of the junior agent incurred at the previous period, that is, we have to impute a cost of $\frac{1}{\delta}w_J$ and $\frac{1}{\delta}w'_J$ instead of w_J and w'_J.

t – 1	t	t + 1	t + 2	…
w_J	w_J	w_J	w_J	…
(w_S, Δ_S)	(w_S, Δ_S)	(w_S, Δ_S)	(w_S, Δ_S)	…
w_J	w'_J	w'_J	w'_J	…
(w_S, Δ_S)	(w_S, Δ_S)	(w'_S, Δ'_S)	(w'_S, Δ'_S)	…

Figure 1: A firm considers changing the contract

The situation is similar if a firm which is currently offering the contract C^{LT} decides to switch to a series of ST contracts: it first changes the contract it offers to the junior agent to be able to fully implement the new strategy in the subsequent period. Also, we face the same situation if a firm is currently offering ST contracts and plans to switch to LT contracts: it needs to change the agreement with the junior agent today but still needs to hire a senior agent through an ST contract to be able to fully implement the change tomorrow. Finally, when a firm switches from ST contracts to another stream of ST contracts in a period, it can do it immediately, without waiting till the subsequent period. Indeed, it can keep hiring junior agents under the same conditions as before (that is, under the lowest salary that the agent is ready to accept). Whether we compute the cost of the junior agent as $\frac{1}{\delta}w_J$ or w_J is not relevant for the comparison of profits in the two strategies, since the firm pays the same cost under both, the old and the new contracts.

Therefore, we can develop the analysis of the (stationary) equilibria of our model by focusing on the profits firms make in one period, provided that we consider the cost of the junior agent as being generated the previous period, that is, as long as we associate a cost of $\frac{1}{\delta}w_J$, instead of w_J, to the junior agent. From now on, we will refer to this level of profits as "a firm's one-period profits" and we will denote $E\widetilde{\pi} = F - \frac{1}{\delta}w_J + pe(R - \Delta_S) - w_S$. A firm has incentives to switch from contract C to contract C' if and only if $E\widetilde{\pi}(C) < E\widetilde{\pi}(C')$.

4 Long-term contracts in equilibrium

Consider a firm that owns a project whose additional value in case of success is $R \in [\underline{R}, \overline{R}]$ and that signs LT contracts with junior workers. At each period t, the firm runs the project with the junior agent that it hires at t and with the senior worker that it hired at period $t-1$. The senior agent has ability p_H with probability q and ability p_L with probability $1-q$, as his ability was unknown at $t-1$. As previously said, the ability of the agent is publicly known before he starts working as a senior; hence, the LT contract signed at $t-1$ may have payments contingent on the ability of the agent when senior.[12]

All workers are ex-ante identical and there are more junior workers than positions to fill. Therefore, at any period there are unemployed junior agents ready to accept any LT contract that provides them with an expected utility equal to their (two-period) outside utility $\underline{U}_J + \delta \underline{U}_S$. Hence, the participation constraint (PC) specifies that the total expected utility the worker obtains in the relationship be at least equal to $\underline{U}_J + \delta \underline{U}_S$.

Following the discussion of the previous section, a candidate LT contract for equilibrium $(w_J, w_H, \Delta_H, w_L, \Delta_L)$ maximizes the firm's one-period profits, also taking into account the ICCs and the limited liability constraints (LLC), that is, it solves

$$\max_{(w_J, w_H, \Delta_H, w_L, \Delta_L)} F - \frac{1}{\delta} w_J + q(p_H e_H (R - \Delta_H) - w_H) + (1-q)(p_L e_L (R - \Delta_L) - w_L)$$

$$\text{s.t. } w_J + \delta \left[q(p_H e_H \Delta_H + w_H - (k e_H)^2) + (1-q)(p_L e_L \Delta_L + w_L - (k e_L)^2) \right] \geq \underline{U}_J + \delta \underline{U}_S$$

$$e_H = \frac{1}{2k^2} p_H \Delta_H, \qquad e_L = \frac{1}{2k^2} p_L \Delta_L$$

$$w_J \geq \underline{w}, \qquad w_H \geq \underline{w}, \qquad w_L \geq \underline{w}.$$

If the contract does not satisfy the previous program, then the firm can deviate by offering a different acceptable LT agreement to junior agents and obtain larger discounted profits. We state the characteristics of the candidate LT contract in Proposition 2, where we denote

$$\widetilde{p} \equiv \sqrt{q p_H^2 + (1-q) p_L^2},$$

$$R_1^{LT} \equiv \frac{2k}{\widetilde{p}} \sqrt{\frac{1}{\delta} (\underline{U}_J - \underline{w}) + \underline{U}_S - \underline{w}} \text{ and } R_2^{LT} \equiv \frac{4k}{\widetilde{p}} \sqrt{\frac{1}{\delta} (\underline{U}_J - \underline{w}) + \underline{U}_S - \underline{w}}.$$

[12] As will be clear later, this flexibility has no effect on the optimal contract. Therefore, at the candidate equilibrium contract, no third party needs to verify the ability of the agent.

Proposition 2 *If firm R is in the set $\mathcal{R}^{LT}$, then it offers the following LT contract:*
Region a^{LT}: If $R < R_1^{LT}$, then[13]

$$C^{LT}(R) = \left(w_J^{LT} = \underline{w}, w_H^{LT} = w_L^{LT} = \frac{1}{\delta}\left(\underline{U}_J - \underline{w}\right) + \underline{U}_S - \frac{1}{4k^2}R^2\widetilde{p}^2, \Delta_H^{LT} = \Delta_L^{LT} = R \right).$$

Region b^{LT}: If $R \in \left[R_1^{LT}, R_2^{LT}\right]$, then

$$C^{LT}(R) = \left(w_J^{LT} = \underline{w}, w_H^{LT} = w_L^{LT} = \underline{w}, \Delta_H^{LT} = \Delta_L^{LT} = \frac{2k}{\widetilde{p}}\sqrt{\frac{1}{\delta}\left(\underline{U}_J - \underline{w}\right) + \underline{U}_S - \underline{w}} \right).$$

Region c^{LT}: If $R > R_2^{LT}$, then

$$C^{LT}(R) = \left(w_J^{LT} = \underline{w}, w_H^{LT} = w_L^{LT} = \underline{w}, \Delta_H^{LT} = \Delta_L^{LT} = \frac{R}{2} \right).$$

We now explain the main characteristics of the LT contract $C^{LT}(R)$. Despite the absence of risk aversion, the moral hazard problem of the senior agent induces an inefficiency due to the presence of limited liability that restricts the capacity of the firm to induce the senior worker to exert a high effort. Therefore, the firm is interested in relaxing the senior agent's limited liability constraint, which explains why it concentrates as much as possible the agent's payments in his second period of life (i.e., the firm pays to a young worker the minimum possible wage: $w_J^{LT} = \underline{w}$.) Young agents accept contracts with a low payoff because of the credible promise to be "well" paid when they are senior. The limited liability constraints also explain why, unless R is very low, workers are paid the minimum salary if the outcome turns out to be a failure: $w_H^{LT} = w_L^{LT} = \underline{w}$.

The impact of limited liability on bonuses and on payoffs obtained by agents and firms differs depending on the profitability of the project R (as well as on the level of agents' reservation utility $\underline{U}_J + \delta\underline{U}_S$, cost of effort k, and "average" probability of success $\widetilde{p}$). Some characteristics are shown in Figure 2.

For high values of R (Region c^{LT}), the optimal bonus depends only on the value of the project. The firm shares half of the value in the event of success because it maximizes profits when the senior agent supplies effort $e_i^{LT} = \frac{1}{4k^2}p_i R$, for $i = H, L$. Given this bonus, the worker ends up with a utility larger than $\underline{U}_J + \delta\underline{U}_S$ (i.e., he obtains informational rents).

[13] In this region, there are other contracts that are also candidates for equilibrium. Any combination of w_J, w_H and w_L that satifies $w_J + \delta\left(qw_H + (1-q)\,w_L\right) + \delta\frac{1}{4k^2}R^2\widetilde{p}^2 = \underline{U}_J + \delta\underline{U}_S$ and such that each variable is higher than $\underline{w}$, is also a candidate as it would give the same profits to the firm.

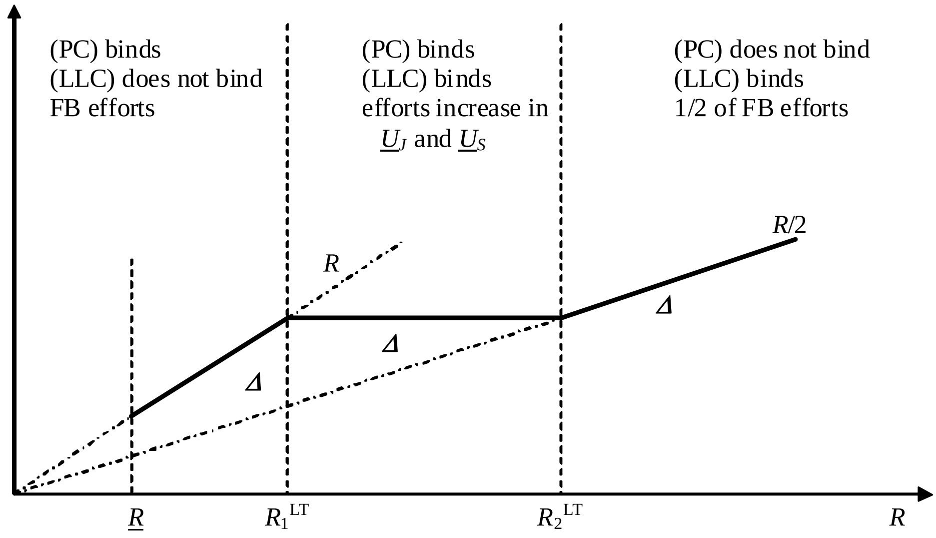

Figure 2: Incentives in the optimal LT contracts

For intermediate values of R (Region b^{LT}), the equilibrium payment scheme also depends on $\underline{U}_J$, $\underline{U}_S$, k and δ, as the participation constraint (together with the limited liability constraint) binds. Given that the firm needs to provide a level of utility of $\underline{U}_J + \delta \underline{U}_S$, it gives it in terms of bonuses, which lead to a senior agent's effort of $e_i^{LT} = \frac{p_i}{k\widetilde{p}}\sqrt{\frac{1}{\delta}\left(\underline{U}_J - \underline{w}\right) + \underline{U}_S - \underline{w}}$, for $i = H, L$.

Finally, firms with low-valued projects (Region a^{LT}) give all the project's returns to the worker (they set $\Delta = R$) in exchange for a fixed payment (a franchise-type contract). Therefore, agents obtain their total outside utility $\underline{U}_J + \delta \underline{U}_S$ and they provide, when senior, the first-best level of effort $e_i^{LT} = \frac{1}{2k^2} p_i R$, for $i = H, L$.

Next corollary provides the expression of the firm's one-period profits for $C^{LT}(R)$.

Corollary 1 *The firm's one-period profits under $C^{LT}(R)$ are:*
Region a^{LT}: If $R < R_1^{LT}$, then

$$E\widetilde{\pi}^{LT}(R) = F - \frac{1}{\delta}\underline{U}_J - \underline{U}_S + \frac{1}{4k^2}R^2\widetilde{p}^2.$$

Region b^{LT}: If $R \in \left[R_1^{LT}, R_2^{LT}\right]$, then

$$E\widetilde{\pi}^{LT}(R) = F + \frac{1}{k}R\widetilde{p}\sqrt{\frac{1}{\delta}\left(\underline{U}_J - \underline{w}\right) + \underline{U}_S - \underline{w}} - \frac{1}{\delta}\left[2\underline{U}_J - (1+\delta)\underline{w}\right] - 2\underline{U}_S.$$

Region c^{LT}: If $R > R_2^{LT}$, then

$$E\widetilde{\pi}^{LT}(R) = F + \frac{R^2\widetilde{p}^2}{8k^2} - \frac{(1+\delta)}{\delta}\underline{w}.$$

The profit function $E\widetilde{\pi}^{LT}(R)$ is continuously differentiable and convex in R.

5 Short-term contracts in equilibrium

All firms signing ST contracts hire similar young workers, as they are indistinguishable ex-ante. Concerning senior workers, they can decide to hire high-ability or low-ability workers.

Consider an equilibrium where some firms sign ST contracts. A fraction of those firms offer contracts to high-ability senior agents. Denote by U_H the (minimum) level of utility that this type of agent obtains at the equilibrium.[14] Similarly, denote by U_L the (minimum) level of utility received by low-ability senior workers. Both U_H and U_L need to be higher than or equal to $\underline{U}_S$. Additionally, given the limited liability constraint and the competition among firms, U_H and, possibly, U_L can be strictly higher than $\underline{U}_S$. Therefore, a junior agent is ready to sign an ST contract that provides a utility level lower than $\underline{U}_J$ as long as the reduction is not higher than the expected extra utility he will obtain when senior. Formally, the salary w_J that the junior agent is ready to accept must satisfy:

$$w_J + \delta\left[qE_H + (1-q)\,E_L\right] \geq \underline{U}_J + \delta\underline{U}_S,$$

where we denote E_H and E_L the expected utility of a high- and a low-ability worker. For example, if all the low-ability workers obtain the same U_L in all the possible jobs, then $E_L = U_L$.

The candidate equilibrium contract of firm R in $\mathcal{R}^H$ ($\mathcal{R}^L$) to a high- (low-) ability senior agent must be the optimal one-period contract for this agent, taking into account that it must grant him a level of utility of at least U_H (U_L); that is, it solves

$$\begin{aligned}
&\max_{(w_i,\Delta_i)} F + p_i e_i\,(R-\Delta_i) - w_i \\
\text{s.t.} \quad & p_i e_i \Delta_i + w_i - (ke_i)^2 \geq U_i \\
& e_i = \frac{1}{2k^2} p_i \Delta_i \\
& w_i \geq \underline{w}
\end{aligned}$$

[14] Given the limited liability constraint, similar senior agents might obtain different utility levels at equilibrium. A firm with a very high R ends up providing its senior agent a utility level higher than U_H as its participation constraint will not be binding (see also, Alonso-Pauli and Pérez-Castrillo, forthcoming).

for $i = H, L$. Next proposition provides the candidate equilibrium contract for those firms, where we use the notation

$$R_{i1}^{ST}(U_i) \equiv \frac{2k}{p_i}\sqrt{U_i - \underline{w}} \text{ and } R_{i2}^{ST}(U_i) \equiv \frac{4k}{p_i}\sqrt{U_i - \underline{w}}.$$

Proposition 3 *If firm R is in the set $\mathcal{R}^i$, with $i \in \{H, L\}$, then it offers the following ST contract to a senior agent:*
Region $a_i^{ST}(U_i)$: If $R < R_{i1}^{ST}(U_i)$, then

$$C_i^{ST}(R, U_i) = \left(w_i^{ST} = U_i - \frac{1}{4k^2}p_i^2R^2,\ \Delta_i^{ST} = R\right).$$

Region $b_i^{ST}(U_i)$: If $R \in \left[R_{i1}^{ST}(U_i), R_{i2}^{ST}(U_i)\right]$, then

$$C_i^{ST}(R, U_i) = \left(w_i^{ST} = \underline{w},\ \Delta_i^{ST} = \frac{2k}{p_i}\sqrt{U_i - \underline{w}}\right).$$

Region $c_i^{ST}(U_i)$: If $R > R_{i2}^{ST}(U_i)$, then

$$C_i^{ST}(R, U_i) = \left(w_i^{ST} = \underline{w},\ \Delta_i^{ST} = \frac{R}{2}\right).$$

In Region $a_i^{ST}(U_i)$, senior agent's effort is the first-best level $e_i^{ST} = \frac{1}{2k^2}p_iR$, while in Region $b_i^{ST}(U_i)$, his effort is lower than the first-best level: $e_i^{ST} = \frac{1}{k}\sqrt{U_i - \underline{w}}$. In these two regions, the agent's expected utility is U_i. Finally, in Region $c_i^{ST}(U_i)$ where the project is very valuable, the senior agent's effort is $e_i^{ST} = \frac{1}{4k^2}p_iR$, for $i = H, L$, and he receives an informational rent. His expected utility in this region is $\underline{w} + \frac{1}{16k^2}p_i^2R^2 > U_i$.

Corollary 2 provides the expression of the firm's one-period profits under $C_i^{ST}(R, U_i)$, denoting w_J^{ST} the equilibrium salary paid to junior agents.

Corollary 2 *A firm R in the set $\mathcal{R}^i$, with $i \in \{H, L\}$ obtains the following one-period profits with $C_i^{ST}(R, U_i)$*
Region $a_i^{ST}(U_i)$: If $R < R_{i1}^{ST}(U_i)$, then $E\widetilde{\pi}_i^{ST}\left(R, w_J^{ST}, U_i\right) = F + \frac{1}{4k^2}p_i^2R^2 - U_i - \frac{1}{\delta}w_J^{ST}$.
Region $b_i^{ST}(U_i)$: If $R \in \left[R_{i1}^{ST}(U_i), R_{i2}^{ST}(U_i)\right]$, then
$E\widetilde{\pi}_i^{ST}\left(R, w_J^{ST}, U_i\right) = F - 2U_i + \underline{w} + \frac{1}{k}p_iR\sqrt{U_i - \underline{w}} - \frac{1}{\delta}w_J^{ST}$.
Region $c_i^{ST}(U_i)$: If $R > R_{i2}^{ST}(U_i)$, then $E\widetilde{\pi}_i^{ST}(R, w_J, U_i) = F - \underline{w} + \frac{1}{8k^2}p_i^2R^2 - \frac{1}{\delta}w_J^{ST}$.
The profit function $E\widetilde{\pi}_i^{ST}\left(R, w_J^{ST}, U_i\right)$ is continuously differentiable and convex in R.

6 Equilibrium matching and equilibrium contracts

The previous sections identify the equilibrium contracts once we know the type of agreements firms offer (that is, once the sets $\mathcal{R}^{LT}$, $\mathcal{R}^{H}$ and $\mathcal{R}^{L}$ are determined) and the levels of utility U_L and U_H that they must guarantee to low- and high-ability agents. In the present section, we characterize equilibria where at least some firms offer ST contracts. Therefore, we identify the distribution of firms in $\mathcal{R}^{LT}$, $\mathcal{R}^{H}$ and $\mathcal{R}^{L}$, the levels U_L and U_H and the minimum salary w_J that firms must offer to juniors under ST contracts.

We look for equilibria where $U_L = \underline{U}_S$. Low-ability workers do not have special skills and the firms will not compete for them.[15] On the other hand, the level of U_H will be determined by the equilibrium conditions, that is, by the (marginal) firm's willingness to pay to attract a high-ability worker instead of either attracting a low-ability one, or signing an LT contract.

We develop the analysis for markets where high-ability workers are not abundant but they make a difference for the firm they work for. That is, we consider environments with many "normal" workers and some "stars". Assumption 1 reflects this idea, together with the reasonable hypothesis that the outside reservation utility of a senior agent is larger or equal to that of a junior worker (part (i)). Assumption 1 (ii) states that the proportion of high-ability agents is small enough. Finally, Assumption 1 (iii) reproduces the idea that the difference among the two types of agent is large enough.

Assumption 1 *The parameters satisfy the following conditions:*
(i) $\underline{U}_S \geq \underline{U}_J$,
(ii) $q < \frac{\delta}{1+2\delta}$,
(iii) $\left(\frac{p_H}{p_L}\right)^2 > 1 + \frac{1}{\delta q}$.

Why may some firms be interested in LT relationships while others prefer to secure high-ability agents through ST contracts? Even more, why would a firm choose a strategy that implies contracting low-ability agents through ST contracts, instead of offering LT contracts and, sometimes, benefiting from high-ability senior agents? The two main equilibrium variables that make firms prefer one or another type of contract are the

[15] However, at equilibrium the measure of senior workers with low ability is the same as the measure of firms looking for them. Therefore, other equilibria may exist where $U_L > \underline{U}_S$ for all low-ability players.

salary of a young worker w_J^{ST} (or rather, the comparison between w_J^{ST} and $\underline{U}_J$) and the difference between the cost of a high- versus a low-ability senior agent, that is, $U_H - U_L$.

The firms that obtain large profits in the event of success, that is, firms with a high R, are ready to pay a high price to always hire a good senior agent given his added value in terms of increased probability of success. Therefore, firms at the right end of the interval $\left[\underline{R}, \overline{R}\right]$ must be those most interested in signing ST contracts to hire high-ability senior agents. Similarly, firms that do not care much about agents' effort, i.e., firms with a low R, pay more attention to the potential savings they can make in a junior's contract if they offer him an ST contract than to the gains obtained through an LT contract, or by securing a high-ability agent. Therefore, firms at the left end of $\left[\underline{R}, \overline{R}\right]$ are the likely candidates to sign ST contracts to hire low-ability senior agents.

Lemma 1 provides a first confirmation of the previous intuitions. It compares the slopes, in terms of R, of the profits obtained from the different types of contract.

Lemma 1 *Under Assumption 1, the slopes of the profit functions satisfy the following relations:*[16]
(a) $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}(R, w_J, \underline{U}_S) < \frac{\partial E\widetilde{\pi}^{LT}}{\partial R}(R)$, *for all* R, w_J;
(b) $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}(R, w_J, \underline{U}_S) < \frac{\partial E\widetilde{\pi}_H^{ST}}{\partial R}(R, w_J, U_H)$, *for all* R, w_J, *and for all* $U_H \geq \underline{U}_S$; *and*
(c) $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R}(R) < \frac{\partial E\widetilde{\pi}_H^{ST}}{\partial R}(R, w_J, U_H)$, *for all* R, w_J, *and for all* $U_H \geq \underline{U}_S$.

A firm's ST profits increase with the value of success R when it hires a low-ability worker. However, this increase is smaller than that of a firm's profits under the optimal LT contract (part (a)). It is also smaller than the rate at which its profits increase if it hires high-ability workers through ST contracts (part (b)). A higher R implies a larger interest in securing the services of a high-ability worker, which explains the previous relations. A similar argument gives the intuition of part (c) in the lemma.

Let us denote by R^o the value that would "balance" the set of firms if all the firms with $R < R^o$ would hire low-ability workers while all the firms with $R \geq R^o$ would hire high-ability workers, that is, R^o is characterized by

$$\frac{G(R^o)}{1 - G(R^o)} \equiv \frac{1-q}{q}.$$

[16] Lemma 1 (a) and 1 (b) do not depend on Assumption 1. However, if Assumption 1 does not hold, then Lemma 1 (c) may fail if $U_H < \underline{U}_H \equiv \left[\frac{qp_H^2 + (1-q)p_L^2}{p_H^2}\right]\left[\frac{1}{\delta}\left(\underline{U}_J - \underline{w}\right) + \underline{U}_S - \underline{w}\right] + \underline{w}$.

Also, we denote $\widehat{R}$ the value that makes the firm indifferent between using LT contracts and hiring low-ability senior workers through ST contracts, when the junior salary is $w_J^{ST} = \underline{w}$, that is, $\widehat{R}$ is characterized by

$$E\widetilde{\pi}^{LT}(\widehat{R}) = E\widetilde{\pi}_L^{ST}(\widehat{R}, \underline{w}, \underline{U}_S).$$

As we check in Claim 1 in the proof of Theorem 1, under Assumption 1 firm $\widehat{R}$ lies in regions a^{LT} and $a_L^{ST}(\underline{U}_S)$. Therefore, we can easily calculate $\widehat{R} : \widehat{R} \equiv \frac{2k}{\sqrt{\delta q\left(p_H^2 - p_L^2\right)}} \sqrt{\underline{U}_J - \underline{w}}$.

We first consider the case where $\widehat{R} \in [\underline{R}, R^o)$, that is, some of the firms in the market have a low-valued project, but there is a relatively high number of firms with valuable projects.

Theorem 1 *Suppose $\underline{R} \leq \widehat{R} < R^o$, and denote R^{oo} the firm such that $qG\left(\widehat{R}\right) = (1-q)G(R^{oo})$. Then, under Assumption 1, an equilibrium exists where*
(i) firms with $R \leq \widehat{R}$ offer ST contracts: $\underline{w}$ to junior workers and $C_L^{ST}(R, \underline{U}_S)$ to low-ability senior workers,
(ii) firms with $R \in \left(\widehat{R}, R^{oo}\right)$ offer the LT contracts $C^{LT}(R)$,
(iii) firms with $R \geq R^{oo}$ offer ST contracts: $\underline{w}$ to junior workers and $C_H^{ST}(R, U_H^{oo})$ to high-ability senior workers, where U_H^{oo} is such that $E\widetilde{\pi}^{LT}(R^{oo}) = E\widetilde{\pi}_H^{ST}(R^{oo}, \underline{w}, U_H^{oo})$,
(iv) junior workers accept both LT contracts that guarantee them $\underline{U}_J + \delta\underline{U}_S$ and ST contracts with $w_J^{ST} = \underline{w}$, and
(v) senior workers accept contracts that guarantee them $\underline{U}_S$.[17]

When R^o is high enough, that is, the population of firms is not concentrated on low levels of R, then, at equilibrium, firms are divided according to three hiring strategies. Firms with low-valued projects use ST contracts and only hire low-ability seniors; firms with a high R also use ST contracts but they only hire high-ability seniors; and firms with intermediary Rs use LT contracts.

The rationale behind Theorem 1 is the following. Firms with more profitable projects give more importance to hiring the high-ability worker, and they offer more to attract them. This increases the expected utility of a junior worker when he accepts the ST contract: if he turns out to be of high ability he will obtain a large utility level. The

[17] At equilibrium, high-ability workers receive a level of utility of, at least, $U_H^{oo} > \underline{U}_S$. However, out of equilibrium, they should be ready to accept lower offers, as long as they guarantee $\underline{U}_S$.

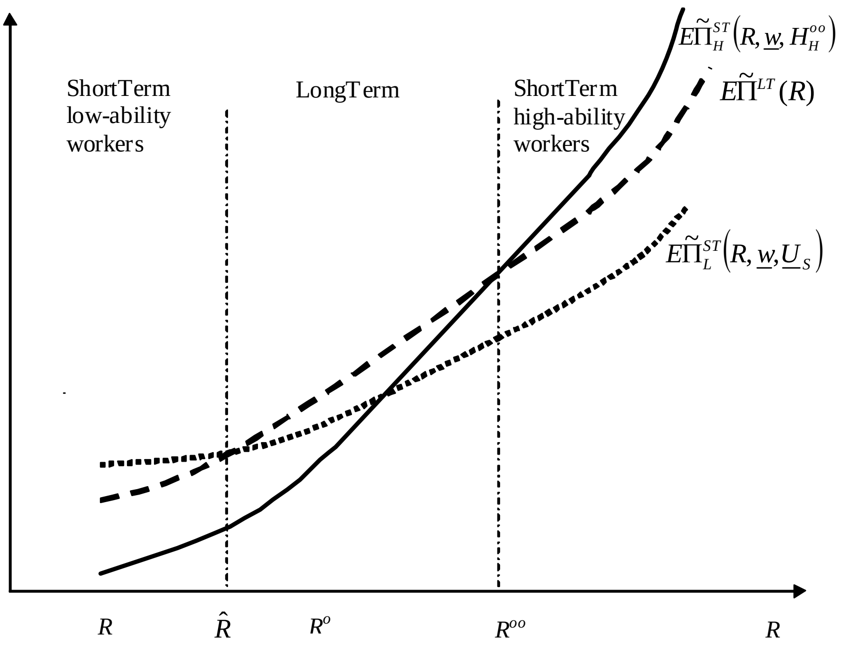

Figure 3: Profit functions at equilibrium

expectation of this potential reward leads workers to accept a wage $w_J = \underline{w}$ when junior which is under their reservation utility $\underline{U}_J$ because they will be compensated in the future (in expected terms) for this sacrifice. Firms with low R take advantage of this reduction in the wage that can be offered to junior workers who sign ST contracts: their value of the project is low enough so that the reduction in the wage of junior agents more than compensates the fact that they always end up hiring low-ability senior workers.

Given the difference in equilibrium salaries between high- and low-ability senior workers, firms with intermediary R do not perceive a large difference between hiring one type or another. Therefore, it is better for them to profit from the additional improvement in efficiency due to the commitment they make through LT contracts.

Figure 3 draws the LT and ST profits, as a function of R, for the equilibrium values for salaries and utility U_H. As shown in Lemma 1, the slope of $E\widetilde{\pi}_H^{ST}(R, \underline{w}, U_H^{oo})$ is always higher than that of $E\widetilde{\pi}^{LT}(R)$, which in turn is higher than the slope of $E\widetilde{\pi}_L^{ST}(R, \underline{w}, \underline{U}_S)$. At equilibrium, the market price that a firm has to pay in order to attract a high-ability worker (U_H^{oo}), is such that the three profit functions cross as shown in Figure 3.

It is worth noting that even though all junior workers are identical when they sign their equilibrium contracts and they perform identical jobs, their expected utility is different depending of the type of contract they are offered. Under an LT contract, a junior worker expects a total utility of $\underline{U}_J + \delta \underline{U}_S$. However, his expected utility if he signs an ST contract with any firm is at least $\underline{w} + \delta \left[q U_H^{oo} + (1-q) \underline{U}_S \right]$, which is strictly higher than $\underline{U}_J + \delta \underline{U}_S$. Therefore, workers that sign ST contracts end up receiving part of the surplus created by the increase in efficiency induced by the optimal sorting of senior agents.

Next, we consider the case where R^o is small (the population of firms has a big concentration of firms with a low R).

Theorem 2 *Suppose $R^o \leq \widehat{R}$. Then, under Assumption 1, an equilibrium exists where*
(i) firms with $R < R^o$ offer ST contracts: $\underline{w}$ to junior workers and $C_L^{ST}(R, \underline{U}_S)$ to low-ability senior workers,,
(ii) all firms with $R \geq R^o$ offer ST contracts: $\underline{w}$ to junior workers and $C_H^{ST}(R, U_H^o)$ to high-ability senior workers, where $U_H^o \equiv \underline{U}_S + \left(\frac{R^o}{2k}\right)^2 (p_H^2 - p_L^2)$,
(iii) junior workers accept ST contracts with $w_J^{STo} = \max\{\underline{U}_J - \delta \left(q E_H + (1-q) E_L - \underline{U}_S \right), \underline{w}\}$, and
(iv) senior workers accept contracts that guarantee them $\underline{U}_S$.

The basic trade-offs between the several contractual forms a firm can choose from do not depend on whether R^o is higher or lower than $\widehat{R}$. Therefore, the intuition behind Theorem 2 is similar to the one behind Theorem 1. However, no firm is interested in offering LT contracts when $R^o \leq \widehat{R}$. ST contracts for low-ability agents provide higher profits than LT contracts for any firm with $R \leq \widehat{R}$ while ST contracts with high-ability agents are better than LT contracts for firms with $R \geq R^o$ (given w_J^{STo} and U_H^o). Therefore, there is no space for the intermediate region where LT contracts are the best alternative.

Theorem 2 presents a situation where, at equilibrium, the market will only be formed by firms offering ST contracts. When $R^o \leq \widehat{R}$, most firms give low value to success since most of them are below $\widehat{R}$. Therefore, they care more about decreasing the cost of junior agents than about the additional incentives provided by LT contracts. In fact, firms with a low R may benefit from the existence of firms with a high R in such a way that even the marginal firm R^o may strictly prefer ST contracts (either with low- or with high-ability workers) than LT contracts! This happens when $E_H > U_H^o$ (and $w_J^{STo} > \underline{w}$) in which case, junior workers accept really low salaries because of the fact that they may migrate, when senior, to a firm with a very high R, which gives them high informational rents.

Figure 4: Types of equilibria as a function of $\widehat{R}$

Finally, we consider the case where $\underline{R}$ is large (or $\widehat{R}$ is small), that is, only firms with high-valued projects are in the market.

Theorem 3 *Suppose $\widehat{R} < \underline{R}$. Then, under Assumption 1, no equilibrium exists where ST contracts are adopted.*

When $\underline{R} > \widehat{R}$, all firms value their projects enough so that none is ready to offer ST contracts to always keep low-ability workers. They would rather offer LT agreements, which ensures them high-ability workers with some probability.

Figure 4 represents the three types of equilibrium configurations, obtained in theorems 1, 2 and 3, as a function of $\widehat{R}$ and R^o.

Is the market equilibrium efficient? In particular, is the assignment of firms to type of contract efficient, or there would be a gain in efficiency by expanding, shrinking, or changing the set of firms that offer ST agreements? It is easy to check that the equilibrium highlighted in theorems 1 and 2 is strictly more efficient than that in Proposition 1. First, firms that choose ST contracts could have chosen LT contracts, therefore their profits are higher (they are strictly higher in the interior of the regions). Second, as we discussed after Theorem 1, workers obtain higher expected utility under ST contracts. Therefore, the use of ST contracts unambiguously leads to a Pareto improvement.

If the efficient assignment of firms to type of contract involves the use of ST contracts, the set of firms will be divided in such a way that those with the lowest R sign ST contracts with low-ability senior agents and firms with the highest R sign ST contracts with high-ability senior agents. By arguments similar to those before, shrinking the set of firms and workers that sign ST contracts at equilibrium cannot be beneficial: everyone involved would lose. On the other hand, expanding the set of firms that use ST contracts at equilibrium can improve efficiency. To see the reason, consider the case where $\widehat{R} \in [\underline{R}, R^o)$, that is, the condition that leads to Theorem 1. If firms in the intervals $\left(\widehat{R}, \widehat{R}+\varepsilon\right]$ and $[R^{oo}-\mu, R^{oo})$ (where ε and μ are small and such that the proportion of firms in the intervals is the same as the rate of low- versus high-ability seniors) switch to ST contracts, then both marginally lose profits. However, the workers strictly obtain higher expected utility. Therefore, the sum of profits and utility increases. This argument suggests that firms have too much incentives, compared to the social optimum, to use LT contracts.

Assumption 1 is a *sufficient* condition that allows a precise separation of markets: when there is a large concentration of firms with a low R, then we should observe only ST contracts while both LT and ST contracts coexist at equilibrium otherwise. However, it is not a necessary condition.

An alternative scenario where Theorems 1 to 3 are easy to replicate is one where senior agents are much more "important" than junior agents and, therefore, irrespective of their ability they have more market value. In this case, $\underline{U}_S$ is much larger than $\underline{U}_J$ and/or the discount rate δ is high.[18]

Assumption 2 *The parameters satisfy the following condition:*

$$\frac{\delta\left(\underline{U}_S-\underline{w}\right)}{\left(\underline{U}_J-\underline{w}\right)}>\frac{1}{\left(p_H^2-p_L^2\right)}\max\left\{\frac{1}{q}p_L^2, \frac{1}{(1-q)}\widetilde{p}^2\right\}.$$

Theorem 4 shows that (Lemma 1 and) Theorems 1 to 3 indeed hold if we replace Assumption 1 by Assumption 2.

Theorem 4 *Suppose Assumption 2 holds. Then:*
(A) if $\underline{R}\leq\widehat{R}<R^o$, the strategies proposed in (i) to (v) from Theorem 1 constitute an equilibrium;
(B) if $R^o\leq\widehat{R}$, the strategies proposed in (i) to (iv) from Theorem 2 constitute an equilibrium; and
(C) if $\widehat{R}<\underline{R}$, no equilibrium exists where ST contracts are used (as in Theorem 3).

[18] In a broad interpretation, the parameter δ might also reflect the ratio of the length of the relationship of a senior agent with the firm versus the length of the relationship of the junior agent. According to this broad interpretation, δ could be larger than 1. However, this would require adapting the model to accomodate for firms with one junior and several senior agents of different cohorts or the other way around.

Our analysis allows us to predict the circumstances under which ST contracts are more likely to emerge ($\underline{R} \leq \widehat{R}$) and those under which only ST contracts exist at equilibrium ($R^o \leq \widehat{R}$). As to the distribution of firms, that affects $\underline{R}$ and R^o, we should expect ST contracts when there are some firms whose projects are not too profitable, so that they are ready to hire through ST agreements to save on the juniors' wage. Moreover, only ST contracts appear at equilibrium when the firms are mostly concentrated on low levels of R. On the other hand, we should observe the coexistence of ST and LT contracts in markets where R^o is high but, simultaneously, $\underline{R}$ is low enough. In this case, there is a relevant fraction of firms with quite profitable projects (i.e., firms for which hiring a good senior really makes a difference) and, simultaneously, there are firms that give little value to the project.

For a given distribution of firms, the equilibrium includes ST contracts when $\widehat{R}$ is high enough. Next corollary identifies the characteristics of the parameters of the model (affecting $\widehat{R}$) that make the presence of, at least, some firms with ST contracts more likely.

Corollary 3 *The existence of short-term contracts at equilibrium is, ceteris paribus, more likely,*
(a) the lower is the difference between high- and low-ability seniors ($p_H^2 - p_L^2$)
(b) the higher is the reservation utility of young workers ($\underline{U}_J$) *and the lower the minimum wage* ($\underline{w}$),
(c) the higher the discount rate (i.e., high δ*), and*
(d) the more costly is the agents' effort (low k*)*

Remember that $\widehat{R}$ is the value that makes the firm indifferent between using LT contracts and ST agreements to hire a low-ability senior, when $w_J^{ST} = \underline{w}$. Why does $\widehat{R}$ decrease when $(p_H^2 - p_L^2)$ increases? If $(p_H^2 - p_L^2)$ is larger, the optimal sorting between senior workers and firms becomes more relevant, which would suggest that ST contracts should prevail more often. However, this is not the case because the relevant question is whether the firm ($\widehat{R}$) that was indifferent between LT and ST contracts with low-ability agents, prefers LT or ST contracts once $(p_H^2 - p_L^2)$ has increased. Given that $w_J^{ST} = \underline{w}$, an increase in $(p_H^2 - p_L^2)$ does not provide any additional saving in costs under ST contracts, which is the reason for a firm to choose an ST strategy to hire low-ability agents. However, it does increase the benefits accruing to those firms if they use LT contracts, as the difference between a high- and a low-ability agent is larger. Therefore, increasing $(p_H^2 - p_L^2)$ provides additional incentives for a firm to prefer an LT contract to an ST contract to hire low-ability agents and, as a consequence, LT contracts are more likely.

A similar argument explains why ST contracts are more frequent when $(\underline{U}_J - \underline{w})$ is higher: the benefits from ST agreements with low-ability agents decrease. Also, a lower δ implies that firms care more about today's cost savings (through ST contracts) than about future gains (from LT agreements); therefore, ST contracts are more profitable. Finally, a higher k means that firms can make less profit out of the same workers, which again favors the presence of ST contracts.

Note that, in general, we can not ensure the effect that an increase in the proportion of high-ability seniors has on the emergence of ST contracting at equilibrium. The reason is that, even if an increase in q reduces the value of $\widehat{R}$, which favors the coexistence of short- and long-term contracts, it also reduces the value of R^o, which works in the opposite direction. Therefore, unless we make an explicit assumption about the distribution of firms in the R space $(G(R))$, we cannot predict the exact effect of q on the market outcome.

Finally, we briefly comment on the hypothesis concerning the set of active firms. We have assumed that the set of firms active in the market is fixed and is measured by the distribution function $G(R)$ on the set $\left[\underline{R}, \overline{R}\right]$. Therefore, we have made the implicit assumption that, irrespective of the market outcome, all firms find it worthwhile to stay active in the market. This assumption can be sustained either because we consider that the worst firm has a sufficiently profitable project (i.e., $\underline{R}$ is high enough) or because the fixed component of the firms' activity (F) is sufficiently high. However, in general, the set of active firms is endogenous; it depends on the profitability of the market.

Suppose that the set of potential firms is distributed in the interval $\left[0, \overline{R}\right]$ according to some distribution function $J(R)$. The set of active firms $\left[\underline{R}, \overline{R}\right]$ will be determined by the condition that, at the market equilibrium, the profits of the firm $\underline{R}$ are zero and the distribution $G(R)$ will be derived from $J(R)$. The level of $\underline{R}$ will depend on the different parameters of the model. We can discuss, for example, how changes in the parameter F affect the set of active firms and the type of contracts signed in the market.

If F is very low, then the minimum $\underline{R}$ (and also R^o) is high. Therefore, $\widehat{R}$ (which does not depend on the distribution of firms) is lower than $\underline{R}$ and, at equilibrium, only LT contracts are signed. When F increases, both $\underline{R}$ and R^o increase until we reach the region where ST contracts appear for low and high values of R. Following the same logic as in Figure 4, additional increases in F lead to larger regions with ST contracts, until only ST contracts exist at equilibrium. Therefore, we should expect a prevalence of ST contracts in those markets where the "fixed" component of the revenue (that is, the component that does not depend on incentives) is large, while we should observe LT contracts in those markets where most of the income comes from work subject to moral hazard. This, in fact, reflects the main trade-off any firm faces. Focusing solely on incentives, LT contracts are better as they can exploit the intertemporal nature of the relationship in order to alleviate the moral hazard problem. On the other hand, ST contracts offer an added flexibility that improves the efficiency of the firm/worker matching. If the market characteristics are such that incentives play an important role in all firms' profits, then LT contracts prevail. However, if an appropriate worker selection is the key issue, then ST contracts are used at equilibrium.

7 Conclusion

We have introduced and analyzed an equilibrium model to discuss advantages and disadvantages of short-term versus long-term contracts in a dynamic environment where senior workers are subject to a moral hazard problem. On the one hand, long-term contracts allow the better provision of incentives because firms can credibly transfer payments from early to late periods in the life of the workers, and this transfer alleviates the incentive compatibility constraint. On the other hand, short-term contracts allow the market to ensure a better matching between agents' ability and firms' needs. Those agents that turn out to have high ability can be hired by firms that can really profit from them.

We solve the model for markets where most workers have a standard ability but a small proportion of them have high productivity; they are stars. Moreover, these stars really make a difference in the firms they work for.

When firms operating very profitable projects[19] identify very talented workers, they are ready to offer very high salaries or bonuses to these stars. High-ability seniors end

[19] Projects where the incentives of the senior play an important role in firms' profits.

up receiving high remuneration when ST contract are in place which, in turn, allows the reduction of the payment to juniors, who foresee the prospects of a very high wage when seniors. Consequently, some firms having less lucrative ventures[20] may not be able to retain the high-talented workers, but they indirectly profit from the existence of such workers as it allows them to hire juniors at a much lower cost.

At equilibrium, we often find that two types of firms use short-term contracts: firms in which the success of the project depends very much on the senior's effort, which always end up hiring high-ability senior workers; and firms whose profits do not depend too much on the effort, which hire low-ability senior workers. Intermediate firms may use long-term or short-term contracts, depending on several market characteristics. We show that coexistence of both types of contract is more likely when there is a relevant fraction of firms with profitable projects, when the reservation utility of young workers is low and the minimum wage is high, when the discount rate is small, when there is a large difference between the productivity of high- and low-ability workers, and when the agents' effort is not too costly.

In addition to the equilibrium with short-term contracts that often exists, there always exists an equilibrium where all firms choose a long-term contracts (see Proposition 1). However, we argue that, in our environment, whenever the equilibrium with short-term contracts and the one with only LT contracts coexist, the former is more "robust" or "sensible" as the latter is a "knife-edge" result. The full long-term outcome is sustained by the fact that, since no other firm is choosing a short-term contract, no firm can profit from the enhanced flexibility that short-term contracts offer. A small amount of firms with low-valued projects and another with high-valued projects have incentives to switch from LT to ST agreements to obtain higher profits.

Appendix

A Proof of Proposition 1

Proof. We first note that, in a situation where all firms sign LT contracts, if a firm follows the strategy of offering ST contracts to its workers, it necessarily hires as senior

[20] Firms where the role of the senior is less important for the outcome.

agent at period t the same agent that it hired as a junior at period $t-1$. Also, the only alternative occupation for the senior agent is to get out of the market, since no other firm is interested in hiring him, independent on his ability. Then, any sequence of ST contracts can be replicated as an LT contract. Therefore, the optimal ST contracts cannot give higher profits than the optimal LT contracts. ■

B Proof of Proposition 2

Proof. Substituting e_H and e_L by their value and multiplying the objective function by δ, the firm's program can be rewritten as:

$$\max_{(w_J, w_H, \Delta_H, w_L, \Delta_L)} \delta F + \delta q \left(\frac{1}{2k^2} p_H^2 \Delta_H (R - \Delta_H) - w_H \right) + \delta(1-q) \left(\frac{1}{2k^2} p_L^2 \Delta_L (R - \Delta_L) - w_L \right) - w_J$$

$$\text{s.t. } w_J + \delta q \left(\frac{1}{4k^2} p_H^2 \Delta_H^2 + w_H \right) + \delta(1-q) \left(\frac{1}{2k^2} p_L^2 \Delta_L^2 + w_L \right) \geq \underline{U}_J + \delta \underline{U}_S \qquad (1)$$

$$w_J \geq \underline{w}, \qquad w_H \geq \underline{w}, \qquad w_L \geq \underline{w}.$$

Let λ, μ, ρ_H and ρ_L be the Lagrange multipliers corresponding to the constraints. The Kuhn-Tucker (first-order) conditions of the above maximization problem include the constraints, and the non-negativity of the multipliers: $\lambda \geq 0$, $\mu \geq 0$, $\rho_H \geq 0$, $\rho_L \geq 0$. The derivatives of the Lagrangian with respect to Δ_H and Δ_L are:

$$\delta q \frac{1}{2k^2} p_H^2 (R - 2\Delta_H) + \lambda \delta q \frac{1}{2k^2} p_H^2 \Delta_H = 0 \qquad (2)$$

$$\delta (1-q) \frac{1}{2k^2} p_L^2 (R - 2\Delta_L) + \lambda \delta (1-q) \frac{1}{2k^2} p_L^2 \Delta_L = 0, \qquad (3)$$

which imply that $\Delta_H = \Delta_L$, which we denote Δ in the rest of the proof.
The derivatives of the Lagrangian with respect to w_J, w_H and w_L are:

$$-1 + \lambda + \mu = 0,$$

$$\begin{aligned} -\delta q + \lambda \delta q + \rho_H &= 0, \\ -\delta(1-q) + \lambda \delta(1-q) + \rho_L &= 0, \end{aligned} \qquad (4)$$

which imply $\mu = 1 - \lambda$, $\rho_H = \delta q (1-\lambda)$ and $\rho_H = \delta(1-q)(1-\lambda)$; therefore, either the three constraints are binding or none is. The last Kuhn-Tucker conditions are:

$$\lambda \left[\underline{w} + \delta \left[q \left(w_H + \frac{1}{4k^2} p_H^2 \Delta_H^2 \right) + (1-q) \left(w_L + \frac{1}{4k^2} p_L^2 \Delta_L^2 \right) \right] - (\underline{U}_J + \delta \underline{U}_S) \right] = 0$$

$$\mu\left(w_J - \underline{w}\right) = 0$$

$$\rho_H\left(w_H - \underline{w}\right) = 0$$

$$\rho_L\left(w_L - \underline{w}\right) = 0.$$

From (2) and (4) we can deduce that:

$$\lambda = 2 - \frac{R}{\Delta} \qquad \text{and} \qquad \rho_H = \delta q\left(\frac{R}{\Delta} - 1\right).$$

We study the different regions where the Kuhn-Tucker conditions may be satisfied:
Case 1: $\lambda > 0$, $\mu > 0$, $\rho_H > 0$, $\rho_L > 0$. Payments when young and in case of failure are $w_J = w_H = w_L = \underline{w}$ and the bonus in case of success is $\Delta = \frac{2k}{\widetilde{p}}\sqrt{\frac{1}{\delta}\left[\left(\underline{U}_J + \delta\underline{U}_S\right) - (1+\delta)\,\underline{w}\right]}$. Finally, this is a candidate only if $\lambda \in [0,1]$, i.e.,

$$R \in \left[\frac{2k}{\widetilde{p}}\sqrt{\frac{1}{\delta}\left[\left(\underline{U}_J + \delta\underline{U}_S\right) - (1+\delta)\,\underline{w}\right]}, \frac{4k}{\widetilde{p}}\sqrt{\frac{1}{\delta}\left[\left(\underline{U}_J + \delta\underline{U}_S\right) - (1+\delta)\,\underline{w}\right]}\right].$$

Case 2: $\lambda = 0$, $\mu > 0$, $\rho_H > 0$, $\rho_L > 0$. Then $w_J = w_H = w_L = \underline{w}$, and $\Delta = \frac{R}{2}$. In this case the participation constraint holds only if $R \geq \frac{4k}{\widetilde{p}}\sqrt{\frac{1}{\delta}\left[\left(\underline{U}_J + \delta\underline{U}_S\right) - (1+\delta)\,\underline{w}\right]}$. (The candidate at the lower bound of this case coincides with the candidate at the higher bound of Case 1.)
Case 3: $\mu = \rho_H = \rho_L = 0$. Then $\lambda = 1$ and $\Delta = R$. We write the participation constraint as

$$w_J + \delta\left(qw_H + (1-q)\,w_L\right) + \delta\frac{1}{4k^2}R^2\widetilde{p}^2 = \underline{U}_J + \delta\underline{U}_S.$$

Any combination of w_J, w_H and w_L that satisfies the previous constraint and such that the three values are larger or equal to $\underline{w}$ constitutes an optimal solution (in particular, the values proposed in the proposition). This can be the case only if $w_J + \delta\left(qw_H + (1-q)\,w_L\right) \geq (1+\delta)\,\underline{w}$, that is $R \leq \frac{2k}{\widetilde{p}}\sqrt{\frac{1}{\delta}\left[\left(\underline{U}_J + \delta\underline{U}_S\right) - (1+\delta)\,\underline{w}\right]}$.
The unique candidate for each value of R is the optimal solution of the firm's maximization program. From the optimal contract in each case, it is immediate to compute agent's effort(s) and utility, and firm's profits. Additionally, easy calculations show that the function $E\pi^{LT}(R)$ is continuously differentiable in R. ■

C Proof of Proposition 3

Proof. Substituting e_i by its value in the firm's program, we can rewrite it as

$$\max_{(w_i,\Delta_i)} F + \frac{1}{2k^2} p_i^2 \Delta_i (R - \Delta_i) - w_i$$
$$\text{s.t.} \qquad \frac{1}{4k^2} p_i^2 \Delta_i^2 + w_i \geq U_i$$
$$w_i \geq \underline{w}.$$

Let λ, ρ be the Lagrange multipliers corresponding to the constraints. The Kuhn-Tucker (first-order) conditions of the above maximization problem include the constraints, and the non-negativity of the multipliers: $\lambda \geq 0$, $\rho \geq 0$. The derivatives of the Lagrangian with respect to w_i and Δ_i are

$$-1 + \lambda + \rho = 0, \tag{5}$$
$$\frac{1}{2k^2} p_i^2 (R - 2\Delta_i) + \lambda \frac{1}{2k^2} p_i^2 \Delta_i = 0. \tag{6}$$

From (5) and (6) we can deduce that:

$$\lambda = 2 - \frac{R}{\Delta_i} \qquad \text{and} \qquad \rho = \frac{R}{\Delta_i} - 1.$$

We study the different regions.

Case 1: $\lambda > 0$, $\rho > 0$. Payment are $w_i = \underline{w}$ and $\Delta_i = \frac{2k}{p_i}\sqrt{U_i - \underline{w}}$. This is a candidate only if $\lambda \geq 0$ and $\rho \geq 0$, i.e., $R \in \left[\frac{2k}{p_i}\sqrt{U_i - \underline{w}}, \frac{4k}{p_i}\sqrt{U_i - \underline{w}}\right]$.

Case 2: $\lambda = 0$, $\rho > 0$. Then $w_i = \underline{w}$, and $\Delta_i = \frac{R}{2}$. In this case the participation constraint holds only if $R \geq \frac{4k}{p_i}\sqrt{U_i - \underline{w}}$.

Case 3: $\rho = 0$. Then $\Delta_i = R$, which implies $\lambda = 1 > 0$. The participation constraint is $\frac{1}{4k^2} p_i^2 R^2 + w_i = U_i$. Therefore, $w_i = U_i - \frac{1}{4k^2} p_i^2 R^2 \geq \underline{w}$ if and only if $R \leq \frac{2k}{p_i}\sqrt{U_i - \underline{w}}$.

The unique candidate for each value of R is the optimal solution of the firm's maximization program. From the optimal contract in each case, it is immediate to compute agent's effort(s) and utility. ■

D Proof of Lemma 1

Proof. We highlight that the three derivatives that we consider in the lemma, $\frac{\partial E\tilde{\pi}_L^{ST}}{\partial R}(R, w_J, \underline{U}_S)$, $\frac{\partial E\tilde{\pi}^{LT}}{\partial R}(R)$, and $\frac{\partial E\tilde{\pi}_H^{ST}}{\partial R}(R, w_J, U_H)$, have a similar shape: they are first linear in R until they reach some R_1 (either $R_{L1}^{ST}(\underline{U}_S)$, or R_1^{LT}, or $R_{H1}^{ST}(U_H)$), then they are constant until they reach a second threshold R_2 and, from R_2 on, they are linear in R again. The proof of the three parts in the lemma is similar. We write a complete proof of part (a) and we point out the main elements of parts (b) and (c).

(a) First, notice that if R lies in both regions $a_L^{ST}(\underline{U}_S)$ and a^{LT}, $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R} = \frac{1}{2k^2}p_L^2 R < \frac{1}{2k^2}\widetilde{p}^2 R = \frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$. The same comparison holds if R lies in both regions $c_L^{ST}(\underline{U}_S)$ and c^{LT}. Additionally, if R lies in both regions $b_L^{ST}(\underline{U}_S)$ and b^{LT}, $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R} = \frac{1}{k}p_L\sqrt{\underline{U}_S - \underline{w}} < \frac{1}{k}\widetilde{p}\sqrt{\frac{1}{\delta}(\underline{U}_J - \underline{w}) + \underline{U}_S - \underline{w}} = \frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$.

Second, if $R_1^{LT} \geq R_{L1}^{ST}(\underline{U}_S)$ (and $R_2^{LT} \geq R_{L2}^{ST}(\underline{U}_S)$), then $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$ is increasing in a larger region of parameters than $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}$ before becoming constant (at a higher level than $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}$ in region $b_L^{ST}(\underline{U}_S)$). Finally, even if $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}$ starts increasing again (i.e., it reaches region $c_L^{ST}(\underline{U}_S)$) before $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$ (because $R_{L2}^{ST}(\underline{U}_S) \leq R_2^{LT}$), it is always lower than the latter, since it is lower even when $R = R_2^{LT}$, given that we have seen that $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R} < \frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$ for any R which lies in both regions $c_L^{ST}(\underline{U}_S)$ and c^{LT}.

Third, suppose $R_1^{LT} < R_{L1}^{ST}(\underline{U}_S)$ (and $R_2^{LT} < R_{L2}^{ST}(\underline{U}_S)$). Given that $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}$ is smaller than $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$ when $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R}$ reaches the region where it becomes constant, and that it is certainly also smaller when it starts increasing again (because $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$ has reached this region before), it is not possible that the two derivatives cross. Therefore, $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R} < \frac{\partial E\widetilde{\pi}^{LT}}{\partial R}$ for any $R > 0$.

(b) If R lies in both regions $a_L^{ST}(\underline{U}_S)$ and $a_H^{ST}(U_H)$, $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R} = \frac{1}{2k^2}p_L^2 R < \frac{1}{2k^2}p_H^2 R = \frac{\partial E\widetilde{\pi}_H^{ST}}{\partial R}$. The same comparison holds in regions $c_L^{ST}(\underline{U}_S)$ and $c_H^{ST}(U_H)$. Also, if R lies in both regions $b_L^{ST}(\underline{U}_S)$ and $b_H^{ST}(U_H)$, $\frac{\partial E\widetilde{\pi}_L^{ST}}{\partial R} = \frac{1}{k}p_L\sqrt{\underline{U}_S - \underline{w}} < \frac{1}{k}p_H\sqrt{U_H - \underline{w}} = \frac{\partial E\widetilde{\pi}_H^{ST}}{\partial R}$. The rest of the proof is identical to the one in part (a).

(c) For R in both regions a^{LT} and $a_H^{ST}(U_H)$ (and similarly in c^{LT} and $c_H^{ST}(U_H)$), $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R} = \frac{1}{2k^2}\widetilde{p}^2 R < \frac{1}{2k^2}p_H^2 R = \frac{\partial E\widetilde{\pi}_H^{ST}}{\partial R}$. If R lies in both regions b^{LT} and $b_H^{ST}(U_H)$, then $\frac{\partial E\widetilde{\pi}^{LT}}{\partial R} = \frac{1}{k}\widetilde{p}\sqrt{\frac{1}{\delta}(\underline{U}_J - \underline{w}) + \underline{U}_S - \underline{w}} < \frac{1}{k}p_H\sqrt{U_H - \underline{w}} = \frac{\partial E\widetilde{\pi}_H^{ST}}{\partial R}$ if and only if $U_H > \frac{\widetilde{p}^2}{p_H^2}\left[\frac{1}{\delta}(\underline{U}_J - \underline{w}) + \underline{U}_S - \underline{w}\right] + \underline{w}$. If this inequality holds, the rest of the proof of Lemma (c) is identical to the one in part (a). A sufficient condition is

$$\underline{U}_S \geq \frac{\widetilde{p}^2}{p_H^2}\left[\frac{1}{\delta}(\underline{U}_J - \underline{w}) + \underline{U}_S - \underline{w}\right] + \underline{w} \tag{7}$$

which, given Assumption 1 (i), is implied by $\delta\left(p_H^2 - \widetilde{p}^2\right) > \widetilde{p}^2$, i.e., $\delta(1-q)\left(p_H^2 - p_L^2\right) > qp_H^2 + (1-q)p_L^2$, or, $\left(\delta(1-q) - q\right)\frac{p_H^2}{p_L^2} > (1+\delta)(1-q)$. Assumption 1 (ii) implies that $\delta(1-$

$q)-q>0$. Therefore, given Assumption 1 (iii), the inequality holds if $(\delta(1-q)-q)\left(1+\frac{1}{\delta q}\right) \geq (1+\delta)(1-q)$, i.e., $\delta(1-q)-q \geq 0$, which closes the proofs. ■

E Proof of Theorem 1

Proof. We do the proof through a series of claims.

Claim 1: If $w_J^{ST} = \underline{w}$, then $\widehat{R} < R_{L1}^{ST}(\underline{U}_S)$ and $\widehat{R} < R_1^{LT}$.

Proof of Claim 1: If the value $\widehat{R}$ that satisfies $E\widetilde{\pi}^{LT}(\widehat{R}) = E\widetilde{\pi}_L^{ST}(\widehat{R}, \underline{w}, \underline{U}_S)$ lies in both regions a^{LT} and $a_L^{ST}(\underline{U}_S)$ (i.e., $\widehat{R} < R_{L1}^{ST}(\underline{U}_S)$ and $\widehat{R} < R_1^{LT}$), then $\widehat{R} = \frac{2k}{\sqrt{\delta q\left(p_H^2 - p_L^2\right)}}\sqrt{\underline{U}_J - \underline{w}}$. Moreover, it is easy to check that each of the inequalities $\widehat{R} < R_{L1}^{ST}(\underline{U}_S)$ and $\widehat{R} < R_1^{LT}$ is equivalent to the following:

$$p_L^2\left(\underline{U}_J - \underline{w}\right) < \delta q\left(p_H^2 - p_L^2\right)\left(\underline{U}_S - \underline{w}\right). \tag{8}$$

Given Assumption 1 (i), (8) is implied by Assumption 1 (ii).

Claim 2: Consider the value $\widehat{U}_H$ such that $E\widetilde{\pi}_L^{ST}\left(\widehat{R}, w_J^{ST}\right) = E\widetilde{\pi}_H^{ST}\left(\widehat{R}, w_J^{ST}, U_H = \widehat{U}_H\right)$. If junior workers anticipate that they will obtain at least $\widehat{U}_H$ when senior if they turn out to be high-ability, then they are ready to accept $w_J^{ST} = \underline{w}$.

Proof of Claim 2: We proceed as follows. We conjecture that $\widehat{U}_H$ is such that $\widehat{R} < R_{H1}^{ST}\left(\widehat{U}_H\right) = \frac{2k}{p_H}\sqrt{\widehat{U}_H - \underline{w}}$, we will compute the corresponding $\widehat{U}_H$ in this region, and then we will show that it is indeed the case that $\widehat{R} < R_{H1}^{ST}\left(\widehat{U}_H\right)$. Therefore, $\widehat{U}_H$ is defined by

$$F - \frac{1}{\delta}w_J^{ST} - \underline{U}_S + \left(\frac{p_L}{2k}\right)^2\widehat{R}^2 = F - \frac{1}{\delta}w_J^{ST} - \widehat{U}_H + \left(\frac{p_H}{2k}\right)^2\widehat{R}^2$$

i.e., $\widehat{U}_H = \underline{U}_S + \left(\frac{1}{2k}\right)^2\left(p_H^2 - p_L^2\right)\widehat{R}^2$ or $\widehat{U}_H = \underline{U}_S + \frac{1}{\delta q}\left(\underline{U}_J - \underline{w}\right)$. For this value, $R_{H1}^{ST}\left(\widehat{U}_H\right) = \frac{2k}{p_H}\sqrt{\underline{U}_S + \left(\frac{1}{2k}\right)^2\left(p_H^2 - p_L^2\right)\widehat{R}^2 - \underline{w}}$. Therefore, $\widehat{R} < R_{H1}^{ST}\left(\widehat{U}_H\right)$ holds if and only if $\widehat{R}^2 < \left(\frac{2k}{p_H}\right)^2\left[\underline{U}_S + \left(\frac{1}{2k}\right)^2\left(p_H^2 - p_L^2\right)\widehat{R}^2 - \underline{w}\right]$, i.e., $p_L^2\widehat{R}^2 < (2k)^2\left(\underline{U}_S - \underline{w}\right)$, which is equivalent to (8). Finally, given $\widehat{U}_H$, and taking into account that $E_H \geq \widehat{U}_H$ and $E_L \geq \underline{U}_S$, a junior worker is ready to accept an ST contract with w_J^{ST} whenever $w_J^{ST} + \delta\underline{U}_S + \delta q\left(\frac{1}{2k}\right)^2\left(p_H^2 - p_L^2\right)\widehat{R}^2 \geq \underline{U}_J + \delta\underline{U}_S$, that is, when $w_J^{ST} \geq \underline{w}$.

Claim 3: $E\widetilde{\pi}_L^{ST}\left(R, w_J^{ST} = \underline{w}, \underline{U}_S\right) > E\widetilde{\pi}_H^{ST}\left(R, w_J^{ST} = \underline{w}, U_H\right)$ for any $U_H \geq \widehat{U}_H$ and for any $R < \widehat{R}$.

Proof of Claim 3: Given the definition of $\widehat{U}_H$ in Claim 3, $E\widetilde{\pi}_L^{ST}\left(\widehat{R}, w_J^{ST}=\underline{w}, \underline{U}_S\right) \geq E\widetilde{\pi}_H^{ST}\left(\widehat{R}, w_J^{ST}=\underline{w}, U_H\right)$ for any $U_H \geq \widehat{U}_H$. Then, the claim follows after Lemma 1 (b).

Claim 4: $E\widetilde{\pi}_L^{ST}\left(R, w_J^{ST}=\underline{w}, \underline{U}_S\right) > \max\left\{E\widetilde{\pi}_H^{ST}\left(R, w_J^{ST}=\underline{w}, U_H^{oo}\right), E\widetilde{\pi}^{LT}(R)\right\}$ for any $R < \widehat{R}$.

Proof of Claim 4: The first inequality follows after Claim 3, also taking into account that $R^{oo} > \widehat{R}$ implies $U_H^{oo} > \widehat{U}_H$. The second inequality follows the definition of $\widehat{R}$ and Lemma 1 (a).

Claim 5: $E\widetilde{\pi}^{LT}(R) \geq \max\left\{E\widetilde{\pi}_H^{ST}\left(R, w_J^{ST}=\underline{w}, U_H^{oo}\right), E\widetilde{\pi}_L^{ST}\left(R, w_J^{ST}=\underline{w}, \underline{U}_S\right)\right\}$ for any $R \in \left[\widehat{R}, R^{oo}\right]$.

Proof of Claim 5: The first part of the inequality follows after the characterization of U_H^{oo} in part (vi) of the theorem, by the property that $U_H^{oo} > \widehat{U}_H$ and Lemma 1 (c). The second part follows the definition of $\widehat{R}$ and Lemma 1 (a).

Claim 6: $E\widetilde{\pi}_H^{ST}\left(R, w_J^{ST}=\underline{w}, U_H^{oo}\right) > \max\left\{E\widetilde{\pi}^{LT}(R), E\widetilde{\pi}_L^{ST}\left(R, w_J^{ST}=\underline{w}, \underline{U}_S\right)\right\}$ for any $R > R^{oo}$.

Proof of Claim 6: By the same argument as in Claim 5, the maximum of the two terms inside the maximization is $E\widetilde{\pi}^{LT}(R)$. Then, the inequality is implied by the characterization of U_H^{oo} in part (vi) of the theorem, by the property that $U_H^{oo} > \widehat{U}_H$ and Lemma 1 (c). ■

F Proof of Theorem 2

Proof. Given that the behavior of the workers is optimal by construction, we prove the theorem if we show that firms' strategies are optimal. We do it through a series of claims.

Claim 1: $w_J^{STo} \leq \underline{U}_J - \delta q\left(\frac{R^o}{2k}\right)^2\left(p_H^2 - p_L^2\right).$

Proof of Claim 1. Given that $E_H \geq U_H^o$ and $E_L \geq \underline{U}_S$, $\underline{U}_J - \delta\left(qE_H + (1-q)E_L - \underline{U}_S\right) \leq \underline{U}_J - \delta q\left(\frac{R^o}{2k}\right)^2\left(p_H^2 - p_L^2\right)$. Moreover $\underline{w} \leq \underline{U}_J - \delta q\left(\frac{R^o}{2k}\right)^2\left(p_H^2 - p_L^2\right)$ because this inequality is equivalent to $R^o \leq \widehat{R}$.

Claim 2: $R^o \leq R_{H1}^{ST}\left(U_H^o\right).$

Proof of Claim 2: $R^o \leq \frac{2k}{p_H}\sqrt{\underline{U}_S + \left(\frac{R^o}{2k}\right)^2\left(p_H^2 - p_L^2\right) - \underline{w}}$ if and only if $R^o \leq \frac{2k}{p_L}\sqrt{\underline{U}_S - \underline{w}} = R_{L1}^{ST}\left(\underline{U}_S\right)$, which is implied by the fact that $R^o \leq \widehat{R}$ and $\widehat{R} \leq R_{L1}^{ST}\left(\underline{U}_S\right)$ (by Claim 1 in the proof of Theorem 1).

Claim 3: $E\pi_H^{ST}\left(R^o, w_J^{STo}, U_H^o\right) = E\pi_L^{ST}\left(R^o, w_J^{STo}\right) \geq E\pi^{LT}\left(R^o\right).$

Proof of Claim 3. Given that $R^o \leq R_{L1}^{ST}(\underline{U}_S)$ and $R^o \leq R_{H1}^{ST}(U_H^o)$, the first equality comes directly from the definition of U_H^o. To prove the inequality, we notice that $R^o \leq R_1^{LT}$, because $R^o \leq \widehat{R}$ and $\widehat{R} \leq R_1^{LT}$ (by Claim 1 in the proof of Theorem 1). Given that $R^o \leq R_1^{LT}$ and $R^o \leq R_{L1}^{ST}(\underline{U}_S)$, the inequality can be written as $-w_J^{STo} + \left(\frac{p_L R^o}{2k}\right)^2 \geq -\underline{w} - \frac{1}{\delta}(\underline{U}_J - \underline{w}) + \left(\frac{\widetilde{p} R^o}{2k}\right)^2$. By Claim 1, a sufficient condition is $-\left(\underline{U}_J - \delta q \left(\frac{R^o}{2k}\right)^2 (p_H^2 - p_L^2)\right) + \left(\frac{p_L R^o}{2k}\right)^2 \geq -\underline{w} - \frac{1}{\delta}(\underline{U}_J - \underline{w}) + \left(\frac{\widetilde{p} R^o}{2k}\right)^2$. This inequality holds because it is equivalent to $R^o \leq \widehat{R}$.

Claim 4: $E\widetilde{\pi}_L^{ST}\left(R, w_J^{STo}\right) > \max\left\{E\widetilde{\pi}_H^{ST}(R, w_J^o, U_H^o), E\widetilde{\pi}^{LT}(R)\right\}$, for any $R < R^o$.

Proof of Claim 4: It follows from Claim 3 and Lemma 1 (a) and (b).

Claim 5: $E\widetilde{\pi}_H^{ST}(R, w_J^o, U_H^o) \geq \max\left\{E\widetilde{\pi}_L^{ST}\left(R, w_J^{STo}\right), E\widetilde{\pi}^{LT}(R)\right\}$, for any $R \geq R^o$.

Proof of Claim 5: It follows from Claim 3 and Lemma 1 (b) and (c). ■

G Proof of Theorem 3

Proof. We recall that $\widehat{R}$ is characterized by $E\widetilde{\pi}^{LT}(\widehat{R}) = E\widetilde{\pi}_L^{ST}(\widehat{R}, \underline{w}, \underline{U}_S)$. If $\widehat{R} < \underline{R}$, then $\widehat{R} < R$ for all $R \in \left[\underline{R}, \overline{R}\right]$. Therefore, Lemma 1 (b) implies $E\widetilde{\pi}^{LT}(R) > E\widetilde{\pi}_L^{ST}(R, \underline{w}, \underline{U}_S)$ for all $R \in \left[\underline{R}, \overline{R}\right]$. It easily follows that $E\widetilde{\pi}^{LT}(R) > E\widetilde{\pi}_L^{ST}(R, w_J, U_L)$ for all $R \in \left[\underline{R}, \overline{R}\right]$, $w_J \geq \underline{w}$ and $U_L \geq \underline{U}_S$. Therefore, at equilibrium, no ST contract can be signed, since it would imply that some firms choose the strategy of keeping low-ability senior workers, which is dominated by the strategy of always offering LT contracts. ■

H Proof of Theorem 4

Proof. The proofs of theorems 1 and 2 and that of Lemma 1 only use Assumption 1 to show that the inequalities (7) and (8) hold. Therefore, we prove theorem 4 if we show that Assumption 2 also imply (7) and (8). We write Assumption 2 as

$$\delta q \left(p_H^2 - p_L^2\right)(\underline{U}_S - \underline{w}) > p_L^2 (\underline{U}_J - \underline{w})$$

and

$$\delta(1-q)\left(p_H^2 - p_L^2\right)(\underline{U}_S - \underline{w}) > \widetilde{p}^2 (\underline{U}_J - \underline{w}).$$

The first inequality corresponds to (8). Moreover, it is easy to check that the second inequality also corresponds to (7) (with strict inequality). ■

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