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Peace Agreements Without Commitment

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PEACE AGREEMENTS WITHOUT COMMITMENT*

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Abstract

In this paper we present a model of war between two rational and completely informed players. We show that in the absence of binding agreements war can be avoided in many cases by one player transferring money to the other player. In most cases, the "rich" country transfers part of her money to the "poor" country. Only when the military proficiency of the "rich" country is sufficiently great, it could be that the "poor" country can stop the war by transfering part of its resources to the "rich" country.

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1. Introduction

War has played an important role in human history. Recently, social scientists have devoted a great deal of attention to show how rational and fully informed players can engage in war, e.g. Bueno de Mesquita (1981), O'Neill (1990), Hirshleifer (1991), Skaperdas (1992), Sánchez-Pagés (2006) and Jackson and Morelli (2007). This approach leaves aside many complications that are relevant to the explanation of wars such as irrational or incompletely informed players, religion, politics, ethnicity, etc. But by focussing on such a stylized world, it captures the core of many conflicts, namely rational interest in some valuable resource.[1]

On the contrary, the question of which kind of agreements can prevent war -which is a wasteful way to settle conflicts- has been relatively neglected. A difficulty here is how to explain the commitment of both parties to the agreed upon course of action. On the one hand, contracts between two sovereign states are not usually legally enforceable.[2] On the other hand, reputation effects can only arise under incomplete information or in infinitely repeated games (Fearon [1995], p. 409).

There is ample historical evidence of such agreements: Roman emperors used to buy peace with invaders, e.g. Alaric was paid 5,000 pounds of gold, 30,000 pounds of silver plus other valuables in return for calling off the siege of Rome in AD 409 (Gibbons [1776-1788], Chap. XXXI). Attila obtained 13.000 pounds of gold during the period AD 440-450 from the eastern provinces of the Roman empire to stop him from invading them (Keegan [1994], p. 183). Both Alaric and Attila knew Rome very well. The first served in the Roman army and the second spent time in Rome as a hostage. The eastern Roman emperor Justinian and its Persian counterpart Chosroes negotiated a long series of agreements, some of which were upheld, e.g. the truce in AD 541 in which the Persians agreed not to attack Byzantine territory for the next five years in return for 5,000 pounds of gold (Evans [1998]). An "everlasting" peace agreement, though,

[1] The connection between war and games was already noticed by Clausewitz ([1832] Chapter 1, end of paragraph 21): "War is akin to a card game".

[2] For instance in Alesina-Spolaore (2005) war may arise from a surprise attack during negotiations.

lasted only for 10 years. Viking kings used to be "bought off" in tenth century England. They also received Normandy from the king of France in return for lifting the siege of Paris (Sykes, 2006, p. 263). Christian kingdoms used to extort the Moorish kingdoms in eleventh century Spain with tributes called *Parias* in return for peace, but two centuries earlier similar tributes were paid the other way around (Nelson [1979]). Tribal wars in Africa were avoided by paying slaves as tribute (Nunn, [2007] p. 6), etc. In many cases, agreements took place among people who knew each other well and with whom similar agreements -not all of them upheld- were struck in the past. Thus, it would be convenient to have an explanation for such agreements that does not rely exclusively on reputation effects.

Consider the following mechanism. Before war is waged, the potentially attacked player (the "prey") gives some of resources to the potentially attacking player (the "predator") with a double target: to compensate him for the expected spoils of war and to make him so rich that he is no longer interested in waging war. Why is this so? Because of two reasons: on the one hand, the prey is slimmer after the transfer so the expected revenue from attacking will be less than before; on the other hand the predator has more to lose after the transfer because he is now richer. What is not so clear is that the prey is better off, since in some cases peace is bought so dearly that war might be preferable. The objective of this paper is to explore the possibility of such agreements in a simple model with two finitely-lived and fully informed players.[3]

The game is played as follows: In the first stage transfers are made. In the second stage players decide simultaneously if they declare war or not. If one of them declares war, war occurs. In the third stage, if there is war, each player decides the war effort. In the last stage the outcome of the war is determined and the winner takes all.[4] Thus our model is close to Clausewitz's (1976) concept of absolute war.[5]

We assume that the probability of winning war is a function of war efforts and two

[3] Thus the question of the distribution of the characteristics of the population, that plays an important role in the analysis of conflicts in Esteban and Ray (1999) does not arise here.

[4] Bueno de Mesquita, Smith, Siverson, and Morrow (2003, p. 417) consider three outcomes following the defeat of a leader (nation): confiscation of resources, to install a puppet king and to alter the institutional arrangements in the defeated country. In our model we only consider the first alternative.

[5] See Smith and Stam (2004) for a model of war more akin to Clausewitz's limited wars.

parameters: the (relative) military proficiency of Player 1 and the responsiveness of the probability of winning war to war efforts. The latter is an inverse measure of the role of chance in war. For simplicity, we assume that a fixed proportion of war efforts can be recovered by the winner, so a fixed proportion of resources are lost in the war. Players are endowed with a resource that can be devoted to war effort or consumed. The resources of Player 1 are larger than those of Player 2 so the first (resp. second) player will be called *the rich* (resp. *poor*) player. Thus in our model there are four parameters: military proficiency, the role of chance in war, the (marginal) cost of war and the inequality of resources between players.

In order to highlight the role of each parameter, we start analyzing in Section 3 the case where both players are equally proficient and the probability of winning the war is proportional to war efforts. In this case, the unique source of asymmetry among players is derived from the differences in resources. We have two kind of results. Firstly, on the occurrence of war if no transfers are made. Secondly, on how war can be avoided by making transfers.[6]

In absence of transfers, only the relatively poor player has incentives to attack when inequality of resources is large and the marginal cost of war is not small. Inequality is a necessary condition for the war to occur. However, it is far from being a sufficient condition. Other factors also play a role, for instance Player 2 must use all his resources in the war but Player 1 does not (we say that Player 2 is constrained but Player 1 is unconstrained). In any other case, peace is an equilibrium outcome. If both players are unconstrained (the marginal cost of war is bigger than 0.5), aggression does not pay because it entails a small expected gain and a large loss of resources. If both agents are constrained, war entails too much destruction because both players commit all their resources to the conflict and since the probability of winning is proportional to war efforts, in the most favorable case (zero marginal cost of war) they can only expect to gain exactly what they had before the war. If only Player 2 is constrained, the role of

[6] In games with incomplete information, transfers may signal a "chicken" attitude of the prey so they may exacerbate the demands of the predator and make war inevitable. We do not discuss whether this effect exists. We just point out that, at least in some cases, there is also a good side of "being chicken" namely that an increase in the wealth of our enemy may make it less aggressive.

inequality of resources is clear, Player 2 attacks in the hope of becoming rich. However, the effect of the marginal cost of war is contrary to what intuition might suggest. A high cost of war may yield incentives to Player 2 to attack since the resources used in war by Player 1 are decreasing with the cost of the war. Thus, when this cost is high, the chance for Player 2 to win the war is greater than when this cost is small.

Transfers avoid war unless inequality of resources is very large. The transfer from the rich to the poor country reduces resource inequality and induces a peaceful behavior in the poor country.[7]

In Section 4, we analyze the consequences of making the probability of winning less responsive to war efforts. We show that this gives even more incentives to the poor player to start a war since from his point of view is just a lottery with a fair outcome. In the limit (when war is just a lottery) war is possible even in the absence of inequality. This is because when the success of war is not very sensitive to war efforts, both players use only a small part of their resources in war and the loot of the winner is considerable. Also in this case, if inequality of resources is not very large, transfers from the rich to the poor country will avoid war. A transfer acts as a costless lottery that leaves both players better off.

The analysis of Sections 3 and 4 gives a good explanation of the *uneven contenders paradox*, raised by Clausewitz (1832), where one weak country initiates war, even though the rich country has a higher probability of winning the war. In the words of Adam Smith (1776, p.659) "An industrious, and upon that account a wealthy nation, is of all nations the most likely to be attacked."

Finally, in Section 5 we analyze the case where the rich country has an advantage in military proficiency. This advantage gives incentives to the rich player to attack in the absence of transfers. For this to occur its military proficiency should be high, the resources of the poor can not be too small (so the potential loot is large) and the marginal cost of war has to be small (so the winner can recoup a large part of war efforts). But in this case, transfers from the poor to the rich would avoid war. A large inequality protects the weak player (Player 2) from an attack by Player 1, since this

[7] An implication of this result is that it may be not a good idea to make your enemy poorer because this may make war unstoppable.

player has little to gain. Thus, in our mode, it is the asymmetry in military proficiency that makes possible that the stronger player initiates the war as in Bueno de Mesquita (1981, pp. 129 and 155).

Summing up, we find that the transfer mechanism avoids war in a large number of cases showing that peaceful agreements may be reached in a world where no commitment is possible. This agrees with the observation of Morrow (2008) that "interstate war is very rare" and that "most disputes are resolved without escalating to war" (op.cit. pp. 11-13).

Relationship with the Literature

Hirshleifer (1991), proposed a model of conflict where poorer combatants often end up improving their position relative to richer ones. He called this the Paradox of Power. This is close to our observation that a high cost of war may yield incentives to the poor player to attack. But our models are different. First Hirshleifer does not study the incentives for declaring war he just assumes that both countries are at war. Second, in his model there is production and the marginal cost of war is one. Finally and most important, he does not discuss the role of transfers to overcome the problem of commitment.

Bueno de Mesquita and Lalman (1992) consider a sequential game with perfect information where countries can make demands on each other. They can grant the demands, challenge the demand or negotiate (with commitment). Under complete information and when domestic factors (such as the ideology of the supporting coalition) do not play any role, war never arises in equilibrium (they call this case "Realpolitik"). This is because they either negotiate or remain at the status quo. However, when domestic factors play a role, war is possible.

Powell (1996) analyzed a dynamic model where "a state that is declining in power is unsure of the aims of a rising state. If those aims are limited, then the declining state prefers to appease the rising state's demands rather than go to war to oppose them" (p.749). These appeasement takes the form of "salami tactics" where little concessions are made over time. Under complete information the policy of appeasement brings peace always (p. 755). There are several differences between our models: Our´s is a model of a "decisive battle", not of a series of small conflicts so appeasement does not make much sense. Also, in his model the aggressor is exogenously chosen to be the rising state.

Bueno de Mesquita, Smith, Siverson and Morrow (2003) consider a model where leaders have to obtain the support of a winning coalition in order to fight a war. After the war is fought "the members of the winning coalition...... decide whether to retain their leader or to defect to a domestic political rival" (p. 265). Their model is somewhat similar to ours: war is modeled as a costly lottery and the probability that a country wins the war is increasing in effort which is a choice variable of leaders. However, the cost of war is fixed (pp. 227-8) and they do not consider the possibility to make transfers that may stop war.

Jackson and Morelli (2007) studies how the decisions to go to war depend on the political bias of the decision makers. Political bias refers to the discrepancy between the interests of decision makers and citizens. They do not distinguish between war effort and resources and thus the consideration of the constraints of resources on war effort, that plays a prominent role in our analysis, is absent there. The center of Jackson and Morelli's paper is the case where both parties can commit. They also give some results on the non commitment case. Firstly, if the decision makers are unbiased, two countries may decide to go to war depending on the responsiveness of the probability of winning the war to the difference in resources. This is very close to our Proposition 4 and 6. Secondly, "it is possible that too high a transfer will lead to war while a lower transfer will avoid a war". This is because the larger transfer will increase the probability that the poor country wins the war. Thirdly, their Proposition 4 asserts that "if the probability of winning is proportional to relative wealths..... two.. countries will never go to war if they can make transfers to each other (even without commitment)". The reason is that if the winning probabilities are proportional no country wants to go to war in the absence of transfers, which is our Proposition 1. Thus, our paper complements and extends their findings in the case of non commitment.

The rest of the paper goes as follows. We present our model in Section 2. In Sections 3, 4 and 5, we analyze the incentives for both players to go to war and when war can be avoided by transfers. Section 6 concludes the paper and suggests some avenues of further research. All the proofs are gathered in an appendix.

2. The Model

There are two players with resources V_1 and V_2. W.l.o.g. we will assume that $V_1 > V_2$. They play the following game.

In the first stage, each player may transfer part of his resources to the other player.

In the second stage, each of them decides whether to declare war on the other player or not. If one of them declares war, war occurs. If both abstain from declaring war, peace results.

In the third stage, if there is peace, the game ends. Payoff to player i is his resource V_i, $i = 1, 2$. If there is a war, each player has to commit part of his resources to the war effort, denoted by e_i, $i = 1, 2$. It is assumed that there is no outside credit and therefore no player can use in the war more than his available resources.[8]

In the fourth stage, war is waged. The outcome is partially determined by nature and partially determined by war efforts.[9] Thus in our model, the wealth of a nation does not translate automatically into military capabilities as in Bueno de Mesquita (1981, p. 102) and Jackson and Morelli (2007).

If p_i is the probability that player $i = 1, 2$ wins the war, we assume that

$$p_1 = \frac{\lambda e_1^\gamma}{\lambda e_1^\gamma + e_2^\gamma} \quad \text{and} \quad p_2 = \frac{e_2^\gamma}{\lambda e_1^\gamma + e_2^\gamma}, \quad \lambda \in (0, \infty), \quad 0 \leq \gamma \leq 1. \tag{2.1}$$

The functions in (2.1) are called *contest success functions (CSF)*. The parameter λ is a measure of the war skills of Player 1. When $\lambda = 1$, we will say that the CSF are *symmetric*. The parameter γ measures the sensitivity of the probability of winning war to the efforts. When $\gamma = 0$, the outcome of war is purely random. When $\gamma = 1$, we will say that the CSF are *proportional*.

A motivation for this functional form is that it seems reasonable to require that the CSF is homogeneous of degree zero, so winning probabilities do not depend on how resources are measured (pounds or francs, number or thousands of soldiers, etc.). Clark and Riis (1998), following Skaperdas (1996), have shown that under certain assumptions the only functional form that is homogeneous of degree zero is precisely the one above.

[8] A good motivation for the assumption that there is only one player by nation is presented by Bueno de Mesquita (1981), p. 28.

[9] See Clausewitz (1832), Chapters 7 and 8 for a brief description of the role of chance in war.

We will assume that there is a winner who takes all, i.e. the war does not end in a stalemate. Assume that a fixed proportion of the war effort, say k, can not be recovered by the winner, with $0 \leq k \leq 1$. The parameter k is the marginal cost of effort.

For simplicity we assume, as it is customary in the literature, that players are risk-neutral.[10] Thus, the payoff of, say Player 1, if he wins the contest is $V_1 + V_2 - k(e_1 + e_2)$ and zero otherwise. Let $V \equiv V_1 + V_2$. Expected payoff of player i, denoted by $E\pi_i$, is

$$E\pi_i = p_i(V - k(e_1 + e_2)). \tag{2.2}$$

Finally we assume that information is complete and that the equilibrium concept is subgame perfection.

The game is characterized by four parameters, V_2/V, k, λ, γ. In order to analyze how the solution of the game depends on those parameters, we will proceed as follows. First, in Section 3 we solve the case where CSF are proportional and symmetric. In Section 4 we consider CSF which are symmetric but non proportional and in Section 5 we consider CSF which are non symmetric but proportional. The analysis in these last two sections complements the one in Section 3 and allows to highlight the role of γ and λ respectively.

3. The Game with Symmetric and Proportional CSF ($\lambda = 1$, $\gamma = 1$)

In this case, the CSF reads

$$p_i = \frac{e_i}{e_1 + e_2},\ i = 1, 2. \tag{3.1}$$

Thus, the expected payoff of player i, is

$$E\pi_i = p_i(V - k(e_1 + e_2)) = \frac{e_i}{e_1 + e_2}(V - k(e_1 + e_2)) = \frac{e_i V}{e_1 + e_2} - ke_i. \tag{3.2}$$

We solve the game backwards. Since no player has to move in the fourth stage, let us begin by analyzing the third stage. Setting $\frac{\partial E\pi_i}{\partial e_i} = 0$, we get:

$$e_1(e_2) = \sqrt{\frac{Ve_2}{k}} - e_2 \text{ and } e_2(e_1) = \sqrt{\frac{Ve_1}{k}} - e_1, \tag{3.3}$$

[10] See Cornes and Hartley (2005) for contests where players are risk averse. In the final section we will discuss how risk aversion may affect our results.

Figure 1 below shows the case where $V/k = 16$. This figure can be interpreted as the best reply function of Player 1 and Player 2 when no agent is constrained. In this case, we readily see that the maximum effort occurs when $e_i = \frac{V}{4k}$ which is also the solution to (3.3) above.

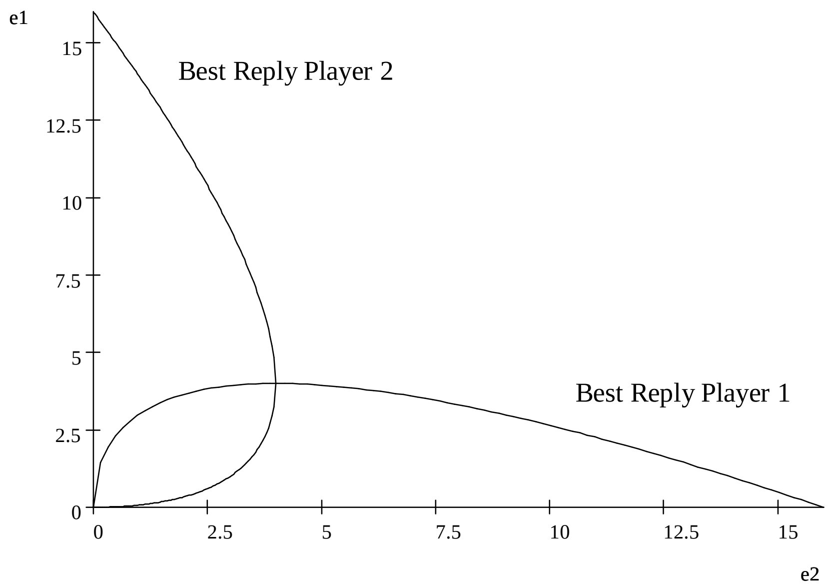


Figure 1

In the interval where Player 1 becomes constrained by his resources, his best reply becomes totally flat (a vertical line in the case of Player 2). Thus, it is clear that we have only the following possibilities: both players are unconstrained; both players are constrained or just one player is constrained. Let us analyze each of these cases in turn.

3.1. Both Players Are Unconstrained

This case arises iff the resources in the hands of each player are larger than the solution to (3.3), i.e. $V \leq 4kV_1$ and $V \leq 4kV_2$, or equivalently,

$$\frac{1}{4k} \leq \frac{V_2}{V} \leq \frac{4k-1}{4k}. \tag{3.4}$$

Notice that this case can only occurs for $k \geq 0.5$. For those values of k, $\frac{4k-1}{4k} \geq 0.5$, and since $V_2 < V_1$, $\frac{V_2}{V} \leq 0.5$. Thus the above inequalities can be summarized as

$$k \geq 0.5, \text{ and } \frac{V_2}{V} \geq \frac{1}{4k}. \tag{3.5}$$

Then, equilibrium occurs at

$$e_1^* = e_2^* = \frac{V}{4k}. \tag{3.6}$$

Payoffs amount to

$$E\pi_1^* = E\pi_2^* = \frac{V}{4}. \tag{3.7}$$

3.2. Player 1 is Unconstrained and Player 2 is Constrained

This case arises iff the resources in the hand of Player 2 are smaller than the solution to (3.3), i.e. $V > 4kV_2$, and the best reply of Player 1 to V_2 ($e_1(V_2)$ described in (3.3)) is smaller than V_1, i.e. $\sqrt{\frac{VV_2}{k}} - V_2 \leq V_1$, or equivalently,

$$\frac{V_2}{V} < \min\{k, \frac{1}{4k}\}. \tag{3.8}$$

Equilibrium occurs at

$$e_1^* = \sqrt{\frac{VV_2}{k}} - V_2 \text{ and } e_2^* = V_2. \tag{3.9}$$

Payoffs amount to

$$E\pi_1^* = V + kV_2 - 2\sqrt{VV_2k} \text{ and } E\pi_2^* = \sqrt{VV_2k} - kV_2. \tag{3.10}$$

3.3. Both Players are Constrained

This case arises iff $\frac{\partial E\pi_i(V_1,V_2)}{\partial e_i} > 0$, which implies,

$$k < \frac{V_2}{V}. \tag{3.11}$$

We see that equilibrium occurs at

$$e_1^* = V_1 \text{ and } e_2^* = V_2. \tag{3.12}$$

$$E\pi_1^* = V_1(1-k) \text{ and } E\pi_2^* = V_2(1-k). \tag{3.13}$$

In Figure 2 below, the increasing line corresponds to $\frac{V_2}{V} = k$, and the decreasing line corresponds to $\frac{V_2}{V} = \frac{1}{4k}$.

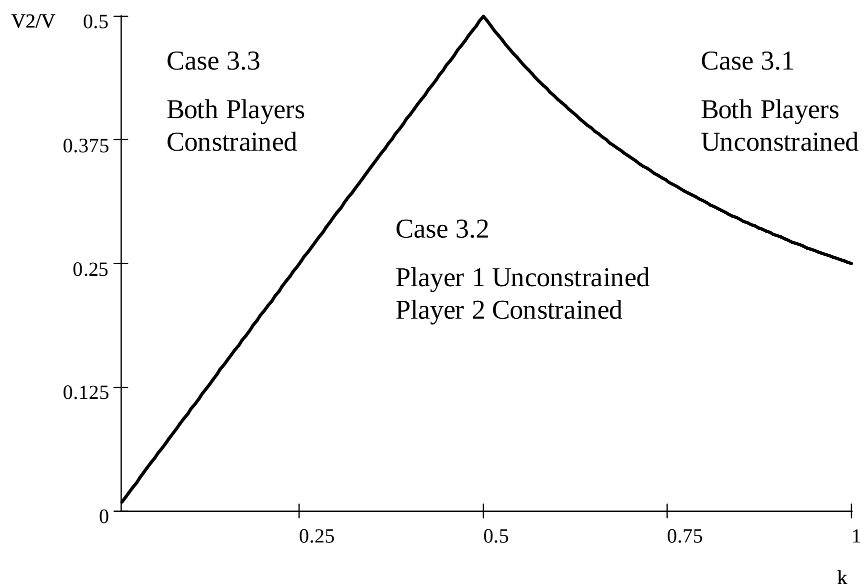


Figure 2

Case 3.1 occurs in the area above the decreasing line. Case 3.2 occurs in the area to the right of the increasing line and below the decreasing line. Finally, Case 3.3 occurs in the area to the left of the increasing line.

We are now ready to analyze the second stage of the game by comparing the expected payoffs obtained under war with the payoffs in the case of peace, assuming that no transfers have been made in the first stage. Then, peace occurs iff $E\pi_i \leq V_i$, $i = 1, 2$.

If both players are unconstrained, $\frac{V}{4} \leq kV_1$, $\frac{V}{4} \leq kV_2$, since $k \leq 1$, $E\pi_i \leq V_i$ for all $i = 1, 2$. Thus, war in this case is not profitable.

If both players are constrained, $E\pi_i = V_i(1 - k)$ for all $i = 1, 2$, which confirms that war is not profitable either. We record these findings in the following proposition.

Proposition 1. *If both players are unconstrained or both players are constrained, no war is declared in equilibrium in the absence of transfers.*

The interpretation of this result is the following: In case 3.1 both players are not very different and the technology of recovery of the spoils of war is not very efficient (i.e. k larger than .5). Therefore, aggression does not pay because it entails a small expected gain and a large loss of resources. War in case 3.3 entails too much destruction because both players commit all their resources to the conflict, and since the probability of winning is proportional to war efforts, in the most favorable case ($k = 0$) they can only expected to gain their initial resources.

The analysis of Case 3.2 is far more interesting. Recall that in this case, $\frac{V_2}{V} \leq \min\{k, \frac{1}{4k}\}$. Peace occurs iff

$$E\pi_1^* = V + kV_2 - 2\sqrt{VV_2k} \leq V_1, \text{ or equivalently } \frac{V_2}{V} \leq \frac{4k}{(1+k)^2}, \quad (3.14)$$

$$\text{and } E\pi_2^* = \sqrt{VV_2k} - kV_2 \leq V_2, \text{ or equivalently } \frac{k}{(1+k)^2} \leq \frac{V_2}{V}. \quad (3.15)$$

The inequality $\frac{V_2}{V} \leq \frac{4k}{(1+k)^2}$ holds in Case 3.2 since $\frac{V_2}{V} \leq \min\{k, \frac{1}{4k}\}$, and $k \leq \frac{4k}{(1+k)^2}$ for all $k \in [0, 1]$. Thus, it is never in the interest of Player 1 to declare war. However, inequality (3.15) not always hold, thus if $\frac{V_2}{V} < \frac{k}{(1+k)^2}$ Player 2 has an incentive to declare war. This result is summarized in the next proposition.

Proposition 2. *Suppose Case 3.2 holds. In the absence of transfers, war is declared by Player 2 iff* $\frac{V_2}{V} < \frac{k}{(1+k)^2}$. *For any other values of the parameters, peace is the equilibrium outcome of the game.*

Proposition 2 says that if Player 1 is sufficiently rich with respect to Player 2, the poor player has an incentive to declare war. The poor player declares war in the hope of becoming rich. But the effect of k is contrary to what intuition might suggest, which is that when the marginal cost of war is high, war would not pay off. This counterintuitive result comes from the fact that the resources committed to war by Player 1 decrease with k and, thus, the larger k the larger is the probability that Player 2 wins the war. This makes the payoff for Player 2 in the case of war increasing in k and explains the result.

In Figure 3 below we picture the function $\frac{V_2}{V} = \frac{k}{(1+k)^2}$ as a dotted line. The area enclosed by the three lines corresponds to parameters for which there is peace. The area below the dotted line corresponds to values of parameters where 2 declares war if no transfers have been made in the first stage.

A rough estimate of the probability that war arises can be obtained by assuming that $k \times \frac{V_2}{V}$ is uniformly distributed in $[0,1] \times [0,0.5]$. If so, the probability of war when no transfer has been made yet, denoted by $\Pr(warNT)$ is

$$\Pr(warNT) = \int_0^1 (\int_0^{\frac{k}{(1+k)^2}} 2dx)dk = 0.386\,29. \tag{3.16}$$

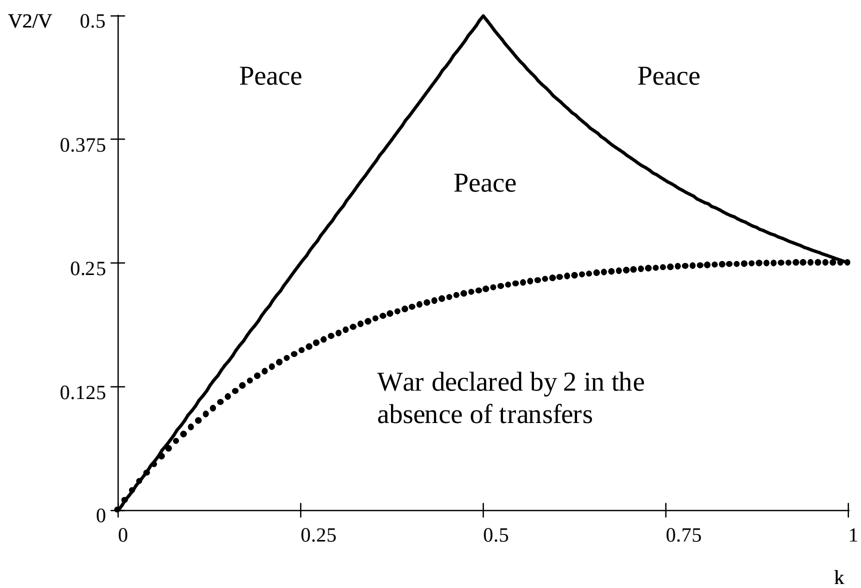


Figure 3

Now we are ready for the analysis of the first stage of the game. In particular we look for a transfer T from Player 1 to Player 2 with the following properties:

1. Before the transfer, equilibrium means war. That is, case 3.2 occurs and war is declared by Player 2, $\frac{V_2}{V} < \frac{k}{(1+k)^2}$.

2. After the transfer, both players are better off than if there had being a war. That is,

$$-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} \leq \frac{V_2 + T}{V} \leq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}, \tag{3.17}$$

where the first (resp. the second) inequality corresponds to the expected value of Player 2 (resp. Player 1) before the transfer being smaller than his resources after the transfer.

3. After the transfer, peace is an equilibrium outcome.

   Either we are in Case 3.1, i.e.,

$$\frac{1}{4k} \leq \frac{V_2 + T}{V}, \; k \geq 0.5, \tag{3.18}$$

   or in Case 3.2 with peace,

$$\frac{k}{(1+k)^2} \leq \frac{V_2 + T}{V} < \min(\frac{1}{4k}, k), \tag{3.19}$$

   or in Case 3.3,

$$k < \frac{V_2 + T}{V} < \frac{1}{4k}. \tag{3.20}$$

   If T satisfies Conditions 1, 2 and 3, we will say that a peace agreement is *feasible.*

In Figure 3 we visualize the possible cases that can occur. For $k = 1$, after the transfer we can only be in Case 3.1, the area above the decreasing line. For $k \in [0.5, 1)$, after the transfer we can be either in Case 3.1 or in Case 3.2, but if Case 3.1 can be achieved with a certain transfer, Case 3.2 can be achieved with a smaller transfer (see Appendix, Lemma 1). For $k \in (0, 0.5)$, after the transfer we can be either in Case 3.2 or in Case 3.3, but if Case 3.3 can be achieved with a certain transfer, Case 3.2 can be achieved with a smaller transfer (see Appendix, Lemma 2). Therefore, for all $k \in (0, 1)$ we only have to explore the existence of a peace agreement such that after the transfer we are in Case 3.2. Summing up, we need to study under what conditions there is a transfer T such that

$$\max(\frac{k}{(1+k)^2}, -k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k}) \leq \frac{V_2 + T}{V} \leq \min(\frac{1}{4k}, k, -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}). \tag{3.21}$$

Let $x_1(k) \in [0, \frac{k}{(1+k)^2})$ be a solution of $\frac{k}{(1+k)^2} = -kx + 2\sqrt{xk}$. $x_1(k)$ exists (see Lemma 4) and it is the key ingredient that allows to characterize the conditions under which a transfer T satisfying (3.21) can exist. This is shown in the following proposition.

Proposition 3. *For each $k \in (0, 1]$, a peace agreement is feasible if and only if $\frac{V_2}{V} > x_1(k)$. The minimal transfer, $\hat{T}$, that avoids war is such that it makes Player 2 be indifferent between war and peace, that is, $\frac{V_2+\hat{T}}{V} = \frac{k}{(1+k)^2}$.*

Figure 4 summarizes the above result. The area below the dot line ($x_1(k)$) represents the area where a peace agreement is impossible. If resource inequality is very large, negotiations cannot avoid war because the minimal transfer that will stop Player 2 to declare war is too expensive for Player 1. Above the $x_1(k)$ line, peace is possible. The transfer that avoids war increases with k, thus reflecting that, as we remarked before, the war effort of Player 1 decreases with k. So a high k increases the probability that Player 2 wins the war and therefore increases the transfer that makes Player 2 peaceful.

It is worth noting that in the worst situation, a peace agreement can be reached when the resources of the poorest player are at least 2.9 per cent of the total resources. Again, an idea of the probability that war will arise when transfers can be made is obtained assuming that $k \times \frac{V_2}{V}$ is uniformly distributed in $[0,1] \times [0,0.5]$. The probability of war with transfers denoted by $\Pr(warT)$ is

$$\Pr(warT) = \int_0^1 (\int_0^{x_1(k)} 2dx)dk = 0.046496.$$

This probability is, approximately, eight times smaller that the probability of war without transfers (0.386 29).

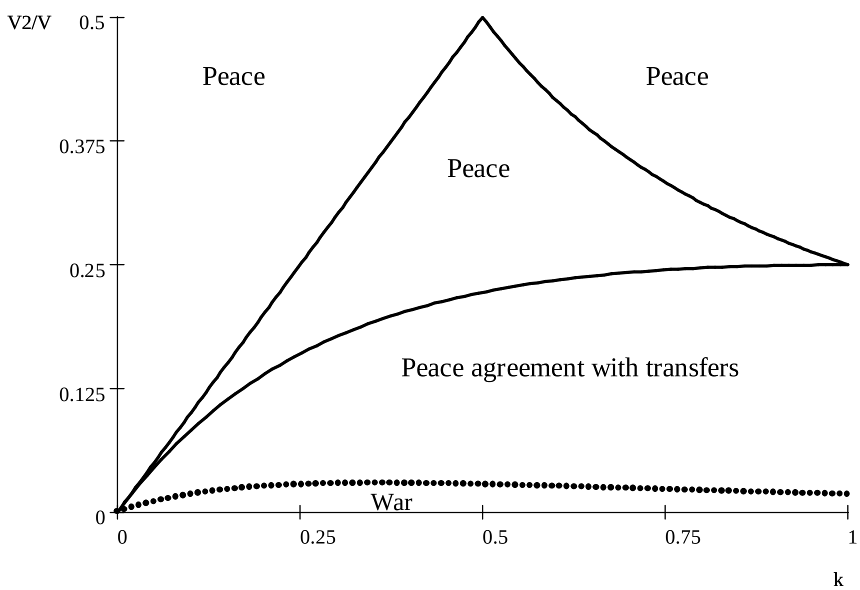


Figure 4

4. The Game with a Symmetric CSF ($\lambda = 1, 0 \leq \gamma \leq 1$)

In this section we consider a generalized symmetric form of the CSF used in the previous section. We will see that symmetry in the CSF implies that the rich player never has an incentive to declare war. But, contrary to what happened in the previous section, there may be an incentive to wage war even when both players are unconstrained and their resources are very similar or when both players are constrained. In the first case, a peaceful arrangement (with the appropriate transfer) is always possible. Unfortunately, in the second case there are instances in which no transfers can stop war.

Suppose that the CSF is of the following form

$$p_i = \frac{e_i^\gamma}{e_1^\gamma + e_2^\gamma},\ i = 1, 2 \ \text{ with } 0 \leq \gamma \leq 1. \tag{4.1}$$

This is the CSF proposed by Tullock (1980) that has been ubiquitously used in the literature. It reduces to the form postulated in Section 3 when $\gamma = 1$. Now expected payoffs for player i, denoted by $E\pi_i$, assuming that war has been declared are

$$E\pi_i = p_i(V - k(e_1 + e_2)) = \frac{e_i^\gamma}{e_1^\gamma + e_2^\gamma}(V - k(e_1 + e_2)). \tag{4.2}$$

Noting that $p_2 = 1 - p_1$ we have that

$$\frac{\partial E\pi_1}{\partial e_1} = \frac{e_2^\gamma e_1^{\gamma-1}\gamma}{(e_1^\gamma + e_2^\gamma)^2}(V - k(e_1 + e_2)) - p_1 k; \tag{4.3}$$

$$\frac{\partial E\pi_2}{\partial e_2} = \frac{e_1^\gamma e_2^{\gamma-1}\gamma}{(e_1^\gamma + e_2^\gamma)^2}(V - k(e_1 + e_2)) - (1 - p_1)k. \tag{4.4}$$

Setting $\frac{\partial E\pi_1}{\partial e_1} = 0, i = 1, 2$, and dividing (4.3) by (4.4) we obtain that $e_1^{\gamma+1} = e_2^{\gamma+1}$ which implies that $e_1 = e_2$. Now (4.3) reads

$$\frac{\partial E\pi_i}{\partial e_i} = \frac{\gamma e_i^{2\gamma-1}}{4e_i^{2\gamma}}(V - 2ke_i) - \frac{k}{2} = 0,\ i = 1, 2. \tag{4.5}$$

The solution to (4.5) is,

$$e_1^* = e_2^* = \frac{\gamma V}{2k(\gamma + 1)} \ \text{ and } \ E\pi_1^* = E\pi_2^* = \frac{V}{2(\gamma + 1)}. \tag{4.6}$$

4.1. Both players are unconstrained

This case arises if the following inequality holds,

$$V_1 \geq \frac{\gamma V}{2k(\gamma+1)} \quad \text{and } V_2 \geq \frac{\gamma V}{2k(\gamma+1)}. \tag{4.7}$$

Since $V_1 > V_2$, if the second inequality holds, the first inequality also holds.
For war to be a rational option, we need the following:

$$\frac{V}{2(\gamma+1)} > V_1 \quad \text{or} \quad \frac{V}{2(\gamma+1)} > V_2. \tag{4.8}$$

We first notice that it is impossible that both inequalities occur, because adding them up we get $\frac{V}{\gamma+1} > V$ which is impossible. This implies that Player 1 has no incentive to go to war because if it had, Player 2 would also have incentives to declare war (since $V_1 > V_2$). Thus, we are left with the case where only the second inequality in (4.8) holds, so the second country has an incentive to go to war. Notice that, contrarily to the case in which $\gamma = 1$, war is now possible: Indeed, the occurrence of war is equivalent to

$$V_2 \geq \frac{\gamma V}{2k(\gamma+1)} \quad \text{and} \quad \frac{V}{2(\gamma+1)} > V_2, \tag{4.9}$$

which is possible whenever $\gamma < k$. Our next result summarizes this discussion.

Proposition 4. *When the CSF are of the form (4.1), and both players are unconstrained, in the absence of transfers, no war is declared for any $\gamma \geq k$. When $\gamma < k$, war is declared by Player 2 for any value of $V_2/V \in [\frac{\gamma}{2k(\gamma+1)}, \frac{1}{2(\gamma+1)})$.*

We see that the occurrence of war depends on several factors. First, the probability of winning the war should not depend very much on war efforts relative to the marginal cost of war, k. For a small γ the poor player has a chance of winning the war without much effort which implies a sizeable loot should war be won. Second, the ratio of the resources of Player 2 with respect to those of Player 1 should not be too high, -because otherwise Player 2 risks a lot- nor too low, because in this case Player 2 is constrained.

An important consequence of this result is that when $\gamma \to 0$, war is possible for any value of V_2/V. In other words, here war is possible even in the absence of inequality.

This is because when the success of war is not very sensitive to war efforts, both players use only a small part of their resources in war and the loot of the winner is considerable.

Let us study equilibrium in the first stage of the game. Consider the minimum transfer that leaves Player 2 indifferent between peace and war, namely

$$V_2 + \hat{T} = \frac{V}{2(\gamma+1)}. \tag{4.10}$$

Notice that $\hat{T}$ is always smaller than V_1 (if it were not, $\frac{V}{2(\gamma+1)} - V_2 > V_1$, or $\frac{V}{2(\gamma+1)} > V$ which is impossible). After the transfer, both players will be better off than if they had had a war, Player 1 will still be unconstrained and no one will have incentives to declare war. The following proposition proves that this is indeed the case.

Proposition 5. *When the CSF are of the form (4.1) and both players are unconstrained, a peace agreement is feasible. The minimal transfer that avoids war is such that it makes Player 2 be indifferent between war and peace, that is* $V_2 + \hat{T} = \frac{V}{2(\gamma+1)}$.

The interpretation of this result is that, as we saw before, war is a rational option for Player 2 when it is a kind of lottery, i.e. the outcome of the war does not depend much on war efforts. But in this case a transfer acts as a costless lottery that leaves both players better off.

4.2. Both Players are Constrained

This case arises iff $\frac{\partial E\pi_i(V_1,V_2)}{\partial e_i} > 0$, or equivalently:

$$\frac{V_2^\gamma \gamma}{(V_1^\gamma + V_2^\gamma)} V(1-k) \quad > \quad V_1 k; \tag{4.11}$$

$$\frac{V_1^\gamma \gamma}{(V_1^\gamma + V_2^\gamma)} V(1-k) \quad > \quad V_2 k. \tag{4.12}$$

Notice first that Player 1 constrained (inequality (4.11)) implies Player 2 constrained (inequality (4.12)).

In this case, $E\pi_i = \frac{V_i^\gamma}{V_1^\gamma + V_2^\gamma} V(1-k)$, $i = 1, 2$. Since the probability of winning for Player 1 is increasing with γ, .

$$\frac{V_1^\gamma}{(V_1^\gamma + V_2^\gamma)} V(1-k) \leq \frac{V_1}{(V_1 + V_2)} V(1-k) = V_1(1-k) \leq V_1, \tag{4.13}$$

which implies that Player 1 has no incentive to declare war.

However, contrary to the case in which $\gamma = 1$, war is now possible. For Player 2 to have an incentive to declare war, $E\pi_i > V_2$ should hold. Combining this inequality with (4.11),

$$1 - \frac{V_2^\gamma}{(V_1^\gamma + V_2^\gamma)} \frac{(1-k)\gamma}{k} < \frac{V_2}{V} < \frac{V_2^\gamma}{(V_1^\gamma + V_2^\gamma)}(1-k). \tag{4.14}$$

Notice first that if $\gamma = 1$, the above inequality never holds (as we proved in Section 3). But when $k = 0$ and $\gamma \in (0, 1)$, the above condition always holds. Firstly because for $k = 0$, both players are constrained no matter how the resources are distributed (see conditions (4.11) and (4.12)). Secondly, $\frac{V_2^\gamma}{(V_1^\gamma + V_2^\gamma)}$ is decreasing with γ, thus the second inequality in (4.14) is trivially satisfied. So in those situations Player 2 always has an incentive to declare war and no transfer will avoid war. Since $k = 0$, the minimal transfer that will stop Player 2 from declaring war should be such that resources of both players are equalized. But then, Player 1 will not be better off because before the transfer he expects in the worse case ($\gamma = 0$) half of the resources. This result is summarized in the following proposition.

Proposition 6. *When the CSF are of the form (4.1), $k = 0$, and $\gamma \in (0, 1)$, war is always declared by Player 2 and there is no transfer that avoids war.*

The interpretation of this result is that when the war is costless, war is a good option because the transfers that would make Player 2 peaceful are too costly for Player 1.

If $k > 0$ we can find values for V_2 for which war is avoidable. In particular, V_2 should be such that the minimal transfer needed to avoid war by Player 2 is less than the total cost of war, kV. Otherwise, Player 1 will not agree on the transfer. This result is shown in the following proposition. Let $\bar{V}_2$ be such that the minimal transfer that makes Player 2 indifferent between war and peace is kV.

Proposition 7. *When the CSF are of the form (4.1), both players are constrained, and (4.14) holds, a peace agreement is feasible if $k > 0$ and $V_2 \geq \bar{V}_2$.*

4.3. Player 1 is Unconstrained and Player 2 is Constrained.

As in the previous cases, we will show here that Player 1 has no incentive to start a war. We can conclude that symmetry of the CSF implies that the rich player never has an incentive to declare war.

This case arises if $V_2 < e_2^*$ and $\frac{\partial E\pi_1(V_1,V_2)}{\partial e_1} < 0$, or equivalently

$$\frac{V_2}{V} < \frac{\gamma}{2k(\gamma+1)}; \text{ and} \tag{4.15}$$

$$\frac{V_2^{\gamma}\gamma}{(V_1^{\gamma}+V_2^{\gamma})}V(1-k) < V_1 k. \tag{4.16}$$

The optimal effort of Player 1 in case of war is the solution of

$$\frac{V_2^{\gamma}\gamma}{(e_1^{\gamma}+V_2^{\gamma})}(V-(e_1+V_2)k) = e_1 k, \tag{4.17}$$

The expected payoff of Player 1 is

$$E\pi_1 = \frac{e_1^{\gamma}}{(e_1^{\gamma}+V_2^{\gamma})}(V-(e_1+V_2)k). \tag{4.18}$$

Since $e_1 < V_1$,

$$\frac{e_1^{\gamma}}{(e_1^{\gamma}+V_2^{\gamma})}(V-(e_1+V_2)k) \leq \frac{V_1^{\gamma}}{(V_1^{\gamma}+V_2^{\gamma})}(V-(e_1+V_2)k). \tag{4.19}$$

Since the probability of winning for Player 1 is increasing with γ,

$$\frac{V_1^{\gamma}}{(V_1^{\gamma}+V_2^{\gamma})}(V-(e_1+V_2)k) \leq \frac{V_1}{(V_1+V_2)}(V-(e_1+V_2)k) < V_1 \tag{4.20}$$

Thus, Player 1 has no incentive to declare war.

We leave the analysis of this case here because it is qualitatively identical to the case when $\gamma = 1$.

5. The Game with Asymmetric and Proportional CSF ($\gamma = 1$, $\lambda \in [1,\infty)$)

In this section we consider an asymmetric CSF in which the rich country has advantage in military proficiency. We show that when both countries are unconstrained no player has an incentive to declare war, no matter how high the military proficiency of the rich country is. When both countries are constrained, in some cases, the rich country would have an incentive to declare war but there is a transfer that will avoid war.

Consider a CSF of the following form

$$p_1 = \frac{\lambda e_1}{\lambda e_1 + e_2} \text{ and } p_2 = \frac{e_2}{\lambda e_1 + e_2}, \quad \lambda \in [1, \infty). \tag{5.1}$$

It reduces to the form postulated in Section 3 when $\lambda = 1$. We assume $\lambda \geq 1$ because in many cases, war proficiency is positively correlated with relative wealth, i.e. countries that are good at producing wealth are also good at producing weapons.

Expected payoffs for Players 1 and 2 when war is declared are

$$E\pi_1 = \frac{\lambda e_1}{\lambda e_1 + e_2}(V - k(e_1 + e_2)) \text{ and } E\pi_2 = \frac{e_2}{\lambda e_1 + e_2}(V - k(e_1 + e_2)). \tag{5.2}$$

From the definition of expected payoffs we obtain

$$\frac{\partial E\pi_1}{\partial e_1} = \lambda \frac{Ve_2 - k(\lambda e_1^2 + 2e_1 e_2 + e_2^2)}{(\lambda e_1 + e_2)^2} \text{ and} \tag{5.3}$$

$$\frac{\partial E\pi_2}{\partial e_2} = \frac{V\lambda e_1 - k(e_2^2 + 2\lambda e_1 e_2 + \lambda e_1^2)}{(\lambda e_1 + e_2)^2}. \tag{5.4}$$

Setting $\frac{\partial E\pi_i}{\partial e_i} = 0$, $i = 1, 2$, we get that

$$e_1 = \frac{-e_2 + \sqrt{e_2^2(1-\lambda) + \frac{V\lambda e_2}{k}}}{\lambda} \text{ and } e_2 = -e_1\lambda + \sqrt{e_1^2 \lambda(\lambda - 1) + \frac{\lambda V e_1}{k}}. \tag{5.5}$$

Solving (5.5) we obtain

$$e_1^* = \frac{V}{2k(1+\sqrt{\lambda})} \text{ and } e_2^* = \frac{V\sqrt{\lambda}}{2k(1+\sqrt{\lambda})}. \tag{5.6}$$

5.1. Both Players Are Unconstrained

In this case,

$$e_1^* = \frac{V}{2k(1+\sqrt{\lambda})} \leq V_1 \text{ and } e_2^* = \frac{V\sqrt{\lambda}}{2k(1+\sqrt{\lambda})} \leq V_2. \tag{5.7}$$

These inequalities above can be written as

$$\frac{\sqrt{\lambda}}{2k(1+\sqrt{\lambda})} \leq \frac{V_2}{V} \leq 1 - \frac{1}{2k(1+\sqrt{\lambda})}. \tag{5.8}$$

Expected payoffs read:

$$E\pi_1 = \frac{V\sqrt{\lambda}}{2(1+\sqrt{\lambda})} \text{ and } E\pi_2 = \frac{V}{2(1+\sqrt{\lambda})}. \tag{5.9}$$

Notice that the total expected payoff under war is $V/2$. So Player 1 has no incentive to declare war because his expected payoff is less than V_1. Player 2 had no incentive to declare war when $\lambda = 1$, so he can not have any incentive to declare war when $\lambda > 1$ because his expected payoff is decreasing with λ.

The following Proposition summarizes this result.

Proposition 8. *When CSF is of the form (5.1) and both players are unconstrained, peace is an equilibrium outcome in absence of transfers.*

5.2. Both Players are Constrained

This case arises iff $\frac{\partial E\pi_i(V_1,V_2)}{\partial e_i} > 0$, or equivalently:

$$VV_2 > k(\lambda V_1^2 + 2V_1V_2 + V_2^2) \text{ and } V\lambda V_1 > k(V_2^2 + 2\lambda V_1V_2 + \lambda V_1^2), \tag{5.10}$$

which can be written as

$$\lambda(\frac{V}{V_2} + \frac{V_2}{V} - 2) + 2 - \frac{V_2}{V} < \frac{1}{k}, \text{ and} \tag{5.11}$$

$$\frac{(\frac{V_2}{V})^2}{\lambda(1-\frac{V_2}{V})} + 1 + \frac{V_2}{V} < \frac{1}{k}. \tag{5.12}$$

Let us see first that if Player 1 is constrained, Player 2 is also constrained, i.e. that (5.11) implies (5.12). Let

$$G(\lambda, \frac{V_2}{V}) \equiv \lambda(\frac{V_2}{V} + \frac{V}{V_2} - 2) + 2 - \frac{V_2}{V}, \text{ and } F(\lambda, \frac{V_2}{V}) \equiv \frac{(\frac{V_2}{V})^2}{\lambda(1-\frac{V_2}{V})} + 1 + \frac{V_2}{V}. \tag{5.13}$$

Since $G(\lambda, \frac{V_2}{V})$ is increasing with λ and $G(1, \frac{V_2}{V}) = \frac{V}{V_2}$, $F(\lambda, \frac{V_2}{V})$ is decreasing with λ and $F(1, \frac{V_2}{V}) = \frac{V}{V_1}$, and $V_1 > V_2$, then for all $\lambda > 1$, $G(\lambda, \frac{V_2}{V}) > F(\lambda, \frac{V_2}{V})$. Thus, both players are constrained if and only if

$$\frac{1}{k} > G(\lambda, \frac{V_2}{V}). \tag{5.14}$$

Expected payoffs read:

$$E\pi_1(V_1, V_2) = \frac{\lambda V_1 V(1-k)}{\lambda V_1 + V_2} \quad \text{and} \quad E\pi_2(V_1, V_2) = \frac{V_2 V(1-k)}{\lambda V_1 + V_2}. \tag{5.15}$$

The probability of winning for Player 2 decreases with the military proficiency of Player 1, so if Player 2 had no incentive to declare war when $\lambda = 1$, he does not have one now. The probability of winning for Player 1 increases with his military proficiency, thus, if the cost of war is not too high relative to the inequality of resources and his military proficiency, Player 1 has an incentive to declare war. The following proposition formally states this result. The condition under which Player 1 will declare war follows directly from $E\pi_1(V_1, V_2) > V_1$.

Proposition 9. *When CSF is of the form (5.1) and both players are constrained, in the absence of transfers war is declared by Player 1 if and only if*

$$k < \frac{V_2}{V} \frac{(\lambda - 1)}{\lambda}, \tag{5.16}$$

Player 2 has no incentive to declare war.

In the next proposition we study the possibility of avoiding war by transfers, in this case from the poor to the rich country.

Proposition 10. *When CSF is of the form (5.1) and both players are constrained, if* $k < \frac{V_2}{V} \frac{(\lambda-1)}{\lambda}$, *a peace agreement is feasible if*

$$\frac{V_2}{V} > \frac{\lambda - 2}{\lambda - 1}. \tag{5.17}$$

The minimal transfer, $\hat{T}$, *that avoids war is such that makes Player 1 be indifferent between war and peace.*

To complete the analysis of this asymmetric case, we would need to study what will happen in the situation where Player 1 is unconstrained and Player 2 is constrained. The analysis of this part is more complicated, and we believe that will not bring any new insights. What is important in the analysis of the asymmetric case is the fact that it is the rich country that has an incentive to declare war, and that a transfer from the poor to the rich will avoid war.

6. Conclusions

In this paper we presented a simple model of war where players are rational, information is complete and there are no binding agreements. We have shown that it is possible to avoid war by transferring resources from one player to another: from the rich to the poor player (Sections 3 and 4) or from the poor to the rich when the military proficiency of the rich is high (Section 5).

Clearly, our model is very simple. In order to have a broader picture, other factors like dynamics, heterogeneous resources and risk-averse players should be considered. Other functional forms of CSF and the cost of war should be tried as well.

To end the paper, we discuss other mechanisms of altering initial conditions to the advantage of one or several players, that have been used in other parts of the literature.[11]

1: Burning money. In some games, the outcome in equilibrium is affected by the capability of a player to destroy her own resources (van Damme [1989], Ben-Porath and Dekel [1992]). This resembles what happens here but the mechanism by which the destruction affects the outcome is very different. In "burning money" games, it is a signal that one of the players is going after a certain payoff in a subgame. In our case, it is a way of reaching a certain subgame.

2: Transfer/Destruction of Endowments. In General Equilibrium models it is sometimes good for a country to transfer goods to another country. This is the so-called "Transfer Paradox" in International Trade (Leontief [1937], Samuelson [1952], [1954], Gale [1974]). The paradox arises because by making a transfer (or even by destroying one´s resources, as in Aumann and Peleg [1974]) agents affect relative prices. Again, our case is different because in our model there is only one good, so relative prices play no role whatsoever. What happens in our case is that transfers affect both the opportunity cost and the expected revenues of war: i.e., once the potential aggressor has been loaded with money she risks too much and can gain very little by going to war.

[11] Another mechanism is when the rich country destroys a part of its own resources without transfering them to the poor. This mechanism is less powerful than the one considered here because it affects relative wealth only in one way, i.e. making the rich country less rich and not making the poor country richer. But it may work in cases in which, by whatever reason, the poor could not receive transfers.

3: Economic Diplomacy. Ponsatí (2004) studies bilateral conflicts that affect the welfare of a third party. The conflict takes the form of a war of attrition, and intervention is modelled as the possibility that the stakeholder aids the agreement with transfers to the contenders. In this case, the source of money is external to the conflict.

4: Patents. Gallini (1984) has shown that an incumbent firm may license its production technology to reduce the incentive of a potential entrant to develop its own, possibly better, technology. If the licensing contract leaves the potential entrant with its expected return from further research, it will have no incentive to engage in further R&D activity. Thus an incumbent might decide to share its market with a potential entrant in order to deter that entrant from engaging in R&D to displace the incumbent. This is akin to the idea that the rich country pays the poor country in order to deter it from attacking. But the model is different from ours since we have the problem of preventing either country from attacking the other.

7. Appendix.

Lemma 1. *Let $k \in [0.5, 1)$. Suppose that a peace agreement is feasible with a transfer $\hat{T}$ such that after the transfer both players are unconstrained. Then, a peace agreement is also feasible with a transfer $T' < \hat{T}$ such that only Player 2 is constrained.*

Proof. Since $k \in [0.5, 1)$, $\min(\frac{1}{4k}, k) = \frac{1}{4k}$. Thus the constraints for T' are $\frac{k}{(1+k)^2} \leq \frac{V_2+T'}{V} < \frac{1}{4k}$. Notice first that $\frac{k}{(1+k)^2} < \frac{1}{4k}$ since $k < 1$, so T' can be chosen between these bounds. Furthermore, since $\frac{1}{4k} \leq \frac{V_2+\hat{T}}{V}$ and $\frac{V_2+T'}{V} < \frac{1}{4k}$, $T' < \hat{T}$ and thus $\frac{V_2+T'}{V} \leq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$. We finally show that T' can be chosen such that $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} \leq \frac{V_2+T'}{V}$. Since $\frac{V_2}{V} < \frac{1}{4k}$, $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} < \frac{1}{4}$, and since $k < 1$, $\frac{1}{4} < \frac{1}{4k}$. Thus, choosing T' such that $\frac{V_2+T'}{V} \simeq \frac{1}{4k}$ will satisfy the desiderata. ■

Lemma 2. *Let $k \in (0, 0.5)$. Suppose that a peace agreement is feasible with a transfer $\hat{T}$ such that after the transfer both players are constrained. Then, a peace agreement is also feasible with a transfer $T' < \hat{T}$ such that only Player 2 is constrained.*

Proof. Since $k \in (0, 0.5)$, $\min(\frac{1}{4k}, k) = k$. Thus the constraints for T' are $\frac{k}{(1+k)^2} \leq \frac{V_2+T'}{V} < k$. Notice first that $\frac{k}{(1+k)^2} < k$, so T' can be chosen between these bounds.

Furthermore, since $k < \frac{V_2+\hat{T}}{V}$ and $\frac{V_2+T'}{V} < k$, $T' < \hat{T}$ and thus $\frac{V_2+T'}{V} \leq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$. We finally show that T' can be chosen such that $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} \leq \frac{V_2+T'}{V}$. Since $\frac{V_2}{V} \leq k < \frac{1}{4k}$, and $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k}$ is increasing for all $\frac{V_2}{V} \in [0, \frac{1}{4k}]$, $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} \leq -k^2 + k < k$. Thus, choosing T' such that $\frac{V_2+T'}{V} \simeq k$ will satisfy the desiderata. ■

Lemma 3. *If before the transfer war occurs,* $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} \leq \frac{k}{(1+k)^2}$.

Proof. Since war occurs, $\frac{V_2}{V} < \frac{k}{(1+k)^2}$. The function $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k}$ is increasing in $\frac{V_2}{V}$ in the interval $[0, \frac{1}{4k}]$, since $\frac{k}{(1+k)^2} < \frac{1}{4k}$, the maximal value in the relevant interval is reached at $\frac{V_2}{V} = \frac{k}{(1+k)^2}$. Thus, $-k\frac{V_2}{V} + \sqrt{\frac{V_2}{V}k} \leq \frac{k}{(1+k)^2}$. ■

Lemma 4. *Let* $k \in (0, 1]$. *There is a solution* $x_1(k)$ *of equation* $\frac{k}{(1+k)^2} = -kx + 2\sqrt{xk}$ *such that* $x_1(k) \in [0, \frac{k}{(1+k)^2})$.

Proof. Let

$$x_1(k) = \frac{(4 - \frac{2k}{(1+k)^2}) - \frac{4}{(1+k)}\sqrt{1+k+k^2}}{2k}. \tag{7.1}$$

It is straightforward to see that $x_1(k)$ so define is a solution of the equation $\frac{k}{(1+k)^2} = -kx + 2\sqrt{xk}$. Suppose that there is $k \in (0, 1]$ such such that $x_1(k) > \frac{k}{(1+k)^2}$. Since $x = \frac{k}{(1+k)^2}$ is the solution of the equation $\frac{k}{(1+k)^2} = -kx + \sqrt{xk}$ and $-kx + \sqrt{xk}$ is increasing in $x \in [0, \frac{1}{4k}]$, $\frac{k}{(1+k)^2} < -kx_1(k) + \sqrt{x_1(k)k}$. But, $-kx_1(k) + \sqrt{x_1(k)k} \leq -kx_1(k) + 2\sqrt{x_1(k)k} = \frac{k}{(1+k)^2}$, which is a contradiction. Therefore, $x_1(k) \leq \frac{k}{(1+k)^2}$. ■

Notice first that if $k \in [0.5, 1)$, $\frac{1}{4k} \leq k$. By Lemma 3 a peace agreement in this case is feasible if and only if

$$\frac{k}{(1+k)^2} < \min(\frac{1}{4k}, -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}). \tag{7.2}$$

Lemma 5. *Let* $k \in [0.5, 1)$. *There is a solution* $x_0(k)$ *of equation* $\frac{1}{4k} = -kx + 2\sqrt{xk}$ *such that* $x_0(k) \in [0, \frac{k}{(1+k)^2})$.

Proof. Let

$$x_0(k) = \frac{-0.5 + 4k - \sqrt{16k^2 - 4k}}{2k^2}. \tag{7.3}$$

It is straightforward to see that $x_0(k)$ so define is a solution of the equation $\frac{1}{4k} = -kx + 2\sqrt{xk}$. Furthermore, $x_0(k)$ is decreasing in k for all $k \in [0.5, 1)$. Since $\frac{k}{(1+k)^2}$ is increasing in $[0.5, 1)$, and $x_0(0.5) = 0.171\,57 < \frac{0.5}{(1+0.5)^2} = 0.222\,22$, $x_0(k) \in [0, \frac{k}{(1+k)^2})$. ■

Lemma 6. *Let $k \in [0.5, 1)$. Then, $x_1(k) < x_0(k)$.*

Proof. Since $\frac{k}{(1+k)^2} < \frac{1}{4k}$ for all $k \in [0.5, 1)$ and $-kx + 2\sqrt{xk}$ is increasing for all $x \in [0, \frac{k}{(1+k)^2})$, $x_1(k) < x_0(k)$. ■

Lemma 7. *Let $k \in [0.5, 1)$. If $\frac{V_2}{V} \geq x_0(k)$, a peace agreement is feasible.*

Proof. Since $-kx + 2\sqrt{xk}$ is increasing for all $x \in [0, \frac{k}{(1+k)^2})$, if $\frac{V_2}{V} \geq x_0(k)$ then $\frac{1}{4k} \leq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$. Since $\frac{k}{(1+k)^2} < \frac{1}{4k}$, condition (7.2) is satisfied, and therefore, a peace agreement is feasible. ■

Lemma 8. *Let $k \in [0.5, 1)$. If $x_1(k) < \frac{V_2}{V} < x_0(k)$, a peace agreement is feasible.*

Proof. Since $\frac{V_2}{V} < x_0(k)$, $-k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k} < \frac{1}{4k}$. Since $x_1(k) < \frac{V_2}{V}$, and $-kx + 2\sqrt{xk}$ is increasing for all $x \in [0, \frac{k}{(1+k)^2})$, $\frac{k}{(1+k)^2} < -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$, thus condition (7.2) is satisfied, and therefore, a peace agreement is feasible. ■

Lemma 9. *Let $k \in [0.5, 1)$. If $\frac{V_2}{V} \leq x_1(k)$, there is no possibility of a peace agreement.*

Proof. Clearly, if $\frac{V_2}{V} \leq x_1(k)$, $\frac{k}{(1+k)^2} \geq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$, and since $x_1(k) < x_0(k)$, $-k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k} < \frac{1}{4k}$. Thus, condition (7.2) is never satisfied. ■

Secondly, notice that if $k \in (0, 0.5)$, $\frac{k}{(1+k)^2} < k < \frac{1}{4k}$. By Lemma 3 a peace agreement in this case is feasible if and only if

$$\frac{k}{(1+k)^2} < \min(k, -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}). \tag{7.4}$$

Lemma 10. *Let $k \in (0, 0.5)$. There is a solution $x_2(k)$ of equation $k = -kx + 2\sqrt{xk}$ such that $x_2(k) \in (0, \frac{k}{(1+k)^2})$.*

Proof. Let

$$x_2(k) = \frac{2 - k - 2\sqrt{1-k}}{k}. \tag{7.5}$$

It is straightforward to see that $x_2(k)$ so defined is a solution of the equation $k = -kx + 2\sqrt{xk}$. Furthermore, $x_2(k)$ is increasing in $(0, 0.5)$, $\lim_{x\to 0} x_2(0) = 0$ which coincides with the value of $\frac{k}{(1+k)^2}$ in $k = 0$, and $x_2(0.5) = 3 - 2\sqrt{2} < \frac{0.5}{(1+0.5)^2} = \frac{2}{9}$. Since $\frac{k}{(1+k)^2}$ is also increasing, it follows that $x_2(k) \in (0, \frac{k}{(1+k)^2})$. ■

Lemma 11. *Let $k \in (0, 0.5)$. Then, $x_1(k) < x_2(k)$.*

Proof. Since $\frac{k}{(1+k)^2} < k$ for all $k \in (0, 0.5)$ and $-kx + 2\sqrt{xk}$ is increasing for all $x \in [0, \frac{k}{(1+k)^2})$, $x_1(k) < x_2(k)$. ■

Lemma 12. *Let $k \in (0, 0.5)$. If $\frac{V_2}{V} \geq x_2(k)$, a peace agreement is feasible.*

Proof. Since $-kx + 2\sqrt{xk}$ is increasing for all $x \in [0, \frac{k}{(1+k)^2})$, if $\frac{V_2}{V} \geq x_2(k)$ then $k \leq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$. Since $\frac{k}{(1+k)^2} < k$, condition (7.4) is satisfied, and therefore, a peace agreement is feasible. ■

Lemma 13. *Let $k \in (0, 0.5)$. If $x_1(k) < \frac{V_2}{V} < x_2(k)$, a peace agreement is feasible.*

Proof. Since $\frac{V_2}{V} < x_2(k)$, $-k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k} < k$. Since $x_1(k) < \frac{V_2}{V}$, and $-kx + 2\sqrt{xk}$ is increasing for all $x \in [0, \frac{k}{(1+k)^2})$, $\frac{k}{(1+k)^2} < -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$, thus condition (7.4) is satisfied, and therefore, a peace agreement is feasible. ■

Lemma 14. *Let $k \in (0, 0.5)$. If $\frac{V_2}{V} \leq x_1(k)$, there is no possibility of a peace agreement.*

Proof. Clearly, if $\frac{V_2}{V} \leq x_1(k)$, $\frac{k}{(1+k)^2} \geq -k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k}$, and since $x_1(k) < x_2(k)$, $-k\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}k} < k$. Thus, condition (7.4) is never satisfied. ■

Proof of Proposition 3. If $k = 1$, notice that it is impossible to be in Case 3.2 or in Case 3.3 after the transfer, so we are left with Case 3.1 as the only possibility for achieving peace. Then, conditions 1, 2, and 3 read $\frac{V_2}{V} < \frac{1}{4}$, $-\frac{V_2}{V} + \sqrt{\frac{V_2}{V}} \leq \frac{V_2+T}{V} \leq -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}$ and $\frac{1}{4} \leq \frac{V_2+T}{V}$. It can be easily shown that for $\frac{V_2}{V} < \frac{1}{4}$, $-\frac{V_2}{V} + \sqrt{\frac{V_2}{V}} \leq \frac{1}{4}$.

Thus, the transfer that brings peace exists if and only if $\frac{1}{4} < -\frac{V_2}{V} + 2\sqrt{\frac{V_2}{V}}$, i.e. if and only if $\frac{V_2}{V} > x_1(1)$, otherwise, is not in the interest of Player 1 to make any transfer.
It follows directly from Lemmas 7, 8 and 9 that a peace agreement for $k \in [0,5,1)$ is feasible if and only if $\frac{V_2}{V} > x_1(k)$.
It also follows directly from Lemmas 12, 13 and 14 that a peace agreement for $k \in (0, 0.5)$ is feasible if and only if $\frac{V_2}{V} > x_1(k)$.
The minimal transfer needed is such that $\frac{k}{(1+k)^2} = \frac{V_2+\hat{T}}{V}$. Notice that for such a transfer Player 2 is indifferent between war and peace, since for $\hat{T}$

$$V_2 + \hat{T} = E\pi_2^* = \sqrt{V(V_2 + \hat{T})k} - k(V_2 + \hat{T}).$$

■

Proof of Proposition 5. Consider the second stage of the game where war would be declared when no transfers are made, but that a transfer $\hat{T}$ such that $V_2 + \hat{T} = V/2(1+\gamma)$ has been made. After the transfer, payoffs for Player 1 in case of peace are

$$V_1 - \hat{T} = V_1 - \frac{V}{2(\gamma+1)} + V_2 = V - \frac{V}{2(\gamma+1)}. \tag{7.6}$$

Assuming that these payoffs are less than those in the case of a war with no transfer,

$$V - \frac{V}{2(\gamma+1)} < \frac{V}{2(\gamma+1)} \Leftrightarrow V < \frac{V}{\gamma+1}, \tag{7.7}$$

which is impossible. Thus, $\hat{T}$ yields an incentive for peace for both players. It is only left to show that after the transfer, Player 1 is not constrained, which amounts to

$$V_1 - \hat{T} \geq \frac{\gamma V}{2k(\gamma+1)} \Leftrightarrow V - \frac{V}{2(\gamma+1)} \geq \frac{\gamma V}{2k(\gamma+1)} \Leftrightarrow k \geq \frac{\gamma}{2\gamma+1}. \tag{7.8}$$

By the previous Proposition, if the relatively poor player had an incentive to go to war before the transfer, $\gamma < k$. But then, $k > \gamma > \frac{\gamma}{2\gamma+1}$, as desired. ■

Proof of Proposition 6. If $k = 0$ and $\gamma \in (0,1)$ both players are constrained and in this situation Player 2 has an incentive to declare war since condition (4.14) holds. The minimal transfer that could avoid war by Player 2 is such that this player has no incentive to declare war after the transfer. That is, once the transfer has been made, the resources into the hands of Player 2, say V_2', should be such that

$$\frac{V_2'}{V} = \frac{V_2'^{\gamma}}{(V_1'^{\gamma} + V_2'^{\gamma})}. \tag{7.9}$$

But for $\gamma \in (0,1)$, the above equation only holds if $\frac{V_2'}{V} = \frac{1}{2}$. But if this is the case, Player 1 is worse off after the transfer than with war because

$$\frac{V_1^{\gamma}}{(V_1^{\gamma} + V_2^{\gamma})} V > \frac{1}{2} V = V_1'. \tag{7.10}$$

A larger transfer will be even worse for Player 1. ■

Proof of Proposition 7. In what follows we developed the necessary steps to prove the Proposition

Step 1. Let T be the minimal transfer that makes Player 2 indifferent between war and peace,

$$\frac{V_2 + T}{V} = \frac{(V_2 + T)^{\gamma}}{(V_1 - T)^{\gamma} + (V_2 + T)^{\gamma}}(1 - k). \tag{7.11}$$

Let us see that T exists and $V_2 + T < V/2$.

Let $f(x) = \frac{x^{\gamma}}{(1-x)^{\gamma}+x^{\gamma}}(1-k)$. Let $x_0 = \frac{V_2}{V}$, by condition (4.14), $x_0 < f(x_0)$. For $x = \frac{1}{2}$, $f(\frac{1}{2}) < \frac{1}{2}$. Since f is increasing in x and continuous, there is a unique $x' \in (x_0, 1/2)$ such that $x' = f(x')$. Let T be such that $\frac{V_2+T}{V} = x'$. Since $x' \in (x_0, 1/2)$, $T > 0$ and $V_2 + T < V/2$. Notice also that T is decreasing in V_2.

Step 2. Player 2 is better off with the transfer.

Since the probability of winning for Player 2 is increasing with V_2,

$$\frac{V_2^{\gamma}}{V_1^{\gamma} + V_2^{\gamma}} V(1-k) < \frac{(V_2 + T)^{\gamma} V}{(V_1 - T)^{\gamma} + (V_2 + T)^{\gamma}}(1 - k) = V_2 + T. \tag{7.12}$$

Step 3. Both players still constrained after the transfer.

Clearly, if Player 1 was initially constrained, he is also constrained after the transfer. Furthermore, since $V_2+T < V/2$ and Player 1 is constrained, Player 2 is also constrained.

Step 4. Neither Player 1, Player 2, has an incentive to declare war after the transfer.

Since both players are constrained, in case of war the expected utility of Player 2 is

$$\frac{(V_2 + T)^{\gamma}}{(V_1 - T)^{\gamma} + (V_2 + T)^{\gamma}}(1 - k). \tag{7.13}$$

and by the definition of T (7.11), Player 2 does not have an incentive for going to war. Neither does Player 1 by the same argument as before the transfer.

Step 5. For all $V_2 \geq \bar{V}_2$, Player 1 is better off after the transfer.

Let us see that the following inequality holds,

$$\frac{V_1^\gamma}{(V_1^\gamma + V_2^\gamma)} V(1-k) \leq V_1 - T = V_1' \tag{7.14}$$

Proving inequality (7.14) is equivalent to proving that

$$V_2' \leq V - \frac{V_1^\gamma}{(V_1^\gamma + V_2^\gamma)} V(1-k). \tag{7.15}$$

Or that

$$\frac{V_2'}{V} \leq k + \frac{V_2^\gamma}{(V_1^\gamma + V_2^\gamma)}(1-k). \tag{7.16}$$

Since $\bar{V}_2$ is such that the minimal transfer that makes Player 2 indifferent between war and peace is kV, and $V_2 \geq \bar{V}_2$, $T \leq kV$. Thus,

$$\frac{V_2'}{V} = \frac{V_2}{V} + \frac{T}{V} \leq \frac{V_2}{V} + k. \tag{7.17}$$

Furthermore, since Player 2 is better off with war if the transfer is not made,

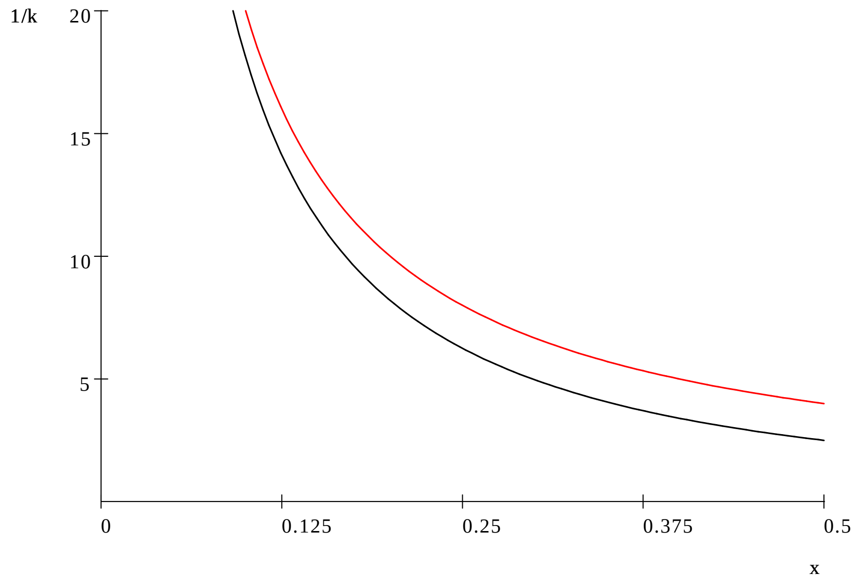
$$\frac{V_2}{V} < \frac{V_2^\gamma}{(V_1^\gamma + V_2^\gamma)}(1-k). \tag{7.18}$$

Which combined with (7.17) gives inequality (7.16) as we wanted to prove.

■

Proof of Proposition 10. It is easy to see that $\frac{V_2}{V} > \frac{\lambda-2}{\lambda-1}$ is equivalent to $G(\lambda, \frac{V_2}{V}) < \frac{\lambda}{(\lambda-1)}\frac{V}{V_2}$. Let us see that we can find a transfer from Player 2 to Player 1 that avoids war.

Consider the following picture where $x \equiv \frac{V_2}{V}$ (this is the case for $\lambda = 2$ where $G(2,x) = x + (2/x) - 2)$ and clearly $G(2,x) < (2/x)$).



The red line is $\frac{1}{k} = \frac{\lambda}{(\lambda-1)x}$ and the black line is $\frac{1}{k} = \lambda(x + \frac{1}{x} - 2) + 2 - x$. The area to the right of both is the area where there is war. Starting from any point in this area, say $(\frac{1}{\bar{k}}, \bar{x})$, define $x' \equiv \frac{\bar{k}\lambda}{\lambda-1}$. Let x' be the new distribution of resources after the transfer has been made. Notice first that both players remain constrained after the transfer. Because of the definition of the new distribution of resources, after the transfer Player 1 has no incentive to declare war. What it is not clear is whether he will accept the transfer. So, we have to see that

$$E\pi_1(V_1, V_2) = \frac{\lambda V_1 V(1-k)}{\lambda V_1 + V_2} \leq V_1' = V - V_2' = V(1 - \frac{\bar{k}\lambda}{(\lambda-1)}). \tag{7.19}$$

The left hand side is clearly increasing in V_1, thus

$$\frac{\lambda V_1 V(1-k)}{\lambda V_1 + V - V_1} < \frac{\lambda V_1' V(1-k)}{\lambda V_1' + V - V_1'} = V_1'. \tag{7.20}$$

Finally, let us prove that Player 2 is also better off. So we have to compare the payoffs of player2 at $x' = V\frac{\bar{k}\lambda}{(\lambda-1)}$ with the payoffs should a war arise, $\frac{V_2 V(1-\bar{k})}{\lambda V - \lambda V_2 + V_2}$. What we want to prove is that

$$\frac{V_2(1-\bar{k})}{\lambda V - \lambda V_2 + V_2} \leq \frac{\bar{k}\lambda}{(\lambda-1)}.$$

Notice first that when $\lambda \to \infty$, the left hand side tends to zero and the right hand side tends to $\bar{k}$. Secondly, when $\lambda \to 1$, the left hand side tends to $V_2(1-\bar{k})$ and the right hand side tends to ∞. Thus, if the inequality were the other way around for some value of λ there must be, generically, two positive solutions to the equation $V_2(1-\bar{k})(\lambda-1) = \bar{k}\lambda(\lambda V - \lambda V_2 + V_2)$. It is easy to see that this equation has, at most, one positive root. Thus, there must be a contradiction. ■

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