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Multi-product firms and product variety*

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Abstract

The goal of this paper is to study the role of multi-product firms in the market provision of product variety. The analysis is conducted using the spokes model of non-localized competition proposed by Chen and Riordan (2007). In the presence of economies of scope the equilibrium configuration includes a small number of multi-product firms who use their product range as a key strategic variable. Product variety under multi-product firms may be higher or lower than under single-product firms. Similarly, from a social point of view the level of variety provided by large multi-product firms may also be excessive or insufficient. A key determinant of the overall provision of variety is the strategic price effect. Under some conditions, firms drastically restrict their product range in order to relax price competition, causing a substantital underprovision of variety.

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Key words: product variety, multiproduct firms, oligopoly, spokes

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1 Introduction

Does the market provide too much or too little product variety? Is the supply of books, CD's, TV programs, furniture or cereals sufficiently diverse to efficiently match the preferences of heterogeneous consumers? Or, on the contrary, do profit maximizing firms tend to produce a disproportionate array of products and incur into excessive costs? The existing theoretical literature clearly suggests that anything can happen, i.e., product diversity may be excessive or insufficient depending on the relative strength of various effects. However, the literature has typically focused on the case of single-product firms. In contrast, in many markets individual firms produce a significant fraction of all varieties. These multi-product firms choose their product range as an additional strategic tool, which may potentially affect the overall provision of variety. Does the presence of multi-product firms reduce or expand product variety with respect to the case of single-product firms? And with respect to the first best? Can an incumbent firm use product proliferation to monopolize the market and prevent entry?

In this paper we address these issues by introducing multi-product firms into the spokes model of non-localized competition proposed by Chen and Riordan (2007). There are several reasons that justify the choice of model; in particular, the spokes model provides a tractable, intuitive and transparent framework to study competition and product variety when neighboring effects are absent, which is increasingly the case in many industries. By considering a continuous approximation of the original model we show that it is possible to study in the same framework alternative market structures, including monopolistic competition (large number of single-product firms, oligopoly (small number of large multi-product firms) and even asymmetric competition (one large, multi-product firm competing with a large number of single-product firms).

The study of product diversity has been typically conducted using three alternative families of models. Many authors have chosen spatial models of localized competition, similar to those proposed by Hotelling (1929) and Salop (1979). However, it is generally agreed that they are not well suited to study either the welfare implications of product variety or multi-product firms. Alternatively, a large fraction of the literature has followed Spence (1976) and Dixit and Stiglitz (1977) (SDS) and assumed the existence of a representative consumer with well defined preferences over all possible varieties. In this set up neighboring effects are absent and typically each variety

is symmetrically placed with respect to the rest. Finally, some authors have used some version of the multinomial logit model, where consumers are statistically identical and their choices are described by logit models.[1] Several papers have recently introduced multi-product firms into CES and nested logit frameworks and have analyzed their role in the provision of product variety. The papers which are most closely related to ours are Ottaviano and Thisse (1999), Ju (2003), and Anderson and de Palma (2006).[2] Below we comment on these papers in more detail.[3]

The spokes model (Chen and Riordan, 2007) extends the Hotelling model to an arbitrary number of varieties in a perfectly symmetric set up. A crucial feature of the model is that each consumer only cares about two varieties, which differ across consumers. The preference space consists of N spokes that start from the same central point. The producer of each potential variety is located in the extreme end of a different spoke. If $N = 2$ then we are in the standard Hotelling set up. As N goes to infinity, and if the number of varieties per firm is small, the model becomes an adequate representation of monopolistic competition. In this paper we take the spokes model one step further by considering multi-product firms. It turns out that the spokes model can accommodate multi-product firms as easily as the SDS model and, moreover, it brings about new insights and useful welfare results.

Tractability of the spokes model is considerably enhanced by assuming that the number of varieties is sufficiently large. In the next section we review the finite model and formulate the continuous approximation. Thus, the product range of a multi-product firm can be treated as a continuous variable. In the same section we also briefly review the solution to the social planner's problem and the free entry equilibrium with single-product firms. These problems were examined in Chen and Riordan (2007). The only novelty is that we provide explicit values for the variety variable in the continuous approximation, which sets the stage for the analysis of multi-product firms.

[1] See, for instance, Perloff and Salop (1985), Besanko et al. (1990), and Anderson and de Palma (1992a)

[2] Earlier papers like Brander and Eaton (1984), Chamsaur and Rochet (1989), and Anderson and De Palma (1992b) are also important milestones in the formal analysis of multi-product firms.

[3] A number of papers have introduced multi-product firms in the SDS framework in order to study the effects of trade liberalization. Some of the prominent and recent papers include Ottaviano et al. (2002), Allanson and Montagna (2005), Nocke and Yeaple (2006), Eckel and Neary (2006), Bernard et al. (2006), and Feenstra and Ma (2007).

Although the main focus of the paper is the effect of competition among multi-product firms, in Section 3 we establish some interesting insights for the monopoly case. First, a protected monopolist may provide a higher level of product variety than the social planner. Excessive diversity occurs when the cost of producing an additional variety is relatively low. In this case, and in contrast to the first best, the optimal monopoly pricing implies that every new variety generates a strong market expansion effect. Second, in contrast to the case of localized competition, an incumbent firm cannot use product proliferation to monopolize the market and prevent entry. In fact, we show that firm size (as measured by the length of its product range) is a competitive disadvantage. Therefore, multi-product firms can only emerge in industries characterized by non-localized competition if economies of scope are sufficiently strong.

Section 4 is the core of the paper and deals with the oligopoly case. The presence of economies of scope implies that only a small number of multi-product firms can survive and each one chooses both product range and prices strategically. As it is standard, the number of firms is endogenous and determined by a non-negative profit condition. In line with the literature, the number of firms in equilibrium is inefficiently high because of the duplication of entry costs. However, in contrast to most of the literature it is not always the case that the overall provision of product variety is insufficient from a social point of view. In fact, the presence of multi-product firms influences the overall provision of product variety through several channels:

a) *Cannibalization:* a multi-product firm internalizes the impact of a new variety on the demand for the other varieties it produces. This effect tends to reduce product diversity.

b) *Appropriability*: The presence of a small number of large multi-product firms is associated with prices which are higher than those set by single-product firms. This effect tends to expand product variety.

c) *Strategic price effect*: an oligopolistic firm anticipates that its product range influences its rival's prices. It turns out that the sign of the strategic price effect is ambiguous. If consumers' reservation prices are not too low then firms find it optimal to restrict product variety in order to relax price competition. For sufficiently low reservation prices the sign of the strategic price effect may be reversed and a firm may find it optimal to expand its product variety in order to raise its rival's prices.

Obviously, these effects are larger when the number of firms is small (when economies of scope are stronger). In fact, the equilibrium with multi-product

firms converges to the equilibrium with single-product firms as the number of firms goes to infinity (as economies of scope vanish).

As shown by Chen and Riordan (2007), product variety chosen by single-product firms may be insufficient or excessive with respect to the first best. Since large multi-product firms may expand or contract the level of product variety selected by single-product firms, then the overall level of product variety chosen by large multi-product firms can also be socially excessive or insufficient depending on parameter values. The size of the discrepancy between the market provision of variety and the first best level is particularly large in the duopoly case, if consumers' reservation prices are not too small, and the fixed cost per variety is relatively low. In the limit case of zero fixed costs per variety, duopolists may choose to produce a relatively low fraction of potential varieties, as low as 50%, even though social efficiency calls for 100%. Thus, in contrast to the case of single-product firms, under duopoly product diversity may be inefficiently low for both low and high values of the fixed cost. For intermediate values anything can happen.

Some of the effects that are present in this paper are similar, or at least related, to those identified by the literature; in particular by Ottaviano and Thisse (1999), Ju (2003), and Anderson and de Palma (2006).

Ottaviano and Thisse (1999) consider a quadratic utility model. A non-desirable feature of their framework is that the optimal prices of a multi-product firm are independent of its product range. In other words, in their set up a firm cannot take advantage of producing a significant fraction of all potential varieties in order to raise its prices. In their model multi-product firms generate less product diversity than single-product firms because of the cannibalization effect. Also, product diversity under multi-product firms is socially excessive only if the fixed cost of producing an additional variety is sufficiently low (the threshold is lower than under monopolistic competition). In contrast, in the current set up when the fixed cost is low then firms find it optimal (in the adequate parameter range) to restrict their product range in order to maintain a friendly price environment, which results in insufficient product variety.

The strategic price effect of our oligopoly game is related to that of Ju (2003), and Anderson and de Palma (2006). In these models a broader product range also induces rivals to price more aggressively. Ju (2003) considers a nested CES utility function and assumes that varieties produced by a single firm are better substitutes for one another than varieties produced by different firms. He identifies an strategic price effect of a nature similar to ours.

Unfortunately, his framework does not allow a clean comparison between the market solution and the first best and hence we cannot compare his results against ours.[4] In Anderson and de Palma (2006) consumers have preferences for both firms and varieties. Consumers' decisions are taken in two steps. First, they choose which firm to purchase from. Second, they choose which products to buy from the selected firm. As a result, a firm with a broader product range becomes more attractive to consumers, which induces a more aggressive price response by rival firms. In this case, in the free entry equilibrium there are too many firms, each one producing too narrow a product range. In contrast (and consistent with Ottaviano and Thisse, 1999), we consider markets where consumers have symmetric preferences for varieties (they do not care who produces what) and hence the impact of multi-product firms on overall product variety can be evaluated more clearly.

The results of this paper also contrast with those obtained in standard spatial models (See, for instance, Schmalensee, 1978; and Bonanno, 1987). In these models, an incumbent firm may find it profitable to monopolize the market by crowding the product space or by choosing the appropriate location. Instead, the current set up suggests that the presence of neighboring effects in those models was crucial for their results. In fact, in the absence of neighboring effects proliferation cannot be an effective entry deterrence mechanism.

2 The spokes model with a large number of varieties

2.1 The set up

We start up by reviewing the spokes model introduced by Chen and Riordan (2007). There are N possible varieties of a differentiated product, indexed by i, $i = 1, ..., N$. The preference space consists of N lines of length $\frac{1}{2}$, also indexed by i, that meet at the centre (spokes network). The producer of variety i is located in the extreme end of line i. A particular variety may or

[4] Minniti (2006) imbeds Ju's framework in a growth setting and conducts the welfare analysis. However, he assumes a continuum of firms and, as a result, the strategic price effect is not present any more. Consistent with Anderson and de Palma(2006), he also finds that in equilibrium there are too many firms, each one providing a too narrow product range.

may not be active (supplied). Supplying a variety involves a fixed cost, f, and constant marginal cost (which, for simplicity, is normalized to 0).

There is a mass $\frac{N}{2}$ of consumers that are uniformly distributed over the spokes network. A consumer located in line i (her favorite brand), at a distance x from the extreme end, obtains a utility of $v - x_i - p_i$ is she buys one unit of variety i at the price p_i (unit transportation cost is normalized to 1). Her second preferred brand, $j \neq i$, is chosen by nature with probability $\frac{1}{N-1}$. If she purchases one unit of variety j at a price p_j then she obtains a utility of $v - (1 - x) - p_j$. Each consumer buys at most one unit of one of the two desired brands. Throughout the paper we assume that $v > 2$.[5]

The spokes framework offers a very useful representation of non-localized competition. We consider the limiting case in which the number of possible varieties, N, goes to infinite, keeping the mass of consumers per variety equal to $\frac{1}{2}$. In this context, we can treat the fraction of active varieties, denoted by k, as a continuous variable.[6]

For an arbitrary value of $k \in (0, 1)$, consumers can be classified in three different groups. A fraction k^2 of consumers have access to their two desired varieties, a fraction $2k(1-k)$ are only able to purchase one of them, and a fraction $(1-k)^2$ have access to none of their desired varieties and drop out of the market.

Considering the pool of consumers that demand variety i, a fraction k of these consumers also have access to their alternative variety, and a fraction $1 - k$ do not.

Throughout the paper we express all variables per unit mass of consumers. For instance, the number of active varieties is $\frac{kN}{\frac{N}{2}} = 2k$ and, hence, total production costs are $2kf$.

2.2 First benchmark: The social planner

For a given value of k, those consumers with access to their two desired varieties should be allocated, from an efficiency point of view, to the closest supplier. Thus, the maximum average surplus generated by this group of

[5] Chen and Riordan (2007) consider a larger parameter range: $v > 1$. By focusing on $v > 2$ we simplify the presentation considerably without affecting the main insights.

[6] Chen and Riordan (2007) solve the model for a finite N and study the limiting properties of the equilibrium as N goes to infinity. In this paper we work directly on the limit economy. It is shown below that, in the case of single-product firms, the two approaches are equivalent.

consumers is $v - \frac{1}{4}$. Consumers with access to only one variety incur in higher average transportation costs, and thus the average surplus generated by this group is $v - \frac{1}{2}$. Finally, consumers without access to any of their desired varieties generate zero surplus. Therefore, total surplus is:

$$W(k) = k^2\left(v - \frac{1}{4}\right) + 2k(1-k)\left(v - \frac{1}{2}\right) - 2kf.$$

The first order condition (objective function is concave) can be written as follows:

$$\frac{dW}{dk} = 2\left\{k\frac{1}{4} + (1-k)\left(v - \frac{1}{2}\right) - f\right\} = 0.$$

The first term represents the preference matching effect. When we activate an additional variety, a fraction k of all consumers that have a taste for that variety already have access to the other desired variety and hence the new variety simply involves lower average transportation costs $\left(\text{from } \frac{1}{2} \text{ down to } \frac{1}{4}\right)$. The second term is the market expansion or aggregate demand effect. A fraction $1-k$ of these consumers had no access to any of their desired varieties and the availability of the additional variety involves an average surplus of $\left(v - \frac{1}{2}\right)$. Since $v - \frac{1}{2} > \frac{1}{4}$ then total surplus created by an additional variety decreases with k.

Thus, the optimal value of k, denoted by k^* , can be computed directly from the first order condition (See Figure 1):

$$k^* = \begin{cases} 0 \text{ if } f \geq v - \frac{1}{2} \\ \frac{v-\frac{1}{2}-f}{v-\frac{3}{4}} \text{ if } \frac{1}{4} \leq f \leq v - \frac{1}{2} \\ 1 \text{ if } f \leq \frac{1}{4} \end{cases}$$

2.3 Second benchmark: competition among single-product firms

Suppose that the number of potential firms is equal to the number of possible varieties, and each firm can produce only one variety. If a firm decides to enter the market and supply variety i then it pays the fixed cost, f, and sets a price, p_i.[7] We restrict attention to equilibria with symmetric prices and

[7] Since each firm is negligible then the timing of decisions does not affect equilibrium behavior.

free entry. Hence, firms make zero profits if $k < 1$, and non-negative profits if $k = 1$. This is a model of monopolistic competition, in the sense that each firm is negligible with respect to the market but it enjoys some market power. This is why we denote equilibrium values with the superscript MC.

Chen and Riordan (2007) focused on the case of single-product firms and also study the limiting properties of the model as the number of varieties goes to infinity. Hence, there are no original results in this subsection and we simply present it as an additional benchmark in order to set the stage for the analysis of multi-product firms (we provide explicit expressions for the equilibrium values of k).

We need to distinguish between two alternative cases.[8] If f is sufficiently high, $f \geq \frac{(v-1)^2}{v}$, then k is low and firms face little competition from their rivals. In equilibrium firms set prices $p^{MC} = v - 1$. From the zero-profit condition we obtain that:

$$k^{MC} = \begin{cases} 2\frac{v-1-f}{v-1}, \text{ if } f \in \left[\frac{(v-1)^2}{v}, v-1\right] \\ 0, \text{ if } f \geq v-1 \end{cases}$$

If f is sufficiently low, $f \leq \frac{(v-1)^2}{v}$, then k is relatively high and competition drives equilibrium prices below $v - 1$. More specifically, $p^{MC} = \frac{2-k}{k}$.[9] As a result, from the zero-profit condition we obtain that:

$$k^{MC} = \begin{cases} 1, \text{ if } f \leq \frac{1}{2} \\ 2 + f - \sqrt{4f + f^2}, \text{ if } f \in \left[\frac{1}{2}, \frac{(v-1)^2}{v}\right] \end{cases}$$

Chen and Riordan (2007) already pointed out that the level of product variety provided by single-product firms may be excessive or insufficient with respect to the first best. The ambiguity comes from the tension between two countervailing effects:

a) *Partial appropriability of the market expansion effect*: an entering firm cannot appropriate all the surplus it generates by expanding aggregate demand. First, it does not price discriminate; second, the price tends to fall as the number of varieties increases (price competition).

[8] Algebraic details are available upon request.

[9] If $v \geq 1 + \frac{(2-k)^2}{2k(1-k)}$ then a firm has incentives to deviate from $p = \frac{2-k}{k}$ and set $p = v - 1$. In this case, a symmetric equilibrium does not exist.

b) *Business stealing (excessive appropriability of the preference matching effect)*: A fraction of the profits of an entrant firm come from stealing customers of established firms, and these profits are higher than the efficiency gains generated by reallocating these consumers.

Thus, the first effect depresses private incentives with respect to social incentives and tend to generate insufficient product variety. In contrast, the second effect works the other way around and tends to generate excessive diversity.

For extreme values of f the spokes model behaves like Salop's. If f is sufficiently low then k is close to one, and the business stealing effect dominates (excessive product variety). Similarly, if f is sufficiently high then k is close to zero, then firms are effectively local monopolists and the partial appropriability effect dominates (insufficient product variety).

For intermediate values of f it is not so easy to track the relative strength of these two effects and anything can happen (at least when v is not too low).

3 Monopoly

Although we are principally concerned with competition among multi-product firms, there are some important insights to be established for the monopoly case. First, a protected monopolist may choose an excessive level of product variety (with respect to the first best). Second, a multi-product firm is at competitive disadvantage vis-a-vis single-product firms. Thus, proliferation is not an effective barrier to entry in the absence of economies of scope. In fact, a multi-product firm can only survive if economies of scope are sufficiently strong.

3.1 Optimal product variety

Consider first the optimal pricing policy for a given k. Given the symmetry of the model it makes sense to focus on symmetric pricing policies: $p_i = p$, for all i. Also, we can restrict attention to prices in the interval $\left[v-1, v-\frac{1}{2}\right]$. First, a monopolist does not find it optimal to set a price below $v-1$, since it cannot attract any additional consumer by lowering the price below this level. Second, given that $v > 1$, it does not find it optimal to set a price above $v - \frac{1}{2}$ and leave consumers located close to the center of the spokes

network unattended. The optimal price schedule (See Appendix for details) is the value of p that maximizes:

$$\pi(p) = \left[k\frac{1}{2} + (1-k)(v-p)\right]pk - fk \tag{1}$$

subject to $p \in \left[v-1, v-\frac{1}{2}\right]$. The solution is given by:

$$p^M(k) = \begin{cases} v - \frac{1}{2}, \text{ if } k \geq \frac{2v-2}{2v-1} \\ \frac{v}{2} + \frac{k}{4(1-k)}, \text{ if } \frac{2v-4}{2v-3} \leq k \leq \frac{2v-2}{2v-1} \\ v-1, \text{ if } k \leq \frac{2v-4}{2v-3} \end{cases} \tag{2}$$

If k is low then it is optimal to set a price sufficiently low, $v-1$, in order to attract all consumers that only have access to their second preferred variety (which are a relatively large fraction of the pool of potential consumers). However, if k is large, most consumers have access to their most preferred variety, and the monopolist finds it optimal to set a higher price, $v-\frac{1}{2}$, and leave consumers with only access to their second preferred variety out of the market. For intermediate values of k the optimal price is somewhere between these two values.

If we plug (2) into (1) then we are ready to compute the monopolist's optimal product line. It turns out (See Appendix for details) that if $v > \frac{7+\sqrt{5}}{4} \approx 2.3$ then it is never optimal to choose a value of k that induces $p^M \in \left(\frac{2v-4}{2v-3}, \frac{2v-2}{2v-1}\right)$.[10] As a result the optimal product variety is given by:

$$k^M = \begin{cases} 0, \text{ if } f \geq v-1 \\ \frac{v-1-f}{v-1}, \text{ if } v-1 \geq f \geq \sqrt{\frac{v-1}{2}} \\ 1, \text{ if } f \leq \sqrt{\frac{v-1}{2}} \end{cases}$$

The next proposition compares the monopoly solution with the first best and the case of single-product firms. (See Figures 1 and 2).

Proposition 1 If $v > \frac{7+\sqrt{5}}{4}$, (i) under monopoly the level of product variety is excessive ($k^M > k^*$) if $f \in \left(\frac{1}{4}, \sqrt{\frac{v-1}{2}}\right)$ and insufficient ($k^M < k^*$) if

[10] If $v \in \left[2, \frac{7+\sqrt{5}}{4}\right]$ then for some values of f the monopolist chooses a price in the interval $\left(\frac{2v-4}{2v-3}, \frac{2v-2}{2v-1}\right)$. The characterization of the monopoly solution in this interval is more convoluted.

$f \in \left(\sqrt{\frac{v-1}{2}}, v-1\right)$, (ii) there exists a threshold $\widehat{f} \in \left[\sqrt{\frac{v-1}{2}}, \frac{(v-1)^2}{v}\right)$ such that under monopoly the level of product variety is higher than under monopolistic competition ($k^M > k^{MC}$) if $f \in \left(\frac{1}{2}, \widehat{f}\right)$ and lower $\left(k^M < k^{MC}\right)$ if $f \in \left(\widehat{f}, v-1\right)$.

The result that a monopolist may provide too much product variety with respect to the first best is somewhat surprising. If f is relatively high then k is low and the social benefits generated by an additional variety come from the market expansion effect. A monopolist can only capture a fraction of the social benefits but has to pay the entire fixed cost. As a result the level of product variety under monopoly is inefficiently low. However, if f is relatively low then k is high, and most of the social benefits generated by an additional variety come from better preference matching (in the first best most consumers have access to at least one of their desired varieties.) However, in this parameter range a monopolist sets a price equal to $v - \frac{1}{2}$ and excludes all consumers without access to their most preferred variety. As a result, when a monopolist introduces an additional variety then it attracts all consumers for which such a variety is their favorite, and from each one it obtains a high surplus $\left(v - \frac{1}{2}\right)$. In other words, the additional variety generates a substantial market expansion effect and no cannibalization.

Also, if f is low single-product firms are subject to intense competition, which results in relatively low prices. Incentives to enter come almost exclusive from the customers stolen from rival firms, although entering firms can only charge them a relatively low price. In contrast, when a monopolist introduces an additional variety then it attracts a similar amount of consumers but it charges them a higher price. As a result the level of product diversity is higher under monopoly than under single-product firms (monopolistic competition).

3.2 Product proliferation as an entry barrier

The main goal of this section is to determine whether or not, in the current set up, size (as measured by the number of varieties) is a source of competitive advantage and whether product proliferation is an effective entry deterrence mechanism. In order to address these issues we consider an incumbent monopolist who anticipates potential competition from a large number of small firms, each one of them able to supply a single variety (a competitive fringe).

In order to focus on the potential strategic effect of a firm's product range, we assume there are no economies of scope.

More specifically, consider the following three-stage game. In the first stage firm L chooses the fraction of varieties, k_L, and pays the fixed costs (per unit mass of consumers) associated to activating these varieties: $2fk_L$. In the second stage, small firms decide whether or not to enter, and hence the mass of small active firms, k_C, is determined. Fixed costs per variety are the same for small and large firms, so that each small firm that chooses to enter pays f. In the third stage, firms simultaneously set the prices for those varieties that have been activated. We rule out the case that the large firm finds it optimal to set $k_L = 1$, which makes further entry physically impossible. That is, we assume that $f > \frac{2v-1}{4}$. We first state the main result of this section.

Proposition 2 There is no equilibrium where the large firm makes strictly positive profits. More specifically, if $f \in \left[\frac{(v-1)^2}{v}, v-1\right]$ then $k_C + k_L = k^{MC}$ and the values of k_C and k_L are undetermined (k_L must be below a certain threshold). If $f \in \left(\frac{2v-1}{4}, \frac{(v-1)^2}{v}\right)$ then $k_L = 0$ and $k_C = k^{MC}$.

The first part of the proposition is immediate. We cannot have an equilibrium where the large firm makes strictly positive profits and small firms make zero profits. The reason is that a small firm can always imitate the large firm and set $p_i = p_L$ and make the same level of profits per variety of the large firm. Thus, if the large firm produces a positive fraction of varieties it must be the case that both the large and small firms set the same price. In the Appendix it is shown that an equilibrium with equal prices exists for those parameter values that under monopolistic competition firms set $p^{MC} = v - 1$. Otherwise, the large firm strictly prefers to drop out of the market. Thus, under non-localized competition the size of a firm, as measured by its product range, tends to be a source of competitive disadvantage. Therefore, multi-product firms can emerge only if economies of scope are sufficiently strong.

In contrast, other standard location models (see, in particular, Schmalensee, 1978, and Bonanno, 1987) predict that the incumbent monopolist may prevent entry either by crowding out the product space or by choosing the right location pattern. In these models competition is localized and the firm producing the new brand competes only with one or two of the existing brands.

Thus, the entrant correctly anticipates that the incumbent firm will react to entry by cutting the price of the competing brands. In contrast, in our set up those neighboring effects are absent and a new brand does not affect the prices of existing brands.

4 Oligopoly

In the previous section we have shown that in the absence of economies of scope multi-product firms are at a competitive disadvantage vis-a-vis single-product firms. In this section we study the market structure that arises endogenously in the presence of a specific form of economies of scope. In equilibrium only a small number of firms are active, and each one produces a significant fraction of the total number of varieties. The aim is to investigate how the strategic incentives of large multi-product firms affect prices and, specially, product diversity.[11]

Suppose that there is a large number of potential firms, all of them ex-ante identical with respect to their production capabilities. Consider the following three-stage game. In the first stage, each firm decides whether or not to enter. If a firm enters then incurs an entry cost, g. The number of active firms, n, becomes public information. In the second stage, active firms simultaneously choose the fraction of potential varieties they wish to supply. In particular, firm i chooses $k_i \in [0,1]$ and incurs an additional costs $k_i f$. Thus, the cost per variety, $f + \frac{g}{k_i}$, decreases with k_i. If there are n active firms then the total fraction of activated varieties is denoted by $k = \sum_{i=1}^{n} k_i$. It will be convenient to denote by k_{-i} the range of varieties supplied by firm i's competitors; i.e., $k_{-i} = \sum_{j \neq i} k_j$. The vector of active varieties, $\{k_i\}_{i=1}^{n}$ becomes public information. In the third stage, firms simultaneously set prices for all active varieties. We focus on symmetric, subgame perfect equilibria, where $k_i = \frac{1}{n}k$ for any $i = 1, ..., n$, and all varieties are sold at the same price. We consider any value of g such that in equilibrium we have $n \geq 2$.

Given firms' decisions about their product range, a fraction k_i^2 of consumers will have access to two varieties supplied by firm i, a fraction $2k_i(1-k)$

[11] Since we follow a similar approach, our results will be comparable to a large strand of the literature, which includes Ottaviano and Thisse (1999), Ju (2003), and Anderson and de Palma (2006). The differences in results can only be explained by the alternative specifications of consumer preferences.

will have access only to one of the varieties supplied by firm i, and a fraction $2k_ik_{-i}$ will have access to one variety supplied by firm i and one variety supplied by firm i's competitors. Hence, firm i enjoys absolute monopoly power with the first two groups of consumers and faces competition for the third group.

For further reference we define

$$\overline{v}(n) \equiv \frac{2n-1}{n-1}$$

Note that $\overline{v}(2) = 3, \overline{v}(n)$ decreases with n and converges to 2 as n goes to infinity. In the first part of this section we focus on the case $v \geq \overline{v}(n)$. In this case no firm has incentives to set a price above $v-1$ (See Appendix). At the end of the section we discuss the case $2 < v < \overline{v}(n)$, where some of the effects are reversed and therefore requires separate consideration.

4.1 The case of high reservation prices

Suppose $v \geq \overline{v}(n)$. In this case, we can write firm i's optimization problem in the third stage as follows: given (k_i, k_{-i}), and the average prices chosen by rival firms, $\widehat{p}_{-i} = \frac{1}{n-1}\sum_{j\neq i} p_j$, firm i chooses the price of its varieties, p_i, in order to maximize:

$$\pi_i = \left[k_i^2 + 2k_i(1-k_i-k_{-i}) + 2k_ik_{-i}\left(\frac{1}{2} + \frac{\widehat{p}_{-i}-p_i}{2}\right)\right]p_i - 2k_if - g$$

subject to $p_i + 1 \leq v$.[12] If the constraint is not binding then firm i's reaction function is:

$$p_i = \frac{2-k}{2k_{-i}} + \frac{\widehat{p}_{-i}}{2}$$

As usual reaction functions are upward sloping (prices are strategic complements). More interesting is the effect of the fraction of varieties supplied by each firm on the optimal price. First, p_i decreases with k_i. The reason is that the fraction of total consumers that have to choose between two varieties supplied by different firms, $\frac{2k_ik_{-i}}{k_i^2+2k_i(1-k)+2k_ik_{-i}}$, increases with k_i. Second, p_i decreases with the range of varieties supplied by firm i's rivals, k_{-i}. A

[12] It will be apparent that we need not worry about deviations such that $p_i \notin [\widehat{p}_{-i}-1, \widehat{p}_{-i}+1]$.

higher k_{-i} reduces the fraction of consumers that can only buy from firm i and raises the fraction of consumers that have two options.

The reaction functions of firm i's competitors are symmetric. Thus, for a given vector $(k_1, ..., k_n)$, in equilibrium firm i sets the following price:

$$p_i = \frac{2-k}{2n-1}\left(\frac{n}{k_{-i}} + \sum_{j \neq i} \frac{1}{k_{-j}}\right) \tag{3}$$

It is important to emphasize that p_i decreases with both k_i and k_{-i}. That is, when firms choose their product range they will take into account the strategic price effect: an increase in a firm's product range reduces its rivals' prices. This effect will explain some of the results below and may also cause large inefficiencies.

Along a symmetric equilibrium path, $k_i = \frac{1}{n}k$ for any $i = 1, ..., n$. Thus, the candidate to equilibrium price can be obtained rewriting equation (3):

$$p = \frac{n}{n-1}\frac{2-k}{k} \tag{4}$$

provided $p \leq v - 1$, which is equivalent to:

$$k \geq \overline{k} \equiv \frac{2n}{(n-1)v+1} < 1 \tag{5}$$

Note that, for any value of k, the equilibrium price decreases with n and converges to the equilibrium price under single-product firms as n goes to infinity. In fact, a multi-product firm can be interpreted as a coalition of single-product firms. Cooperation allows the coalition to raise their profits by raising the price above the level prevailing in the equilibrium with single-product firms. Take the case $n = 2$ and $k = 1$. In this case, one half of consumers are able to choose between two varieties supplied by different firms, but the other half are trapped and can only choose between two varieties supplied by the same firm. This is why a large multi-product firm can exploit its monopoly power and charge higher prices than single-product firms.

Let us begin the search for equilibrium candidates in the region of parameter values where condition (5) does not hold; i.e., a region of parameters that yield in equilibrium $k \leq \overline{k}$. In this case, firms are expected to set prices equal to $v - 1$ and make profits:

$$\pi_i = k_i\left(2 - k_i - k_{-i}\right)(v-1) - 2k_i f - g$$

This is precisely firm i's objective function in the second stage, which is concave in k_i. From the first order condition with respect to k_i, evaluated at $k_i = \frac{1}{n}k$, we obtain the level of product variety in the candidate equilibrium:

$$k = \frac{2n}{n+1}\frac{v-1-f}{v-1} \tag{6}$$

provided $k \leq \overline{k}$, which is equivalent to:

$$f \geq \overline{f} \equiv \frac{(n-1)v-n}{(n-1)v+1}(v-1)$$

Thus, if $f \geq \overline{f}$ the equilibrium fraction of active varieties in oligopoly, $k^O(n)$, is given by equation (6).[13]

Let us turn our attention to the case $f < \overline{f}$. Potentially, condition (5) may fail: $k > \overline{k}$. In this case, prescribed prices are given by equation (4).[14] By plugging equilibrium prices in the profit function we obtain firm i's payoff, in the range that satisfies $k \geq \overline{k}$:

$$\pi_i(k_1, ..., k_n) = k_i k_{-i}\left(\frac{2-k}{2n-1}\right)^2\left(\frac{n}{k_{-i}} + \sum_{j\neq i}\frac{1}{k_{-j}}\right)^2 - 2k_i f - g$$

The first derivative evaluated at $k_i = \frac{1}{n}k$ is:

$$\frac{d\pi_i}{dk_i} = \alpha(k,n)\frac{(2-k)^2}{k} - 2f \tag{7}$$

where

$$\alpha(k,n) \equiv \frac{n(2n-3)(2-k)-(2n-1)2k}{(2n-1)(n-1)(2-k)}$$

If $k > \frac{2n(2n-3)}{2n^2+n-2}$ then $\alpha(k,n) < 0$ and $\frac{d\pi_i}{dk_i} < 0$ for all f. Also, α decreases with k, is lower than 1, and it tends to 1 as n goes to infinity.

[13] Clearly, no firm finds it profitable to select such a large product range that induces its rivals to set prices below $v-1$.

[14] We need to check that no firm has incentives to deviate from $p = \frac{n}{n-1}\frac{2-k}{k}$ and set $p = v-1$. A sufficient condition is $v \leq 1 + \frac{(2-k)^2}{2k}\min\left\{\frac{1}{1-k}, \frac{9}{6-5k}\right\}$. This is analogous to the condition required in the case of single-product firms (See footnote 8 and also Chen and Riordan, 2007).

Let us consider the value of v, denoted by $\underline{v}(n)$, such that $\overline{k} = \frac{2n(2n-3)}{2n^2+n-2}$. That is:

$$\underline{v}(n) \equiv \frac{2n^2 - n + 1}{(2n-3)(n-1)}$$

If $v \leq \underline{v}(n)$, given that $\overline{k}$ decreases with v, then $\frac{2n(2n-3)}{2n^2+n-2} < \overline{k}$ and therefore for all $k \geq \overline{k}$, $\frac{d\pi_i}{dk_i} < 0$ and all firms find it optimal to set $\frac{\overline{k}}{n}$ for any value of f.

Also, let us consider the value of f, denoted by $\underline{f}$, such that $\frac{d\pi_i}{dk_i}\left(\overline{k}\right) = 0$. That is:

$$\underline{f} \equiv \alpha(\overline{k})\frac{(2-\overline{k})^2}{2\overline{k}}$$

If $v > \underline{v}(n)$ then $\frac{2n(2n-3)}{2n^2+n-2} > \overline{k}$ and $\alpha(k,n) > 0$, which opens the possibility to the existence of an equilibrium where $k > \overline{k}$. It turns out that in the case $n = 2$ second order conditions are not satisfied and hence if $f < \underline{f}$ then no symmetric equilibrium exists. However, for $n > 2$ second order conditions are always satisfied and if $f < \underline{f}$ then k^O is implicitly given by equalizing equation (7) to zero (provided the solution satisfies $k \leq 1$).

Note that $\underline{v}(2) = 7, \underline{v}(n)$ decreases with n, and $\underline{v}(n) < \overline{v}(n)$ if and only if $n > 3$.

All this discussion is summarized in the following lemma. In the Appendix we provide some technical details as well as the value of the threshold, f_a.

Lemma 1 *Whenever a symmetric equilibrium exists: (i) k^O is given by equation (6) if $f \in \left[\overline{f}, v-1\right]$; (ii) $k^O = \overline{k}$ if $f \in \left[\underline{f}, \overline{f}\right]$; (iii) k^O follows from equalizing equation (7) to zero if $f \in \left[\max\{0, f_a\}, \underline{f}\right]$;* and *(iv)* $k^O = 1$, if $f \in [0, f_a]$.

If we compare k^O with k^{MC} the following pattern emerges (See Figure 3 and Appendix for details).

Proposition 3 Under oligopoly, if $v \geq \overline{v}$, the fraction of varieties supplied in a symmetric equilibrium is lower than under monopolistic competition $\left(k^O < k^{MC}\right)$ for either relatively low or relatively high values of f, and higher $\left(k^O > k^{MC}\right)$ for intermediate values of f.

The differential incentives to introduce product variety by oligopolistic and monopolistically competitive firms depend on the relative weight of three effects:

a) *Cannibalization:* when a multi-product firm introduces a new variety it anticipates that some of the buyers are recruited from the customers of the firm.

b) *Appropriability/competition:* for any number of active varieties equilibrium prices tend to be higher under oligopoly than under monopolistic competition.

c) *Strategic price effect:* a multi-product firm anticipates that a wider product range induces a more aggressive pricing behavior from its rivals.

The first and the third effects work in favor of lower product diversity under oligopoly. The second effect works in the opposite direction as firms can appropriate a larger fraction of the surplus created.

For relatively high values of f, so that k^{MC} is very low, both the strategic price effect and the competition effect are non-operative since prices are equal to the monopoly level under both market structures. In this case the cannibalization effect dominates and product variety is lower under oligopoly. In contrast, if f is relatively low so that k^{MC} is close to 1, the strategic price effect dominates, since oligopolistic firms are not willing to expand their product range at the cost of triggering a price war. However, for intermediate values of f, the competition effect dominates and oligopolistic firms introduce more product variety than monopolistic firms because they can charge higher prices and appropriate a large fraction of total surplus.

Let us now compare k^O and k^* (See Figure 4 and Appendix for details). If $n \leq 5$ then we have that k^O and k^* cross each other twice at different values of f, and then $k^O < k^*$ for either relatively low or relatively high values of f, and $k^O > k^*$ for intermediate values of f. If $n > 5$ then there are two possibilities. If v sufficiently high then k^O and k^* cross each other three times, and $k^O < k^*$ for high values of f, and $k^O > k^*$ for low values of f. Instead, if v is relatively low then k^O and k^* cross each other only once and there is excessive variety for low values of f and insufficient variety for high values of f. In sum, we obtain the following result (See Appendix for details):

Proposition 4 Under oligopoly, if $v \geq \overline{v}(n)$, the fraction of varieties supplied in a symmetric equilibrium is insufficient $\left(k^O < k^*\right)$ for relatively high values of f, but the sign of the inefficiency is ambiguous for low and intermediate values. In particular, for relatively low values of f there are two cases: (i) if $n < 6$ then the fraction of varieties supplied in a symmetric equilibrium is insufficient, but (ii) if $n \geq 6$ then it is

excessive $\left(k^O < k^*\right)$.

If f is relatively high then the behavior of multi-product firms is similar to that of single-product firms and they provide an inefficiently low level of product variety because they cannot price discriminate and hence are only able to appropriate a fraction of the market expansion effect. On the top of this effect there is the cannibalization effect which further reduces product variety. However, if f is relatively low then the sign of the inefficiency depends very much on market structure. If n is low the strategic price effect dominates and firms refrain from expanding their product range in order to relax price competition, and as a result product variety is insufficient. If n is relatively high then the business stealing effect dominates (like in the case of single-product firms) and multiproduct-firms produce an excessive level of product variety. For intermediate values of f anything can happen.

It is important to note that the strategic price effect may cause large inefficiencies. For example, consider the case $n = 2$, $v = \underline{v}(2) = 7$, and $f = 0$. In this case, social efficiency requires all potential varieties being produced. In contrast, in the market solution only one half of all potential varieties is provided. Hence, the strategic price effect may cause a substantial underprovision of product diversity.

Finally, let us consider the endogenous determination of the number of firms (the first stage of the game). In equilibrium it must be the case that all active firms make non-negative profits and that if any additional firm decides to enter then it would make negative profits. Let g_2 be the value of g for which in equilibrium with $n = 2$ firms make zero profits. In the Appendix, we prove the following result:

Lemma 2 There exists a strictly decreasing sequence $\{g_n\}_{n=2}^{\infty}$ that converges to 0 as n goes to infinity, such that $n \geq 2$ is the maximum number of firms that make non-negative profits in equilibrium if and only if $g \in (g_{n+1}, g_n]$. In other words, for any value of $g \in [0, g_2]$ the equilibrium number of firms is given by a single-valued, step function, $n(g)$, which decreases with g.

Remark 1 The equilibrium number of firms is inefficiently high.

Obviously, a social planner would dictate that a single firm produces the efficient number of varieties in order to avoid the duplication of entry costs (in order to exploit economies of scope).

4.2 The case of low reservation prices

Finally, we turn our attention to a region of the parameter space, $2 < v < \overline{v}(n)$, where some of the effects analyzed above are reversed. In this region firms never set prices below $v-1$,[15] but they may choose to set prices above $v-1$, and not to serve some potential customers in submarkets where consumers can buy either one of the firms' varieties or nothing. However, firms will never choose to set prices above $v-\frac{1}{2}$ (See Appendix). In the main text we focus our discussion on the case $n=2$ in order to make the presentation more transparent, and postpone to the Appendix the discussion of the general case.

In the third stage, firm i chooses p_i in order to maximize:

$$\pi_i = \left[k_i^2 + 2k_i\left(1-k_i-k_j\right)\left(v-p_i\right) + 2k_ik_j\left(\frac{1}{2}+\frac{p_j-p_i}{2}\right)\right]p_i - 2k_if - g$$

subject to $p_i \in \left[v-1, v-\frac{1}{2}\right]$; $i,j=1,2, i\neq j$. If these constraints are not binding then firm i's optimal price is given by:

$$p_i = \frac{k_i+k_j+2v(1-k_i-k_j)+k_jp_j}{2\left(2-2k_j-k_j\right)}$$

Since firm j has a similar reaction function, then we can write firm i's equilibrium price as follows:

$$p_i = \frac{[k+2v(1-k)][2(2-k)-k_j]}{8(2-k)(1-k)+3k_ik_j}, \quad i,j=1,2, i\neq j$$

Note that in this case p_i increases with both k_i and k_j. In other words, in this region of the parameter space the sign of the strategic price effect is reversed. An expansion of a firm's product range induces its rivals to set a higher price. That is, more product variety implies a more relaxed price environment. The intuition is the following. If k_j is very low, then firm i pays a great deal of attention to those submarkets where consumers can either buy one of its varieties or nothing. Since v is relatively low then firm i has incentives to moderate its pricing (set a price equal to $v-1$) and serve all these consumers. As k_j increases then firm i puts less weight on these submarkets and more weight on submarkets where consumers can choose

[15] Suppose they do. Then, from equation (4), $p \geq \frac{n-1}{n} \geq v-1$, provided $v \leq \overline{v}(n)$.

between varieties supplied by different firms. In those submarkets firm i does not have to attract consumers located at the other end of the segment, but only consumers located in the middle. As a result firm i finds it optimal to set a higher price.[16]

Thus, in the second stage, firms have incentives to expand their product range in order to relax price competition in the subsequent pricing stage. As a result, the strategic price effect, together with the competition effect, may dominate the cannibalization effect. Consequently, and in contrast to the case where $v \geq \overline{v}(n)$, for relatively low values of f product variety under oligopoly may be higher than under monopolistic competition (and hence excessive from a social viewpoint).

In order to illustrate this possibility in the Appendix we consider the case $n = 2, v = \frac{5}{2}$. We show that if $f \leq \frac{3}{4}$, then there exists a symmetric equilibrium with $k_i = \frac{1}{2}$, $i = 1, 2$. Hence, if $f \in \left(\frac{1}{2}, \frac{3}{4}\right]$ we have that $k^O = 1 > k^{MC} > k^*$.

5 Concluding remarks

In this paper we have examined the role of multi-product firms in the market provision of product variety. The spokes model provides a very useful set up to compare the product diversity generated by alternative market structures in industries where neighboring effects can be neglected.

We have shown that multi-product firms are in a competitive disadvantage vis-a-vis single-product firms and they will emerge only if economies of scope are sufficiently strong. Thus, in the absence of neighboring effects product proliferation is not an effective entry deterrence mechanism.

Under economies of scope the equilibrium configuration includes a small number of multi-product firms who use their product range as a key strategic variable. It turns out that product variety may be higher or lower than in the case of single-product firms; moreover, large multi-product firms may provide too little or too much variety with respect to the first level. However, for a relevant range of parameter values, oligopolistic firms drastically restrict their product range in order to relax price competition. As a result, product diversity may be substantially lower than the efficient level.

[16] The result that more product variety may imply higher prices is analogous to the result found in the monopoly case (Section 3) and to the "price rising entry" discussed by Chen and Riordan (2007) in the case of a finite number of single-product firms.

These results contribute to a better understanding of the impact of multi-product firms and nicely complement those obtained in the SDS and logit models. Moreover, on a more methodological spirit, the analysis indicates that the spokes set up is sufficiently flexible to accommodate multi-product firms and hence it reinforces the idea that the model proposed by Chen and Riordan (2007) is indeed a significant development within the family of spatial models.

6 References

Allanson, P. and C. Montagna (2005), Multi-product Firms and Market Structure: An Explorative Application to the Product Life Cycle, International Journal of Industrial Organization 23(7-8), 587-598.

Anderson, S. and A. de Palma (2006), Market Performance with Multi-product Firms, Journal of Industrial Economics 54(1), 95-124.

Anderson, S. and A. de Palma (1992a), The Logit as a Model of Product Differentiation, Oxford Economic Papers 44, 51-67.

Anderson, S. and A. de Palma (1992b), Multiproduct Firms: a Nested Logit Approach, Journal of Industrial Economics 40, 261-276.

Bernard, A., S. Redding and P. Schott (2006), Multi-Product Firms and Trade Liberalization, mimeo.

Besanko, D., M. Perry, and S. Lerman (1990), The Logit Model of Monopolistic Competition: Brand Diversity, Journal of Industrial Economics 38, 397-415.

Bonanno, G. (1987), Location Choice, Product Proliferation and Entry Deterrence, Review of Economic Studies 54 (1), 37-45.

Brander, J.A. and J. Eaton (1984), Product Line Rivalry, American Economic Review 74, 323-333.

Chamberlain, E. (1933), The Theory of Monopolistic Competition, Harvard University Press, Cambridge MA.

Chamsaur, P. and J.C. Rochet (1989), Multiproduct Duopolists, Econometrica 57, 533-557.

Chen, Y. and M. Riordan (2007), Price and Variety in the Spokes Model, The Economic Journal, vol 117 (522), 897-921.

Dixit, A. and J. Stiglitz (1977), Monopolistic Competition and Optimum Product Diversity, American Economic Review 67 (3), 297-308.

Eckel, C. and P. Neary (2006), Multi-Product Firms and Flexible Manufacturing in the Global Economy, mimeo.

Feenstra, R. and J. Ma (2007), Optimal Choice of Product Scope for Multiproduct Firms Under Monopolistic Competition, NBER working paper No. 13703.

Hotelling, H. (1929), Stability in Competition, The Economic Journal 39 (153), 41-57.

Ju, J. (2003), Oligopolistic Competition, Technology Innovation, and Multiproduct Firms, Review of International Economics 11, 346-359.

Minniti, A. (2006), Multi-product Firms, R&D, and Growth, The B.E. Journal of Macroeconomics 6, 1-44.

Nocke, V. and S. Yeaple (2006), Globalization and Endogenous Firm Scope, mimeo.

Ottaviano G.P. and J.F. Thisse (1999), Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity, CEPR Discussion Paper 2151.

Ottaviano G.P., T. Tabucchi and J.F. Thisse (2002), Agglomeration and Trade Revisited, International Economic Review 43 (2), 409-435.

Perloff,J. and S. Salop (1985), Equilibrium with Product Differentiation, Review of Economic Studies 52, 107-120.

Salop, S. (1979), Monopolistic Competition with Outside Goods, The Bell Journal of Economics 10(1), 141-156.

Schmalensee, R. (1978), Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry, Bell Journal of Economics 9 (2), 305-327.

Spence, M. (1976), Product Selection, Fixed Costs and Monopolistic Competition, Review of Economic Studies 43 (2), 217-235.

7 Appendix

7.1 Proof of Proposition 1

Given the optimal prices stated in the main text, there are three alternative candidates to the optimal value of k. Option (a) consists of setting $p = v - 1$. In this case profits are equal to $\left(1 - \frac{k}{2}\right) k\,(v-1) - kf$. Hence, the optimal value of k can be computed directly from the first order condition:

$$k = 1 - \frac{f}{v-1}$$

Note that this is an optimal pricing policy only if $k \leq \frac{2v-4}{2v-3}$, which implies that this is a candidate only if $f \geq \frac{v-1}{2v-3}$. Maximum profits that can be obtained by setting $p = v - 1$ are given by:

$$\pi^a(f) = \frac{(v-1-f)^2}{2(v-1)}$$

Option (b) consists of setting a price $p = \frac{v}{2} + \frac{k}{4(1-k)}$, which is optimal provided $k \in \left[\frac{2v-4}{2v-3}, \frac{2v-2}{2v-1}\right]$. In this case profits are:

$$\pi(k) = k(1-k)\left[\frac{v}{2} + \frac{k}{4(1-k)}\right]^2 - kf$$

It can be checked that $\pi'''(k) > 0$. In other words, $\pi'(k)$ is a convex function. Also, $\pi'\left(\frac{2v-4}{2v-3}\right) = \frac{v-1}{2v-3} - f$. Thus, if $f > \frac{v-1}{2v-3}$ then there is no local maximum in the interval $\left(\frac{2v-4}{2v-3}, \frac{2v-2}{2v-1}\right)$. However, if $f < \frac{v-1}{2v-3}$ then there is a unique local maximum, $\widetilde{k}(f)$, although it is not possible to compute analytically the associated level of profits $\pi^b(f)$. However, using the envelope theorem we know that $\frac{d\pi^b}{df} = -\widetilde{k}(f) > -1$.

Finally, option (c) consists of setting $p = v - \frac{1}{2}$. In this case profits are equal to $\frac{k}{2}\left(v - \frac{1}{2}\right) - kf$. Hence, if $f < \frac{2v-1}{4}$ it is optimal to set $k = 1$ and make profits:

$$\pi^c(f) = \frac{2v-1}{4} - f$$

Next, we compare $\pi^a(f)$ and $\pi^c(f)$. These two functions cross each other at a single point $f = \sqrt{\frac{v-1}{2}}$. Option (a) dominates if f is above the threshold and option (c) dominates if f is below. We can rule out option (b) if we place an additional constraint on parameter values. Note that $\pi^b(f) > \pi^a(f)$ only if $f < \frac{v-1}{2v-3}$, and in this case $\frac{d\pi^b}{df} > -1$. If we consider values of v higher or equal than $\frac{7+\sqrt{5}}{4}$ (≈ 2.3), then $\frac{v-1}{2v-3} \leq \sqrt{\frac{v-1}{2}}$. Thus, if $f > \sqrt{\frac{v-1}{2}}$ then option (a) is preferred to option (c), and (b) is not even a candidate. If $f \in \left(\frac{v-1}{2v-3}, \sqrt{\frac{v-1}{2}}\right)$ then option (c) is preferred to option (a), and again option (b) is not a candidate. Finally if $f \leq \frac{v-1}{2v-3}$ then option (c) is preferred to both options (a) and (b). In order to check this, define $\Psi(f) \equiv \pi^c(f) - \pi^b(f)$. First, $\Psi\left(\frac{v-1}{2v-3}\right) \geq 0$. Second, $\Psi'(f) = -1 + \widetilde{k}(f) < 0$.

Therefore, $\pi^c(f) > \pi^b(f)$ for all $f < \frac{v-1}{2v-3}$. Thus, we have computed explicitly k^M in the case $v \geq \frac{7+\sqrt{5}}{4}$.

The comparison between k^M and k^* is straightforward. It is also immediate to check that k^M never crosses the linear portion of k^{MC} $\left(f > \frac{(v-1)^2}{v}\right)$. However, it may cross the non-linear portion. In particular if $v \geq 5$, then there are two solutions to the equation:

$$\frac{v-1-f}{v-1} = 2 + f - \sqrt{4f + f^2}$$

which are given by:

$$f = \frac{v-1}{2v-1}\left[v - 2 \pm \sqrt{(v-2)^2 - (2v-1)}\right].$$

If we denote by f^+ and f^- the highest and the lowest solution respectively, then it turns out that $f^- < \sqrt{\frac{v-1}{2}}$. Also, $f^+ > \sqrt{\frac{v-1}{2}}$ if and only if v is higher than a threshold $\widehat{v}$, $\widehat{v} > 5$. This proves part (ii) of the proposition and also that the threshold value, $\widehat{f}$, is equal to $\sqrt{\frac{v-1}{2}}$ if $v < \widehat{v}$ and to f^- otherwise.

7.2 Proof of Proposition 2

Suppose that, in the third stage of the game, firm L sets $p_L > v - 1$. Then a small firm i sets p_i in order to maximize:

$$\pi_i = \left[(1-k)(v-p_i) + \frac{1 + k_C p_C + k_L p_L - p_i}{2}\right] p_i$$

provided $p_i, p_C \in \left[v-1, v-\frac{1}{2}\right]$. It turns out that $\frac{d\pi_i}{dp_1}(p_i = p_C = p_L) < 0$. Suppose now that $p_L < v-1$. Then a small firm i sets p_i in order to maximize:

$$\pi_i = \left(1 - k + \frac{1 + k_C p_C + k_L p_L - p_i}{2}\right) p_i$$

The first order condition evaluated at $p_i = p_C = p_L$ implies that $p_L = \frac{3-2k}{2-k}$. Also, firm L chooses p_L in order to maximize:

$$\pi_L = k_L\left(1 - k_L + \frac{k_L}{2} + k_C \frac{1 + p_C - p_L}{2}\right) p_L$$

It turns out that $\frac{d\pi_L}{dp_L}$ evaluated at $p_i = p_C = p_L = \frac{3-2k}{2-k}$ is strictly positive. Therefore, the only possible equilibrium with equal prices involves $p_C = p_L = v - 1$. This is the case if $f \in \left[\frac{(v-1)^2}{v}, v-1\right]$. In this interval, under monopolistic competition $k^{MC} \leq \frac{2}{v}$ and $p^{MC} = v - 1$. The presence of a large firm does not change the aggregate outcome. From the monopoly case we know that if $k_L < 1 - \frac{\sqrt{\frac{v-1}{2}}}{v-1}$ then the large firm will still set $p_L = v - 1$. The large firm is indifferent about any k_L that induces $p_L = v - 1$, but it will never set a value k_L large enough to induce $p_L > v - 1$ in the third stage. Finally, if $f \in \left[\frac{2v-1}{4}, \frac{(v-1)^2}{v}\right]$, then we know from the monopolistic competition case that if everyone else sets a price equal to $v-1$, then a small firm wants to deviate and set a price below $v - 1$, and hence a large firm cannot make non-negative profits in this environment.

7.3 Proof of Lemma 1

Consider first the case $\underline{v}(n) \geq v > \overline{v}(n)$. In the region where $f \in [\overline{f}, v-1]$, we have $k^O < \overline{k}(n)$. Note that since $v > \overline{v}(n)$ then $\overline{k}(n) < 1$. We need to check that in this region the only symmetric equilibrium involves $p = v - 1$. Let us consider prices above $v - 1$. Firm i chooses p_i in order to maximize:

$$\pi_i = \left[k_i^2 + 2k_i\left(1 - k_i - k_{-i}\right)\left(v - p_i\right) + 2k_i k_{-i}\left(\frac{1}{2} + \frac{\widehat{p}_{-i} - p_i}{2}\right)\right] p_i$$

subject to $v - \frac{1}{2} \geq p_j \geq v - 1$ for any $j = 1, ..., n$. Suppose these constraints are not binding. Then, firm i's optimal price is given by:

$$p_i = \frac{k + 2\left(1-k\right)v + k_{-i}\widehat{p}_{-i}}{2\left(2 - k_i - k\right)}$$

If $p_j = v - 1$ for all $j \neq i$, then the optimal price, p_i, will be given by:

$$p_i = \frac{v}{2} + \frac{k_i}{2\left(2 - k_i - k\right)} < v - 1$$

and the slope of the reaction function is less than one. Hence, there is no symmetric equilibrium with prices above $v - 1$.

If $v \geq \underline{v}(n)$ with $n = 2, 3$ then there is a candidate for a symmetric equilibrium, which is given by equalizing equation (7) to zero. It turns out that for

$n = 2$ second order conditions are not satisfied; $\frac{\partial \pi_i^2}{\partial k_i^2}\left(k_i = \frac{k_{-i}}{n-1} \geq \frac{\overline{k}}{n}\right) > 0$, and hence there is no symmetric equilibrium if $f < \underline{f}$. However, if $n > 2$ then it can be shown (the proof is available upon request) that $\frac{\partial \pi_i^2}{\partial k_i^2}\left(k_i \leq \frac{k_{-i}}{n-1} \geq \frac{\overline{k}}{n}\right) < 0$, and hence first order conditions are sufficient to characterize the symmetric equilibrium.

Finally, let us define the threshold value, f_a. Evaluating equation (7) at $k = 1$ and equalizing to zero yields the following value of f:

$$f_a = \frac{1}{2}\frac{2n^2 - 7n + 2}{2n^2 - 3n + 1}$$

Note that $f_a < 0$ for $n = 3$, $0 < f_a < \frac{1}{2}$ for $n \geq 4$, and f_a tends to $\frac{1}{2}$ as n goes to infinity. Thus, if $f_a > 0$ then $k^O = k^{MC} = 1$ if $f \in [0, f_a]$.

7.4 Proof of Proposition 3

We show that there exist thresholds f_a, f_b and f_c such that the following holds: (i) when $\underline{v}(n) > v > \overline{v}(n)$ for $n = 2, 3$, $k^O < k^{MC}$ if $f \in [0, f_b) \cup (f_c, v-1)$, and $k^O > k^{MC}$ if $f \in (f_b, f_c)$; and (ii) when $v > \max\{\overline{v}(n), \underline{v}(n)\}$ for $n \geq 3$, $k^O < k^{MC}$ if $f \in [\max\{0, f_a\}, \overline{f}) \cup (f_c, v-1)$, and $k^O > k^{MC}$ if $f \in (f_b, f_c)$.

It turns out that $f_a < \frac{1}{2} < f_b < \overline{f} < f_c < \frac{(v-1)^2}{v}$, and in the limiting case where n goes to infinity, $f_a(n)$ tends to $\frac{1}{2}$, $f_b(n)$ to $\frac{(v-1)^2}{v}$,$\overline{f}(n)$ to $\frac{(v-1)^2}{v}$ as well, and $k^O(n)$ to k^{MC}.

Consider first that $\underline{v}(n) > v > \overline{v}(n)$ for $n = 2, 3$. The reader should remember that in this case k^O is given by equation (6) if $f \geq \overline{f}$, and $k^O = \overline{k}$ if $f \leq \overline{f}$. Also, $k^{MC} = 2\frac{v-1-f}{v-1}$ if $f \in \left[\frac{(v-1)^2}{v}, v-1\right]$, and $k^{MC} = 2 + f - \sqrt{4f + f^2}$, if $f \in \left[\frac{1}{2}, \frac{(v-1)^2}{v}\right]$. First of all, $\overline{f} < \frac{(v-1)^2}{v}$ and $k^{MC}\left(f = \frac{(v-1)^2}{v}\right) < \overline{k}$. Second, if we look for crossing points between k^{MC} and k^O for values of $f < \overline{f}$, then we find only one, f_b, which is given by:

$$f_b = \frac{(n-1)^2(v-1)^2}{n[(n-1)v + 1]}$$

Note that $\frac{1}{2} < f_b < \overline{f}$ for all $v > \overline{v}(n)$. Since $k^{MC}(f)$ is a decreasing function in this interval then we have that $k^{MC} > k^O$ if $f \in [0, f_b)$ and $k^{MC} < k^O$ if $f \in \left(f_b, \overline{f}\right]$. Finally, since $k^{MC}(f)$ is not only decreasing but also a convex

function, then there is another crossing point between k^{MC} and k^O, denoted by f_c, such that $\overline{f} < f_c < \frac{(v-1)^2}{v}$, which is implicitly given by $k_c = k^O(f_c)$ where f_c satisfies $\frac{(2-k_c)^2}{2k_c} = (v-1)\left[1 - \frac{n+1}{2n}k_c\right]$. Hence, if $f \in \left[\overline{f}, f_c\right)$ then $k^{MC} < k^O$ and if $f \in (f_c, v-1)$ then $k^{MC} > k^O$.

Consider now $v > \max\{\overline{v}(n), \underline{v}(n)\}$ for $n \geq 3$. If $f < \underline{f}$ then k^O follows from equalizing equation (7) to zero. The only difference is that if $f_a > 0$ (where f_a has been defined in the proof of Lemma 1) then $k^O = k^{MC} = 1$ if $f \in [0, f_a]$. Let us consider $f \in \left[f_a, \underline{f}\right]$, given that $\alpha(k) < 1$ then abusing notation we can write $f^{MC} = \frac{(2-k)^2}{2k} > \alpha(k)\frac{(2-k)^2}{2k} = f^O$. That is, for all $f \in \left[f_a, \underline{f}\right]$ then $k^{MC} > k^O$. The results are the same as in the previous case if $f > \underline{f}$.

7.5 Proof of Proposition 4

We show that there exist thresholds f_d, f_e, f_f, f_g, f_h, β such that: (i) when $\underline{v}(n) > v > \overline{v}(n)$ for $n = 2, 3$, $k^O < k^*$ if $f \in [0, f_d) \cup \left(f_e, v - \frac{1}{2}\right)$, and $k^O > k^*$ if $f \in (f_d, f_e)$; (ii) when $v > \max\{\overline{v}(n), \underline{v}(n)\}$ and $n = 3, 4, 5$ then $k^O < k^*$ if $f \in (\max\{f_a, 0\}, f') \cup \left(f_e, v - \frac{1}{2}\right)$, and $k^O > k^*$ if $f \in (f', f_e)$, where f' is either f_d or f_h (iii) when $\beta > v > \max\{\overline{v}(n), \underline{v}(n)\}$ for $n > 5$, $k^O < k^*$ if $f \in (f_f, v - \frac{1}{2})$, and $k^O > k^*$ if $f \in (\frac{1}{4}, f_f)$; and (iv) when $v > \beta$ for $n > 5$, $k^O < k^*$ if $f \in (f_g, f_h) \cup (f_f, v - \frac{1}{2})$, and $k^O > k^*$ if $f \in (\frac{1}{4}, f_g) \cup (f_h, f_f)$.

It turns out that: (i) f_d and f_e are such that $f_d < \overline{f} < f_e$; (ii) f_f is such that $\overline{f} < f_f < v - 1$; (iii) f_g and f_h are such that $\frac{1}{2} < f_g < f_h < \overline{f}$; (iv) $\beta(n)$ is real-valued if and only if $n > 5$, increases with n, and approaches $\frac{5}{2} + \sqrt{2}$ as n goes to infinity.

Let us first compare k^O with k^* under $\underline{v}(n) > v > \overline{v}(n)$ for $n = 2, 3$. If $\frac{1}{4} \leq f \leq \overline{f}$, $k^O = \overline{k}$, which equals k^* at the value of f given by

$$f_d = \frac{2(n-1)v - 3n + 1}{2[(n-1)v + 1]}(v-1)$$

where $\frac{1}{4} < f_d < \overline{f}$. For $\overline{f} < f \leq v - \frac{1}{2}$, there is another crossing point at $f = f_e$, which is given by:

$$f_e = \frac{2(n-1)v - 2n + 1}{2(n-1)v - n + 2}(v-1)$$

Next, consider $v > \max\{\overline{v}(n), \underline{v}(n)\}$ for $n \geq 3$. In the proof of Lemma 1 we gave the exact value of f_a, where $k^O(f_a) = 1$. Note that f_a is higher than

$\frac{1}{4}$ if and only if $n > 5$. This implies that if $n = 3, 4, 5$, $k^O(f)$ goes through the point $(f_a, 1)$ and a is decreasing and convex function. This function may cross k^* at either f_d (whenever $\underline{f} < f_d$) or at a lower level of f $\left(\text{whenever } \underline{f} > f_d\right)$, denoted by f_h. In either case, there are also three different regions like in the previous case, with insufficient variety for low and high values of f, and excessive variety for intermediate values.

Finally, if $n > 5$, then either k^O and k^* cross each other twice or none in the interval $k \in \left[\overline{k}, 1\right]$. In particular, we are looking for values of k such that

$$\alpha(k)\frac{(2-k)^2}{2k} = k\left(v - \frac{1}{4}\right) + (1-2k)\left(v - \frac{1}{2}\right)$$

The two roots to this equation are

$$k = \frac{2(2n^2 - 3n + 1)v + 6n^2 - n - 5 \pm \sqrt{w(n,v)}}{4(2n^2 - 3n + 1)v - 2n^2 + 11n - 7}$$

which are real valued if and only if $w(n, v) > 0$, where

$$\begin{aligned} w(n,v) \quad \equiv \quad & (2n-1)\left[4(2n^3 - 5n^2 + 4n - 1)v^2 + \right. \\ & \left. -4(10n^3 - 33n^2 + 28n - 5)v + 34n^3 - 101n^2 + 108n - 25\right] \end{aligned}$$

In turn, $w(n, v) > 0$ and if $v > \beta(n)$, where

$$\beta(n) = \frac{10n^3 - 33n^2 + 28n - 5 + 2\sqrt{2}\sqrt{4n^6 - 36n^5 + 99n^4 - 115n^3 + 57n^2 - 9n}}{4n^3 - 10n^2 + 8n - 2}$$

Note that $\beta(n)$ is real valued for all $n > 5$, increases with n, and tends to $\frac{5}{2} + \sqrt{2}$ as n goes to infinity.

Consequently, the two roots above are real valued whenever $v > \beta(n)$, and then the lower of those roots is greater than $\overline{k}$ and the upper root is less than 1. We denote by f_g and f_h the two values of f at which k^O and k^* cross each other in the interval where $\overline{k} < k < 1$. Thus, if $v < \beta$ then $k^O > k^*$ if $f \in \left(\frac{1}{4}, f_e\right)$, and $k^O < k^*$ if $f \in \left(f_e, v - \frac{1}{2}\right)$. However, if $v > \beta$, then $k^O > k^*$ if $f \in \left(\frac{1}{2}, f_g\right) \cup (f_h, f_e)$ and $k^O < k^*$ if $f \in (f_g, f_h) \cup \left(f_e, v - \frac{1}{2}\right)$.

7.6 Proof of Lemma 2

In equilibrium profits have different expressions depending on the value of f (See Lemma 1). Let us consider the following notation. If $f \in \left[\overline{f}, v - 1\right]$ then

we can write profits as $\pi_1(f,n)-g$. If $f \in \left[\underline{f}, \overline{f}\right]$ then we write $\pi_2(f,n)-g$. Finally, if $f \in \left[0, \underline{f}\right]$ we write $\pi_3(f,n)-g$. Plugging the optimal value of k in the profit function we obtain:

$$\pi_1(f,n) = \frac{v-1}{n}\left(\frac{2n}{n+1}\frac{v-1-f}{v-1}\right)^2$$

$$\pi_2(f,n) = \frac{\overline{k}}{n}\left(2-\overline{k}\right)(v-1) - 2f\frac{\overline{k}}{n}$$

where $\overline{k}$ is given by equation (5).

$$\pi_3(f,n) = \frac{\left(2-k^O\right)^2}{n-1} - \frac{2k^O}{n}f$$

where k^O follows from equalizing equation (7) to zero. Note that $\frac{\partial \pi_i}{\partial n} < 0$ for all $i = 1,2,3$. Hence, if we compute the value of g such that n firms make zero profits, g_n, then provided f belongs to the same interval $g_{n+1} < g_n$. However, as n changes thresholds also change, and hence the equilibrium may be given by a different $\pi_i(f,n)$. In particular, both $\overline{f}$ and $\underline{f}$ increase with n.

Let us denote by $\lambda(k,f,n) \equiv \frac{v-1}{n}k(2-k) - \frac{2f}{n}k$. In fact, $\lambda\left(k^O,f,n\right) = \pi_1(f,n)$, where k^O is given by equation (6). Similarly, $\lambda\left(\overline{k},f,n\right) = \pi_2(f,n)$, where $\overline{k}$ is given by equation (5). It turns out that $\frac{\partial \lambda}{\partial k}\left(k^O,f,n\right) < 0$ and $\frac{\partial^2 \lambda}{\partial k^2} > 0$. That is, If $f \in \left[\overline{f}, v-1\right]$, then $k^O < \overline{k}$, and hence $\pi_1(f,n) \geq \pi_2(f,n)$. Suppose there is a value n' for which $f \in \left[\overline{f}, v-1\right]$, and for $n'+1$ we have that $f \in \left[\underline{f}, \overline{f}\right]$. In this case $\pi_2(f,n'+1) < \pi_2(f,n') < \pi_1(f,n')$, and hence strict monotonicity is preserved.

Similarly, let us denote by $\theta(p,k,f,n) = \frac{p}{n}k(2-k) - \frac{2f}{n}k$. In fact, $\theta\left(v-1,\overline{k},f,n\right) = \pi_2(f,n)$, and $\theta\left(p^O,k^O,f,n\right) = \pi_3(f,n)$, where p^O is given by equation (4) and k^O follows from equalizing equation (7) to zero. Clearly, if $f \in \left[\underline{f}, \overline{f}\right]$ $\pi_2(f,n) \geq \pi_3(f,n)$, since prices are higher and product variety is lower. In other words, in this interval an individual firm does not want to expand its product range above $\frac{\overline{k}}{n}$, when its rivals choose this level. The move to a higher product range by all firms is even more detrimental to profits. Now suppose there is a value n' for which $f \in \left[\underline{f}, \overline{f}\right]$, and for $n'+1$ we have that $f \in \left[0, \underline{f}\right]$. In this case $\pi_3(f,n'+1) < \pi_3(f,n') < \pi_2(f,n')$, and hence strict monotonicity is once again preserved.

Finally, the same argument can be applied in case of a possible switch from $f \in \left[\overline{f}, v-1\right]$ to $f \in \left[0, \underline{f}\right]$.

7.7 Low reservation prices: $v < \overline{v}(n)$

In the third stage, firm i chooses p_i in order to maximize:

$$\pi_i = \left[k_i^2 + 2k_i\left(1 - k_i - k_{-i}\right)\left(v - p_i\right) + 2k_i k_{-i}\left(\frac{1}{2} + \frac{\widehat{p}_{-i} - p_i}{2}\right)\right] p_i - 2k_i f - g$$

subject to $p_i \in \left[v - 1, v - \frac{1}{2}\right]$. As shown in the monopoly case, no firm has ever incentives to set prices above $v - \frac{1}{2}$. Also, if $v \leq \overline{v}\left(n\right)$ then firms never want to set prices below $v - 1$. If these constraints are not binding then firm i's optimal price is given by:

$$p_i = \frac{k_i + k_{-i} + 2v(1 - k_i - k_{-i}) + k_{-i}\widehat{p}_{-i}}{2\left(2 - 2k_i - k_{-i}\right)}$$

We can write firm i's equilibrium price as follows:

$$p_i = \frac{(n-1)[k + 2v(1-k)][1 + k_{-i}\theta(k_1, ..., k_n, n)]}{4(n-1) - (2n-1)k_i - (2n-3)k}$$

where

$$\theta(k_1, ..., k_n, n) \equiv \frac{\displaystyle\sum_{j=1}^{n} \frac{1}{4(n-1) - (2n-1)k_j - (2n-3)(k_j + k_{-j})}}{1 - \displaystyle\sum_{j=1}^{n} \frac{k_{-j}}{4(n-1) - (2n-1)k_j - (2n-3)(k_j + k_{-j})}}$$

so that p_i increases with both k_i and k_{-i}.

Let us focus on the case the strategic price effect is the strongest: $n = 2$ and a particular parameter value: $v = \frac{5}{2}$. Consider the following equilibrium candidate $k_1 = k_2 = \frac{1}{2}$. In this case, $p_i = 2 = v - \frac{1}{2}$. A sufficient condition for $k = 1$ to be part of a symmetric equilibrium is $f \leq \frac{3}{4}$. Let us consider a deviation by firm 1. Note that if $k_1 < 1$ then $p_2 > p_1 \geq \frac{3}{2}$.

$$\frac{d\pi_1}{dp_1}\left(k_2 = \frac{1}{2}\right) = p_1\left[3 - p_1 + \frac{1}{2}\left(p_2 - p_1\right) + 8k_1\left(p_1 - 1 + \frac{1}{16}\frac{\partial p_2}{\partial k_1}\right)\right] - 2f$$

Since $3 - p_1 \geq 1, p_2 > p_1 > \frac{3}{2}, \frac{\partial p_2}{\partial k_1} \geq 0$, then $\frac{d\pi_1}{dp_1} \geq \frac{3}{2} - 2f$. Hence, if $f \leq \frac{3}{4}$ then $k = 1$ is an equilibrium of the duopoly game. In other words, if $f \in \left(\frac{1}{2}, \frac{3}{4}\right]$ then $k^O > k^{MC} > k^*$.

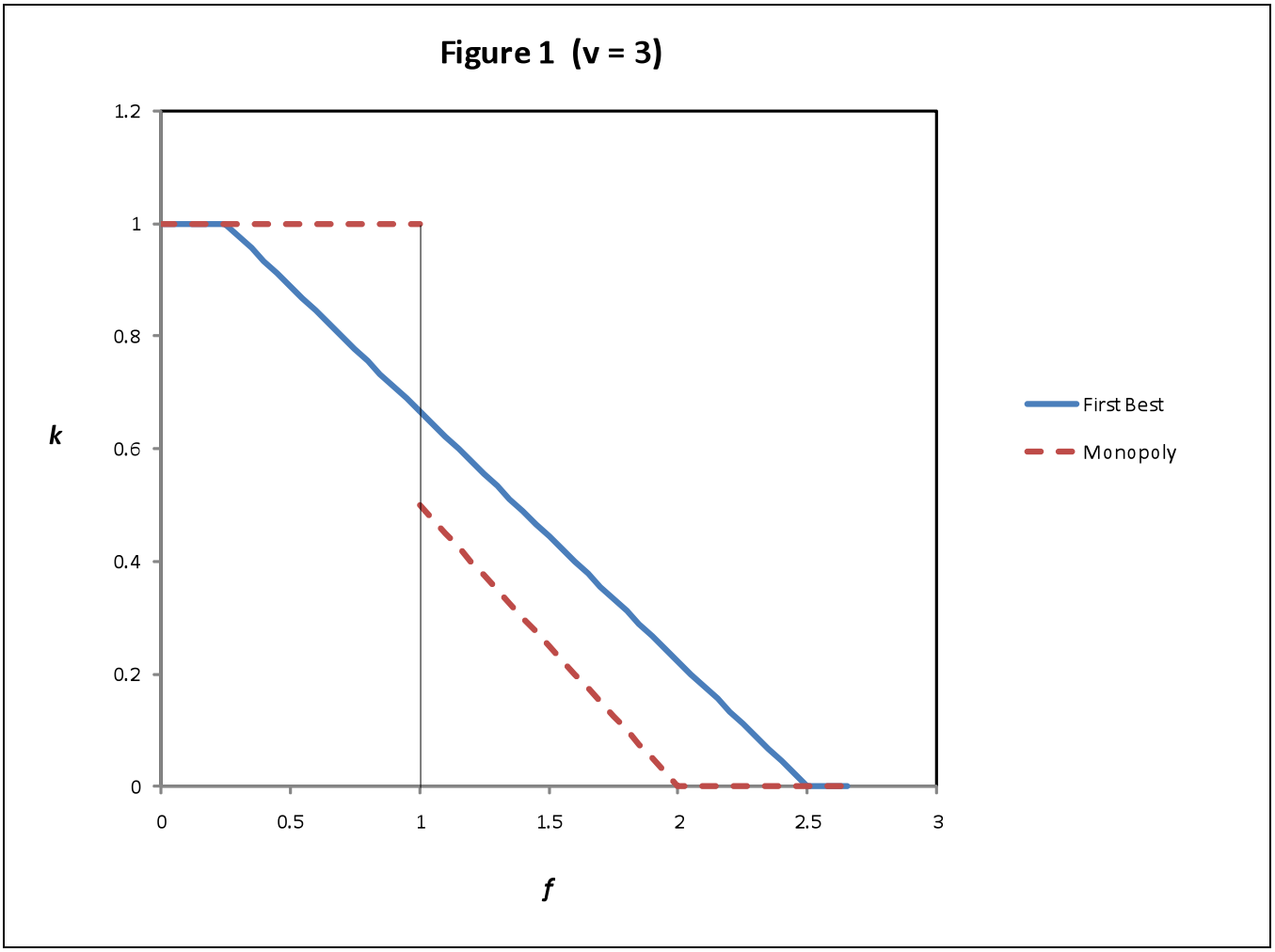
Figure 1 (v = 3)
1.2
1
0.8
0.6
0.4
0.2
0
k
0
0.5
1
1.5
2
2.5
3
f
First Best
Monopoly

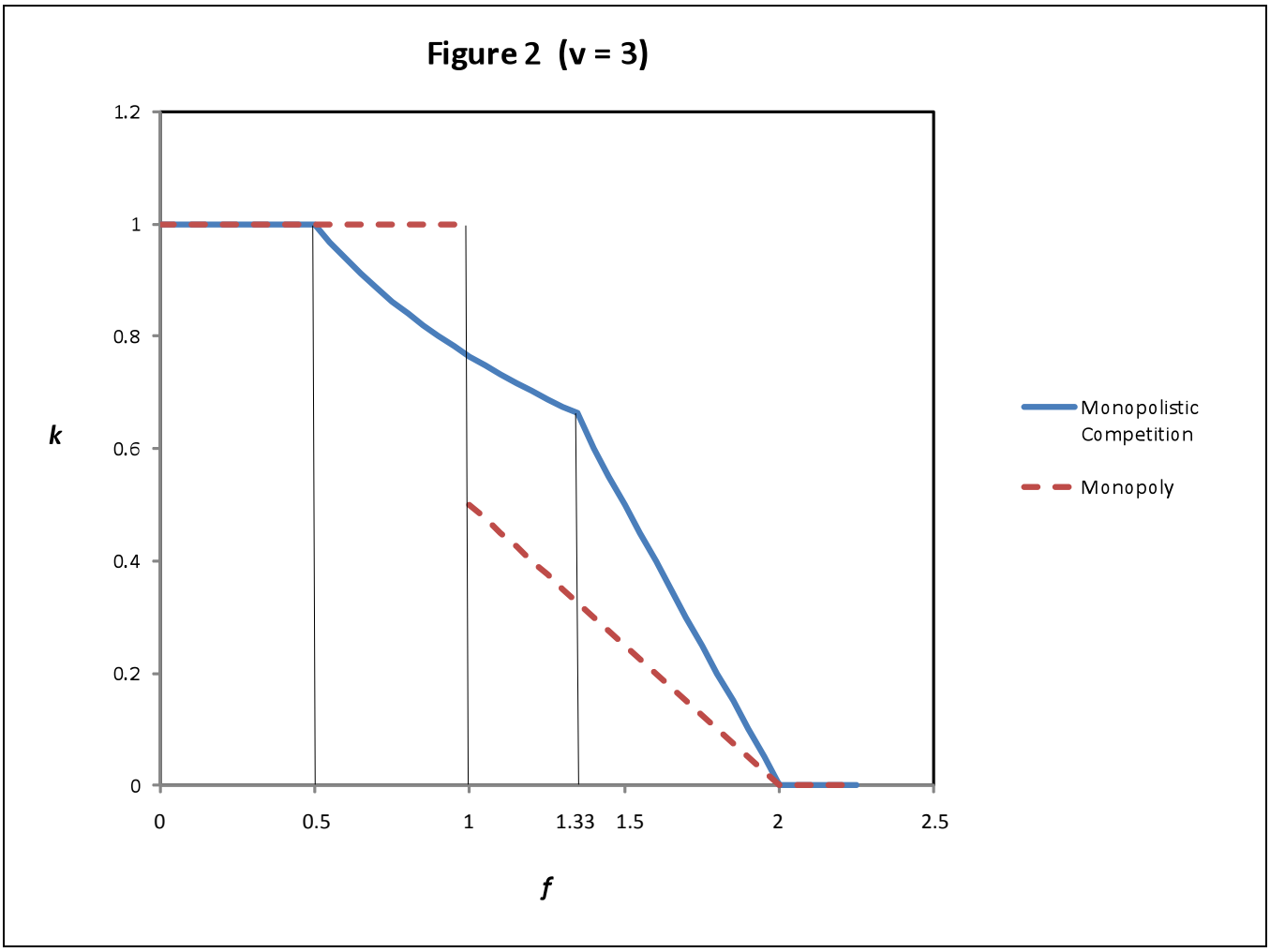
Figure 2 (v = 3)
1.2
1
0.8
0.6
0.4
0.2
0
k
0
0.5
1
1.33
1.5
2
2.5
f
Monopolistic Competition
Monopoly

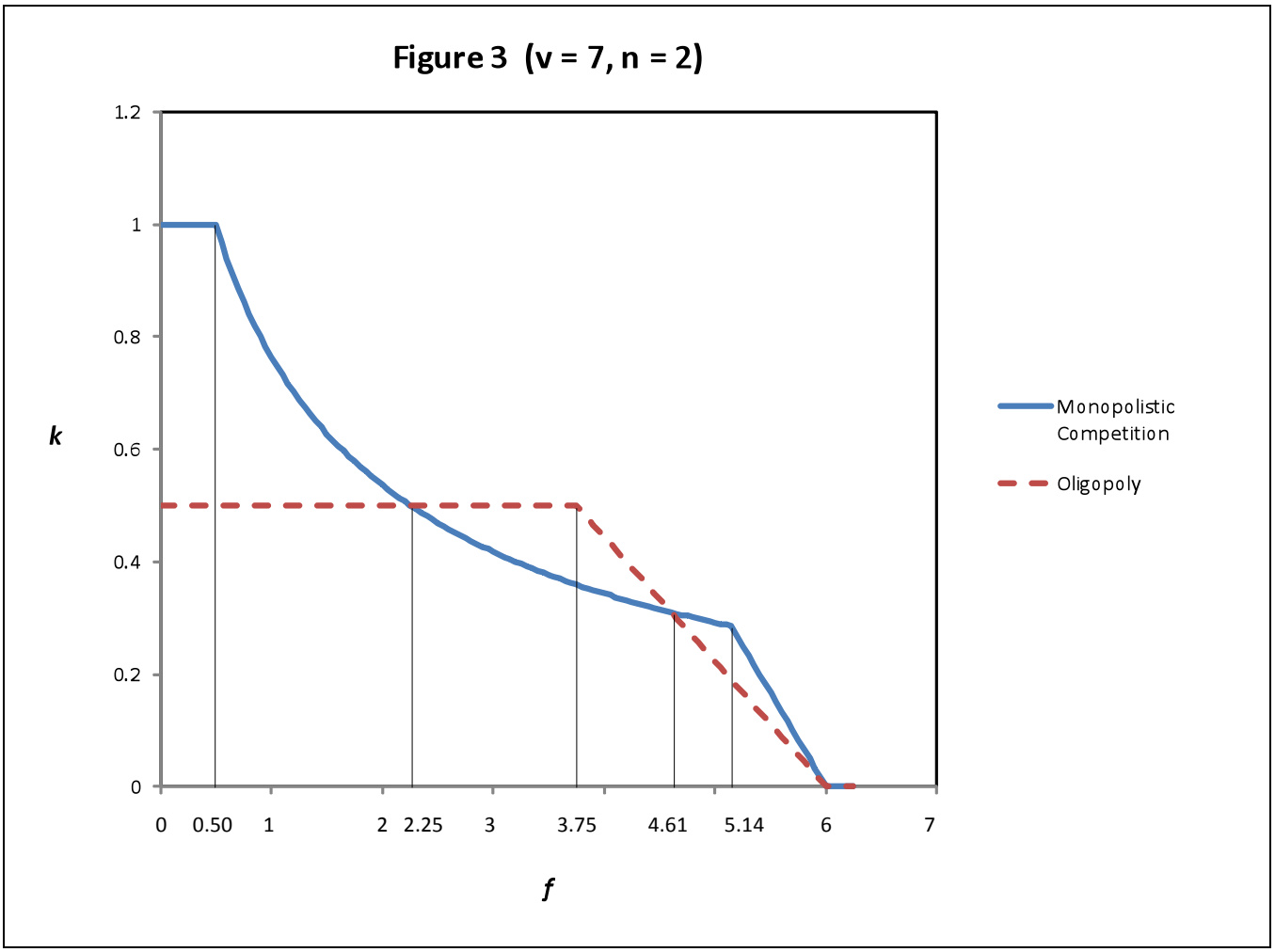
Figure 3 (v = 7, n = 2)
1.2
1
0.8
0.6
0.4
0.2
0
k
0
0.50
1
2
2.25
3
3.75
4.61
5.14
6
7
f
Monopolistic Competition
Oligopoly

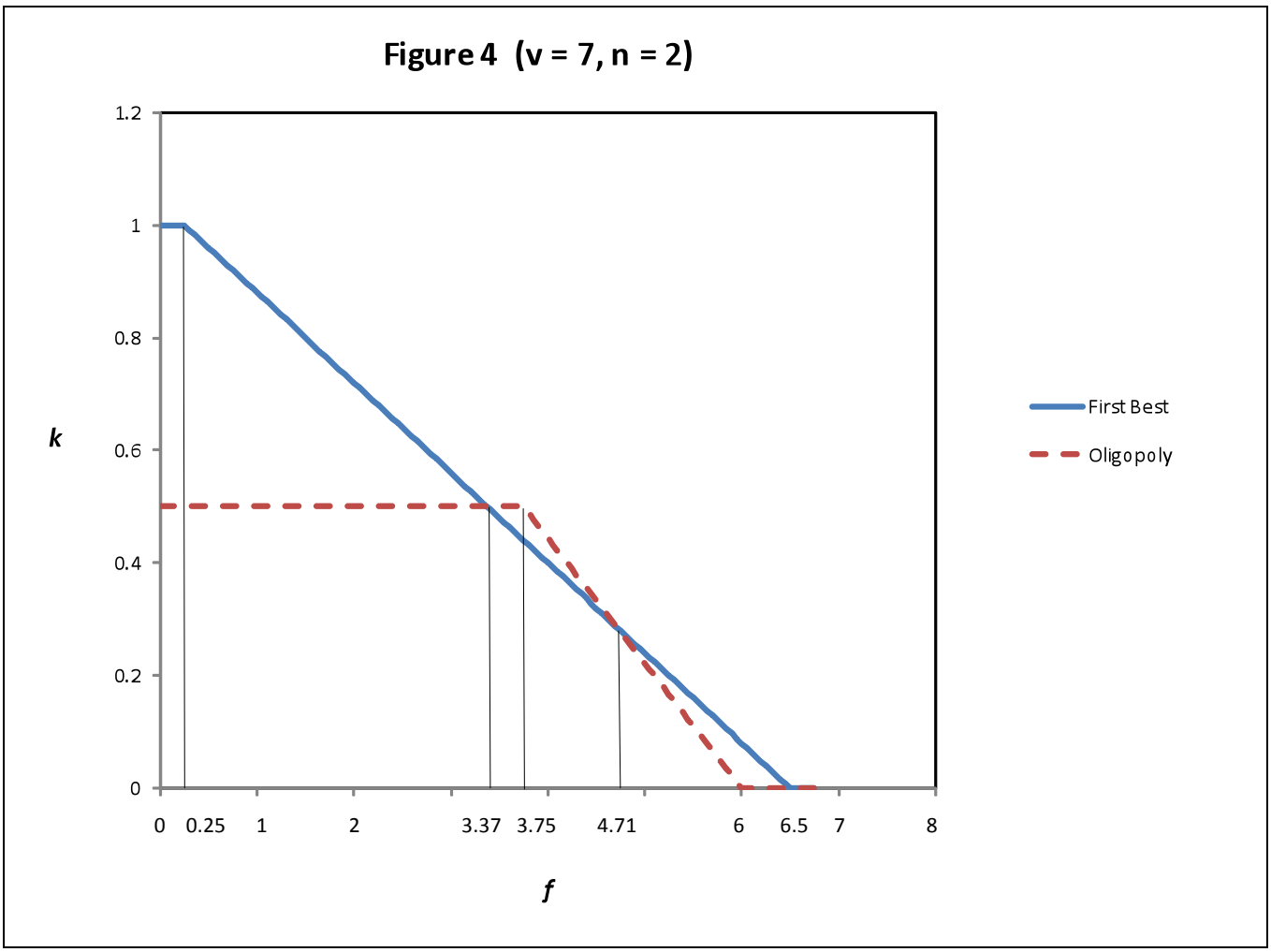
Figure 4 (v = 7, n = 2)
1.2
1
0.8
0.6
0.4
0.2
0
k
0
0.25
1
2
3.37
3.75
4.71
6
6.5
7
8
f
First Best
Oligopoly