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**The Informational Value of Incumbency**

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# The Informational Value of Incumbency\*

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## Abstract

This paper proposes an argument that explains incumbency advantage without recurring to the collective irresponsibility of legislatures. For that purpose, we exploit the informational value of incumbency: incumbency confers voters information about governing politicians not available from challengers. Because there are many reasons for high reelection rates different from incumbency status, we propose a measure of incumbency advantage that improves the use of pure reelection success. We also study the relationship between incumbency advantage and ideological and selection biases. An important implication of our analysis is that the literature linking incumbency and legislature irresponsibility most likely provides an overestimation of the latter.

Key words: Incumbency, information, candidate quality, selection bias, ideology.

JEL Classification Number(s): D72, D78.

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## 1. Introduction

It seems a well established fact that incumbent political parties enjoy some advantage in the reelection. For instance on U.S. House elections, Gelman and King (1990) and Levitt and Wolfram (1997) find that incumbency confers an advantage that ranges from 7 to 10 percent of the two-party vote. Lee (forth.) finds that incumbency has a significant *causal* effect on raising the probability that a *party* will be successful in a re-election bid. The effect of incumbency is on the order of 0.40 in probability.

The potential causes that has been suggested to explain incumbency advantage are based on the hypothesis that incumbents utilize policies and actions to raise their chances of re-election: redistricting (Levitt and Wolfram 1997, Cox and Katz 2002), seniority (McKelvey and Reizman 1992), lack of collective responsibility (Fiorina 1989), informational advantages (Krehbiel and Wright 1983), access to campaign resources (Goodliffe 2001, Jacobson 2001), franking, casework, federal pork, position taking opportunities, etcetera (Cain, Ferejohn and Fiorina 1987, Fiorina 1989, Ansolabehere, Snyder and Stewart 2000). This approach has raised concerns about the representativity of elections and the accountability of representatives. These concerns lie behind (if they have not instigated) many recent political reforms like term limits or tighter controls of campaign financing.

Ansolabehere and Snyder (2002) extend the study to elections to all state executives and find incumbency advantage magnitudes similar (if not higher) than for legislators. They argue then that "the theories of incumbency advantage based on redistricting, legislative irresponsibility, pork barrel politics, and other features of the legislatures do not explain the incumbency advantage", asking for new approaches not based on the irresponsibility of legislators.

In this paper we propose a theoretical model that explains incumbency advantage without recurring to the collective irresponsibility of legislatures. The distinctive feature of our approach is to exploit the informational value of incumbency which offers information about politicians in office not available from challengers. We assume two parties, the incumbent and the challenger, that compete for election in a majority voting system. Each party is represented by a candidate. Parties differ in their ideology and candidates differ in their ability. The size of the effective budget controlled by a candidate in office depends positively on her ability. In order to eliminate the strategic use of policies to affect the probabilities of reelection, we follow Alesina (1988) and assume that parties cannot make credible promises, forcing candidates to implement their ideal policy. Although policies do not directly affect the probabilities of reelection, and hence do not have a strategic use, policies act as a *noisy* signal about the ability of the incumbent, not available for the challenger. Voters use this information to update their beliefs and, in this sense, vote retrospectively *à la Downs* (Fiorina 1981, Downs 1957). With all these ingredients, any advantage from incumbency generated in our framework must come from the noisy information about the quality of the candidate.

Finally, an important point in our analysis is how to measure incumbency advantage. Suppose that voters are so *ideologically* closer to the current incumbent, that they prefer a low ability candidate from that party to a high ability candidate from the challenger party. Then the incumbent party would win every single election without enjoying any real incumbency advantage, since it would also win the election were it the challenger. Thus, we depart from the use of the mere probability of winning and propose to measure incumbency advantage as the difference between the probability of winning of the current incumbent minus its probability of winning were it the challenger. This measure captures the increase in the probability of winning directly derived from holding office.

We obtain the following results. Incumbency offers voters extra information about

the type of the candidate currently in power. But only high type candidates will benefit from this extra information. Low type candidates, on the other hand, would be better off if their type were not revealed. Therefore, incumbency may be advantageous or disadvantageous depending on the distribution of types (proposition 1). Furthermore, the more informative the signal provided by being in power, the larger the effect of incumbency (proposition 2). This implies that incumbency advantage obtains even after eliminating any strategic use of policies. The presence of an ideological bias in favor or against the incumbent mitigates the importance of the candidate's type, reducing the informational value of incumbency. When incumbency is advantageous, it reaches its maximum value in the absence of an ideological bias and, for relevant values, decreases as the bias increases. A similar argument holds when incumbency is disadvantageous (proposition 3). Finally, we show that as candidates accumulate terms in office, they enjoy a higher probability of reelection, offering a selection based argument for the prevalence of positive incumbency advantage (proposition 4). We emphasize that this increase in their probability of winning does not result from voters appreciation of their experience, but from a raise in voters' confidence that the incumbent is in fact a high ability candidate.

The main implication of our results is that if being in power provides information about the personal characteristics of politicians, then we should revise estimations of incumbency advantage which, most likely, overestimate legislature irresponsibility.

The paper is organized as follows. Section 2 presents the model and introduces our measures for the probability of reelection and incumbency advantage. The informational value of incumbency is the focus of section 3. Section 4 explains the role of ideology, and particularly its relevance for distinguishing between the probability of reelection and incumbency advantage. In section 5 we let incumbents have a history of many terms in office.

## 2. The Model

We have a continuum of voters and two parties,  $R$  and  $D$ . Each party is represented by a candidate with ability  $\theta \in \Theta = \{\theta_H, \theta_L\}$ ,  $\theta_L < \theta_H$ . We interpret the ability as the capacity of the candidate to transform the budget into public goods and transfers. Given a public budget  $A$ , a candidate with ability  $\theta$  effectively controls a budget  $\theta A$ . We will refer to  $\theta A$  as the *effective budget*. Citizens know  $A$  but ignore the ability  $\theta$  of the candidate. Hence, we can normalize  $A = 1$  and let  $\theta \in \mathbb{R}_{++}$ . Let  $f(\theta)$  be the common prior on candidates' ability.

The incumbent, whose representative has ability  $\theta$ , decides how to distribute his effective budget  $\theta$  between a public good  $g$ , and a per capita transfer  $t$ , such that  $g + t = \theta$ .

Voters observe imperfectly policies  $t$  and  $g$ . They get a noisy observation,  $B \equiv \theta + \varepsilon$ , where  $\varepsilon$  is a random variable normally distributed with  $E[\varepsilon] = 0$  and  $var[\varepsilon] = \sigma^2$ . Notice that the distribution of  $B$  depends on the ability of the candidate. Let  $H(B|\theta)$  be the conditional distribution of  $B$ , with  $E[B|\theta] = \theta$  and  $var[B|\theta] = \sigma^2$ . Let  $h(B|\theta)$  be the associated density function. Voters use the observation of  $B$  as a signal providing noisy information about the incumbent's type. There are many possible origins for this noise. It may arise because the returns of the public goods may not have fully realized by election time or because the actual level of public goods may depend not only on the quantity of resources devoted but also on external factors affecting the environment. For the analysis, the relevant feature of this noise is that the observed policy does not reveal the type of the incumbent but it acts as an external signal of her ability.

At election time voters decide whether to reelect the incumbent candidate or to choose the challenger. We assume, without loss of generality, that party  $D$  is the incumbent.

## Parties' Preferences

Parties have preferences over policies represented by the quasi-linear utility

$$u_J(t, g) = t + \beta_J v(g), \quad J = D, R, \quad (2.1)$$

with  $\beta_D \neq \beta_R$ . Thus, parties differ in their valuation of the public good. We take the standard assumptions  $\beta_J > 0$ ,  $v'(g) > 0$ ,  $v''(g) \leq 0$ ,  $\lim_{g \rightarrow 0} v'(g) = +\infty$  and  $\lim_{g \rightarrow +\infty} v'(g) = 0$ . The limit properties guarantee the existence of a positive and finite “ideal” level of public good for each party. Let  $g_J^*$  be such that  $v'(g_J^*) = \frac{1}{\beta_J}$ . Thus  $g_J^*$  is the ideal level of public good for party  $J$ . It follows from quasi-linearity that, given an effective budget  $\theta$ , party  $J$ 's optimal allocation between public good and transfers is given by

$$\begin{aligned} g_J^*(\theta) &= \min\{g_J^*, \theta\} \text{ and} \\ t_J^*(\theta) &= (\theta - g_J^*(\theta)). \end{aligned} \quad (2.2)$$

Although it is not crucial for the analysis, it simplifies the exposition to assume that even low types provide transfers.<sup>1</sup> This reduces the difference between types to the amount of the transfer. Parties' preferences are common knowledge, but recall that the ability of a candidate is private information.

A distinctive feature of our description of incumbency is that it wants to abstract from any strategic use of implemented policies that could affect the chances of reelection. For this purpose we follow Alesina (1988) and assume that politicians cannot make credible promises. Also recall that only the size of the effective budget (the realization of  $B$ ), and not the particular policies, carry information about the ability of candidates. It follows then that, once in power, candidates implement their ideal policy. Therefore, party  $K$  obtains the following utility when a party  $J$ 's candidate with ability  $\theta$  is in

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<sup>1</sup>This is equivalent to assume a sufficiently high budget for the provision of the public good.

power:

$$u_K^J(\theta) = u_K(t_J^*(\theta), g_J^*(\theta)). \quad (2.3)$$

### Voters' Preferences

Voters differ in their valuation of the public good. Voter  $i$ 's preferences are represented by

$$u_i(t, g) = t + \beta_i v(g), \quad (2.4)$$

with  $v'(g) > 0$ ,  $v''(g) \leq 0$ ,  $\lim_{g \rightarrow 0} v'(g) = +\infty$  and  $\lim_{g \rightarrow +\infty} v'(g) = 0$ . Let  $\beta_i$  be distributed in  $[\beta^0, \beta^1]$  according to the continuous distribution  $F$ . Denote the median voter by  $m$ , that is  $F(\beta_m) = 1/2$ .

Voters want to maximize their utility from the implemented policies. Given that they only have imperfect information about the ability of the candidates, voters decide to reelect the incumbent if they expect a higher utility from her reelection than the expected utility from the policy that a challenger would implement. In this sense, voters vote (at least in part) retrospectively *à la* Downs, that is, they observe the history of candidates in power to derive information about their types (Fiorina 1981). For the electoral outcome, it will suffice to restrict attention to the median voter  $m$ .

Let  $u_m^J(\theta) = u_m(t_J^*(\theta), g_J^*(\theta))$  be the utility that the median voter obtains from a candidate of party  $J$  with ability  $\theta$ . Let  $\mu(\theta|\hat{B})$  be a probability distribution representing voters' beliefs about the type of the incumbent when the observed external signal is  $\hat{B}$ . Formally,

$$\mu(\theta|\hat{B}) = \frac{h(\hat{B}|\theta) f(\theta)}{\sum_{\theta' \in \Theta} h(\hat{B}|\theta') f(\theta')}. \quad (2.5)$$

Then, the median voter's expected utility from reelecting the incumbent is

$$E_\mu(u_m^I|\hat{B}) = \sum_{\theta' \in \Theta} u_m^I(\theta') \mu(\theta'|\hat{B}). \quad (2.6)$$



Assuming that all challengers are newcomers randomly chosen from the initial distribution of types, the expected utility from the challenger, given that voters use the common prior on the distribution of types, is

$$E_f(u_m^C) = \sum_{\theta' \in \Theta} u_m^C(\theta') f(\theta'). \quad (2.7)$$

### Probability of Reelection and Incumbency Advantage

Voters will elect the candidate that offers the highest expected utility. Therefore, the incumbent gets reelected if  $E_\mu(u_m^I | \hat{B}) \geq E_f(u_m^C)$ . Observe that candidates differing in their ability will face different probabilities of reelection, since the distribution of  $B$  is type-dependent and an incumbent may need a *favorable* realization of  $B$  to be reelected. In particular, the probability of reelection for a type  $\theta$  candidate is

$$\rho^I(\theta) = \Pr[E_\mu(u_m^I | B) - E_f(u_m^C) \geq 0 | \theta]. \quad (2.8)$$

We can exploit the properties of the noise and rewrite the probability of winning in a simpler way. This is done in the following lemma whose proof (as all other results) is provided in the appendix.

**Lemma 1.** *There exists a value  $B^I$  such that*

$$\rho^I(\theta) = \begin{cases} 0 & \text{if } E_f(u_m^C) \geq u_m^I(\theta_H) \\ \Pr(B \geq B^I | \theta) & \text{if } u_m^I(\theta_H) > E_f(u_m^C) > u_m^I(\theta_L) \\ 1 & \text{if } E_f(u_m^C) \leq u_m^I(\theta_L) \end{cases} \quad (2.9)$$

Moreover, the probability of reelection for a high type is never lower than the probability of reelection for a low type.

As the proof of Lemma 1 shows, the value of  $B^I$  is implicitly defined as

$$\mu(\theta_H | B^I) = \frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)}. \quad (2.10)$$

Next, we define the *ex-ante* probability that an incumbent party  $I \in \{R, D\}$  wins as

$$\pi^I \equiv \rho^I(\theta_H) f(\theta_H) + \rho^I(\theta_L) f(\theta_L). \quad (2.11)$$

Suppose that the electorate prefers a low ability candidate from party  $D$ , the incumbent, than a high ability candidate from party  $R$ . Then, we would observe party  $D$  winning every single election (see (2.9)). However, we claim that party  $D$  does not enjoy any real incumbency advantage because, were this party the challenger, it would still win. Therefore, holding office does not confer any advantage to this party. As the example illustrates, the probability of winning may be a misleading measure of incumbency advantage. Here we propose a measure of incumbency advantage that captures the increase in the probability of winning directly derived from holding office. We let party  $D$ 's incumbency advantage to be the difference between its probability of winning when it is the incumbent ( $\pi^D$ ), and its probability of winning were it the challenger (measured as the probability that an incumbent party  $R$  would lose,  $1 - \pi^R$ ). Therefore, we define party  $D$ 's **incumbency advantage** as

$$IA^D = \pi^D - (1 - \pi^R). \quad (2.12)$$

Observe that  $IA^D$  can take negative values, representing the possibility that a party may be harmed by incumbency. Since incumbency provides information about the type of the incumbent politician, if politicians are more likely to be of the bad type, more information will hurt more often than help their ex-ante probability of being elected. In this sense we can talk of positive incumbency advantage as well as negative incumbency advantage or incumbency disadvantage.

### 3. The Informational Value of Incumbency

The main point of this paper is to highlight the role of incumbency as a source of information about politicians. Recognizing the informational value of incumbency implies that current estimations of legislature irresponsibility (the source of our concerns) by incumbency advantage are almost surely biased. Namely, if high ability politicians are the most frequent type, incumbency advantage overestimates legislature irresponsibility, since the information revealed by incumbency would explain some –potentially all– the observed advantage.

In order to concentrate on the relevance of the information provided by incumbency, and following the empirical analysis of Lee (forth.), we make parties *ex-ante comparable* by assuming that  $E_f(u_m^I) = E_f(u_m^C)$ . Therefore, we abstract from any asymmetry between the parties, except from one being the incumbent and the other being the challenger. This makes the electorate indifferent between the two parties unless they have extra information about the incumbent. The results of this section hold, with qualifications, under the more general case when the electorate favors one of the parties. We postpone the analysis of the more general case to the next section.

When parties are *ex-ante* comparable, the probability of reelection is independent of the party holding office, that is  $\pi^D = \pi^R$ . It follows from (2.12) that incumbency advantage is  $IA^I = 2\pi^I - 1$ , and hence an increasing function of the probability of winning. Therefore, both incumbency advantage and probability of victory move together. Incumbency offers voters extra information about the type of the candidate currently in power. But only high type candidates will benefit from this extra information. Low type candidates, on the other hand, would be better off if their type were not revealed. From the incumbent party's point of view, the information revealed from incumbency works in its favor if the fraction of high type candidates is larger than the fraction of low type candidates,  $f(\theta_H) > f(\theta_L)$ , and against it otherwise.

**Proposition 1.** *Let  $E_f(u_m^I) = E_f(u_m^C)$ . If  $f(\theta_H) > f(\theta_L)$ , incumbency advantage is positive, revealing the beneficial impact of information for the incumbent. If  $f(\theta_H) < f(\theta_L)$ , incumbency advantage is negative.*

As the external signal becomes more informative (that is, as it facilitates the identification of the incumbent politician's type), it increases incumbent party advantage if and only if high types are more frequent than low types. In the limit, if incumbency were informative enough to reveal the type of the politician, voters would always reelect high ability candidates and kick out low ability ones.

**Proposition 2.** *Let  $E_f(u_m^I) = E_f(u_m^C)$ .*

- i) *If  $f(\theta_H) > f(\theta_L)$ , a more informative external signal (a smaller  $\sigma$ ) implies a larger incumbency advantage.*
- ii) *If  $f(\theta_H) < f(\theta_L)$ , a more informative external signal implies a larger incumbency disadvantage.*

It is worth emphasizing that as voters become better informed about the ability of governing candidates, incumbency advantage (if positive) increases without raising concerns about legislature irresponsibility. In particular, according to Proposition 2, improvements in voters' information about the activity of governing politicians that make more difficult for low ability candidates to hide their type can explain some – potentially all– the observed increase in incumbency advantage. Hence, any analysis of incumbency advantage should control for the informational value of incumbency before making any kind of recommendation.

#### **4. Incumbency Advantage and the Ideological Bias of the Electorate**

In the previous section we made parties *ex-ante* comparable. However, electorates are normally ideologically closer to one of the parties, which implies that voters may prefer

the least skill candidate when it runs for the preferred party. How much skill voters are willing to trade off for closer policies, that is, how much they are willing to pay for the supply of the public good to be closer to their ideal quantity, will depend on the ideological gap between parties and the difference between high and low skills. In an extreme case, if the electorate showed a large ideological preference for one of the parties, this party's candidate would be repeatedly elected and incumbency advantage would be zero, since electoral victory would have nothing to do with its incumbency status.

To study incumbency advantage in the presence of an ideological bias and to determine the relevant values of the latter, we need first to introduce some more notation. We say that the electorate shows an ideological bias towards party  $J$  if the median voter is ideologically closer to party  $J$  than to party  $K$ , that is  $|\beta_J - \beta_m| < |\beta_K - \beta_m|$ . This is equivalent to saying that her expected utility from  $J$  is higher than her expected utility from the other party. Define  $\Delta_f = E_f(u_m^I) - E_f(u_m^C)$  as the *ideological bias* of the electorate towards the incumbent party given that types are distributed according to  $f$ , and interpret  $\Delta_f < 0$  as a bias against the incumbent. Let  $IA_f^J(\Delta_f)$  be the incumbency advantage of party  $J$  when it faces an ideological bias  $\Delta_f$  and types are distributed according to  $f$ . Obviously, for very large and positive (negative)  $\Delta_f$  the incumbent always wins (loses) and incumbency advantage is zero.

The following proposition shows that incumbency advantage reaches its largest magnitude in the absence of an ideological bias and, within reasonable values, it falls in absolute value as the ideological bias becomes more salient. The intuition is simple. An ideological bias dilutes the importance of the ability of the candidate in determining the electoral outcome, and hence reduces the informational value of incumbency. Figure 4.1 illustrates this. When a party benefits from incumbency (Figure 4.1.a), an ideological bias, even in its favor, reduces its advantage since the ability of the candidate loses importance. On the other hand, when the proportion of low types is larger than the

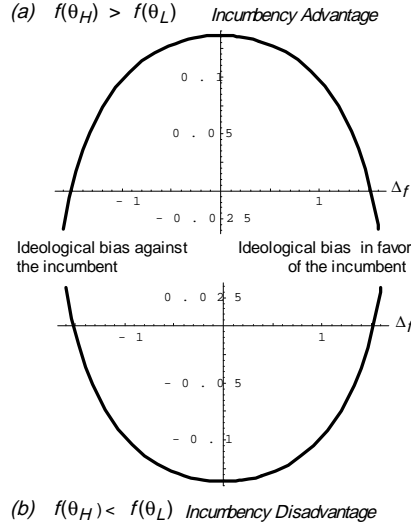


Figure 4.1: *Incumbency advantage* ( $IA^D$ ) as a function of the ideological bias ( $\Delta_f$ ) when (a) high types are more frequent than bad types, and when (b) bad types are more frequent than good types. Values for the parameters in the example:  $\theta_H = 5$ ,  $\theta_L = 1/2$ ,  $\sigma = \frac{\theta_H - \theta_L}{2} = 9/4$ , (a)  $f(\theta_H) = 3/5$ , (b)  $f(\theta_H) = 2/5$ .

proportion of high types (Figure 4.1.b), the incumbent party is harmed by the extra information and hence a larger bias reduces its "disadvantage" from incumbency.

**Proposition 3.** *Incumbency advantage attains its maximum in absolute value in the absence of an ideological bias:  $|IA_f^J(0)| > |IA_f^J(\Delta_f)|$  for all  $\Delta_f \neq 0$ .*

Moreover, for  $|\Delta_f| \leq .95 f(\theta_L)(\theta_H - \theta_L)$  and  $\theta_H - \sigma \geq \theta_L + \sigma$ , if

i)  $f(\theta_H) > f(\theta_L)$  incumbency advantage decreases as the bias in favor or against the incumbent increases.

ii)  $f(\theta_H) < f(\theta_L)$  incumbency advantage increases as the bias in favor or against the incumbent increases.

Although the impact of incumbency is always the largest in the absence of an ide-

ological bias, its monotonicity needs some qualification, which explains the conditions in the previous proposition. Take for instance the case  $f(\theta_H) > f(\theta_L)$ . The proof of Proposition 3 shows that high type candidates benefit relatively less from a bias in favor than low type candidates suffer from a bias (of the same magnitude) against, and vice-versa. Since a bias in favor of the current incumbent implies a bias against the current challenger were it the incumbent,<sup>2</sup> it may happen that an incumbent facing a large bias in its favor is not reelected with probability one, while the current challenger (with a bias against) would lose for sure were it the incumbent. In this case, incumbency advantage is negative and depends only on the probability of reelection of the current incumbent:  $IA^I(\Delta_f) = \pi^I(\Delta_f) - 1$ . Therefore, as the bias increases, the probability of victory of the current incumbent increases, and so does incumbency advantage. We exclude as irrelevant those cases where the probability of victory of one party is independent of the candidate's type and concentrate on situations where voters face a trade-off between ideology and the ability of candidates. It follows from Lemma 1 that one party loses with probability one if it run as the incumbent when  $|\Delta_f| \geq \bar{\Delta}_f = f(\theta_L) (A\theta_H - A\theta_L)$ . Thus we restrict the range of  $\Delta_f$  to the interval  $[-.95\bar{\Delta}_f, .95\bar{\Delta}_f]$ .<sup>3</sup>

## 5. Incumbency Advantage and Selection

As already pointed out, incumbency may prove to be advantageous or disadvantageous for parties presenting candidates for reelection. However, if high ability candidates are more likely to survive than low ability candidates, we should expect to observe the prevalence of a positive incumbency advantage.

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<sup>2</sup>Recall that measuring incumbency advantage requires considering the hypothetical situation where the current challenger would be the incumbent.

<sup>3</sup>The bounds on the range of  $\Delta$  represent a sufficient condition for the monotonicity of  $IA$  when  $\theta_H - \sigma \geq \theta_L + \sigma$ . These specific conditions respond to the particularities of the Normal distribution and specifically of its concavity-convexity and long tails.

An argument for the *selection* of high type candidates has to do with the evolution of incumbency advantage as candidates accumulate terms in office.<sup>4</sup> Incorporating the possibility of accumulating terms requires adapting our notation. Let  $\mu_t(\theta)$  represent the probability that the incumbent candidate is of type  $\theta$  given that she won in the previous  $t - 1$  elections. Observe that  $\mu_1(\theta) = f(\theta)$ , since the incumbent is running for reelection for the first time. Using Lemma 1, if a candidate of type  $\theta$  won in the previous election was because the realization of  $B$  was such that  $B \geq B_{t-1}^I$ , which provides extra information about the type of the candidate. We will assume that voters do not keep track of all past realizations of  $B$  but, instead, they just know that the incumbent candidate has won the last  $t - 1$  elections. Thus,  $\mu_t(\theta)$  can be written as

$$\mu_t(\theta) = \frac{\Pr(B \geq B_{t-1}^I | \theta) \mu_{t-1}(\theta)}{\sum_{\theta' \in \Theta} \Pr(B \geq B_{t-1}^I | \theta') \mu_{t-1}(\theta')}. \quad (5.1)$$

Notice that, since  $\Pr(B \geq B_{t-1}^I | \theta_H) > \Pr(B \geq B_{t-1}^I | \theta_L)$ , if an incumbent candidate has been reelected for  $t - 1$  terms,  $\mu_t(\theta_H) > \mu_{t-1}(\theta_H)$ . That is, the belief that a candidate is of a high type increases with her terms in office.

At election time,  $\mu_t(\theta | B_t)$  represents voters' belief that an incumbent who has been in power for  $t$  terms is of type  $\theta$  when the realization of  $B$  is  $B_t$ . By Bayes rule,

$$\mu_t(\theta | B_t) = \frac{h(B_t | \theta) \mu_t(\theta)}{\sum_{\theta' \in \Theta} h(B_t | \theta') \mu_t(\theta')}. \quad (5.2)$$

Likewise, the *ex-ante* probability that an incumbent party wins the election for a  $t$ -th time can be expressed as

$$\pi_t^I = \rho_t^I(\theta_H) \mu_t(\theta_H) + \rho_t^I(\theta_L) \mu_t(\theta_L). \quad (5.3)$$

where  $\rho_t^I(\theta)$  represents the probability of reelection for a type  $\theta$  candidate and its computation is a straightforward extension of Lemma 1.

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<sup>4</sup>This point is related to the literature that finds a positive correlation between incumbency advantage and seniority or political experience (McKelvey and Reizman 1992).



When measuring the incumbency advantage of party  $D$ , represented by a candidate who has been in office for  $t$  terms, we choose to compare its current probability of winning with its probability of winning had party  $R$  been a new incumbent.<sup>5</sup> That is

$$IA_t^D = \pi_t^D - (1 - \pi_1^R). \quad (5.4)$$

Since a good realization of  $B$  is necessary to be reelected and high type candidates will on average obtain better realizations of  $B$ , the fraction of high type candidates running for reelection for the  $t$ -th time increases as  $t$  increases. That is, high type candidates are more likely to survive a larger number of consecutive contests. On the other hand, since the belief that a candidate is of a high type also increases with the terms in office (5.1), voters will lower the required threshold  $B^I$ , increasing the probability of reelection for all types. Hence, as the following proposition shows, both the probability of reelection and incumbency advantage increase with the number of terms in office.

**Proposition 4.** *The probability of reelection and incumbency advantage increase with the number of terms in office.*

Observe that, although candidates get more easily reelected the larger their tenure, this increase in their probability of winning does not result from voters appreciation of their experience. In our context, tenure gets rewarded because it increases the confidence that the incumbent is in fact a high type and allows voters to ease her reelection by lowering the required threshold  $B^I$ .

Finally, a strand of the literature has emphasized a different selection argument based on the strategic retirement of candidates. This approach argues that incumbents decide

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<sup>5</sup>Alternatively, we could have measured  $D$ 's incumbency advantage as  $\pi_t^D - (1 - \pi_t^R)$ , that is taking as the hypothetical reference scenario the case when  $R$  was the incumbent for  $t$  periods. This approach would increase the magnitude of incumbency advantage, but would not affect the results of the paper.

to retire when their electoral prospects seem particularly low, producing an upward bias in conventional estimates of incumbency advantage (Cox and Katz 2002, Engstrom and Monore 2002). It is easy to incorporate this argument in our framework by assuming that parties, unlike voters, learn the ability of candidates once in power and can choose to retire an incumbent from running for reelection if they would rather be represented by a newcomer. For simplicity, take parties to be ex-ante comparable. They they would always retire low ability candidates and voters would learn that candidates running for reelection are of high ability. Hence high ability candidates would face a probability of victory equal to one, while low ability candidates would be replaced by a new candidate and the incumbent party would win with probability one half. Therefore, the ex-ante probability of victory for an incumbent party would be  $\pi^I = f(\theta_H) + \frac{1}{2}f(\theta_L)$  and so  $IA = 2\pi^I - 1 = f(\theta_H) > 0$ . That is, positive incumbency advantage obtains even when low type candidates are more numerous.

## 6. Concluding Remarks

For the past thirty years, scholars have been increasingly concerned about the advantage that incumbent politicians seem to enjoy and its implications for the representativity of elections and the accountability of representatives. In this paper we have explained incumbency advantage by means of the informational value of being in power questioning the causality relation between high reelection rates and legislature irresponsibility.

One of the novelties of the paper is a new measure of incumbency advantage that differs from the mere observation of incumbents' probability of victory. We show that the probability of victory of incumbents may be prove to be misleading and argue that in order to capture the real impact of incumbency in the probability of reelection, we need to compare the current situation with the case where the incumbent run as a challenger. Our analysis also shows that if we want to focus on the probability of reelection

of current incumbents, we need to make parties *ex-ante* comparable (that is, controlling for any ideological advantage/disadvantage that the incumbent party may enjoy). For, only in this case, there is a one-to-one relationship between incumbency advantage and the probability of victory. This is in fact the approach taken in the empirical analysis of Lee(forth.), who obtains an incumbency advantage of 40%. But, as our paper emphasizes, the connection between this finding and legislature irresponsibility is still inconclusive, for it may be (totally or partially) explained by the information attached to incumbency.

Finally, since candidate ability plays a central role in our approach, it is important to elaborate on it. Our view of candidate's ability corresponds to candidate quality in Stone et al. (2004) and Krasno and Green (1988). A candidate's ability or quality is "inherent to the individual candidate, prior to and distinct from that candidate's performance [...] It is a resource a candidate brings to his or her campaign" (Stone et al. 2004, p.480). In particular, candidate quality differs from electoral prospects, that is, candidate's subjective probability of winning. The important finding for our approach is that incumbent quality affects reelection prospects. Actually, Stone, Maisel, and Mestas show that incumbent personal quality has a strong effect on reelection prospects "both indirectly by enhancing their strategic resources, and directly because personal quality has value to voters and other in the district who determine the incumbent's prospects to continuing in office" (p.487). Therefore, reinforcing the importance of candidates' quality. Thus our argument, in short, emphasizes that, because incumbency is a source of information about candidate quality not available to the challenger, and quality affects reelection prospects, we can expect incumbency to affect candidates' chances of winning without raising (at least partially) concerns on competition and representation of elections.

## Appendix

**Proof of Lemma 1.** We know that  $B$  conditional to  $\theta$  is normally distributed. Therefore,  $h$  satisfies the monotone likelihood ratio property (that is, if  $\theta > \theta'$ , then  $h(B|\theta)/h(B|\theta')$  is strictly increasing in  $B$ ),  $\lim_{B \rightarrow -\infty} (h(B|\theta_H)/h(B|\theta_L)) = 0$  and  $\lim_{B \rightarrow +\infty} (h(B|\theta_H)/h(B|\theta_L)) = +\infty$ . It follows from the monotone likelihood ratio property that  $\mu(\theta_H|B)$  is strictly increasing in  $B$ .

First, if  $u_m^I(\theta_H) > E_f(u_m^C) > u_m^I(\theta_L)$ , then

$$0 < \frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)} < 1.$$

Let  $B^I$  be implicitly defined as

$$\mu(\theta_H|B^I) = \frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)}.$$

The existence and uniqueness of  $B^I$  are guaranteed by the monotonicity of  $\mu$  and the limit conditions above. Thus, the probability of reelection for a type  $\theta$  incumbent (2.8), can be written as

$$\rho^I(\theta) = \Pr(B \geq B^I|\theta).$$

Secondly, if  $E_f(u_m^C) \geq u_m^I(\theta_H)$ , then

$$\frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)} > 1,$$

thus, for all possible realizations of  $B$ ,

$$\mu(\theta_H|B) < \frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)},$$

which implies that the incumbent never gets reelected,  $\rho^I(\theta) = 0$ .

Thirdly, if  $E_f(u_m^C) \leq u_m^I(\theta_L)$ , then

$$\frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)} < 0,$$

thus, for all possible realization of  $B$ ,

$$\mu(\theta_H|B) \geq \frac{E_f(u_m^C) - u_m^I(\theta_L)}{u_m^I(\theta_H) - u_m^I(\theta_L)},$$

which implies that the incumbent always gets reelected,  $\rho^I(\theta) = 1$ .

Finally, the monotone likelihood ratio property implies that  $\Pr(B \geq B^I|\theta_H) > \Pr(B \geq B^I|\theta_L)$ . ■

### Proof of Proposition 1

Since both parties are *ex-ante* comparable, expression (2.10) can be written as  $\mu(\theta_H|B^I) = f(\theta_H)$ , which implies that  $B^I$  is implicitly defined by  $h(B^I|\theta_H) = h(B^I|\theta_L)$ . Since the conditional distribution of  $B$  follows a Normal,  $B^I = \frac{\theta_H + \theta_L}{2}$ . Furthermore, in this case  $\Pr(B \geq B^I|\theta_L) = 1 - \Pr(B \geq B^I|\theta_H)$ . Thus, the probability of reelection for the incumbent party can be written as:

$$\pi^I = (f(\theta_H) - f(\theta_L)) \Pr(B \geq B^I|\theta_H) + f(\theta_L). \quad (6.1)$$

We also know that  $IA^I = 2\pi^I - 1$ . Then, rearranging terms

$$IA^I = (f(\theta_H) - f(\theta_L))(2 \Pr(B \geq B^I|\theta_H) - 1).$$

Because  $\Pr(B \geq B^I|\theta_H) > 1/2$ , the sign of  $IA^I$  coincides with the sign of  $(f(\theta_H) - f(\theta_L))$  as we wanted to prove. ■

### Proof of Proposition 2.

*i)*  $f(\theta_H) > f(\theta_L)$ .

Denote by  $h(B|\theta; \sigma)$  the conditional density function of  $B$  when the variance is  $\sigma^2$ . Given that both parties are *ex-ante* comparable, expression (2.10) can be written as  $\mu(\theta_H|B^I) = f(\theta_H)$ , which implies that  $B^I$  is implicitly defined by  $h(B^I|\theta_H; \sigma) = h(B^I|\theta_L; \sigma)$ . Since the conditional distribution of  $B$  follows a Normal,  $B^I = \frac{A(\theta_H + \theta_L)}{2}$

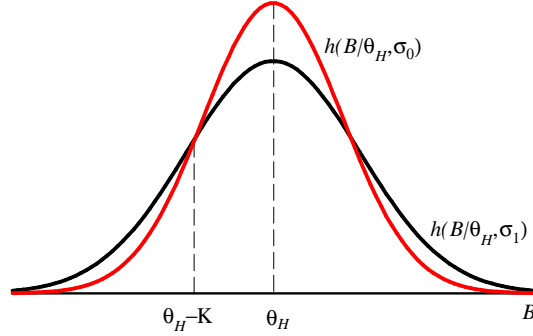


Figure 6.1: Companion figure to the proof of proposition 2. Two distributions of the external signal  $B$  for a high type candidate. The distribution with a lower dispersion  $\sigma_0$  ( $< \sigma_1$ ) represents a more informative signal.

and is independent on  $\sigma$ . Furthermore, in this case  $\Pr(B \geq B^I | \theta_L; \sigma) = 1 - \Pr(B \geq B^I | \theta_H; \sigma)$ . Thus, the probability of reelection can be written in terms of  $\sigma$  as

$$\pi^I(\sigma) = (f(\theta_H) - f(\theta_L)) \Pr(B \geq B^I | \theta_H; \sigma) + f(\theta_L). \quad (6.2)$$

It is enough to show that  $\Pr(B \geq B^I | \theta_H; \sigma)$  is decreasing with  $\sigma$ .

Take  $\sigma_0 < \sigma_1$ , and let  $K$  be such that  $h(\theta_H - K | \theta_H; \sigma_0) = h(\theta_H - K | \theta_H; \sigma_1)$ . Refer to Figure 6.1 to follow the proof. For all  $B \leq \theta_H - K$ ,  $h(B | \theta_H; \sigma_0) \leq h(B | \theta_H; \sigma_1)$ , and for all  $B \in [\theta_H - K, \theta_H]$ ,  $h(B | \theta_H; \sigma_0) \geq h(B | \theta_H; \sigma_1)$ .

First, if  $B^I \leq \theta_H - K$ ,  $H(B^I | \theta_H; \sigma_0) < H(B^I | \theta_H; \sigma_1)$ , which implies that  $\Pr(B \geq B^I | \theta_H; \sigma_0) > \Pr(B \geq B^I | \theta_H; \sigma_1)$ .

Secondly, if  $B^I \in [\theta_H - K, \theta_H]$ , write  $H(B^I | \theta_H; \sigma)$  as

$$H(B^I | \theta_H; \sigma) = \frac{1}{2} - \int_{B^I}^{\theta_H} h(B | \theta_H; \sigma) dB.$$

Since  $h(B | \theta_H; \sigma_0) \geq h(B | \theta_H; \sigma_1)$  for all  $B^I \leq B \leq \theta_H$ , then  $H(B^I | \theta_H; \sigma_0) < H(B^I | \theta_H; \sigma_1)$ , which implies that  $\Pr(B \geq B^I | \theta_H; \sigma_0) > \Pr(B \geq B^I | \theta_H; \sigma_1)$ .

We have shown then that  $\Pr(B \geq B^I | \theta_H; \sigma)$  is decreasing in  $\sigma$ , concluding the proof of

this part.

ii)  $f(\theta_H) < f(\theta_L)$ .

Let  $g(\theta_H) = 1 - f(\theta_H)$ . From (6.2), the reelection probability for a distribution of types  $f$  can be expressed in terms of  $\sigma$  as

$$\pi_f^I(\sigma) = (f(\theta_H) - f(\theta_L)) \Pr(B \geq B^I | \theta_H; \sigma) + f(\theta_L). \quad (6.3)$$

Thus,

$$\pi_f^I(\sigma) = -(g(\theta_H) - g(\theta_L)) \Pr(B \geq B^I | \theta_H; \sigma) + 1 - g(\theta_L) = -\pi_g^I(\sigma) + 1.$$

Since  $g(\theta_H) > g(\theta_L)$ , we know from part (i) that  $\pi_g^I(\sigma)$  is decreasing in  $\sigma$ . Therefore  $\pi_f^I(\sigma)$  is increasing in  $\sigma$ , proving that the incumbency disadvantage decreases with  $\sigma$ . ■

We introduce the following notation and provide a new lemma for the proof of Proposition 3.

Given a distribution of types,  $f$ , and the ideological bias of the electorate towards party  $D$ ,  $\Delta_f = E_f(u_m^D) - E_f(u_m^R)$ , rewrite the right hand side of (2.10) as

$$K_f^D(\Delta_f) = f(\theta_H) - \frac{\Delta_f}{\theta_H - \theta_L}, \quad (6.4)$$

if party  $D$  is the incumbent, and as

$$K_f^R(\Delta_f) = f(\theta_H) + \frac{\Delta_f}{\theta_H - \theta_L}, \quad (6.5)$$

if party  $R$  is the incumbent, where we have used that  $u_m^D(\theta_H) - u_m^D(\theta_L) = u_m^R(\theta_H) - u_m^R(\theta_L) = \theta_H - \theta_L$ . By Lemma 1, there exist  $B_f^J(\Delta_f)$ ,  $J \in \{D, R\}$ , such that the probability of reelection for party  $J$  can be written as  $\rho^J(\theta) = \Pr(B \geq B_f^J(\Delta_f) | \theta)$ . Since  $B$  conditional to  $\theta$  is normally distributed, we can compute explicitly  $B_f^J(\Delta_f)$ ,  $J \in \{D, R\}$ , from 2.10 as

$$B_f^J(\Delta_f) = \frac{\theta_H + \theta_L}{2} + \frac{\sigma^2}{\theta_H - \theta_L} \ln \left[ \frac{K_f^J(\Delta_f)}{1 - K_f^J(\Delta_f)} \frac{f(\theta_L)}{f(\theta_H)} \right]. \quad (6.6)$$

**Lemma 2.** Let  $IA_f^D(\Delta_f)$  be the incumbency advantage of party  $D$  when types are distributed according to  $f$ . Let  $g(\theta_L) = 1 - f(\theta_L)$ , then  $IA_g^D(\Delta_g) = -IA_f^D(\Delta_f)$ .

**Proof.** Recall that  $u_m^D(\theta_H) - u_m^D(\theta_L) = u_m^R(\theta_H) - u_m^R(\theta_L) = \theta_H - \theta_L$ . Since  $g(\theta_L) = 1 - f(\theta_L)$ , then  $\Delta_f = \Delta_g$ . In what follows we will use  $\Delta$  as the bias towards party  $D$ . Using Lemma 1, write incumbency advantage of party  $D$  as

$$\begin{aligned}
IA_f^D(\Delta) &= (f(\theta_H)\rho^D(\theta_H) + f(\theta_L)\rho^D(\theta_L)) - & (6.7) \\
&\quad (f(\theta_H)(1 - \rho^R(\theta_H)) + f(\theta_L)(1 - \rho^R(\theta_L))) \\
&= f(\theta_H) (\Pr(B \geq B_f^D(\Delta)|\theta_H) - \Pr(B \leq B_f^R(\Delta)|\theta_H)) + \\
&\quad f(\theta_L) (\Pr(B \geq B_f^D(\Delta)|\theta_L) - \Pr(B \leq B_f^R(\Delta)|\theta_L)),
\end{aligned}$$

where  $B_f^D(\Delta)$  and  $B_f^R(\Delta)$  are defined in (6.6).

It follows from (6.4) and (6.5) that  $K_g^D(\Delta) = 1 - K_f^R(\Delta)$  and  $K_g^R(\Delta) = 1 - K_f^D(\Delta)$ . Therefore,  $B_g^D(\Delta) = \theta_H + \theta_L - B_f^R(\Delta)$  and  $B_g^R(\Delta) = \theta_H + \theta_L - B_f^D(\Delta)$ . And consequently,  $\Pr(B \geq B_g^D(\Delta)|\theta_H) = \Pr(B \leq B_f^R(\Delta)|\theta_L)$ ,  $\Pr(B \geq B_g^D(\Delta)|\theta_L) = \Pr(B \leq B_f^R(\Delta)|\theta_H)$ ,  $\Pr(B \leq B_g^R(\Delta)|\theta_H) = \Pr(B \geq B_f^D(\Delta)|\theta_L)$ , and  $\Pr(B \leq B_g^R(\Delta)|\theta_L) = \Pr(B \geq B_f^D(\Delta)|\theta_H)$ . Using these four equalities into (6.7) we obtain  $IA_f^D(\Delta) = -IA_g^D(\Delta)$ .

■

### Proof of Proposition 3.

Since the distribution of types  $f$  is fix throughout the proof, we omit the subscript  $f$ . From (6.4) and (6.5)  $K^D(-\Delta) = K^R(\Delta)$ . Then  $B^D(\Delta) = B^R(-\Delta)$  (6.6), and  $IA^D(\Delta) = IA^D(-\Delta)$  (6.7). Thus, it suffices to concentrate on  $IA^D(\Delta)$  for  $\Delta \geq 0$ . We will also omit the argument  $\Delta$  for all functions except when necessary for the exposition.



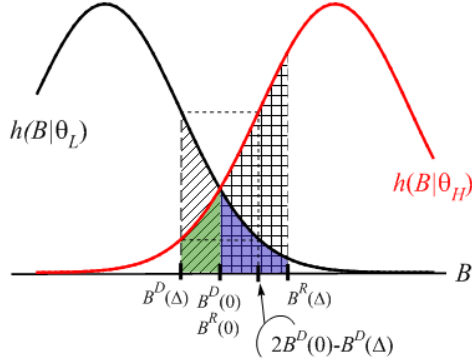


Figure 6.2: Companion figure to the proof of Proposition 3. Reelection threshold values of the signal for each party in the presence of an ideological bias  $\Delta$ ,  $B^D(\Delta)$  and  $B^R(\Delta)$ . In the absence of an ideological bias both parties face the same threshold  $B^D(0) = B^R(0)$ . Observe that  $h(B^D(\Delta)|\theta) = h(2B^D(0) - B^D(\Delta)|\theta)$ .

First, we need to show that  $|IA(0)| > |IA(\Delta)|$  for all  $\Delta > 0$ . Let  $f(\theta_H) > f(\theta_L)$  and refer to Figure 6.2. Using Lemma 1, write

$$\begin{aligned}
 IA^D(\Delta) - IA^D(0) &= f(\theta_H) [\Pr(B^D(\Delta) \leq B \leq B^D(0)|\theta_H) - & (6.8) \\
 &\Pr(B^R(0) \leq B \leq B^R(\Delta)|\theta_H)] + \\
 &f(\theta_L) [\Pr(B^D(\Delta) \leq B \leq B^D(0)|\theta_L) - \\
 &\Pr(B^R(0) \leq B \leq B^R(\Delta)|\theta_L)],
 \end{aligned}$$

where  $B^D(\Delta)$  and  $B^R(\Delta)$  are defined in (6.6), and hence  $B^D(0) = B^R(0) = \frac{\theta_H + \theta_L}{2}$ . Each of the two bracketed terms in expression (6.8) can be interpreted as the difference between the gain in the probability of victory from a bias in favor and the loss from a bias against for a high and low type, respectively. Since  $H(B|\theta_H)$  is a translation to

the right of  $H(B|\theta_L)$  in the amount  $\theta_H - \theta_L$ , and using the symmetry properties of the Normal density,

$$\Pr(B^D(\Delta) \leq B \leq B^D(0)|\theta_H) = \Pr(B^D(0) \leq B \leq 2B^D(0) - B^D(\Delta)|\theta_L).$$

It is easy to show that  $f(\theta_H) > f(\theta_L)$  implies that  $B^D(0) - B^D(\Delta) < B^R(\Delta) - B^R(0)$ . Therefore

$$\Pr(B^D(0) \leq B \leq 2B^D(0) - B^D(\Delta)|\theta_L) < \Pr(B^R(0) \leq B \leq B^R(\Delta)|\theta_L),$$

implying that the gain in the probability of victory for a high type from a bias in favor is smaller than the loss from a bias against for a low type. Similarly, the gain in the probability of victory for a low type from a bias in favor is smaller than the loss from a bias against for a high type. Going back to expression (6.8),  $IA^D(\Delta) < IA^D(0)$ .

The proof for  $f(\theta_L) > f(\theta_H)$  follows directly from the previous case and Lemma 2. Thus  $IA^D(\Delta) > IA^D(0)$  in this case.

Now we are left with showing the monotonicity of  $IA$  as a function of  $\Delta$  under the conditions stated in the proposition.

*i)  $f(\theta_H) > f(\theta_L)$ .*

Rewrite party  $D$ 's incumbency advantage as

$$IA^D(\Delta) = 1 - f(\theta_H)(H(B^D|\theta_H) + H(B^R|\theta_H)) - f(\theta_L)(H(B^D|\theta_L) + H(B^R|\theta_L)),$$

and take derivatives with respect to  $\Delta$ . Using that  $\mu(\theta_H|B^D) = K^D$ ,  $\mu(\theta_H|B^R) = K^R$  and rearranging terms we get

$$\frac{\partial IA^D(\Delta)}{\partial \Delta} = -\frac{\partial B^D}{\partial \Delta} \frac{h(B^D|\theta_H)f(\theta_H)}{K^D} - \frac{\partial B^R}{\partial \Delta} \frac{h(B^R|\theta_H)f(\theta_H)}{K^R}.$$

From the definition of  $B^D$  and  $B^R$  (6.6),

$$\frac{\partial B^D}{\partial \Delta} = -\frac{\sigma^2}{K^D(1 - K^D)}, \text{ and } \frac{\partial B^R}{\partial \Delta} = \frac{\sigma^2}{K^R(1 - K^R)}.$$

For all relevant values of  $\Delta$ ,  $\frac{\partial B^D}{\partial \Delta} < 0$  and  $\frac{\partial B^R}{\partial \Delta} > 0$ . Thus,  $IA^D$  is decreasing if and only if

$$\frac{-\frac{\partial B^D}{\partial \Delta} K^R}{\frac{\partial B^R}{\partial \Delta} K^D} \leq \frac{h(B^R|\theta_H)}{h(B^D|\theta_H)},$$

or equivalently,

$$\ln \left[ \left( \frac{K^R}{K^D} \right)^2 \frac{(1-K^R)}{(1-K^D)} \right] \leq \ln \frac{h(B^R|\theta_H)}{h(B^D|\theta_H)}. \quad (6.9)$$

The right-hand-side of (6.9) equals

$$\ln \frac{h(B^R|\theta_H)}{h(B^D|\theta_H)} = \frac{1}{2\sigma^2} (B^R - B^D)(2\theta_H - (B^R + B^D)). \quad (6.10)$$

From the definitions of  $B^D$  and  $B^R$ ,

$$\begin{aligned} \frac{1}{2\sigma^2} (B^R - B^D) &= \frac{1}{2(\theta_H - \theta_L)} \ln \frac{K^R(1-K^D)}{K^D(1-K^R)}, \text{ and} \\ 2\theta_H - (B^R + B^D) &= (\theta_H - \theta_L) + \frac{\sigma^2}{\theta_H - \theta_L} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right]. \end{aligned}$$

Thus, rewrite (6.10) as

$$\begin{aligned} \ln \frac{h(B^R|\theta_H)}{h(B^D|\theta_H)} &= \frac{1}{2} \ln \frac{K^R(1-K^D)}{K^D(1-K^R)} + \\ &\quad \frac{\sigma^2}{2(\theta_H - \theta_L)^2} \ln \frac{K^R(1-K^D)}{K^D(1-K^R)} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right]. \end{aligned}$$

Since  $f(\theta_H) > f(\theta_L)$ ,<sup>6</sup> and  $\sigma \leq \frac{1}{2}(\theta_H - \theta_L)$ ,

$$\frac{\sigma^2}{2(\theta_H - \theta_L)^2} \ln \frac{K^R(1-K^D)}{K^D(1-K^R)} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right] \geq \frac{1}{8} \ln \frac{K^R(1-K^D)}{(1-K^R)K^D} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right],$$

Therefore,

$$\ln \frac{h(B^R|\theta_H)}{h(B^D|\theta_H)} \geq \frac{1}{2} \ln \frac{K^R(1-K^D)}{K^D(1-K^R)} + \frac{1}{8} \ln \frac{K^R(1-K^D)}{(1-K^R)K^D} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right].$$

Going back to (6.9),  $IA^D$  is decreasing in  $\Delta$  if

$$\ln \left[ \left( \frac{K^R}{K^D} \right)^2 \frac{(1-K^R)}{(1-K^D)} \right] \leq \frac{1}{2} \ln \frac{K^R(1-K^D)}{K^D(1-K^R)} + \frac{1}{8} \ln \frac{K^R(1-K^D)}{(1-K^R)K^D} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right],$$

which, after rearranging terms, becomes

$$\frac{3}{2} \ln \frac{K^R(1-K^R)}{(1-K^D)K^D} \leq \frac{1}{8} \ln \frac{K^R(1-K^D)}{(1-K^R)K^D} \ln \left[ \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \right].$$

Since

$$1 \geq \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \geq \frac{K^R(1-K^R)}{(1-K^D)K^D},$$

then, we only need to show that for all relevant values of  $\Delta$

$$\ln \frac{K^R(1-K^D)}{(1-K^R)K^D} \leq 12. \tag{6.11}$$

Let  $Q = \frac{K^R(1-K^D)}{(1-K^R)K^D}$ . It is easy to show that, for all relevant values of  $\Delta$ ,  $Q$  is an increasing

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<sup>6</sup>Observe that  $f(\theta_H) > f(\theta_L)$  implies

$$\frac{K^R(1-K^D)}{(1-K^R)K^D} \geq 1, \text{ and } \frac{(1-K^D)(1-K^R)}{K^R K^D} \left( \frac{f(\theta_H)}{f(\theta_L)} \right)^2 \leq 1.$$

function of  $\Delta$ . Thus, it suffices to show that (6.11) holds for the largest ideological bias under consideration,  $\Delta_{\max}$ . Let  $\Delta_{\max} = k f(\theta_L)(\theta_H - \theta_L)$ , with  $k$  close to 1. Then

$$Q(\Delta_{\max}) = \frac{f(\theta_H) (1 - k^2) + k + k^2}{f(\theta_H) (1 - k^2) - k + k^2},$$

which is a decreasing function of  $f(\theta_H)$ . Thus, in order to show that (6.11) holds, it is enough to concentrate on the lowest bound of  $f(\theta_H) = 1/2$ :

$$Q(\Delta_{\max}) \Big|_{f(\theta_H)=\frac{1}{2}} = \frac{\frac{1}{2} (1 - k^2) + k + k^2}{\frac{1}{2} (1 - k^2) - k + k^2} \leq e^{12}, \text{ for all } k \leq .995.$$

We can conclude then that, for  $f(\theta_H) > f(\theta_L)$ ,  $IA^D$  is decreasing in  $\Delta$  for all  $\Delta \leq 0.995 f(\theta_H) (\theta_H - \theta_L)$ .

ii)  $f(\theta_H) < f(\theta_L)$ .

Now we need to show that  $IA_f^D(\Delta)$  is increasing for all  $\Delta \geq 0$ . Let  $g(\theta_H) = 1 - f(\theta_H) > g(\theta_L)$ . By lemma 2  $IA_g^D(\Delta) = -IA_f^D(\Delta)$ . From case 1,  $IA_g^D(\Delta)$  is decreasing in  $\Delta$ . Then  $IA_f^D(\Delta)$  is increasing in  $\Delta$ , and we conclude the proof. ■

**Proof of Proposition 4.** A  $t$ -term incumbent has been reelected for  $t - 1$  times, which implies that the last realization of  $B$  was larger than  $B_{t-1}^I$ . Voters will reelect the incumbent whenever the realization of  $B$  is larger than  $B_t^I$ . Since  $\mu_t(\theta_H) > \mu_{t-1}(\theta_H)$ ,  $B_t^I < B_{t-1}^I$ . Thus,

$$\begin{aligned} \rho_t(\theta_H) &= \rho_{t-1}(\theta_H) + \Pr(B_t^I \leq B \leq B_{t-1}^I | \theta_H) \\ \rho_t(\theta_L) &= \rho_{t-1}(\theta_L) + \Pr(B_t^I \leq B \leq B_{t-1}^I | \theta_L) \end{aligned} \tag{6.12}$$

The probability of winning for the incumbent increases with respect to the previous election if

$$\rho_t(\theta_H) \mu_t(\theta_H) + \rho_t(\theta_L) \mu_t(\theta_L) > \rho_{t-1}(\theta_H) \mu_{t-1}(\theta_H) + \rho_{t-1}(\theta_L) \mu_{t-1}(\theta_L), \tag{6.13}$$

or, equivalently

$$\begin{aligned} & (\rho_t(\theta_H) - \rho_t(\theta_L))\mu_t(\theta_H) + \rho_t(\theta_L) - \rho_{t-1}(\theta_L) > \\ & (\rho_{t-1}(\theta_H) - \rho_{t-1}(\theta_L))\mu_{t-1}(\theta_H). \end{aligned}$$

It follows from (6.12) that

$$\begin{aligned} & (\rho_t(\theta_H) - \rho_t(\theta_L))\mu_t(\theta_H) + \rho_t(\theta_L) - \rho_{t-1}(\theta_L) = \\ & (\rho_{t-1}(\theta_H) - \rho_{t-1}(\theta_L))\mu_t(\theta_H) + \\ & \Pr(B_t^I \leq B \leq B_{t-1}^I | \theta_H)\mu_t(\theta_H) + \\ & \Pr(B_t^I \leq B \leq B_{t-1}^I | \theta_L)\mu_t(\theta_L). \end{aligned}$$

Since  $\mu_t(\theta_H) > \mu_{t-1}(\theta_H)$ , inequality (6.13) holds.

Finally, since  $\pi_1^R$  does not depend on  $t$ , it follows from the definition of incumbency advantage (2.12) that  $IA_t^D$  is increasing in  $t$ . ■

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