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**Global Bifurcations, Credit Rationing
and Recurrent Hyperinflations**

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Global Bifurcations, Credit Rationing and Recurrent Hyperinflations[*]

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Abstract

This paper proposes an alternative explanation to recurrent hyperinflations other than bounded rationality by explicitly considering the global dynamics of an economy with credit market frictions. In this paper we show that hyperinflations are self-generated and are manifestations of the underlying global dynamic properties of an economy with perfect foresight rational agents that face credit rationing. Moreover, we find that economies that are more credit constrained are more likely to experience recurrent hyperinflations.

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1 Introduction

Although the long run relationship between money and prices is a phenomenon that has received a lot of attention by macroeconomists during the last century, the study of hyperinflations is rather new. One of the first studies on the subject can be attributed to Cagan's (1956) seminal work. He proposes a money demand function based on asset-market considerations in order to study inflation dynamics, the money supply and inflationary finance. He finds that in hyperinflationary periods movements in prices are much greater than movements in real variables [8].[1]

Within the general equilibrium framework, Sargent (1986) shows that when governments are unable to either reduce their fiscal deficits or finance them through capital, high seignorage is required and high inflation rates are unavoidable [20]. On the other hand, Sargent and Wallace (1987) generate a standard Laffer curve with two stationary rational expectations equilibria where hyperinflations can occur as speculative equilibria converging to the high inflation steady state. Their paper explains how inflation can grow even though seignorage is stable [21].[2] Similarly, Eckeisten and Leiderman (1992) explain the very large inflation rates in Israel with an ever increasing Laffer curve [10].

Some other work on hyperinflations has incorporated expectations of a monetary reform to explain the behavior of certain economic variables during hyperinflations. Bental and Eckstein (1990) consider an economy where agents know in advance the whole future path of government policies. In particular, agents know the date and composition of any stabilization package which requires the budget to be balanced and keep the price level fixed [3]. In the same spirit,

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[1]Households adjust their real balances according to expected inflation.

[2]In the same spirit, Evans and Yarrow (1981) [11] and Bruno and Fisher (1990) [6] interpret hyperinflations as a situation in which the economy converges towards a high inflation steady state along the inflation tax Laffer curve.

Paal (2000) examines the possibility that pre-stabilization rates of inflation in Hungary after World War II resulted from the anticipation of strict credit controls in the post-stabilization period [18].

Finally, some authors have explored hyperinflations with learning and moving away from rationality. Auernheimer (1976) [2], and Kiguel (1986) [16] show that in order to obtain a hyperinflationary process one needs to assume adaptive expectations. In other words, within Cagan's framework, large budget deficits can result in hyperinflations only when agents make systematic mistakes in forecasting inflation. More recently, Marcet and Nicolini (2004) construct a general equilibrium model of bounded rational learning to account for the observations of recurrent hyperinflations in the eighties [17].

To date only models with quasi rational agents are able to generate recurrent hyperinflations. An alternative explanation to recurrent hyperinflations is to consider the global properties of an economy with frictions. The overlapping generation framework is a rich source of interesting dynamics, endogenous cycles, chaos, bifurcations or sunspot equilibria that can help explain recurrent hyperinflations. Within this spirit, Boyd and Smith (1998) explore how the presence of credit market frictions, whose severity is endogenous, affects capital and inflation dynamics [4]. The authors find that if any monetary steady state equilibria exist, there are generally two of them. One of these equilibria has a low capital stock and output level, and it is necessarily a saddle. The other steady state has a high capital stock and output level; either it is necessarily a sink, or its stability properties depend on the rate of money creation. If the high capital stock steady state is not a sink for all rates of money growth, then increases in the rate of money growth can induce a Hopf bifurcation. Hence dynamical equilibria can display damped oscillations as well as limit cycles. The corresponding local properties of this economy are not able to generate inflation dynamics consistent with hyperinflations.

In particular, the local properties of the Boyd and Smith (1998) economy can only generate quasi-periodic hyperinflations when very high steady state inflation rates are considered. As a consequence, the resulting time series for the inflation rate are "too" cyclical. Furthermore, the local properties are not able to predict isolated hyperinflations for moderately high inflation rates. Moreover, Boyd and Smith (1998) are only able to examine local inflation dynamics for a very small set of initial conditions; i.e, only those near the monetary steady state. As a result, transition dynamics leading to recurrent hyperinflations can not be analyzed. Finally, when considering a local analysis the potential time series patterns of the inflation rate can not be greatly affected by the existence of other equilibria.

The goal of this paper is to show that global dynamic properties of an economy with credit market frictions, the Boyd and Smith (1998) model, are consistent with the qualitative behavior of recurrent hyperinflations. We take Cagan's view that hyperinflations are indeed self-generated and that they are manifestations of some underlying global stability properties of the economy. In particular, we believe that the relevant dynamic properties to consider are the global ones. In order to fully examine this possibility, we explore the evolution of a credit constrained economy while providing a comprehensive description of the corresponding geometric structures of the economy as it moves away from the steady states. We can then study the time series patterns of the inflation rate nonlocally, allowing the possibility of other steady state to affect its dynamics. In order to analyze these possibilities we employ algorithms suggested by Gomis-Porqueras and Haro (2003) which help characterize the underlying manifolds of this credit constrained economy [13].

In this paper we show that recurrent hyperinflations are consistent with the global properties of an economy with perfect foresight rational agents that face credit rationing. In this environment the introduction of money reduces the capital stock but when the economy is

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credit constrained this has the effect of reducing the steady state return on savings. Thus, the reduction in the per capita capital stock forces wages to decrease and hence borrowers are able to provide less internal financing for their investment project, lowering the expected real return on loans. Thus, the introduction of an additional asset diverts some savings away from capital accumulation. Furthermore, high rates of inflation will reduce savings, increasing the attractiveness of borrowing which worsens access to the credit market. Thus, higher rates of inflation exacerbate credit market frictions, and can increase the rationing of credit.

Within this framework, we find that for relatively low money growth rates the time series patterns of the inflation rate are relatively confined, with the exception of sudden bursts followed by periods of deflation and price stability; converging finally to the steady state inflation rate. As the money growth rate increases, the predicted time series for the inflation rate becomes richer. The magnitudes of these sudden bursts tend to be smaller and more frequent. Thus the potential for recurrent hyperinflations increases whenever an economy faces credit market frictions and has large money growth rates. Finally, we find that economies with more severe credit market frictions tend to experience more recurrent hyperinflations.

2   The Model

The basic goal of this paper is to understand the asymptotic behavior of an iterative process while providing a comprehensive description of the geometric structures arising from a dynamic model of a credit constrained economy. The underlying structure of this economy is based on a standard monetary growth model where borrowing is subject to a costly state verification, the Boyd and Smith economy (1998). By introducing credit market frictions –such as costly state verification problems that have an endogenous degree of severity– we are able to predict

that high inflation rates lead to more severe credit rationing, indeterminacy of equilibria and endogenous arising volatility. We believe that these features can help explain inflation dynamics observed during recurrent hyperinflations when global properties of the economy are considered.

Following Boyd and Smith (1998), the economy examined in this paper consists of an infinite sequence of two-period lived overlapping generations. Within each generation, there are "potential borrowers" and "lenders". At each date a single final good is produced using a Cobb-Douglas production function. All agents are endowed with one unit of labor, which is supplied inelastically and there is no labor endowment when old. Furthermore, all agents only care about old age consumption.

Potential borrowers have access to a linear stochastic technology for converting t goods into date $t+1$ capital investments. Each potential borrower has one unit of investment project which can only be operated at a scale, q. In order to finance capital investments, which are subject to a costly state verification *a la* Williamson (1986) [23], they need to borrow from lenders who can not access this technology. Loan contracts are offered by borrowers and are either accepted or rejected by intermediaries (lenders) which take deposits, make loans, and monitor the project as required by the contracts they accept. Thus, loan contract offers must satisfy the expected return constraint for lenders:

$$\int_{A_t} [R_t(z)b_t - \rho_{t+1}\gamma]g(z)dz + x_t b_t \int_{B_t} g(z)dz \geq r_{t+1}b_t;$$

where A_t (B_t) is the set of project returns for which verification (does not) occurs, z is the return to the linear stochastic technology for converting t goods into $t+1$ capital with a probability density $g(z)$, b_t is the amount borrowed, ρ_t is the rental rate of capital, R_t is the promised payment if $z \in A_t$, γ is the verification cost in terms of capital and x_t the uncontingent payment when $z \in B_t$. Moreover, the loan contract must also satisfy the appropriate incentive constraint;

i.e, $R_t(z) \leq x_t$ for $z \in A_t$.

The solution to the problem faced by borrowers is to offer a standard debt contract; where the borrower repays principal plus interests if this is feasible, if not the borrower defaults on the loan, the lender verifies the project return and retains any proceeds from it. Within this environment, it is possible that among ex ante identical borrowers some will receive credit and some will not. Borrowers who are denied credit will have no mechanism for bidding up the expected return to a lender, and hence will be unable to bid credit away from funded borrowers. Under credit rationing, the expected return on bank deposits is given by:

$$r_{t+1} = \frac{q\rho_{t+1}}{b_t} \left(\eta - \frac{\gamma}{q}G(\eta) - \int_0^\eta G(z)dz \right)$$

where η is the maximum expected return to a lender and $G(z)$ is the probability distribution of z. Potential borrowers prefer borrowing to lending under credit rationing iff:

$$\rho_{t+1}\phi \geq r_{t+1} = \psi\frac{\rho_{t+1}}{q - w(k_t)}$$

where $\psi = q\left(\eta - \gamma/qG(\eta) - \int_0^\eta G(z)dz\right)$, $\phi = \hat{z} - \gamma/qG(z)$ with $\hat{z}$ being the average return on the linear stochastic technology, and $w(k_t)$ are young wages.

Imposing market clearing for the capital stock k_t, the no arbitrage condition between capital and money, and assuming an intensive production function given by $f(k) = Bk^\beta$, the resulting evolution for k_t and m_t for this economy is given by the following planar system:

$$k_{t+1} = \phi(w(k_t) - m_t) = \phi(B(1-\beta)k_t^\beta - m_t) \tag{1}$$

$$m_{t+1} = \sigma\frac{m_t}{\Pi(k_t, m_t)} = \frac{\sigma\psi B\beta}{k_{t+1}^{1-\beta}(q - B(1-\beta)k_t^\beta)} m_t, \tag{2}$$

where σ denotes the money growth rate, Π is the inflation rate, B is a positive parameter and $0 < \beta < 1$.[3]

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[3]Note then that young wages are given by $w(k) = B(1-\beta)k^\beta$.

Throughout the paper, we examine economies such that the per capita capital stock k_t and real balances m_t satisfy $0 \leq m_t < w(k_t) < q$, so that intermediation and external finance is required, and money is valued in the economy. These inequalities describe the natural domain in which the economy is defined.[4]

The straight line $m=0$ is invariant, and the 1-dimensional dynamics of the nonmonetary equilibria is described by $k_{t+1} = \phi w(k_t)$. The conditions on wages guarantees that the economy without money has a unique capital steady state k_0, which is an attracting steady state for the 1-dimensional dynamics. For the 2-dimensional dynamics, we have that this nonmonetary equilibria is given by $z_0 = (\phi w(k_0), 0)$ which is located at the natural boundary $m=0$. On the other hand, the monetary steady state equilibria, $m > 0$, are implicitly defined by the following conditions:

$$k = \phi(w(k) - m) \tag{3}$$

$$\Pi(k, m) = \sigma; \tag{4}$$

suggesting that the steady state rate of inflation is given by the money growth rate, σ. In particular, the steady state inflation rate solves the equation $\Sigma(k) = \sigma$; where $\Sigma(k)$ is the continuous function given by:

$$\Sigma(k) = \begin{cases} \dfrac{1}{\psi} \dfrac{q - w(k)}{f'(k)} & \text{if } k > 0 \ , \\[2mm] 0 & \text{if } k = 0 \ . \end{cases} \tag{5}$$

Note that the function $\Sigma(k)$ vanishes for $\hat{k} = w^{-1}(q)$ and has a unique maximum at $k_c = f^{-1}(q)$, defining a critical steady state inflation rate $\sigma_c = \Sigma(k_c) = \frac{1}{\psi} f^{-1}(q)$. If $\sigma < \sigma_c$, there are two monetary steady states: a high steady state capital stock denoted by $z_1 = (k_1, m_1)$ and a low steady state capital stock denoted by $z_2 = (k_2, m_2)$, where $k_1 > k_2$ and $m_i = w(k_i) - \frac{1}{\phi} k_i$ for $i = 1, 2$. On

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[4]The state of the economy is completely characterized by the per capita capital stock k and real balances m. From now on, we will denote the state of the economy as $z = (k, m)$.

the other hand, if $\sigma=\sigma_c$ both steady states coincide, and if $\sigma>\sigma_c$ there are no economically feasible steady states. This suggests then that the government has an upper bound on how much money can be injected into the economy without losing its value; i.e, $\sigma\leq\sigma_c$. Furthermore, the rate of inflation, $\Pi(k,m)$, is a continuous function within the natural boundaries of the problem, $0<m<w(k)<q$, and it vanishes at the natural boundaries $q=w(k)$ and $m=w(k)$.

The existence of the costly state verification implies that the introduction of money reduces the capital stock. In particular, the reduction in the per capita capital stock forces wages to decrease and hence borrowers are able to provide less internal financing for their investment project. Moreover, high rates of inflation will reduce the attractiveness of saving and increase the attractiveness of borrowing, thus worsening access to the credit market. As a result, higher rates of inflation exacerbate credit market frictions, and can increase the rationing of credit.

In terms of local dynamics, this economy is able to generate non-convergence phenomena and indeterminacy of monetary equilibria. Monetary equilibria display non monotonic dynamics so that the economy oscillates while converging to the steady state.[5] A local analysis of this economy can not generate inflation dynamics qualitatively consistent with recurrent hyperinflations. In particular, the local properties of the Boyd and Smith (1998) economy can only generate quasi-periodic hyperinflations when very high steady state inflation rates are considered. As a consequence the resulting time series for the inflation rate are then "too" cyclical. Moreover, the resulting local properties are not able to predict isolated hyperinflations for moderately high inflation rates and they are only able to examine inflation dynamics for a very small set of initial conditions; i.e, only those near the monetary steady state. As a result, transition dynamics leading to recurrent hyperinflations can not be analyzed. Finally, when considering a local analysis the potential time series patterns of the inflation rate can

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[5]We refer to the original paper by Boyd and Smith (1998) for further details.

not be greatly affected by the existence of other steady state equilibria.

In this paper we explicitly consider the possibility that the presence of money and financial markets are a potential source of indeterminacies and endogenous volatility, as suggested by Friedman (1960) [12] or Keynes (1936) [15]. In order to study complex dynamics a bifurcation analysis of the credit rationed economy is warranted.[6] Bifurcation problems usually involve systems where the linearization of the original system has a very large, and possibly infinite dimensional, stable part and a small number of "critical" modes which change from stable to unstable as the bifurcation parameter exceeds a threshold. At this point a drastic change occurs in a large portion of the phase space, which results in complex predicted time series.[7] Finally, global bifurcation theory requires more than linearizing the dynamic equations at the steady state. The stable and unstable manifolds have to be computed.

3   Computing Global Dynamics

There are several numerical methods, used in the dynamical systems literature, that let us study the evolution of an economy as we move away from the steady state in an environment where economic agents do not face uncertainty.[8] With these techniques, one can learn more

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[6]Earlier applications of bifurcation theory in economics include de Vilder (1996) in a 2-dimensional overlapping generation model with productive investment and capital accumulation where agents behave rationally and markets are competitive [9], Brock and Hommes (1997) in a heterogeneous agent cobweb model with evolutionary strategy switching [5], and Pintus, Sands and de Vilder (2000) in an infinite horizon economy with capital-labor substitution [19].

[7]Bifurcations represent a qualitative change in the dynamics caused by a change of one or more parameters of the dynamical system. As a result, bifurcations are able to qualitatively change the orbit structure of a given economy.

[8]In the dynamical system literature, local analysis refers to a linear description of the system and global analysis refers to a nonlinear description.

about the nonlinear properties of the stable and unstable manifolds of a given dynamical system as a point in the phase space moves away from the steady state(s). Once we contemplate nonlinear properties, the corresponding phase portrait of the system can be quite complicated because of possible intersections of the stable and unstable manifolds. This situation can not be observed when performing a local analysis. It is thanks to these nonlinearities that we can capture new dynamical phenomena. In particular, when studying global dynamics we may find strange attractors, suggesting that the iteration of virtually any point on it eventually leads to seemingly random behavior. Therefore, a trajectory of a point between invariant sets predicts large jumps. One can also find *wandering cycles* which are situations associated with manifolds that are quite folded. The corresponding predicted time series allows for some degree of periodicity. These types of phenomena observed when performing a global analysis have the potential to qualitatively describe inflation dynamics during recurrent hyperinflations.

A first step towards a better understanding of the dynamical system is to identify its invariant objects, and study their changes when varying the parameters of the dynamical system. An invariant object is described by a subset of the phase space that is invariant under the action of the dynamical system. The invariant objects are preserved through time and give us an idea of the possible predicted time series. Among the invariant objects of a dynamical system, the steady states (and periodic orbits) and their invariant manifolds are extremely important.[9] In general these manifolds are nonlinear. In particular, the stable manifold of a steady state contains all the points that under the iteration of the map tend to it. On the other hand, points on the unstable manifold of a steady state may tend to a stable cycle or an attracting closed curve or a chaotic attractor.

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[9]We use the word manifold since in general this mathematical object is not necessarily a curve, it can represent a piece of a line, rectangle, etc, that folds into $\mathbb{R}^n$ as a curve or as a surface, etc.

We are now going to briefly describe the parameterization method for invariant manifolds associated to fixed points.[10] The underlying idea behind these algorithms is to find the non-linear properties of these manifolds by exploiting an invariance equation that analytically and implicitly defines them. The natural invariant condition is the one that describes the steady state, but instead of considering a single point, we consider an invariant object parameterized by functions. We can then explore the nonlinear properties of the manifold through successive local approximations of the invariance equation by considering the corresponding Taylor series expansion. These expansions describe the object in a fundamental domain around the steady state with high accuracy. We can then iterate this domain, capturing the nonlinear aspects of the manifold.

In order to clarify ideas, let us define a discrete dynamical system in $\mathbb{R}^n$. It is a pair (X, F), where X is a subset of $\mathbb{R}^n$ that contains all the variables describing the system, and $F:X \to \mathbb{R}^n$ is a map that describes the motion of the system as a function of time t and the initial conditions $X_0=(x_0^1, \ldots, x_0^n)$. Typically, a dynamical system is given by a system of equations such as these:

$$X_{t+1} = F(X_t). \tag{6}$$

Very often the dynamical system depends also on parameters $\alpha=(\alpha^1, \ldots, \alpha^p) \in \mathbb{R}^p$. The study of the changes in the qualitative structure of the solutions of the dynamical system when parameters change is known as a *bifurcation analysis* of the model. Bifurcations involving changes in the stability properties of steady states and periodic orbits are known as *local bifurcations*. On the other hand, bifurcations characterized by collisions of stable and unstable

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[10]In order to find more detailed information regarding the different algorithms that can be used to compute the associated non local manifolds of a dynamical system, we refer the reader to Gomis-Porqueras and Haro (2003) and the references therein. For instance, see [22, 7].

manifolds of steady states and periodic orbits are referred as *global bifurcations*.

With the linear approximation, the stable and unstable eigenspaces (corresponding to the eigenvalues of modules smaller and bigger than one, respectively), do not offer enough information to study transitional dynamics during recurrent hyperinflations. The geometric invariant objects associated with these linear subspaces trap the contracting and expanding dynamics of the stable and unstable manifolds, $\mathcal{W}^s$ and $\mathcal{W}^u$, respectively. Moreover, these stable and unstable manifolds are invariant manifolds which emanate from the steady state(s).

Since the model we are studying is 2-dimensional, the invariant manifolds of a steady state are 1-dimensional, that is, these are invariant *curves*. Thus, we are going to consider only 1-dimensional invariant manifolds, but most of the following methods can also be extended to higher dimensional manifolds. Let $F : X \subset \mathbb{R}^n \to \mathbb{R}^n$ be the map describing the dynamical system, and suppose that the origin is a steady state(s).[11] The topological behavior near the steady state(s) is fully explained by the matrix $A{=}\mathrm{D}F(0)$, provided that this matrix does not contain eigenvalues in the unit circle (of modulus 1) as asserted by the Hartman-Grobman theorem. In many instances, one can get differential or analytic equivalence using the normal forms theorems of Poincare, Dulac and Siegel (see [1] or [14] for a detailed description of the theorems). Our goal is to obtain a 1-dimensional invariant nonlinear manifold $\mathcal{W}$ parameterized by the function $W{=}W(\tau)$, where $W{:}I \subset \mathbb{R}{\to}\mathbb{R}^n$, so that the motion of the parameter is just a multiplication by the corresponding eigenvalue λ.[12] Since any point in the invariant manifold will always be on the manifold under the action of the map, the resulting

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[11] We can always achieve this situation by just translating the coordinates to the steady state(s).

[12] For the stable manifold we consider the eigenvalue that has absolute value less than one, for the unstable manifold we consider the eigenvalue with absolute value larger than one.

invariance equation for the dynamical system is given by:[13]

$$F(W(\tau)) = W(\lambda \tau) . \tag{7}$$

Note that the 0-order equation is $F(W(0))=W(0)$; i.e. $W(0)=0$, which says that the steady state(s) belongs to the manifold. Now if we take derivatives on both sides of the invariance equation and set $\tau=0$, we obtain the following 1-order equation:

$$ADW(0) = \lambda DW(0). \tag{8}$$

Hence, $DW(0)$ must be an eigenvector of the linearized system whose eigenvalue is λ. So we have that $\det(A - \lambda I)=0$, and $DW(0)=w_1$, where $Aw_1=\lambda w_1$. We can now obtain higher order approximations of the manifold by considering Taylor series expansions around the steady state. Recall that our goal is to find recursively the unknown vectors, w_k $\forall k>1$, which are given by:

$$W(\tau) = w_1\tau + w_2\tau^2 + w_3\tau^3 + \ldots \tag{9}$$

In order to find these unknown vectors w_k we have to consider Taylor series expansions of the invariance equation. The left-hand side of the invariance, equation (7), up to order k is given by:[14]

$$F(W(\tau)) = F_{\leq k}(W_{<k}(\tau)) + Aw_k\tau^k + \ldots \tag{10}$$

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[13]Intuitively, when F is an invertible map, the unstable manifold is a *curve* through the steady state which is mapped to itself by F^{-1}. All points on the unstable manifold tend to the steady state under the iteration of F^{-1}. In other words, they tend to the steady state as if time moved backwards. However, in general F^{-1} may not be a single valued function; i.e, F may be a noninvertible map. In this case, at least one inverse exists, say F_k^{-1}, such that all points of the unstable manifold tend to the steady state under the iteration of F_k^{-1}.

[14]We introduce some notation: for instance, in the expression $W(\tau) = W_{<k}(\tau) + w_k\tau_k + \ldots$, $W_{<k}(\tau)$ is the $(k-1)$-order approximation of W, $w_k\tau^k$ is the k-order term and '$\ldots$' correspond to terms of higher order.

and the right-hand side is given by:

$$W(\lambda\tau) = W_{<k}(\lambda\tau) + w_k\lambda^k\tau^k + \dots. \tag{11}$$

Collecting terms of the same order, we can find the kth-order vector of unknown coefficients of equation (7) by matching polynomials up to order k. The resulting linear equation is given:

$$(A - \lambda^k I)\tau^k w_k = W_{<k}(\lambda\tau) - (F_{\leq k}(W_{<k}(\tau)))_{\leq k} = z_k\tau^k, \tag{12}$$

where z_k is a known vector of smaller order terms from the manifold and smaller and current order terms from the dynamic equations. Solving for the unknown vector w_k, we obtain the following expression:

$$w_k = (A - \lambda^k I)^{-1}z_k, \tag{13}$$

provided that the matrix $(A-\lambda^k I)$ is invertible. Notice that if we had previously computed the diagonal form (or, if not possible, the Jordan form) of the matrix A, the calculation of the unknown vector can be greatly simplified.[15] Note that this is a recursive procedure since it makes use of previously computed coefficients. In particular, we know that the fixed point is at the origin, which corresponds to the constant term $w_0=0$, and the eigenvalues of the Jacobian corresponds to the linear term, w_1. Thus, we can exploit this linear information to find higher order terms in a recursive manner.

A possible analytical and computational obstruction of this algorithm is the fact that $(A - \lambda^k I)$ may be a singular matrix for a certain $k>1$. Under those circumstances, we say that λ is a resonant eigenvalue of order k. This is certainly not the case, for instance, when we are computing the stable and unstable manifolds of a saddle-type steady state. For example, in a 2-dimensional map the Jacobian matrix, A, has two eigenvalues $0<|\lambda_1|<1 <|\lambda_2|$ and hence

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[15]This algorithm is global in the sense that one can compute all the Taylor coefficients up to the first resonance, a situation where we find the smallest value of k for which $A - \lambda^k I$ is not invertible.

$\lambda_1 \neq \lambda_2^k$ and $\lambda_2 \neq \lambda_1^k$ for any $k > 1$. As a result, $(A - \lambda^k I)$ is always invertible $\forall k$. This algorithm also works when we have an attracting node steady state with $0 < |\lambda_1| < |\lambda_2| < 1$ for $\lambda = \lambda_1$, because $\lambda_2 \neq \lambda_1^k$ for any k,[16], and for $\lambda = \lambda_2$ provided that $\lambda_2^k \neq \lambda_1$ for any k.[17]

Notice that in numerical applications one has to truncate the Taylor series approximation, and the accuracy is of the order of the first dropped term. These approximations are good in a relatively larger neighborhood of the steady state when compared to the ones obtained by linearization.

4   Global Bifurcations: A Parameterized Example

In order to study the global dynamic properties associated with a credit rationed economy computer simulation is required. The algorithm that we have presented in the previous section let us compute the associated manifolds for any parameterized credit constrained economy given by equations (1) and (2).

The specific example that we present in this section follows the original Boyd and Smith choice of parameter values, ϕ=1.8, ψ=0.35, B= 2, β=0.5, q=2 which allow us to study the temporal evolution of the economy as the money growth rate increases; i.e, $\sigma \in (\frac{1}{2}, 4)$. The dynamical properties associated with this example are not exclusive to this particular choice of parameter values. Our numerical analysis in different regions of the parameter space yield qualitatively similar phenomena. For illustration purposes we present the global dynamic properties associated with the Boyd and Smith choice of parameter values, which generate time series that are qualitatively consistent with recurrent hyperinflations.

A first step is to consider the local bifurcations of the steady states; i.e, the study of the

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[16]In this case, we are computing the fast stable manifold associated with fixed point.

[17]In this case, we are computing the slow stable manifold associated with fixed point.

stability properties. This is summarized in Figure 1. The description of local and global bifurcations are described in the Figures 2, 3,4 and 5. These figures display the invariant objects computed with the algorithms described in the previous section.[18]

For low money growth rates, $\sigma < \sigma_t = 1.0286$, there are only two economically feasible steady states, the nonmonetary steady state, z_0, which is an attracting node and the low capital stock steady state, z_2, which is a saddle.[19]

As the money growth rates increases a new monetary equilibrium is feasible, the high capital stock steady state, z_1, see Figure 1. When $\sigma = \sigma_t$, $z_0 = z_1$ with $k_1 = 3.24$ and $m_1 = 0.00$, and there is a transcritical bifurcation. That is, z_0 and z_1 change their stability: z_0 becomes a saddle and z_1 becomes an attracting node, see Figure 2.

After a node-focus transition at $\sigma = \sigma_a = 1.1672$, the high capital stock steady state z_1, with $k_1 = 3.1296$ and $m_1 = 0.0304$, becomes an attracting focus. The stability of the steady state does not change, thus this is not a local bifurcation.

The drastic changes in the properties of the high capital stock steady state z_1 as we increase σ decrease the potential number of initial conditions for which the economy would converge to the nonmonetary equilibria z_0; reducing then hyperinflations a la Sargent and Wallace, see Figure 2.

As we further increase the money growth rate, the associated manifolds of the steady states start to fold. In particular, when $\sigma = 1.20$, $\sigma = 1.50$ and $\sigma = 2.00$, the associated unstable manifold of the nonmonetary steady state z_0 rolls up to z_1, see Figure 2 and Figure 3.

As we increase the money growth rate, the unstable manifold of the nonmonetary steady state z_0 escapes the natural domain. Before this phenomenon occurs, the unstable manifold of z_0 and the stable manifold of z_2 intersect each other, resulting in an heteroclinic bifurcation

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[18]The thick black curve in Figures 2, 3, 4 and 5 denotes the boundary of natural domain for the system.

[19]The high capital stock steady state, z_1, has negative real balances.

when $\sigma \in [2.19, 2.20]$, see Figure 3. Thus, for some values of σ in this interval, the unstable manifold of z_0 intersects the stable manifold of z_2.[20] Thus, there are heteroclinic tangles, so that the invariant manifolds are folded onto each other. However, in this model such tangles are extremely small and the corresponding transition occurs for a very small range of values of σ. This heteroclinic intersection allows for quite different dynamical behavior in regions of the phase space that are fairly distant from the monetary steady states. We then expect quite different inflation dynamics around this bifurcating money growth rate. This sort of phenomenon can not be observed when performing a local analysis. This finding then highlights the importance of considering properties of other steady state equilibria when studying inflation dynamics.

Similarly, as we further increase the money growth rate, the manifolds become highly nonlinear and folded. In particular, when $\sigma=2.30$, the unstable manifold associated with the low capital steady state z_2 rolls up to the high capital steady state, z_1, see Figure 4. This nonlinearity of the manifolds suggests the possibility of large jumps for economies that are between folds. This is consistent with transition dynamics during recurrent hyperinflations.

When $\sigma=2.40$ the unstable manifold of z_2 also escapes the natural domain, and a new invariant object appears: an invariant circle. This object is a repelling closed curve, see Figure 4. This closed curve defines a trapping region, that is the attracting set of z_1, where all the points inside this curve evolve to the high capital stock steady state z_1 and inflation converges asymptotically to its steady state value. On the other hand, the majority of points

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[20]In general, intersections between the invariant manifolds of different saddle type steady states are called *heteroclinic intersections*, and the intersection points are *heteroclinic points*. Notice that an heteroclinic point $x \in \mathcal{W}^s_{x_1} \cap \mathcal{W}^u_{x_2}$ tends to distinct saddle points. When iterating forward, it tends to x_1, and when backwards it tends to x_2. On the other hand, when $x_1 = x_2$, then the intersection between the manifolds and the intersection points are called *homoclinic*.

near the invariant curve but outside the trapping region slowly oscillate away from that region.

This dynamical phenomena involves the intersections between the stable an unstable manifolds of z_2, *homoclinic intersections*, which occurs when $\sigma \in (2.32, 2.33)$, as we see in Figure 4. Further numerical analysis of this *homoclinic bifurcation* shows that it is produced for values when $\sigma \in [2.322344, 2.322345]$, for which an homoclinic tangle exists. However, the corresponding angle of intersection between the manifolds is extremely small and the tangles are not visible. Moreover, the range of values of the parameter σ at which this tangle exists is so small that it is not observationally relevant.

After this bifurcation the stable manifold of z_2 rolls up such invariant curve. The resulting motion looks cyclical, and after a transient period it tends to the low monetary steady state z_2. These dynamical properties are also consistent with what we observe during recurrent hyperinflations.

If we further increase the money growth rate, the invariant circle vanishes in a subcritical Hopf-Neimark-Sacker bifurcation when $\sigma = \sigma_h = 2.4712$, since z_1 with $k_1 = 1.8702$ and $m_1 = 0.3285$ becomes a repelling focus. Thus, the inflation rate slowly oscillates away from the high capital stock steady state z_1, see Figure 5. This type of bifurcation occurs, in a planar system, when eigenvalues are complex conjugates with modulus one. The change in stability is accompanied by the appearance of closed curves for parameter values on one side of the bifurcation point. In our model this is precisely when the repelling curve disappears.

After the Hopf-Neimark-Sacker bifurcation the high capital monetary steady state z_1 is a repelling focus, and the other two steady states z_0 and z_2 are saddle. Moreover, there is an invariant curve that starts at z_1 and ends at z_2, which is the stable manifold of z_2. The points in such a curve move forward in time towards z_2 and backwards to z_1.

If we further increase the money growth rate, $\sigma = \sigma_r = 2.8558$, we find a repelling focus-node

transition in z_1 with k_1=1.0443 and m_1=0.4417. Finally, when $\sigma=\sigma_{sn}$=2.857143 we find a saddle-node bifurcation at $z_1 = z_2$ with k_1=1.0000 and m_1=0.4444. In planar systems such as this one, a saddle-node bifurcation occurs when one of the eigenvalues is one and the other is different from one in absolute value, and there is the merging and disappearance of two hyperbolic equilibria, one attracting (or repelling) and a saddle.

As we can see, in this credit rationed economy bifurcations and changes in the stability properties among different monetary steady states are more frequently observed when the money growth rate increases, see Figure 1. Thus, the potential for richer time series patterns are going to be associated with higher money growth rates. Furthermore, in all the cases examined, the low steady state capital stock, z_2, does not change its stability being always a saddle point. As a result, the majority of interesting dynamics are going to be generated by the dynamic properties of the high capital stock steady state, z_1, and the global interaction between the different steady states.

It is apparent from these numerical explorations that once we contemplate the nonlinear properties of the dynamical system, the corresponding phase space can become quite complicated because of the possible intersections of the stable and unstable manifolds. It is thanks to the nonlinearities of these manifolds that we can capture new dynamical phenomena not observed when performing a linear analysis. In particular, this credit constrained economy is able to generate manifolds that "fold" as the money growth rate increases, predicting then large jumps in the inflation rate and frequent periods of instability. In order to present these possibilities more transparently, we calculate the different predicted time series of the inflation rate as the money growth increases. The initial conditions considered are k^o=2.00 and m^o=0.05 and m^o=0.20, which are depicted by the R and G lines in Figure 6. Furthermore, in order to highlight the importance of the nonlinearities of the manifolds, we consider an initial

condition that is near the stable manifold of the low capital stock steady state z_2, which we depict by the B line in Figure 6.

Our numerical experiments show that for relatively low money growth rates, that is $\sigma=1$ and $\sigma=1.5$, the time series patterns of the inflation rate are relatively confined, with the exception of sudden bursts in the inflation rate away from the steady state, followed by a period of deflation and price stability. Finally, the economy converges to the steady state inflation rate. This type of transition dynamics during a hyperinflation can only be observed when performing a global analysis of the economy. Furthermore, these bursts in inflation rates are only associated with initial conditions that are near the stable manifold of the low capital stock z_2, see Figure 6. This finding then suggests that hyperinflations in countries with low inflation rates are less likely since there is a limited set of initial conditions that are able to generate sudden bursts in the inflation rate but they can not be ruled out.

As the money growth rate increases, the predicted time series for the inflation rate becomes more complex and interesting. In particular, when $\sigma=2$ and $\sigma=2.20$ the potential for sudden bursts in the inflation rate is greater, the variation in the inflation rate increases with the constant money growth rate and recurrent hyperinflations are less sensitive to initial conditions. Moreover, the magnitude of the sudden bursts in the inflation rate tends to be smaller in comparison to economies with lower money growth rates but they are more frequent. Furthermore, these recurrent hyperinflations are not confined to initial conditions near the stable manifold of the low capital stock z_2. Now the set of initial conditions that yield such behavior is larger than in the previous case z_2. The magnitude of the sudden bursts in the inflation rate are smaller for initial conditions that are not close to the stable manifold of the low capital steady state, z_2.[21] Finally, when the value of the money growth rate is greater than one corresponding

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[21]The higher the steady state inflation rate the more likely recurrent hyperinflations are going to be since

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to the homoclinic bifurcation range, the sudden bursts in the inflation rate tend to be more periodic.

All these new rich time series patterns are a direct result of the nonlinear properties of the stable and unstable manifolds of this dynamic model of a credit constrained economy. These findings then suggest that the potential set of initial conditions that can yield recurrent hyperinflations substantially increases when credit market frictions are important and governments have high inflation rates. Since higher inflation rates exacerbate credit market frictions and can increase the rationing of credit, hyperinflations are more likely to be observed in economies that are more credit rationed.

An important insight from the costly state verification literature is that the ability to contribute to internal financing raises the expected return that a borrower can offer, reducing the credit constraint. In order to see how important are these credit market frictions in explaining recurrent hyperinflations given a certain inflation rate, we study how the associated manifolds of this economy change as the role for internal financing becomes less important. We can then compare the time series patterns between different credit constrained economies. For any given money growth rate we find that economies that are more credit constrained but otherwise identical, are more likely to experience more recurrent hyperinflations. Thus, hyperinflations can occur in economies where there is a relatively low steady state inflation rate but the credit market is highly rationed. This finding then highlights the importance of considering credit market frictions when studying hyperinflation dynamics.

the set of initial conditions generating these sudden bursts are much larger.

5   Conclusions

The goal of this paper is to propose an alternative to bounded rationality when studying recurrent hyperinflations by explicitly considering credit market frictions. In this paper we take Cagan's view that hyperinflations are self-generated and are the manifestations of the underlying global dynamic properties of the economy. In particular, we consider the temporal evolution of an economy with perfect foresight rational agents that face credit rationing as the economy moves away from the steady states.

We find that for relatively low steady state inflation rates the time series patterns of the inflation rate are relatively confined, with the exception of sudden bursts followed by a period of deflation and price stability; converging finally to the steady state inflation rate. As the steady state inflation rate increases, the magnitude of the sudden bursts in the inflation rate tends to be smaller and more frequent. Finally, we show that economies that are more credit rationed, other things being equal, tend to experience more recurrent hyperinflations. This finding highlights the importance of not just considering inflation rates and fiscal imbalances but also examining the credit market frictions when studying recurrent hyperinflations.

References

[1] D.K. Arrowsmith and C.M. Place. *An Introduction to Dynamical Systems*. Cambridge U. Press, 1990.

[2] L. Auernheimer. The effects of inflationary finance on stability: A theoretical analysis. *Southern Economic Journal*, 42:502–507, 1976.

[3] B. Bental and Z. Eckstein. The dynamics of inflation with constant deficit under expected regime change. *Economic Journal*, 100:1245–1260, 1990.

[4] J. Boyd and B. Smith. Capital market imperfections in a monetary growth model. *Economic Theory*, 11:241–273, 1998.

[5] W.A. Brock and C. Hommes. A rational route to randomness. *Econometrica*, 65:1059–1095, 1996.

[6] M. Bruno and S. Fisher. Seigniorage, operating rules and the high inflation trap. *Quarterly Journal of Economics*, 105:353–374, 1990.

[7] Xavier Cabré, Ernest Fontich, and Rafael de la Llave. The parameterization method for invariant manifolds. I. Manifolds associated to non-resonant subspaces. *Indiana Univ. Math. J.*, 52(2):283–328, 2003.

[8] P. Cagan. *The Monetary Dynamics of Hyperinflation. In: Friedman, M. (ed) Studies in the Quantity Theory of Money.* The University of Chicago Press, Chicago, 1956.

[9] R. de Vilder. Complicated endogenous business cycles under gross substitutability. *Journal of Economic Theory*, 71:416–442, 1996.

[10] Z. Eckstein and L. Leiderman. Seigniorage and the welfare cost of inflation. *Journal of Monetary Economics*, 29:389–410, 1992.

[11] J. Evans and G. Yarrow. Some implications of alternative expectation hypotheses in the monetary analysis of hyperinflations. *Oxford Economic Papers*, 33:61–80, 1981.

[12] M. Friedman. *A Program for Monetary Stability.* Fordham University Press. New York, 1960.

[13] P. Gomis-Porqueras and A. Haro. Global dynamics in macroeconomics: An overlapping generations example. *Journal of Economic Dynamics and Control*, 27:1941–1959, 2003.

[14] A. Katok and B. Hasselblat. *Introduction to Modern Theory of Dynamical Systems.* Cambridge U. Press, 1995.

[15] M. Keynes. *Essays in Persuasion.* Harcourt, Brace and Co. New York, 1932.

[16] M. Kiguel. Deficit e inflación. *Revista de Desarrollo Económico*, 26:256–268, 1986.

[17] A. Marcet and J. Nicolini. Recurrent hyperinflations and learning. *American Economic Review*, 93:1476–1498, 2004.

[18] B. Paal. The destabilizing effects of a successful stabilization: A forward looking explanation of the second Hungarian hyperinflation. *Economic Theory*, 15:599–630, 2000.

[19] P. Patrick, D. Sands, and R. de Vilder. On the transition from local regular to global irregular fluctuations. *Journal of Economic Dynamics and Control*, 24:247–272, 2000.

[20] T. Sargent. *The End of the Four Big Inflations. In: Rational Expectations and Inflation.* St. Martin's Press, New York, 1986.

[21] T. Sargent and N. Wallace. *Inflations and the Government Budget Constraint. In: Razin, A. Sadka, E. (eds) Economic Policy in Theory and Practice.* Harper and Row, New York, 1987.

[22] Carles Simó. On the analytical and numerical approximation of invariant manifolds. In Daniel Benest and Claude Froeschlé, editors, *Les Mthodes Modernes de la Mecnique Cleste (Course given at Goutelas, France, 1989)*, pages 285–329, Paris, 1990. Editions Frontières.

[23] S. Williamson. Costly monitoring, financial intermediation, and equilibrium credit rationing. *Journal of Monetary Economics*, 18:159–179, 1986.

Appendix
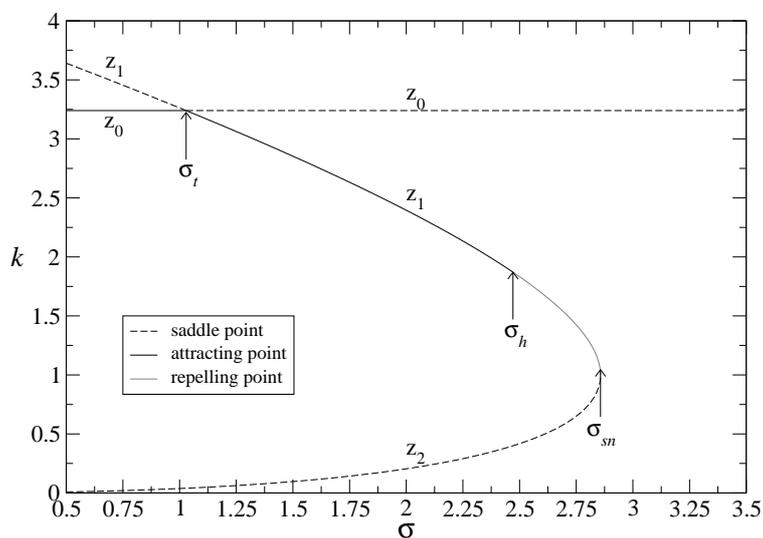
Figure 1: Local bifurcation diagram (stability of steady states).
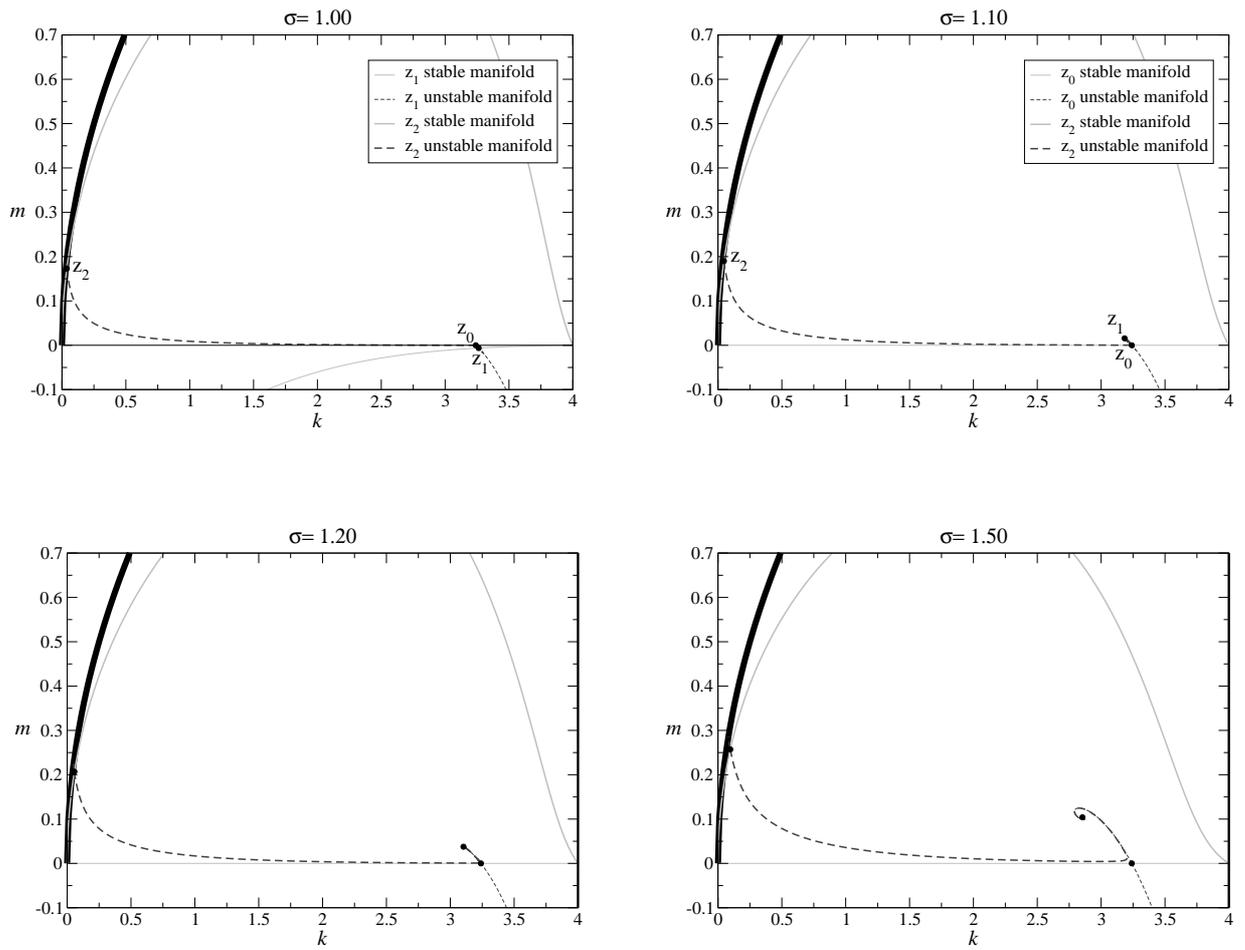
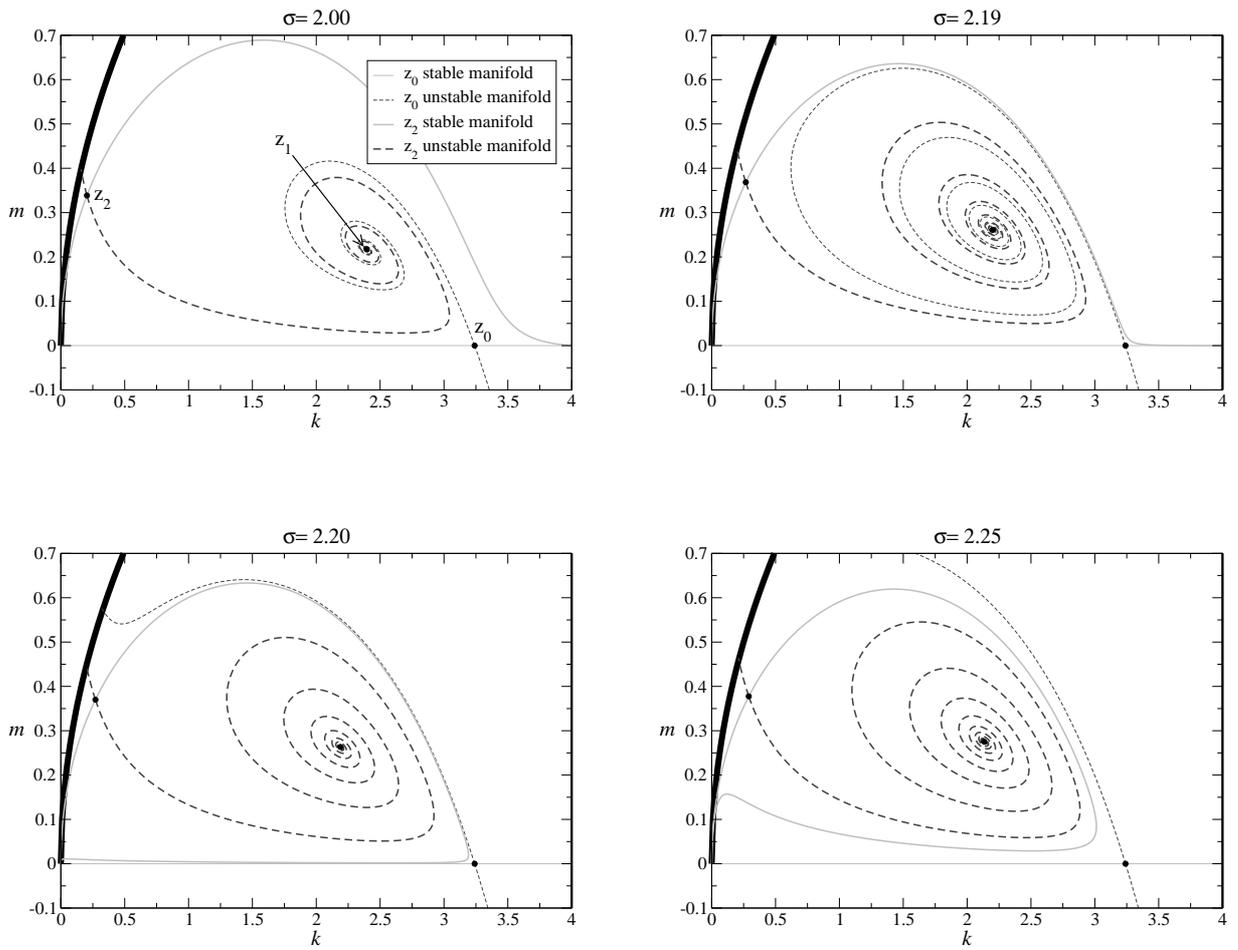Figure 2: Invariant manifolds corresponding to a Transcritical bifurcation.

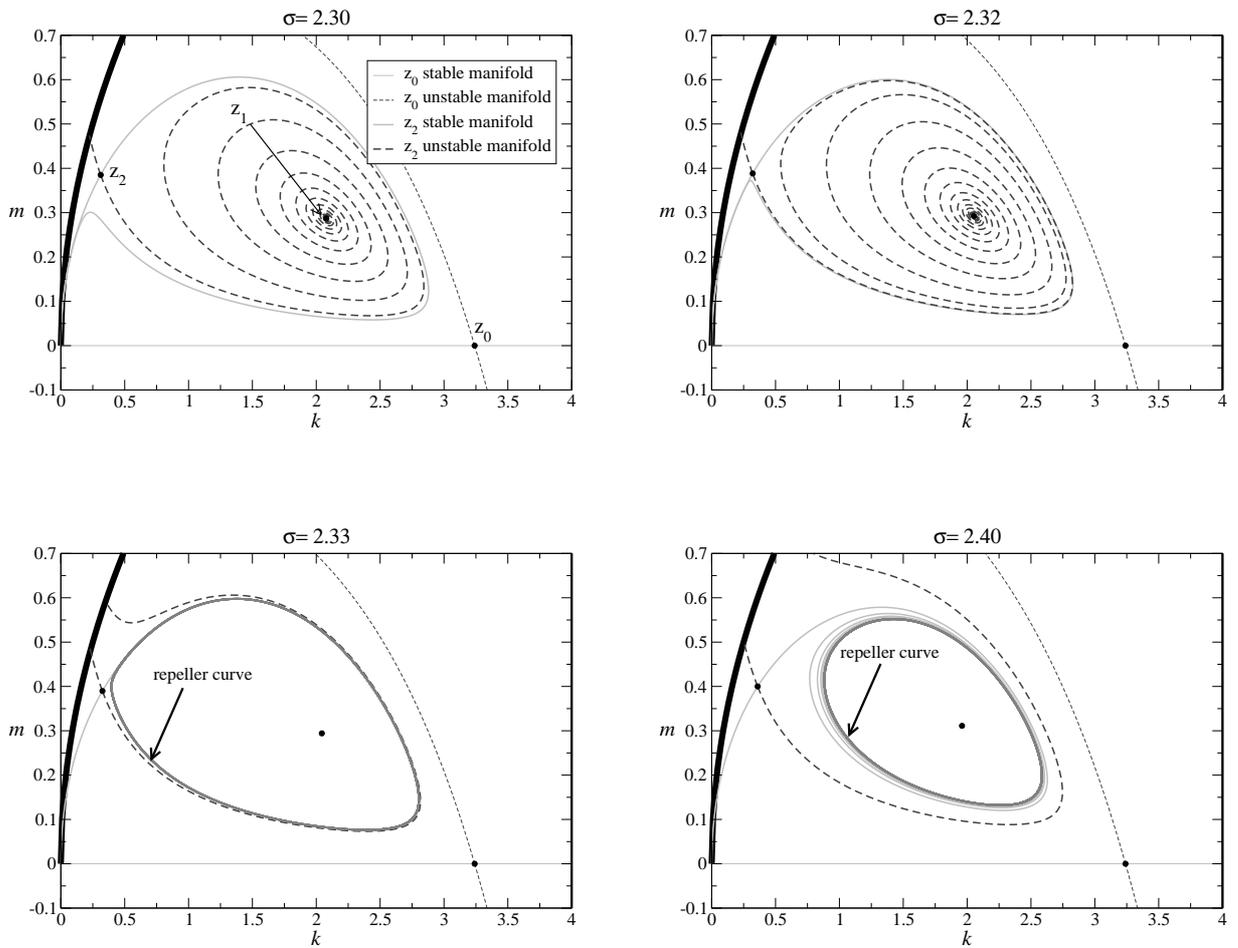Figure 3: Invariant manifolds corresponding to an Heteroclinic bifurcation.

Figure 4: Invariant manifolds corresponding to an Homoclinic bifurcation and a repelling invariant curve.
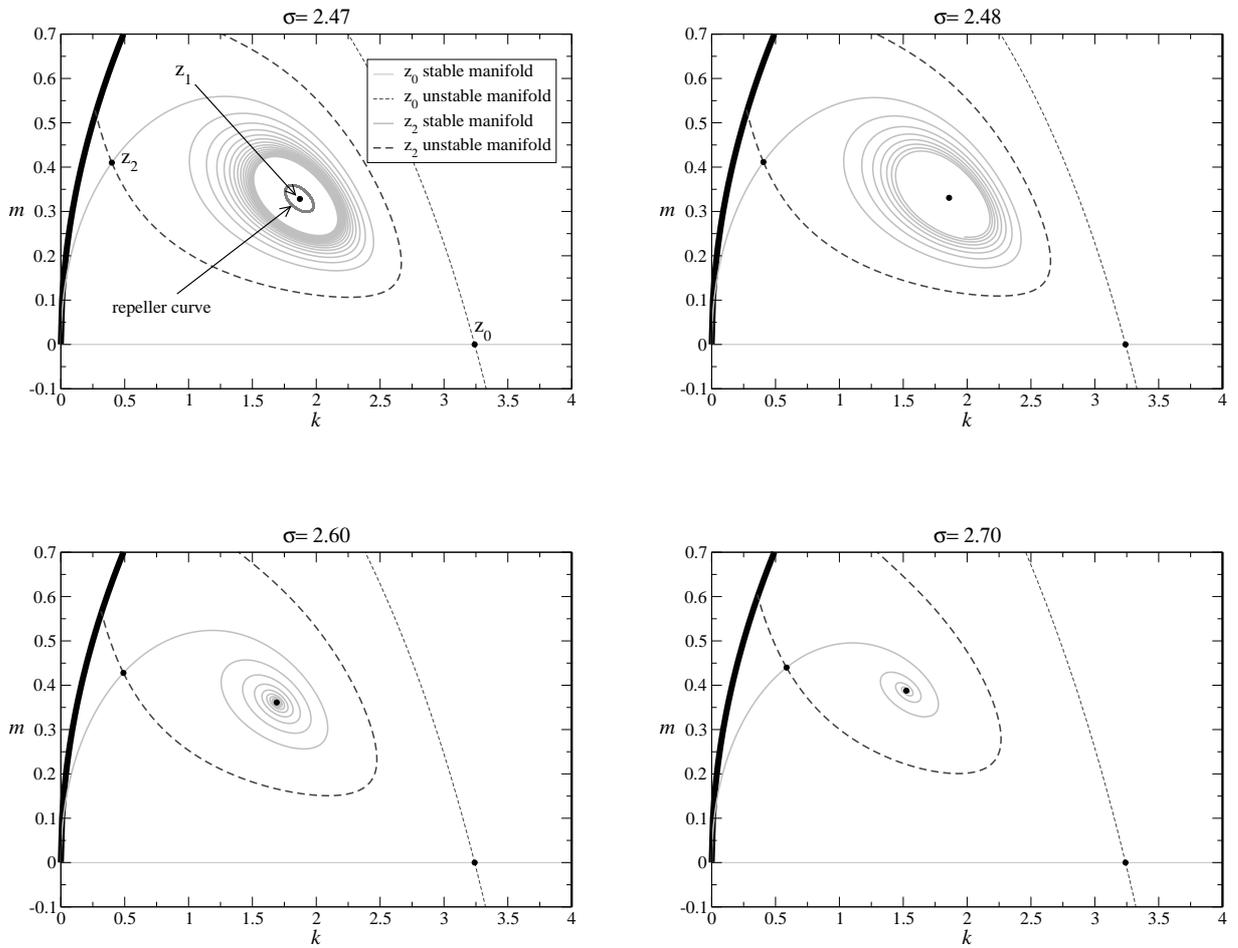
Figure 5: Invariant manifolds corresponding to a Hopf bifurcation and destruction of the repelling invariant curve.
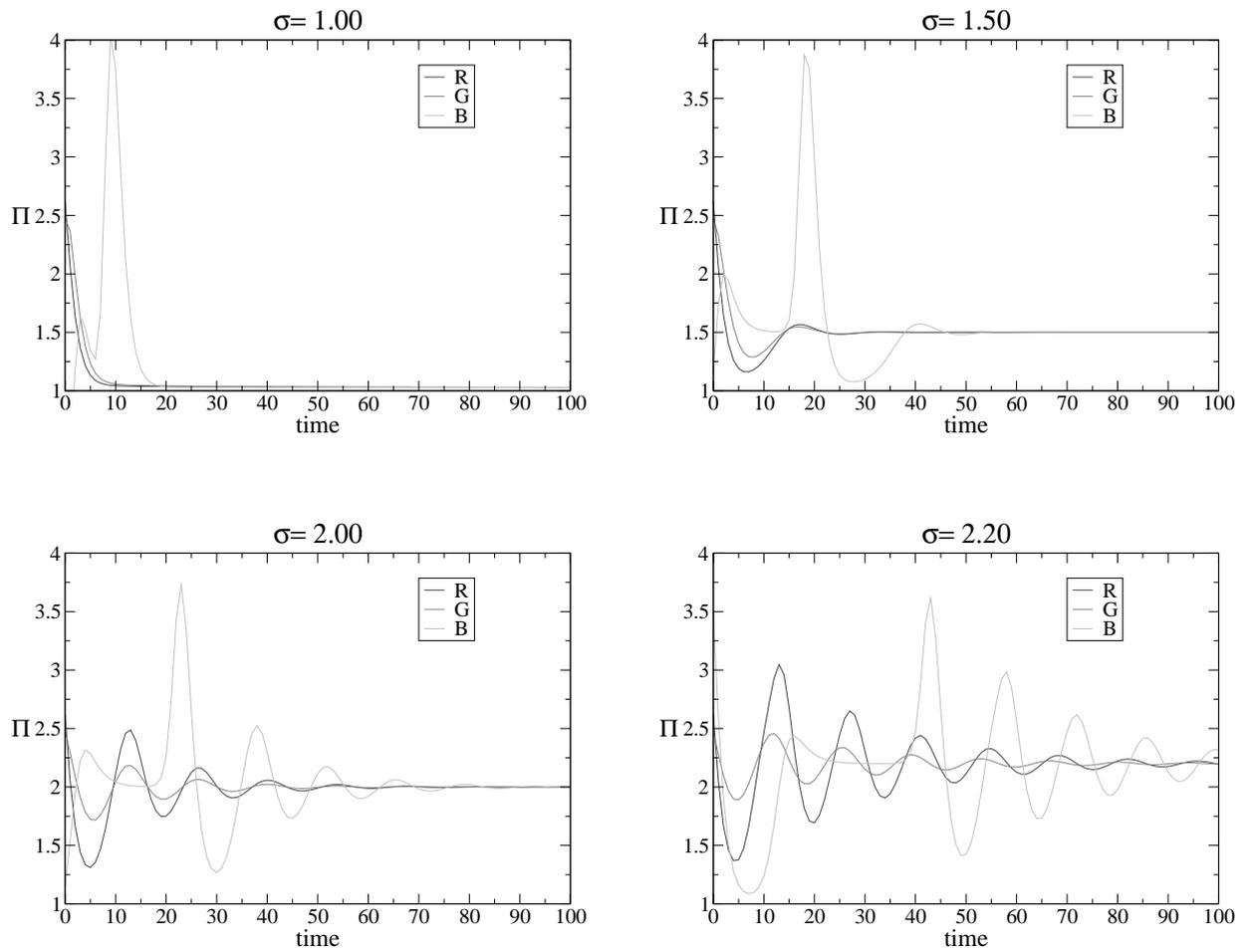
Figure 6: Evolution of the inflation rate for different money growth rates.