Centre de Referència en Economia Analítica

Barcelona Economics Working Paper Series

Working Paper nº 236

Reputation and Rhetoric in Elections

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September 2005

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ABSTRACT

We analyze conditions under which campaign rhetoric may affect the beliefs of the voters over what policy will be implemented by the winning candidate of an election. We develop a model of repeated elections with complete information in which candidates are purely ideological. We analyze an equilibrium in which voters' strategies involve a credible threat to punish candidates who renege of their campaign promises, and all campaign promises are believed by voters, and honored by candidates. We characterize the maximal credible campaign promises and obtain that the degree to which promises are credible in equilibrium is an increasing function of the value of a candidate's reputation.

1. Introduction

Politicians seeking office make promises. This is presumably done in the belief that the promises will alter voters' beliefs about the policies the politician will implement if he is elected, and about the capabilities of the politician. The flip side of the coin is that these promises may later come back to haunt an office holder seeking re-election, so candidates must temper their promises in anticipation of future elections. This paper presents a model in which these effects arise as equilibrium phenomena.

We focus on one aspect of the role of such rhetoric in political campaigns, that we refer to as *credible commitment*, and study it using an infinitely repeated version of the one-dimensional spatial model, where candidates have policy preferences that change over time. With sufficiently patient voters and candidates, there are many equilibria. We characterize the range of credible promises that candidates can commit to. Rhetoric, in the form of promises provides a mechanism for voters to select among multiple equilibria in a repeated game, much like a focal point. In this sense, credible rhetoric solves a coordination problem that arises naturally in the context of multi-principal agency problems, where the many principals must somehow converge on a common rule in order to effectively control the agent. Campaign promises affect voters' expectations about what policies will be chosen by an elected official and they provide a benchmark for voters to link policy decisions with future re-election. In the absence of such public announcement, it is hard to imagine how voters would be able to magically come to a common agreement about what constitutes acceptable performance by an elected official.

The difficulty with the argument that campaign statements are a mere act of promising, or pledging, to carry out a particular policy is that they are cheap talk. That is, fixing all actions of all participants, no payoffs differ when messages alone are changed. Consider, then, a problem in which there is a single election in which candidates vie for office. Suppose candidates are purely ideological, that is, that they receive no direct payoff from holding office, but care only about the policy chosen. In this environment, any candidate who is elected will choose the policy alternative that he most prefers, regardless of any campaign promise that might have been made. Consequently, if voters have rational expectations, no campaign promise can alter voters' beliefs about what action will be taken by a candidate if he is elected. If there were any statement that did alter beliefs in a way that increased the probability of election for a candidate, the candidate would make such a statement regardless of what he intended to do if elected. Hence, no campaign statement can convey information that alters the chance of election.[1]

When we move from the case of a single election to multiple elections, campaign

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[1]See Harrington (1992) for an elaboration of this argument.

promises may be costly because voters can condition their strategies on these promises in the repeated game. Voters may vote differently in future elections if a candidate promises to do something if elected, but subsequently reneges on that promise. Simply put, voters may punish a candidate for reneging on campaign promises by voting him out of office. In this way the promises serve a coordinating role for voters. Under certain conditions, threats of such punishment can support an equilibrium in which campaign promises are kept, and in which voters' beliefs about what a candidate will do if elected are affected by campaign promises. There is a potential problem, however, with voters behaving on the basis of "retrospective" assessments of candidates: at the time of the next election, the future choices that the candidate might make could look far better than those of his opponent. Threats to vote candidates out of office regardless of the circumstances may not be credible, or in other words, strategies employing such threats are dominated. Despite the fact that these strategies are dominated, they are often used to justify the assumption that politicians can commit to platforms or policies prior to an election.

We present and analyze a dynamic model in which candidates make campaign promises, and voters use those promises to form beliefs about the policies the candidate will choose, if elected. We analyze equilibria of the model in which some promises will be kept, even when the promised policy differs from the elected candidate's ideal point, because of fear of voter reprisal. However, unlike the retrospective punishments described above, punishment in our model is prospective. Voters discipline candidates by believing some promises a candidate makes as long as that candidate has never reneged on a promise in the past. Once he reneges, no future promises will be believed. Candidates only make promises they intend to keep, and keep those promises if elected. In other words, we consider only subgame perfect equilibria.

Modelling campaign rhetoric in this way has advantages beyond simply avoiding dominated strategies. The incentive to fulfill campaign promises is based on the threat that future promises will not be believed; the cost to a candidate of this punishment is finite. Consequently, promises to carry out policies that are known to be anathema to the candidate will not be believed, since it will be understood that the gain from reneging will outweigh the cost in lost credibility.[2] Thus, unlike models that simply assume that candidates can commit, we find that there typically will be policies that candidates can commit to (credibly), but other policies that they cannot commit to. In addition, the precise modelling of the source of a candidate's ability to alter voters' beliefs about what he will do if elected, permits an analysis of how the magnitude of his credibility is affected by circumstances such as the probability of being elected, the expected duration of his political career, his opponent, etc.

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[2] Think, for example, of the skepticism that greeted Bob Dole's promise to cut taxes after a long history of arguing against the wisdom of this.

1.1. Related literature

As mentioned above, much of the work on campaigns has followed Downs (1957) in assuming, implicitly or explicitly, that candidates could commit to platforms or policies they would implement if elected. Ferejohn (1986), (and Barro (1973)), consider a repeated principal agent model of sequential elections in which the threat of being thrown out of office reduces the incentives for shirking while in office. Candidates are identical and have no policy preferences, and they are judged by their past performance, rather than any campaign promises or commitments they might make. Austen-Smith and Banks (1989) explore a two period variation of this principal agent model. Candidates propose performance goals during the election, and achievement of these goals depends on a combination of effort and luck. They look at the subset of implicit contracts where voters discipline the incumbent by a quadratic scoring rule that compares actual performance to the incumbent's performance goal. Wittman (1990) analyzes a model with politicians facing an infinite sequence of elections with unchanging ideal points. He characterizes the equilibrium between the candidates when they are restricted to choosing the same policy each period. This differs from our model in two ways: voters play no active role in that model, and candidates never compare the costs and benefits in carrying out the policies, so issues of rhetoric or credibility do not enter the model. Banks and Duggan (2002) analyze a dynamic, multidimensional policy model without rhetoric, and characterize equilibria in terms of simple strategies. In each period, the incumbent faces a random opponent; they show existence of an equilibrium in which an individual votes for the incumbent if his utility meets a critical threshold, which is determined endogenously. There is no consideration of rhetoric or prospective evaluations of candidates. Duggan and Fey (2002) investigate properties of the set of equilibria with infinitely repeated elections and complete information, with office-motivated candidates and without rhetoric. In their model there is no issue of candidate credibility or retrospective voting, since candidates are purely office motivated and therefore are indifferent over which policy they actually implement if elected.

This earlier work either ignored the effect of a politician's performance in office on the chances of reelection, or considered only office-motivated candidates. Most of the work that embodies retrospective assessment leaves out any possibility of campaign rhetoric. Our contribution is to model political campaigns by ideological candidates who make campaign promises, with voters who are fully rational in the degree to which the promises can be believed.

The outline of the paper is as follows. In the next section, we focus on the case in which there are (potentially) infinite elections and complete information. In this case we show how candidates may (rationally) choose to maintain a reputation for fulfilling campaign promises. We do this initially for the case in which candidates have linear utility functions. We next analyze several extensions, including the effect that concavity

in utility has on the set of believable promises. We end with a brief discussion of our results.

2. Sequential elections

With an infinite horizon, promises can be credible in equilibrium as long as reputation has a value. Of course, promises can always be broken - and will be broken - if it is in the interest of the candidate to do so. Promises are kept only because it is in the interest of the candidate to do so, since the future payoffs are different for the candidate when he keeps his promise than when he does not. Promises may change voters' beliefs about the choices that candidates will make if elected because voters understand that it is sometimes in a candidate's selfish interest to fulfill his promises, even when there is a short-run gain from reneging. Voters also understand that the threat of future punishment is not sufficient to deter all reneging: some promises may be so far from a candidate's preferred outcome that the short-run gain from reneging is sufficiently high that a candidate will relinquish his electoral future. In short, the ability of a candidate to alter voters' beliefs is not a "technological" given, but rather, is an equilibrium phenomenon.

We assume complete information: voters know candidates' preferences over policies perfectly at the time they vote.[3] We assume that at each election candidates' reputation may be either good or bad: candidates with good reputations are candidates who have never reneged in the past and candidates with bad reputations are those who have reneged on a promise sometime in the past.[4] Voters believe only promises of candidates who have a good reputation and never believe any promise of candidates who have a bad reputation. After each election, a winning candidate with a good reputation compares the one time benefit of reneging on any promise he may have made with the value of maintaining his reputation by fulfilling the promise. Candidates with a bad reputation choose their optimal policy independent of their promises. Voters predict that candidates with a bad reputation will implement their ideal policy regardless of any promises, and that candidates with a good reputation will fulfill any promise that is not too costly to carry out, that is, for which the benefit of reneging is less than the decrease in their continuation payoffs if they renege. These strategies comprise a subgame perfect equilibrium. If there is no uncertainty, candidates do not make promises they do not intend to keep since with complete information, voters can predict they will renege and the promise will not influence their voting.[5]

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[3]We will discuss later a variant of the model in which candidates preferences are not known with certainty at the time of the election.

[4]Reputations need not have this "all-or-nothing" property; we discuss below richer possibilities of how past behavior can affect reputation.

[5]Uncertainty (symmetric between voters and candidates) about what alternatives will arise between

Candidates will be able to change voters' beliefs about the policy they will undertake as long as the discount factor is large enough. That is, as long as the future has sufficient value, candidates will carry out their promises when it is not too costly to do so. If there is a positive (expected) value to being elected in each of the future periods, the value to retaining a good reputation goes to infinity as the discount factor goes to one. For high enough value to retaining a good reputation, all promises will be kept (hence, believed by voters).

In these models there will always be one equilibrium in which campaign rhetoric is irrelevant: all candidates make random promises, and for all messages they hear, voters do not alter their beliefs about a candidate's type or the choices he will make if elected. Candidates choose their most preferred policy if elected. Here, the only information relevant to voters is the candidate's choice: their predictions of choice in the second period are independent of any campaign promises, and hence reneging on campaign promises cannot affect voting in the second election.

What is interesting, however, is that in addition to this uninformative equilibrium, there may be equilibria in which voters *do* change their beliefs about candidates and their voting behavior on the basis of campaign promises.

Rhetoric matters if and only if candidates' payoffs if they renege on their campaign promises are different from the payoffs they obtain if they fulfill their promises. That is, we obtain different election outcomes following a failure to fulfill a promise than after a promise has been fulfilled. For the outcome of future elections to differ following fulfillment or nonfulfillment of promises, voters' strategies must depend on the relationship between a campaign promise and the policy choice of a candidate: voters' actions must depend on rhetoric.

In general, candidates will *not* be able to induce all possible beliefs in voters. We consider this a very important feature of our approach. In our model, it is *endogenous* which promises will be made, believed, and fulfilled when both candidates and voters are fully rational. Each candidate will have available to him a subset of the possible beliefs voters might have about his policy choices if elected. It is important to note that the sets of beliefs that candidates can induce in voters are typically quite different, since they depend on voters' initial beliefs about the candidates, including their discount factors, δ, utility functions, etc.[6]

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the time of voting and the time at which the alternatives to the promise action are known would change this. In that case one would expect that in equilibrium some promises will not be kept when the benefits of reneging outweigh the value of reputation.

[6]This construction provides a rational explanation for the exogenous cost of commitment assumed in Banks (1990), for example.

2.1. The model

There are two candidates, L and R, who compete in all elections. At each election, the structure of the game is as follows:

Campaign stage: both candidates simultaneously make an announcement. Each candidate has to decide between making a promise about the policy he will implement in case he wins the election or sending a message devoid of promises.

Voting stage: each voter votes for the candidate who maximizes their expected utility, which depends on the policy that he or she believes will be implemented after the election.

Office stage: the winner of the election implements a policy.

Candidates and voters derive utility only from the policy implemented. We assume that the utility an agent obtains from each election is represented by

$$u_i(x) = -|x - x_i|.$$

where x_i represents the ideal point of agent i.

The policy space is represented by the interval $[-1, 1]$. We assume that the ideal point of the median voter is the same at all elections, and normalized to be $x_m = 0$.

Elections take place over time. Voters simply vote in each election for the candidate whose predicted policy choice is most preferred.[7] Candidates discount future payoffs with a discount factor $\delta \in [0, 1)$. The discount factor represents the weight that future payoffs have on candidates' total utility. We have in mind an interpretation of δ that combines both time preference and the probability that a candidate will run for office in the future. For example, we can think of it as $\delta = \lambda\beta$, where $\lambda \in [0, 1]$ represents the probability that the candidate will run for office in any period, and $\beta \in [0, 1)$ represents time preference. Since the value of δ is less than one, elections that are further away in the future have less effect on the total utility of the candidate than earlier elections.

We assume that the policy preferences of the two candidates change at each election. In particular, we assume that at each election the ideal point of candidate L is $x_L \in [-1, 0]$, given by an independent random draw from a uniform probability distribution over $[-1, 0]$. Similarly at each election the ideal point of candidate R is $x_R \in [0, 1]$, given by an independent random draw from a uniform probability distribution over $[0, 1]$. Candidates' ideal points are drawn independently of each other and of past draws before each election.

Candidates know the preferences of the median voter, and at the beginning of each electoral period, voters and candidates learn the ideal points of both candidates for that period.

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[7]We rule out the possibility that voters will "punish" candidates when it is not in their interest to do for the same reasons that attention is restricted in games to subgame perfect equilibria.

A candidate's strategy selects for each one period game a pair (p, x) where $x \in [-1, 1]$ represents the policy the candidate implements in case he wins the election, and $p \in [-1, 1] \cup \{\emptyset\}$ represents the announcement that the candidate makes at the campaign stage (either a promised policy or nothing). Formally, we may define a promise by the exact policy that will be implemented, in which case, if a candidate promises policy $x \in [0, 1]$, he will break his promise only if he implements $x' \neq x$. We may also think of a promise as the worst policy that will be implemented according to the median voter's preferences, that is if a candidate promises policy $x \in [0, 1]$, he will only break his promise if he implements $x' \in (x, 1]$. In our model these definitions are equivalent.

Before deciding their vote, voters may update their beliefs about the candidates' policy choices in case they win the election, given the announcements made at the campaign stage. Given their beliefs, voters decide to vote for the candidate that maximizes their expected utility.

Since voters know the candidates' ideal points, we assume that in the absence of promises, voters believe that candidates will choose their ideal point if elected. After the campaign stage voters may update their beliefs about the policy choices the candidates would make if elected. Voters decide rationally whether to believe the campaign promises or not. Voters will only believe a promise if honoring it is compatible with the candidate's incentives after the election. Thus, even though campaign promises do not affect the payoffs of any of the agents, they may affect their decisions.

2.2. Credible commitment with rhetoric

We describe an equilibrium of this repeated game in which campaign promises matter, in the sense that different promises imply different strategy choices, and therefore lead to different payoffs. In this equilibrium, voters will believe the maximal set of incentive compatible promises, that is, promises that the candidate would have an incentive to fulfill should he be elected. For a candidate with discount factor δ, we will show that there is a number $d(\delta)$ such that voters will believe promises made by the candidate if and only if the distance between the candidate's ideal point and his promise is not greater than $d(\delta)$. In the equilibrium we describe, voters will believe all promises from a candidate for which the distance from the candidate's ideal point is not greater than $d(\delta)$ if the candidate has never reneged on a promise and will believe no promise if he has ever reneged (that is, implemented a policy other than a promised policy).[8] If the candidate makes a promise that is not incentive compatible or if he makes no promise voters believe that he will implement his ideal point.

These strategies essentially treat candidates as one of two types. At each election we

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[8]There are other equilibria that can be thought of as intermediate cases in which voters believe some, but not all, promises that are incentive compatible. The equilibria in these cases will look like the equilibrium we describe, with a smaller d, that is, voters believe fewer promises.

may have candidates with a good reputation, who have never reneged on any promises and whose (incentive compatible) promises will be believed by voters, and candidates with a bad reputation, who have reneged on a promise at some time in the past, and independently of what promises they make at the campaign stage, voters will believe that if they win the election they will implement their ideal point.

After the election the winner implements the policy that maximizes his expected payoffs, taking into account that the voters' strategies for future elections might depend on the candidate's promises and choice. Thus at this stage, candidates will compare the gains and costs of reneging. The gains from reneging are represented by the instantaneous increase in their utility produced by deviating from their promised policy, choosing instead their ideal point. The costs of reneging are reflected in their expected payoffs from future elections: the difference between the future expected payoffs for a candidate with a good reputation and a candidate with a bad reputation. A candidate will only renege on a promise if the instantaneous gain is larger than his future expected loss.

In the equilibrium we describe, candidates will only make incentive compatible promises and they will fulfill the promises they make. Therefore, voters will believe the promises that are made and the winner will be the candidate who is able to promise a policy closer to the median voter's ideal point. The winning candidate must promise a policy that is at least as attractive to the median voter as his opponent's policy. If the losing candidate promises a policy that is consistent with incentive compatibility and as close as possible to the median voter's ideal point, the winning candidate will have to promise a policy that is at least as close to the median voter's ideal point.[9] Since we assume that the candidates' ideal points are on opposite sides of the median voter's ideal point, when the winner makes promises closer to the median voter's ideal point, the losing candidate's utility increases. The candidates' strategies in the equilibrium we describe have the losing candidate promising the policy closest to the median voter's ideal point that is consistent with incentive compatibility, and the winning candidate making a promise that is equally close.

Formally, the strategies for the equilibrium described are:

Candidates' strategies:

(i) If neither candidate has ever reneged on a promise, the candidate whose ideal point is further from the median voter's ideal point promises the policy that is closest to the median voter's ideal point consistent with incentive compatibility. The candidate whose ideal point is closer to the median voter's ideal point promises a policy that is

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[9]If candidate A is promising the policy that is as close as possible to the median voter's ideal point and it is consistent with incentive compatibility, and candidate B has an incentive compatible promise that is closer, candidate B will win the election. However, the set of incentive compatible promises that are strictly preferred by the median voter is open. We assume that candidate B is the winning candidate in this case.

equally attractive to the median voter. If elected, both candidates fulfill their promise.

(ii) If both candidates have reneged on a promise in the past, both candidates promise to implement the median voter's ideal point. If elected, they implement their own ideal point.

(iii) If one candidate has reneged on a promise but the other candidate has never reneged, the candidate who has reneged promises to implement the median voter's ideal point. If elected, he implements his own ideal point. The candidate who has not reneged promises a policy that is as attractive to the median voter as the opponent's ideal point, if such a promise is incentive compatible. If that policy is not incentive compatible, he promises his ideal point. If elected, he fulfills his promise.

Voters' strategies:

Each voter casts his or her vote for the candidate whose expected policy, if elected, maximizes the voter's utility. Voters' beliefs are as follows.

(i) Voters believe that incentive compatible promises of candidates who have never reneged on a promise will be fulfilled.

(ii) Voters believe that a candidate who makes a promise that is not incentive compatible will implement his ideal point.

(iii) Voters believe that a candidate who has reneged on a promise in the past, will implement his ideal point.

Proposition 1: *The strategies described above constitute an equilibrium. The promises believed and fulfilled in equilibrium with linear utility functions are those within a distance $d^D(\delta)$ of the candidates' ideal points, where*

$$d^D(\delta) = \begin{cases} 0 & if\ 0 \leq \delta \leq \frac{1}{2} \\ \frac{3}{2}\left(1 - \sqrt{\frac{4-5\delta}{3\delta}}\right) & if\ \frac{1}{2} \leq \delta \leq \frac{3}{4} \\ 1 & if\ \frac{3}{4} \leq \delta \leq 1 \end{cases}$$

The proof is in the appendix.

The distance $d^D(\delta)$ characterizes an equilibrium with the maximal range of incentive compatible promises. We obtain that, in the equilibrium we have analyzed, candidates who have never reneged on a promise fulfill all the promises they make, and voters believe these promises: both candidates maintain a good reputation over time. There is a continuum of equilibria with similar characteristics: for all $d \leq d^D(\delta)$, there is an equilibrium in which voters believe promises up to a distance d away from the candidate's ideal point.

Our analysis yields some simple but interesting comparative statics. Notice that the maximal promise believed in equilibrium is an increasing function of the discount factor, since $\frac{\partial d^D(\delta)}{\partial \delta} = \frac{1}{\delta^2}\sqrt{\frac{3\delta}{4-5\delta}} \geq 0$. Thus, as the discount factor increases, the value of reputation (the cost of reneging) increases, and it implies that larger promises will be kept and believed in equilibrium.

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In general, we should expect to see that candidates with high probability running for office in the future are more likely to fulfill their promises and voters are more likely to believe promises from these candidates. Thus promises are more likely to be believed at the same time that candidates are less likely to make them.

Similarly, all else equal, younger candidates are more likely to fulfill their promises, since they have a longer time horizon to consider, and thus their reputation is more valuable. However, there may be things like seniority effects that cause younger candidates to have smaller chances of being elected in the future. This would work in the opposite direction.

Note that the expected value of maintaining a good reputation for a candidate is the same independently of whether his opponent has a good or a bad reputation, that is

$$v_{GG}\left(d^{S}\left(\delta\right)\right) - v_{BG}\left(d^{S}\left(\delta\right)\right) = v_{GB}\left(d^{S}\left(\delta\right)\right) - v_{BB}\left(d^{S}\left(\delta\right)\right).$$

That the value of a good reputation is independent of the opponent's reputation is due to the assumed linearity of the utility functions.

We also analyze the effects of maintaining a good reputation on the welfare of the median voter. The median voter's expected utility from each election as a function of the credible promises in equilibrium is given by:

$$u_{GG}\left(d\right) = -\frac{1}{3} + d^{2}\left(1 - \frac{2}{3}d\right) > -\frac{1}{3}$$

$$u_{BB}\left(d\right) = u_{GB}\left(d\right) = u_{BG}\left(d\right) = -\frac{1}{3}$$

With $\frac{\partial u_{GG}(d)}{\partial d} > 0$ for $0 \leq d \leq 1$. Thus, the median voter is better off when both candidates have a good reputation because all promises are made toward the median voter's ideal point. In equilibrium, both candidates have a good reputation and the utility of the median voter increases with the size of the set of credible promises.

The probability that a voter is better off when candidates can make credible promises than when no promises are credible decreases with the absolute value of the ideal point of the voter. In particular this implies that the voter most favored by the credibility of promises is the median voter ($x_m = 0$). Voters with ideal points at the extremes of the policy space obtain the same expected utility when both candidates have a good reputation as when both candidates have a bad reputation. The reason is that for each realization of the candidates' ideal points such that a voter's utility decreases when some promises are credible, there is another realization (symmetric) of the candidates' ideal points such that the voters' utility increases by the same amount when promises are credible. Thus, voters' utility can only increase with the size of the set of credible promises.

2.3. Extension to concave utility functions

Up to now we have assumed that the utility function of the candidates was linear with respect to the distance between their ideal point and the implemented policy. In this section we will assume that this function is concave. Formally we assume that for all i

$$U_i\left(x\right) = -\left|x_i - x\right|^k$$

where $k \geq 1$ measures the degree of concavity, that is, the larger the value of k the larger the degree of concavity. A candidate with a strictly concave utility function, $k > 1$, suffers more than candidate with a linear utility function ($k = 1$) from the implementation of policies that are far away from his ideal point. In a sense, the degree of concavity of the utility function is a measure of the intensity of the candidate's political preferences.

We should expect that the value of maintaining a good reputation for a candidate is larger the larger the degree of concavity of his preferences, since his utility loss from losing an election increases with the degree of concavity, while his utility when he wins (even with a promise different from his ideal point) is affected less. In this section we replicate the above analysis of the equilibrium with rhetoric when candidates' utility functions are concave. We assume that both candidates' utility exhibit the same degree of concavity. We find that the set of credible campaign promises is larger the higher the degree of concavity of the candidates' utility functions.

Proposition 2: *The strategies described in section 3.2 constitute an equilibrium. The promises believed and fulfilled in equilibrium with concave utility functions are those within a distance $\widetilde{d}^D\left(\delta, k\right)$ of the candidates' ideal points, where*

$$\widetilde{d}^D\left(\delta, k\right) = \begin{cases} 0 & if & \delta = 0 \\ 0 < \widetilde{d}^D\left(\delta, k\right) < 1 & if & 0 < \delta < \dfrac{1}{1 + \frac{2^k\left[3 - \left(1 - d^S\right)^{k+2}\right] - k - 3}{(k+1)(k+2)}} \\ 1 & if & \dfrac{1}{1 + \frac{2^k\left[3 - \left(1 - d^S\right)^{k+2}\right] - k - 3}{(k+1)(k+2)}} \leq \delta \end{cases}$$

and

$$\frac{\partial\left(\widetilde{d}^D\left(\delta, k, \widetilde{d}^S\right)\right)}{\partial k} \geq 0.$$

The proof is in the appendix.

When we assumed that the candidates' utility functions were linear, we saw that their expected utilities were unaffected by the kind of reputation that they had as long as both candidates had the same kind of reputation. When both candidates have a good reputation, it is equally likely that a given candidate will be helped or hurt by his reputation. When a candidate's ideal point is closer to the median voter, he will win

whether both candidates have a good or a bad reputation. When both candidates have a good reputation, in equilibrium he will make a promise, and hence be worse off than if both have had a bad reputation, in which case he could have won by promising his ideal point. On the other hand, if his opponent's ideal point is closer to the median voter, this candidate benefits from having a good reputation. With linear utility functions, these exactly offset, and the candidates' expected utility when both candidates have a good reputation is the same as when neither does.

With concave utility functions, this is no longer the case. When both candidates have a good reputation, the equilibrium policies enacted will be closer to the median voter than they would be if both candidates had a bad reputation. This convergence toward the median voter is beneficial to candidates, however, with strictly concave utility functions. When a candidate is forced to move his policy choice toward the median voter's ideal point because both candidates have a good reputation, the loss is not as large as the gain he gets from his opponent's doing the same thing. Hence, with concave utility functions, candidates' expected utility is larger when both candidates have a good reputation than when both candidates have a bad reputation, and the greater the degree of concavity, the greater the difference between the two.

Candidates' welfare increases in our model because of the policy convergence that a good reputation generates. The effect is similar to the welfare increase that results from policy convergence in Alesina (1988) and Dixit, Grossman and Gul (2000). In those papers, policy convergence arises through tacit cooperation between two parties that moderate their policies when in office. Although the welfare benefits in these papers, as in our paper, are due to policy convergence, the policy convergence that we obtain when we assume linear utility functions stems from the interactions between the voters and the candidates, rather than between the candidates themselves.

2.4. Extension to random median voters

In the model analyzed in the previous sections of this paper we assume that the ideal points of the candidates change from election to election and that the ideal point of the median voter does not change over time. These assumptions can be interpreted as if voters had stable preferences but the issues changed from election to election. For instance, in one period the main campaign issue, and therefore the candidates' promises, are on tax reform, the next election the issue is abortion, etc. At each election the ideal point of the median voters is normalized to be zero, and the candidates' ideal points are different reflecting the different relative positions of all agents for each specific issue. In one sense, this can be thought of as a model of short-term policies.

In this section we describe an alternative model in which the policies can be thought of as long-term policies. Here we assume that candidates' ideal points are fixed at all elections, and that the ideal point of the median voter changes across elections. This

variation of the model can be interpreted as the candidates having long run, stable ideal points over some policy, say income distribution. The assumption that the ideal point of the median voter is random captures the idea that the median voter may change over time due to demographic changes or that individual voters' preferences may change due to changes in the economy.

Consider the following variant of the model described previously, where the ideal points of the candidates are $x_L = 0$ and $x_R = 1$ at each election, and the ideal point of the median voter m at each election is an independent realization of a uniform random variable on the interval $[0, 1]$. Notice that here the ideal points of the candidates are not independent, in contrast to what was assumed in the previous sections.

Proposition 3: *The strategies described in section 3.2 constitute an equilibrium. The promises believed and fulfilled in equilibrium with a random median voter are those within a distance $d^D(\delta)$ of the candidates' ideal points, where*

$$d^D(\delta) = \begin{cases} 0 & if & \delta \leq \frac{2}{3} \\ 2\frac{3\delta-2}{\delta} & if & \frac{2}{3} \leq \delta \leq \frac{4}{5} \\ 1 & if & \frac{4}{5} \leq \delta \end{cases}$$

The proof is in the appendix.

In this case, we also obtain that the maximal promise depends on the discount factor in a very natural way: when the discount factor is very small, no promises are believed in equilibrium; for larger values of the discount factor more promises are believed in equilibrium, and when the discount factor is sufficiently large, all promises are believed.

Thus, the results obtained with this alternative formalization of the two candidate electoral competition are qualitative the same as the results we found when we assumed that the candidates' ideal points were randomly determined at each election and the median voter's ideal point was fixed at all elections.

The welfare effects in this case are similar to those in the previous section. As in that case, the median voter is strictly better off when candidates have reputations. When the candidates have linear utility functions, they are equally well off when both or neither have reputations; with strictly concave utility functions, they will be better off when both have reputations than when neither does.

2.5. Extension to multidimensional policy space

The logic and intuition of the results above carries over to the multidimensional policy case. We discuss here the two dimensional case where voters and candidates have Euclidean preferences, where each voter or candidate has an ideal point with utility a decreasing function of Euclidean distance from their ideal point. The voters' ideal points are distributed such that there is a global median. For concreteness, suppose there is a continuum of voters, with ideal points uniformly distributed on a rectangle, so the global median is simply the individual whose ideal point is the center of the rectangle.

For any two policies chosen by the candidates, each voter will prefer the policy that is closer to his or her ideal point. The policy that is preferred by the median voter will be preferred by a majority of the voters (that is, by more than one half of the voters).

In each period, the candidates ideal points are drawn from the uniform distribution on the rectangle; the candidates' ideal points are independent of each other and independent across elections. As before, suppose that each candidate can make promises and those promises will be believed if they are incentive compatible and if the candidate has never reneged on a promise in the past, and will not be believed if he has ever reneged. It is straightforward to see that for a candidate who has never reneged, there is a maximum distance from his ideal point that he can promise that is incentive compatible, with that distance determined by the net present value of being able to make promises in future periods.

The Nash equilibrium of the repeated game of promises between candidates follows the same logic as in one dimension. As in one dimension, generally one of the candidates is closer to the median and the other is further away. Call the latter candidate the loser. There can't be an equilibrium with the loser making a less than maximal promise when they both have good reputations, for exactly the same reason as in one dimension. The equilibrium is illustrated in figure 9. On the equilibrium path, the loser adopts the closest possible credible policy to the median voter, along the line segment between the loser's ideal point, B, and the median voter's ideal point, V. The policy is labeled B'. The other candidate "matches" by adopting the (unique) policy on the line segment between that candidate's ideal point, A, and V. The policy is labeled A'. Although indifferent, the voter votes for A', using the same justification as in the one dimensional case.[10]

The two dimensional case has an interesting feature that is absent in one dimension. If there were a different timing structure in the game, the losing candidate might gain by making a promise that induces his opponent to counter-promise something other than A' that the loser prefers to A'. With simultaneous moves, such manipulation is not possible, but it would be if B could irrevocably make a promise before A.

Figure 10 illustrates a case in which the optimal "manipulative" promise for the losing candidate is *not* the maximal incentive compatible promise that he could make. Suppose B' is the largest incentive compatible promise that B can make, inducing A to make promise A'. But inducing A to make promise A' is *not* the best that B can do. As A makes promises closer to V to win the election, the outcome moves from point A (if A could win without making any promise) toward V's ideal point. The move along this line initially increases B's utility, as movements along the line AV initially lead to outcomes that are closer to B. However, once A makes promises greater than M, the

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[10]That is, A could make an arbitrarily small move toward V in his promise, which B could not credibly match. This would make the voter strictly prefer candidate A to candidate B. Rather than try to model this formally, we simply assume the "better" candidate wins in case of a tie.

move along the line AV leads to outcomes that are further from B, thus B is worse off. If B were to scale down his promise to B'' (prior to A making his promise), A's response would be to scale back *his* promise to M, which is the point on the line AV that gives B the highest utility.

From this, it is easy to see that with this kind of manipulation, there may be no promises at all.[11] If the lines AB and AV form an obtuse angle, any movement along the line AV reduces B's utility, hence the loser would be better off making no promise, rather than making the equilibrium promise. The analog of this in the one dimensional case is when both candidates are on the same side of the median voter. In the absence of any promises, the candidate whose ideal point is closer to the median will win the election. The only promises that the candidate whose ideal point is further from the median voter's ideal point can make that have any effect are promises of outcomes that are closer to the median voter's ideal point than his rival's ideal point, but movements in this direction are to the candidate's detriment.

The arguments above carry over generally to any number of dimensions, provided a global median exists. If a global median fails to exist, then there is still always an equilibrium in which no promises are made (and none believed), and the winning candidate simply implements his ideal point. This suggests that equilibria with small but positive promises will exist when δ is positive.

2.5.1. Asymmetry of issues

The discussion above treats the case in which all parties have circular indifference curves. Suppose the issues are not symmetric, that is, that the voters' utility functions are given by $U(x,y) = -a^2(x-\bar{x})^2 - b^2(y-\bar{y})^2$. One can make a change of variables with $\tilde{x} = \frac{x}{a}$ and $\tilde{y} = \frac{y}{b}$, so that $U(\tilde{x},\tilde{y}) = -(\tilde{x}-\bar{x})^2 - (\tilde{y}-\bar{y})^2$. If it is assumed that the voters' ideal points in the space of changed variables is uniform, the analysis above carries over more or less intact. As before voters will choose the candidate whose (incentive compatible) promise is closer to their ideal point. There is one difference, however. If the candidates do not have the same utility function as the voters (i.e., if the a and b parameters in their utility function are not the same as in the voters' utility functions), their indifference curves after the change of variables will not be circles, but rather ellipses. The promises that will be made by a candidate will still be on the locus of tangencies of the indifference curves of the candidate with the indifference curve of the voter, but this locus will no longer be a straight line.

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[11]This kind of manipulation is in the spirit of the Stackelberg equilibrium. Of course, the *Nash* equilibrium of the simultaneous move game will always have the loser making a maximal promise, as shown before.

3. Discussion

There are several features of this model that deserve further discussion.

Interpretation of discounting and time preferences: As the value of the discount factor decreases, the value of future payoffs also decreases, and therefore reputation becomes less valuable, and fewer promises will be credible in equilibrium. Hence, reputation is most valuable to candidates who have a higher probability of running for reelection and who have a higher probability of winning should they run. Since reputation is more valuable to such candidates, their promises are consequently more credible.

A particularly interesting consequence of this is that, all else equal, two candidate systems have an advantage over multi-candidate systems. In the latter, the average candidate clearly has lower chance of being elected in future elections, and hence has lower value for maintaining a reputation. This lower value of reputation makes fewer promises credible, with the result that there will be less mediating effect of credible promises and, hence, implemented policies with more candidates.[12]

The effect of candidate ideology on credibility: How does intensity of candidates' ideology affect the credibility of the candidates? Our results above assumed that the candidates' ideal points were uniformly distributed on the unit interval. Imagine instead environments in which there is more polarization between the candidates as captured by distributions of ideal points that put greater weight on points further from the median voter. The parameter d measured the magnitude of candidates' credibility in section 2.2 above; we are interested in whether this parameter would increase or decrease when there is greater polarization as described above.

Suppose we symmetrically change the distributions of the candidates' ideal points, putting greater weight on points further from the median voter and less on points nearer. As before, it will still be the case that a candidate is more likely to win an election when his reputation is intact than when he has lost his reputation. The candidate whose reputation is intact benefits from this. Sometimes that benefit will come about when the candidates ideal points are relatively close to the median voter's, and sometimes when they are farther away from the median voter. The magnitude of the benefit of the reputation will be greater when the ideal points are further away, simply because the distance between the ideal points is larger in this case. But then the effect of an increase in ideological intensity is to put greater probability on those cases where the benefit is larger, hence the value of having a reputation is greater with the increase.

The increased value of having a reputation when there is greater ideological intensity translates into an increase in the potential credibility. Not all promises are typically believed by voters; what they will (can) believe is limited by what the candidate has to lose by reneging after being elected. Anything that increases the value of maintaining

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[12]We thank Abhijit Banerjee for this observation.

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one's reputation increases the loss to the candidate should he renege, and consequently, increases the magnitude of the promises that he will have an incentive to keep.

Uncertainty: Suppose that between the voting stage and the office stage the policy preferences of the winner suffer a shock that changes the candidate's ideal point with some positive probability. In the case analyzed in the previous section, all promises made by a candidate during the campaign were fulfilled in equilibrium. Adding uncertainty about the candidates' preferences alters this: we will then have that some promises that are believed in equilibrium will not be fulfilled. Furthermore, larger probability of shocks on candidates' preferences should also imply a lower future expected value from maintaining a good reputation (since with positive probability it will be lost in any case), thus a lower value of reputation (lower cost of reneging), and therefore in equilibrium we will obtain a smaller d : fewer promises will be credible.

Alternative Punishment strategies: We have assumed that voters' punishment of candidates who renege is extreme: after a candidate reneges once voters keep the punishment of not believing any of his promises for all future elections. There are other equilibria in which voters' punishment is less extreme. We could think that after a candidate reneges once, voters apply the same punishment to the candidate for a finite number of periods, and believe his incentive compatible promises afterwards. Since the future expected payoffs if he reneges will be higher in equilibrium we will obtain a lower value for maintaining reputation, and therefor a smaller d, that is, fewer promises will be credible.

Forward Looking Voters: We have assumed voters are myopic, and do not take into account the future. If voters, like candidates, discount the future, then the equilibria we have characterized continue to be equilibria. Voters will still in each period choose the candidate who offers the preferred platform, since there is no link between what platform is promised, and/or enacted, except to the extent that promises are broken. Some additional equilibria are possible if the voters not only discounted the future but were more strategic than in our model. In particular, they can induce candidates to make even better promises. In our model, the winning (more moderate) candidate only has to offer the median voter a platform with the same utility as that offered by the losing (more extreme) candidate. A strategic forward-looking voter might adopt a strategy that requires the more moderate candidate to make an even better (credible) promise, or suffer a bad reputation, with the losing (more extreme) candidate still always making the maximum promise, as in our analysis. In equilibrium, this would reduce the value to a candidate of maintaining a reputation, and therefore would decrease the maximal promise that can be sustained, so the voters would face a trade-off. On the one hand, they would receive the maximal promise from the more moderate candidate every period, whereas in our model, they get strictly less than that each period since the moderate candidate only matches the extreme candidate's promise. On the other

hand, the maximal promise is less, so when the two candidates have ideal points that are nearly the same distance from the median voter, the median voter is worse off. There are many subgame perfect equilibria of this sort, and in principle one could compute the best equilibrium from the standpoint of the (strategic) median voter.

Standard preferences: Voters and candidates in our model have standard preferences, that is, they care about the policies that will be chosen. One might argue that there are honest politicians and dishonest politicians, and that voters have a preference for honest politicians. It is certainly plausible that politicians differ in the degree to which prefer to keep their word, and that voters care about this. Candidates' behavior in a model that incorporated these ingredients would be similar to their behavior in our model: they would hesitate to make promises that they did not intend to keep, and voters would be less likely to vote for candidates who have reneged on promises in the past.

A disadvantage of such a model is the introduction of additional arguments in candidates' and voters' utility functions. It is relatively easy to explain a particular phenomenon by adding parameters to a model. Formal modelling has been successful because a single parsimonious model is able to account for a wide range of phenomena. Even if one thought that candidates do have preferences for keeping their word and voters do have a taste for honest politicians, it is valuable to know the extent to which the campaign promises that politicians make and fulfill can be understood within the standard model without adding these.

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5. Appendix

Proof of Proposition 1

In order to find equilibrium strategies for the two candidates we will consider three different cases: when both candidates have a bad reputation, when only one of the candidates has a good reputation, and when both have a good reputation.

Suppose that both candidates have a bad reputation. In this case, given that voters do not believe any promises (other than the candidates' ideal points) the cost of reneging is zero since no promises will be believed in any case, therefore at the 'office stage' all candidates will always implement their ideal points. Similarly, given that the only promise that is incentive compatible for the candidates is their own ideal point, it is optimal for the voters not to believe any other promise. Thus, we have that at each election the winner will be the candidate whose ideal point is closer to the ideal point of the median voter (zero) and the policy implemented after the election will be his ideal point. In this case, the expected payoff (prior to the realization of the candidates' ideal points) for each candidate at each election is given by (see figure 1):

$$v_{BB} = \int_0^1 \int_{-1}^{-x_R} u_L\left(x_R\right) dx_L dx_R + \int_0^1 \int_{-x_R}^0 u_L\left(x_L\right) dx_L dx_R = -\frac{1}{2}.$$

Now suppose that candidate R has a bad reputation, which means that voters will believe that he will implement his ideal point, and candidate L has a good reputation, that is, voters believe all promises he makes that are consistent with the incentive compatibility constraints.

We start by assuming that voters believe all promises made by candidate L that are less than a distance d from his ideal point. Then, solving for the equilibrium strategies, we will find the maximal d that is consistent with incentive compatibility.

If $-x_L < x_R$, candidate L wins by promising his ideal point. In this case, he does not need to make any promises, and obtains the maximal possible utility.

If $-x_L > x_R$, candidate L loses if he does not make any promise or if he cannot credibly promise a policy that is closer to the ideal point of the median voter than x_R. In this case candidate R wins the election and implements x_R. Otherwise, candidate L may credibly promise a policy $-x_R$ that, for the median voter is at least as good as x_R. Making a promise that allows him to win the election is a better strategy for L than allowing R to win, since he gets a higher utility even if he decides to fulfill his promise:

$$u_L\left(-x_R\right) = x_L + x_R > u_L\left(x_R\right) = x_L - x_R.$$

Thus, in equilibrium candidate L promises policy $-x_R$. Voters will believe him only if he has a good reputation, and if implementing $-x_R$ is incentive compatible for candidate L, that is, if the gain he obtains from fulfilling his promise in terms of future

expected payoffs is larger than the cost of reneging. In this case candidate L wins the election[13].

The cost of reneging is the difference between his future expected payoff if he maintains a good reputation, and his future expected payoff if he loses his reputation, given that candidate R does not have a good reputation.

Let $v_{GB}(d)$ denote the one-election expected utility for a candidate that has a good reputation when his opponent has a bad reputation. Similarly let $v_{BB}(d)$ denote the one-election expected utility for each candidate when both have a bad reputation. Thus, given the assumptions of our model they yield to (see figure 2):

$v_{GB}(d) = \int_0^{1-d} \int_{-1}^{-x_R-d} u_L(x_R) dx_L dx_R + \int_0^{1-d} \int_{-x_R-d}^{-x_R} u_L(-x_R) dx_L dx_R +$

$\int_{1-d}^1 \int_{-1}^{-x_R} u_L(-x_R) dx_L dx_R + \int_0^1 \int_{-x_R}^0 u_L(x_L) dx_L dx_R = -\frac{1}{6} - \frac{(1-d)^3}{3}$

$v_{BB}(d) = \int_0^1 \int_{-1}^{-x_R} u_L(x_R) dx_L dx_R + \int_0^1 \int_{-x_R}^0 u_L(x_L) dx_L dx_R = -\frac{1}{2}$

Given the one-election expected payoffs, we can compute the expected future payoffs for a candidate with a good reputation, given that his opponent has a bad reputation:

$$V_{GB}(d;\delta) = \sum_{t=1}^{\infty} \delta^t v_{GB}(d).$$

Similarly the future expected payoffs for a candidate with a bad reputation given that his opponent also has a bad reputation are:

$$V_{BB}(d;\delta) = \sum_{t=1}^{\infty} \delta^t v_{BB}(d).$$

Thus we obtain the cost of reneging as a function of the maximal promise believed by voters and the discount factor. Let $C^S(d;\delta)$ denote the cost of reneging. Then we have that

$$C^S(d;\delta) = V_{GB}(d;\delta) - V_{BB}(d;\delta) = \frac{\delta}{1-\delta} \frac{1}{3} \left(1 - (1-d)^3\right).$$

The gain from reneging: the maximal gain a candidate may obtain from reneging of a promise is d, that is the maximal difference in utility between implementing the policy he promised and implementing his ideal point. Therefore, it is an optimal strategy for candidate L to fulfill all promises that are at most at a distance d from his ideal point, where d satisfies $d \leq C^S(d;\delta)$

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[13]Observe that when candidate L promises $-x_R$ the median voter is indifferent between the two candidates. We assume that when a voter is indifferent between the two candidates he votes for the unconstrained candidate.

It is also an optimal strategy for the voters to believe all promises that are at most at a distance d from the candidate's ideal point, with d such that $d \leq C^S(d; \delta)$, since in equilibrium they will be fulfilled.

We denote by d^S the value of d that solves

$$d = C^S(d; \delta).$$

d^S is the maximal promise that a candidate will always fulfill, and it is also the maximal promise that voters will believe.

Since $\frac{\partial C^S(d)}{\partial d} = \frac{\delta}{1-\delta}(1-d)^2 \geq 0$ and $\frac{\partial C^S(0)}{\partial d} = \frac{\delta}{1-\delta}$ we have that in equilibrium (see figure 3):

   i) for $\delta \leq \frac{1}{2}$ we must have $d^S = 0$, no promises are believed

   ii) for $\frac{1}{2} < \delta < \frac{3}{4}$ we must have $0 < d^S < 1$, some promises may be believed

   iii) for $\frac{3}{4} \leq \delta \leq 1$ we must have $d^S = 1$, all promises may be believed.

Thus the promises that in equilibrium may be believed and fulfilled are:

$$d^S(\delta) = \begin{cases} 0 & if\ 0 \leq \delta \leq \frac{1}{2} \\ \frac{3}{2}\left(1 - \sqrt{\frac{4-5\delta}{3\delta}}\right) & if\ \frac{1}{2} \leq \delta \leq \frac{3}{4} \\ 1 & if\ \frac{3}{4} \leq \delta \leq 1 \end{cases}$$

Notice that since $\frac{\partial^2 C^S(d)}{\partial d^2} = -\frac{2\delta}{1-\delta}(1-d) \leq 0$ we have that the cost of reneging is a concave function. This is intuitively plausible since a candidate only benefits from an increase of the set of credible promises, that is, an increase in $d^S(\delta)$, when his ideal point is more than a distance $d^S(\delta)$ from the median voter's ideal point, and the probability of this event is lower the larger the value of $d^S(\delta)$.

Now consider the case in which both candidates have a good reputation. Let $v_{GG}(d)$ denote the one election expected utility for a candidate that has a good reputation when both candidates have a good reputation. Similarly let $v_{BG}(d)$ denote the one election expected utility for a candidate who has a bad reputation when his opponent has a good reputation. As before we start by assuming that voters believe all promises that are at most a distance d away from the ideal point of the candidate. We then look for a function $d^D(\delta)$ that characterizes the maximal promise that candidates will fulfill and voters will believe if both candidates have a good reputation. When both candidates have a good reputation, that is, both candidates can make credible promises, the maximal promise that is incentive compatible could be different than the one we found in the case in which only one candidate can make credible promises. Given the assumptions of our model, we have (see figure 4):

$$v_{GG}(d) = \int_0^{1-d} \int_{-1}^{-x_R-d} u_L(x_R)\, dx_L dx_R + \int_d^1 \int_{-x_R+d}^0 u_L(x_L)\, dx_L dx_R +$$

$$\int_0^d \int_{-d}^0 u_L(0)\, dx_L dx_R + \int_d^1 \int_{-x_R}^{-x_R+d} u_L(-x_R+d)\, dx_L dx_R +$$
$$\int_{-1}^{-d} \int_{-x_L-d}^{-x_L} u_L(-x_L-d)\, dx_R dx_L = -\tfrac{1}{2}$$

$$v_{BG}(d) = \int_0^1 \int_{-1}^{-x_R} u_L(x_R)\, dx_L dx_R + \int_{d^S}^1 \int_{-x_R}^{-x_R+d^S} u_L(-x_L)\, dx_L dx_R +$$
$$\int_0^{d^S} \int_{-x_R}^0 u_L(-x_L)\, dx_L dx_R + \int_{d^S}^1 \int_{-x_R+d^S}^0 u_L(x_L)\, dx_L dx_R = -\tfrac{5}{6} + \frac{\left(1-d^S\right)^3}{3}$$

In this case the future expected payoff for a candidate who has a good reputation when the other candidate also has a good reputation is:

$$V_{GG}(d;\delta) = \frac{\delta}{1-\delta} v_{GG}(d) = -\frac{1}{2}\frac{\delta}{1-\delta}.$$

Observe that when both candidates have a good reputation, their payoffs are independent of the size of the set of credible promises. This is due to the linearity of the candidates' utility functions: in expectation the increase in utility that a candidate receives because his opponent can make promises compensates for the lose in utility he obtains from fulfilling his promises. Similarly, the future expected payoff for a candidate who has a bad reputation when his opponent has a good reputation is

$$V_{BG}(d;\delta) = \frac{\delta}{1-\delta} v_{BG}\left(d^S\right) = \frac{\delta}{1-\delta}\left(-\frac{5}{6} + \frac{\left(1-d^S\right)^3}{3}\right).$$

Observe that the expected future payoff for a candidate with a bad reputation when his opponent has a good reputation is a function of the maximal promise that voters believe when only one candidate can make promises, that is the value $d^S(\delta)$ that we found for the previous case, while the expected future payoffs for a candidate with a good reputation when his opponent also has a good reputation is independent of d. Thus when both candidates have a good reputation the cost of reneging for a candidate is given by

$$C^D(d;\delta) = V_{GG}(d;\delta) - V_{BG}(d;\delta) = \frac{\delta}{1-\delta}\frac{1}{3}\left(1 - \left(1-d^S\right)^3\right).$$

Comparing this cost with the results found for the case in which only one candidate has a good reputation we conclude that (see figure 5):

$$C^D(d;\delta) = C^S\left(d^S;\delta\right) = d^S(\delta).$$

That is, the cost of reputation when both candidates have a good reputation equals the value of maintaining a good reputation for a candidate when his opponent has a bad reputation, therefore it is equal to the maximal promise that voters believe when only one candidate has a good reputation. This implies that we must have $d^D(\delta) = d^S(\delta)$, that is, if both candidates have a good reputation, the maximal promises that are going

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to be fulfilled by candidates and believed by voters in equilibrium are the same as in the case in which only one candidate has a good reputation. ♦

Proof of Proposition 2

We first consider the case in which both candidates have a bad reputation. As before, since no promises are ever believed by voters, the cost of reneging is zero and therefore at the office stage all candidates always implement their ideal point. At each election the winner will be the candidate whose ideal point is closer to the median voter's ideal point. The expected payoff (prior to the realization of the candidates' ideal points) for each candidate at each election is given by:

$$\widetilde{v}_{BB}(k) = \int_0^1 \int_{-1}^{-x_R} -(x_R - x_L)^k \, dx_L dx_R = \frac{1 - 2^{k+1}}{(k+1)(k+2)}$$

Observe that the expected payoff in this case is strictly decreasing with the degree of concavity of the candidates' utility function:

$$\frac{\partial \widetilde{v}_{BB}(k)}{\partial k} = \frac{2^{k+1}[2k+3-(k+1)(k+2)\ln 2]-(2k+3)}{(k+1)^2(k+2)^2} < 0$$

Now suppose that candidate L has a good reputation and candidate R has a bad reputation. As before, we first assume that voters believe all promises made by candidate L that are less than a distance d from his ideal point, and we then determine the maximal d that is consistent with incentive compatibility.

The gain from reneging: the maximal gain that a candidate may obtain from reneging on a promise is d^k, that is, the maximal difference in utility between implementing the promised policy and implementing his ideal point.

The cost of reneging is the difference between his future expected payoff if he maintains a good reputation, and his future expected payoff if he loses his reputation, given that candidate R has a bad reputation. In this case we have that the one-election expected utility for candidate L in this case is:

$$\widetilde{v}_{GB}(d;k) = \int_0^{1-d} \int_{-1}^{-x_R-d} -(x_R-x_L)^k \, dx_L dx_R + \int_0^{1-d} \int_{-x_R-d}^{-x_R} -(-x_R-x_L)^k \, dx_L dx_R$$
$$+ \int_{1-d}^1 \int_{-1}^{-x_R} -(-x_R-x_L)^k \, dx_L dx_R = \frac{-\frac{1}{2}(2-d)^{k+2}-\frac{3}{2}d^{k+2}+1}{(k+1)(k+2)} - \frac{d^{k+1}(1-d)}{k+1}$$

As before, given the one-election expected payoffs, we can compute the expected future payoffs for a candidate with a good reputation given his opponent reputation, and then compute the cost of reneging as the difference between them:

$$\widetilde{C}^S(d;\delta,k) = \widetilde{V}_{GB}(d;\delta,k) - \widetilde{V}_{BB}(d;\delta,k) = \sum_{t=1}^{\infty} \delta^t \left[\widetilde{v}_{GB}(d;k) - \widetilde{v}_{BB}(k)\right].$$

When both candidates' utility functions are concave we have that the cost of reneging is given by the following expression:

$$
\begin{aligned}
\widetilde{C}^S (d; \delta, k) &= \frac{\delta}{1 - \delta} \left(\widetilde{v}_{GB} (d; k) - \widetilde{v}_{BB} (k) \right) \\
&= \frac{\delta}{1 - \delta} \left[\frac{\frac{1}{2} \left[2^{k+2} - (2 - d)^{k+2} - 3d^{k+2} \right]}{(k + 1)(k + 2)} - \frac{d^{k+1}(1 - d)}{k + 1} \right]
\end{aligned}
$$

Therefore, it is optimal for candidate L to fulfill all promises that are at most a distance d from his ideal point, where d satisfies:

$$
d^k \leq \widetilde{C}^S (d; \delta, k) .
$$

It is also optimal for the voters to believe all promises that are at most a distance d that satisfy the previous inequality, since in equilibrium they will be fulfilled.

Observe that the cost of reneging is increasing with the amount of promises believed by voters[14]:

$$
\frac{\partial \widetilde{C}^S (d; \delta, k)}{\partial d} = \frac{\delta}{1 - \delta} \left[\frac{\frac{1}{2} \left[(2 - d)^{k+1} - d^{k+1} \right]}{(k + 1)} - d^k (1 - d) \right] \geq 0.
$$

The cost of reneging is also a concave function of the amount of promises believed by voters:

$$
\frac{\partial^2 \widetilde{C}^S (d; \delta, k)}{\partial d^2} = \frac{\delta}{1 - \delta} \left[-\frac{1}{2} (2 - d)^k + \frac{1}{2} d^k - k d^{k-1} (1 - d) \right] \leq 0.
$$

On the other hand, the gains from reneging, d^k, are an increasing and convex function of the amount of promises believed by voters.

Since $\widetilde{C}^S (0; \delta, k) = 0$ and

$$
\widetilde{C}^S (1; \delta, k) = \frac{\delta}{1 - \delta} \frac{2^{k+1} - 2}{(k + 1)(k + 2)} \leq 1 \ \ iff \ \ \delta \leq \frac{1}{1 + \frac{2^{k+1} - 2}{(k+1)(k+2)}} .
$$

This implies that the cost of reneging and the gains from reneging intersect at most at one single point when $d \in [0, 1]$. Thus, there is a value of d for which $d^k = C^S (d; \delta, k)$,

---

[14] Since $\frac{\partial \widetilde{C}^S (d; \delta, 1)}{\partial d} = \frac{\delta}{1-\delta} (1 - d)^2 \geq 0$ and $\frac{\partial \left(\frac{\partial \widetilde{C}^S (d; \delta, k)}{\partial d} \right)}{\partial k} = \frac{\delta}{1-\delta} \left[\frac{\frac{1}{2}(2-d)^{k+1}[(k+1)\ln(2-d)-1] + \frac{1}{2}d^{k+1}[1-(k+1)\ln d]}{(k+1)^2} - d^k (1 - d) \ln d \right] > 0$

which determines the maximal promise believed by voters. Let $\widetilde{d}^S$ denote this value. As before we have that (see figure 6):

$$
\begin{array}{ccc}
\widetilde{d}^S\left(\delta,k\right)=0 & if & \delta=0 \\
0<\widetilde{d}^S\left(\delta,k\right)<1 & if & 0<\delta<\frac{1}{1+\frac{2^{k+1}-2}{(k+1)(k+2)}} \\
\widetilde{d}^S\left(\delta,k\right)=1 & if & \delta\geq\frac{1}{1+\frac{2^{k+1}-2}{(k+1)(k+2)}}
\end{array}
$$

Observe that when candidates' utility functions are strictly concave, there are always some promises different from the candidates' ideal points that are believable by voters, as long as the discount factor is greater than zero. And as in the linear case, when the discount factor increases, the set of believable promises also increases, since $\frac{\partial\widetilde{C}^S(d;\delta,k)}{\partial\delta}=\frac{\widetilde{C}^S(d;\delta,k)}{1-\delta}\geq0$. Finally, if the discount factor is sufficiently large, all promises are incentive compatible.

We can also show that the maximal promise believed by voters increases with the degree of concavity of the candidates' utility function, that is,

$$
\frac{\partial\widetilde{d}^S\left(\delta,k\right)}{\partial k}\geq0
$$

since the cost of reneging for each value of d increases with the degree of concavity we have that[15]

$$
\frac{\partial\widetilde{C}^S\left(d;\delta,k\right)}{\partial k}\geq0
$$

and, on the other hand, the gain from reneging decreases with the degree of concavity

$$
\frac{\partial\left(d^k\right)}{\partial k}=d^k\ln d\leq0.
$$

Now consider the case in which both candidates have a good reputation. We first compute the one-election expected payoffs for a candidate that has a good reputation and then a bad reputation, given that the opponent has a good reputation. We assume that voters believe promises from either candidate that are at most distance d from the candidate's ideal point, and we look for a function $\widetilde{d}^D\left(\delta,k\right)$ that characterizes the maximal promise that candidates will fulfill (and, hence, voters will believe) given that both

---

[15]This is true since:
1) $\frac{\partial\widetilde{C}^S(0;\delta,k)}{\partial d}$ increases with k
2) $\widetilde{C}^S\left(1;\delta,k\right)$ increases with k
3) $\frac{\partial\widetilde{C}^S(d;\delta,k)}{\partial d}$ increases with k

candidates have a good reputation. The one-election expected payoff for a candidate with a good reputation when his opponent has also a good reputation is:

$$\widetilde{v}_{GG}\left(d;k\right) = \int_0^{1-d}\int_{-1}^{-x_R-d} -\left(x_R-x_L\right)^k dx_L dx_R + \int_0^d\int_{-d}^0 -\left(-x_L\right)^k dx_L dx_R$$

$$+ \int_d^1\int_{-x_R}^{-x_R+d} -\left(-x_R-x_L+d\right)^k dx_L dx_R + \int_{-1}^{-d}\int_{-x_L-d}^{-x_L} -\left(-2x_L-d\right)^k dx_R dx_L$$

$$= \frac{-\frac{1}{2}\left[(2-d)^{k+2}+d^{k+2}\right]+1}{(k+1)(k+2)} + \left(\frac{d}{2}-1\right)\frac{d^{k+1}}{k+1} - \frac{d}{2}\frac{(2-d)^{k+1}}{k+1}$$

When computing the expected utility for a candidate with a bad reputation when his opponent has a good reputation, we need to take into account that the set of promises that voters believe in this case is given by the function $\widetilde{d}^S\left(\delta,k\right)$ found above:

$$\widetilde{v}_{BG}\left(\widetilde{d}^S\left(\delta,k\right);k\right) = \int_0^1\int_{-1}^{-x_R} -\left(x_R-x_L\right)^k dx_L dx_R + \int_{\widetilde{d}^S}^1\int_{-x_R}^{-x_R+\widetilde{d}^S} -\left(-2x_L\right)^k dx_L dx_R$$

$$+ \int_0^{\widetilde{d}^S}\int_{-x_R}^0 -\left(-2x_L\right)^k dx_L dx_R = \frac{\frac{1}{4}2^{k+2}\left[\left(1-\widetilde{d}^S\right)^{k+2}-3\right]+1}{(k+1)(k+2)}$$

As before, given the one-election expected utilities we find the value of the future expected payoffs, and the cost of reneging as

$$\widetilde{C}^D\left(d;\delta,k\right) = \widetilde{V}_{GG}\left(d;\delta,k\right) - \widetilde{V}_{BG}\left(d;\delta,k\right) = \frac{\delta}{1-\delta}\left[\widetilde{v}_{GG}\left(d;k\right) - \widetilde{v}_{BG}\left(\widetilde{d}^S\left(\delta,k\right);k\right)\right]$$

Using the previous expressions we obtain the cost of reneging as a function of the size of the set of credible promises when the two candidates have a good reputation, for each maximal amount of credible promises when only one candidate has a good reputation:

$$\widetilde{C}^D\left(d;\delta,k,\widetilde{d}^S\right) = \frac{\delta}{1-\delta}\left[\frac{-\frac{1}{2}\left[(2-d)^{k+2}+d^{k+2}\right]-\frac{1}{4}2^{k+2}\left[\left(1-\widetilde{d}^S\right)^{k+2}-3\right]}{(k+1)(k+2)} + \left(\frac{d}{2}-1\right)\frac{d^{k+1}}{k+1} - \frac{d}{2}\frac{(2-d)^{k+1}}{k+1}\right]$$

First notice that for all $\widetilde{d}^S\left(\delta,k\right) > 0$ if voters believe no promises other than the candidates' ideal points (when both candidates have a good reputation), the cost of reneging is still positive (and recall that $\widetilde{d}^S = 0$ only when $\delta = 0$):

$$\widetilde{C}^D\left(0;\delta,k,\widetilde{d}^S\right) = \frac{\delta}{1-\delta}\left[\frac{2^k\left[1-\left(1-\widetilde{d}^S\right)^{k+2}\right]}{(k+1)(k+2)}\right] > 0$$

This implies that the cost of losing a good reputation for one of the candidates, when both have a good reputation might be positive, even if no promises are being believed by voters. This can happen if some promises are believed by voters only when a single candidate has a good reputation. The reason for this anomaly is that if a candidate

were to lose his reputation they would revert to the state in which only one candidate has a good reputation, that is a state in which the amount of credible promises is given by $\widetilde{d}^S > 0$. In that state the candidate with a bad reputation is worse off than when both have good reputations, even if no promises are believed in that case.

Furthermore, we have that the cost of reneging in this case is increasing with the size of the set of believable promises[16]:

$$\frac{\partial \widetilde{C}^D(d;\delta,k)}{\partial d} = \frac{\delta}{1-\delta}\left[\frac{d}{2}(2-d)^k - \frac{1}{2}(2-d)d^k\right] \geq 0$$

We can also show that for low values of d, $\widetilde{C}^D(d;\delta,k)$ is a convex function of d, and as d increases $\widetilde{C}^D(d;\delta,k)$ becomes a concave function:

$$\frac{\partial^2\left(\widetilde{C}^D(d;\delta,k)\right)}{\partial d^2} = \frac{\delta}{1-\delta}\left[1 - \frac{d}{2}(k+1)\right]\left[(2-d)^{k-1} - d^{k-1}\right]$$

And $\frac{\partial^2\left(\widetilde{C}^D(d;\delta,k)\right)}{\partial d^2} \leq 0$ if and only if $d \geq \frac{2}{k+1}$.

For a given value of k the maximal credible promise, denoted by $\widetilde{d}^D(\delta,k)$ is given by the largest value of d that satisfies (see figure 7):

$$\widetilde{C}^D(d;\delta,k) \geq d^k.$$

In this case we also have that the size of the set of credible promises increases with the value of the discount factor, if $\widetilde{d}^D(\delta,k) > 0$:

$$
\begin{array}{ccc}
\widetilde{d}^D(\delta,k) = 0 & if & \delta = 0 \\
0 < \widetilde{d}^D(\delta,k) < 1 & if & 0 < \delta < \dfrac{1}{1 + \frac{2^k\left[3 - \left(1 - d^S\right)^{k+2}\right] - k - 3}{(k+1)(k+2)}} \\
\widetilde{d}^D(\delta,k) = 1 & if & \delta \geq \dfrac{1}{1 + \frac{2^k\left[3 - \left(1 - d^S\right)^{k+2}\right] - k - 3}{(k+1)(k+2)}}
\end{array}
$$

Finally, we have that the cost of reneging for all d is an increasing function of k, that is,[17]

---

[16]Since $\frac{\partial\left(\widetilde{C}^D(d;\delta,1)\right)}{\partial d} = 0$ and

$\frac{\partial\left(\frac{\partial\left(\widetilde{C}^D(d;\delta,k)\right)}{\partial d}\right)}{\partial k} = \frac{\delta}{1-\delta}\left[\frac{d}{2}(2-d)^k\ln(2-d) - \frac{1}{2}(2-d)d^k\ln d\right] \geq 0$

[17]This is true since:
1) $\widetilde{C}^D(0;\delta,k)$ increases with k
2) $\widetilde{C}^D(1;\delta,k)$ increases with k
3) $\frac{\partial\widetilde{C}^D(d;\delta,k)}{\partial d}$ increases strictly with k for all $d \in (0,1)$.
Then we must have that if $k < k'$ then for all $d < d'$
$\widetilde{C}^D(d;\delta,k') - \widetilde{C}^D(d;\delta,k) < \widetilde{C}^D(d';\delta,k') - \widetilde{C}^D(d';\delta,k)$

$$\frac{\partial C^D (d; \delta, k)}{\partial k} \geq 0.$$

and

$$\frac{\partial \left(\widetilde{d}^D \left(\delta, k, \widetilde{d}^S \right) \right)}{\partial k} \geq 0.$$

Since we have already shown that the gain from reneging for all d decreases with k, we obtain that the value of the maximal credible promise increases as k gets larger. ◆

Proof of Proposition 3

Consider first the case in which one the candidates has a good reputation (L) while the other candidate has a bad reputation (R).

In this case, we have that the expected payoff from one election for candidate L are:

$$v_{BB} (d) = \frac{1}{2} u (0) + \frac{1}{2} u (1) = -\frac{1}{2}$$

$$v_{GB} (d) = \frac{1}{2} u (0) + \int_{\frac{1}{2}}^{\frac{1+d}{2}} u (2m - 1) \, dm + \frac{1-d}{2} u (1) = -\frac{1}{2} + \frac{d}{2} \left(1 - \frac{d}{2} \right)$$

Thus the cost of reneging when the opponent has a bad reputation is

$$C^S (d) = V_{GB} (d; \delta) - V_{BB} (d; \delta) = \frac{\delta}{1 - \delta} \frac{d}{2} \left(1 - \frac{d}{2} \right)$$

Since the maximal gain from reneging is d we have that the maximal promise that is incentive compatible is (see figure 8):

$$d^S (\delta) = \begin{cases} 0 & if & \delta \leq \frac{2}{3} & 0 & \delta \leq \frac{2}{3} \\ 2 \frac{3\delta - 2}{\delta} & if & \frac{2}{3} \leq \delta \leq \frac{4}{5} & 2 \frac{3\delta - 2}{\delta} & \frac{2}{3} \leq \delta \leq \frac{4}{5} \\ 1 & if & \frac{4}{5} \leq \delta & 1 & \delta \geq \frac{4}{5} \end{cases}$$

As before the maximal promise that is credible in equilibrium when only one candidate has a good reputation is an increasing function of the discount factor. For small values of the discount factor $\left(\delta \leq \frac{2}{3} \right)$ no promises are believed, and for large values all promises are believed $\left(\delta \geq \frac{4}{5} \right)$.

Now consider the case in which the two candidates have a good reputation. The expected payoffs from one election for candidate L are:

$$v_{GG} (\delta) = \frac{1-d}{2} u_L (0) + \int_{\frac{1-d}{2}}^{\frac{1}{2}} u_L (2m - 1 + d) \, dm +$$

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$$\int_{\frac{1}{2}}^{\frac{1+d}{2}} u_L\left(2m-d\right) dm + \frac{1-d}{2} u_L\left(1\right) = -\frac{1}{2}$$

$$v_{BG}\left(\delta\right) = -1 - v_{GB}\left(\delta\right) = -\frac{1}{2} - \frac{d^S}{2}\left(1 - \frac{d^S}{2}\right)$$

Thus the cost of reneging in this case is:

$$C^D\left(d;\delta\right) = V_{GG}\left(d;\delta\right) - V_{BG}\left(d;\delta\right) = \frac{\delta}{1-\delta}\frac{d^S}{2}\left(1 - \frac{d^S}{2}\right) = d^S\left(\delta\right)$$

Therefore, in this case we will also have that $d^D\left(\delta\right) = d^S\left(\delta\right)$, that is the maximal credible promise when both candidates have good reputation coincides with the maximal credible promise that a candidate can make when his opponent has a bad reputation. ♦
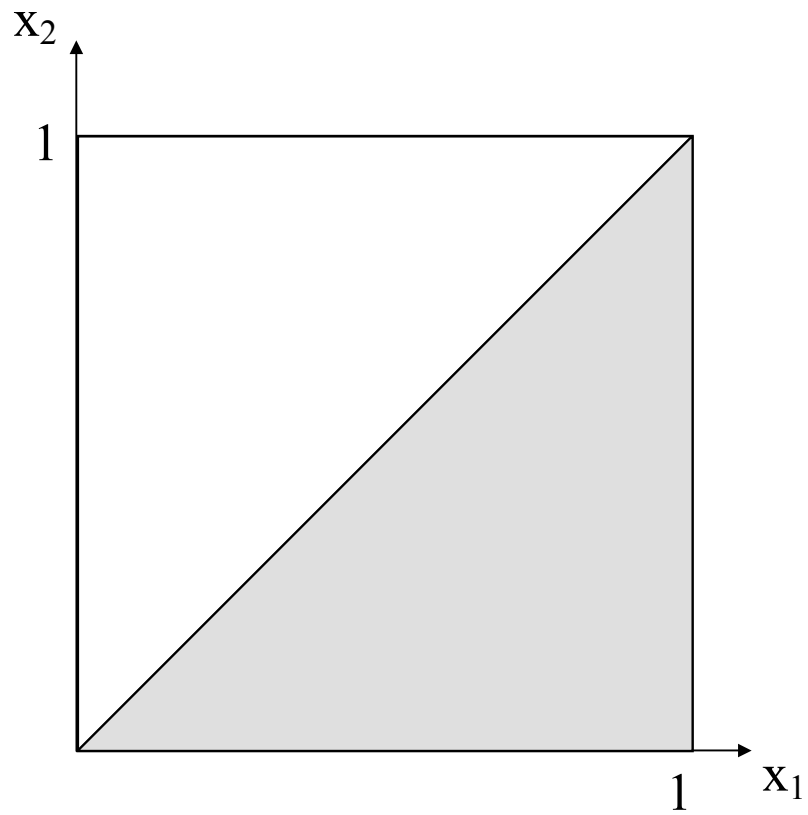
Figure 1: Both have bad reputation.
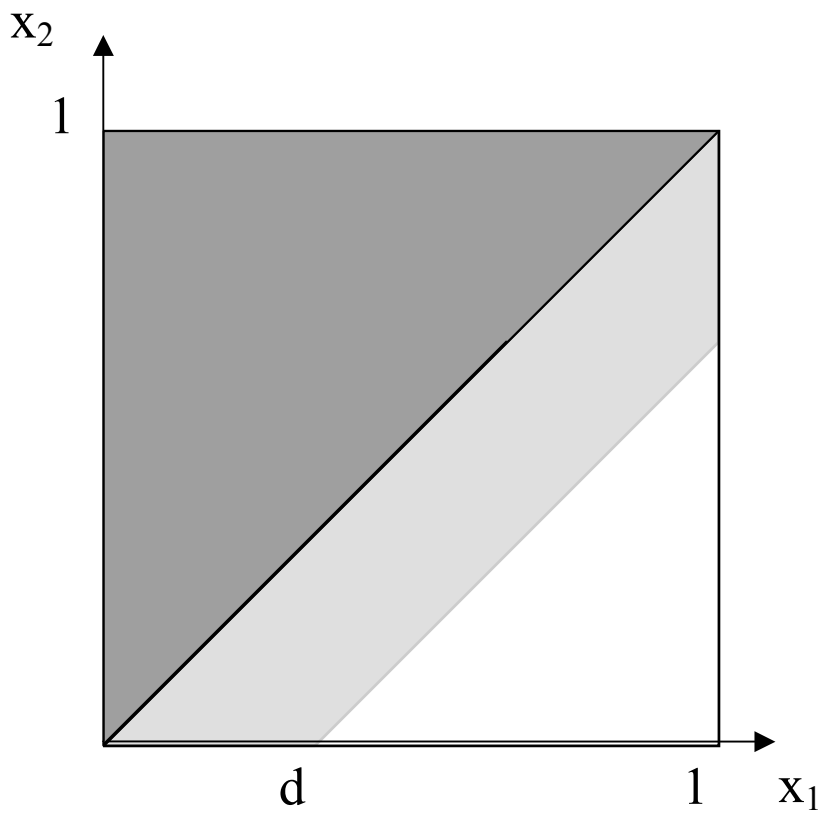
Figure 2: Only 1 has good reputation.
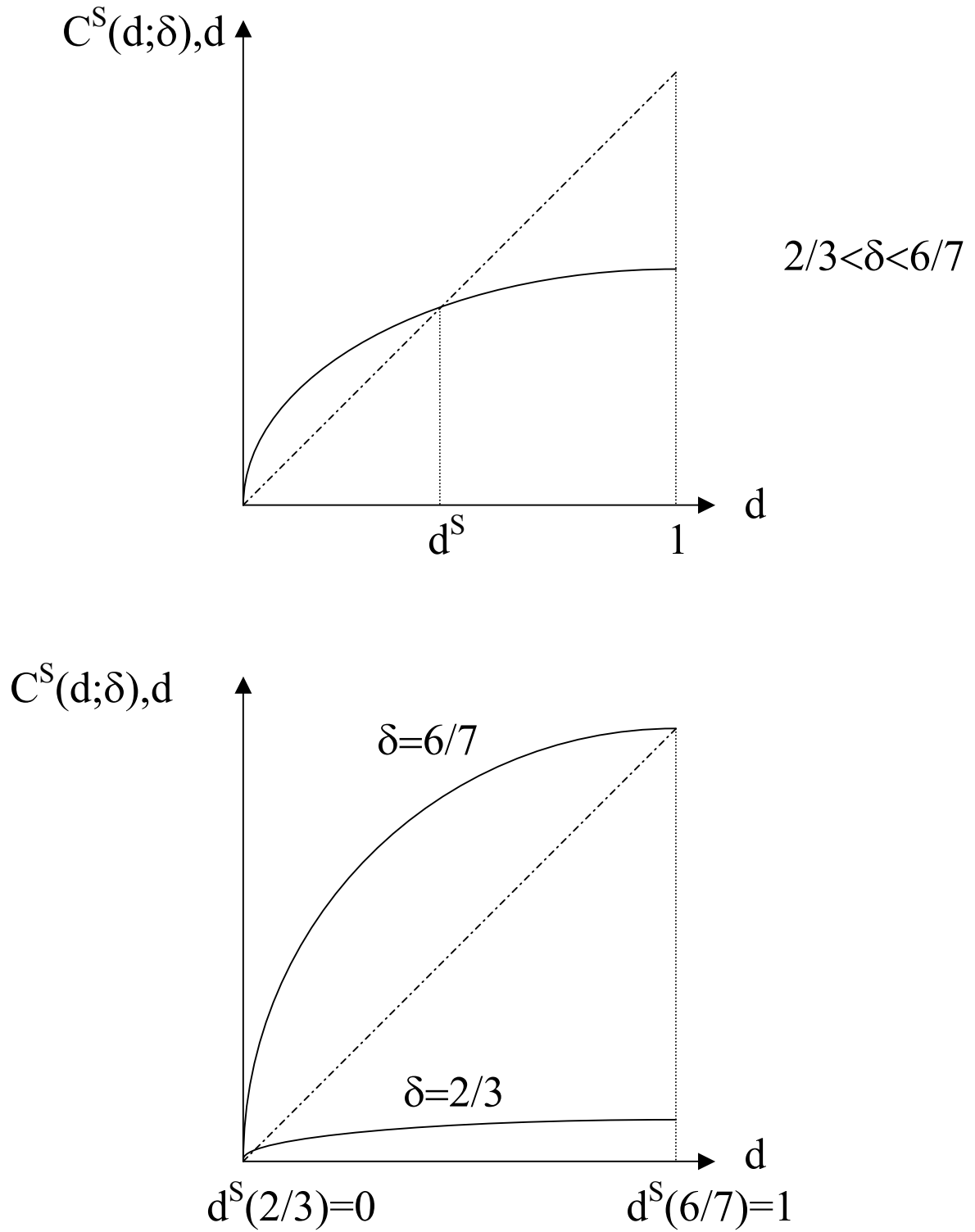
Figure 3: Only 1 has good reputation.
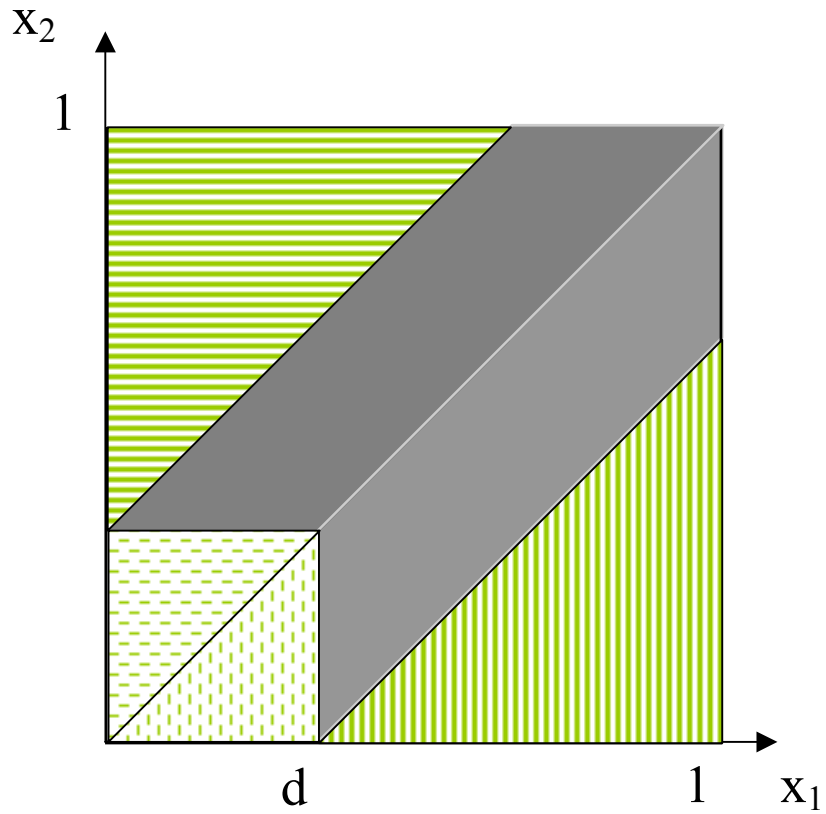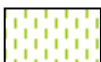
Figure 4: Both have good reputation.
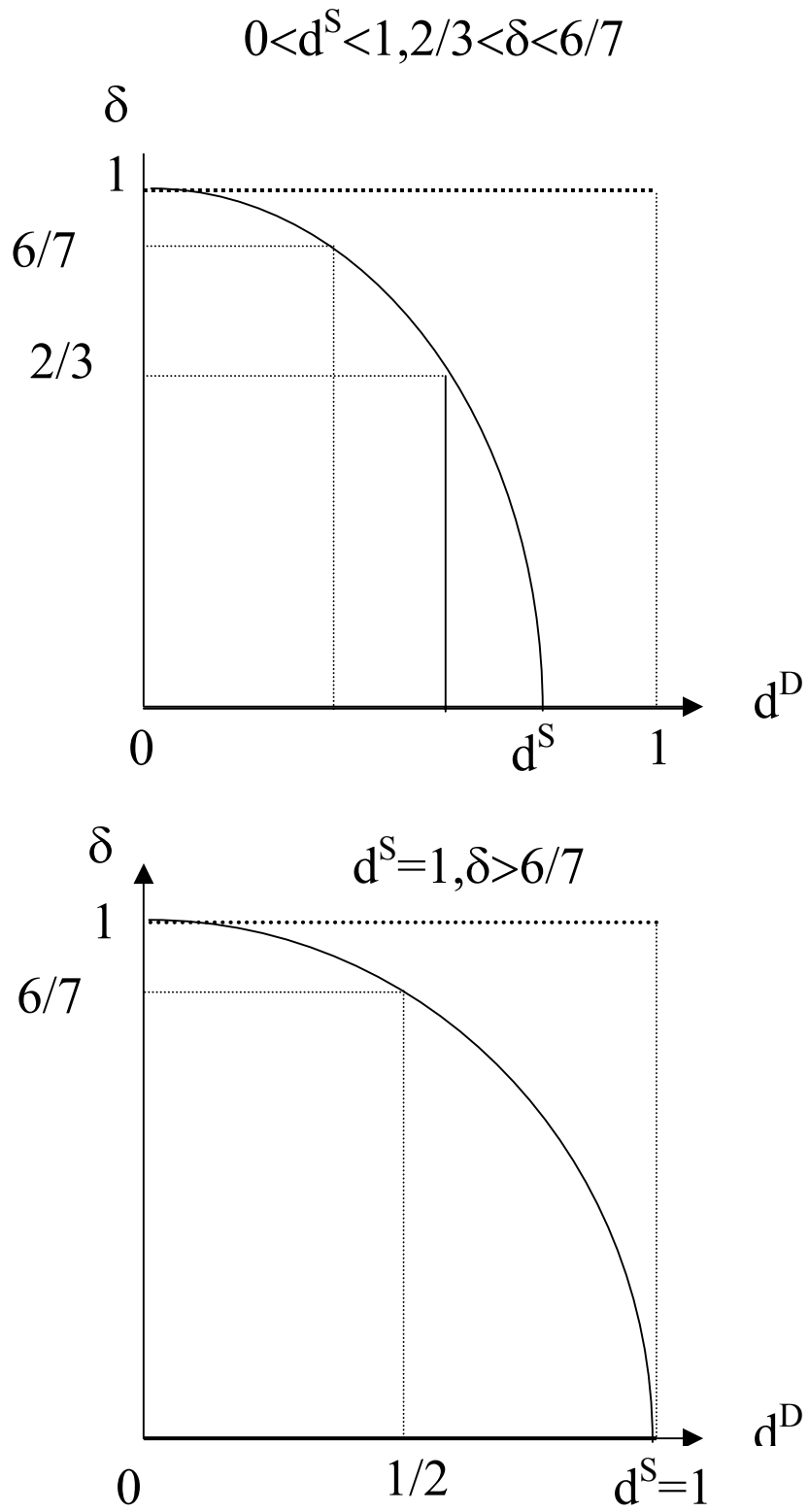
Figure 5: Both have good reputation.
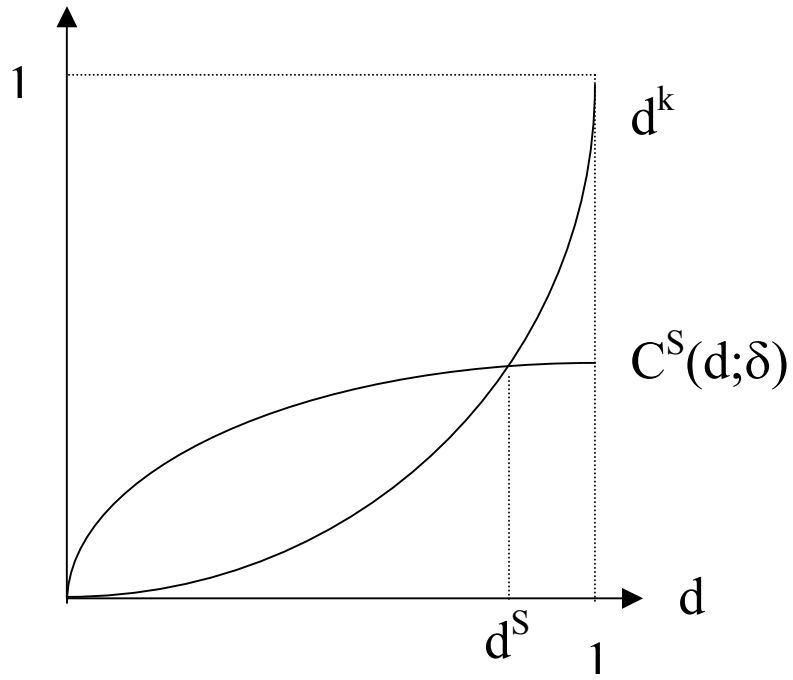
Large δ
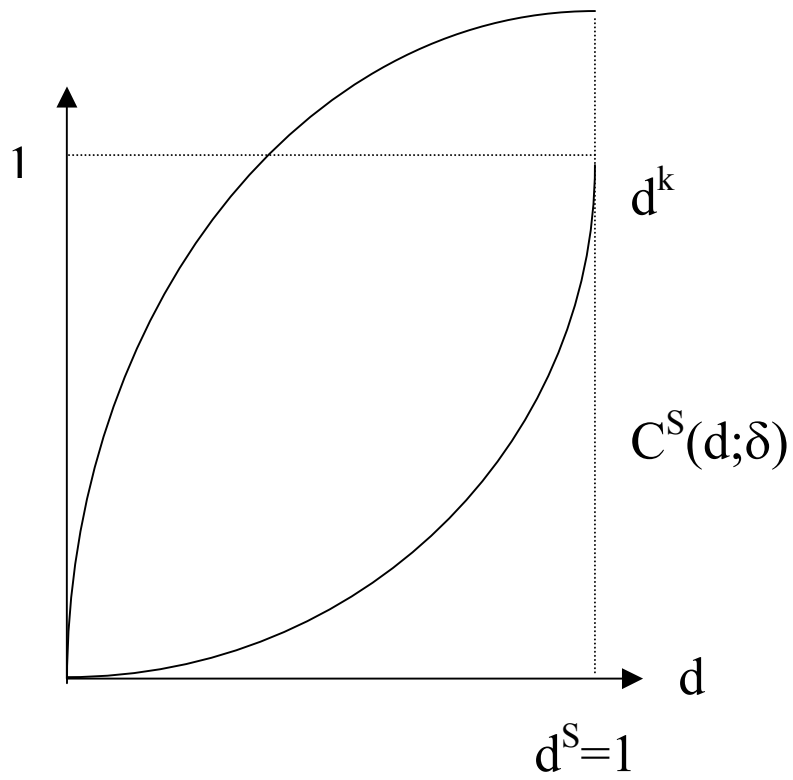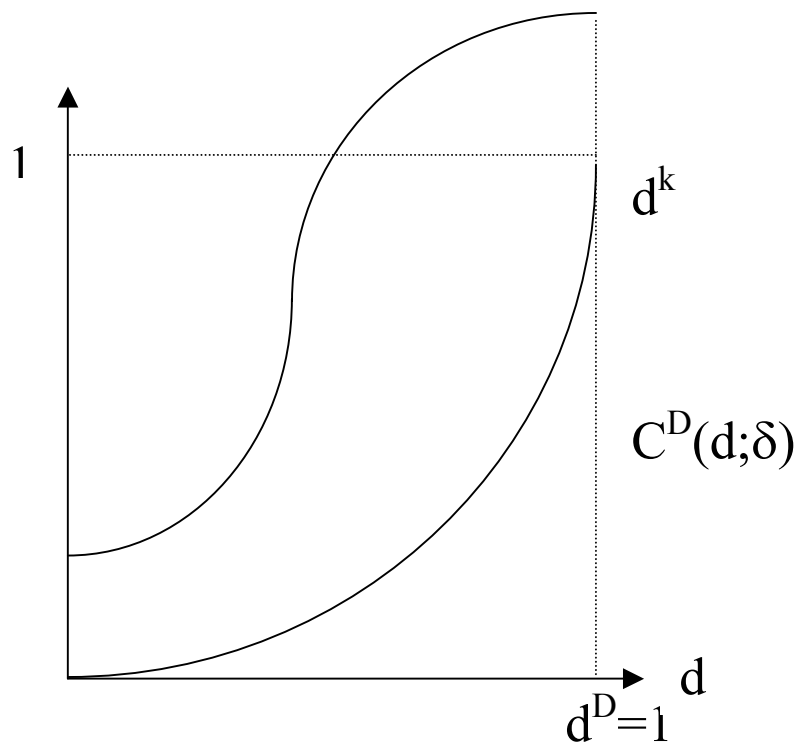

Figure 6: Only L has good reputation.
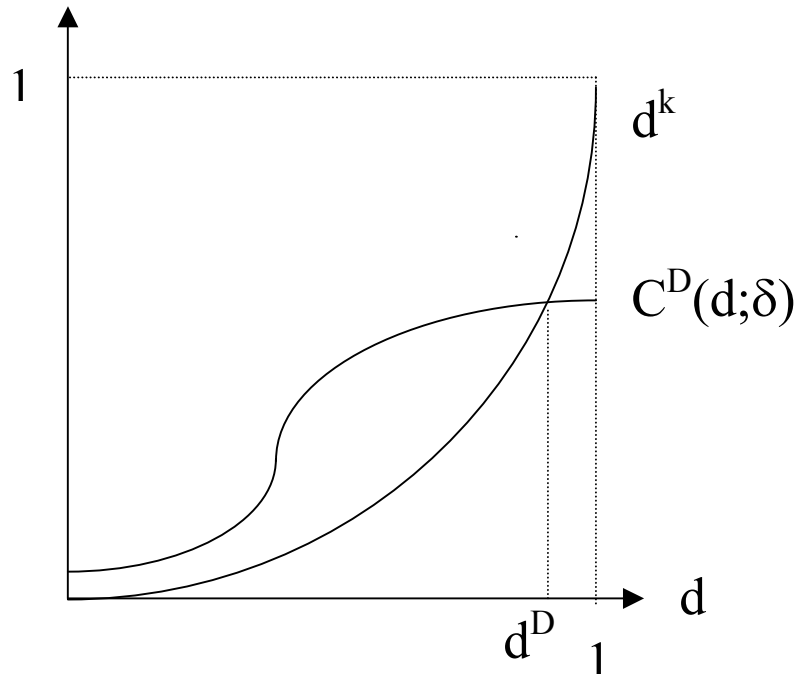
Large δ

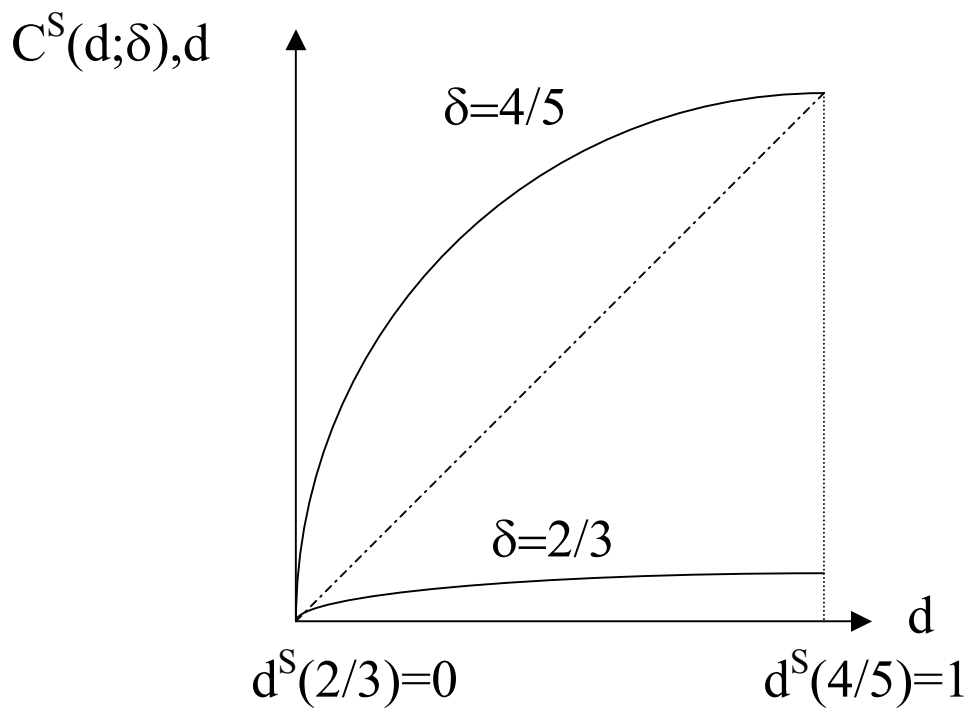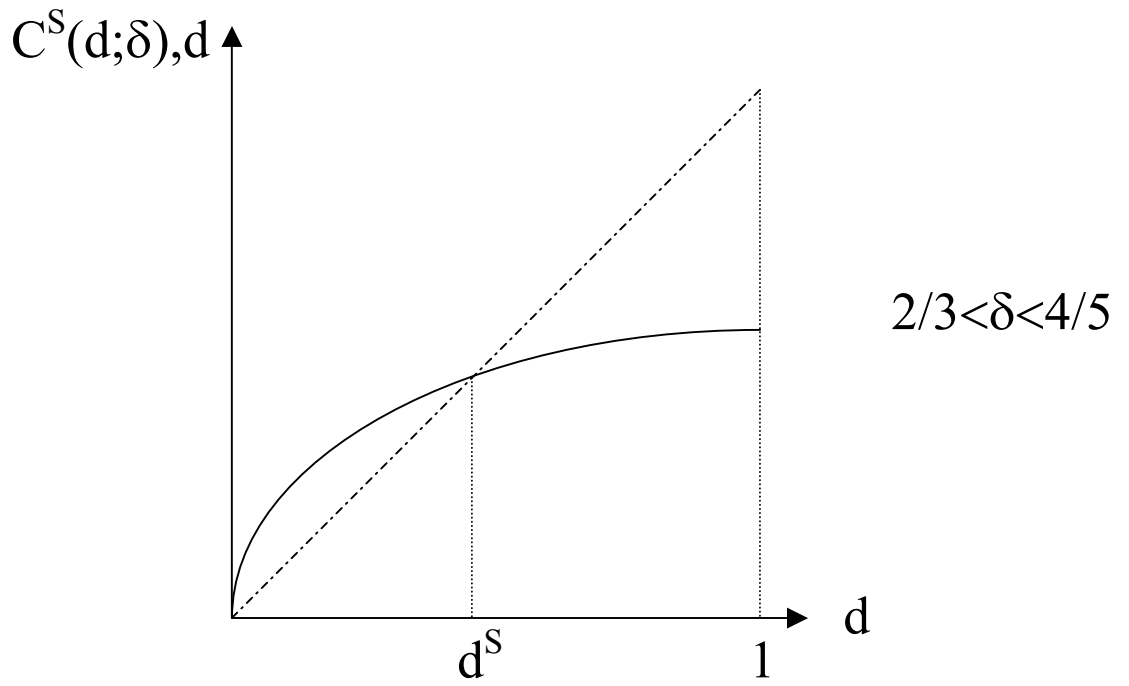Figure 7: Both have good reputation.

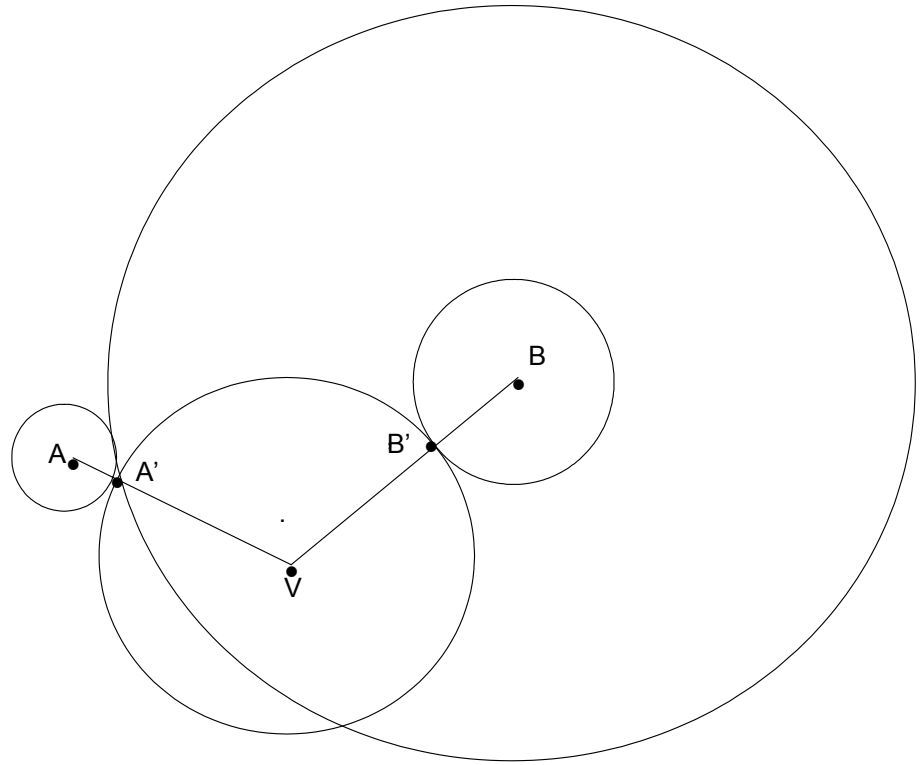Figure 8: Random median voter.
Only L has good reputation.

Figure 9: Equilibrium with Simultaneous
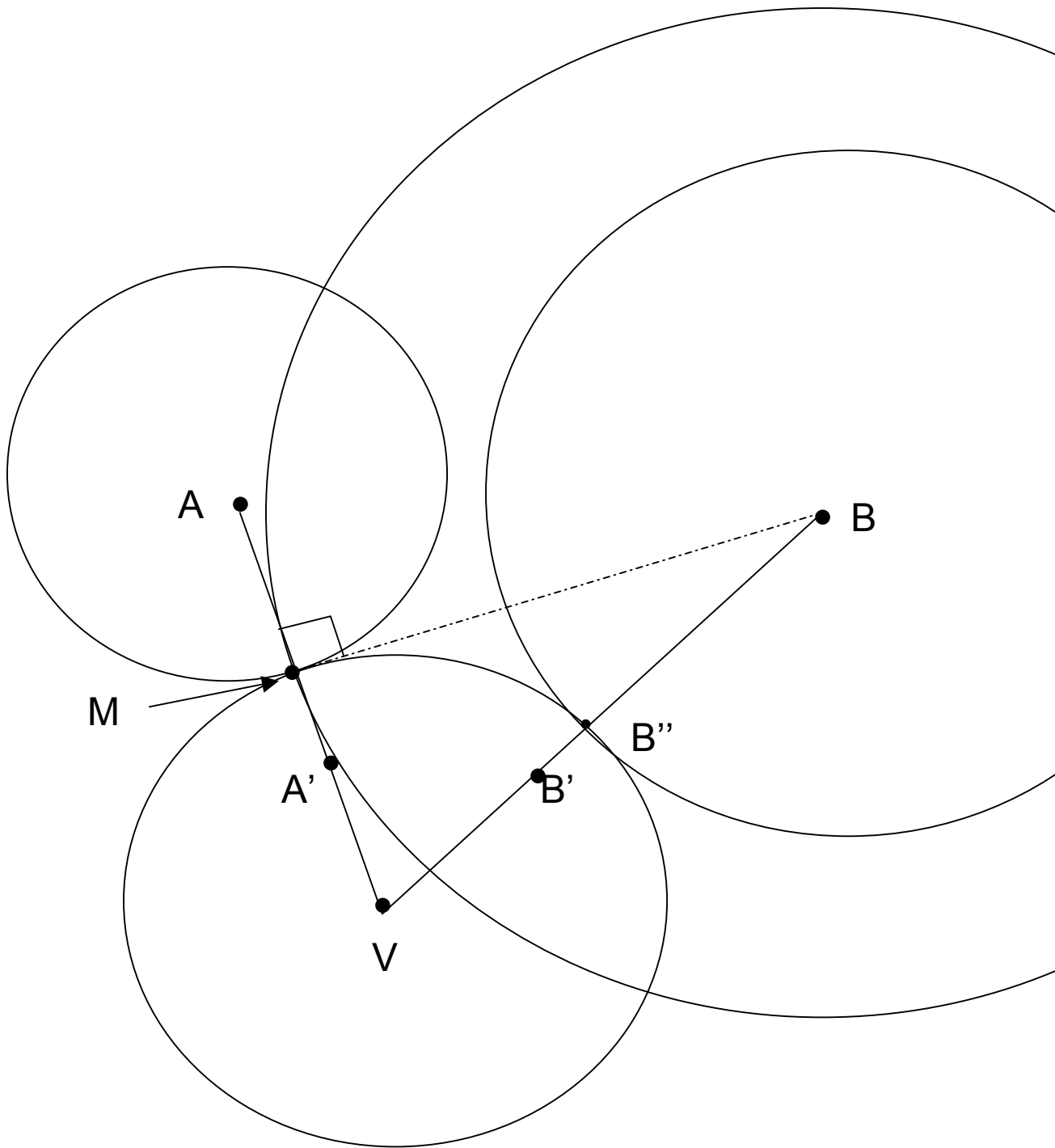Moves in a two dimensional Policy Space.

Figure 10: Equilibrium with Sequential
Moves in a Two dimensional Policy Space.