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Understanding the Case of Porto Alegre**

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Abstract

Participatory Democracy is a process of collective decision making that combines elements from both Direct and Representative Democracy: Citizens have the power to decide on policy and politicians assume the role of policy implementation. The aim of this paper is to understand how Participatory Democracy operates, and to study its implications over the behavior of citizens and politicians and over the final policy outcomes. To this end, we explore a formal model inspired by the experience of Participatory Budgeting implemented in the Brazilian city of Porto Alegre, that builds on the model of meetings with costly participation by Osborne, Rosenthal, and Turner (2000).

JEL codes: D7, H0, R5.

Keywords: Participatory Democracy, Retrospective voting, assembly, legislator.

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"We are not selling the illusion of the direct democracy in the Greek plaza which, let us bear in mind, was not the democracy of the all but the democracy of the best."

Olivio Dutra, first Workers Party mayor of Porto Alegre.

1   Introduction

Participatory democracy is a process of collective decision making that combines elements from both direct and representative democracy: Citizens have the power to decide on policy and politicians assume the role of policy implementation. The electorate can monitor politicians' performance simply by comparing citizens' proposals with the policies actually implemented. As a result, the discretion of politicians is severely constrained. In this system, the extent to which citizens can affect policy and determine social priorities is directly aligned with the degree to which they choose to involve themselves in the process.[1]

Real life experiences of participatory democracy have mainly materialized in processes of "Participatory Budgeting" at the city level. This is the case of nearly two hundred Brazilian municipalities where direct democracy, in the form of popular assemblies, coexists with formal political parties and local elections: citizens have to make a budget proposal but they also have to elect the city executive and legislative bodies. Other participatory systems have been implemented at the state level in Rio Grande del Sul (Brazil) and in West Bengal and Kerala (India), and at the school level in Chicago, through the Local School Councils[2].

The aim of this paper is twofold: First, to analyze a theoretical model of participatory democracy. We want to understand how this institutional arrangement operates, and to study its influence over the behavior of citizens

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[1] Although inspired by earlier figures such as Rousseau or John Stuart Mill, the first theoretical formulations of participatory democracy were made during the 70s by Pateman (1970) and MacPherson (1977). An excellent discussion of the main features of this model of democracy can be found in Held (1987).

[2] For a more detailed description of these cases, see Fung and Wright (2001) and the references therein.

and politicians and over the final policy outcomes. To this end, we explore a formal model inspired by the experience of the Brazilian city of Porto Alegre, the most successful and lasting real example of Participatory Budgeting.

Our second goal is precisely to discuss the application of this system to the particular case of Porto Alegre. The results of the formal analysis allow us to construct an explanation for the political events ocurred in that city during the period in which the Participatory Budgeting system has been operating.

Participatory democracy and the experience of Porto Alegre must be of interest to economists for several reasons. On the one hand, Economics has exclusively focused on more traditional models of democracy like direct democracy and, above all, representative democracy, since the latter is the prevailing system in the Western world. But the success of the mentioned real life examples of participatory democracy on both economic and social grounds[3] demands an evaluation of its potential virtues and flaws. The present paper represents a first step in that direction. On the other hand, two facts occurred in Porto Alegre escape obvious rational explanation: the substantial participation at meetings, despite the explicit costs of attendance, and that the policies implemented under this system have consistently followed the citizens' proposals. Our formal analysis sheds light on these issues. In particular, we characterize participation at the meetings, in terms of preferences of the attendees, and offer a theoretical explanation for the increasing participation rates observed in Porto Alegre. We also offer an analysis of the government's incentives to fulfill the society's claims under this system.

1.1   Overview of the model

The process of Participatory Budgeting in Porto Alegre can be described as an annual cyclical process that consists of three stages: a deliberation stage, a negotiation stage, and a monitoring stage. In the deliberation stage citizens may participate in assemblies to decide on the investment priorities of their

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[3]On the positive social and economic consequences of participatory democracy, see Fung (2001), Santos (1998) and the report on Participatory Budgeting of the Inter-American Development Bank (2003).

neighborhood and to vote for the representatives who will present and defend the assemblies' decisions in front of the city government. In the negotiation stage, the city government and the representatives from all neighborhoods determine the city investment plan. After the Municipal Budget has been approved by the city government, the representatives of each neighborhood monitor the execution of the investment plan[4]. Hence, the main institutional characteristics of this system are (i) the combination of elements of representative democracy -elected municipal bodies- with elements of direct democracy -assemblies; and (ii) the high degree of accountability due to the direct involvement of citizens in the process.

In order to take these features on board, our analysis builds on the model of pure direct democracy by Osborne, Rosenthal, and Turner (2000) (referred to as ORT henceforth). There, the members of a society decide independently whether to attend, at a cost, or not to a meeting, where the policy decision taken will be a compromise among the attendees' ideal positions. Attendance is based on a cost-benefit calculation: citizens compare the cost of participation with the impact that their presence will have on the compromise. We extend the ORT model by considering the existence of a representative or *legislator* who is in charge of policy implementation. The legislator can choose freely the policy to be implemented. We assume that the legislator has her own preferences over policies and she also cares about reelection.

Notice that the roles played here by citizens and representatives differ from the roles they play in a standard model of representative democracy. In our model of participatory democracy, citizens are the first ones to move by making a policy proposal, and representatives have to react to it, deciding whether to implement it or not. In a standard model of representative democracy the policy decision is made by the elected representatives and the electorate reacts to it, approving or disapproving the policy choice with their vote in future elections[5].

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[4] A more detailed description of this process may be found in Appendix 1.

[5] In order to emphasize this difference we can draw an analogy to Stackelberg's model of oligopoly: in the model of participatory democracy citizens play the role of the leader and representatives play the role of the follower, while in a model of representative democracy citizens are followers and representatives are leaders. The analogy is not perfect though,

Even though the process of Participatory Budgeting is a yearly process, citizens only reelect legislators every 4 years. Thus it is natural to assume that, when electing the legislator, citizens vote retrospectively[6]: before a formal election takes place, citizens evaluate a legislator according to her performance in each of the budgeting decisions that took place during the legislative period. In order to simplify the model, we consider a reduced form game in which one period policy choice represents the whole set of decisions taken by a legislator during a legislature.

In particular, we assume that citizens reward those legislators that implement policies proposed by the citizens' assembly by reelecting them; and punish those who do not take the assembly's proposals into account by not reelecting them. This retrospective voting rule seems to fit well the behavior of citizens involved in a process of participatory democracy.[7] By agreeing on a policy proposal at the assembly, citizens resolve the conflict of interests due to the heterogeneity of their policy preferences. And, at the time of deciding on the reelection of the legislator, they only need to evaluate his performance by comparing the policies they proposed and the policy choices of the legislator. Thus, society's preferences with respect to the performance of the legislator may be considered to be homogeneous[8]. Nevertheless, since we assume that both, citizens and the legislator, care about the policy im-

[6]There exists some theories of voting which suggest that voters base their decision on past performance of parties. This literature starts with Downs' (1957) theory according to which, parties' past performance is the cheapest way for voters to predict future performance. It continues with the reward-punishment theory proposed by Key (1966), and with the empirical studies by Fiorina (1981), among others. Other models and applications of retrospective voting include: Barro (1973), Ferejohn (1986), Alesina and Rosenthal (1989), Austen-Smith and Banks (1989), and Persson and Tabellini (2000).

[7]For instance, in Porto Alegre there are committees formed by elected delegates whose function is to supervise the implementation of the budget. Since they have the right to ask the city government for detailed explanations on each investment work, any deviation that cannot be explained by sound technical or economic criteria may have straightforward electoral consequences.

[8]An alternative justification for this assumption comes from a veil-of-ignorance-like argument: Uncertainty can make citizens ignore their future opinions in the forthcoming budgeting decisions. Hence, they will prefer to keep in office a legislator that follows citizens' proposals even though at a given period, some of them would have been better off had the legislator ignored the corresponding proposal.

plemented, the conflict between the legislator's and the citizens's preferences allows for the possibility of a variety of equilibria.[9] Hence, citizens in this model punish the legislator if they do not approve of her performance by not coordinating their votes for her in future elections; otherwise they reward the legislator by reelecting her. This assumption captures the high degree of accountability in participatory democracy mentioned above.

More specifically, we model the system of participatory democracy as a game in three stages. In the first stage, each citizen decides whether to attend or not to attend a meeting in which a policy proposal will be decided. In the second stage, citizens who attend the meeting come out with a policy representing their interests and their delegates make a proposal to the legislator, aimed at making her implement the assembly's choice. In the third stage, the legislator decides the policy to be implemented, taking into account the reduced form game that includes her chances of reelection.

In order to find the Subgame Perfect Equilibrium strategies of this game, we analyze the optimal choices of the players by backward induction. First, we analyze the optimal reaction of the legislator in terms of policy choices, to a given proposal made by the delegates. Then, we analyze the optimal proposal of the assembly's delegates, for a given distribution of preferences of the attendees, and taking into account the optimal reaction of the legislator. Finally, we analyze the optimal decision of each citizen regarding whether to attend the meeting, given an optimal play of all agents in the continuation of the game.

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[9]The important differences between the present paper and the recent developments in the theory of political agency make comparisons difficult. First, these papers often study voters' decision problems from a moral hazard or an adverse selection perspective (see Ferejohn (1986) and Besley and Case (1995) respectively) or by combining both (Banks and Sundaram, 1998). On the contrary, in our model voters can perfectly monitor the legislator actions and preferences. But beyond this, the retention rule in this literature is often imposed, as in Ferejohn (1986), and always set costlessly. In our case, although a rule is given, its actual enforcement depends upon citizens costly participation in the meeting.

1.2 Overview of the results

Following the intuition of the retrospective voting models, our first result shows that the set of policies that can be implemented in equilibrium is a subset of the policy space that contains the ideal point of the legislator. That is, the legislator will only implement policies that are close to her ideal point up to a maximal compromise policy, at which point the legislator is indifferent between jeopardizing her reelection by implementing her ideal policy, or guaranteeing her reelection by satisfying the assembly. We show that the more the legislator cares about holding office the larger is the set of policies that can be implemented in equilibrium. The intuition is clear: a legislator who does not care so much about policy is willing to accept proposals further from her ideal point in order to guarantee a sure win in a future election. On the other hand, the softer is the threat of punishment, the smaller the size of the set of implementable policies. That is, a legislator who believes that her chances of being reelected will be very low unless she follows the policy proposed by the assembly, will be willing to implement a larger set of policies.

As in the ORT model, if the cost of attending the meeting is high enough there is a unique equilibrium in which nobody attends the meeting, but otherwise in equilibrium there is always some attendance.

In our model, the legislator's ideal policy plays a role similar to the default policy in ORT: it is the policy selected if no citizen attends the meeting. There exists, however, a crucial difference. In our model, the legislator has *decision power* and in equilibrium she will never compromise more than what is needed to ensure her reelection. We find that in equilibrium only citizens who are far enough from the legislator's ideal point attend the meeting. But they are not necessarily extreme in the usual (spatial) sense: when the legislator has extreme policy preferences, citizens who are moderate relative to the spectrum of tastes in society may actually have strong incentives to participate. *Moderation becomes thus a relative concept.*

The results we obtain show the relevance of two features: 1) the *alignment* between the policy preferences of the legislator and the policy preferences of

society; and 2) the degree of *extremism* of the legislator.

On the one hand, we show that when the most preferred outcome to the society lies relatively close to the legislator's ideal point, that is, when society and legislator's preferences are aligned, the policy implemented in equilibrium is the assembly's most preferred policy. This equilibrium exists only for a certain subset of distribution of preferences.

On the other hand, we find that for any distribution of preferences if the legislator is extremist relative to the society's preferences there is an equilibrium in which the legislator implements the maximal compromise policy: on her left if her ideal point is on the right hand side of the policy space and on her right if her ideal point is on the left hand side of the policy space. In this equilibrium only one citizen attends the meeting: with an ideal point to the left of the legislator if the legislator is rightist, and with an ideal point to the right of the legislator if the legislator is leftist. This result is driven by the combination of two facts: 1) the legislator will never compromise more than what is necessary to guarantee her reelection, and 2) each citizen that attends the meeting is assumed to have an impact on the proposed policy. Therefore, in our model, only one citizen, even a moderate, may be enough to force an extreme legislator to a maximal compromise.

This result should not be taken literally. A real life interpretation of each citizen as defined in our model could be a community association. In fact, today about six hundred community associations are established and active in Porto Alegre[10], and the members of these associations account for a large proportion of the participation in the popular assemblies. Since it makes sense to identify each such community association with a particular position in the policy space, we can interpret our extreme result on attendance as an equilibrium in which only members of one community association attend the meeting. It still refers to a very low participation result but not an implausible one: it does not refer to one individual having a large impact on the legislator's choice but to the members of an organized group forcing the

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[10]The high number of politically active community associations is one feature that explains the introduction of the system of Participatory Budgeting in Brazil.

legislator to a maximal compromise policy.[11]

We show that the two equilibria described above are the only pure strategy equilibria that may be obtained in our model. If the legislator is extremist and her preferences are not aligned with society's the maximal compromise equilibrium is unique. If the legislator is extremist and her preferences are aligned with society's, for some distribution of preferences we have another equilibrium in which the policy outcome coincides with the most preferred choice of the assembly. Finally, if the legislator is relatively moderate and her preferences are not aligned with society's we find that there is no equilibrium in pure strategies. Therefore, with polarized or extremist societies and moderate legislators, the process of participatory democracy may generate unstable outcomes.

The rest of the paper proceeds as follows. The next section describes the formal model. Section 3 presents the sequential derivation of optimal policy choices. Section 4 analyzes the citizens' choice on participation in the assemblies. Section 5 analyzes the evidence from Porto Alegre in the light of the results obtained in the previous sections. Finally, Section 6 contains some concluding remarks. All proofs can be found in Appendix 2.

2   The model

The policy space is continuous and one dimensional, and represented by the interval $[0, 1]$. There is a finite number N of citizens with single-peak preferences over the interval $[0, 1]$. The citizens' ideal points are distributed according to a probability distribution $F(\theta)$ with support in $[0, 1]$. We will assume, without loss of generality, that there is always at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$.

Given that the high degree of involvement of citizens in the process of Participatory Budgeting in Porto Alegre has been induced by the strong activism of community associations, and since, by definition, their members share a common interest, we can also think of citizens in our model as different

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[11]In fact, it is a common practice for associations to send one or two members to the assembly meetings and to take turns.

community associations.

At the first stage of the game, citizens have to decide whether to attend a meeting in which a policy will be proposed. Attendance implies that their opinion will be taken into account in the elaboration of that policy proposal, but it also involves an individual cost $0 < c < \frac{1}{2}$. This may include transportation costs, the opportunity cost of the time spent in the assembly or the cost of identifying the own preferences. The welfare of a citizen i with ideal point θ_i depends on the policy implemented and on whether he attends the meeting and it is given by the following expression:

$$V_i(x, a_i) = -|x - \theta_i| - a_i c,$$

where x is the policy implemented, and a_i represents the decision of citizen i on whether to attend the meeting: if $a_i = 1$, i attends the meeting and pays a cost c, if $a_i = 0$, i does not attend the meeting and pays no cost.

Following the functioning of the Participatory Budgeting system in Porto Alegre, where citizens rank and select their priorities, we assume that the citizens who attend the meeting aggregate their preferences according to some previously fixed aggregation rule and that the policy selected by that rule is the one that the assembly would like to see implemented. Let X denote the list of ideal points of those citizens who attend the meeting, $|X|$ the number of attendees, and let $\theta^*(X)$ denote the assembly's most preferred policy. The aggregation rule we consider is

$$\theta^*(X) = \begin{cases} median(X) & \text{if } |X| \text{ is odd} \\ \frac{m_1 + m_2}{2} & \text{otherwise} \end{cases}$$

where m_1 and m_2 are the two medians' ideal points when $|X|$ is even.[12]

Given a distribution of ideal points of the citizens, $F(\theta)$, let θ^{**} denote society's most preferred policy defined according to the corresponding aggre-

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[12]We are of course aware of the fact that with an even number of attendees the choice of this policy cannot be rationalized by a voting process within the assembly. However, this assumption allows us to pin down a unique compromise regardless of the number of participants .

gation rule. That is, θ^{**} would be the assembly's decision if all individuals participate in the assembly. Notice that the policy chosen by the assembly at the meeting does not depend on $F(\theta)$, but on the distribution of ideal points of the citizens who decide to attend the meeting, X.

After the assembly, a proposal, considered as the "general will", is transmitted to a *legislator* who is in charge of implementing the final policy. In Porto Alegre, two bodies formed by delegates elected by citizens at the assemblies are in charge of doing this. Hence, we assume the existence of an intermediate body of *delegates* between the citizens and the legislator who elaborates the policy proposal, denoted by x^*. We assume that the delegates' utility function coincides with the assembly's one so they try to induce the legislator to implement a policy as close as possible to θ^*.[13] This implies that the policy proposal to the legislator, x^*, does not need to coincide with the assembly's most preferred policy θ^*; it is an strategic choice that takes into account the distortion introduced by the legislator.

The welfare of the legislator depends on her own policy preferences and on the probability of being reelected, and it is represented by a convex combination as follows:

$$V_L(x^*, x) = (1 - \alpha)P(x^*, x) - \alpha \left| x - \theta_L \right|.$$

where $\theta_L \in [0, 1]$ represents the ideal point of the legislator, $\alpha \in [0, 1]$ is an exogenous parameter that represents the intensity of the policy preferences of the legislator relative to her preferences for holding office. From the point of view of the legislator, $P(x^*, x)$ is interpreted as the probability with which she will be reelected. It depends on the amount of support that she will be able to obtain from the population, which in turn depends on whether the citizens approve of her performance. As mentioned, citizens judge the legislator' performance according to how close her choice x is from the general will represented by their proposal to her, x^*. In particular, we assume that the probability of reelection $P(x^*, x)$ is a step function of the distance between

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[13]See footnote 16 on the plausibility of this assumption.

the implemented policy x and the mandate of the citizens' assembly x^*:

$$P(x^*, x) = \begin{cases} 1 & \text{if } |x - x^*| \leq B \\ \varepsilon & \text{otherwise} \end{cases}$$

where $B > 0$ is the degree of discretion of the legislator, that may account for financial or technical circumstances unforeseen by the citizens. In participatory democracy, legislators still control this knowledge and have privileged access to it.[14] So if the difference between the policy proposed by the delegates and the policy implemented by the legislator is not larger than this degree of discretion B, citizens will approve the legislator's performance and they will likely reelect her in future elections. Otherwise, the reelection of the legislator is compromised and we assume that she will only win future elections with probability ε, with $0 < \varepsilon < 1$. We assume that ε takes small values, reflecting the high degree of accountability of the participatory process[15].

Finally, if nobody attends the meeting the legislator can implement her ideal point and she is reelected with probability one.
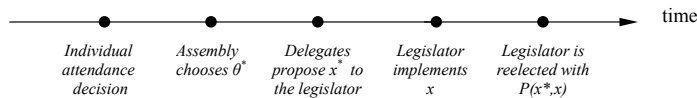

Figure 2: Timing of the game.

So far, we have constructed a game in three stages. In the first stage, citizens decide whether to attend or not to attend the meeting. In the second stage citizens who attend the meeting choose a policy and the delegates make a proposal to the legislator. In the third stage the legislator decides which policy will be finally implemented.

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[14] In Porto Alegre, there have been substantial efforts to train delegates and councillors aiming to reduce this inevitable degree of discretion.

[15] The discrete form for $P(x^*, x)$ can be dropped without qualitatively changing the results. Section 6 contains a discussion on the effects of employing continuous functional forms.

3 Optimal policy choices

In order to find the Subgame Perfect Equilibrium strategies of this game, we analyze in this Section the optimal choices of the players by backward induction, starting with the optimal reaction of the legislator, in terms of policy choices, to a given proposal made by the delegates. Then, we analyze the optimal proposal of the assembly's delegates for a given distribution of preferences of the attendees, taking into account the optimal reaction of the legislator. Finally, we characterize the equilibrium policy choice for any pair of legislator's and assembly's policy preferences, (θ_L, θ^*).

3.1 The optimal choice of the legislator

In order to choose the policy that will be finally implemented, x, the legislator maximizes her payoff function, given a policy proposed by the delegates, x^*.

$$\max_x \; V_L(x^*, x) \;\; = \;\; (1-\alpha)P(x^*, x) - \alpha\, |x - \theta_L|$$

$$\text{where} \quad P(x^*, x) \;\; = \;\; \begin{cases} 1 & \text{if } |x - x^*| \leq B \\ \varepsilon & \text{otherwise} \end{cases}$$

Let us define $b = \frac{(1-\alpha)}{\alpha}(1-\varepsilon)$. Notice that $(1-\varepsilon)$ represents the probability with which the legislator is not reelected when the citizens feel deceived, and $\frac{(1-\alpha)}{\alpha}$ represents the value of holding office. Thus, b represents a measure of the cost that the legislator has to pay when she is punished by the electorate. Straightforward computations allow us to characterize the best response of the legislator That is,

$$x\left(x^*\right) = \begin{cases} x^* + B & \text{if } B \leq |x^* - \theta_L| \leq b + B \;\; and \;\; x^* \leq \theta_L \\ x^* - B & \text{if } B \leq |x^* - \theta_L| \leq b + B \;\; and \;\; x^* \geq \theta_L \\ \theta_L & \text{otherwise} \end{cases}$$

The intuition behind this function can be easily grasped: The legislator is forced to face a trade-off when maximizing his objective function.

Consider first the case in which the delegates' proposal is very close to

the legislator's ideal point, i.e. $|x^* - \theta_L| < b + B$. In that case, the legislator can implement her ideal point without compromising her reelection. Similarly, when $|x^* - \theta_L| > B$, the delegates' proposal is so far away from the legislator's ideal point, that she prefers to ignore the proposal even though that implies jeopardizing reelection. Only in the remaining case the proposal of the delegates is far enough from the legislator's ideal point, so she cannot implement her ideal point without compromising her approval, but it is close enough for the legislator to prefer to compromise and thus guarantee a sure victory in the future.

Hence, the optimal policy choice of the legislator will be the legislator's ideal point θ_L, if the proposal of the assembly is not further than a distance B from it, or if the legislator cares mostly about policy (b is sufficiently small). Otherwise the legislator will choose a policy that is located exactly B away from the proposal of the assembly.

3.2 The optimal choice of the delegates

The policy proposal x^* is made by a small group of delegates. Since they are elected by the assembly we assume they are committed to force the legislator to implement θ^* or the closest possible policy[16]. But they are aware of the preferences of the legislator so they choose their policy proposal x^* strategically, knowing that the only policies that can be finally implemented are either the legislator's ideal point or policies that are exactly B away from their proposal. The optimal policy choice of the delegates can be then

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[16]The strength of the bond between the delegates and the regions they represent has been a source of certain unrest in Porto Alegre: CIDADE, a NGO monitoring the process, has reported problems and discussions about delegates who allegedly took positions without consulting their constituencies or who failed to report back decisions at higher levels. Nevertheless, the strong accountability and the reelection rules employed have tempered the potential for collusion and corruption. Surveys show that over the 85% of the population thinks that delegates "always" or "most of the time" represent properly the assemblies' interests (CIDADE, 2002).

characterized by the following function:

$$
x^* \left(\theta^* \right) =
\begin{cases}
\theta_L - b - B & \text{if } \theta^* \leq \theta_L - b \\
\theta^* - B & \text{if } \theta_L - b \leq \theta^* \leq \theta_L \\
\theta^* + B & \text{if } \theta_L \leq \theta^* \leq \theta_L + b \\
\theta_L + b + B & \text{if } \theta_L + b \leq \theta^*
\end{cases}
$$

In the second and third cases, the assembly's most preferred policy is very close to the legislator's ideal point, i.e. $|\theta_L - \theta^*| \leq b$, and the delegates can induce the legislator to implement the assembly's most preferred policy by making proposals B away from θ^*.[17] Otherwise, the best the delegates can do is to induce a compromise. The best choice in this case is to propose a policy that makes the legislator indifferent between implementing her ideal point and jeopardizing the next election, and implementing the compromise policy that still assures her approval, that is, a policy of maximal compromise.

3.3 The equilibrium policy choice

By combining the optimal choices of delegates and legislator we can characterize the policies that will be implemented in equilibrium as a function of the legislator's ideal point and the most preferred policy of the assembly. The next Proposition presents the most immediate consequence of introducing a legislator's choice in a process of pure direct democracy: not all policies are implementable in equilibrium.

Proposition 1 *In equilibrium $x \in [\theta_L - b, \theta_L + b]$.*

This result could be seen as equivalent to imposing an exogenous restriction on the policy space; and it is so, strictly speaking. But these restrictions are not arbitrary. As in the retrospective voting models, they come from the legislator's preferences over policies and reelection. Note that the size of the set of implementable policies depends positively on $b = \frac{(1-\alpha)}{\alpha}(1 - \varepsilon)$. And

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[17]Note that the role of B ultimately boils down to the level of distortion that citizens need to introduce in their proposals. Therefore, it might be set to zero.

this is in turn increasing on the value that the legislator attaches to holding office (α), and decreasing in the probability with which the legislator's performance is approved independently of the policy implemented (ε).

These relations are intuitive: legislators who do not care so much about policy are willing to accept proposals further from their ideal point in order to stay in office. By the same token, a legislator who believes that her chances of being reelected will be very low unless she follows the policy proposed by the assembly, will be ready to compromise. In fact, if citizens could commit to a certain degree of punishment, in terms of the probability represented by ε, their optimal choice should be $\varepsilon = 0$, the maximal degree of accountability.

Finally, the combination of all the previous derivations and assumptions allow us to derive a function mapping any pair of legislator's and assembly's policy preferences, (θ_L, θ^*) into a unique policy outcome x:[18]

$$
x\left(\theta^*, \theta_L\right) = \begin{cases} \theta^* & \text{if } \theta^* \in [\theta_L - b, \theta_L + b] \\ \theta_L - b & \text{if } \theta^* < \theta_L - b \\ \theta_L + b & \text{if } \theta^* > \theta_L + b \end{cases}
$$

When the policy most preferred by the assembly is relatively close to the legislator's ideal point, the policy finally implemented coincides exactly with the preferences of the assembly. Otherwise, citizens cannot induce the legislator to implement their most preferred policy. They can at most induce a compromise between the policy preferences of the assembly and the legislator. Moreover, the less the legislator cares about policy and the higher the probability with which she is being punished by the citizens, the smaller the chances are that the assembly can achieve its most preferred policy. In fact, if the values of α and ε are sufficiently small (so that b is large enough) the set of implementable policies may be the whole policy space.

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[18]The derivation of this function can be found in the proof of Proposition 1.

4 Endogenous participation

In this Section, we finally analyze citizens' attendance decisions, taking as given the equilibrium policy choice derived above that embeds the optimal play of all agents in the continuation of the game.

In this subgame, an equilibrium is a list of values for $\{a_1, ..., a_N\}$ with $a_i \in \{0, 1\}$ such that for all i, a_i is a best response against $\{a_1, ..., a_{i-1}, a_{i+1}, ..., a_N\}$. We show that when the cost of attendance is large enough, relative to the parameters of the objective function of the legislator, there is a unique equilibrium in which nobody attends the meeting.

Proposition 2 (Non Attendance) *If $c > b$ there is a unique equilibrium in which nobody attends the meeting.*

As in ORT model, here citizens perform a cost benefit analysis in order to decide whether to attend the meeting. If the cost of attending is larger than the benefit they will obtain from the impact that their presence at the meeting will have on the final policy, they decide not to attend. We have assumed that if nobody attends the meeting the legislator can implement her ideal point without compromising her future reelection. Thus, in this model the size of b represents the maximal effect that any citizen can have on the policy implemented. Clearly, when the cost of attendance is larger than b nobody has any incentive to attend the meeting.

Next we show that when this is not the case, that is, if the cost of attendance is small enough, in equilibrium there must be some positive attendance.

Proposition 3 *If $c < b$, then no-attendance is not an equilibrium.*

In fact, the proof of Proposition 3 shows not only that nobody attending is not an equilibrium, but also that if nobody attends the meeting any citizen whose ideal point is more than a distance c away from the legislator's ideal point would be better off attending. The intuition of this result comes again from the cost-benefit analysis that citizens perform: any citizen who attends the meeting can induce the legislator to implement either the citizen's ideal point or a maximal compromise policy (a policy that is a distance b away

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from the legislator's ideal point). In both cases, if the ideal point of the citizen is more than a distance c away from θ_L, the benefit for the citizen is larger than c, which is the cost of attending the meeting.[19]

Next we characterize some equilibrium strategies when $c < b$, that is, when there are some citizens who attend the meeting in equilibrium.

4.1   Aligned moderate legislators

Let us assume that the distribution of ideal points of society, $F(\theta)$, is symmetric around the legislator's ideal point. This will better illustrate the cost-benefit analysis citizens have to perform when making their attendance choices. In this case, the legislator's policy preferences qualify as moderate and aligned with society's. Notice that when we assume a distribution of citizens' ideal points that is symmetric around θ_L, we must have that $\theta_L = \frac{1}{2}$ given that we have assumed that there is at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$.

Given the symmetry assumed in the structure of the game, we will look for equilibria with symmetric distributions of attendees, that is, an equilibrium strategy profile $\{a_1, ..., a_N\}$ such that the distribution of the ideal points of the attendees is symmetric around θ^*. Since we must have $\theta_L = \frac{1}{2}$, in this equilibrium the policy preferred by the assembly coincides with the ideal point of the legislator, with the implemented policy, and with the most preferred policy of the society, i.e. $\theta^* = \theta_L = x = \theta^{**} = \frac{1}{2}$.

Next, we show that when the cost of attendance is relatively small, and the distribution of the ideal points of the citizens is symmetric around θ_L we can completely characterize equilibrium strategies in this case.

Proposition 4 (Equilibrium with a Symmetric Distribution) *For any finite number of citizens with distribution of ideal points symmetric around* $\theta_L = \frac{1}{2}$*, if $c < b$, there is an equilibrium in which $X = \{\theta_i : |\theta_L - \theta_i| > c\}$ and $x = \frac{1}{2}$.*

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[19]When $c = b$ some citizens (or all, depending on the distribution of ideal points of the population) could be indifferent between attending and not attending the meeting when nobody else is attending. Existence of equilibrium in this case would depend on how indifferences are resolved.

On the one hand, citizens who are close to the legislator's ideal point $(|\theta_L - \theta_i| < c)$ become the median if they were to attend, but the improvement of the policy outcome would be less than c. Hence, they prefer to stay home. On the other hand, citizens who are far away from the legislator's ideal point lose more than c from withdrawing. Thus, in this equilibrium all citizens whose ideal points are more of a distance c away from the legislator's ideal point will attend the meeting[20]. Note that in this extremely symmetric scenario, our results become a particular case of the result obtained by ORT for more general utility functions, since in this case the legislator's ideal point is equivalent to the default policy of their model.

4.2 Biased legislators, skewed populations

Next we analyze the attendance equilibrium strategies when the ideal point of the legislator takes values other than $\frac{1}{2}$ and the distribution of society's preferences is not necessarily symmetric. Recall that we have assumed that there is at least one citizen with ideal point $\theta_i = 0$ and at least one citizen with ideal point $\theta_i = 1$. Therefore, independently of the shape of the distribution, the fact that the legislator's ideal point is away from $\frac{1}{2}$ introduces some asymmetries. In this case, two factors arise as critical: *how aligned* the legislator is with society's preferences and *how extremist* (in the spatial sense) she is with respect to $\frac{1}{2}$.

First we analyze the case in which the legislator is relatively extremist with respect to $\frac{1}{2}$, that is, $\left|\theta_L - \frac{1}{2}\right| > b - c$. In this case we find that for any distribution of citizens' ideal points there is an equilibrium in which the legislator implements the maximal compromise policy on her left if her ideal point is to the right of $\frac{1}{2}$, and she implements the maximal compromise policy on her right if her ideal point is to the left of $\frac{1}{2}$. Furthermore, only one

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[20]If there was $\theta_i = \frac{1}{2} - c$ and $\theta_{i'} = \frac{1}{2} + c$, in a symmetric equilibrium they both would have a weak preference to attend, and a weak preference not to attend, in case both are attending and also in case only one of them attends. Furthermore, if they both would attend, then in equilibrium all $\theta_j \notin \left(\frac{1}{2} - c, \frac{1}{2} + c\right)$ would have a weak preference for attending. If they both would not attend, then all $\theta_j \notin \left(\frac{1}{2} - c, \frac{1}{2} + c\right)$ would have a strong preference for attending. Thus, in this case there is also an equilibrium in which only $\theta_i = \frac{1}{2} - c$ and $\theta_{i'} = \frac{1}{2} + c$ attend.

citizen attends the meeting in this equilibrium: a leftist one if the legislator is rightist, and a rightist one if the legislator is leftist.

Proposition 5 (Maximal Compromise Equilibrium) *If $c < b$:*

(a) $x = \theta_L + b$ is an equilibrium outcome if and only if $\theta_L < \frac{1}{2} - (b - c)$. Moreover, only one citizen with ideal policy $\theta_i > \max\{\theta_L + b, 2(\theta_L + b - c)\}$ attends.

(b) $x = \theta_L - b$ is an equilibrium outcome if and only if $\theta_L > \frac{1}{2} + b - c$. Moreover, only one citizen with ideal policy $\theta_i < \min\{\theta_L - b, 2(\theta_L - b + c) - 1\}$ attends.

Observe that this result holds for any distribution of the citizens' ideal points. When the legislator is extremist, citizens far from the legislator's ideal point have a strong incentive to participate. Moreover, one citizen is enough to induce the maximal compromise. But in this particular case, there is no response from those citizens who are extremist *and* close to the legislator since that maximum compromise is not far enough from them.

With extreme legislators we have thus an equilibrium in which only one citizen or association attends the meeting. This result can be thought of as an extreme case of ORT *Low participation* result. But it also challenges ORT *Non participation of the moderates* result: If the legislator preferences are too extreme, citizens in the center of the political spectrum have strong incentives to participate and moreover they are able to force her to implement the maximal compromise outcome in their favor. Note that this type of equilibria arise even if the legislator's policy preferences are aligned with the view of a majority of society.

Observe that the previous Proposition also implies that when the legislator's policy preferences are not extreme, in the spatial sense, a maximal compromise policy is never implemented in equilibrium.

Corollary 1 (Moderate legislator) *If $\left|\theta_L - \frac{1}{2}\right| \leq b - c$, in equilibrium we must have $x \in (\theta_L - b, \theta_L + b)$ and therefore $x(\theta^*, \theta_L) = \theta^*$.*

If the legislator is relatively moderate, in equilibrium the choice of the legislator must be a policy in the interior of the set of implementable policies.

And given the optimal play of the delegates and the legislator in the continuation of the game, we know that the latter will implement the assembly's ideal policy. Moreover, citizens located at both extremes have high stakes in the meeting, and the policy proposed will be relatively centrist.

Proposition 6 *Suppose that $c < b$ and $x \in (\theta_L - b, \theta_L + b)$ is the equilibrium outcome: if i does not attend then θ_i leaves the same number of attendees' ideal points on each side.*

In equilibrium, we have two sets of ideologically extreme attendees separated by one set of non-attendees. The most preferred policy of the assembly, according to the median, will always be the average of the two attendees surrounding the set of non-attendees. Let $\theta_l = \max \{\theta_i \in X : \theta_i < \theta^* (X)\}$ denote the most moderate leftist attendee, and let $\theta_r = \min \{\theta_i \in X : \theta_i > \theta^* (X)\}$ denote the most moderate rightist attendee. Then, the most preferred policy by the assembly according to the median is given by $\theta^* (X) = \frac{\theta_l + \theta_r}{2}$.

Indeed, we can completely characterize the set of citizens who attend the meeting in equilibrium: exactly all those citizens whose ideal points are further than c from the policy outcome.

Proposition 7 (Non-participation of the represented) *If $c < b$ and $x \in (\theta_L - b, \theta_L + b)$ is the equilibrium outcome, then i attends if and only if $|\theta_i - x| > c$.*

Since incentives to attend are given by the impact of attendance decisions on the final outcome, those citizens already represented by the legislator prefer to stay home. Thus, we generalize the result on attendance provided in ORT. Note that this Proposition and the two previous ones, are generalizations of Proposition 4 to values of the legislator's ideal point different from $\frac{1}{2}$, and to any distribution of the citizens' policy preferences.

Observe that the results stated in Propositions 6 and 7 hold for any value of the legislator's ideal point. Therefore, the interior equilibrium described here may exist for any location of the legislator's ideal point. These results offer a broader picture of ORT *Non participation of the moderates* result:

Those citizens who are already represented by the predicted outcome or by the legislator's ideology, have no incentive to participate and will not attend the meeting. But they are not necessarily moderates in the political spectrum. Interestingly enough, this is consistent with the observed lack of participation of unions in the Porto Alegre's Participatory Budgeting process. Marquetti (2000) argues that one of the reasons explaining this is that unionists feel that they are already represented in the Participatory Budgeting by the leftist leaning of the municipal administration.

Observe that the conditions to be satisfied in an interior equilibrium are rather strong:

(i) There must be an identical number of attendees on both sides of the set of non-attendees, so $\theta^{*}(X) = \frac{\theta_{l}+\theta_{r}}{2}$;

(ii) All attendees must be at more than a distance c of the policy outcome.

What these two conditions imply is that the existence of an interior equilibrium will depend largely on the shape of the distribution of the ideal points of the population. Furthermore, they impose stronger restrictions on the set of implementable policies.

Proposition 8 *If $c < b$ and $x \in (\theta_{L} - b, \theta_{L} + b)$ is an equilibrium outcome, then $x \in (\theta_{L} - b + c, \theta_{L} + b - c)$.*

Next we show that a necessary condition for existence of an interior equilibrium for any value of the legislator's ideal point is that the policy preferences of the legislator must be aligned with those of society.

Proposition 9 (Alignment is needed) *If $c < b$ and $\theta^{**} \notin (\theta_{L} - b, \theta_{L} + b)$ then there is no interior equilibrium.*

If society's most preferred policy θ^{**} does not belong to the set of implementable policies then an interior equilibrium does not exist. But when it does, the policy implemented will coincide with the most preferred policy of the assembly, and will be close to the most preferred policy of society as well.

Nevertheless, the previous results have a rather negative implication about existence of this type of equilibrium:

Corollary 2 (Equilibrium existence) *If $c < b$ and $\theta^{**} \notin (\theta_L - b, \theta_L + b)$ there is no equilibrium when the legislator is moderate. When the legislator is extremist, maximal compromise is the unique equilibrium.*

Therefore, when the preferences of the legislator are relatively moderate, the alignment between the legislator's and society's preferences is a necessary condition for the existence of an equilibrium in pure strategies. The following example illustrates the non-existence of equilibrium in pure strategies when the median of society is not well aligned with the ideal point of the legislator and the legislator is moderate.

Example: Non-existence of equilibrium

Consider a society with only three citizens located at 0, $\frac{1}{3}$, and 1. Suppose that the legislator is located at $\frac{3}{5}$ and suppose that $b = \frac{1}{5}$. In this case, the ideal point of the median cannot be implemented in equilibrium. Suppose that the cost of attending is $c < \frac{1}{10}$. We will show that there is no pure strategy equilibrium.
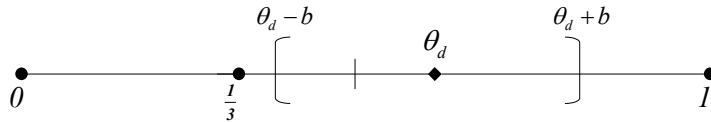

Figure 3: Non-Existence of Equilibrium.

It is clear that all citizens want to attend when nobody is attending, since the attendance cost c is smaller that the impact each one of them has on the final outcome. If all of them were attending, the median of the assembly would be $\frac{1}{3}$, and the final outcome $\theta_L - b = \frac{2}{5}$. Since the legislator will not compromise beyond that, the citizen located at 1 would be better off withdrawing. If the only attendees were the two leftist citizens, any of them would be better off not attending. If the two extremist citizens were the ones attending the meeting, then the citizen at $\frac{1}{3}$ would prefer to attend since his

utility would increase by $\frac{1}{2} - (\theta_L - b) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} > c$. Finally, if $\frac{1}{3}$ and 1 were the ones to attend, the citizen at 0 would prefer to attend since his utility would increase by $\frac{2}{3} - (\theta_L - b) = \frac{2}{3} - \frac{2}{5} = \frac{4}{15} > \frac{1}{10} > c$. Hence, there is no equilibrium in pure strategies.

We have shown that in a system of participatory democracy, citizens, under certain conditions, may obtain their most preferred policy in equilibrium. But we have also shown that otherwise the policy outcomes may imply instability. Next we characterize general conditions on the distribution of citizens' ideal points that provide the necessary alignment and guarantee the existence of an interior equilibrium.

Let us define the set:

$S = \{x \in (\theta_L - b, \theta_L + b) : x = \frac{\theta_l(x) + \theta_r(x)}{2}$ for some $\theta_l(x)$ and $\theta_r(x)$

$\quad$ s.t. $\theta_l(x), \theta_r(x) \in (\theta_L - b, \theta_L + b)$ and $|\{\theta_i : \theta_i \leq \theta_l(x)\}| = |\{\theta_i : \theta_i \geq \theta_r(x)\}|\}$.

From previous results we know that an equilibrium outcome must belong to the set of implementable policies, $[\theta_L - b, \theta_L + b]$, and that an interior equilibrium outcome has to be the average of the ideal points of the two attendees delimiting the set of non-attendees. Thus, the policies contained in S are all candidates to equilibrium outcomes. Notice that if $\theta^{**} \notin (\theta_L - b, \theta_L + b)$ then the set S is empty. The next Proposition derives a sufficient condition under which the elements of S can be implemented in equilibrium.

Proposition 10 *If $c < b$, then for each $x \in S$ there exist $\underline{c}(x)$ and $\overline{c}(x)$ such that if $c \in (\underline{c}(x), \overline{c}(x))$, x is an equilibrium outcome.*

These condition ensures the stability of the division of citizens into "extreme" sets of attendees separated by a set of non-attendees. It comes from Proposition 8: the cost should be sufficiently small to offer to relatively extreme citizens enough incentives to attend the meeting but it must be high enough to discourage citizens with ideal points relatively close to the equilibrium policy outcome from attending. In that case, the assembly's most preferred policy can be implemented in equilibrium. Recall that in this case the assembly most preferred policy is close to society's most preferred policy.

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5 Revisiting Porto Alegre

In this Section, we analyze, in the light of the results obtained above, the political events witnessed in Porto Alegre since the Participatory Budgeting experience started. Our claim is that the two types of equilibria characterized in the previous sections represent two different scenarios observed in the city.

One of the main tasks of the popular assemblies in Porto Alegre is to set a consensual rank of priorities within districts and a list of hierarchical demands inside each priority. There are a total of thirteen issue areas available: Basic Sanitation, Land use regulation (that includes Housing policy), Transportation, Social Service, Education, Health, Paving, City organization, Leisure, Sports, Culture, Environment and Economic Development.

At the early stages of the implementation of the Participatory Budgeting system, participation was very low (see Figure 1 below) and the priorities selected at the assemblies referred to issues of interest to the working classes: all the investment budget was devoted to cover basic needs. Table 1 displays the three top priorities selected in the period 1992-2005. It shows that three areas received the main interest of the population: Paving (that also includes public illumination and garbage collection), Basic Sanitation (water supply and sewage disposal) and Land Use Regulation (house production, relocation of families living in slums). These priorities simply permuted their rankings in the priority list during the first half of that period.

Year	1st Priority	2nd Priority	3rd Priority
2005	Housing	Education	Social Services
2004	Housing	Social Services	Education
2003	Housing	Education	Paving
2002	Housing	Education	Paving
2001	Paving	Housing	Basic Sanitation
2000	Housing Policy	Paving	Health
1999	Basic Sanitation	Paving	Housing Policy
1998	Paving	Housing Policy	Basic Sanitation
1997	Housing Policy	Paving	Basic Sanitation
1996	Paving	Basic Sanitation	Land use regulation
1995	Paving	Land use regulation	Basic Sanitation
1994	Land use regulation	Paving	Basic Sanitation
1993	Basic Sanitation	Paving	Land use regulation
1992	Basic Sanitation	Education	Paving

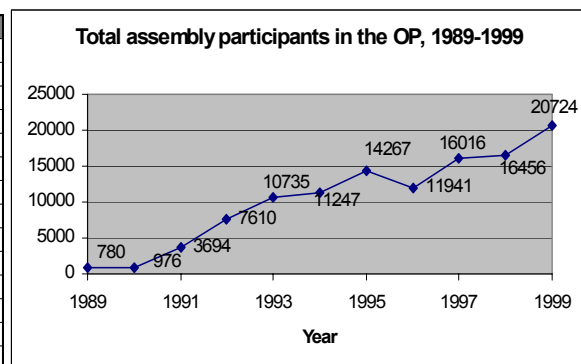Source: Prefeitura Municipal de Porto Alegre (2005)

Table 1

Figure 1

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In that case, the best interpretation of our unidimensional policy space is the population's wealth distribution, as it can be thought as a representation of the citizens' preferences over basic needs. When policy preferences are given by the wealth distribution, the legislator is expected to be extreme with respect to the population, since elected representatives normally belong to high-income segments, specially in Brazil where barriers to entry into politics are important. Our prediction in this case is given by the maximal compromise equilibrium: very few citizens with preferences opposed to the legislator's attend the meeting and they force the legislator to implement a policy close to their most preferred one.

As a result, the more pressing needs were effectively alleviated: In 1989 in Porto Alegre only 49% of the population was covered by basic sanitation; this percentage rose to 85% in 1996. Over the same period, the proportion of households with water supply reached the 99.5% and the number of students enrolled in elementary schools increased by 190%; five times more housing units were constructed in the period 1993-96 than in the period 1986-88.[21]

At later stages of the implementation of the Participatory Budgeting system, once basic needs had been covered, the priorities selected shifted to issues that were also attractive to middle class citizens like Education, Health and Social Services. The composition of the attendance to the meetings changed: The percentage of participants in assemblies with up to 4 minimum wages fell from 62% in 1995 to 54% in 2000 (the average household income in Porto Alegre was thirteen minimum wages in 1998)[22]. In our model, this can be thought of as a change of the relevant policy space, and therefore, as a change in the relevant distribution of the citizens' preferences. Thus, it is reasonable to think that the legislator's policy preferences became more aligned with those of the population. Our prediction in that case is given by the interior equilibrium when the policy preferences of the legislator and

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[21] The extent to what these improvements can be attributed to the process of Participatory Budgeting or to the changes in the city goverment is still subject to debate. But the evidence shows that they took place against the overall trend in Brazil as a whole (see Marquetti, (2000) and the mentioned report by the Inter-Amercican Development Bank (2003)).

[22] See Marquetti (2000) and CIDADE (2002).

those of the society are aligned: Participation is expected to be higher and the policy implemented should be close to the most preferred policy by the assembly. The political scenario in Porto Alegre seems to confirm this, as suggested by: 1) the rising figures of participants (see Figure 1), and 2) the surveys on the citizens' level of satisfaction with the process: in 1995, 60.3% of the citizens replied that the people who participate in the process either "always" or "most of the time" were decisive on public policies; in 2002 this percentage rose to the 69% (CIDADE, 2002).

Casual evidence also shows a pattern of confrontation within assemblies that resembles the opposition of blocks of extremists that characterize the interior equilibrium of our model. Santos (1998) reports, that once the lack of physical infrastructures was attenuated, there were increasing levels of conflict of interests (regarding culture and leisure issues) between the participants at the early (regional) meetings, typically coming from low income groups, and those who attend the later (thematic) meetings, more educated and wealthy.

6   Concluding Remarks

In this paper, we have proposed a model of participatory democracy inspired by the system of participatory budgeting implemented in the city of Porto Alegre. This experiment, now extended to many other cities worldwide, shows that a participatory system at the local level is indeed possible and can successfully, but not without problems, govern large communities.

We have analyzed participatory democracy by introducing a legislator, with the role of policy implementation, in a formal model of direct democracy, and we have shown that this political system is characterized by a relative autonomy between citizens and the legislator: the former will not be able to implement any policy, and the latter has to acquire calculated compromises.

We have found two different kinds of equilibria: a maximal compromise equilibrium and an interior equilibrium. The former equilibrium always exists when the preferences of the legislator are extreme with respect to the policy space. In this equilibrium only one citizen (or community association),

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with preferences opposed to those of the legislator's, attends the meeting and the policy outcome is relatively moderate. An interior equilibrium only exists when the preferences of the legislator are aligned with the majoritarian views of the population. This equilibrium may exist with an ideologically extreme or moderate legislator, but only for a certain subset of distributions of citizens' ideal points. In this equilibrium the policy implemented is the most preferred by the assembly and it is close to the policy most preferred by society. The number of attendees depends on the distribution of the preferences of society, and the composition of the assembly is characterized by two groups of equal size and opposed preferences.

The evidence from Porto Alegre can be explained by these two different scenarios: the lack of alignment between citizens and representatives concerning basic needs could have induced the low levels of participation observed at first, as predicted by our maximal compromise equilibrium. When the population demands shifted from basic infrastructures, alignment increased and, as predicted by our interior equilibrium, both the level of participation and of confrontation among citizens rose; moreover, the policies finally implemented seemed to match better the citizens' proposals.

One natural question about the robustness of our results refers to our assumption on the legislator's probability of reelection. But the two kinds of equilibria obtained are not an artifact of our specification. For instance, suppose that the continuous function $P\left(x^*, x\right) = 1 - \left(x - x^*\right)^2$ represents the probability with which the legislator is reelected as a function of the assembly's proposal and of the implemented policy. It is easy to show that the optimal decision of the legislator given a policy proposal x^* is $x = \left(1 - \alpha\right)x^* + \alpha\theta_L$. And given the optimal choice of the delegates we have that the policy implemented in equilibrium will be always in a neighborhood of the legislator's ideal point: $\left[\alpha\theta_L, \left(1 + \alpha\right)\theta_L\right]$. In particular, for $\theta^* < \theta_L$ in equilibrium we will have $x = \theta^*$ whenever $\theta^* \geq \alpha\theta_L$; and $x = \alpha\theta^*$ whenever $\theta^* < \alpha\theta_L$. Similarly, for $\theta^* > \theta_L$ in equilibrium we will have $x = \theta^*$ whenever $\theta^* \leq \left(1 + \alpha\right)\theta_L$; and $x = \left(1 + \alpha\right)\theta^*$ whenever $\theta^* > \left(1 + \alpha\right)\theta_L$. Thus, the results are qualitative identical to the ones we obtain with the step function. Therefore, the discontinuity obtained in our characterization

of equilibria is not implied by the probability of reelection we employ but rather by the non-alignment of preferences between legislator and society.

Another question refers to the plausibility of having a legislator whose preferences are not aligned with society's. It may seem natural to expect that only the legislators with desirable preferences will be selected in the long run, so alignment will be ultimately achieved. This would probably be the case under perfect competition in the political arena. But in the market of political candidates, barriers to entry abound. It is very rare for representatives to come from average or below-average income levels. Entering political competition often entails some costs, mostly financial. If preferences over policies are given by wealth, then it is not reasonable to expect preferences within the pool of potential candidates to be similar to those of the society as a whole.

Two final remarks are in order. The main institutional features of participatory democracy may seem striking at first glance: only the opinions of those who participate at the meetings are taken into account. On the contrary, in a representative democracy everybody's opinion can be heard through the simple act of voting (surely, casting a vote is cheaper than attending an assembly). This comparison is misleading for two reasons: First, as mentioned, many of the participants in the assemblies are representing families or communities associations so it is not correct to think that only a few thousand opinions are heard. Another reason is that, as the low level of turnout in Western democracies indicates, this alleged advantage of representative democracy is doubtful: it is well known that the fraction of population systematically disenfranchised from the representative processes are the less educated and wealthy, precisely those segments overwhelmingly represented at the assemblies in Porto Alegre (see Appendix 1).

Finally, our model is not able to generate enough observables that would allow us to perform comparisons between different political systems. An avenue for further research would be to relax some of the assumptions of the model in order to be able to perform these comparisons. In Brazil, almost two-hundred municipalities have adopted a Participatory Budgeting process but they are still a minority. There is certainly room for applying the present

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model to the ongoing research comparing economic performances across these cities and over time.

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A   Appendix 1: Participatory Budgeting in Porto Alegre

During the last decade, Participatory Budgeting has been implemented in more than one hundred Brazilian cities, including some state capitals such as Porto Alegre (with a population of 1.4 million inhabitants), Belo Horizonte (2 million) and Recife (1.5 million). There are two elements that explain the introduction of this system in Brazil during this period: 1) the new Brazilian Constitution of 1988 that modified the balance of political power in the country and signalled a political and administrative decentralization, and 2) the increasing number of active community associations that were politically involved.

Porto Alegre's system of Participatory Budgeting (*Orçamento Participativo*), referred to as OP, is the best known and most successful experiment of local management based on participatory democracy. It was introduced in 1989 when the Workers Party (*Partido dos Trabalhadores,* PT henceforth) won the local elections[23].

The OP is a pyramidal system whose main elements are: the regional and thematic assemblies, the Fora of Delegates, and the Council of the OP (COP). Regional assemblies, called *rodadas,* take place in each of the sixteen regions of the city during April and May. These assemblies are the principal forums for popular participation; they are totally open and any citizen may attend them. In these meetings, each region evaluates the executive's performance, defines its priorities and demands, and elects delegates for the Forum of Delegates and councillors for the COP. Prior to the *rodada,* preparatory meetings organized by the community take place.[24]

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[23]The description of Participatory Budgeting in Porto Alegre builds on Santos (1998) and Marquetti (2000) and (2003).

[24]Until 2002 there were two rounds of *rodadas*; the second round was supressed because

Public scrutiny and control of the municipal government is the main issue at the early meetings. The municipality accounts for the implementation of the previous year Investment Plan. In the ensuing meetings, discussions focus on setting a consensual regional rank of priorities and a list of hierarchical demands inside each priority. Each region selects five out of the mentioned thirteen issue areas available. All decisions are taken by majority rule. The choices of each region are ranked according to three criteria: lack of the service or infrastructure, population, and regional and city ranking of the priority.

Thematic assemblies (*tematicas*) take place along with the *rodada* and cover six areas (Health and Social Welfare, Transportation and Circulation, City Organization and Urban Development, Culture and Leisure, Education and Economic Development and Taxation). Participation depends on the interest that citizens might have in the area. Decisions are also taken by majority rule.

The Fora of Delegates is formed by about one thousand delegates. They are elected during the popular meetings according to criteria based on the number of participants. Their role is to serve as intermediaries between the COP and the citizens. They supervise the implementation of the budget and inform the population. Delegates are typically leaders of community organizations, so citizens not integrated in these structures are hardly elected.

Finally, the COP is a body composed of 44 councilors: two councilors for each region assembly (32), two for each thematic assembly (10), one representative of the Residents Association Union of Porto Alegre, and one from the City Hall's Attendants Labor Union. It is constituted in July of each year and its role is to design and submit to the city government a detailed budget proposal based on the priorities determined in the regional assemblies, and to monitor the execution of the approved public works.

The OP is an example of participatory democracy because it reconciles direct democracy, embedded in associations and meetings, and representative democracy at the urban level. Assemblies coexist with two elected bodies who hold the formal municipal power: The Mayoralty or executive body

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it was increasingly seen as redundant.

(*Prefeitura*) and the Chamber of Deputies or legislative body (*Câmara de Vereadores*). The COP submits the budget proposal to the Chamber of Deputies who has total autonomy to amend or defeat it. However, since the proposal has been approved by citizens, assemblies, and community organizations, the political cost of turning it down is very high, and the Chamber has never done it.

The relationship between the "formal" elected representatives and popular movements has not been without problems. In fact, the conflict between them has been one of the main political issues in Porto Alegre. On the one hand, the OP has been criticized because the PT community leaders seem to have helped the party to "capture" the process by making regions' political agenda fit into the PT's one. On the other hand, the executive has been accused of abusing its privileged position when resorting to "technical reasons" in order to challenge the budget proposal; councillors and delegates have reported that they have been denied relevant information by the city technical staff in some occasions. This problem was serious enough to prompt the COP to start training seminars.

Nevertheless, the city has witnessed a remarkable improvement regarding the behavior of the politicians and community leaders who, as in the rest of Brazil, were used to clientelism. Now, the city councilors and potential candidates face a more informed population and more politicized grass-root organizations, so there is little space for the "gift exchange" that characterizes clientelism and corruption. The high degree of accountability of the administration has reduced corruption and rent seeking behavior.

One remarkable success of the OP has been the massive participation of those segments of the population typically disengaged from the institutions of representative democracy. This can be seen in Figure 2 showing the income profile of both participants and inhabitants of Porto Alegre. A large majority of participants in the OP structures has a household income below the average. Since the average household income in Porto Alegre in 1998 was thirteen minimum wages (Marquetti, 2000), it becomes clear that the less wealthy citizens are overrepresented in the OP. Results regarding the education level of the participants follow the same pattern: 56.5% of the par-

ticipants have completed less than 8 years of schooling. Finally, middle-class segments participate more in the OP at higher levels, so the composition of the COP is closer to a random sample of the population.
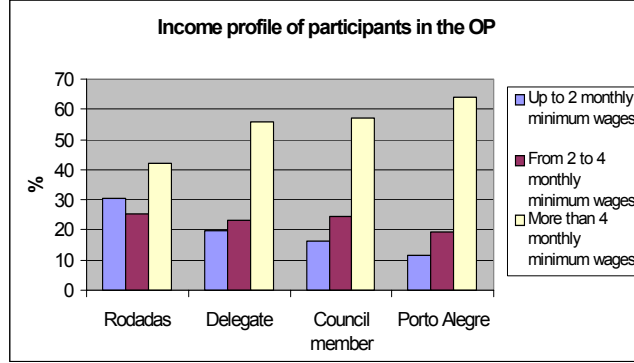
Income profile of participants in the OP

Figure 2

B Appendix 2: Proofs

Proof of Proposition 1: Implementable Policies in Equilibrium.

In order to find the policies implemented by the legislator in equilibrium we combine the optimal choice functions of the legislator and the assembly.

$$
x\left(x^*\right) = \begin{cases} x^* + B & if \ B \leq |x^* - \theta_L| \leq b + B \ \ and \ \ x^* \leq \theta_L \\ x^* - B & if \ \ B \leq |x^* - \theta_L| \leq b + B \ \ and \ \ x^* \geq \theta_L \\ \theta_L & \text{otherwise} \end{cases}
$$

$$
x^*\left(\theta^*\right) = \begin{cases} \theta_L - b - B & if \ \ \theta^* \leq \theta_L - b \\ \theta^* - B & if \ \theta_L - b \leq \theta^* \leq \theta_L \\ \theta^* + B & if \ \theta_L \leq \theta^* \leq \theta_L + b \\ \theta_L + b + B & if \ \theta_L + b \leq \theta^* \end{cases}
$$

We have to consider four different cases:

(i) if $\theta^* \leq \theta_L - b$, then the best response of the assembly is to propose $x^* = \theta_L - b - B$. In this case, $\theta_L - x^* = b + B$ and $x^* \leq \theta_L$, thus the legislator

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implements $x = x^* + B = \theta_L - b$.

(ii) if $\theta_L - b \leq \theta^* \leq \theta_L$, then the best response of the assembly is to propose $x^* = \theta^* - B$. In this case, $B = \theta^* - x^* \leq \theta_L - x^* = \theta_L - \theta^* + B \leq b + B$ and $x^* \leq \theta_L$, thus the legislator implements $x = x^* + B = \theta^*$.

(iii) if $\theta_L \leq \theta^* \leq \theta_L + b$, then the best response of the assembly is to propose $x^* = \theta^* + B$. In this case, $B = x^* - \theta^* \leq x^* - \theta_L = \theta^* + B - \theta_L \leq b + B$ and $x^* \geq \theta_L$, thus the legislator implements $x = x^* - B = \theta^*$.

(iv) if $\theta_L + b \leq \theta^*$, then the best response of the assembly is to propose $x^* = \theta_L + b + B$. In this case, $\theta_L - x^* = b + B$ and $x^* \geq \theta_L$, thus the legislator implements $x = x^* - B = \theta_L + b$.

Therefore, in equilibrium the legislator will implement the most preferred policy of the assembly only when it belongs to the interval $[\theta_L - b, \theta_L + b]$, otherwise the only policies that will be implemented in equilibrium are $\theta_L - b, \theta_L + b$. ∎

Proof of Proposition 2: Non Attendance.

First we show that when $c > b$ there is an equilibrium in which nobody attends the meeting.

Suppose that nobody attends the meeting. In this case the legislator will implement her ideal point, thus $x = \theta_L$. Consider a citizen i such that $\theta_i \leq \theta_L - b$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i| = \theta_i - \theta_L$. If he were to attend, he would be the only participant. Since his ideal point is not within the interval $[\theta_L - b, \theta_L + b]$ the best he could obtain is $x = \theta_L - b$, thus his payoff would be $V_i(\theta_L - b, 1) = -|\theta_L - b - \theta_i| - c = \theta_i - \theta_L + b - c$. Since $c > b$ we have that $V_i(\theta_L, 0) > V_i(\theta_L - b, 1)$, so he is better off not attending.

Now consider a citizen i such that $\theta_L - b \leq \theta_i \leq \theta_L$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i|$. If he were to attend, he would be the only participant. Since his ideal point is within the interval $[\theta_L - b, \theta_L + b]$ he would be able to obtain his ideal point θ_i as the policy output, thus his payoff would be: $V_i(\theta_i, 1) = -|\theta_i - \theta_i| - c = -c$. Since $c > b > |\theta_L - \theta_i|$, we have that $V_i(\theta_L, 0) = -|\theta_L - \theta_i| > V_i(\theta_i, 1) = -c$, so he is better off not attending. Using a symmetric argument we can also prove that all citizens with $\theta_i \geq \theta_L$ are better off not attending the meeting when

nobody else is attending.

Next we will show that nobody attending the meeting is the unique equilibrium of the game when $c > b$.

Let $x(X) = x(\theta^*(X))$ denote the policy implemented as a function of the set of citizens who attend the meeting, X, given that citizens and legislator are playing their best responses, the last two stages of the game.

Observe that for any set of attendees in equilibrium X, and for any $\theta_i \in X$ we must have that $V_i(x(X), 1) = -|x(X) - \theta_i| - c > V_i(x(X - \{\theta_i\}), 0) = -|x(X - \{\theta_i\}) - \theta_i|$, that is $|x(X) - x(X - \{\theta_i\})| > c$. Since $x \in [\theta_L - b, \theta_L + b]$ we have that:

(i) If $\theta_i \leq \theta_L$, this condition implies that $\theta_L - b \leq x(X) \leq \theta_L + b - c < \theta_L$.

(ii) If $\theta_i \geq \theta_L$, this condition implies that $\theta_L < \theta_L - b + c \leq x(X) \leq \theta_L + b$.

These two conditions cannot hold simultaneously. These implies that in equilibrium we must have that either $X = \{\theta_i : \theta_i \leq \theta_L\}$ or $X = \{\theta_i : \theta_i \geq \theta_L\}$. Suppose that $X = \{\theta_i : \theta_i \leq \theta_L\}$, then since all $\theta_i \geq \theta_L$ do not attend the meeting we must have that $x(X) \leq \theta_L$ and $x(X - \{\theta_i\}) \leq \theta_L$ for all $\theta_i \in X$. But this implies that $|x(X) - x(X - \{\theta_i\})| < b < c$ which contradicts the condition that makes attendance optimal. Similarly we can prove that $X = \{\theta_i : \theta_i \geq \theta_L\}$ leads to a contradiction. Therefore, if $c > b$ there cannot be an equilibrium in which some citizens attend.∎

Proof of Proposition 3: Attendance in Equilibrium.

We will show that if nobody attends the meeting any citizen with ideal point θ_i such that $|\theta_i - \theta_L| > c$ would be better off attending the meeting. Since we have assumed that there is at least a citizen with $\theta_i = 0$ and at least a citizen with $\theta_i = 1$, we will always have someone that has a profitable deviation. Suppose that $c < b$ and nobody attends the meeting, that is, $X = \varnothing$. Consider a citizen with θ_i such that $b > |\theta_L - \theta_i| > c$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i|$. If he were to attend, he would be the only participant. Since his ideal point is within the interval $[\theta_L - b, \theta_L + b]$ he would be able to obtain his ideal point θ_i as the policy output, thus his payoff would be $V_i(\theta_i, 1) = -|\theta_i - \theta_i| - c = -c$. Since $c < |\theta_L - \theta_i|$, we have that $V_i(\theta_L, 0) = -|\theta_L - \theta_i| < V_i(\theta_i, 1) = -c$. Therefore, he would be better off attending.

Now consider a citizen with θ_i such that $\theta_i \leq \theta_L - b$. Since he does not attend the meeting his payoff is $V_i(\theta_L, 0) = -|\theta_L - \theta_i| = \theta_i - \theta_L$. If he were to attend, he would be the only participant. Since his ideal point is not within the interval $[\theta_L - b, \theta_L + b]$ the best policy he could obtain is $x = \theta_L - b$, thus his payoff would be $V_i(\theta_L - b, 1) = -|\theta_L - b - \theta_i| - c = \theta_i - \theta_L + b - c$. Since $c < b$ we have that $V_i(\theta_L, 0) < V_i(\theta_L - b, 1)$. Therefore, he would be better off attending.

Similarly we can prove that any citizen with $\theta_i \geq \theta_L + b$ would be better off attending the meeting. Therefore, nobody attending the meeting cannot be an equilibrium.∎

Proof of Proposition 4: Symmetric Equilibrium.

We are assuming that $F(\theta_i)$ is symmetric around $\theta_L = \frac{1}{2}$ and $c < b$. We will show that there is a symmetric equilibrium in which $X = \{\theta_i : |\theta_i - \theta_L| > c\}$, by proving that all $\theta_i \in X$ are better off attending and all $\theta_i \notin X$ are better off not attending.

Suppose that $X = \{\theta_i : |\theta_i - \theta_L| > c\}$ and consider a citizen $\theta_i \in X$ such that $\theta_i < \frac{1}{2} - c$. Since he attends the meeting his payoff is $V_i(\frac{1}{2}, 1) = -|\theta_i - \frac{1}{2}| - c = \theta_i - \frac{1}{2} - c$. If he would not attend his utility would be $V_i(x(X - \{\theta_i\}), 0) = -|\theta_i - x(X - \{\theta_i\})| = \theta_i - x(X - \{\theta_i\})$. Thus he is better off attending if and only if $x(X - \{\theta_i\}) > \frac{1}{2} + c$. We will show that this is always the case.

Since $x(X - \{\theta_i\}) = \min\{\theta^*(X - \{\theta_i\}), \frac{1}{2} + b\}$, and we have that $\theta^*(X - \{\theta_i\}) = median(X - \{\theta_i\}) > \frac{1}{2} + c$ and $b > c$, then we must have that $x(X - \{\theta_i\}) > \frac{1}{2} + c$.

Now consider a citizen $\theta_i \notin X$ such that $\frac{1}{2} - c < \theta_i < \frac{1}{2}$. Since he does not attend his utility is $V_i(\frac{1}{2}, 0) = -|\theta_i - \frac{1}{2}| = \theta_i - \frac{1}{2}$. If he was to attend he would obtain $V_i(x(X \cup \{\theta_i\}), 1) = -|\theta_i - x(X \cup \{\theta_i\})| - c = -c$ since in this case he would become the median of the assembly, that is, $\theta^*(X \cup \{\theta_i\}) = median(X \cup \{\theta_i\}) = \theta_i$ and since $\theta_i \in [\theta_L - b, \theta_L + b]$ his ideal point would be implemented $x(X \cup \{\theta_i\}) = \theta_i$. Thus, he is better off not attending because $\frac{1}{2} - c < \theta_i$.∎

Proof of Proposition 5: Maximal Compromise equilibrium.

We will only prove part (a). The proof of part (b) is almost identical and

is left to the reader.

First we show that if $x(X) = \theta_L + b$ is the equilibrium outcome, then X must be a singleton. Suppose that the equilibrium outcome is $x(X) = \theta_L + b$. Then for all $\theta_i \in X$ such that $\theta_i < \theta_L - b$ we have that $x(X - \{\theta_i\}) = x(X)$, therefore we must have that $V_i(x(X - \{\theta_i\}), 0) > V_i(x(X), 1)$. Therefore, they would be better off not attending and in equilibrium we must have that $X \subseteq \{\theta_i : \theta_i \geq \theta_L + b\}$.

Given that, for all $\theta_i \in X$ such that $\theta_i \geq \theta_L + b$ we have that
$$x(X - \{\theta_i\}) = \begin{cases} x(X) & if\ |X| > 1 \\ \theta_L & if\ |X| = 1 \end{cases}$$
This implies that $V_i(x(X - \{\theta_i\}), 0) > V_i(x(X), 1)$ as long as $|X| > 1$. Therefore, they would be better off not attending as long as $|X| > 1$. Therefore, the only possibility for equilibrium is $|X| = 1$.

Next we show that there is an equilibrium with $x(X) = \theta_L + b$ iff $\theta_L < \frac{1}{2} - b + c$. We already know that if $x(X) = \theta_L + b$ is the equilibrium outcome, we must have $X = \{\theta_i\}$ with $\theta_i \geq \theta_L + b$. Observe that if a citizen θ_i with $\theta_i \geq \theta_L + b$ is the only citizen who attends the meeting his best proposal is $x^* = \theta_L + b + B$, the policy implemented is $x = \theta_L + b$ and he obtains $V_i(\theta_L + b, 1) = -|\theta_i - \theta_L - b| - c = \theta_L + b - \theta_i - c$. If he was not to attend he would obtain $V_i(\theta_L, 0) = -|\theta_i - \theta_L| = \theta_L - \theta_i$. Thus he is better off attending since $b > c$.

Suppose that $X = \{\theta_i\}$ with $\theta_i \geq \theta_L + b$. Observe that for any $\theta_j \geq \theta_L + b, \theta_j \neq \theta_i$ we have that $x(X \cup \{\theta_j\}) = \theta_L + b$, thus they would gain nothing by attending, and they would have to pay the cost. Therefore, they are all better off not attending.

Now consider a θ_j with $\theta_j < \theta_L + b$. If we show that $\theta_j = 0$ prefers not to attend the meeting, we will have that all $\theta_j < \theta_L + b$ prefer not to attend.

If $\theta_j = 0$, if he does not attend the meeting he obtains $V_0(x(X), 0) = -|\theta_L + b| = -\theta_L - b$; if he were to attend the policy outcome would be $x(X \cup \{0\}) = \max\{\frac{\theta_i}{2}, \theta_L - b\}$ and thus he would obtain

$$V_0(x(X \cup \{0\}), 1) = -\left|\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\}\right| - c = -\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} - c$$

We have that he is better off not attending iff $-\theta_L - b > -\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} - c$ iff $\theta_L + b - c < \max\left\{\frac{\theta_i}{2}, \theta_L - b\right\}$.

Notice that if $\max\left\{\frac{\theta_i}{2}, \theta_L - b\right\} = \theta_L - b$ the inequality is never satisfied. Thus it is necessary and sufficient that we have $\frac{\theta_i}{2} > \theta_L + b - c$, that is $\theta_i > 2(\theta_L + b - c)$. And there would be such a citizen if and only if $2(\theta_L + b - c) < 1$ iff $\theta_L < \frac{1}{2} - b + c$.

Thus we have shown that there is an equilibrium with $x(X) = \theta_L + b$ iff $\theta_L < \frac{1}{2} - b + c$. And in this equilibrium we have that $X = \{\theta_i\}$ with $\theta_i > \max\{\theta_L + b, 2(\theta_L + b - c).\}$.

Similarly we could show that there is an equilibrium with $x(X) = \theta_L - b$ iff $\theta_L > \frac{1}{2} + b - c$. And in this equilibrium we would have that $X = \{\theta_i\}$ with $\theta_i < \min\{\theta_L - b, 2(\theta_L - b + c) - 1\}$. $\blacksquare$

Proof of Proposition 6: Symmetric Attendance.

This result is directly implied by the two following lemmata with their symmetric counterparts.

Lemma 1: *For any distribution of citizens' ideal points, if $c < b$, the equilibrium outcome satisfies $x(X) \in (\theta_L - b, \theta_L + b)$, and there is a $\theta_i < x(X)$ (or a $\theta_i > x(X)$) that is not attending, then he must leave either the same number of attendees at each side or one more attendee to her right (left).*

Proof of Lemma 1:

Suppose that $\theta_i < x(X)$ is not attending, and there are k attendees on her left and $k + l$ attendees on her right with $l > 1$. Let $\theta_1, ..., \theta_l$ denote the first l attendees on the right of θ_i, then $x(X) = median(\theta_1, ..., \theta_l)$.

Suppose that l is odd. Then we have that $x(X) = \theta_{\frac{l+1}{2}}$ and $x(X \cup \theta_i) = x\left(X - \theta_{\frac{l+1}{2}+1}\right) = \max\left\{\frac{\theta_{\frac{l+1}{2}-1} + \theta_{\frac{l+1}{2}}}{2}, \theta_L - b\right\}$. But we need $x(X) - x\left(X - \theta_{\frac{l+1}{2}+1}\right) > c$ because $\theta_{\frac{l+1}{2}+1}$ is attending, and $x(X) - x(X \cup \theta_i) < c$ because θ_i is not attending. Therefore we have a contradiction .

Now suppose that l is even. Then we have that $x(X) = \frac{\theta_{\frac{l}{2}} + \theta_{\frac{l}{2}+1}}{2}$ and $x(X \cup \theta_i) = x\left(X - \theta_{\frac{l}{2}+1}\right) = \max\left\{\theta_{\frac{l}{2}}, \theta_L - b\right\}$. But we need $x(X) - x\left(X - \theta_{\frac{l}{2}+1}\right) > c$ because $\theta_{\frac{l}{2}+1}$ is attending, and $x(X) - x(X \cup \theta_i) < c$ because θ_i is not attending. Therefore we have a contradiction. Thus, we have proved that in

equilibrium we must have $l \leq 1.\blacksquare$

Lemma 2: *For any distribution of citizens' ideal points, if $c < b$, the equilibrium outcome satisfies $x(X) \in (\theta_L - b, \theta_L + b)$, and there is a $\theta_i < x(X)$ that is not attending, then all θ_j such that $\theta_i < \theta_j \leq x(X)$ are not attending.*

Proof of Lemma 2:

Suppose that $\theta_i < x(X)$ is not attending and there is a θ_j with $\theta_i \leq \theta_j < x(X)$ that is attending. By the previous lemma θ_i must leave either k attendees on both sides or k attendees to his left and $k + 1$ attendees to his right.

In the first case, it implies that θ_j must leave k attendees to his left and $k - 1$ attendees to his right. Which would imply that $x(X) = median(X) < \theta_j$. A contradiction since $\theta_j < x(X)$.

In the second case, it implies that θ_j must leave k attendees to his left and k attendees to his right. Which would imply that $x(X) = median(X) = \theta_j$. A contradiction since $\theta_j < x(X).\blacksquare$

Proof of Proposition 7: Non-participation of the represented.

We will show that if any $\theta_i < x(X) - c$ does not attend the meeting, the equilibrium conditions are not satisfied. A similar reasoning can be used to show the symmetric counterpart for any $\theta_i > x(X) - c$.

Suppose that there is a $\theta_i \notin X$ such that $\theta_L - b < \theta_i < x(X) - c$. His utility is $V_i(x(X), 0) = -|\theta_i - x(X)| = \theta_i - x(X) < -c$. Since he is not attending the meeting, by Proposition 6 there must be half of the attendees' ideal points to his right and half to his left θ_i. If he was to attend he would become the median of the attendees and $x(X \cup \{\theta_i\}) = \theta_i$, thus his utility would be $V_i(x(X \cup \{\theta_i\}), 1) = -|\theta_i - x(X \cup \{\theta_i\})| - c = -c$. Therefore, he would be better off attending, and this is a contradiction.

Now suppose that there is a $\theta_i \notin X$ such that $\theta_i < \theta_L - b < x(X) - c$. His utility is $V_i(x(X), 0) = -|\theta_i - x(X)| = \theta_i - x(X)$ and since he is not attending the meeting, by Proposition 6 he must also have that half of the attendees' ideal points to his right and half to his left. Thus, if he was to attend he would become the median of the attendees and $x(X \cup \{\theta_i\}) = \theta_L - b$. Thus his utility would be $V_i(x(X \cup \{\theta_i\}), 1) = -|\theta_i - x(X \cup \{\theta_i\})| - c =$

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$\theta_i - (\theta_L - b) - c > \theta_i - (x(X) - c) - c = \theta_i - x(X)$. Therefore, he would be better off attending, and this is a contradiction.■

Proof of Proposition 8:

Suppose that $x(X) \in (\theta_L - b, \theta_L + b)$. From Proposition 6 we know that we must have $x(X) = \frac{\theta_l + \theta_r}{2}$ for some θ_l and θ_r that are attending the meeting. Since θ_l and θ_r are attending the meeting, by Proposition 7 we must also have that $\theta_l, \theta_r \notin (x(X) - c, x(X) + c)$.

Since θ_r is attending his utility is $V_r(x(X), 1) = -|\theta_r - x(X)| - c = x(X) - \theta_r - c$. If θ_r was not attending, θ_l would be the median of the attendees, thus $\theta^*(X - \{\theta_r\}) = \theta_l$. Since $\theta_l \leq x(X) - c < \theta_L - b$ we have that the policy that legislator would implement in this case is $x(X - \{\theta_r\}) = \theta_L - b$. Thus his utility in this case would be $V_r(x(X - \{\theta_r\}), 0) = -|\theta_r - \theta_L + b| = \theta_L - b - \theta_r$. Since we assumed that $x(X) - c < \theta_L - b$, we have that θ_r would be better off not attending the meeting, which is a contradiction. Similarly we can prove that in equilibrium we must have $x(X) + c < \theta_L + b$. ■

Proof of Proposition 9: Alignment is needed.

By Proposition 7 we know that in equilibrium all citizens with ideal points to the left of $x(X) - c$ will attend, and by Proposition 8 we know that $\theta_L - b < x(X) - c$. This implies that when $\theta^{**} < \theta_L - b$ we have more than half of the population to the left of $x(X) - c$, and they are all attending the meeting. Combining Propositions 7 and 8 there should be exactly half of the attendees to the left of $x(X) - c$. This is a contradiction. ■

Proof of Proposition 10:

Suppose that $x \in S$, that is, $x = \frac{\theta_r + \theta_l}{2}$ for some θ_l and θ_r such that $|\{\theta_i : \theta_i \leq \theta_l\}| = |\{\theta_i : \theta_i \geq \theta_r\}|$. Observe that if θ_l and θ_r attend the meeting their payoffs are $V_R(x(X), 1) = V_L(x(X), 1) = \frac{\theta_l - \theta_r}{2} - c$. If one of them decides not to attend his payoff is $V_i(x(X - \{\theta_i\}), 0) = \theta_l - \theta_r$ for $i = L, R$. Thus, in order to have both citizens attending in equilibrium we must have $\frac{\theta_l - \theta_r}{2} - c > \theta_l - \theta_r$. Hence we need c to satisfy the following condition $c < \overline{c} = \frac{\theta_r - \theta_l}{2}$. Observe that $\overline{c} < b$. If θ_l and θ_r are attending, then all the other citizens such that either $\theta_i \leq \theta_l$ or $\theta_i \geq \theta_r$ are better off attending.

Next we will show that all citizens such that $\theta_l < \theta_i < \theta_r$, are better off not attending the meeting: Let θ_{l+1} denote the ideal point of the voter

next to θ_l on his right and let θ_{r-1} denote the ideal point of the voter next to θ_r on his left. If they are not attending the meeting their payoffs are $V_{L+1}\left(x\left(X\right),0\right)=-\left|\theta_{l+1}-\frac{\theta_l+\theta_r}{2}\right|$ and $V_{R-1}\left(x\left(X\right),0\right)=-\left|\theta_{r-1}-\frac{\theta_l+\theta_r}{2}\right|$. If one of them decides to attend his payoffs is $V_i\left(x\left(X\cup\{\theta_i\}\right),1\right)=-c$ for $i=l+1,r-1$. Thus, in order to have both not attending in equilibrium we must have $\left|\theta_{l+1}-\frac{\theta_l+\theta_r}{2}\right|<c$ and $\left|\theta_{r-1}-\frac{\theta_l+\theta_r}{2}\right|<c$. Hence we need c to satisfy the following condition $c>\underline{c}=\max\left\{\left|\theta_{l+1}-\frac{\theta_l+\theta_r}{2}\right|,\left|\theta_{r-1}-\frac{\theta_l+\theta_r}{2}\right|\right\}$. Observe that $0\leq\underline{c}<\overline{c}$. If θ_{l+1} and θ_{r-1} are better off not attending, then so are all the other citizens such that either $\theta_i\in[\theta_{l+1},\theta_{r-1}]$. Thus, if $c\in(\underline{c},\overline{c})$ there is an equilibrium with $X=\{\theta_i:\theta_i\leq\theta_l\}\cup\{\theta_i:\theta_i\geq\theta_r\}$ and $x\left(X\right)=\frac{\theta_r+\theta_l}{2}$. $\blacksquare$