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**Are loyalty-rewarding pricing
schemes anti-competitive?**

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Are loyalty-rewarding pricing schemes anti-competitive?*

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Abstract

Many economists and policy analysts seem to believe that loyalty-rewarding pricing schemes, like frequent flyer programs, tend to reinforce firms' market power and hence are detrimental to consumer welfare. The existing academic literature has supported this view to some extent. In contrast, we argue that these programs are business stealing devices that enhance competition, in the sense of generating lower average transaction prices and higher consumer surplus. This result is robust to alternative specifications of the firms' commitment power and demand structures, and is derived in a theoretical model whose main predictions are compatible with the sparse empirical evidence.

JEL Classification numbers: D43, L13

Key words: repeat purchases, switching costs, price commitment, coupons.

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1 Introduction

In some markets sellers discriminate between first time and repeat buyers using a variety of instruments. For instance, manufacturers have been offering repeat-purchase coupons for a long time. That is, they provide a coupon along with the product purchased, which consumers can use to obtain a discount in their next purchase of the same product. Recently, firms have designed more sophisticated pricing schemes to reward loyalty. For example, most airlines have set up frequent-flyer programs (FFPs) that offer registered travelers free tickets or free class upgrades after a certain number of miles have been accumulated.[1] Similar programs are also run by car rental companies, supermarket chains, hotels, and other retailers.

What are the efficiency and distributional effects of these loyalty-rewarding programs? Do they enhance firms'mark et power? Should competition authorities be concerned about the proliferation of those schemes?

Loyalty programs can perhaps be interpreted as a form of price discrimination analogous to quantity and bundled discounts. In particular, in the context of vertical relations, it has been recognized that loyalty discounts offered by manufacturers when selling to retailers, which are very often buyer-specific, may serve the same purpose as other vertical control practices, such as tying and exclusive dealing, and hence they have been subject to scrutiny by anti-trust authorities.[2]

However, the analogy with quantity and bundled discounts is, at best, only part of the story. In all the above examples the time dimension seems crucial. In particular, these programs involve some commitment capacity (sellers restrict their future ability to set prices) and they affect the pattern of repeat purchases (current demand depends on past sales). It is precisely this dynamic aspect which is the main focus of this paper. In other words, our aim is not to undertake a complete analysis of loyalty rewards. Instead, we restrict attention to single product markets (exclud-

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[1]Frequent flyer programs seem to be more popular than ever. In fact, according to The Economist (January 8th, 2005, page 14) "the total stock of unredeemed frequent-flyer miles is now worth more than all the dollar bills in circulation around the world". The same article also mentions that unredeemed frequent flyer miles are a non-negligible item in some divorce settlements!

The reader can visit www.webflyer.com for more detailed information on the volume and specific characteristics of some of these programs.

[2]See, for instance, Kobayashi (2005).

ing bundled discounts) with inelastic demand (excluding static non-linear pricing). Moreover, we focus on markets for final consumption goods, and hence neglect all the issues associated with vertical relations.

Regarding the dynamic aspect of loyalty programs, it is important to note that the specific details of the examples given above vary substantially. In particular, repeat buyers do not always know in advance the actual transaction price. For instance, in the case of FFPs, frequent travelers may gain the right to "buy" a ticket at zero price, but they can also use these miles to upgrade the ticket, in which case the net price is left undetermined ex-ante.[3] In the case of repeat-purchase coupons, discounts can take various forms (proportional, lump-sum, or even more complex), and again there is no specific commitment to a particular price.

Many economists and policy analysts seem to believe that loyalty programs are anti-competitive, in the sense that they benefit firms and hurt consumers. Unfortunately, the empirical evidence currently available is scarce. In the marketing literature one can find somewhat weak evidence on the influence of loyalty programs on the pattern of repeat purchases.[4] In some cases the evidence refers to industries (for instance, grocery retailing) where loyalty programs have an important bundling component.

The most important evidence for our purposes comes from the air transport industry. FFPs were first introduced by major US airlines immediately after deregulation and they were interpreted as an attempt to isolate themselves from competition. Very recently, Lederman (2003) reported significant effects of FFPs on market shares. In particular, she showed that enhancements to an airline's FFP, in the form of improved partner earning and redemption opportunities, are associated with increases in the airline's market share. Moreover, those effects are larger on routes that depart from airports at which the airline is more dominant. She interprets these results as indicating that FFP reinforces firms' market power. Our analysis challenges this interpretation.

From a theoretical point of view, some of these issues have been approached by Cairns and Galbraith (1990), Banerjee and Summers (1987) and Caminal and Matutes (1990) (CM, hereafter). Cairns and Galbraith

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[3]Airlines also impose additional restrictions, like blackout dates, that are sometimes modified along the way.

[4]See, for instance, Sharp and Sharp (1997) and Lal and Bell (2003). The introduction of a loyalty program by a particular firm tends to increase its market share, although its effect on profitability is less clear.

(1990) showed that, under certain circumstances, FFP-type policies could be an effective barrier to entry. We believe that this insight is essentially correct, but this is only one dimension of the problem. The last two papers focused on symmetric, multiperiod duopoly models and characterized loyalty-rewarding policies as endogenous switching costs. On the one hand, because of these policies consumers are partially locked-in, and hence they may remain loyal even when switching is ex-post efficient. On the other hand, their effect on consumer welfare is less straightforward. Banerjee and Summers (1987) did show that lump-sum coupons are likely to be a collusive device and hence consumers would be better off if coupons were forbidden. However, CM argued that the specific form of the loyalty program might be crucial. In particular, if firms are able to commit to the price they will charge to repeat buyers, then competition is enhanced and prices are reduced. However, in their model lump-sum coupons tend to relax price competition, a result very much in line with those of Banerjee and Summers (1987). Hence, the desirability of such programs from the point of view of consumer welfare seemed to depend on the specific details, which in practice may be hard to interpret. Moreover, the emphasis on symmetric duopoly and on restricting the analysis to an arbitrary subset of commitment devices was probably misleading.[5] [6]

In this paper we try to make progress by introducing several innovations. Firstly, we extend the standard Hotelling model to allow for a large number of monopolistically competitive firms. Market structure is particularly crucial in determining the dynamic effects of loyalty-rewarding schemes. In oligopoly, a firm's commitment to the price for repeat purchases influences future profits through two different channels: (i) consumer demand (lock-in effect) and (ii) future prices set by rivals (strategic effect). The size of the latter effect is maximized in a symmetric duopoly, but it is negligible if the number of firms is large. In order to understand the relative role of these two channels, it is helpful to study the limiting case (monopolistic

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[5]More recently, Kim et al. (2001) have also studied a duopoly model where firms can offer lump-sum discounts. The novelty is that firms can choose the nature of those discounts (cash versus non-cash). They show that firms may have incentives to offer 'inefficient' cash rewards (higher unit reward cost for the firm than a free product of the firm). In either case reward programs weaken price competition.

[6]There is also recent literature on the effect of bundled loyalty discounts. See, for example, Gans and King (2004) and Greenlee and Reitman (2005). These models are static.

competition) where the strategic effect has been shut down completely.

Our second innovation has to do with the set of commitment devices. We start by studying firms' incentives to commit to prices for repeat purchases. However, firms may not have access to such a commitment technology; or, even if they do, they might prefer not to use it, perhaps because they are uncertain about future demand or costs. In this case, instead of restricting attention to lump-sum coupons, we allow firms to choose the discounting rule. It turns out that the equilibrium discounting rule is simple but quite different from lump-sum discounts.

Thirdly, we study the interaction between endogenous and exogenous switching costs. In particular, we ask whether firms have more or less incentives to introduce loyalty rewarding schemes whenever consumers are already partially locked-in for exogenous reasons. In other words, we ask whether endogenous and exogenous switching costs are complements or substitutes.

Fourthly, we extend the analysis beyond the two-period framework (where firms actually compete for a single generation of consumers), and consider an overlapping generation set up. In this context, it is reasonable to assume that firms are unable to discriminate between different types of newcomers. In other words, former customers of rival firms and consumers that have just entered the market must be treated equally.

Finally, we study the role of firms' relative sizes, in order to contrast the predictions of the model with the existing empirical evidence, and discuss some other issues more informally, such as consumer horizon, partnerships, and entry.

This paper provides an unambiguous message: loyalty rewarding pricing schemes are essentially business-stealing devices that enhance competition, in the sense that average prices are reduced and consumer welfare is increased. The introduction of a loyalty program is a dominant strategy for each firm (provided these programs involve sufficiently small administrative costs) but in equilibrium all firms lose (prisoner's dilemma). This result is robust, in particular, to different specifications of the firms' commitment power, and to alternative demand structures. Moreover, the predictions of our theory are compatible with the empirical evidence reported by Lederman (2003). As mentioned above, she shows that the introduction (or an enhancement) of an airline's FFP raises its market share. Such a link is also present in our model. Lederman goes on and argues that this empirical fact is the result of the FFP enhancing the firm's market power. Our

theory challenges this interpretation and claims that the use of FFPs may actually signal fiercer competition among airlines.[7] Lederman (2003) also shows that the positive effect of the firm's FFP on its market share is relatively larger for large firms. The predictions of the asymmetric version of our model are also consistent with these results. Large firms are relatively protected from the pro-competitive effects of FFP (the reduction in profits is relatively smaller for larger firms), but nevertheless all firms would prefer that loyalty rewards were forbidden.

In the next section we present the two-period benchmark model. As mentioned above the model accommodates a large number of monopolistically competitive firms in an otherwise standard Hotelling framework. A key feature of the model is that consumers are uncertain about their future preferences. If, alternatively, preferences were stable over time, then repeat buyers would only care about the present value of prices but not about their time sequence. In contrast, under uncertain preferences, a firm can raise sales and profits by setting a higher current price and committing to a lower future price (rewarding consumer loyalty).

Section 3 contains a preliminary discussion of the main effects. In particular, it studies the optimal strategy of a single firm when rivals are myopic and play the equilibrium strategy of the one-shot game. It is shown that the firm which is allowed to discriminate between first-time and repeat buyers has incentives to commit to a price equal to marginal cost for repeat purchases. The reason is twofold. Firstly, such a pricing rule maximizes the value of the firm-customer relationship, since consumers go back to the same supplier every time their reservation price is above the firm's opportunity cost. Secondly, the firm is able to appropriate all the rents generated by such a commitment through a higher first period price.[8] [9] The firm's commitment creates a negative externality on other firms (a

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[7]To the best of our knowledge there is no systematic evidence on the effect of FFPs on firm profitability. Lederman (2003) constructs an index of the average fare charged by each airline. These indices do not seem to include the zero price tickets used by frequent flyers. She shows that an enhancement of the airline's FFP raises its own average fare, which is again compatible with the predictions of our model.

[8]With the first period price consumers purchase a bundle: one unit of the good in the first period plus an option to buy another unit of the good in the second period at a predetermined price.

[9]The reasons behind marginal cost pricing for repeat purchases are analogous to those in Crémer (1984), which was a model of experience goods. See also Bulkley (1992) for a similar result in a search model, and Caminal (2004) in cyclical goods model.

business-stealing effect) which will also be present when we let other firms use the same commitment technology.

In Section 4 we present the equilibria of the two-period model under alternative strategy sets. In one case (full commitment game) we allow all firms to commit in the first period not only to the price for repeat purchases but also to the second period price for newcomers. This is a useful benchmark. In the other, more realistic case (partial commitment game) firms can only commit to the price for repeat purchases, and the second period price for newcomers is chosen in the second period. We show that the equilibrium strategies of the first game are time inconsistent. Nevertheless, the time inconsistency problem has only a minor impact on prices and payoffs. In both cases, firms choose to commit to marginal cost pricing for repeat buyers and, as a result, average prices are lower and consumer welfare is higher than in the case in which firms are unable to commit to any future price.[10]

Under some circumstances firms may not be able or may not wish to commit to the price for repeat purchases. In Section 5 we show that commitment to a simple discounting rule (a combination of proportional and lump-sum discounts) is equivalent to committing to future prices for both repeat buyers and newcomers. Therefore, as a first approximation, coupons are actually equivalent to price commitment. In other words, the focus of the previous literature on lump-sum coupons was highly misleading, especially in combination with the strategic commitment effect present in duopoly models.

In Section 6 we pay attention to the interactions between exogenous and endogenous switching costs. As discussed in Klemperer (1995), switching consumers often incur in transaction costs (closing a bank account) or learning costs (using a different software for the first time). Such switching costs are independent of firms' decisions. If firms can use loyalty-rewarding pricing schemes then average prices and firm profits decrease with the size of these exogenous switching costs. The same result occurs when firms cannot discriminate between repeat buyers and newcomers, although the mechanism is completely different. We also show that the presence of exogenous switching costs reduces firms' incentives to introduce artificial switching

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[10]However, from a social point of view, those commitment strategies distort the ex-post allocation of consumers and average transportation costs increase. In our model with inelastic demand total surplus depends exclusively on transportation costs. In a more general model lower average prices would imply higher total surplus.

costs. That is, when consumers are relatively immobile for exogenous reasons the ability of loyalty rewarding pricing schemes to affect consumer behavior is reduced.

In Section 7 we embed the benchmark model in an overlapping generations framework in order to consider the more realistic case where firms cannot distinguish between consumers that just entered the market and consumers with a history of purchases from rival firms. More specifically, firms set for each period a price for repeat buyers (those who bought in the past from the same supplier) and a regular price (for the rest). We show that there is a stationary equilibrium with features similar to those of the benchmark model. In particular, average prices are also below the case in which firms cannot commit to the price of repeat purchases. The main difference with the benchmark model is that firms set the price for repeat buyers above marginal cost (but below the regular price). The reason is that the regular price is not only the instrument to collect the rents generated by a reduced price for repeat purchases, but is also the price used to attract consumers who previously bought from rival firms. Hence, firms are not able to capture all these rents and hence are not willing to maximize the value of the long-run customer relationship.

In Section 8 we discuss several extensions, including the existence of firms with different relative sizes. Section 9 concludes.

2   The benchmark model

This is essentially a two-period Hotelling model extended to accommodate an arbitrary number of firms and, at the limit, it can be interpreted as a monopolistic competition model.

There are n firms (we must think of n as a large number) each one produces a variety of a non-durable good. Both firms and varieties are indexed by i, $i = 1, ..., n$. Firms are located in the extremes of n spokes of length $\frac{1}{2}$, which start from the same central point. Demand is perfectly symmetric. There is a continuum of consumers with mass $\frac{n}{2}$ uniformly distributed over the n spokes. Each consumer derives utility from only two varieties and the probability of all pairs is the same. Thus, the mass of consumers who have a taste for variety i is 1, and $\frac{1}{n-1}$ have a taste for varieties i and j, for all $j \neq i$. Consumer location represents the relative valuation of both varieties. In particular, a consumer who has a taste for

varieties i and j, and is located at $x \in \left[0, \frac{1}{2}\right]$ of the ith spoke, obtains a utility equal to $R - tx$ from consuming one unit of variety i and $R - t(1 - x)$ from consuming one unit of variety j. As usual we assume that R is sufficiently large, so that all the market is served in equilibrium.

If $n = 2$ then this is the classic Hotelling model. If $n > 2$ firm i competes symmetrically with the other $n - 1$ firms. If n is very large the model resembles monopolistic competition, in the sense that each firm: (i) enjoys some market power, and (ii) is small with respect to the market, even in the strong sense that if one firm is ejected from the market then no other firm is significantly affected.

In practice this model works exactly the same as the standard, two-firm Hotelling model, although interpretation is different. In the current model, a representative firm is located at one extreme of the $[0, 1]$ interval and 'the market'at the other. Consumers with a preference for the variety supplied by the representative firm are uniformly distributed over the interval, although in each location consumers are heterogeneous with respect to the name of the alternative brand. At the same time these consumers represent a very small fraction of the potential customer base of any rival firm. As a result, the representative firm correctly anticipates that its current actions have a negligible effect on its rivals' market shares and hence they will not affect their future actions.

An important feature of the model is that consumers are uncertain about their future preferences. More specifically, each consumer derives utility from the same pair of varieties in both periods, although her location is randomly and independently chosen in each period. Thus, consumers' uncertainty refers only to their future relative valuations of the two varieties.[11]

Marginal production cost is $c \geq 0$. In this class of models, in equilibrium the absolute margin, $p - c$, is independent of c. Hence, typically there is no loss of generality in normalizing $c = 0$. In fact, setting $c = 0$ does not make any difference in most of this paper. The exception is Section 5, where we analyze discounts. If we set $c = 0$ then a proportional coupon of 100% would be equivalent to a commitment to marginal cost pricing. However, if we allow for $c > 0$ then a proportional coupon alone is generally not sufficient to achieve the desired outcome.

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[11] At given prices, a consumer may prefer today to travel with a particular airline, given her destination and available schedules. However, the following week the same consumer may prefer to fly a different airline as travel plans change.

Both firms and consumers are risk neutral and neither of them discount the future. Thus, their total expected payoff at the beginning of the game is just the sum of the expected payoffs in each period.

This model is related to the "spokes" mo del of Chen and Riordan (2004). The main difference is that in their model all consumers have a taste for all varieties. In particular, a consumer located at x, $x \in \left[0, \frac{1}{2}\right]$, of the ith spoke pays transportation cost tx if she purchases from firm i, and $t(1-x)$ if she buys from any firm $j \neq i$. Hence, firms are not small with respect to the market, in the sense that an individual firm is able to capture the entire market by lowering its price sufficiently. Thus, their model can be interpreted as a model of non-localized oligopolistic competition, rather than a model of monopolistic competition.

3   Preliminaries

Let us consider the case $t = 1$ and $c = 0$ and suppose that only one firm can discriminate in the second period between old customers (those who bought from that firm in the first period) and newcomers (those consumers who patronized other firms), while the rest cannot tell these two types of consumers apart. In equilibrium non-discriminating firms will set the price of the static game in both periods, i.e., if we let subscripts denote time periods then we have $\bar{p}_1 = \bar{p}_2 = 1$. Let us examine the alternatives of the firm which is able to price discriminate. In case such a firm does not use its discriminatory power, then it will find it optimal to imitate its rivals and set $p_1 = p_2 = 1$. It will attract a mass of consumers equal to one half in each period, and hence it will make profits equal to $\frac{1}{2}$ in the first period, and the same in the second, i.e., $\frac{1}{4}$ from repeat buyers and $\frac{1}{4}$ from new customers.

Suppose instead that the discriminating firm commits in the first period to a pair of prices (p_1, p_2^r), where p_1 is the price charged for the first period good, and p_2^r is the price charged in the second period only to repeat buyers. In this case we are assuming that the ability to commit is only partial, since the firm cannot commit to the second period price for newcomers . In fact, the discriminating firm will also charge a price $p_2^n = 1$ to new customers in the second period, since the market is fully segmented and the firm will be on its reaction function. The firm's commitment is an option for consumers, who can always choose to buy from rival firms in the second period. Thus, p_1 is in fact the price of a bundle, one unit of the good in the first period

plus the option to repeat trade with the same supplier at a predetermined price.

We can now ask what is the value of p_2^r that maximizes the joint payoffs of the discriminating firm and its first period customers. Clearly, the answer is $p_2^r = 0$, i.e., marginal cost pricing for repeat buyers. In other words, the optimal price, from the point of view of the coalition of consumers and a single firm, is the one that induces consumers to revisit the firm if and only if consumers' willingness to pay in the second period is higher than or equal to the firm's opportunity cost. Moreover, the discriminating firm will in fact be willing to set $p_2^r = 0$ because it can fully appropriate all the rents created by a lower price for repeat buyers. More specifically, if the firm does not commit to the price for repeat buyers then a consumer located at x who visits the firm in the first period will obtain a utility $U^{nc} = R - 1 - x + R - 1 - \frac{1}{4}$. That is, she expects to pay a price equal to 1 in both periods, but expected transportation costs in the second period are $\frac{1}{4}$. Instead, if the firm commits to $p_2^r = 0$ then the same consumer gets $U^c = R - p_1 - x + R - \frac{1}{2}$. That is, in the first period she pays the price p_1 but in the second period with probability 1 the consumer will buy from the same supplier (maximum transportation cost is equal to the price differential) and pay the committed price $p_2^r = 0$ and the expected transportation cost $\frac{1}{2}$. Hence, independently of their current location, consumers' willingness to pay has increased by $\frac{3}{4}$ because of the commitment to marginal cost pricing for repeat buyers ($U^c - U^{nc} = \frac{3}{4} + 1 - p_1$). Hence, the first period demand function of the discriminating firm has experienced an upwards parallel shift of $\frac{3}{4}$. Thus, if the firm were to serve half of the market (same market share as in the equilibrium without price discrimination) then $p_1 = \frac{7}{4}$. As a result, profits from customers captured in the first period would be equal to $\frac{7}{8}$ (which is higher than the level reached in the absence of discrimination, $\frac{3}{4}$) and those from newcomers in the second period would be $\frac{1}{4}$ in the second period (equal to the level reached in the absence of discrimination). Summarizing, commitment to $p_2^r = 0$ reduces the average price paid by repeat buyers ($\frac{7}{8}$ instead of 1), but increases sales (reinforces consumer loyalty) at the expense of rival firms. As a result, if the firm were to serve half of the market, profits of the discriminating firm increase by $\frac{1}{8}$.[12]

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[12]In fact, the optimal first period price is $p_1 = \frac{13}{8}$, which is lower than $\frac{7}{4}$. This implies that the first period market share is higher than one half, and total profits are equal to

The intuition about the incentives to commit to marginal cost pricing for repeat buyers is identical to that provided by Crémer (1984).[13] In contrast to Crémer's results, the seller's commitment to marginal cost pricing for repeat buyers does not make any consumer worse off. The seller enhances consumer loyalty by offering a sequence of prices, which decreases over time, that are lower on average (to compensate for higher average transportation costs). Summarizing, when a single firm commits to the price for repeat buyers then, on the one hand, consumer surplus increases and, on the other hand, this creates a negative externality to rival firms (a business stealing effect).[14]

Most of these intuitions will be present in all the games that will be analyzed below, where all firms are allowed to price discriminate between old customers and newcomers. Strategic complementarities will exacerbate the effects described in this section and as a result consumers will be better off than in the absence of price discrimination although overall efficiency will be reduced (higher transportation costs).

At this point it is important to note that marginal cost pricing is part of the equilibrium strategy only under specific circumstances. Our benchmark model includes some special assumptions. One of them is that the first period price is paid only by a new generation of consumers who have just entered the market and face a two-period horizon. As a result, all the rents created by marginal cost pricing in the second period can be fully appropriated by the firm through the first period price. This is why the firm is willing to offer a contract that includes marginal cost pricing in the second period. In Section 7 we discuss in detail the importance of this assumption. For now it may be sufficient to think of the case in which a fraction of first period revenues are taxed away. In this case, the firm cannot fully appropriate all the rents and as a result p_2^r will be set above marginal costs, but below the price charged to newcomers.

$\frac{145}{128}$ (profits increase by $\frac{17}{128}$ because of the commitment to $p_2^r = 0$).

[13]See also Bulkley (1992) and Caminal, (2004) for the same result in different set-ups.

[14]In fact, the firm would like to sell the option to buy in the second period at a price equal to marginal cost, separately from the first period purchase. However, transaction costs associated to such a marketing strategy could be prohibitive. Ignoring those transaction costs, the firm would charge a price equal to $\frac{3}{4}$ for the right to purchase at a price equal to zero in the second period and a price $p_1 = 1$ for the first period purchase. The entire potential customer base would buy such an option and hence total profits would be $\frac{5}{4}$ which is above the level reached by selling the option to first period buyers only, $\frac{145}{128}$.

4 Symmetric commitment to the price of repeat purchases

4.1 The full commitment game

Let us start with a natural benchmark. Suppose that each firm sets the three prices (p_1, p_2^r, p_2^n)simultaneously in the first period (same notation as previous section.[15] If we denote the average prices set by rival firms with bars, then second period market shares among repeat buyers and newcomers, x_2^r, x_2^n, are given respectively by:

$$x_2^r = \frac{t + \overline{p}_2^n - p_2^r}{2t} \tag{1}$$

and

$$x_2^n = \frac{t + \overline{p}_2^r - p_2^n}{2t} \tag{2}$$

Finally, the first period market share, x_1, is given by:

$$p_1 + tx_1 + x_2^r \left(p_2^r + \frac{tx_2^r}{2}\right) + (1 - x_2^r) \left(\overline{p}_2^n + \frac{t(1 - x_2^r)}{2}\right) = \tag{3}$$

$$= \overline{p}_1 + t(1 - x_1) + x_2^n \left(p_2^n + \frac{tx_2^n}{2}\right) + (1 - x_2^n) \left(\overline{p}_2^r + \frac{t(1 - x_2^n)}{2}\right)$$

A firm's optimization problem consists of choosing (p_1, p_2^r, p_2^n) in order to maximize the present value of profits:

$$\pi = (p_1 - c) x_1 + x_1 x_2^r (p_2^r - c) + (1 - x_1) x_2^n (p_2^n - c) \tag{4}$$

The next proposition summarizes the result (some computational details are given in Appendix 11.1):

Proposition 1 *There is a unique symmetric Nash equilibrium of the full commitment game, which is described in the second column of Table 1.*

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[15]In this case, since firms set all prices at the beginning of the game, the characteristics of the equilibrium are independent of the number of firms. In other words, the current model with monopolistic competition is equivalent to the standard duopoly model.

The first column of Table 1 shows the equilibrium of the game in which firms cannot discriminate between repeat buyers and newcomers. In this case all prices in both periods are equal to $c+t$, all market shares are equal to $\frac{1}{2}$, and hence total surplus is maximized (the allocation of consumers is ex-post efficient). If we compare the first two columns we note that:

Remark 1 *In the equilibrium under full commitment consumers are better off and firms are worse off than in the absence of commitment.*

Finally, when firms can discriminate between repeat buyers and newcomers total surplus is lower because of the higher transportation costs induced by the endogenously created switching costs.

Thus, the possibility of discriminating between repeat buyers and newcomers makes the market more competitive with average prices dropping far below the level prevailing in the equilibrium without discrimination. Firms offer their first period customers an 'efficient' contract, in the sense of maximizing their joint payoffs, which includes a price equal to marginal cost for their repeat purchases in the second period. Such a loyalty rewarding scheme heightens the competition for customers in the second period and induces firms to charge relatively low prices for newcomers. Since firms make zero profits from repeat purchases but also low profits out of second period newcomers, their fight for first period customers is only slightly more relaxed than in the static game. The other side of the coin is that consumers' valuation of the option included in the first period purchase is relatively moderate. All this is reflected in first period prices which are only slightly above the equilibrium level of the static game.

It is important to note that p_2^n is above the level that maximizes profits from newcomers in the second period (see below). The reason is that by committing to a higher p_2^n the firm makes the offer of their rivals less attractive, i.e., from equation 3 we have that $\frac{dx_1}{dp_2^n} > 0$.

4.2 The partial commitment game

In the real world firms sometimes sign (implicit or explicit) contracts with their customers, which include the prices prevailing in their future transactions. However, it is more difficult to find examples in which firms are able to commit to future prices that apply to new customers.

Let us consider the game in which firms choose (p_1, p_2^r) in the first period, and p_2^n is selected in the second period after observing x_1 and $\bar{p}_{2.}^r$.

The next result shows that the equilibrium strategies of Proposition 1 are not time consistent (intermediate steps are specified in Appendix 11.2).

Proposition 2 *There is a unique subgame perfect and symmetric Nash equilibrium of the partial commitment game, which is described in the third column of Table 1.*

The equilibrium of the partial commitment game also features marginal cost pricing for repeat buyers, since the same logic applies. However, the equilibrium value of p_2^n is now lower than that of the full commitment game. The reason is that p_2^n is chosen in the second period in order to maximize profits from second period newcomers. Hence, firms disregard the effect of p_2^n on the first period market share. In this case, since firms obtain higher profits from newcomers, competition for first period customers decreases, which is reflected in higher first period prices. As a result:

Remark 2 *In the equilibrium of the partial commitment game consumers are better off and firms are worse off than in the absence of commitment.*

Remark 3 *Both consumers and firms are better off under partial commitment than under full commitment.*

Thus, the time inconsistency problem has only a minor effect on the properties of the equilibrium. Moreover, the payoff of a particular firm increases with its own commitment capacity but decreases with the commitment capacity of its rivals.

Our model can be easily compared with the duopoly model analyzed in CM. In fact, the only difference is that in the current model firms cannot influence the future behavior of their rivals. In other words, the strategic commitment effect is missing. As a result, firms wish to commit to marginal cost pricing for repeat buyers since this is the best deal it can offer their customers. On the other hand, in the equilibrium of the duopoly game, firms commit to a price below marginal cost for repeat buyers. The reason is that if a duopolist cuts p_2^r below marginal costs this has a second order (negative) effect on profits, but it also induces its rival to set a lower $\overline{p}_2^n$ in the second period, which has a first order (positive) effect on profits, since $\frac{dx_1}{d\overline{p}_2^n} < 0$.

5 Commitment to a linear discount

There might be many reasons why firms may not be able to commit to a fixed price for repeat buyers. Even if they can they may choose not to do so, perhaps because of uncertainty about future cost or demand parameters. In fact, in some real world examples we do observe firms committing to discounts for repeat buyers while leaving the net price undetermined. In this section we consider the same deterministic benchmark model used above but with different strategy spaces. In particular, we allow firms to commit to linear discounts for repeat buyers instead of committing to a predetermined price. Below we also discuss the role of uncertainty.

Suppose that in the first period firms set (p_1, v, f), where v and f are the parameters of the discount function:

$$p_2^r \equiv (1 - v) p_2 - f \tag{5}$$

Thus, v is a proportional discount and f is a fixed discount. In the second period firms set the regular price, p_2.

We show that there exist an equilibrium of this game that coincides with the symmetric equilibrium of the full commitment game of Section 4.1. Thus, in our model a linear discount function is a sufficient commitment device. By fixing the two parameters of the discount function firms can actually commit to the two prices, p_2^r and p_2^n.

More specifically, in the second period firms choose p_2 in order to maximize second period profits:

$$\pi_2 = x_1 x_2^r (p_2^r - c) + (1 - x_1) x_2^n (p_2 - c)$$

where p_2^r is given by equation 5. The first order condition characterizes the optimal price:

$$x_1 (1 - v) \left(x_2^r - \frac{p_2^r - c}{2t} \right) + (1 - x_1) \left(x_2^n - \frac{p_2 - c}{2t} \right) = 0$$

If other firms set the prices given by Proposition 1, and $x_1 = \frac{1}{2}$, then it is easy to check that it is optimal to set those same prices provided $v = \frac{4}{5}$ and $f = \frac{2}{15} t - \frac{4}{5} c$. Thus, using such a pair of (v, f) a firm can implement the desired pair of second period prices. Consequently, given that other firms are playing the prices given by Proposition 1, the best response for an individual firm consists of using such a linear discount function and the

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value of p_1 given also in Proposition 1, which results in $x_1 = \frac{1}{2}$. The next proposition summarizes this discussion.

Proposition 3 *There exist an equilibrium of the linear discount game that coincides with the equilibrium of the full commitment game.*

Hence, in our deterministic model there is no difference between price commitment and coupon commitment, at least as long as firms can use a combination of proportional and lump-sum coupons. This equivalence result suggests that the emphasis of the existing literature on lump-sum coupons was probably misleading. However, two remarks are in order. First, in practice it may not be so easy to use a combination of proportional and fixed coupons, as some consumers may be confused about the actual discounting rule. Second, firms may be uncertain about future demand and/or cost conditions. Let us discuss these two issues in turn.

In the absence of uncertainty and if firms feel that they should use one type of coupons exclusively then they will attempt to use the type that performs better as a commitment device, which depends on parameter values. For instance, if c is approximately equal to $\frac{t}{6}$ then proportional discounts alone will approximately implement the payoffs of the full commitment game (the optimal value of f is approximately zero). Actually, in a broad set of parameters, proportional discounts are better than lump-sum discounts at approximating full commitment strategies. We illustrate this point in Appendix 11.3.

In the duopoly model of CM firms prefer committing to p_2^r than committing to a lump-sum discount. Our point here is that if commitment to p_2^r is not feasible or desirable then firms are likely to prefer proportional discounts to lump-sum discounts.

In order to compare the role of lump-sum coupons under oligopoly and monopolistic competition, in Appendix 11.4 we compute the symmetric equilibrium of the game with lump-sum coupons, i.e. firms set (p_1, f) in the first period and p_2 in the second. In this case we have that $p_2^r = p_2 - f$. It turns out that in equilibrium $f > 0$, firm profits are below the equilibrium level of the static game, but above the level obtained in the equilibrium of the partial commitment game. The ranking of these three games in terms of consumers surplus is the reverse. In other words, firms are better off if they are restricted to use lump-sum coupons instead of being allowed to commit to the price for repeat purchases. Nevertheless, the use of lump-sum coupons makes the market more competitive than in cases where no

commitment device is available. The reason is that lump-sum coupons are a poor commitment device and hence the business stealing effect is moderate but present. Under duopoly (CM) firms are better off using lump-sum coupons than in the absence of any commitment device, just because of the strategic commitment effect; that is, coupons imply a commitment to set a high regular price in the future which induces the rival firm to set a higher future price. It is this Stackelberg leader effect that made coupons a collusive device in CM.[16]

If firms are uncertain about future market conditions then they face the usual trade-off between commitment and flexibility. Suppose first, that firms are uncertain about future marginal costs. In this case the ex-ante optimal, full contingent pricing rule involves both p_2^r and p_2^n exhibiting the same sensitivity with respect to the realization of the marginal cost variable. Thus, in terms of the optimal discounting rule, flexibility calls for a zero proportional discount. In fact, if uncertainty is so great that it is the dominant effect then the optimal discounting rule probably involves a small v. Let us now consider the case of firm-specific demand shocks. For instance, suppose that in the second period a new generation of consumers enter the market and their distribution over different brands is random. In this case, the ex-ante optimal, full contingent pricing rule involves a fixed p_2^r and a variable p_2^n. Thus, in terms of the optimal discounting rule, flexibility calls for a large proportional discount in order to disentangle p_2^r from changes in p_2^n.

Summarizing, uncertainty about future market conditions clearly breaks the equivalence between price and coupon commitment. However, its impact on the equilibrium discounting rule is difficult to ascertain and probably depends on the dominant source of uncertainty. Perhaps, we could explain the prevalence of lump-sum discounts in some real world markets on the basis of the relative strength of cost uncertainty. In this case, the commitment power of the discounting rule would be rather limited but nevertheless the use of lump-sum coupons would be a signal of fiercer competition among firms, at least as long as the number of firms is not too small and the strategic commitment effect is not sufficiently strong.

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[16] In the Appendix we discuss the intuition behind the difference between the duopoly and the monopolistic competition cases in more detail.

6 Interaction between endogenous and exogenous switching costs

Suppose that consumers incur an exogenous cost s if they switch suppliers in the second period. Let us assume that s is sufficiently small for optimal strategies to be given by interior solutions. If firms can use loyalty rewarding pricing schemes, what is the effect of exogenous switching costs on market performance? Does such a natural segmentation of the market increase or decrease firms'incen tives to introduce artificial switching costs?

Let us introduce exogenous switching costs in the partial commitment game of Section 4.2. That is, firms choose (p_1, p_2^r) in the first period, and p_2^n in the second period after observing x_1 and $\overline{p}_2^r$. The only difference is that now, those consumers that switch suppliers in the second period pay s. Therefore, second period market shares become:

$$x_2^r = \frac{t + \overline{p}_2^n + s - p_2^r}{2t}$$

$$x_2^n = \frac{t + \overline{p}_2^r - s - p_2^n}{2t}$$

Similarly, first period market shares are implicitly given by:

$$p_1 + tx_1 + x_2^r \left(p_2^r + \frac{tx_2^r}{2}\right) + (1 - x_2^r)\left(\overline{p}_2^n + s + \frac{t(1 - x_2^r)}{2}\right) =$$

$$= \overline{p}_1 + t(1 - x_1) + x_2^n \left(p_2^n + s + \frac{tx_2^n}{2}\right) + (1 - x_2^n)\left(\overline{p}_2^r + \frac{t(1 - x_2^n)}{2}\right)$$

Proposition 4 *The unique subgame perfect and symmetric Nash equilibrium of the partial commitment game with exogenous switching costs includes $p_1 = c + \frac{9t}{8} + \frac{s^2 - 2st}{8t}$, $p_2^r = c$, $p_2^n = c + \frac{t}{2} - \frac{s}{2}$. As a result, $x_1 = \frac{1}{2}$, $x_2^r = \frac{3}{4} + \frac{s}{4t}$, $x_2^n = \frac{1}{4} - \frac{s}{4t}$. Total profits per firm are $\pi = \frac{5t}{8} + \frac{s^2 - 2st}{8t}$, and consumer surplus per firm is $CS = R - c - \frac{29t^2 - 6st + 5s^2}{32t}$.*

Hence, exogenous switching costs do not affect the price for repeat buyers but they reduce p_1 and p_2^n. Therefore, they reduce average prices and

firm profits. The intuition goes as follows. For the same reasons as in Section 4, firms have incentives to commit to marginal cost pricing for repeat buyers. However, because of the exogenous switching costs, in the second period firms find it more difficult to attract consumers who previously bought from rival firms. As a result, they choose to set a lower second period regular price and the fraction of switching consumers decreases. Since second period profits from newcomers are reduced, firms are more willing to fight for consumers in the first period and hence find it optimal to set a lower first period price. Thus, even though consumers are partially locked-in for exogenous reasons and hence the market is even more segmented, profits fall.

Note, however, that in the absence of price discrimination, since all consumers change location, profitability also decreases with switching costs.[17] However, the mechanism is quite different. In the absence of price discrimination, switching costs affect prices through two alternative channels. On the one hand, in the second period a firm with a higher first period market share finds it profitable to set a higher price in order to exploit its relatively immobile customer base. As a result, first period demand will be more inelastic, since consumers expect that a higher market share translates into a higher second period market price and hence are less responsive to a price cut. This effect pushes first period prices upwards. On the other hand, firms make more profits in the second period out of their customer base, so incentives to increase the first period market share are higher. This effect pushes prices downwards. It turns out that the second effect dominates.

Therefore, the presence of price commitment affects the impact of exogenous switching costs. If firms commit to the second period price for repeat buyers, then this is equivalent to a commitment not to exploit locked-in consumers. Hence, the price sensitivity of first period consumers is unaffected. Nevertheless, firms'incen tives to fight for first period market share increase in both cases, which turns out to be the main driving force.

Let us now turn to the question of how exogenous switching costs affect the incentives to introduce loyalty rewarding pricing schemes. Suppose that committing to the price of repeat purchases involves a fixed transaction cost. For instance, these are the costs airlines incur in running their frequent flier programs (advertising, recording individual purchases, etc.).

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[17]This result holds under both monopolistic competition and duopoly (Klemperer, 1987).

The question is how the maximum transaction cost firms are willing to pay is affected by s.

The main intuition can already be obtained by considering the case of large switching costs. If s is sufficiently large then consumers will never switch in the second period, i.e., $x_2^r = 1$, $x_2^n = 0$. In this case, it is redundant to introduce endogenous switching costs, since they do not affect consumer allocation in the second period, which implies that consumers and firms only care about $p_1 + p_2$ and not about the time sequence. Hence, in this extreme case, it is clear that the presence of exogenous switching costs leaves no room for loyalty rewarding pricing schemes.

For low values of s the comparative static result provides a similar insight. As s increases, consumers switch less frequently and hence the effectiveness of price commitment to induce consumer loyalty is reduced. More precisely, if no other firm commits to p_2^r the net gain from committing to $p_2^r = c$ decreases with s. Similarly, if all other firms commit to $p_2^r = c$ the net loss from not committing also decreases with s (See Appendix 11.5 for details). In other words, exogenous and endogenous switching costs are imperfect substitutes.

7 An overlapping generations framework

In many situations firms may find it difficult to distinguish between consumers who have just entered the market and consumers who have previously bought from rival firms. In order to understand how important this assumption was in the analysis of the benchmark model we extend it to an infinite horizon framework with overlapping generations of consumers, in the spirit of Klemperer and Beggs (1992).[18]

Time is also a discrete variable, but now there is an infinite number of periods, indexed by $t = 0, 1, 2, ...$ Demand comes from overlapping generations of the same size. Each generation is composed of consumers who live for two periods and have the same preference structure as the one described in Section 2. Thus, besides the greater number of periods, the main difference with respect to the benchmark model is that in this section we assume that firms are unable to discriminate between first period (young) consumers and second period (old) consumers that previously patronized rival firms. Firms set two prices for each period: p_t, the price they charge

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[18]See also To (1996) and Villas-Boas (2004).

to all consumers who buy from the firm for the first time[19], and p_t^r, they price they charge to repeat buyers.

Thus, profits in period t are given by:

$$\pi_t = (p_t - c)\left[x_t + (1 - x_{t-1})\, x_t^n\right] + x_{t-1}\left(p_t^r - c\right) x_t^r$$

where x_t, x_t^r, x_t^n, as in previous sections, stand for the firm's period t market share with young consumers, old consumers loyal to the firm, and new customers of the old generation, respectively, which are given by:

$$x_t = \frac{1}{2t}\left\{\overline{p}_t - p_t + x_{t+1}^n\left(p_{t+1} + \frac{x_{t+1}^n}{2}\right) + \left(1 - x_{t+1}^n\right)\left(\overline{p}_{t+1}^r + \frac{1 - x_{t+1}^n}{2}\right) - \right.$$

$$\left. -x_{t+1}^r\left(p_{t+1}^r + \frac{x_{t+1}^r}{2}\right) - \left(1 - x_{t+1}^r\right)\left(\overline{p}_{t+1} + \frac{1 - x_{t+1}^r}{2}\right)\right\} \tag{6}$$

$$x_t^r = \frac{t + \overline{p}_t - p_t^r}{2t} \tag{7}$$

$$x_t^n = \frac{t + \overline{p}_t^r - p_t}{2t} \tag{8}$$

These equations are analogous to equations 3, 1, and 2, respectively. The firm's payoff function in period 0 is:

$$V_0 = \sum_{t=0}^{\infty} \beta^t \pi_t \tag{9}$$

where $\beta \in (0, 1)$ is the discount factor. We will focus later on the limiting case of $\beta \to 1$.

Let us first deal with the full commitment case. Thus, given the sequence of current and future prices set by the rivals, $\{\overline{p}_t, \overline{p}_t^r\}_{t=0}^{\infty}$, the price for repeat buyers set in the past, p_0^r, and the past market share with young consumers, x_{-1}, an individual firm chooses $\{p_t, p_{t+1}^r\}_{t=0}^{\infty}$ in order to maximize 9. We focus on the stationary symmetric equilibria, for the limiting case of $\beta \to 1$. The result is summarized below (See Appendix 11.6 for details):

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[19] At the end of this section we discuss the consequences of relaxing such a restriction on the set of strategies and allowing firms to offer a menu of contracts to induce newcomers to self-select.

Proposition 5 *In the unique stationary symmetric equilibrium $c + t > p > c + \frac{t}{2} > p^r > c$.*

Thus, the flavor of the results is very similar to the one provided by the benchmark model. Firms have incentives to discriminate between repeat buyers and newcomers, which creates artificial switching costs, and nevertheless consumers are better off than in the absence of such discrimination. The reason is that treating repeat buyers better than newcomers only has a business stealing effect and as a result the market becomes more competitive, in the sense that average prices are lower than in the absence of such discrimination (i.e., in the equilibrium of the static game).

The main difference with respect to the benchmark model is that in the current set up p^r is set above marginal cost. In the two-period model p_1 was the only instrument used by the firm to collect the rents created by setting a lower price to repeat buyers in the second period. Since an individual firm could fully appropriate all these rents, it was also willing to commit to marginal cost pricing in the second period, which maximizes the joint surplus of the firm and its customers. In the current framework, the regular price p_t is not only paid by young consumers but also by old newcomers. Thus, if p_t increases in order to capture the rents created by a lower p^r_{t+1} then the firm loses old newcomers. As a result, the firm does not find it profitable to maximize the joint surplus of the firm and young consumers and set the price for repeat purchases equal to marginal cost. Nevertheless, such a price is still lower than the regular price.

In this section we have dealt so far with the case of unlimited commitment capacity. It would probably be more realistic to grant firms more limited commitment power. Firms can sometimes sign long-run contracts with current customers, but it is much more unlikely that they can commit to future prices for newcomers. Thus, alternatively, we could have assumed that in period t firms can set their regular price, p_t, and the price to be charged to repeat buyers in the next period, p^r_{t+1}. We conjecture that the Markov equilibria of such partial commitment game differs from that of the full commitment game. The reason is twofold. First, under partial commitment firms set p_t after x_{t-1} has already been determined. This is analogous to the game of Section 4.2. Thus, firms do not take into account that a higher p_t makes the offers of their rivals less attractive and hence raises x_{t-1}. Therefore, under partial commitment regular prices will tend to be lower. Second, under partial commitment demand by young consumers

becomes more elastic. A lower p_t implies a larger x_t, which implies that the firm's incentives to attract in period $t+1$ old consumers that are currently trading with its rivals are reduced. As a result, p_{t+1} will be expected to be higher, which in turn increases x_t further. Therefore, the higher elasticity of demand induces firms to set lower regular prices. Hence, both effect push regular prices downwards.

On the other hand, lower regular prices imply that firms are less able to capture the rents associated to reduced prices for repeat buyers, which will tend to raise the price for repeat purchases. That is, we conjecture that, under partial commitment, the stationary symmetric equilibrium will be characterized by a lower p and a higher p^r than under full commitment. As occurred in Section 4, restricting firms ability to commit to future prices for newcomers has a quantitative effect on equilibrium prices, but the main qualitative features of equilibrium are independent of it.

In this section firms are restricted to a common price for young and old newcomers. Alternatively, firms could offer a menu of contracts and let these two types of consumers separate themselves. The contract targeted to old newcomers could simply offer a single price for the current transaction, p_t^n. The contract targeted to the young could include a price for the current transaction, p_t, and a price for the next period if the customer remains loyal, p_{t+1}^r. In a separating equilibrium prices must satisfy two incentive compatibility constraints, which implies that neither type has incentives to imitate the other type. If neither of these two constraints is binding then firms face fully segmented markets and hence equilibrium prices must be those of Section 4.2. In other words, in this case the overlapping generations structure would be redundant. It turns out that if a stationary equilibrium exists then it is separating. It is immediate that firms have incentives to set a lower price for old newcomers (who in turn have access to reduced prices if they remain loyal to their previous suppliers). Moreover, in such an equilibrium one of the incentive compatibility constraints is binding. Hence, allowing firms to offer newcomers a menu of contracts does have an effect on equilibrium prices, although we conjecture that the qualitative properties are the same as in the game where firms are restricted to setting a common price for all newcomers.

8 Discussion

In this section we discuss the role of various assumptions and consider different extensions.

8.1 Consumer horizon

If we let consumers live for more than two periods, then consumers might be able to accumulate claims to different loyalty programs (might join more than one FFP). This could reduce the potential lock-in effect of loyalty programs. However, if rewards are properly designed (that is, if rewards are a convex function of the number of purchases), then these programs would still involve significant switching costs for consumers and the same qualitative effects should be obtained.[20]

8.2 Heterogenous patterns of repeat purchases

Let us consider the two-period game of Section 4.1 with the following variation. There are two types of consumers: frequent flyers, who purchase in both periods, and occasional flyers, who only purchase in one period. In order to maintain total demand constant we could let first period occasional flyers be replaced in the second period by a different generation of the same size. First, if firms cannot discriminate between these two types of consumers then p_2^n will be higher than in Proposition 1. As a result, profits in the second period from newcomers who are frequent flyers will be lower, and hence competition for frequent flyers in the first period will be relaxed. Nevertheless, in the first period frequent flyers will be sensitive to the commitment to a lower price for repeat purchases and hence their willingness to pay will be higher than that of occasional travelers. Hence, firms may be able to discriminate between these two types of consumers by offering a menu of contracts, as discussed at the end of the previous section.

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[20]Fernandes (2001) studies a model where consumers live for three periods. Unfortunately, he restricts attention to a particular kind of reward. In particular, consumers obtain a lump-sum coupon with the first purchase, which must be used in the next purchase with the same supplier. In this extreme example, consumers' lock-in effects are minimized.

8.3   Entry

In this paper we have characterized loyalty programs as a business-stealing device provided there is sufficient competition (the market is fully served). However, in markets where there is room for entry, incumbents may use loyalty programs as a barrier to entry. The existence of a large share of consumers with claims to the incumbents' loyalty program may be sufficient to discourage potential entrants.[21]

8.4   Partnerships

Recently airlines have formed FFP partnerships. On the one hand, those partnership enhance the FFP program of each partner by expanding earning and redemption opportunities. On the other hand, they may affect the degree of rivalry. Those observers that interpret FFP as enhancing firms' market power have a hard time understanding the formation of partnerships of domestic airlines who compete head to head on the same routes. In their view those partnerships appear to increase airline substitutability and hence they are likely to reduce profits[22] In contrast, we claim that FFP are business-stealing devices. Hence, partnership between directly competing firms may relax competition by colluding on less generous loyalty rewards. A rigorous analysis of these issues is beyond the scope of this paper, but some intuition can be provided. Consider the duopoly model analyzed in CM. In the non-cooperative equilibrium firms offer loyalty-rewarding policies (commit to a lower price for repeat buyers) and as a result industry profits are lower. Hence, firms would like to collude and agree to cancel these programs even if they choose regular prices non-cooperatively. This type of collusion can be implemented by forming a partnership and setting a common and negligible reward system (setting $p_2^r = c + t$) that would apply to all customers independently of which firm they patronized in the first period. In this case, the reward system does not affect the allocation of consumers in the first period, and in equilibrium first period prices are equal to $c + t$ (the one-period equilibrium price). In an oligopoly with more than two firms the effect of a partnership would be less drastic, but still each pair of firms would like to commit not to steal consumers from each other through loyalty programs, although they still wish to lure consumers

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[21]See Cairns and Gailbraith (1990).
[22]See Lederman (2003), Section VI.

from their rivals. As a result, we conjecture that direct rivals still have incentives to form partnerships, and they result in less generous loyalty rewards and higher industry profits.

8.5   Relative sizes

In order to study the effect of firm size we need to go back to an oligopoly model. Let us consider the duopoly model of CM with an asymmetric distribution of consumers. In particular, there are two firms located at the extreme points of the Hotelling line. A proportion α of consumers are located at 0, and a proportion $1 - \alpha$ are uniformly distributed over $[0, 1]$. Thus, the firm located at 0 is the large firm. Consumer location is independent across periods. Therefore, the large firm's commitment to p_2^r is more valuable to any consumer than the small firm's commitment to the same p_2^r because they anticipate that repeating a purchase at the large firm is more likely than at the small firm.[23]

Unfortunately, an analytical solution of this asymmetric game is not feasible and we need to turn to numerical simulations. We have focused on the case that α is sufficiently small, so that in equilibrium the small firm is able to attract a positive mass of newcomers (competition is effective) in the second period. For simplicity, we have also restricted to the full commitment game: firms can commit in the first period to the price for repeat purchases as well as to the price for second period newcomers (no strategic commitment effect). We have checked (See Appendix 11.7) that both firms loose with the introduction of loyalty programs, but the large firm looses relatively less, because its market share increases as consumers attach a higher value to the large firm's program.

Thus, the empirical evidence reported by Lederman (2003) indicating that the impact of an airline's FFP on its market share is relatively more important for large firms is perfectly compatible with our model. However, it is not obvious that such a fact implies that FFP's enhance airlines market power. In fact our model proposes the opposite interpretation. In our view, large airlines are relatively protected from the pro-competitive effects of FFP, but all airlines loose in absolute terms with the introduction of FFPs.

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[23]Redemption oportunities of an airline's FFP increases with its size: number of destinations, frequency of flights, etc.

9   Concluding remarks

The answer we provide to the title question is rather sharp. Loyalty rewarding pricing schemes are essentially a business stealing device, and hence reduce average prices and increase consumer welfare. Such a pro-competitive effect is likely to be independent of the form of commitment (price level versus discounts). Therefore, competition authorities need not be particularly concerned about these pricing strategies.

Our model focuses on the case of a single product market with inelastic demand. This set up has allowed us to concentrate on the intertemporal link of prices and purchases, which seems crucial in most loyalty programs. However, in some cases these programs also have a multiproduct dimension. In fact, some of them (like in grocery retailing) are designed in such a way that rewards are a combination of static bundled discounts and the intertemporal commitment device that we emphasize in this paper. Hence, the current analysis can be viewed as a building block for a more general model of loyalty rewards.

From an empirical point of view there are many important questions that need to be posed. In the real world, we observe high levels of dispersion in the size and characteristics of loyalty rewarding pricing schemes. What are the factors that explain these cross-industry differences? One possible answer is transaction costs. Discriminating between repeat buyers and new consumers can be very costly, as sellers need to somehow keep track of individual history of sales. Those transaction costs are likely to vary across industries, both in absolute value and also relative to the mark up. This might explain some fraction of the cross-industry variations in loyalty-rewarding pricing schemes. Unfortunately, it is not obvious which proxies of industry-specific transaction costs are available.

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11   Appendix

11.1   Proposition 1

The first order conditions of the firm's optimization problem are given by:

$$\frac{d\pi}{dp_1} = x_1 - \frac{M}{2t} = 0$$

$$\frac{d\pi}{dp_2^r} = x_1 x_2^r - \frac{x_2^r M}{2t} - \frac{x_1 \left(p_2^r - c\right)}{2t} = 0$$

$$\frac{d\pi}{dp_2^n} = \left(1 - x_1\right) x_2^n + \frac{x_2^n M}{2t} - \frac{\left(1 - x_1\right)\left(p_2^n - c\right)}{2t} = 0$$

where $M \equiv p_1 - c + x_n^r \left(p_2^r - c\right) - x_2^n \left(p_2^n - c\right)$ and x_2^r, x_2^n and x_1 are given by equations 1-3 in the text. In a symmetric equilibrium we have that $x_1 = \frac{1}{2}$, $x_2^r = 1 - x_2^n$. Plugging these conditions into the first order conditions and solving the system we obtain the strategies stated in the proposition.

If we denote the elements of the Hessian matrix by H_{ij}, then evaluated at the first order conditions we have that $H_{11} = -\frac{1}{t}$, $H_{22} = -\frac{17}{18t}$, $H_{33} = -\frac{13}{18t}$, $H_{12} = -\frac{5}{6t}$, $H_{13} = H_{23} = 0$. Hence, the matrix is negative semidefinite and second order conditions are satisfied.

11.2   Proposition 2

In the second period the firm chooses p_2^n in order to maximize second period profits, which implies that:

$$p_2^n = \frac{t + \overline{p}_2^r + c}{2}$$

After plugging this expression into equation 3, the firm chooses (p_1, p_2^r) in order to maximize 4. The first order conditions are:

$$\frac{d\pi}{dp_1} = x_1 - \frac{M}{2t} = 0$$

$$\frac{d\pi}{dp_2^r} = x_1 x_2^r - \frac{x_2^r M}{2t} - \frac{x_1 (p_2^r - c)}{2t} = 0$$

Evaluating these conditions at a symmetric equilibrium and solving, we obtain the strategies stated in the proposition.

The elements of the Hessian matrix evaluated at the first order conditions are $H_{11} = -\frac{1}{t}$, $H_{12} = -\frac{3}{4t}$, $H_{22} = -\frac{13}{16t}$. Hence, second order conditions are satisfied.

11.3   The commitment capacity of lump-sum coupons

Suppose that other firms have set $\overline{p}_2^r = c$ and $\overline{p}_2^n = c + \frac{2t}{3}$. Then the best response in the first period is to set exactly these prices. Instead, consider a firm that arrives at the second period with $x_1 = \frac{1}{2}$ and a lump-sum coupon f. Then such a firm would choose p_2 in order to maximize:

$$\pi_2 = \frac{1}{2} \{(p_2 - f - c) x_2^r + (p_2 - c) x_2^n\}$$

where

$$x_2^r = \frac{t + \overline{p}_2^n - p_2 + f}{2t}$$

$$x_2^n = \frac{t + \overline{p}_2^r - p_2}{2t}$$

If f is large, then the solution includes $x_2^n = 0$ and the outcome is dominated from the ex-ante point of view by $f = 0$. If f is not too large the solution is interior and the ex-post optimal prices will be given by:

$$p_2^r \equiv p_2 - f = \frac{2t}{3} + c - \frac{f}{2}$$

$$p_2^n \equiv p_2 = \frac{2t}{3} + c + \frac{f}{2}$$

Thus, as f increases p_2^r gets closer to the optimal ex-ante response, but p_2^n is driven further away from its ex-ante optimal value. Therefore, there is no value of f that allows the firm to commit to a pair of prices close to the best response.

11.4 Equilibrium with lump-sum coupons

For arbitrary prices and market shares the second period optimization problem provides the following first order condition:

$$p_2 = \frac{t + c + \overline{p}_2 + 2x_1 f - (1 - x_1)\overline{f}}{2}$$

In the first period, firms choose (p_1, f) in order to maximize first period profits. The first order conditions are:

$$\frac{d\pi}{dp_1} = x_1 - \frac{M}{2t + \frac{(f + \overline{f})(2f + \overline{f})}{4t}} = 0$$

$$\frac{d\pi}{df} = -\frac{x_1 (1 - x_1)\left(2f + \overline{f}\right)}{2t} + M \frac{\overline{p}_2 + t - c + f(2 - 4x_1) + \overline{f}(1 - 3x_1)}{8t^2 + \left(f + \overline{f}\right)\left(2f + \overline{f}\right)} = 0$$

where $M \equiv p_1 - c + x_n^r (p_2 - f - c) - x_2^n (p_2^n - c)$. If we evaluate these conditions at the symmetric allocation, then we have that $p_1 = c + t$, $p_2 = c + \frac{4t}{3}$, $f = \frac{2t}{3}$. Thus, profits are $\pi = \frac{8t}{9}$, and consumer surplus per firm is $CS = R - c - \frac{43t}{36}$.

If we compare the equilibrium under monopolistic competition and duopoly (CM) then we observe that both coupons and second period prices are the same in both games, but the first period under duopoly is $p_1 = c + \frac{13t}{9}$, which is far above the first period price of the monopolistic competition equilibrium. The intuition is the following. Under duopoly the elasticity of the first period demand with respect to the first period price is higher than under monopolistic competition. The reason is that a higher first period market share (because of a lower first period price) induces the rival firm to set a lower second period price, since it has more incentives to attract new customers. Such a lower expected second period price makes the first period offer of the rival firm more attractive, which in turn reduces the

increase in first period market share. As a result, such a reduction in the price elasticity of demand induces firms to set a higher first period price.

Strategic commitment has two separate effects of different signs on the level of coupons, and it turns out that they cancel each other. On the one hand, a higher coupon induces the rival firm to set a lower second period price, which has a negative effect on second period profits. Hence, duopolists would tend to set lower coupons. On the other hand, a higher coupon involves a commitment to set lower prices for repeat buyers, which increases first period demand. If the first period price is higher then the increase in first period profits brought about by a higher coupon is heightened. Hence, through this alternative channel, duopolistic firms would tend to set higher coupons. In our model both effects cancel each other out and coupons are the same under both duopoly and monopolistic competition and therefore, second period prices are also the same.

11.5 Substitutability between endogenous and exogenous switching costs

Suppose that only one firm can commit to p_2^r. Then, analogously to Klemperer (1987), non-discriminating firms set:

$$p_1 = c + t - s + \frac{s^2}{2t}$$

$$p_2 = c + t$$

and make profits:

$$\pi = t - \frac{s}{2} + \frac{s^2}{4t} \tag{10}$$

The discriminating firm will optimally set:

$$p_1 = c + \frac{13t}{8} + \frac{13s^2 - 20st}{32t}$$

$$p_2^r = c$$

33

$$p_2^n = c + t - \frac{s}{2}$$

As a result profits will be:

$$\pi^c = \frac{145t}{128} + \frac{-1312st^3 + 920s^2t^2 - 72s^3t + 81s^4}{2048t^3} \tag{11}$$

The net benefit from committing (the difference between 11 and 10) decreases with s (provided s is not too large).

Suppose now that all firms commit and set the equilibrium strategies of Proposition 4. If one firm does not commit then it will optimally set:

$$p_1 = c + \frac{431t^4 - 104t^3s + 178t^2s^2 + 27s^4}{520t^3 + 48t^2s + 72ts^2}$$

$$p_2 = c + \frac{161t^3 - 23t^2s + 11s^2t - 21s^3}{260t^2 + 24st + 36s^2}$$

As a result profits will be:

$$\pi^{nc} = \frac{1221t^4 - 372t^3s + 190t^2s^2 - 52ts^3 + 37s^4}{2080t^3 + 192t^2s + 288ts^2} \tag{12}$$

The net loss from not committing (the difference between profits obtained in the equilibrium of Proposition 4 and 12) decreases with s.

11.6   Proposition 4

The first order conditions with respect to p_t and p_t^r are respectively:

$$\beta^t \left\{ x_t + (1 - x_{t-1}) x_t^n - (p_t - c) \frac{2 - x_{t-1}}{2t} + [(p_t^r - c) x_t^r - (p_t - c) x_t^n] \frac{dx_{t-1}}{dp_t} \right\} +$$

$$+\beta^{t-1} \left\{ (p_{t-1} - c) \frac{dx_{t-1}}{dp_t} \right\} = 0$$

$$\beta^t \left\{ x_{t-1} \left[x_t^r - \frac{p_t^r - c}{2t} \right] + [(p_t^r - c) x_t^r - (p_t - c) x_t^n] \frac{dx_{t-1}}{dp_t^r} \right\} + \beta^{t-1} \left\{ (p_{t-1} - c) \frac{dx_{t-1}}{dp_t^r} \right\} = 0$$

34

From equations 6 to 8:

$$\frac{dx_{t-1}}{dp_t} = \frac{x_t^n}{2t}$$

$$\frac{dx_{t-1}}{dp_{t-1}} = -\frac{1}{2t}$$

$$\frac{dx_{t-1}}{dp_t^r} = -\frac{x_t^r}{2t}$$

If we evaluate these first order conditions at a symmetric and stationary equilibrium $(x_t = \frac{1}{2}, x_t^r = 1 - x_t^n)$ with $\beta = 1$, then we get:

$$t(2 - x^r) - \frac{3}{2}(p - c) + (p + p^r - 2c) x^r (1 - x^r) = 0 \tag{13}$$

$$t + p - 2p^r + c - \frac{p + p^r - 2c}{2t^2}(t + p - p^r) = 0 \tag{14}$$

where

$$x^r = \frac{1}{2} + \frac{p - p^r}{2t}$$

If $p^r = c$, the value of p that satisfies equation 13 is in the interval $\left(c + \frac{t}{2}, c + t\right)$. Also, p increases with p^r for all $p^r > c$. On the other hand, the equation implicitly characterized by equation 14 goes through the points $(p^r = c, p = c + t)$ and $\left(p^r = p = c + \frac{t}{2}\right)$ and is decreasing in this interval. Therefore, there is a solution of the system in this interval, which proves the proposition.

11.7   An asymmetric duopoly model

Two firms are located at the opposite extremes of the $[0, 1]$ interval. A proportion α of consumers are located at 0 and a proportion $1-\alpha$ are uniformly distributed over the interval$[0, 1]$. The rest is exactly as in the benchmark model, including the fact that the location of individual consumers across periods is independent.

The following notation corresponds to the firm located at 0:

p_1 - first period price

p_2^r - second period price for repeat buyers

p_2^n - second period price for newcomers

x_1 - location of the indifferent consumer in the first period

x_2^r - location of the indifferent consumer in the second period among those who patronized the large firm in the first period

x_2^n - location of the indifferent consumer in the second period among those who patronized the small firm in the first period

$m_1 = \alpha + (1 - \alpha)x_1$ - first period market share

$m_2^r = \alpha + (1 - \alpha)x_2^r$ - second period market share among first period customers

$m_2^n = \alpha + (1 - \alpha)x_2^n$ - second period market share among non-customers

We denote with bars the variables set by the rival firm (the one located in 1).

11.7.1   Static game

Suppose that firms have no commitment capacity. Since there is no intertemporal link, the unique subgame perfect equilibrium of this game consists of repeating the equilibrium strategies of the static game. Hence, in this section we do not need time subscripts. The indifferent consumer is located at:

$$x = \frac{t + \overline{p} - p}{2t}$$

Profits of the two firms in each period are, respectively:

$$\pi = m(p - c)$$
$$\overline{\pi} = (1 - m)(\overline{p} - c)$$

The equilibrium prices, market share and total profits are given by:

$$p = c + t\frac{3 + \alpha}{3(1 - \alpha)}$$
$$\overline{p} = c + t\frac{3 - \alpha}{3(1 - \alpha)}$$
$$m = \frac{3 + \alpha}{6}$$

$$\pi = t\frac{(3+\alpha)^2}{9(1-\alpha)}$$
$$\overline{\pi} = t\frac{(3-\alpha)^2}{9(1-\alpha)}$$

11.7.2 Full commitment game

Now, suppose that firms have full commitment capacity; they can commit to both the price for repeat buyers and the price for newcomers. The expression of the second period indifferent consumers are given by, respectively:

$$x_2^r = \frac{t + \overline{p}_2^n - p_2^r}{2t}$$
$$x_2^n = \frac{t + \overline{p}_2^r - p_2^n}{2t}$$

The first period indifferent consumer, x_1, is determined by the following equation:

$$tx_1 + p_1 + \alpha p_2^r + (1-\alpha)x_2^r(p_2^r + x_2^r\frac{t}{2}) + (1-\alpha)(1-x_2^r)[\overline{p}_2^n + (1-x_2^r)\frac{t}{2}]$$
$$= t(1-x_1) + \overline{p}_1 + \alpha p_2^n + (1-\alpha)x_2^n(p_2^n + x_2^n\frac{t}{2}) + (1-\alpha)(1-x_2^n)[\overline{p}_2^r + (1-x_2^n)\frac{t}{2}]$$

Total profits of each firm are as follows:

$$\pi = m_1(p_1 - c) + m_1 m_2^r(p_2^r - c) + (1-m_1)m_2^n(p_2^n - c)$$
$$\overline{\pi} = (1-m_1)(\overline{p}_1 - c) + (1-m_1)(1-m_2^n)(\overline{p}_2^r - c) + m_1(1-m_2^n)(\overline{p}_2^n - c)$$

First order conditions cannot be solved analytically. Therefore, we have run a set of simulations. Note that some parameters are qualitatively irrelevant in both the static and the full commitment game. First, absolute margins are independent of c, and hence profits and market shares are independent of c. Thus, there is no loss of generality on setting $c = 0$. Second, it is easy to show that profits and absolute margins are proportional to t, and market shares are independent of t. Hence, for our purposes we can normalize $t = 1$.

11.7.3 Simulations

The next table reports the results of the numerical simulations for different values of parameter α. We have chosen values of α that are sufficiently small so that all solutions are interior (all market shares are positive). The main conclusions are the following. Firstly, the large firm (the one located in 0) loses relatively less with the introduction of commitment. In fact, the higher α, the higher the difference between the relative losses of the two firms. Secondly, the first period market share of the large firm also increases with the presence of commitment. Finally, as expected, in all the simulations we obtain that $p_2^r = 0$ (marginal cost pricing for repeat buyers).

	π	$\overline{\pi}$	p_1	p_2^n	$\overline{p}_1$	$\overline{p}_2^n$	m_1	m_2^r	m_2^n
$\alpha = 0.02$									
Static	1.034	1.006	1.027	1.027	1.013	1.013	0.503		
Commitment	0.638	0.609	1.148	0.696	1.119	0.664	0.505	0.835	0.168
Relative loss (%)	38.3	39.5							
$\alpha = 0.04$									
Static	1.070	1.014	1.056	1.056	1.028	1.028	0.507		
Commitment	0.666	0.607	1.187	0.727	1.128	0.662	0.510	0.838	0.170
Relative loss (%)	37.7	40.1							
$\alpha = 0.06$									
Static	1.107	1.022	1.085	1.085	1.043	1.043	0.510		
Commitment	0.696	0.606	1.228	0.759	1.137	0.660	0.515	0.840	0.173
Relative loss (%)	37.1	40.7							
$\alpha = 0.08$									
Static	1.146	1.030	1.116	1.116	1.058	1.058	0.513		
Commitment	0.727	0.604	1.270	0.793	1.146	0.658	0.520	0.843	0.175
Relative loss (%)	36.5	41.4							
$\alpha = 0.1$									
Static	1.186	1.038	1.148	1.148	1.074	1.074	0.517		
Commitment	0.760	0.602	1.314	0.829	1.156	0.656	0.525	0.845	0.177
Relative loss (%)	35.9	42.0							

TABLE 1

	No commitment	Full commitment	Partial commitment
p_1	$c + t$	$c + \dfrac{10}{9}t$	$c + \dfrac{9}{8}t$
p_2^r	$c + t$	c	c
p_2^n	$c + t$	$c + \dfrac{2}{3}t$	$c + \dfrac{1}{2}t$
x_1	$\dfrac{1}{2}$	$\dfrac{1}{2}$	$\dfrac{1}{2}$
x_2^r	$\dfrac{1}{2}$	$\dfrac{5}{6}$	$\dfrac{3}{4}$
x_2^n	$\dfrac{1}{2}$	$\dfrac{1}{6}$	$\dfrac{1}{4}$
p	t	$\dfrac{11}{18}t$	$\dfrac{5}{8}t$
CS	$R - c - \dfrac{5}{4}t$	$R - c - \dfrac{33}{36}t$	$R - c - \dfrac{29}{32}t$