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Abstract

This paper points out an empirical puzzle that arises when an RBC economy with a job matching function is used to model unemployment. The standard model can generate sufficiently large cyclical fluctuations in unemployment, or a sufficiently small response of unemployment to labor market policies, but it cannot do both. Variable search and separation, finite UI benefit duration, efficiency wages, and capital all fail to resolve this puzzle. However, both sticky wages and match-specific productivity shocks help the model reproduce the stylized facts: both make the firm's flow of surplus more procyclical, thus making hiring more procyclical too.

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1 Introduction

A model of real business cycles with matching (RBCM) is a natural candidate for exploring many dynamic policy issues. Postulating a job matching function helps us give a coherent analysis of unemployment and its response to labor market policies. Moreover, several authors, starting with Merz (1995) and Andolfatto (1996), have claimed that endogenizing unemployment by means of a matching function improves the fit of real business cycle models. Thus it is tempting to use the RBCM framework to measure the costs of business cycles, to measure the purported benefits of output stabilization, to ask whether unemployment benefits should be constant over time, or to ask whether the government should attempt to limit job loss during recessions.

These questions interest us. But when we tried to build an RBCM model to address them, we quickly encountered problems with the RBCM framework which previous literature has not pointed out. For our purposes, we hoped to calibrate our model to be consistent both with business cycle facts and with the effects of labor market policies. We found it easy to choose parameters to make the cyclical fluctuation in unemployment as large in the model as it is in the data, or to make the response of unemployment to a change in the unemployment insurance (UI) benefit as small in the model as it is in the data. But no parameterization permits the standard RBCM model to reproduce both these features of the data; improving the fit over the business cycle makes the fit worse with respect to labor market policies, and *vice versa*. Similar conclusions hold for the volatilities of employment, vacancies, and the probability of job finding.

Productivity shocks and unemployment benefits both act on the unemployment rate through their effect on the surplus associated with employment. Therefore, as we demonstrate analytically in a simple benchmark RBCM model, there is a close relationship between the volatility of unemployment over the cycle, and the responsiveness of unemployment to UI benefits. We show that the model's predictions for these two aspects of unemployment variability are seriously inconsistent with the data. We then go on to show numerically that the problem remains even in more complicated and realistic versions of the model, and is also present but undiagnosed in previous papers.

However, we also propose two possible solutions to the problem: sticky wages, or embodied technological progress. As in Shimer (2004) and Hall (2004, 2005), we find that sticky wages help because they make firms' share of surplus more procyclical, allowing hiring to vary more at business cycle frequencies without greatly changing the long run effects of policies. We show that embodied (that is, match-specific) technological change also increases the cyclicality of the match surplus, especially for the firm, so that we can get a similar effect without arbitrarily imposing wage rigidities.

1.1 Stylized facts

We begin by discussing the empirical evidence about the two aspects of unemployment variation our paper addresses: the cyclical volatility of unemployment, and the response of unemployment to labor market policy.

Unemployment over the business cycle

Employment varies less than output over the business cycle, but unemployment, by the same token, is highly volatile relative to its low mean. In seasonally-adjusted US quarterly data from 1951:1 to 2003:1, the mean unemployment rate is 0.0567.[1] Detrending with the HP filter (using $\lambda = 1600$), we find that the standard deviation of the unemployment rate is 0.00743; that is, the standard deviation is $\frac{0.00743}{0.0567} = 13.1\%$ of the mean. By contrast, using the same numbers, the standard deviation of the employment rate is only $\frac{0.00743}{1-0.0567} = 0.787\%$ of its mean.

Similarly, if we consider the log of the unemployment rate, we find that its standard deviation after HP filtering is $\sigma_U = 0.135$. The standard deviation of HP-filtered log GDP in our data is $\sigma_Q = 0.0165$, so by this measure, unemployment fluctuates much more than output: the ratio σ_U/σ_Q equals 8.18. Other authors roughly agree; Merz (1995) finds that $\sigma_U/\sigma_Q = 6.11$,[2] while Greenwood, Gomes, and Rebelo (2002) find $\sigma_U/\sigma_Q = 7.68$.[3] Moreover, HP filtering with $\lambda = 1600$ removes much of the variation in unemployment that might usually be considered cyclical. Before HP filtering, the standard deviation of the log of the (seasonally adjusted) unemployment rate is 0.282 (more than twice the value after HP filtering). Thus by looking at HP-filtered data we are being conservative in our analysis of the cyclicality of unemployment.

Several related series also show high cyclical volatility. We find that the log of median unemployment spell duration has a standard deviation of 0.128 after HP filtering, which is 7.77 times the variability of output; likewise, Greenwood et. al. (2002) state that duration is 6.87 times as variable as output. In our data, the HP-filtered log of vacancies (help-wanted advertising) has standard deviation $\sigma_V = 0.140$, so that $\sigma_V/\sigma_Q = 8.49$; Merz (1995) reports $\sigma_V/\sigma_Q = 7.31$, while Andolfatto (1996) states that this ratio is

[1] We use quarterly US data, or monthly US data aggregated to quarterly frequency, from the St. Louis Fed's FRED database. We use the series GDPC1 for our measure of real output, UNRATE for the unemployment rate, HELPWANT for vacancies, UEMPMED for median unemployment duration. When HP filtering, we always set $\lambda = 1600$ for comparability with most related papers.

[2] Merz uses US quarterly data, 1959:1-1988:2, in logs, HP filtered with $\lambda = 1600$.

[3] Greenwood et. al. use US quarterly data, 1954:1-1991:2, seasonally adjusted, logged, and HP-filtered with $\lambda = 1600$.

greater than 9. The variability of workers' probability of job finding is also similar; Shimer (2005) shows that the coefficients of variation of unemployment, vacancies, and workers' probability of job finding are 0.188, 0.183, and 0.17, respectively.[4]

In our effort to model the cyclical fluctuations of the labor market, we have found that two other stylized facts help to distinguish between competing models. Cole and Rogerson (1999) report that job creation is four times as volatile as employment, and job destruction is six times as volatile as employment.[5] The negative correlation between vacancies and unemployment (the "Beveridge curve") is also a decisive feature of the data. After HP filtering, we find that the correlation between the log unemployment rate and log vacancies is -0.933. Merz (1995) finds that this correlation is -0.95, while Shimer (2005) reports -0.90.

Labor market policy and unemployment

A large literature has documented the negative effect of unemployment benefits on employment. Many microeconomic studies have regressed reemployment probabilities or unemployment durations on the UI benefit replacement ratio. Layard, Nickell, and Jackman (1991) review this literature and conclude that the consensus range of estimates for the elasticity of unemployment duration with respect to the unemployment benefit is from 0.2 to 0.9. Atkinson and Micklewright (1991) come to similar conclusions.

While we simplify the discussion by focusing on the effects of the replacement ratio, similar quantitative results come from studies of other labor market policies. Meyer (1990) measures the rise in workers' probability of job finding as UI benefits expire. Meyer (1995) summarizes policy experiments in which quick job finding was rewarded with a lump sum payment. These papers' estimates indirectly shed light on the effects of the benefit level too. Costain (1997) argues that Meyer's (1995) results imply that a one percentage point rise in the replacement rate should cause a 0.023 percentage point rise in unemployment, which is an elasticity of 0.17 (assuming an initial unemployment rate of 6% and a replacement ratio of 45%).

For our purposes, though, what is really interesting is the general equilibrium effect of UI. Layard, Nickell, and Jackman (1991), in a cross-country regression for the OECD, report that a one percentage point rise in the UI replacement ratio results in a highly

[4]US data, 1951-2001, quarterly averages of seasonally adjusted monthly data, expressed as ratio to HP trend. Shimer includes more of the cyclical variation of unemployment by setting the HP parameter at $\lambda = 100000$.

[5]US quarterly manufacturing data, from LRD database, 1972:2-1988:4, in logs, seasonally adjusted, and HP filtered with $\lambda = 1600$.

significant 0.17 percentage point rise in unemployment. Given the median unemployment rate of 8% and median replacement ratio of 60% in their sample, this works out to a semielasticity of 2.1 or an elasticity of 1.3. Similarly, Scarpetta (1996) finds a rise of 0.13 percentage points. More recently, with more data, Layard and Nickell (1999, henceforth LN99) find that the semielasticity of unemployment with respect to the unemployment insurance replacement ratio is 1.3, with a standard error of 0.5.[6] That is, a rise in the replacement ratio by one percentage point increases the log of the unemployment rate by 0.013, an elasticity of 0.78. The literature review of Disney (2000) reports similar numbers. Thus, as we should expect, the general equilibrium effects of UI on unemployment appear moderately larger than the partial equilibrium effects on workers' unemployment durations. We will take the LN99 semielasticity estimate as our main point of reference.

In general equilibrium as in partial equilibrium, evidence about other policy instruments reinforces the claim that the effects of unemployment benefits are significant but not very big. Solon (1985) documents the fall in unemployment duration after the imposition of a tax on UI benefits in the US. Roughly summarizing his results, imposing a 25% tax on a 50% UI benefit decreased unemployment durations of one quarter by 1.2 weeks, implying an elasticity of unemployment to benefits of approximately 0.4. LN99 also studies the effects of changing the *duration* of UI benefits, and reports that the semielasticity of the unemployment rate with respect to benefit duration is 0.1. We will see below that we can test the matching model using this variable too. Regardless of whether we consider the benefit level or the benefit duration, we run into the same problem: the RBCM framework can reproduce the effects of labor market policy on unemployment only if it understates the cyclical variability of unemployment. Quantitatively, the mismatch between policy effects and cyclical variability is similar with both policy variables.

1.2 Related literature

Two influential studies, Merz (1995) and Andolfatto (1996), showed that including a matching function improves the fit of RBC models by increasing the persistence of fluctuations. Obviously, it also allows them to compare the model to a wider variety of labor market data. Andolfatto claims success in matching the volatility of employment, though he does not address the volatility of unemployment and underpredicts the volatility of vacancies. Merz is fairly successful with all three variables; her model generates

[6]Regression of log unemployment on replacement ratio and other labor market policy variables, for 20 OECD countries, treating averages for 1983-88 and 1989-94 as separate observations.

about 80% of the observed standard deviation of US employment and unemployment (relative to output), and about 60% of that of vacancies.

However, other studies report difficulties with the model. Cole and Rogerson (1999) show that it is hard to reproduce the negative correlation and persistence of job creation and job destruction observed in the data, though they do better if they assume a high baseline unemployment rate (around 15%). Millard, Scott, and Sensier (1997) find that the fluctuations of employment and unemployment in their RBCM model are too small and insufficiently persistent. Shimer (2005) documents in detail the difficulty of obtaining sufficiently volatile unemployment, vacancies, and especially labor market tightness in the RBCM model. Shimer (2004) and Hall (2004, 2005) argue that sticky wages are the key to unemployment volatility.

Like those of Shimer and Hall, our paper argues that there is a serious inconsistency between the standard RBCM model and the data. However, we feel that an important element is missing in their argument, because their claim that unemployment is insufficiently variable in the RBCM model is not true in general: in fact, it is specific to the particular calibration they choose. Shimer and Hall both assume that workers' cost of working is low compared to their productivity, so that the match surplus is large. We point out that when this restriction is removed, it is easy to make unemployment volatile. If the surplus is small on average, then a small fall in labor productivity may eat up a large proportion of the surplus, so that realistic productivity fluctuations generate substantial volatility in vacancies, unemployment, and tightness. In fact, this is how Merz (1995) succeeded in making these series volatile in her simulations.

Some macroeconomists will agree with Shimer and Hall's assumption that the match surplus is large. But undoubtedly others, accustomed to frictionless models where workers' marginal product equals the marginal cost of working, will be more skeptical. Therefore we feel it is important to test the model in a more general way that is independent of any particular calibration. This test is possible because the RBCM model has simultaneous implications for business cycle variability and the effects of policies. This is where we locate the real problem with the standard model: if the match surplus is small enough to reproduce business cycle effects, then the model greatly exaggerates the effects of policy. Moreover, we also consider factors absent in the papers of Shimer and Hall, such as variable search intensity, that might be expected to help the model fit data. And in the end, we do find a way of resolving the model's failure without resorting to nominal rigidities. A flexible-price, optimal RBCM model can fit the unemployment data well if technology shocks include a match-specific or cohort-specific component, as they would in the case of embodied technological progress.

With respect to the impact of policies, many authors have used matching models to examine the effects of UI benefits; Pissarides (2000), p. 233, lists some of these papers. Two studies that use matching models to explain cross-country differences in unemployment are Millard and Mortensen (1996) and Ljungqvist and Sargent (1998). One of the few studies that attempts to model both the cyclical volatility of unemployment and the response of unemployment to unemployment benefits is Greenwood, Gomes, and Rebelo (2002). Their model does not fall into the class considered here, because it has no matching function. Interestingly, though, it suffers from the same failing as the RBCM models we analyze. It does well at business cycle frequencies, but reports a much larger response of unemployment to UI benefits than that found in the data.

2 The model

Our general model is a version of the standard RBCM model, as spelled out in Pissarides (2000) and elsewhere. We simplify by leaving out capital; including it would be likely to reinforce our result that RBCM models exaggerate policy effects relative to cyclical volatility, since capital can more easily adjust to long term policy changes than to short term business cycle fluctuations.[7] In hopes of finding a successful version of the model, we generalize in several ways: we allow productivity to vary across matches, and we allow separation rates and bargaining power to vary too.

2.1 Values and surpluses

Let Z be a random shock to the productivity of the economy, and let z be the value of this shock at the time when a given job was formed. We consider a process for the marginal product of labor y that allows the output of a match to depend on its vintage:

$$y(z, Z) = 1 + \alpha_Z Z + \zeta(1 - \alpha_Z)z \tag{1}$$

In the usual RBC specification ($\alpha_Z = 1$), aggregate productivity fluctuates because tecnology shocks immediately affect all matches. But alternatively, technological progress could require the creation of new jobs. In that case, productivity would have a match-specific or cohort-specific component, which would be consistent with Devereux's (2003)

[7]We also simplify by ignoring two other generalizations that are unlikely to resolve the dilemma that interests us. One might want to consider procyclical unemployment benefits (since benefits are usually computed as a fraction of the wage) or procyclical hiring costs (since the cost of hiring may consist mostly of labor time). However, these factors would only make firms' hiring expenditure *less* procyclical, so they are not likely to help resolve the puzzle that concerns us.

evidence that workers tend to find persistently better matches in booms than in recessions. Setting $\alpha_Z = 0$ attributes all fluctuations in aggregate productivity to this cohort-specific component. The parameter ζ allows us to adjust the impact of the cohort-specific shock z relative to the aggregate shock Z.

It is well known that in matching models without a capital stock, surpluses and most decision variables are independent of the unemployment rate. Without mentioning unemployment, we can write transition probabilities in terms of labor market tightness, which in turn depends on productivity. To save on notation, we immediately impose these restrictions by writing the value and policy functions in terms of their appropriate state variables. Later we point out why these restrictions are valid.

The value for a employed worker, $W^E(z, Z)$, satisfies

$$W^E(z, Z) = w(z, Z) + \beta E_{Z'|Z}\left[(1 - \delta(z, Z))W^E(z, Z') + \delta(z, Z)W^U(Z')\right] \tag{2}$$

Note that we generalize to allow the separation rate δ to depend on productivity. We will see that the probability of finding a job can be written as $p(S, \theta)$, where S is search effort and θ is labor market tightness. The value $W^U(Z)$ of unemployment is:

$$W^U(Z) = \max_S \left\{b - h(S) + \beta E_{Z'|Z}\left[p(S, \theta(Z))W^E(Z', Z') + (1 - p(S, \theta(Z)))W^U(Z')\right]\right\} \tag{3}$$

Here b represents the unemployment benefit, though in general it should also be understood to capture other costs of working, such as disutility costs. S is the intensity of job search and $h(S)$ are the costs of searching. Most of the time we will fix $S = 1$ and $h(S) = 0$, but we will also investigate the effect of varying search intensity.

The workers' surplus is defined as the difference between the values of employment and unemployment; it satisfies

$$\begin{aligned}\Sigma^W(z, Z) &= W^E(z, Z) - W^U(Z) \\ &= w(z, Z) - b + h(S(Z)) + \beta E_{Z'|Z}\left[(1 - \delta(z, Z))\Sigma^W(z, Z') - p(S(Z), \theta(Z))\Sigma^W(Z', Z')\right]\end{aligned} \tag{4}$$

where $S(Z)$ denotes the optimal search at Z. If search is endogenous, then it obeys the first-order condition

$$h'(S(Z)) = \beta \frac{\partial p(S(Z), \theta(Z))}{\partial S} E_{Z'|Z}\Sigma^W(Z', Z') \tag{5}$$

The value to the firm of a filled job, $J(z, Z)$, satisfies the recursive equation

$$J(z, Z) = \Sigma^F(z, Z) = y(z, Z) - w(z, Z) + \beta(1 - \delta(z, Z))E_{Z'|Z}J(z, Z') \tag{6}$$

Unlike a worker's job acceptance decision, filling a job is assumed (as usual) to have no opportunity cost in terms of lost hiring opportunities. Therefore (6) shows that the surplus $\Sigma^F(z, Z)$ associated with a filled job is the same as the value of that job. Firms offer new jobs until the expected profits associated with a vacancy are zero. If the probability of a filling a job is $p^F(S, \theta)$, then the zero profits condition is:

$$\kappa = p^F(S(Z), \theta(Z)) E_{Z'|Z} \Sigma^F(Z', Z') \tag{7}$$

where κ is the flow cost of maintaining a vacancy.

The wage is determined by the Nash bargaining condition

$$\frac{\Sigma^W(z, Z)}{\Sigma^F(z, Z)} = \frac{\mu(z, Z)}{1 - \mu(z, Z)} \tag{8}$$

Here we generalize again, by letting bargaining power μ vary with the aggregate state.

2.2 The labor market

We assume that total matches M are given by

$$M = \gamma V^{1-\lambda} U^{\lambda} S \tag{9}$$

where V is total vacancies, and U is unemployment. Tightness is defined as $\theta \equiv V/U$ so that it depends on unemployment U rather than effective search US, which is unobservable. Matching probabilities are thus functions of tightness and search:

$$p(S, \theta) = \frac{M}{U} = \gamma \theta^{1-\lambda} S \tag{10}$$

and

$$p^F(S, \theta) = \frac{M}{V} = \frac{p(S, \theta)}{\theta} \tag{11}$$

Equ. (10) implicitly provides a metric for search effort, saying that the individual probability of finding a job is proportional to search.

Note that equations (4), (5), (6), (7), (8), (10), and (11) are seven equations that determine the seven functions $\Sigma^W(z, Z)$, $S(Z)$, $\Sigma^F(z, Z)$, $\theta(Z)$, $w(z, Z)$, $p(S, \theta)$, and $p^F(S, \theta)$, without reference to unemployment U. Thus it is reasonable to look for a solution of these equations that is independent of U.

When we incorporate the dynamics of employment and unemployment in our model, we must note that $\alpha_Z < 1$ implies a distribution of matches with different productivities. To deal with this effect in the simplest possible way, in Section 4 where we allow $\alpha_Z < 1$ we will assume that productivity follows a two-state Markov process, taking a low value

Z^{LO} or a high value Z^{HI}. We then distinguish between the fraction of the labor force in matches with low productivity, N_t^{LO}, and the fraction matched with high productivity, N_t^{HI}. Total employment plus unemployment must sum to one:

$$N_t + U_t \equiv N_t^{HI} + N_t^{LO} + U_t = 1 \tag{12}$$

If we write total matches at time t as $M_t \equiv \gamma\theta(Z_t)^{1-\lambda}S(Z_t)U_t$, then the three labor market state variables follow the dynamics

$$N_{t+1}^{HI} = (1-\delta(Z^{HI}, Z_t))N_t^{HI} + M_t\mathbf{1}(Z_{t+1} = Z^{HI}) \tag{13}$$

$$N_{t+1}^{LO} = (1-\delta(Z^{LO}, Z_t))N_t^{LO} + M_t\mathbf{1}(Z_{t+1} = Z^{LO}) \tag{14}$$

$$U_{t+1} = \delta(Z^{LO}, Z_t)N_t^{LO} + \delta(Z^{HI}, Z_t)N_t^{HI} - M_t + U_t \tag{15}$$

where $\mathbf{1}(x)$ is an indicator function equalling 1 if statement x is true, and 0 if x is false.

Here we see that total job destruction is $D_t = \delta(Z^{HI}, Z_t)N_t^{HI} + \delta(Z^{LO}, Z_t)N_t^{LO}$. Finally, given that there is no capital stock, aggregate output Q_t is:

$$Q_t = (1-U_t)(1+\alpha_Z Z_t) + \zeta(1-\alpha_Z)(N_t^{HI}Z^{HI} + N_t^{LO}Z^{LO}) \tag{16}$$

3 Unemployment volatility: cycles and policies

We now consider the simplest and most standard version of this model, in which labor productivity is just $y = 1 + Z$, and the separation rate δ and bargaining power μ are constants.[8] For this case, we can characterize the dynamics explicitly, and demonstrate how the variability of labor market variables over the business cycle is related to their variability in response to UI policy changes.

Define total surplus as $\Sigma_t \equiv \Sigma_t^F + \Sigma_t^W$. Summing equations (4) and (6), and using the fact that the worker's share of surplus is μ, we see that Σ must satisfy

$$\begin{aligned}\Sigma_t &= y_t - b + h(S_t) + \beta(1-\delta)E_t\Sigma_{t+1}^F + \beta(1-\delta-p_t)E_t\Sigma_{t+1}^W \\ &= y_t - b + h(S_t) + \beta(1-\delta-\mu p_t)E_t\Sigma_{t+1}\end{aligned} \tag{17}$$

where $h(S) = 0$ if search is exogenous. In addition, we have the zero profit condition

$$\kappa = p_t^F E_t J_{t+1} = p_t^F(1-\mu)E_t\Sigma_{t+1} \tag{18}$$

[8]To simplify notation, we now use the time subscript t to denote dependence on the aggregate state Z_t (and also on U_t where appropriate).

In equations (17) and (18), $p_t = \gamma S_t \theta_t^{1-\lambda}$ and $p_t^F = \gamma S_t \theta_t^{-\lambda}$ depend only on tightness θ_t and search effort S_t. Thus when search is exogenous, (17) and (18) suffice to determine total surplus Σ_t and tightness θ_t.

In the endogenous search case, the first-order condition (5) plus the zero profit condition (18) allow us to eliminate search in favor of tightness:

$$\frac{\kappa\theta_t}{\gamma(1-\mu)} = \frac{h'(S_t)S_t}{\beta\gamma\mu} \tag{19}$$

Since $h(S)$ is convex, (18) says that search and tightness are positively related: people search harder when job market conditions are good. We call the relation $S(\theta)$, with elasticity $\eta_\theta^S(\theta) \equiv (1 + h''(S(\theta))S(\theta)/h'(S(\theta)))^{-1}$. In what follows, we will assume that search costs are small on average, but that $h(S)$ is very convex, so that job finding is relatively inelastic in response to θ. These restrictions suffice for existence of a unique equilibrium, and as we will see below, large search costs or highly elastic search effort would have counterfactual implications.

Steady state

In the nonstochastic steady state (indicated by dropping the subscript t), equations (17) and (18) give two different expressions for Σ. Substituting for p and p^F, we have:

$$\Sigma = \frac{\kappa\theta^\lambda}{\gamma S(1-\mu)} = \frac{y-b+h(S)}{1-\beta(1-\delta-\mu\gamma S\theta^{1-\lambda})} \tag{20}$$

If S is exogenous, then the left-hand side is increasing in θ, and the right-hand side is decreasing in θ, so there exists a unique steady state for θ and Σ.

In the case of endogenous search, we assume S is sufficiently inelastic so that

$$\lambda^* \equiv \lambda - \eta_\theta^S(\theta) > 0 \tag{21}$$

This suffices to make the left-hand side of (20) increasing in θ. Furthermore, the right-hand side is decreasing if h is sufficiently small but S is sufficiently inelastic; this then suffices for a unique steady state equilibrium.

We can now use (20) to derive comparative statics for θ in terms of b. Let hats represent changes in the log of the steady state. Then we have:

$$\lambda\hat{\theta} - \hat{S} = -\frac{b}{y-b+h(S)}\hat{b} - \left(\frac{\beta\mu p}{1-\beta(1-\delta-\mu p)}\right)[(1-\lambda)\hat{\theta}+\hat{S}] + \frac{h(S)}{y-b+h(S)}\eta_S^h(S)\hat{S} \tag{22}$$

where $\eta^h_S(S) \equiv h'(S)S/h(S)$. We now simplify, using the formula (20) for steady state surplus Σ, and we write the equations in terms of $\hat{p} = (1-\lambda^*)\hat{\theta}$. We obtain:

$$\eta^p_b \equiv \frac{\hat{p}}{\hat{b}} = -\frac{1-\lambda^*}{\lambda^*}\left(\frac{b}{y-b+h}\right)\left(\frac{1-\beta+\beta\delta+\beta\mu p}{1-\beta+\beta\delta+\beta\mu p/\lambda^* - h\eta^h_S\eta^S_\theta/(\lambda^*\Sigma)}\right) < 0 \tag{23}$$

The steady state effect of b on unemployment is approximately the opposite of the effect on the job finding probability p. In steady state,

$$\delta(1-U) = pU \tag{24}$$

which implies

$$\eta^U_b \equiv \frac{\hat{U}}{\hat{b}} = -(1-U)\frac{\hat{p}}{\hat{b}} > 0 \tag{25}$$

Equations (23) and (25) show that $\lambda^* > 0$ is necessary for the negative effect of UI on unemployment that is observed in the data; this justifies assumption (21).

Dynamics

Now consider the dynamics. Suppose that $y_t = 1 + Z_t$ is AR1 in logs:

$$\tilde{y}_{t+1} = \rho\tilde{y}_t + \epsilon_{t+1} \tag{26}$$

where ϵ is *i.i.d.* with $E_t\epsilon_{t+1} = 0$, and $\rho \in (0,1)$. (All variables with tildes signify log deviations from steady state, and unadorned variables are steady state values or constants.) If we linearize the surplus dynamics (17) and the zero profit condition (18) and impose saddle path stability, we find an explicit formula for the dynamics of the job-finding probability, in terms of the productivity shock:

$$\frac{\tilde{p}_t}{\tilde{y}_t} = \frac{1-\lambda^*}{\lambda^*}\left(\frac{y}{y-b+h}\right)\left(\frac{1-\beta+\beta\delta+\beta\mu p}{1/\rho-\beta+\beta\delta+\beta\mu p/\lambda^* - h\eta^h_S\eta^S_\theta/(\lambda^*\Sigma)}\right) \tag{27}$$

It is the close resemblance between (23) and (27) that enables us to test the model. Intuitively, the effects of changes in y and b are similar (though of opposite sign) since they both act on vacancy formation through their effects on the size of surplus. For comparability with LN99, we state the implications of the model in terms of semielasticities instead of elasticities.[9] To keep our results unit-free, we calculate semielasticities with respect to the unitless variable $\xi \equiv b/y$, the steady state ratio of the unemployment

[9]The other crucial reason to state our results in terms of the semielasticity is that it is invariant to any unobserved disutility component in b. In contrast, an estimate of the *elasticity* with respect to b or ξ changes depending on what portion of b we assume consists of UI benefits rather than work disutility.

benefit to the marginal product of labor. For ease of expression, we call ξ the "replacement ratio", even though the correct definition is b/w. In steady state, the difference is small, and we have verified numerically that the quantitative impact of working with b/y instead of b/w is trivial. Thus we define $\epsilon^p_\xi \equiv \eta^p_\xi/\xi \equiv \eta^p_b/\xi$, which we call the semielasticity of job finding with respect to the replacement ratio. Using (23) and (27), we obtain:

Proposition 1. The dynamic elasticity of the probability of job finding with respect to productivity, and the long-run semielasticity of the probability of job finding with respect to the replacement ratio ξ, have the following ratio in absolute value:

$$\left|\frac{\tilde{p}_t/\tilde{y}_t}{\epsilon^p_\xi}\right| \quad = \quad \left(\frac{1-\beta+\beta\delta+\beta\mu p/\lambda^* - h\eta^h_S\eta^S_\theta/(\lambda^*\Sigma)}{1/\rho-\beta+\beta\delta+\beta\mu p/\lambda^* - h\eta^h_S\eta^S_\theta/(\lambda^*\Sigma)}\right) \quad \leq \quad 1 \tag{28}$$

This ratio equals one if and only if $\rho = 1$, that is, if productivity shocks are permanent. This makes sense, because it says that a *permanent* 1% change in labor productivity has the same effect on hiring (with opposite sign) as a permanent change of the unemployment benefit by an amount equal to 1% of labor productivity. For any $\rho < 1$, the ratio is strictly less than one. Endogenous search does not alter this ratio if $\rho = 1$, and it makes the ratio smaller if $\rho < 1$, because the search term $h\eta^h_S\eta^S_\theta/(\lambda^*\Sigma)$ decreases the numerator proportionally more than the denominator.

The simplicity of Proposition 1 is helpful, but to address familiar data it will be better to focus on the unemployment rate U instead of the job-finding probability p. Turning to the dynamics of U, we have:

$$U_{t+1} \quad = \quad U_t + \delta(1-U_t) - \gamma S_t\theta_t^{1-\lambda}U_t \tag{29}$$

In the appendix we calculate the ratio of the standard deviations of the logs (the usual business cycle volatility measure) of unemployment and the technology shock, which we can then compare to the semielasticity $\epsilon^U_\xi \equiv \partial \log U/\partial\xi$ of unemployment with respect to the replacement ratio. Using the notation $\sigma_x \equiv \sqrt{\mathrm{Var}(\tilde{x}_t)}$, we obtain:

Proposition 2. The relative standard deviation of log unemployment to log output, and the long-run semielasticity of unemployment with respect to the replacement ratio ξ, have the following ratio:

$$\frac{\sigma_U/\sigma_Q}{\epsilon^U_\xi} \quad = \quad \frac{(\sigma_y/\sigma_Q)(\sigma_U/\sigma_y)}{\epsilon^U_\xi} \quad = \quad \left|\frac{\tilde{p}_t/\tilde{y}_t}{\epsilon^p_\xi}\right|\left(\frac{\delta(U+\rho(U-\delta))}{(2U-\delta)(U+\rho(\delta-U))}\right)^{\frac{1}{2}}\frac{\sigma_y}{\sigma_Q} \tag{30}$$

We have seen that the first term is strictly less than one unless technology shocks are permanent. The second term is less than or equal to one if $U > \delta$, which is true if and only if $\delta + p < 1$. Thus this restriction is satisfied unless we choose an inappropriately long period (a Cobb-Douglas matching model like this is not well behaved if periods are so long that transition probabilities are near one). The last term is less than one in the data, and it cannot exceed one in our model except in the irrelevant case of a large positive correlation between y and U. Thus for any sensible parameters, all three terms in Proposition 2 are weakly less than one, strictly so in the case of the last term.

Returning to the data, we have seen estimates of the ratio of standard deviations of log unemployment and log output ranging from 6.11 (Merz 1995) to 8.18 (our calculations from the FRED database). LN99's estimate of the semielasticity of unemployment with respect to the replacement ratio is 1.3, with standard error 0.5. Thus the ratio in Proposition 2 is around six in the data, while the model implies that it should be substantially less than one. Even if we interpret LN99's results more generously, by considering the whole 95% confidence interval for their semielasticity estimate, the ratio is still off by at least a factor of three.

We must emphasize that this rejection of the model is independent of the mean level of unemployment U, since both the numerator and denominator of the ratio in Proposition 2 state the variability of unemployment as a proportion of its mean. In the numerator, σ_U is approximately the standard deviation of unemployment divided by U. In the denominator, ϵ_ξ^U is approximately $U^{-1}\partial U/\partial \xi$. Hence U^{-1} cancels. Thus we need not be concerned (as in Cole and Rogerson 1999) that the success of our model depends on how we calibrate mean U.[10]

4 Numerical extensions

We have seen that our simplified model exaggerates UI policy effects relative to the cyclical variation in unemployment. This cannot be resolved by tinkering with parameters, since the upper bound in Proposition 2 is independent of calibration. However, we must still ask whether some generalization of the model might fit better. Therefore we now turn to numerical simulations of the general model from Section 2.

For concreteness, we start by calibrating the model in terms of a conservative interpretation of LN99's results. Our first calibration is chosen to produce a semielasticity

[10]Equivalently, this says the model would still be rejected if we studied *levels* of unemployment, instead of logs of unemployment, since when we cancel out U^{-1} Proposition 2 becomes a statement about the variability of unemployment in *levels*.

of unemployment with respect to the unemployment benefit of 2, which is in the upper range of their confidence interval, and roughly equals the largest point estimate we have seen (that of Layard et. al., 1991).

4.1 Benchmark parameters

The productivity shock Z follows a two-state Markov process, taking the values $Z^{LO} = -0.018$ and $Z^{HI} = 0.018$. The benchmark parameterization assumes that all matches have equal productivity ($\alpha_Z = 1$). The probability that Z remains unchanged from one period to the next is denoted ρ_Z. We simulate the model at weekly frequency, but report results aggregated to quarterly frequency. In our benchmark parameterization, we impose an approximate yearly persistence of $\bar{\rho}_Z \equiv 2/3$, implying business cycles lasting roughly six years, by assuming that Z remains unchanged from one week to the next with probability $\rho_Z \equiv \bar{\rho}_Z^{1/52} \approx 0.9922$.[11]

The elasticity of total matches to unemployment is set to $\lambda = 0.5$, consistent with Blanchard and Diamond (1989). We assume an efficient benchmark equilibrium, setting $\mu = 0.5$ (Hosios 1990). We calibrate an annual job loss rate of approximately $\bar{\delta} \equiv 25\%$ by setting the weekly probability of job loss to $\delta \equiv \bar{\delta}/52$. This is reasonable for the US, though separation rates are higher for the most unstable classes of jobs and workers. To get a discount factor of $\bar{\beta} \equiv 95\%$ annually, we set the weekly discount factor to $\beta \equiv \bar{\beta}^{1/52}$. The matching efficiency and vacancy cost parameters γ and κ are reset in each simulation so that steady state unemployment is always $U = 0.06$ (again, a US calibration) and so that a vacancy lasts two weeks on average. Vacancy duration is just a normalization: doubling it would mean doubling vacancies, reducing κ by half, and adjusting γ to keep total matches, total vacancy costs, and job finding probabilities unchanged. In addition to constant δ and μ, the benchmark specification assumes exogenous search intensity ($h = 0$ and $\eta^h_S = \infty$).

On average, the Markov process spends equal time in good and bad states, so mean productivity y is 1. We set $b = 0.745$ for the benchmark parameterization; that is, the cost of working is 74.5% of the mean marginal product of labor. This parameter is crucial, because a larger b implies a smaller match surplus, which makes unemployment and vacancies more volatile. Intuitively, if b is large, then firms own a highly leveraged claim on the productivity process y, so that small variations in y or b motivate big changes in hiring. In fact, (27) shows that as b approaches $y + h$, the variance of job finding goes

[11] Although this is less persistence than many business cycle models assume, we prefer this calibration because longer cycles would make our results more sensitive to the HP filter.

to infinity: so clearly, the RBCM model cannot be rejected on grounds of insufficient unemployment volatility alone. Our choice of $b = 0.745$ sets the semielasticity ϵ_b^U to two (our conservative interpretation of the LN99 results). This b may seem high, and is much larger than Shimer (2005) assumes. But in the structure of our model, b includes more than unemployment benefits; it also includes any utility costs (or any other costs) of working.[12] These costs are presumably nontrivial. Table 1 shows the results for this parameterization, together with the rest of our simulations.

Benchmark results: importance of the size of the of surplus

The first line of Table 1 shows the benchmark results. All relative standard deviations and correlations refer to data aggregated to quarterly frequency, and results are HP-filtered with parameter $\lambda = 1600$.[13]

In line 1, the long run semielasticity of unemployment with respect to the unemployment benefit is $\epsilon_\xi^U = 2.00$ by construction. But this parameterization yields insufficient variability of log unemployment over the business cycle, with $\sigma_U/\sigma_Q = 1.40$, when this ratio is over six in the data. The punchline is that $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ equals 0.70, far too low for consistency with the data, and also well below our analytical upper bound of one. Similar results hold for the probability of job finding p: the business cycle variability is $\sigma_p/\sigma_Q = 1.61$ (too low), while the semielasticity ϵ_ξ^p is -2.13 (approximately correct; not shown in table). The cyclical variability of vacancies $\sigma_V/\sigma_Q = 3.23$ is also too low.

As we mentioned above, the way to make unemployment more variable is to impose a larger b, so that the surplus is smaller and more volatile. With the benchmark value $b = 0.745$, the total surplus Σ equals 45.2% of the mean quarterly marginal product of labor. In line 2 we raise the cost of working to 90% of the mean marginal product of labor ($b = 0.90$), which shrinks the joint surplus Σ to just 17.7% of the mean quarterly marginal product of labor. This raises σ_U/σ_Q to 3.16, an improvement but still less than in US data (to actually match the data we need $b = 0.95$). However, unemployment also becomes more responsive to the UI benefit, so that $\epsilon_\xi^U = 5.41$ now far exceeds the estimates in the literature. In fact, the key ratio $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ gets worse, falling to 0.58. Thus we see the main tradeoff: we can make the model more volatile to better match cyclical data, or less volatile to better match labor market data, but the two goals are at odds with each other; and in relative terms the tradeoff is worse when b is large.

[12] In the US, benefits average 44% of the wage for newly unemployed workers, according to Engen and Gruber (1994).

[13] The filter has little effect. Without filtering, σ_Q rises from 1.62 to 1.86, but the ratio σ_U/σ_Q, which is more important, only changes from 1.40 to 1.42.

In line 3, we go in the opposite direction and decrease b to 0.4, which is Shimer's (2005) calibration, and amounts to assuming that the only cost of working is the loss of the unemployment benefit.[14] Total surplus Σ is now 106.3% of the mean quarterly marginal product of labor. The unemployment semielasticity ϵ_ξ^U falls to 0.82, and the cyclical volatility of unemployment falls to $\sigma_U/\sigma_Q = 0.62$. Thus this parameterization not only produces insufficient cyclical volatility: it is slightly too inelastic to match even the (small) observed effects of the UI benefit.

Before moving to other versions of the model, we try several parameter changes. In line 4, we set $\bar{\rho}_Z = 75\%$, so that cycles are more persistent, lasting roughly eight years. In line 5, we increase the separation rate to $\bar{\delta} = 40\%$ annually; this would be a reasonable calibration for the US if we chose to focus on relatively unstable jobs and workers. In line 6, we lower the elasticity of matching with respect to unemployment to $\lambda = 0.3$, with worker bargaining power $\mu = 0.5$, while in line 7 we lower μ to 0.3, with $\lambda = 0.5$. Though there are mild changes in some statistics, the ratio $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ is robust to a wide range of parameter changes, staying close to 0.7 in all these experiments.

4.2 Variable separation and variable search

Davis, Haltiwanger, and Schuh (1996) offer evidence that job destruction is strongly countercyclical. Therefore, it is important to ask whether variation in separation rates would change our results. The usual model of variable separation (Mortensen and Pissarides 1994) posits a match-specific productivity shock, so that workers and firms separate when their joint surplus becomes negative. Instead, for simplicity, we just assume an exogenous separation rate that depends negatively on the aggregate technology shock, which is essentially what the model of Mortensen and Pissarides implies. We set $\bar{\delta}(Z^{LO}) = 0.25 * 1.15 = 0.2875$ and $\bar{\delta}(Z^{HI}) = 0.25/1.15 \approx 0.2174$, so that $\bar{\delta}$ varies by $\pm 15\%$, depending on Z. We see in line 8 that this variation in separation brings the cyclicality of unemployment close to that in the data: σ_U/σ_Q rises to 5.89. The semielasticity of unemployment with respect to ξ changes only slightly, so the ratio $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ improves, rising to 2.79.

The problem is that this way of resolving the conflict destroys the Beveridge curve: the correlation between unemployment and vacancies switches sign to $\rho_{U,V} = 0.95$. The fact that variable separation helps increase unemployment volatility, but eliminates the Beveridge curve, has also been noted by Cole and Rogerson (1999) and Shimer (2005).

[14] Hall (2004) assumes that b is 35% of the firm's flow of surplus (in a new match), so his b is even smaller than that of Shimer.

Second, although unemployment becomes more variable, the probability of job finding now varies less: the ratio σ_p/σ_Q falls from 1.61 with the baseline parameters to 1.40 with variable separation. This contradicts the data of Section 1, which showed that job finding has roughly the same percentage variability as unemployment. Third, the amount of variation in the separation probability needed here is too large. The relative standard deviation of job destruction to employment, σ_D/σ_N, is now 13.51, well above Cole and Rogerson's (1999) figure of six. (In the benchmark, it is exactly one by construction.)

Lines 9 and 10 allow for variable search effort, first considering the relatively inelastic case $\eta^h_S = 4$ and then the more elastic case $\eta^h_S = 2$. Variable search effort makes unemployment more cyclical because (as we saw in Section 2) search rises when productivity is high. With $\eta^h_S = 4$, we have $\sigma_U/\sigma_Q = 2.75$, while with $\eta^h_S = 2$, we match cyclical data quite well, reaching $\sigma_U/\sigma_Q = 5.31$. However, the semielasticity of unemployment with respect to the replacement rate ξ rises even more, so that the ratio $(\sigma_U/\sigma_Q)/\epsilon^U_\xi$ falls to 0.66 when $\eta^h_S = 4$, and to 0.59 when $\eta^h_S = 2$. As our analytical calculations indicated, endogenous search only makes the tradeoff worse. Also, sufficiently elastic search effort again destroys the Beveridge curve: with $\eta^h_S = 2$, we have $\rho(U,V) = -0.17$.[15]

4.3 Finite UI benefit duration

Another issue that might matter for our results is that we have assumed that unemployment benefits continue as long as a person remains unemployed. UI benefits might affect unemployment less if instead they eventually expired, and this could help the model better match the data. The easiest way to model finite benefit duration is to make benefits expire with probability ϕ per period, so that their expected duration is $D \equiv 1/\phi$. Then there are three possible labor market states: employed, unemployed with benefits, and unemployed without benefits. The employed workers' Bellman equation (2) is unchanged.[16] Restricting ourselves to exogenous search effort ($S = 1$, $h(S) = 0$), equation (3) for the value of unemployment with benefits is replaced by

$$W^U(Z) = b + \beta E_{Z'|Z} \left\{ p(1,\theta(Z))W^E(Z',Z') + (1 - p(1,\theta(Z)))[(1-\phi)W^U(Z') + \phi W^X(Z')] \right\} \tag{31}$$

Here $W^X(Z)$ is the value of an unemployed worker without benefits, given by

$$W^X(Z) = b_0 + \beta E_{Z'|Z} \left\{ p(1,\theta(Z))W^E(Z',Z') + (1 - p(1,\theta(Z)))W^X(Z') \right\} \tag{32}$$

[15] Merz (1995) also finds that variable search effort acts against the Beveridge curve.

[16] This means we are assuming that workers are eligible for unemployment benefits from the moment of matching. Otherwise we would need to define another labor market state (employed without benefits) with a different wage.

Note that here, for the first time, we must distinguish between the UI benefit $b - b_0$ itself (which expires at rate ϕ) and the disutility cost of working b_0. For consistency with the US, we set $b - b_0 = 0.4$.

We first consider a mean benefit duration of six months, which is the US norm. Shorter benefit duration increases the surplus $\Sigma^W = W^E - W^U$ associated with employment (this is still the relevant surplus for the Nash wage equation), so with the baseline $b = 0.745$ the volatility of unemployment over the cycle is greatly decreased. Therefore, in line 11 of the table we instead set $b = 0.87$, bringing the volatility of unemployment roughly back to its value in the benchmark model of line 1. The implications are very similar to those of the benchmark model: the effect of benefits on unemployment is reasonable, but the cyclical variability of unemployment is much too small, so the key ratio $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ only increases from 0.7 in line 1 to 0.79 in line 11. In line (12), we instead assume benefits last two years, which is the median duration reported for European countries in LN99. We again adjust b, this time to 0.80, to keep unemployment volatility in line with the benchmark model. Results are again similar.

Thus finite benefit duration leaves our main results essentially unchanged. However, it also gives us an additional way to test the model, because LN99 also study the effects of the duration of benefits on the unemployment rate. Their Table 15 reports that the semielasticity of unemployment with respect to benefit duration is $\epsilon_D^U = 0.1$. Since the cyclical variability of unemployment is roughly 7, the ratio $(\sigma_U/\sigma_Q)/\epsilon_D^U$ should be around 70. Instead, the final column of lines 11 and 12 reports values of 7.33 and 11.27, respectively: σ_U/σ_Q is too small compared with the effect of duration on the unemployment rate. Thus considering finite benefit duration reinforces our claim that the standard RBCM framework understates cyclical volatility relative to the effects of policies.

4.4 Sticky wages

We have seen that higher b means higher percentage variation in the firm's surplus over the cycle, increasing the variability of hiring and unemployment. Another obvious way to make the firm's surplus volatile would be to impose some form of wage stickiness, as has been emphasized recently by Shimer (2004) and Hall (2004, 2005). Furthermore, it seems natural to assume that sticky wages are only a short run phenomenon, so that they should have less influence on the long run impact of the UI benefit.

Again, we choose an easy *ad hoc* way of making wages sticky. We assume that workers' bargaining power varies negatively with the technology shock, so that workers get a larger share of surplus in recessions. This stabilizes the wage over the cycle, and thus destabilizes the firm's hiring incentives. In line 13 we assume that the worker's bargaining power increases (decreases) by 15% when the aggregate technology shock is low (high). This amount of variation in bargaining power suffices to raise σ_U/σ_Q to 5.67, roughly consistent with the data. The semielasticity ϵ_ξ^U hardly changes, so that $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ increases to 2.73.

This does not seem like an unreasonable degree of wage stickiness: the ratio of the standard deviations of log wages and log output is now $\sigma_w/\sigma_Q = 0.59$. This is better than the figure of 0.91 in the baseline model, though still not as low as in the data; for example, Merz (1995) reports $\sigma_w/\sigma_Q = 0.37$ for the US. Therefore, sticky wages seem a potentially promising way of improving the model's fit. But obviously they are controversial, and debate goes on about possible justifications for wage stickiness.

One possible microfoundation for wage stickiness is an "efficiency wage". Here, if we follow Shapiro and Stiglitz (1984) by assuming a constant probability of observing shirking behavior, firms should offer workers a constant surplus just sufficient to prevent shirking. Thus in line 14 we report a version of our model where the Nash bargaining condition (8) is replaced by an equation that fixes a constant surplus for the worker at all times (equal to the average surplus in the benchmark model of line 1). While the cyclical variability of unemployment increases, the semielasticity of U with respect to ξ increases even more, so that $(\sigma_U/\sigma_Q)/\epsilon_\xi^U$ falls to 0.64. The problem is that the incentive problem alters wages not only in the short run, but also in the long run. Imposing a constant surplus for the worker makes hiring incentives fall sharply with the replacement ratio, so that our efficiency wage model fits less well than our *ad hoc* sticky wage model, in which wages adjust flexibly to long run changes in UI.

4.5 Cohort-specific technology shocks

Finally, we argue that a form of embodied technological change could also help solve the puzzle that concerns us. If technological progress is embodied in new capital, and requires the hiring of new workers with different skills, then technology shocks should affect new matches without changing the productivity of old ones. Such a specification has the advantage that, unlike our baseline model, it makes the wages for new hires more procyclical than those for continuing jobs. This is a well-established empirical fact (see for example Bils 1985 and Bowlus 1995). There is also plenty of direct evidence that workers find higher-quality jobs in booms than in recessions, from data on movements across sectors (Heckman and Sedlacek 1985), job tenure (Bowlus 1995), and the characteristics of workers and jobs (Devereux 2003). Again, this suggests that productivity should have a match-specific or cohort-specific component.

So now we set $\alpha_Z = 0$, making the productivity of each match specific to the time of its creation. Since shocks no longer affect all matches equally, the persistence of aggregate output increases, and we therefore decrease the persistence $\bar{\rho}_Z$ of the shock from 0.67 to 0.6 annually. We also initially set $\zeta = 1$, so that the cohort-specific shock has the same impact as the aggregate shock did; and we lower b slightly to 0.7, to keep ϵ^U_ξ near its target level of two. This simple change of specification, which we call the "cohort-specific benchmark" in the table, more than suffices to solve the problem. In line 15, we find that $\sigma_U/\sigma_Q = 9.66$, higher than in the data, while $\epsilon^U_\xi = 1.79$ is slightly decreased, so that the ratio $(\sigma_U/\sigma_Q)/\epsilon^U_\xi$ rises to 5.40.[17] Thus, the cyclical variability of unemployment is no longer problematic. The job finding probability also varies more: with filtering, $\sigma_p/\sigma_Q = 11.32$.

When technology shocks are disembodied ($\alpha_Z = 1$) and thus immediately affect all matches, workers and firms know that a high match productivity Z may fall before separation, while a low Z may rise before separation. In contrast, in the embodied ($\alpha_Z = 0$) case, the match productivity z will be unchanged until separation; other things equal, this increases the difference in value between high and low productivity matches, making hiring respond more strongly to the aggregate state. Since employment now varies more relative to output, we also find that the average productivity across matches varies less compared with output than it does in the baseline model. That is, σ_y/σ_Q falls from 0.92 in the benchmark case of line 1 to 0.54 in the cohort-specific benchmark of line 15. This also improves the model's fit, since the relative standard deviation of labor productivity compared with output is only 0.68, according to Merz' (1995) data.

A disadvantage of this new specification is that the standard deviation of log output is now too low, falling to $\sigma_Q = 0.85$. But we will fix this by reparameterization below. A more serious problem is that even though labor productivity now varies less, wages vary more: σ_w/σ_Q more than triples from its benchmark value in line 1, which is already too high. The reason is that even though a technological improvement leaves the productivity of existing matches unchanged, it nonetheless raises all workers' outside options, and thereby their wages.[18] We must emphasize here that matching models do not actually tie down the wage process. Matching models only specify how the surplus

[17] Since fluctuations are more persistent under this specification, the results are now more sensitive to the HP filter. Without filtering, we have $\sigma_U/\sigma_Q = 7.86$ instead of 9.66, but this is still sufficient to match the data.

[18] Our parameterizations ensure that the outside option never rises enough to make separation optimal. But if we allowed a wider range of productivities, and endogenized separation like Mortensen and Pissarides (1994), then old matches might sometimes separate in response to a positive technology shock. This would raise the volatility of of job destruction and vacancies, while making unemployment and job finding probabilities somewhat less volatile.

is split between the firm and worker, and more than one wage process (including, for example, an implicit contract model that keeps the wage of *existing* matches fixed) is consistent with the model's implications for the behavior of the surplus. Therefore we may not want to reject this model on the basis of its wage implications. However, those who wish to take wage data literally may prefer the sticky wage model of line 13.

Since output varies across matches, it now seems especially important to allow for variable separation, depending on the match-specific productivity shock. Thus in line 16 we assume that separation rises or falls by 6% when $z = Z^{LO}$ or $z = Z^{HI}$, respectively. The variability of unemployment increases again, to $\sigma_U/\sigma_Q = 10.02$, and there is little change in the semielasticity ϵ_ξ^U. The relative standard deviation of job destruction compared with employment rises from 1.00 to $\sigma_D/\sigma_N = 2.08$. Furthermore, since productivity now varies across matches, imposing a relation between productivity and separation does less damage to the Beveridge curve than it did in line 8: unemployment and vacancies remain strongly negatively correlated, with $\rho_{U,V} = -0.68$.

To further improve the fit, we next raise ζ to 1.6. By increasing the impact of cohort-specific shocks, this raises the variability of log output, which is too low in line 15. Since this makes σ_U/σ_Q rise even more, when it is already too high, at this point we can afford to go to the intermediate case $\alpha_Z = 0.5$ where technology shocks have both aggregate and cohort-specific effects. This parameterization (without variable separation) is shown in line 17, and both $\sigma_Q = 1.39$ and $\sigma_U/\sigma_Q = 5.36$ fit quite well.

Finally, in line 18 we allow the separation rate to vary by $\pm 10\%$, which gives our most successful simulation. The ratio σ_U/σ_Q rises to 6.43, while the semielasticity of unemployment with respect to the replacement ratio ξ remains nearly unchanged at $\epsilon_\xi^U = 1.80$, and the ratio $(\sigma_U/\sigma_Q)/\epsilon_\xi^U = 3.58$ is consistent with the data. Again, variable separation combined with embodied technological change has little adverse impact on the Beveridge curve: the correlation between unemployment and vacancies is now -0.60. Also, as in the data, the variability of job finding is similar to that of U, at $\sigma_p/\sigma_Q = 6.79$. The relative standard deviation of job destruction compared with employment is now $\sigma_D/\sigma_N = 3.36$, lower than the figure of six reported by Cole and Rogerson (1999), but a big improvement relative to the case of constant δ where it is exactly one. Furthermore, the relative standard deviation of labor productivity $\sigma_y/\sigma_Q = 0.60$ now fits well, and the biggest problem is again the high variability of the wage, $\sigma_w/\sigma_Q = 2.43$.

5 Matching in business cycle models with capital

Up to now we have simplified our calculations by forgetting about physical capital. We have argued that this is probably unimportant for the issue at hand, but to be sure, we now consider the models of Merz (1995) and Andolfatto (1996), where capital is included. While these papers reported some success in modeling the cyclical behavior of the labor market, when we recalculate their steady states to measure the effects of unemployment insurance, we find that they suffer from the same problem as our benchmark model: insufficient cyclical volatility compared with policy effects.

5.1 The model of Andolfatto (1996)

To understand both models it is helpful to start by looking at the surplus. In Andolfatto's case, we calculate that the match surplus is equal to only 17.3% of mean quarterly labor productivity— close to the lowest surplus we considered, in line 3 of Table 1.[19] This suggests that the labor market in his model should be quite volatile.

At first glance, Andolfatto's labor market appears to work well. His Table 1 shows that the percentage variability of employment is 0.51 times that of output, compared with 0.67 in his data. However, this hides a surprising failure to explain unemployment, because of an unusual calibration. Andolfatto sets the mean employment rate to 57%, so that the mean unemployment rate is 43%. Unlike many matching papers (including this one) which ignore the "out of the labor force" state, Andolfatto treats any person over 16 years of age who is not working as unemployed. This goes far beyond some authors, such as Cole and Rogerson (1999) or den Haan et. al. (2000), who have claimed that it is helpful to work with a broader definition of unemployment.

Thus Andolfatto's calibration grossly overstates the number of people looking for a job, by including all pensioners, students, and homemakers as inputs to the matching function. Any given standard deviation of log employment therefore corresponds to a smaller standard deviation of log unemployment in Andolfatto's calibration than it would if baseline unemployment were lower. With his numbers, we find:

$$\frac{\sigma_U}{\sigma_Q} \quad = \quad \frac{1-U}{U}\frac{\sigma_N}{\sigma_Q} \quad = \quad \left(\frac{0.57}{0.43}\right)0.51 \quad = \quad 0.68 \tag{33}$$

[19] In Andolfatto's notation, from $qJ = \kappa$ and $J = \alpha\Sigma$ we get the total surplus as $\Sigma = \kappa/(q\alpha) = 0.105/(0.9 * 0.6) = 0.194$ in units of quarterly output. (This is equal to μ, the shadow value of a match, divided by the marginal utility of consumption.) The marginal productivity of labor is $(1-\theta)y/n = 0.64/0.57 = 1.123$, so that match surplus is equal to 17.3 percent of quarterly marginal productivity.

roughly ten times lower than our reading of the US data, based on the usual definition of unemployment.

Furthermore, even if we choose to ignore the low variability of unemployment in Andolfatto's model, it also implies insufficient variation in other labor market variables. For vacancies, Andolfatto's model yields $\sigma_V/\sigma_Q = 3.2$, compared to 9 in his data. This means that the percentage variability of vacancies is about 4.4. The variability of tightness is only slightly higher (4.6 percent) since unemployment hardly varies. Using $1-\lambda = 0.6$ in Andolfatto's parameterization, the variability of workers' job finding probability is $0.6 * 4.6 = 2.8$ percent. This is about twice the variability of output, while in the data the probability of job finding varies about seven times as much as output.

Andolfatto's model has no unemployment benefits, but since they are equivalent to work disutility in these models, we are able to calculate their effect on his steady state. We mimic a one percentage point increase in the UI replacement ratio by raising the utility of the non-employed by one percent of the mean marginal product of labor, scaled by the marginal utility of consumption. We calculate that the semielasticity of unemployment with respect to the replacement ratio is 2.41 in Andolfatto's model, almost twice LN99's point estimate. But given Andolfatto's interpretation of unemployment, a one percent increase in $\log U$ means a 0.43 *percentage point* increase in unemployment; that is, a one percentage point rise in the replacement ratio increases unemployment by 2.41*0.43=1.04 percentage points, about six times higher than the slope estimate of Layard, Nickell, and Jackman (1991). Seen in this way, Andolfatto's labor market is both insufficiently volatile over the business cycle, and excessively volatile in response to UI; the punchline for his paper is $(\sigma_U/\sigma_Q)/\epsilon^U_\xi = \frac{0.68}{2.41} = 0.28$.

5.2 The model of Merz (1995)

Merz (1995) comes close to fitting the variability of unemployment and job finding probability in US data. With her benchmark specification, the standard deviation of log unemployment over that of log output is 4.77,[20] and for the job finding probability it is 5.41. However, if we back out the effect of the unemployment benefit in the same way we did for Andolfatto, we see that the model exaggerates the sensitivity of unemployment to benefits. For her model, the semielasticity of unemployment with respect to the replacement ratio is 6.54. The statistic $\frac{\sigma_U}{\sigma_Q}\frac{1}{\epsilon^U_\xi}$ is therefore 0.73, so Merz' model fails by roughly the same factor as our benchmark model in Section 4.1.

[20]This is the result of our own calculation and differs slightly from the number in Merz' Table 2.

When we calculate the joint match surplus in Merz' model, it turns out to be only 1.69 percent of mean quarterly marginal product of labor— ten times smaller than anything we have seen before. Thus Merz achieves sufficient volatility to match business cycles only by assuming an almost negligible surplus, and in doing so exaggerates the response to the unemployment benefit.

If anything, the surprising aspect of Merz' results is how little fluctuation she obtains, given the tiny surplus she assumes. The explanation lies in the fact that she defines the surplus differently from all the other papers we have discussed. Most matching models assume that the marginal disutility of work is constant along the extensive margin (increases in employment) even if it is increasing along the intensive margin (increasing marginal disutility as hours per job increase, like Andolfatto assumes). In contrast, Merz assumes that surplus accrues to a family with a continuum of members, and that marginal disutility of work is increasing as more family members find jobs. At the margin in her equilibrium, the disutility from one more job almost equals the wage income from that job, so the surplus is extremely small. To us, the usual formulation seems more appropriate, since typical households contain only one or two earners, each of whom may have a large inframarginal gain when they find a job.

5.3 Other models with capital

Den Haan *et. al.* (2000) study an RBCM model with an endogenous separation decision. They are successful in explaining variations in job creation and destruction, and find that the interaction between job destruction and investment helps amplify shocks. Their results are consistent with our finding that we can make matching volatile by varying separation over the cycle. However, our calculations suggest that their model will fail to generate a Beveridge curve. They do not report the correlation between vacancies and unemployment in their paper.[21]

Gomes et. al. (2001) simulate a business cycle model in which individuals search for jobs. It is not entirely comparable with the models we are analyzing, because there is no matching function. Instead, the distribution of job offers is exogenous, making their model a dynamic extension of McCall's (1970) partial equilibrium search model. They successfully reproduce the cyclical fluctuations of unemployment. However, they state that a rise in the replacement ratio from 0.5 to 0.7 makes unemployment increase from 6.1% to 13.9%, which is a semielasticity of 6.49, at least three times too large to

[21] Fujita (2003) explores several promising extensions of the den Haan *et. al.* RBCM model, including lags in vacancy creation, temporary layoffs, and startup costs for new matches, and finds that these extensions help generate a Beveridge curve.

be consistent with the data. Thus their model suffers from the same problem as the RBCM models we have discussed here.

6 Conclusions

A model of real business cycles and matching implies that match formation depends on the surplus available to the matched pair. Procyclical employment fluctuations occur if match surplus rises in booms, and increased unemployment benefits drive down employment by decreasing match surplus. The standard RBCM model implies a close relationship between these two aspects of employment variability, which is strongly at odds with data. To fit business cycle data, the surplus must be small enough so that productivity shocks have a big effect on vacancies; but to reproduce the observed effects of policies, the surplus must be large enough so that the unemployment benefit has a small effect on vacancies. We have shown analytically that these two requirements cannot be reconciled in a baseline version of the model. We have shown numerically that this result is robust to endogenous search, endogenous separation, finite benefit duration, and efficiency wages; we have also argued that capital, variable benefits, and variable hiring costs are unlikely to resolve the puzzle; and we have argued that the problem would be more severe if we chose not to HP filter the unemployment data.

Embodied technological change can help reconcile these two implications of the model, because it makes the surplus accruing to the firm substantially more procyclical, so that hiring, unemployment, and the worker's job-finding probability all fluctuate more. Sticky wages have a similar impact on the firm's surplus, so they also help increase cyclical variability without affecting the response to labor market policy.

Future research will have to determine whether the standard model's inconsistency with the results of empirical work comes mainly from the RBC mechanism, from the matching function, from the bargaining setup, or from econometric difficulties in estimating the effects of labor market policy. Perhaps one of the variants of the model which we have proposed will prove to be a satisfactory framework for labor market analysis. But for now the most important conclusion is that we must be cautious about using models calibrated to reproduce business cycles as laboratories for labor market policy experiments.

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Appendix: Linearized dynamics

First we linearize the zero profit condition (18):

$$\lambda\tilde{\theta}_t - \tilde{S}_t \quad = \quad \lambda^*\tilde{\theta}_t \quad = \quad E_t\tilde{\Sigma}_{t+1} \tag{34}$$

and the dynamics (17) of the surplus:

$$\tilde{\Sigma}_t \quad = \quad \frac{y}{\Sigma}\tilde{y}_t + \beta(1-\delta-\mu p)E_t\tilde{\Sigma}_{t+1} - \beta\mu p\tilde{p}_t + \frac{h}{\Sigma}\eta_S^h\tilde{S}_t \tag{35}$$

These equations can be simplified by writing $\tilde{p}_t$ and $\tilde{S}_t$ in terms of $\tilde{\theta}_t$ and $E_t\tilde{\Sigma}_{t+1}$, as follows: $\tilde{p}_t = (1-\lambda^*)\tilde{\theta}_t$, and $\tilde{S}_t = \eta_\theta^S\tilde{\theta}_t$, and $\tilde{\theta}_t = \frac{1}{\lambda^*}E_t\tilde{\Sigma}_{t+1}$. The following matrix system summarizes the dynamics:

$$\begin{pmatrix} E_t\tilde{y}_{t+1} \\ E_t\tilde{\Sigma}_{t+1} \end{pmatrix} = \begin{pmatrix} \rho & 0 \\ -\frac{y}{\Sigma}\left[\beta\left(1-\delta-\frac{\mu p}{\lambda^*}\right)+\frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]^{-1} & \left[\beta\left(1-\delta-\frac{\mu p}{\lambda^*}\right)+\frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]^{-1} \end{pmatrix}\begin{pmatrix} \tilde{y}_t \\ \tilde{\Sigma}_t \end{pmatrix} \tag{36}$$

The eigenvalues are $0 < \rho < 1$ and $\left[\beta\left(1-\delta-\frac{\mu p}{\lambda^*}\right)+\frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]^{-1}$. The second eigenvalue is greater than one with exogenous search.[22] We restrict our analysis of endogenous search to the case of sufficiently inelastic search so that this eigenvalue remains greater than one; thus the system is saddle-path stable, and has a unique equilibrium. The eigenvector associated with the stable eigenvalue can be written as $(1\ \ x)'$, where

$$x \equiv \frac{y}{\Sigma\left(1-\rho\left[\beta\left(1-\delta-\mu\frac{p}{\lambda^*}\right)+\frac{h\eta_S^h\eta_\theta^S}{\Sigma\lambda^*}\right]\right)} \tag{37}$$

Using the steady state surplus equation (20), this can be written as

$$x = \left(\frac{y}{y-b+h}\right)\left[\frac{1-\beta+\beta\delta+\beta\mu p}{1-\rho\beta+\rho\beta\delta+\rho\beta\mu p/\lambda^*-\rho h\eta_S^h\eta_\theta^S/(\Sigma\lambda^*)}\right] \tag{38}$$

Saddle path stability implies that x is the elasticity $\tilde{\Sigma}_t/\tilde{y}_t$. Thus, in terms of the observable variable $\tilde{p}$, we have:

$$\tilde{p}_t = (1-\lambda^*)\tilde{\theta}_t = \frac{1-\lambda^*}{\lambda^*}E_t\tilde{\Sigma}_{t+1} = \frac{1-\lambda^*}{\lambda^*}\rho x\tilde{y}_t \tag{39}$$

Again we see that our assumption (21) of sufficiently inelastic search so that $\lambda^* > 0$ is essential to make the model consistent with data: (39) shows that job finding is negatively related to labor productivity if $\lambda^* < 0$.

Now using formula (38) for x, we obtain equation (27), which is used to derive Proposition 1.

For Proposition 2, we linearize the dynamics (29) of unemployment, to obtain:

$$\tilde{U}_{t+1} = (1-\delta)\tilde{U}_t - \delta\left(\frac{1-U}{U}\right)(1-\lambda^*)\tilde{\theta}_t - \delta\left(\frac{1-U}{U}\right)\tilde{U}_t \tag{40}$$

On the saddle path, we have:

$$\tilde{\theta}_t = \frac{1}{\lambda^*}E_t\tilde{\Sigma}_{t+1} = \frac{1}{\lambda^*}\rho\tilde{\Sigma}_t = \frac{1}{\lambda^*}\rho x\tilde{y}_t \tag{41}$$

[22] We assume periods are short enough that p is small, so that this eigenvalue is positive.

so the dynamics of U become $\tilde{U}_{t+1} = A\tilde{U}_t - B\tilde{y}_t$, where we define $A \equiv (U-\delta)/U$ and $B \equiv \delta((1-U)(1-\lambda^*)/(U\lambda^*))\rho x$. This implies:

$$\mathrm{Var}(\tilde{U}_t) = \frac{B^2(1+\rho A)}{(1-A^2)(1-\rho A)}\mathrm{Var}(\tilde{y}_t) \tag{42}$$

which simplifies to:

$$\frac{\sigma_U}{\sigma_y} \equiv \sqrt{\frac{\mathrm{Var}(\tilde{U}_t)}{\mathrm{Var}(\tilde{y}_t)}} = \rho x(1-U)\left(\frac{1-\lambda^*}{\lambda^*}\right)\sqrt{\frac{\delta(U+\rho(U-\delta))}{(2U-\delta)(U+\rho(\delta-U))}} \tag{43}$$

This equation, together with the formula (38) for x, and the formula (23) for the steady state comparative statics, gives us Proposition 2.

	Parameters	Results									
		σ_Q	$\rho_{Q_{-1}}$	$\frac{\sigma_y}{\sigma_Q}$	$\frac{\sigma_w}{\sigma_Q}$	$\frac{\sigma_p}{\sigma_Q}$	$\frac{\sigma_U}{\sigma_Q}$	$\rho_{U,V}$	ϵ_ξ^U	$\frac{\sigma_U}{\sigma_Q}\frac{1}{\epsilon_\xi^U}$	$\frac{\sigma_U}{\sigma_Q}\frac{1}{\epsilon_D^U}$
1)	Benchmark	1.62	0.84	0.92	0.91	1.61	1.40	-0.80	2.00	0.70	-
2)	$b = 0.9$	1.84	0.86	0.81	0.91	3.65	3.16	-0.80	5.41	0.58	-
3)	$b = 0.4$	1.54	0.84	0.96	0.93	0.72	0.62	-0.80	0.82	0.76	-
4)	$\bar{\rho}_Z = 0.75$	1.55	0.87	0.91	0.93	1.68	1.48	-0.83	2.07	0.71	-
5)	$\bar{\delta} = 0.4$	1.64	0.84	0.91	0.94	1.69	1.52	-0.89	2.00	0.76	-
6)	$\lambda = 0.3$	1.70	0.85	0.88	0.95	2.37	2.06	-0.66	2.89	0.71	-
7)	$\mu = 0.3$	1.61	0.84	0.92	0.82	1.54	1.33	-0.80	2.04	0.65	-
8)	δ varies with Z by $\pm 15\%$	2.31	0.88	0.64	1.00	1.40	5.89	0.95	2.11	2.79	-
9)	$\eta_S^h = 4$	1.78	0.85	0.83	0.95	3.18	2.75	-0.60	4.14	0.66	-
10)	$\eta_S^h = 2$	2.21	0.87	0.67	1.01	6.14	5.31	-0.17	8.98	0.59	-
11)	Benefits last 6 months, $b = 0.87$	1.59	0.84	0.94	0.94	1.60	1.45	-0.86	1.83	0.79	7.33
12)	Benefits last 2 years, $b = 0.80$	1.61	0.84	0.92	0.92	1.64	1.44	-0.82	1.96	0.73	11.27
13)	μ varies with Z by $\pm 15\%$	2.27	0.87	0.65	0.59	6.55	5.67	-0.80	2.08	2.73	-
14)	Efficiency wages	1.76	0.85	0.84	0.82	2.97	2.58	-0.80	4.00	0.64	-
15)	Cohort-specific benchmark	0.85	0.93	0.54	3.64	11.32	9.66	-0.77	1.79	5.40	-
16)	Cohort-specific, δ varies with z by $\pm 6\%$	0.97	0.94	0.47	3.56	11.07	10.02	-0.68	1.80	5.56	-
17)	Cohort-specific, $\zeta = 1.6$, $\alpha_Z = 0.5$	1.39	0.88	0.67	2.29	6.28	5.36	-0.77	1.77	3.02	-
18)	Cohort-specific, $\zeta = 1.6$, $\alpha_Z = 0.5$, δ varies with z by $\pm 10\%$	1.55	0.89	0.60	2.43	6.79	6.43	-0.60	1.80	3.58	-

Notes:

Benchmark: $\alpha_Z = 1$, $Z = \pm 0.018$, $\bar{\rho}_Z = 2/3$, $\bar{\beta} = 0.95$, $\bar{\delta} = 0.25$, $\lambda = \mu = 0.5$, $b = 0.745$, $\eta_S^h = \infty$

Cohort-specific benchmark: $\alpha_Z = 0$, $Z = \pm 0.018$, $\zeta = 1$, $\bar{\rho}_Z = 0.6$, $\bar{\beta} = 0.95$, $\bar{\delta} = 0.25$, $\lambda = \mu = 0.5$, $b = 0.7$, $\eta_S^h = \infty$

σ_x: standard deviation of $\log x$ (quarterly)

$\rho_{U,V}$: correlation between $\log U$ and $\log V$ (quarterly)

$\rho_{Q_{-1}}$: annual first order serial correlation of $\log Q$

η_x^y: elasticity of y w.r.t x

ϵ_x^y: semielasticity of y w.r.t x

Table 1: Numerical results