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Optimal Unemployment Insurance in a Matching Equilibrium: The Role of Congestion and Thick-Market Externalities

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Abstract.
This paper characterises optimal unemployment insurance (UI) and optimal tax policy in an equilibrium matching framework where job search effort is unobserved by the Planner. Policy design takes into account congestion externalities (greater job search effort by an individual worker reduces the re-employment rate of other competing job seekers) and thick market externalities (greater job creation rates raise the welfare of the unemployed by making it easier to find work). A key contribution is that it characterises the optimal equilibrium trade-off between higher job creation rates (via lower employment taxes) and a more generous UI program.

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1 Introduction.

Unemployment insurance (UI) programs seek to insure risk averse workers against the costs of being unemployed. Following Shavell and Weiss (1979), there is a large micro-literature which characterises optimal UI programs in frictional markets where worker job search effort is unobserved by the Planner. A partial approach, however, can generate misleading policy implications. For example equilibrium implies there are displacement effects, where distorting UI payments to increase search effort potentially makes it harder for other job seekers to find employment (e.g. Davidson and Woodbury (1993), Meyer (1995)). Using the Diamond/Mortensen/Pissarides matching approach, this paper assesses how congestion and thick market externalities affect the design of optimal UI policy.

A key insight of the optimal UI literature is that the Planner aims to smooth worker consumption across spells of unemployment while also keeping the expected duration of unemployment small. In the standard micro-approach, the Planner reduces average unemployment spells by inducing greater job search effort. But a more effective way of reducing the expected duration of unemployment may be to increase equilibrium job creation rates. A numerical simulation makes the issue clear. Cahuc and Lehmann (2000) calibrate the matching framework to the French economy and discuss how UI payments should vary with duration. Their calibration yields an unemployment rate of 12% and average unemployment spells of over a year. We show that optimal policy implies the employment tax should be cut by 40%.[1] By almost doubling equilibrium job creation rates, this tax cut approximately halves the average unemployment spell. As laid-off workers have much higher re-employment rates, budget balance requires only a modest fall in the level of UI payments. A central contribution of this paper, therefore, is that it characterises the optimal equilibrium trade-off between higher job creation rates (via lower employment taxes) and a more generous UI program.

Hosios (1990) considers market efficiency in a steady state matching equilibrium with no discounting and risk neutral workers. This paper extends that analysis to strictly risk averse workers and incomplete insurance (as job search effort is unobserved). Like Davidson and Woodbury (1997), Fredriksson and Holmlund (2001), this paper considers the optimal UI problem in a steady state matching equilibrium.[2] An

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[1] there are a few minor differences - see Section 5 for details.

[2] but also see Acemoglu and Shimer (1999) who consider efficient unemployment insurance in a directed search equilibrium.

important difference, however, is that this paper also considers optimal tax policy.

Thick market externalities play a central role in the analysis. Given the unemployed necessarily have incomplete insurance (otherwise they choose zero job search effort), raising job creation rates increases welfare not only via the thick market externality - it becomes easier for workers to escape (under-insured) unemployment - it also increases worker re-employment rates and so reduces the operating cost of the UI program. Reflecting this thick market externality, many governments use regional aid to promote investment in depressed regions and so reduce unemployment there. The Utilitarian Planner here maximises worker welfare not only by offering an optimal UI program but also by paying job creation subsidies to firms.

Although thick market externalities play an important role in the analysis, congestion externalities instead play no role. A congestion externality arises as an increase in a worker's job search effort crowds out the matching probabilities of competing job seekers. But note that if a worker increases his job search effort today he is more likely to exit the market, and exit today implies he will not crowd out tomorrow's job seekers. The dynamic framework used here takes such inter-cohort externalities into account. The model finds that conditional on market tightness, expected total search effort by the laid-off worker (i.e. total search effort made between being laid-off and re-employed) is unaffected by any change in the UI profile. The choice of UI profile only determines how search effort and unemployment is distributed across durations. As UI policy cannot change expected total search effort by the laid-off worker, congestion externalities are not a factor in its choice.

A central feature of this paper (and the search literature in general) is that a UI program is not wage neutral. For example the sequential search approach shows that by raising the continuation value of unemployment, a UI program raises the reservation wage of workers (e.g. Mortensen (1977), van den Berg (1990), Shimer and Werning (2003)) and equilibrium then implies firms offer higher wages (e.g. Burdett and Mortensen (1998), Albrecht and Vroman (2005)). Coles and Masters (2003) show the same effect arises when wages are instead determined by strategic bargaining while unemployed. For tractability, however, this paper uses the Nash bargaining approach to determine wages.[3] Cahuc and Lehmann (2000) motivate this approach

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[3]Shimer (2005), Costain and Reiter (2005)) argue that the matching framework fits the business cycle better if wages are 'sticky' rather than determined by Nash bargaining. Note, however, that most of the analysis in this paper holds wages fixed. The only impact of Nash bargaining is that it implies a hold-up problem and optimal policy uses job creation subsidies to correct investment incentives.

by assuming union level bargaining, where the union notes that if the firm should lay-off workers, those workers are then entitled to UI. Fredriksson and Holmlund (2001) instead argue that wages are subject to renegotiation constraints, and the employed worker's threatpoint is the value of being laid-off. Both papers argue that the Planner might reduce UI payments at short durations to lower the value of being laid-off (and so reduce wages), and increase UI payments at long durations to provide more efficient insurance for the longer-term unemployed. Thus UI payments might optimally increase with duration. That argument essentially considers a second best problem where raising the value of being laid-off pushes down equilibrium job creation rates (through higher wages). Here the Planner uses job creation subsidies to target this distortion.

The paper finds that the optimal UI and tax policy problem can be solved recursively in 3 separate steps. The first step describes optimal UI given the wage level w, employment tax T and market tightness θ. We show this first step is equivalent to maximising the employed worker's value of being laid-off subject to a budget constraint. The standard micro-approach is therefore consistent with optimal UI policy, even though the Planner is Utilitarian, there are congestion and thick market externalities and the UI program distorts wages. The second step then describes the efficient employment tax T^* and market tightness θ^* given the wage level w and optimal UI. This step determines the optimal trade-off between more generous UI and faster re-employment rates. We relate this efficiency condition to the Hosios condition for market efficiency when workers are risk neutral. The final step then determines the optimal wage level w^* (given optimal UI and efficient market tightness). Given the 'hold-up' problem implied by Nash bargaining, the paper shows an appropriate mix of employment taxes and job creation subsidies exists which implements the optimum.

The paper is structured as follows. The next section describes the worker's job search problem and the welfare function of the Utilitarian planner. Section 3 describes the policy optimum when wages are exogenous and section 4 describes the optimum when wages are determined by Nash bargaining. Section 5 considers a numerical simulation using the specification of Cahuc and Lehmann (2000). Section 6 discusses these results and the related micro-literature on optimal UI and Section 7 concludes.

2 Model

A Utilitarian Planner insures risk averse workers against unemployment risk in a matching equilibrium. There is the standard moral hazard problem - the job search effort of an individual job seeker is not observed by the Planner. A second market failure is that the labour market is not competitive and so the UI program distorts wages. Following Pissarides (2000) wages are determined by Nash bargaining. Fredriksson and Holmlund (2001) motivate this assumption by arguing wages are renegotiated when employed, where the worker's threatpoint is then the value of being laid-off. Cahuc and Lehmann (2000) instead assume union level bargaining, where the union notes that if the firm should layoff workers, those workers are then entitled to UI. Assuming wages are instead determined by sequential bargaining while unemployed results in a much less tractable wage structure (see Coles and Masters (2003) who assume job search effort is fixed).

As integrating the optimal UI approach into an equilibrium framework is complex, we focus on the Shavell and Weiss (1979) problem. In particular the Planner commits to an unemployment benefit profile $b(.)$ where, assuming time is continuous, $b(t)$ is the flow UI payment to a worker with unemployment duration t. Hopenhayn and Nicolini (1997) shows that a more efficient program exists where the Planner awards an unemployed worker a bonus - an income tax deduction - should the worker quickly find employment. We focus on the simpler scheme for two reasons. The main reason is that it is the one typically used by governments and we can then identify more clearly how optimal UI policy interacts with optimal tax policy. The second is that Meyer (1995) argues strongly against the adoption of UI bonus systems.[4] We shall discuss in Section 6 the Hopenhayn and Nicolini (1997) approach and its implications for optimal UI.

An important simplifying assumption is that workers do not use a savings strategy and so the unemployment insurance program is the sole source of insurance against layoff risk. We discuss in section 6 the related optimal UI literature with hidden

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[4]Meyer (1995) points out that:

"The key drawback [of a re-employment bonus system is its effect] on the size of the claimant population. A re-employment bonus system makes the first trip to the UI office much more valuable for claimants as they become eligible for a bonus payment if they find a job quickly. Such a payment is most valuable to someone who plans to start a job soon and may not currently believe filing for UI is worth the trouble. Using estimates of the effects of benefits on filing rates, I show that changes in initial filing could eliminate or reverse the positive effects of a bonus (system)."

savings; e.g. Werning (2002), Kocherlakota (2004). Allowing savings behaviour in an equilibrium framework is particularly complex as each worker then uses a precautionary savings strategy, accumulating assets while employed and dissaving while unemployed, implying a worker's net wealth position evolves endogenously over time. Costain (1997) argues that a UI scheme is welfare improving to the extent that it reduces the precautionary motive to save while employed. As tractability requires ruling out savings behaviour, the results need to be interpreted with care. We discuss these issues further in Section 6.

The UI program levies an employment tax T and in return offers each laid-off worker a sequence of UI payments which are conditioned on unemployment duration. Given that profile of UI payments, each unemployed worker chooses an optimal job search strategy to maximise expected utility. The Planner has a zero rate of time preference and so chooses policy parameters to maximise a Utilitarian welfare function in a steady state [e.g. Hosios (1990)]. To ensure the Planner and worker preferences are compatible, we shall focus on the optimal job search rules in the limit as worker discount rates $r \to 0$.

Time is continuous and has an infinite horizon. Throughout we shall only consider steady states. There is a unit mass of identical workers who live forever. We let $b(\tau) \geq 0$ denote the flow UI payment to a worker with unemployment duration τ and $k(\tau) \geq 0$ denote the worker's search effort at that duration. The worker's flow payoff while unemployed at duration τ is $u(b(\tau)) - c(k(\tau))$ where $c(.)$ is the cost of search. Assume u is strictly increasing, strictly concave and twice differentiable for all $b > 0$ and that $c(.)$ is strictly convex and twice differentiable with $c(0) = 0$. To avoid corner solutions we assume $\lim_{b \to 0} u'(b) = \infty$ and $c'(0) = 0$. Assuming $u(0)$ is finite (workers with no UI do not starve) we can normalise $u(0) = 0$.

Given search effort $k \geq 0$, the worker obtains employment according to a Poisson process with parameter $k\gamma$. $\gamma > 0$ characterises how easy it is to find work and plays a most important role in what follows. Note, γ will be determined endogenously as part of a steady state matching equilibrium (see Claim 2 below). Also note that a non-degenerate steady state requires search effort $k(\tau) > 0$ at long enough durations (otherwise unemployment is an absorbing state). There are job destruction shocks, where a job is destroyed at rate $\delta > 0$ whereupon the worker becomes unemployed with duration $\tau = 0$. We assume there is no private insurance market.

The Planner chooses UI benefit profile $b(.)$, job creation subsidies s and an employment tax T which must satisfy budget balance. The equilibrium matching framework

used is a standard Pissarides (2000) framework. We describe that framework below. Here we first describe optimal job search by workers given their market environment.

2.1 Optimal job search.

Given policy parameters $P = \{b(.), s, T\}$ and a steady state, the value of being unemployed at duration τ using an optimal job search strategy is properly denoted $V_u(\tau|P)$, while $k(\tau|P)$ denotes the optimal job search effort at duration τ and $V^E(P)$ denotes the value of being employed. To reduce notational complexity, however, we subsume reference to P in these functions. In continuous time and given $r > 0$, standard recursive arguments imply V_u satisfies the Bellman equation

$$rV_u - \frac{dV_u}{dt} = \max_k \left[u(b) - c(k) + \gamma k[V^E - V_u]\right] \tag{1}$$

where the flow value of being unemployed at duration t equals the flow payoff $u(b(t)) - c(k)$ while unemployed plus the expected capital gain by finding work, $\gamma k[V^E - V_u]$. Search effort k is chosen optimally. As c is convex, optimal job search at duration t implies

$$
\begin{aligned}
c'(k) &= \gamma[V^E - V_u] \text{ if } V_u \le V^E \\
k &= 0 \text{ otherwise}
\end{aligned}
\tag{2}
$$

where the worker's trade-off is between costly search effort and an increased probability of finding work.

The aim is to characterise optimal job search in the limit as $r \to 0$. To do this, integrate (1) to get

$$V^E - V_u(t) = \int_t^\infty e^{-\int_t^\tau [r+\gamma k(x)]dx} \left[rV^E - u(b(\tau)) + c(k(\tau))\right] d\tau,$$

where $k(.)$ satisfies (2). Defining the survival probability

$$\Psi(t) = e^{-\int_0^t \gamma k(x)dx}, \tag{3}$$

which is the probability a laid-off worker is still unemployed by duration t, implies

$$V^E - V_u(t) = \int_t^\infty \frac{\Psi(\tau)}{\Psi(t)} e^{-r(\tau-t)} \left[rV^E - u(b(\tau)) + c(k(\tau))\right] d\tau.$$

Now let $V^0 = V_u(0)$ denote the value of being laid-off and note that the flow value of employment is then

$$rV^E = u(w) + \delta[V^0 - V^E] \tag{4}$$

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where δ is the rate of job destruction. Using this to substitute out rV^E in the previous expression yields:

$$V^E - V_u(t) = \int_t^\infty \frac{\Psi(\tau)}{\Psi(t)} e^{-r(\tau-t)} \left[u(w) + \delta[V^0 - V^E] - u(b(\tau)) + c(k(\tau)) \right] d\tau.$$

If search effort is strictly positive at long durations, $\Psi(.)$ then decays exponentially and this integral remains bounded as $r \to 0$ (search effort plays the role of discounting). Claim 3 below establishes that search effort $k(.)$ increases with duration with an optimal UI profile and the above integral therefore remains finite as $r \to 0$.

Claim 1. Optimal Job Search.
Conditional on policy P and in the limit as $r \to 0$, optimal job search implies $V_u(.)$ is given by

$$V^E - V_u(t) = \int_t^\infty \frac{\Psi(\tau)}{\Psi(t)} \left[u(w) + \delta[V^0 - V^E] - u(b(\tau)) + c(k(\tau)) \right] d\tau \qquad (5)$$

where

$$V^E - V^0 = \frac{1}{1 + \delta \int_0^\infty \Psi(t)dt} \int_0^\infty \Psi(t) \left[u(w) - u(b(t)) + c(k(t)) \right] dt. \qquad (6)$$

and k, Ψ are given by (2) and (3).

Claim 1 follows directly from the previous analysis, where $V^E - V^0$ is identified by putting $t = 0$ in (5). Although V^E and $V_u(.)$ become unboundedly large as $r \to 0$, the difference $V^E - V_u(t)$ remains finite and the optimal job search rule (2) is well defined. Note that conditional on being unemployed at duration t, $\Psi(\tau)/\Psi(t)$ is the probability the worker is still unemployed at duration $\tau > t$ using optimal job search. The integrand in (5) is the expected loss in flow utility by being unemployed at duration $\tau > t$ rather than re-employed on wage w and re-entitled to full UI coverage. Given this characterisation of optimal job search by the unemployed, we now turn to the Planner's problem.

2.2 The Utilitarian Planner's Problem.

We consider a standard steady state matching framework (see Pissarides (2000)) where worker job search is given by Claim 1. Vacancies V are determined by a free entry rule where $a > 0$ is the flow cost of holding an unfilled vacancy, p is the revenue flow given a filled vacancy and w is the wage paid. There are matching frictions where

matching between the unemployed workers and vacancies is described by a matching function $M(K, V)$ where K is aggregate search effort. $M(.)$ is an increasing concave function with constant returns. Let $\theta = V/K$ denote labour market tightness and define $m(\theta) \equiv M(1, \theta)$ which is an increasing, concave function. Given the Planner's policy choice $P = \{b(.), s, T\}$ and optimal job search, Claim 2 describes the resulting steady state and the balanced budget constraint.

Claim 2. Given policy $P = \{b(.), s, T\}$ and wage w, a free entry equilibrium with optimal job search and budget balance implies:

(i) labour market tightness θ where

$$a - s = \frac{m(\theta)}{\theta} \frac{p - w - T}{\delta};$$ (free entry)

(ii) $\gamma = m(\theta)$;

(iii) steady state employment

$$E = \frac{1}{1 + \delta \int_0^\infty \Psi(\tau) d\tau};$$

(iv) employment tax

$$T = \frac{\delta \theta}{m} s + \delta \int_0^\infty \Psi(\tau) b(\tau) d\tau.$$ (budget balance)

where $\Psi(.)$ is determined by Claim 1.

The proof of Claim 2 is in the appendix. As is well known, the free entry rule determines equilibrium labour market tightness θ. Note that by offering job creation subsidy $s > 0$, the Planner potentially increases market tightness θ. Of course budget balance requires that such subsidies are financed by an employment tax T. Also note that the worker's search parameter γ depends only on labour market tightness, where $\gamma = m(\theta)$. The Planner thus has two ways of improving worker welfare. One is to increase UI benefits and so cushion the utility loss by being unemployed. The second is to subsidise job creation rates and so make it easier for the unemployed to find work.

As firms necessarily make zero expected profit, the Utilitarian Planner's objective is to maximise the welfare function

$$W = Eu(w) + \delta E \int_0^\infty \Psi(\tau)[u(b(\tau)) - c(k(\tau))] d\tau.$$

The first term describes the flow welfare of the employed, while the second describes the flow welfare of the unemployed; $\delta E \Psi(\tau) dt$ is the measure of unemployed workers with duration in interval $[\tau, \tau + dt]$ and $[u(b(\tau)) - c(k(\tau))]$ is their corresponding utility flow. This welfare function describes aggregate flow welfare in a steady state with optimal job search. Using Claim 2 to substitute out E implies

$$W = \frac{u(w) + \delta \int_0^\infty \Psi(\tau)[u(b(\tau)) - c(k(\tau))]d\tau}{1 + \delta \int_0^\infty \Psi(\tau)d\tau}$$

and (6) then implies

$$W = u(w) + \delta[V^0 - V^E];$$

i.e. the Utilitarian Planner's objective is equivalent to maximising the flow value of employment.

The next two sections choose P to maximise this welfare function where workers use optimal job search. It considers two cases, one where wages are exogenous and independent of policy, one where wages are determined by Nash bargaining and so policy affects the wage outcome.

3 The Utilitarian Planner's Problem with Exogenous Wages.

Assuming a fixed wage $w \in (0, p)$, the Planner's problem is

$$\max_{P=\{b(.),s,T\}} u(w) + \delta[V^0 - V^E]$$

subject to the steady state conditions:

$$a - s = \frac{m(\theta)}{\theta} \frac{p - w - T}{\delta}, \qquad \text{(free entry)}$$

$$T = \frac{\delta \theta}{m} s + \delta \int_0^\infty \Psi(\tau) b(\tau) d\tau, \qquad \text{(budget balance)}$$

where, conditional on P and $\gamma = m(\theta)$, optimal job search determines Ψ, k and $V^0 - V^E$ (see Claim 1).

First note that job creation subsidies s are ineffective in this problem. If the Planner increases job creation subsidies s, say to increase labour market tightness in [free entry], then [budget balance] requires a perfectly offsetting increase in the

employment tax T which leaves market tightness unchanged. As s has no impact on the worker job search problem, there is no loss in generality by setting $s = 0$ (but note that this result only holds when wages are exogenously fixed).

We now break this optimisation problem into two parts. First suppose the Planner's optimum implies tax T_0 and [free entry] then implies market tightness $\theta = \theta_0$. Conditional on (T_0, θ_0) satisfying [free entry], the first part considers the optimisation problem

$$\max_{b(.)} \ u(w) + \delta[V^0 - V^E]$$

subject to

$$\int_0^\infty \Psi(\tau)b(\tau)d\tau = T_0/\delta \qquad \text{(budget balance)}$$

where Ψ and $V^0 - V^E$ are determined by optimal job search with $\gamma = m(\theta_0)$. Note this optimisation problem is equivalent to Shavell and Weiss (1979) as it maximises the value of being laid-off (V^0) subject to a budget constraint.[5] Claim 3 describes this optimum and so identifies maximised welfare payoff $W^*(w, T_0, \theta_0)$. The second step then considers T_0 and θ_0 to maximise $W^*(.)$ subject to the free entry constraint.

3.1 Step 1 - Optimal UI (given T_0, θ_0).

Given $w \in (0, p)$ and conditional on (T_0, θ_0), the proof of claim 3 (given in the Appendix) solves the programming problem

$$W^*(w, T_0, \theta_0) = \max_{b(.)} \ u(w) + \delta[V^0 - V^E]$$

subject to

$$\int_0^\infty \Psi(\tau)b(\tau)d\tau = T_0/\delta \qquad \text{(budget balance)}$$

and optimal job search with $\gamma = m(\theta_0)$.

We identify the optimal policy using techniques described in Burdett and Coles (2003) as that approach yields a useful condition for the comparative statics that

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[5]Note $rV^E = u(w) + \delta[V^0 - V^E]$ implies

$$u(w) + \delta[V^0 - V^E] \equiv \frac{r}{r + \delta}\left[u(w) + \delta V^0\right].$$

As w is fixed, then for any $r > 0$ (no matter how small) the objective is equivalent to maximising V_0.

follow.[6] To use that approach, let $S(t) = V^E - V_u(t)$ denote re-employment surplus at duration t and let $S_0 = V^E - V^0$ denote the loss of utility by being laid off. Substituting out $V^E - V_u(t)$ in Claim 1, optimal job search implies re-employment surplus S evolves with duration according to the differential equation

$$\dot{S} = \gamma k^* S - [u(w) - \delta S_0 - u(b) + c(k^*)] \tag{7}$$

where $k^* = k^*(S, \gamma)$ is optimal job search effort and is defined by

$$
\begin{aligned}
c'(k^*) &= \gamma S \text{ while } S \geq 0; \\
k^* &= 0 \text{ otherwise.}
\end{aligned}
\tag{8}
$$

Ψ then evolves according to the differential equation

$$\dot{\Psi} = -\gamma k^* \Psi \tag{9}$$

with initial value $\Psi(0) = 1$. It is also useful to define

$$B(t) = \int_t^\infty \frac{\Psi(\tau)}{\Psi(t)} b(\tau) d\tau.$$

Conditional on optimal job search, $B(t)$ is the continuation cost of further UI entitlement given an unemployed worker with current duration t; i.e. $\Psi(\tau)/\Psi(t)$ is the probability the worker remains unemployed at duration $\tau > t$ and so receives $b(\tau)$. Note that $B(0) \equiv \int_0^\infty \Psi(\tau) b(\tau) d\tau$ and so budget balance is equivalent to $B(0) = T_0/\delta$. Differentiation also establishes that $B(.)$ evolves according to

$$\dot{B} = \gamma k^* B - b \tag{10}$$

Claim 3 now describes the optimal UI profile. Let $b^*(.|w, T_0, \theta_0)$ denote the optimal UI profile given (w, T_0, θ_0) and $B^*(t)$ denote the corresponding continuation cost $B(t)$, $S^*(t)$ denote the re-employment surplus $S(t)$ and $\Psi^*(t)$ the survival probability $\Psi(t)$ given the optimal policy.

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[6]Burdett and Coles (2003) consider optimal wage tenure contracts which maximise firm profit given workers quit according to an optimal job search strategy. The firm's profit maximisation problem is formally equivalent to the Planner's cost minimsation problem. Stevens (2004) argues that a more efficient wage contract exists where workers must pay a severance fee to the firm on quitting. Her argument parallels Hopenhayn and Nicolini (1997) who show that UI programs are more efficient if exit payments are allowed. She also shows that an efficient contract occurs where a worker pays a one-off entry fee at duration $\tau = 0$. This parallels the argument made in section 6, that the Planner should make a one-off severance payment when the worker becomes unemployed.

Claim 3. Given an exogenous wage $w \in (0, p)$ and (T_0, θ_0) satisfying [free entry], the optimal UI profile b^* and corresponding continuation payoffs B^*, S^* are solutions to the differential equation system $\{b, B, S\}$:

$$\left[\frac{-u''(b)}{u'(b)^2}\right] \dot{b} = -\gamma \frac{\partial k^*}{\partial S} B \tag{11}$$

$$\dot{B} = \gamma k^* B - b \tag{12}$$

$$\dot{S} = \gamma k^* S - [u(w) - \delta S_0 - u(b) + c(k^*)] \tag{13}$$

subject to the boundary conditions:

(i) $\lim_{t \to \infty}(b, B, S) = (0, 0, \overline{S})$ where $\overline{S}$ is defined by

$$\gamma k^*(\overline{S})\overline{S} = \left[u(w) - \delta S_0 - u(0) + c(k^*(\overline{S}))\right],$$

(ii) the initial condition $B(0) = T_0/\delta$,

and the fixed point condition $S_0 = S(0)$.

Proof is in the Appendix.

Equations (12) and (13) describing B, S correspond to (7) and (10) above. (11) is the central result in claim 3 as it describes how UI payments vary optimally with duration. If search effort were perfectly inelastic; i.e. $\partial k^*/\partial S \equiv 0$, (11) implies a flat UI profile, $b(.) = b_0$, would be optimal. But c'' finite implies $\frac{\partial k^*}{\partial S} > 0$ and (11) then implies b^* is strictly decreasing with duration as described in Shavell and Weiss (1979).

Integrating (11) over $[0, t]$ yields the optimal insurance condition:

$$\frac{u'(b^*(0|.))}{u'(b^*(t|.))} = 1 - u'(b^*(0|.)) \int_0^t \gamma \frac{\partial k^*}{\partial S} B^*(\tau) d\tau. \tag{14}$$

Marginally reducing UI benefit $b^*(t|.)$ at duration $t > 0$, marginally increases the re-employment surplus, $S^*(\tau)$, at all durations $\tau < t$. By inducing greater search effort, $\gamma \frac{\partial k^*}{\partial S}(S^*(\tau))$ describes the resulting increase in the exit rate of benefit claimants at those durations. As worker exit at duration τ saves the Planner $B^*(\tau)$ in further UI, the integral in (14) measures the marginal fall in the operating cost of the UI program by marginally reducing UI payment $b^*(t|.)$. This return then distorts optimal UI away from a flat UI profile. We discuss further this optimal insurance condition in section 6.

Solving for the optimal UI profile is complicated by re-entitlement effects. At any duration t, optimal job search depends on the value of being re-entitled to full UI coverage; i.e. $S(.)$ depends on S_0. This payoff has to be solved for recursively. Specifically, as duration t becomes large, UI benefits become arbitrarily small and surplus $S(t)$ converges to $\overline{S}$ consistent with $b = 0$, where $\overline{S}$ depends on S_0. Note that the limit point $(b, B, S) = (0, 0, \overline{S})$ is a stationary point of the differential equation system (11)-(13). The optimal UI profile described in Claim 3 is identified using backward induction: conditional on some value for $S_0 > 0$, start at the stationary point $(0, 0, \overline{S})$ and iterate the system $\{b, B, S\}$ back along the saddle-path implied by (11)-(13). The iteration stops when $B = T_0/\delta$ as that defines the starting point, $t = 0$, of the optimal UI profile (where budget balance is satisfied). As the expected re-entitlement payoff S_0 must equal actual payoff, then S_0 is tied down by the fixed point condition $S_0 = S(0)$. Due to the non-linearity of the differential equation system, we simply assume that such a fixed point always exists, which is equivalent to assuming the optimisation problem has a maximum. In numerical simulations, finding this fixed point was straightforward and existence was never a problem.

3.2 Step 2 - Optimal T and θ (given optimal UI).

Given $w \in (0, p)$, and (θ, T) satisfying [free entry], Claim 3 describes the optimal UI profile $b^*(.)$ and corresponding continuation payoffs $S^*(.), B^*(.)$. This implies maximised welfare function $W^*(w, T, \theta)$. This step now chooses T and θ to maximise $W^*(.)$ subject to the free entry constraint. Given w fixed, Claim 5 below describes the isopayoff frontier $W^*(w, T, \theta) = \overline{W}$. As optimality implies the slope of this contour equals the slope of the free entry constraint, Theorem 1 then describes the policy optimum.

Identifying the isopayoff frontier $W^*(w, T, \theta) = \overline{W}$ is a comparative static exercise based on the equations given in Claim 3. The analysis is much simplified by the following insight.

Claim 4. No discounting implies S and B evolve along the optimal UI profile according to

$$\dot{S} = -u'(b)\dot{B}. \tag{15}$$

Proof - see (28) in the proof of Claim 3 and use (12).

Claim 4 describes an efficiency condition - the worker's re-employment surplus,

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$S(t) = V^E - V_u(t)$, increases one-for-one with the decrease in the continuation cost $B(t)$ along the optimal path. To use this condition, recall that the optimal UI program is the saddle path of the differential equation system (11)-(13) where the starting point, $t = 0$, is identified where $B = T_0/\delta$. The saddle path then identifies $b^*(.)$ and also $S^*(0)$. Suppose now we have a slightly more generous UI budget: $T = T_0 + dT$ where $dT > 0$. Given the same S_0, the optimal UI program is the same saddle path except we continue the backward iteration process a little bit longer, so that $dB = dT/\delta$. As Claim 4 implies $dS(0) = -u'(b^*(0))dB$ along the optimal path at $t = 0$, the higher UI budget implies worker welfare increases by

$$d[V^0 - V^E] = -dS(0) = \frac{u'(b^*(0))}{\delta}dT. \tag{16}$$

As this variation changes the re-entitlement payoff S_0, the overall effect of T on aggregate welfare $W^*(.)$ is more complicated. Fortunately changes in re-entitlement payoffs play no role in what follows as we consider compensating variations $(dT, d\theta)$ which hold welfare $W^* = \overline{W}$. As welfare $W \equiv u(w) - \delta S_0$, these variations also imply $S_0 = (u(w) - \overline{W})/\delta$ is fixed. (16) then describes the direct effect of $dT > 0$ on worker welfare, given a compensating variation $d\theta$ which holds welfare constant. Claim 5 now describes that compensating variation.

Claim 5. Fix $w \in (0, p)$ and a (θ, T) satisfying [free entry]. Given the optimal policy as described in Claim 3, then $W^*(w, \theta + d\theta, T + dT) = \overline{W}$ implies $(d\theta, dT)$ must satisfy:
$$\left[\frac{dW^\theta}{d\theta} - u'(b^*(0|.))\frac{dB^\theta}{d\theta}\right]d\theta + \left[\frac{u'(b^*(0|.))}{\delta}\right]dT = 0 \tag{17}$$

where $\frac{dW^\theta}{d\theta}, \frac{dB^\theta}{d\theta}$ are given by

$$\frac{dW^\theta}{d\theta} = m'(\theta)\int_0^\infty \Psi(t)k(t)S^*(t)dt; \tag{18}$$

$$\frac{dB^\theta}{d\theta} = -m'(\theta)\int_0^\infty \left[\int_0^t k(x)dx\right]\Psi(t)b^*(t)dt - \gamma\int_0^\infty \left[\int_0^t \frac{dk^\theta(x)}{d\theta}dx\right]\Psi(t)b^*(t)dt \tag{19}$$

and

$$\frac{dk^\theta(x)}{d\theta} = -\frac{m'(\theta)}{c''(k(x))}\int_x^\infty \frac{\Psi(\tau)}{\Psi(x)}\dot{S}^*(\tau)d\tau$$

is the change in job search effort at duration x.
Proof is in the Appendix.

Although these expressions are ugly, the intuition is straightforward. (17) describes the variation $(d\theta, dT)$ which holds welfare W^* constant. To understand this condition, note that $d\theta$ has two effects on maximised welfare.

(i) There is a thick market externality: $\frac{dW^\theta}{d\theta} > 0$, as described by (18), describes the direct increase in worker welfare through greater market tightness.

Optimal job search generates capital gain $\gamma k(t) S^*(t)$ at each duration t. An increase in market tightness implies $d\gamma = m'(\theta) d\theta$ and $m'(\theta) k(t) S^*(t) d\theta$ then describes the increase in worker surplus at duration t through greater market tightness. By integrating over all durations and weighting by survival probabilities, $\frac{dW^\theta}{d\theta}$ describes the thick market externality - the increase in the value of being laid off through increased market tightness.

(ii) Changing market tightness changes the operating cost of the UI program, denoted $dB^\theta/d\theta$.

$dB^\theta/d\theta$ as described by (19) has two terms. Greater market tightness increases the exit rates of workers and so directly reduces the cost of the UI program. The second term takes into account that greater market tightness changes search incentives. As optimal search effort is given by $c'(k) = \gamma S$, one might have expected that an increase in market tightness leads to greater search effort. However there is a countervailing effect - greater market tightness reduces S as it is then easier to find work. In general the effect of market tightness on job search effort is ambiguous. Optimal UI and Claim 5, however, imply $\frac{dk^\theta(x)}{d\theta} < 0$ and so search effort falls with an increase in market tightness. This should not be too surprising for optimal search effort $k \to 0$ as $\theta \to \infty$ as only a little effort is then required to generate a large arrival rate of offers.

(17) identifies the required iso-welfare condition as $[dW^\theta/d\theta]d\theta$ describes the increase in welfare through the thick market externality, which is offset by the budget effect, where $\left[\frac{dT}{\delta} - \frac{dB^\theta}{d\theta} d\theta\right]$ is the net increase in the UI budget and $u'(b^*(0|.)) \left[\frac{dT}{\delta} - \frac{dB^\theta}{d\theta} d\theta\right]$ is the corresponding welfare gain (by Claim 4).

(17) describes the slope $dT/d\theta$ of the isowelfare constraint, $W^* = \overline{W}$. As we are maximising $W^*(.)$ subject to the free entry constraint, optimality requires that the slope of the iso-welfare constraint must equal the slope of the [free entry] constraint, which is

$$\frac{dT}{d\theta} = -\frac{m - \theta m'}{m\theta}[p - w - T].$$

Hence we have established the following Theorem.

Theorem 1. Given $w \in (0, p)$, optimal policy P^* is a pair $(b^*(.), T^*)$ and a market tightness θ^* where

(i) b^* is described by Claim 3 with $T_0 = T^*$ and $\theta_0 = \theta^*$, and

(ii) (T^*, θ^*) are the solutions to the pair of equations for (T, θ) :

$$m'(\theta) \int_0^\infty \Psi(t) k(t) S^*(t) dt - u'(b^*(0)) \frac{dB^\theta}{d\theta} = \frac{m - \theta m'}{m\theta} u'(b^*(0)) \frac{p - w - T}{\delta} \qquad (20)$$

$$a = \frac{m(\theta)}{\theta} \frac{p - w - T}{\delta}. \qquad \text{(free entry)}$$

(20) describes the optimal trade-off between increasing market tightness (which directly raises the welfare of the unemployed through the thick market externality and indirectly by reducing the cost of the UI program), and the insurance value of the UI program (where higher market tightness requires a lower employment tax T and hence less generous UI). The next section shows how this condition is related to the Hosios condition for market efficiency when wages are determined by Nash bargaining. Before considering that problem, however, it is useful to consider what wage w maximises the Utilitarian welfare payoff.

We define the first best problem as the case when the Planner can also set w; i.e. the Planner's first best problem is

$$\max_{w, P} \ u(w) + \delta[V^0 - V^E]$$

subject to the same constraints. The first order condition characterising optimal w is surprisingly straightforward.

Theorem 2. The First Best.

The first best is a policy $P^{**} = \{b^{**}(.), T^{**}\}$ where P^{**} is the policy described in Theorem 1 with $w = w^*$, and $w^* = b^{**}(0)$.

Proof is in the Appendix.

Along with optimal policy described in Theorem 1 (given w), the first best implies optimal co-insurance between employed workers and newly laid-off workers; i.e. the laid-off worker receives initial UI payment $b^*(0) = w$. As UI payments are strictly decreasing with duration the first best implies $V^E - V^0 > 0$ and so unemployed workers choose strictly positive search effort. Due to the non-linearity of the differential

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equation system and the recursive definition of the re-entitlement effect, we do not prove existence of the first best market outcome and simply assume the maximum exists. We now consider optimal policy when wages are not set by the Planner but instead determined by bargaining.

4   The Planner's Problem with endogenous wages.

The wage w is now determined by Nash bargaining where the firm has bargaining power $\alpha \in (0, 1)$, the worker's threatpoint is the value of being laid-off V^0 and a free entry equilibrium implies the firm's threatpoint is zero profit.[7] This implies the Planner problem

$$\max_P \ u(w) + \delta[V^0 - V^E]$$

subject to

$$\alpha \frac{V^E - V^0}{u'(w)} = (1 - \alpha)\frac{p - w - T}{\delta}, \qquad \text{(Nash Bargaining)}$$

$$a - s = \frac{m(\theta)}{\theta}\frac{p - w - T}{\delta}, \qquad \text{(free entry)}$$

$$T = \frac{\delta\theta}{m}s + \delta \int_0^\infty \Psi(\tau)b(\tau)d\tau, \qquad \text{(budget balance)}$$

and Ψ, k and $V^0 - V^E$ are determined by optimal job search with $\gamma = m(\theta)$.

Although formally a complicated problem, the solution is straightforward - the Planner implements the first best outcome as described in Theorem 2. To see this note that in the previous section, the job creation subsidy s was arbitrarily set equal to zero. Given wage $w = w^*$, the previous section implies there is a *continuum* of optimal policies - all policies $b = b^{**}$, $T = T^{**} + \frac{\delta\theta^*}{m}s$ and job creation subsidy s yield the same optimal payoff (as variations in s imply a perfectly offsetting change in the

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[7]Assuming $r > 0$ (small) the Nash bargaining approach implies

$$w^{Nash} = \arg\max_w \left[\frac{u(w) + \delta V^0}{r + \delta} - V^0\right]^{1-\alpha} \left[\frac{p - w - T}{r + \delta}\right]^{\alpha}$$

where $V^E = [u(w) + \delta V^0]/(r + \delta)$. Solving for w^{Nash} and letting $r \to 0$ implies [Nash bargaining] as stated in the optimisation problem.

employment tax T). But job creation subsidies play a crucial role given the 'hold-up' problem as implied by Nash bargaining.

The first best as described in Theorem 2 is implemented by $b = b^{**}$, $T = T^{**} + \frac{\delta\theta}{m}s$ and s as long as Nash bargaining implies the negotiated wage $w = w^*$. The Nash bargaining rule therefore ties down the optimal employment tax

$$T = p - w^* - \frac{\alpha}{1 - \alpha}\frac{\delta[V^E - V^0]}{u'(w^*)}$$

which ensures negotiated wage $w = w^*$, and the corresponding job creation subsidy is

$$\frac{\delta\theta^*}{m}s = T - T^{**} = p - w^* - T^{**} - \frac{\alpha}{1 - \alpha}\frac{\delta[V^E - V^0]}{u'(w^*)},$$

where w^*, T^{**}, θ^* and $V^E - V^0 > 0$ are all determined by the first best problem.

There are several insights. First note that $V^E - V^0 > 0$ in the first best implies $p - w^* - T > 0$ (by Nash bargaining) and so the employment tax is sufficiently low that firms with filled vacancies always make positive profit (and do not prefer to close down).

The sign and the magnitude of the job creation subsidy depends on the bargaining power of firms. Suppose for example that $\alpha = 0$, implying workers have all the bargaining power. Then optimal policy implies employment tax $T = p - w^*$ where w^* is the first best wage level. As workers have all the bargaining power they extract all the match surplus and so negotiate $w = p - T$, which, by design, equals w^*. The [free entry] condition then implies job creation subsidy $s = a$. As firms make no profit ex-post, the Planner fully subsidises all job creation investments and so ensures market tightness $\theta = \theta^*$.

As firm bargaining power increases, firms extract more rents from the match and so, ceteris paribus, wages tend to fall and job creation rates tend to increase. The Planner responds by lowering the job creation subsidy and lowering the employment tax. The former stops job creation rates rising above the optimal level, the latter increases match rents and so stops wages falling below the first best level and the economy remains at the first best.

The job creation subsidy becomes negative when firm bargaining power is high enough. In this case the Planner taxes job creation rates and uses those funds to lower the employment tax. Indeed $\alpha \to 1$ implies T becomes negative; the Planner uses a job creation tax to subsidise employment.

Before turning to a numerical simulation, it is insightful to relate the efficiency condition in Theorem 1 to the Hosios rule for market efficency. Assuming risk neutral

workers and noting that $b^* = 0$ solves (11) in Claim 3, the efficiency condition (20) in Theorem 1 and the Nash bargaining rule then imply[8]

$$m'\frac{V^E - V^0}{m} = \frac{m - \theta m'}{m\theta}u'(b^*(0))\frac{\alpha}{1-\alpha}\frac{V^E - V^0}{u'(w)}.$$

This yields the Hosios condition for market efficiency: absent of policy intervention and with risk neutral workers, the market achieves full efficiency if and only if $\alpha = \theta m'/m$; i.e. the firm's bargaining power must equal the elasticity of the matching function with respect to vacancies. With strictly risk averse workers and incomplete insurance, however, this efficiency condition (as written in Theorem 1) instead describes the optimal trade-off between more generous UI (higher $b^*(0)$) and greater market tightness.

5 Numerical Simulation

Cahuc and Lehmann (2000) (C/L from now on) calibrate this matching model to the French economy.[9] The purpose of this Section is not to critique their parameterization nor is it to propose French policy. Rather the aim is to demonstrate how optimal UI policy interacts with optimal tax policy.

Their specification is:[10]

$$M(K,V) = K^{1/2}V^{1/2},$$

$$c(k) = k^{4.35},$$

$$\alpha = 0.5, \delta = 0.1, p = 1, a = 0.4.$$

$\alpha = 0.5$ implies symmetric bargaining powers. As α also equals the vacancy elasticity in the matching function, Hosios (1990) implies no policy intervention would be optimal if workers were risk neutral. The time reference is a year and so $\delta = 0.1$ implies annual job turnover of approximately 10%.[11] C/L reported a French unemployment

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[8]note, $\int_0^\infty e^{-\gamma kt}k\overline{S}dt = \frac{\overline{S}}{\gamma} \equiv \frac{V^E - V^0}{m}$

[9]Fredriksson and Holmlund (2001) perform related simulations but assume utility function $u(b, T - k)$ which is not consistent with the model used here.

[10]Cahuc and lehmann (2000) specify search disutility e as the worker's choice variable and $\pi(e)$ as the corresponding matching probability. This is a renormalisation of the model with $e \equiv c(k)$ and $\pi(e) = k$. As they specify $\pi(e) = e^{0.23}$, this implies $c(k) = k^{1/0.23}$.

[11]The proportion of jobs destroyed per calendar year is $1 - e^{-\delta} = 9.5\%$.

rate of 12%. Note that $\delta = 0.1$ and the steady state turnover condition $\delta E = \overline{\lambda} U$ implies average worker re-employment rate $\overline{\lambda} = 0.73$. This suggests average unemployment spells of around $1/\overline{\lambda} = 16$ months.[12] As such unemployment spells seem rather long, it should not be surprising that optimal policy results in significantly lower durations. Note also that the search cost function implies search effort is highly inelastic - C/L choose this so that this specification "reproduces the microeconomic evaluation of the elasticity of unemployment durations with respect to unemployment benefits". This specification implies UI payments fall relatively slowly with duration.

There are three main differences. C/L assume:

(i) a logarithmic utility function while here we adopt a CRRA utility function $u(w) = w^{1-\sigma}/(1 - \sigma)$ with $\sigma = 0.95$, which implies $u(0)$ is finite and so the saddle path described in Claim 3 is well defined;

(ii) a 5% annual discount rate while here we have no discounting;

(iii) a discrete time framework with time period equal to a year and a two-step UI program where workers receive b_H in the first year of unemployment and b_L thereafter. The discrete time framework implies worker search effort is fixed in each year rather than continually updated.

C/L report wage $w = 0.85$ and their calibration sets UI payments at $b_H = 0.68$, $b_L = 0.51$. Assuming firms face gross labour costs $w(1 + \hat{\tau})$, where $\hat{\tau}$ is the rate of income tax, their calibration implies $\hat{\tau} = 0.1$. We initially consider employment tax $T_0 = \hat{\tau} w = 0.085$ so that, ceteris paribus, the amount of tax collected per match is unchanged.

Given this economy we now solve numerically the conditions described in Claim 3, Theorem 1 and Theorem 2. Table 1 reports the optimal UI profile (at yearly intervals) depending on the assumed tax policy. Table 2 describes the corresponding tax policy and steady state market outcomes.

The first column in these tables describes the market outcomes as described in claim 3 given employment tax $T_0 = 0.085$ and wage $w = 0.85$. The second column describes the market outcomes as described in Theorem 1 when the employment tax T^* is set optimally (and $w = 0.85$). The third column then describes the first best as described in Theorem 2.

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[12]Search effort dispersion in fact implies this is a lower bound - the average completed spell exceeds 16 months.

Table 1: Optimal UI by Tax Policy

Duration (years)	Claim 3	Theorem 1	First Best
	$b^*(.)$	$b^*(.)$	$b^{**}(.)$
0	1.24	1.24	0.87
1	0.72	0.64	0.54
2	0.51	0.43	0.39
3	0.39	0.31	0.29
4	0.31	0.24	0.23
5	0.25	0.19	0.18

Table 2: Optimal Tax Policy and Market Outcomes.

	Claim 3	Theorem 1	First Best
wage	0.85	0.85	0.87
tax	$T_0 = 0.085$	$T^* = 0.066$	$T^{**} = 0.050$
tightness [V/K]	2.64	4.41	3.98
unemployment	9.4%	7.4%	7.0%

Column 1 in Table 1 describes the optimal UI policy given $w = 0.85$ and $T_0 = 0.085$. Note that UI payments are initially very high, fall quickly at short durations but very slowly at long durations. Although $b^*(0) > w$ is inconsistent with the first best, it is not inconsistent with the conditions of claim 3 and reflects a relatively generous UI budget.[13] UI payments fall sufficiently quickly in Column 1 that $S^*(0) = 0.187$ is strictly positive and so laid off workers choose positive search effort. In contrast to the calibrated UI scheme $(b_H, b_L) = (0.68, 0.51)$, the optimal scheme implies much higher UI payments in the first year and much lower UI payments from year 2 onwards. The corresponding steady state unemployment rate is 9.4%.

Taking the wage $w = 0.85$ as exogenous, Column 2 describes the effect of optimal tax policy (given optimal UI). Optimality implies the Planner reduces the employment tax from $T_0 = 0.085$ to $T^* = 0.066$. This tax cut and [free entry] yields a large increase in market tightness and a corresponding increase in γ (from 1.62 to 2.21). As described in Theorem 1, the large increase in θ raises worker welfare through the thick market externality. It also implies higher exit rates out of unemployment and

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[13]This might introduce other moral hazard problems - a worker might accept a job offer but delays starting work while UI payments are higher than wages. Note this problem does not arise in the first best (where $b^{**}(t) < w^*$ for all $t > 0$).

so reduces the operating cost of the UI program. Despite the significant budget cut, the higher exit rates implies the fall in the level of UI payments is moderate. Indeed, optimal UI as described in Column 2 yields an initial UI payment $b^*(0)$ which is the same as in Column 1, but UI payments fall more quickly with duration. This reflects (11), where a higher γ implies UI payments fall more quickly. The intuition is that by making it easier for the unemployed to find work, a higher γ implies the Planner frontloads UI payments even more. The higher market tightness implies steady state unemployment falls to 7.4%.

The first best was identified using a grid search over w and welfare was maximised at $w^* = 0.8703$. Note that $b^{**}(0)$ in Column 3 implies $b^{**}(0) = w^*$ at this optimum. This confirms not only Theorem 2, that optimal co-insurance implies $b^{**}(0) = w^*$, but also that the computed numerical solutions are accurate. Note that the first best implies a further cut in the UI budget with $T^{**} = 0.050$, implying a 40% cut in the UI budget from the baseline case.

The optimal tax policy which implements the first best depends on the assumed bargaining power. With risk neutral workers and no UI, Nash bargaining with power $\alpha = 0.5$ implements a wage which induces efficient search and optimal market tightness (Hosios (1990)). By raising the worker's threatpoint in the Nash bargaining rule, a UI scheme pushes wages higher and so pushes down equilibrium job creation rates. For these parameter values, optimal tax policy implies the Planner raises the employment tax to $T = 10\%$, and the resulting tax surplus $T - T^{**}$ finances optimal job creation subsidies $s/a = 25\%$.

C/L rules out job creation subsidies by assumption and argues that UI payments might then increase with duration - the Planner reduces UI payments at short durations to reduce the value of being laid-off, which then lowers equilibrium wages and leads to an increase in equilibrium job creation rates. It is perhaps surprising, then, that the original wage, $w = 0.85$, is not 'too high' given optimal policy. The excessively high unemployment rate (12%) was instead due to the tax rate being too high. Relative to column 1, optimal policy implies a 40% cut in the funding tax to $T^{**} = 5\%$. By increasing equilibrium job creation rates, the unemployment rate falls to 7%. More importantly for optimal UI, the average unemployment spell is now around 9 months rather than 16 months. Although the lower tax rate implies the UI program has to be less generous, doubling worker re-employment rates implies the fall in the level of UI payments is relatively modest. Over the first year of unemployment, the optimal UI profile $b^{**}(.)$ initially exceeds $b_H = 0.68$ and falls to around $b_L = 0.51$

by the end of it. b^{**} is only significantly lower than the baseline case (b_H, b_L) from year one onwards. But as the expected unemployment spell is now less than a year, the corresponding welfare loss is relatively small.

6   Discussion and Related Literature.

In a matching equilibrium with no discounting, the Planner's problem dichotomises into 3 distinct problems which are solved recursively:

(I) Claim 3 describes the optimal UI profile given wage w, employment tax T and $\gamma = m(\theta)$. As typically assumed in the micro-literature, that problem is equivalent to maximising the value of being laid-off subject to a budget constraint.

(II) Theorem 1 describes the efficient employment tax T^* and market tightness θ^* given the market wage and the optimal UI profile.

(III) Theorem 2 describes the optimal wage level given optimal UI and efficient tax and market tightness. The first best is then implemented by an appropriate mix of employment taxes and job creation subsidies.

A surprising insight is that congestion externalities play no role in the analysis. A congestion externality arises when an increase in a worker's job search effort crowds out the matching probabilities of competing job seekers. But note that if a worker increase his job search effort today, he is more likely to exit the market and exit today implies he will not crowd out tomorrow's job seekers. The steady state framework takes such inter-cohort congestion effects into account. To see why congestion externalities play no role, consider *any* UI profile $b(.)$ consistent with a non-degenerate steady state (i.e. one that implies $\lim_{t\to\infty} \Psi(t) = 0$ and so unemployment is not an absorbing state). Note then that

$$k^{total} = \int_0^\infty \Psi(t)k(t)dt$$

is the total expected search effort of a newly laid-off worker using an optimal job search strategy. As $\Psi(t) \equiv \exp[-\int_0^t \gamma k(x)dx]$, this expression is directly integrable and

$$k^{total} = \frac{1}{\gamma}[1 - \lim_{t\to\infty} \Psi(t)] = \frac{1}{\gamma}$$

in a non-degenerate steady state. As $\gamma = m(\theta)$ then conditional on market tightness θ, k^{total} is independent of the UI profile $b(.)$. The insight is that lowering UI payments at long durations generates greater search effort at short durations but this then

lowers the worker's survival probability $\Psi(t)$. The overall effect of any variation in $b(.)$ on k^{total} is zero. As market tightness is separately determined by the [free entry] condition, $b(.)$ cannot change expected total search effort by the laid-off worker and so congestion externalities are not a factor in its choice. This insight clearly holds for extended optimal UI problems (e.g. Hopenhayn and Nicolini (1997), Werning (2002), Kocherlakota (2004)): given any policy design and corresponding job search rule $k(.)$, total expected search effort of a laid-off worker is always $1/\gamma$.

(14) implies the optimal sequence of UI payments satisfies

$$\frac{u'(b^*(0|.))}{u'(b^*(t|.))} = 1 - u'(b^*(0|.)) \int_0^t \gamma \frac{\partial k^*}{\partial S} B^*(\tau) d\tau. \tag{21}$$

This condition reveals the essential externality in a matching equilibrium - workers ignore when searching for work that finding a job saves the Planner $B(\tau)$ in further UI payments. By marginally reducing UI payment $b^*(t|.)$, the Planner induces greater search effort at all durations $\tau < t$ and the integral in (21) then describes the resulting fall in the cost of the UI program. Of course this externality also distorts the Planner's choice of market tightness - increasing exit rates by increasing market tightness is an alternative way of reducing the cost of this externality.

Hopenhayn and Nicolini (1997) introduce an important variation to this theme - they suppose that re-employed workers must pay an income tax premium which depends on the length of their previous spell of unemployment. They show that this added policy option generates large welfare gains. Their simulation is particularly insightful as it shows that the optimal income tax premium levied at each duration rises one for one with previous UI receipts. Their optimal scheme is therefore not unlike a loans program with an initial one-off severance payment. Such a scheme is efficient as it internalises the externality identified in (21): as the worker must repay any further UI receipts (through future taxes), remaining unemployed does not extract further rents from the Planner. Hence worker job search is efficient. The loan scheme is efficient as the Planner can use it to smooth consumption during the period of unemployment.

Unfortunately establishing that a loan scheme is optimal when workers use a savings strategy is not a straightforward extension. Conditions such as (21) no longer apply as a savings strategy implies consumption at duration t is not $b^*(t|.)$. Indeed wealth effects imply a variation in $b^*(t|.)$ affects search at all durations and the optimality conditions are consequently much more complicated. Werning (2002), Kocherlakota (2004) consider layoff insurance when workers can save out of their UI payments.

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Werning (2002) argues using examples that UI payments might optimally increase with duration, while Kocherlakota (2004) provides an example where the Werning first-order condition approach is not valid and in that example, optimal UI payments are constant with duration. These papers simplify the optimal UI problem by assuming workers are liquidity constrained when laid-off. But this rules out precautionary savings strategies, where employed workers accumulate savings while employed and dissave while unemployed. Costain (1997) argues that when workers use optimal savings strategies, a UI scheme is valuable precisely because it mitigates the need for workers to use a precautionary savings strategy. Indeed, Engen and Gruber (2001) find that UI schemes have significant effects on precautionary savings behaviour.

The micro-literature on optimal UI with hidden savings is an important and ongoing research issue. As argued by Hopenhayn and Nicolini (1997), however, and as suggested by condition (21) and the arguments of Costain (1997), intuition here suggests that a one-off severance payment at duration $t = 0$ may be close to optimal as:

(i) such a scheme internalises the job search externality identified by (21) and so induces efficient job search effort [noting that congestion externalities play no relevant role];

(ii) the severance payment compensates the worker for his/her drop in permanent income by being laid-off, and

(iii) workers use a privately optimal savings strategy to smooth consumption over time.

Of course a lump-sum severance payment scheme implies incomplete insurance against an extended spell of unemployment. But this is a necessary feature of an optimal UI program - it is only through being under-insured that the unemployed worker chooses positive job search effort. Given such a UI scheme, the Planner's macro-tradeoff is between higher job creation rates and more generous severance payments.

7   Conclusion.

This paper has characterised optimal UI policy in a matching equilibrium. As the numerical simulation makes clear, an important issue for optimal policy is ensuring efficient market tightness. In particular raising market tightness not only makes the unemployed better off via the thick market externality it also reduces the cost of

the UI program by increasing worker re-employment rates. Policies which ignore this effect can result in low job creation rates because of high taxes, and high taxes because worker re-employment rates are low (jobs are scarce) and the UI program is then costly to operate and taxes have to be high to fund it.

The paper has also shown that job creation programs, considered here as a subsidy on job creation costs, may play an important policy role. This policy reflects the underlying hold-up problem, that firms first invest in capital before hiring workers. If worker bargaining power is high and so workers extract 'too many' match rents, the Planner undoes the hold-up problem by taxing employment and uses those tax revenues to subsidise job creation investments by firms. Of course the Pissarides (2000) framework assumes the cost of an unfilled vacancy is the advertising cost $a > 0$. This suggests job creation costs may be minor. It is interesting, however, that the matching function approach plays only a minor role in the analysis above. The [free entry] rule determines market tightness

$$\theta = \widehat{\theta}(\frac{p - w - T}{\delta(a - s)})$$

and the worker matching parameter $\gamma = m(\theta)$, describing how easy it is to find work, is of the form:

$$\gamma = \widehat{\gamma}(\frac{p - w - T}{\delta(a - s)})$$

where $\widehat{\gamma}$ is a positive and increasing function.[14] It is this latter object which is central to the analysis. In principle, $\frac{p-w-T}{\delta(a-s)}$ describes a firm's return to capital investment (it is profit flow divided by the (annuitised) capital investment cost). By increasing this return, either by reducing the employment tax T or subsidising investment s, the Planner increases equilibrium job supply. Such a policy then makes it easier for the unemployed to find work. This raises aggregate welfare given that workers are better off employed rather than unemployed and underinsured.

8 Appendix

Proof of Claim 2.

(i) Free entry of vacancies implies

$$a - s = \frac{M(K, V)}{V} \frac{p - w - T}{\delta}$$

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[14]The matching function specified in the simulation implies $\widehat{\gamma}(.)$ is linear.

and constant returns to matching implies the stated equation for tightness θ.

(ii) Aggregating over individual worker exit rates implies

$$\gamma K = M(K, V)$$

and constant returns to matching then implies $\gamma = m(\theta)$.

(iii) Steady state implies unemployment level U is given by

$$U = \delta E \int_0^\infty \Psi(\tau) d\tau$$

where $\Psi(.)$ is the probability a laid off worker is still unemployed at duration τ. As $E + U = 1$, this implies the equation for E.

(iv) Budget balance requires

$$ET = sV + \delta E \int_0^\infty \Psi(\tau) b(\tau) d\tau.$$

Dividing both sides by K and noting that steady state turnover implies $\delta E = \gamma K$ (as both equal M), this constraint is equivalent to the one stated in the Claim. This completes the proof of Claim 2.

Proof of Claim 3. Rather than consider the programming problem stated in the text, it is more convenient to consider the dual problem:

$$\min_{b(.) \geq 0} \int_0^\infty \Psi(t) b(t) dt \tag{22}$$

subject to

$$u(w) + \delta[V^0 - V^E] = W^* \tag{23}$$

where $\Psi(.)$ and $[V^0 - V^E]$ are determined by optimal job search. As $b(.) = 0$ implies a strictly positive welfare payoff, we restrict attention to $W^* > 0$.

Note, (23) is satisfied if and only if $S(0) = (u(w) - W^*)/\delta$. This optimisation problem is equivalent to choosing $b(.)$ to minimise (22) where optimal job search implies (Ψ, S) evolve according to the differential equations

$$\dot{\Psi} = -\gamma k^* \Psi$$

$$\dot{S} = \gamma k^* S - [W^* - u(b) + c(k^*)]. \tag{24}$$

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and subject to initial values $\Psi(0) = 1$ and $S(0) = (u(w) - W^*)/\delta$. This problem can be solved using the Maximum Principle. Define the Hamiltonian

$$H = \Psi b + x_\Psi \left[-\gamma k^* \Psi \right] + x_S \left[\gamma k^* S - \left[W^* - u(b) + c(k^*) \right] \right]$$

where x_Ψ and x_S denote the costate variables associated with the state variables Ψ, S. The Maximum principle implies the necessary conditions for a minimum at any duration t are:

$$
\begin{aligned}
\text{either } b &= 0 \text{ and } \frac{\partial H}{\partial b} \geq 0 \Leftrightarrow x_S \geq 0 \\
\text{or } b &> 0 \text{ and } \Psi + u'(b) x_S = 0 \Leftrightarrow x_S < 0 \text{ and } x_S = -\Psi/u'(b);
\end{aligned}
\tag{25}
$$

where

$$\dot{x}_\Psi = -b + \gamma k^* x_\Psi$$

$$\dot{x}_S = x_\Psi \gamma \Psi \frac{\partial k^*}{\partial S} - x_S \gamma k^*$$

and (Ψ, S) satisfy the differential equations and initial values described above.

As the dynamic optimisation problem is autonomous and there is no discounting, optimality implies $H = 0$ at all durations (Leonard and Long (1992, p298)). Hence the relevant transversality condition for this problem is

$$H = \Psi b + x_\Psi \dot{\Psi} + x_S \dot{S} = 0. \tag{26}$$

Integrating the differential equation for $\dot{x}_\Psi$, using Ψ as an integrating factor, implies

$$x_\Psi(t) \Psi(t) = \int_t^\infty \Psi(\tau) b(\tau) d\tau + A$$

where A is the constant of integration. Hence $x_\Psi(t) \equiv B(t) + A/\Psi(t)$ where $B(t)$ is the continuation cost as defined in the text.

The Inada condition $u'(0) = \infty$ implies $b = 0$ is never optimal at any finite duration. As (25) then implies $x_S = -\Psi/u'(b)$, differentiation implies

$$\dot{x}_S = -\dot{\Psi}/u'(b) + \Psi u''(b) \dot{b}/[u'(b)^2].$$

The differential equation for x_S above now yields:

$$\left[\frac{-u''(b)}{u'(b)^2} \right] \dot{b} = -\gamma \frac{\partial k^*}{\partial S} [B(t) + \frac{A}{\Psi(t)}] \tag{27}$$

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Completing the proof requires using the transversality condition (26) to show that $A = 0$ along the optimal trajectory. Using the above, (26) yields the additional restriction:

$$\dot{S} = u'(b) \left[b - \gamma k^*[B + A/\Psi(t)]\right] \tag{28}$$

To set up a contradiction argument, we establish the following Claim.

Claim A1. The optimal program implies b, S are uniformly bounded and $\Psi(t) \rightarrow 0$ as $t \rightarrow \infty$.

Proof of Claim A1: As $S(0)$ is fixed, the worst off we can make an unemployed worker is to set $b(.) = 0$. This implies $S = \overline{S} > 0$ as defined in the Claim and $b(.) \geq 0$ then implies $S(.) \leq \overline{S}$. Now consider any phase where $S < 0$. As this implies $k^* = 0$, (28) then implies S must be non-decreasing. Hence S is uniformly bounded, where $S \in [0, \overline{S}]$ for all t if $S(0) \geq 0$ and $S \in [S(0), \overline{S}]$ for all t if $S(0) < 0$.

b must be uniformly bounded for if it becomes arbitrarily large it implies $S < 0$ (as $b > w$) for t large enough. But $k^* = 0$ for such t then implies $\dot{\Psi} = 0$ and thus $\int_0^\infty \Psi(t)b(t)dt = \infty$, which contradicts a minimum.

$\Psi \rightarrow 0$, otherwise this requires $k^* \rightarrow 0$ for t large enough, which in turn requires $b(.)$ sufficiently large and we again have $\int \Psi(t)b(t)dt = \infty$. This completes the proof of Claim A1.

Armed with claim A1, we can now prove $A = 0$ using a contradiction argument. Suppose $A \neq 0$. As Claim A1 implies $\Psi \rightarrow 0$ as $t \rightarrow \infty$, there are two possible cases:

(i) suppose $A \neq 0$ and that $[B + A/\Psi(t)]$ becomes unboundedly large in absolute value as t becomes large. But Claim A1 and (28) then imply $k^* \rightarrow 0^+$ as $t \rightarrow \infty$, in which case $S \rightarrow 0^+$. (24) now implies $b \rightarrow \overline{b}$ where $u(\overline{b}) = W^* > 0$. But (27) then requires $[B + A/\Psi(t)] \rightarrow 0$ in this limit which contradicts $[B + A/\Psi(t)]$ becoming unboundedly large.

(ii) suppose $A \neq 0$ and that $[B + A/\Psi(t)]$ remains finite as t becomes large. As this implies B becomes unboundedly large, this can only occur if $k^* \rightarrow 0^+$, in which case $S \rightarrow 0^+$. (24) again implies $b \rightarrow \overline{b}$ where $u(\overline{b}) = W^* > 0$ and (27) then implies $[B(t) + \frac{A}{\Psi(t)}] \rightarrow 0$ as $t \rightarrow \infty$. But (28) implies S is then strictly increasing in this limit which contradicts $S \rightarrow 0^+$. Hence $A = 0$.

$A = 0$ and (27) imply (11) in Claim 3. As (11) implies $b(.)$ decreases with duration, then $b \geq 0$ implies $b(.)$ must converge to some limit value $\underline{b} \geq 0$. (11) then implies $B \rightarrow 0$ in this limit and so $\underline{b} = 0$. As $u(0)$ is finite, S converges to limiting value $\overline{S}$ identified where $b = 0$ and $\dot{S} = 0$. This completes the proof of claim 3, noting that $W^* \equiv u(w) - \delta S(0)$.

Proof of Claim 5.

Consider an arbitrarily small perturbation $(dT, d\theta)$ which holds maximised welfare constant, $W^* = \overline{W}$, and so implies the re-entitlement effect, $S_0 = V^E - V^0 = [u(w) - \overline{W}]/\delta$ is fixed. The following describes the first order welfare effects of this perturbation. As b^* is chosen optimally, the Envelope Theorem implies we can ignore variations in $b^*(.)$ as such variations only generate second order welfare effects.

Hence given $b^*(.)$ and $\overline{W}$ fixed, let $S(t) = S^*(t) + dS^\theta(t)$ denote the worker's re-employment surplus at duration t using optimal job search given increased market tightness $\theta + d\theta$. Claim 1 implies

$$S(t) = \int_t^\infty e^{-\int_t^\tau \gamma k(x)dx} \left[\overline{W} - u(b^*(\tau)) + c(k(.))\right] d\tau. \tag{29}$$

As job search effort is chosen optimally, the Envelope Theorem implies variations in $k(.)$ only generate second order welfare effects on surplus $S(.)$. Noting $d\gamma = m'(\theta)d\theta$, the first order effect of an increase in market tightness on $S(t)$ is therefore:

$$\frac{dS^\theta(t)}{d\theta} = \int_t^\infty [-\int_t^\tau m'(\theta)k(x)dx]e^{-\int_t^\tau \gamma k(x)dx} \left[\overline{W} - u(b^*(\tau)) + c(k(\tau)))\right] d\tau. \tag{30}$$

Integrating by parts[15] this condition simplifies to

$$\frac{dS^\theta(t)}{d\theta} = -m'(\theta) \int_t^\infty \frac{\Psi(\tau)}{\Psi(t)} k(\tau)S^*(\tau)d\tau$$

and hence

$$\frac{dS^\theta(0)}{d\theta} = -m'(\theta) \int_0^\infty \Psi(t)k(t)S^*(t)dt$$

describes the direct change in $S(0)$ through higher market tightness.

Given $b^*(.)$ and $\overline{W}$ fixed, let $B(0) = B^*(0) + dB^\theta$ denote the first order change in the cost of the UI program given increased market tightness $\theta + d\theta$. Recall that

$$B(0) = \int_0^\infty e^{-\int_0^t \gamma k(x)dx} b^*(t)dt. \tag{31}$$

where optimal job search is given by $c'(k) = \gamma S$ (while $S > 0$). There are two first order effects: an increase in market tightness increases γ and also changes search effort $k(.)$. The first order change in search effort at duration t by an increase in market tightness, denoted $dk^\theta(t)$, is given by

$$\begin{aligned} c''(k)dk_\theta(t) &= S^*(t)d\gamma + \gamma dS^\theta(t) \\ &= d\gamma \left[S^*(t) - \frac{1}{\Psi(t)} \int_t^\infty \Psi(\tau)\gamma k(\tau)S^*(\tau)d\tau \right]. \end{aligned}$$

---

[15] use (13), which implies $\overline{W} - u(b) + c(k) = \gamma k S - \dot{S}$, and then note $\frac{d}{d\tau}[-\Psi(\tau)S(\tau)] = \Psi(\tau)[\gamma k S - \dot{S}]$.

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Integrating by parts then yields[16]

$$\frac{dk^\theta(t)}{d\theta} = -\frac{m'(\theta)}{c''(k(t))} \int_t^\infty \frac{\Psi(\tau)}{\Psi(t)} \dot{S}(\tau) d\tau.$$

The total first order effect of an increase in market tightness on UI cost is therefore:

$$dB^\theta = -\int_0^\infty [\int_0^t k(x)d\gamma + \gamma dk_\theta(x)]dx]\Psi(t)b^*(t)dt$$

and so

$$\frac{dB^\theta}{d\theta} = -m'(\theta) \int_0^\infty [\int_0^t k(x)dx]\Psi(t)b^*(t)dt - \gamma \int_0^\infty [\int_0^t \frac{dk^\theta(x)}{d\theta}dx]\Psi(t)b^*(t)dt.$$

The perturbation $(d\theta, dT)$ yields a net budget surplus at $t = 0$ equal to $(\frac{dT}{\delta} - \frac{dB^\theta}{d\theta}d\theta)$. Claim 4 in the text implies this budget surplus yields first order welfare payoff

$$dS(0) = -u'(b^*(0)) \left[\frac{dT}{\delta} - \frac{dB^\theta}{d\theta}d\theta\right].$$

Hence the perturbation $(dT, d\theta)$ holds $S(0)$ constant, and hence welfare $W = u(w) - \delta S(0)$ constant, if and only if

$$\frac{dS^\theta(0)}{d\theta}d\theta - u'(b^*(0)) \left[\frac{dT}{\delta} - \frac{dB^\theta}{d\theta}d\theta\right] = 0$$

which implies the claim.

Proof of Theorem 2.

The Planner's first best problem is

$$\max_{w,P} \ u(w) + \delta[V^0 - V^E]$$

subject to the steady state conditions:

$$a = \frac{m(\theta^*)}{\theta^*}\frac{p - w - T}{\delta}, \qquad \text{(free entry)}$$

$$T = \delta \int_0^\infty \Psi^*(\tau)b^*(\tau)d\tau, \qquad \text{(budget balance)}$$

and optimal job search with $\gamma = m(\theta)$.

---

[16] use $\frac{d[-\Psi]}{d\tau} = \gamma k \Psi$.

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Given an arbitrary $w \in (0, p)$, Theorem 1 describes optimal policy $P^*(w)$. Consider the following policy variation: the Planner (i) increases the market wage to $w + dw$, (ii) holds the UI profile $b(.)$ fixed at b^* as described in Claim 3 (given wage w), and (iii) changes the employment tax dT to hold market tightness constant at $\theta = \theta^*$. As this policy variation does not necessarily imply budget balance we introduce a slack parameter so that the budget balance constraint is rewritten as

$$\delta \int_0^\infty \Psi^*(\tau)b^*(\tau)d\tau = T + \xi, \tag{32}$$

and ξ is a transfer from outside of the economy. Suppose we in addition choose ξ so that the total change in W is zero; i.e. $dW = 0$. If a policy variation (dw, dT, ξ) exists which holds welfare constant and $\xi < 0$ then such a variation must describe a policy improvement as it generates a tax surplus.

Given such a perturbation, Claim 1 implies

$$S(t) = \int_t^\infty e^{-\int_t^\tau \gamma k(x)dx} \left[\overline{W} - u(b^*(\tau)) + c(k(.))\right] d\tau \tag{33}$$

As b^*, market tightness and $\overline{W}$ are all held fixed, this perturbation implies there is no change in $S(.)$ along the UI profile. As $W = u(w) + \delta[V^0 - V^E]$, Claim 4 implies the total first order change in welfare via this perturbation is simply

$$dW = u'(w)dw + \delta \frac{u'(b^*(0))}{\delta}[dT + \xi].$$

Holding market tightness fixed and [free entry] requires $dT = -dw$. Holding welfare fixed $dW = 0$ implies ξ must satisfy

$$\xi = \frac{u'(b^*(0)) - u'(w)}{u'(b^*(0))}dw.$$

If $w < b^*(0)$, then choosing $dw > 0$ implies $\xi < 0$ and this perturbation generates strictly positive surplus. Conversely if $w > b^*(0)$ then choosing $dw < 0$ implies $\xi < 0$ and this perturbation also generates positive surplus. Hence optimality implies the necessary condition $w = b^*(0)$ which completes the proof of Theorem 2.

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