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Education, Matching and the Allocative Value of Romance

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Abstract

Societies are characterized by customs governing the allocation of non-market goods such as marital partnerships. We explore how such customs affect the educational investment decisions of young singles and the subsequent joint labor supply decisions of partnered couples. We consider two separate matching paradigms for agents with heterogeneous abilities - one where partners marry for money and the other where partners marry for romantic reasons orthogonal to productivity or debt. These generate different investment incentives and therefore have a real impact on the market economy. While marrying for money generates greater investment efficiency, romantic matching generates greater allocative efficiency, since more high ability individuals participate in the labour market. The analysis offers the possibility of explaining cross-country differences in educational investments and labor force participation based on matching regimes.

Keywords: Education, participation, matching, marriage, cohabitation.

JEL Classification: I21, J12, J16, J41.

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1 Introduction

- 'Flora must get herself a job, he said, then she would not have to go to India. "We girls are not brought up to do jobs," said Tashie. "We are brought up to marry".' [Mary Wesley (1990), *A Sensible Life*, p169].

- 'The second benefit to be expected from giving to women the free use of their faculties... would be that of doubling the mass of mental faculties available for the higher service of humanity.... Mental superiority of any kind is at present everywhere so much below the demand; there is such a deficiency of persons competent to do excellently anything that requires any considerable amount of ability to do; that the loss to the world, by refusing to make use of one half of the whole quantity of talent it possesses, is extremely serious.' [John Stuart Mill (1869), *The Subjection of Women*, p82-3.]

Unlike the labour market where employer-employee partnerships are characterised by wages paid, partners in marriage do not hire or buy each other. Some cultures have bride prices or dowries but the practice is rare in developed economies. With no price formation in the marriage market, the matching process generates externalities which distort real economic activity. For example Cole, Mailath and Postlewaite (1992) describe a marriage market where males accumulate wealth to attract brides, and the resulting tournament distorts savings rates away from the competitive level. Fernandez, Guner and Knowles (2005) model the interactions between household matching, inequality, and per capita income, and show that equilibrium sorting of spouses by skill type is an increasing function of the skill premium. In this paper we explore how the type of matching in the marriage market affects the education choices of young singles and the joint labour supply decisions of married couples. We consider two separate paradigms - one where partners marry for money (where partners value current wealth and expected future earnings) and one where partners marry for romantic reasons, where matching is driven by skin-deep attributes that are orthogonal to productivity and debt. The different matching paradigms generate different investment incentives for young singles and therefore have a real impact on the market economy.[1]

There are two central aspects to the model. The first is that, if both partners work in the workplace and have children, they have to organise and pay for private childcare. They therefore make a joint workplace participation decision, which trades off a second salary against

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[1] Throughout this paper we use the word marriage to refer to any stable cohabiting partnership, regardless of whether or not it is formalized by the state.

private childcare costs. Traditionally it has been the female partner who opts to be the home-maker. But given the decline in heavy manual labour and that men are arguably equally capable of caring for small children, it is not obvious that female labour market participation rates need be less than male rates. Indeed as female attendance rates at universities are now comparable to male rates in most OECD countries, it seems plausible that female labour force participation rates might converge to male rates in the future.[2] A central interest of the paper is understanding when equilibrium may be asymmetric, where one sex has higher education and labour market participation rates, even though the distributions of male and female abilities are assumed to be the same.

A second aspect of the model is that young adults make their investments in education prior to meeting their future partner. Education is here defined to be post-compulsory schooling. It is not the learning of basic literacy and numeracy skills which we assume is provided to all by compulsory schooling. Although education potentially improves parental skills, it seems reasonable that the principal reason for investing in an expensive education is to increase workplace human capital and so command a higher wage in the labour market. Of course greater earning power attracts more potential partners in the marriage market but the downside is that high education debts makes one a less attractive proposition.

For reasons explained in the paper, labour market participation choices endogenously generate increasing returns to education and so imply a co-ordination problem. It is not optimal for each partner to invest in an intermediate level of education. As education is expensive, it may also not be optimal that they both invest in a full education, particularly if one expects to be raising children. When making an investment choice, a single person not only has to anticipate the education decision of his/her future partner but also takes into account how it affects his/her marriage prospects.

Our paper shows that when people marry for money, equilibrium implies perfect positive assortative matching: men and women of the same ability match with each other. Equilibrium also implies that the investment strategies are efficient; that is, conditional on a partnership,

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the education choices are jointly efficient. Those investments, however, need not be symmetric across the sexes. For example, when the return to education is small, an equilibrium exists where all men invest in education and participate with probability one, while no women invest in education and are likely to spend their time childrearing. In this equilibrium, a woman maximises her marital prospects by avoiding debt (or builds up a dowry if education costs are interpreted as foregone earnings). If the return to education is instead high, then the highest ability women also invest in education. As positive assortative matching implies the partner of such a high ability woman will also be a high flyer, she expects to pay for private childcare.

We show that romantic matching, where the matching allocation is instead orthogonal to real economic variables, leads to quite different investment behaviour. A symmetric equilibrium always exists where the higher ability types of both sexes invest in education. But unlike the case when partners marry for money and education rates are at least 50% (because all men are always educated), education rates with romantic matching may be very low. In particular, when the return to education is low, it is very costly for both partners to be educated: they have to pay for two education debts while trying to raise a family. By co-ordinating the investment plans of singles, the 'marrying for money' equilibrium avoids the two-debt problem. There is no such co-ordination with symmetric romantic matching, and a low return to education then implies low education rates as singles avoid the two-debts problem.

We identify a condition establishing when the symmetric equilibrium with romantic matching is unstable, whereupon asymmetric equilibria exist. In an asymmetric equilibrium, the education rates and participation rates of men may be much higher than women's. Using simulations, we show that equilibria with romantic matching may be more efficient than those where partners marry for money. Although investment is efficient in the marrying for money case, the matching allocation implies there are many high ability women who do not participate in the labour market. This potential loss to society concerned John Stuart Mill in 1869, as the quotation at the start of this paper illustrates. In contrast, romantic matching generates a different allocation of partners that increases average workplace productivity.

Section 2 of the paper describes the model and defines equilibrium. It also describes the first best allocation and investment strategies which maximise total value. Section 3 describes

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equilibrium when partners marry for money and section 4 describes romantic equilibria. Using simulations, in section 5 we compare the equilibrium outcomes and make welfare comparisons.

2 The Model

We consider a two-period, two-sided matching model in which there is a unit mass of men and women. In the first period individuals make education choices which not only determine their second period productivities but also their asset values (education is costly, both in terms of direct education costs and foregone earnings). In the second period, men and women match in the partnership or marriage market and subsequently make joint workplace participation decisions with their partners.

Each individual is endowed with a sex $s = m, w$ and an ability $a \in [0, \overline{a}]$ where A denotes the population distribution of underlying abilities and is the same for both sexes. A is differentiable, contains no mass points and has a connected support. In the first period, each individual chooses an education level $e \in [0, \overline{e}]$. Education here is interpreted as post-compulsory. It is not the learning of basic literacy and numeracy skills which we assume is provided to all by the compulsory school education system. For simplicity we assume additional education investment e increases productivity in the workplace but does not affect capabilities at home production. In particular, an individual with ability a, who invests in e education, will have second period productivity $\alpha = a + e$ in the workplace. Assume the cost of education is $c_0 e$, where $0 < c_0 < 1$ and is the same for all. Note that $\overline{e} < \infty$ describes a ceiling level of education.[3]

In the second period, each individual is fully described by their sex and (α, a), where α is their workplace productivity and $c_0[\alpha - a]$ is their education debt. All matches are heterosexual and so each matched pair is described by their joint attributes (α, α', a, a').

Given a matched pair (α, α', a, a'), they make a joint labour supply decision. Assuming a competitive labour market, if only one partner works in the workplace and that worker has productivity α, the couple enjoy joint income α. If both partners work in the workplace, they jointly earn $\alpha + \alpha' - x$ where x is the opportunity cost of workplace participation of the second

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[3] As the model finds there are increasing returns to education, this cost structure simplifies the second order condition problem.

partner. We shall think of x as the cost of private childcare when both parents participate in the workplace.[4] If grandparents are not around, x also includes the (idiosyncratic) psychic cost of hiring a person from outside the family to look after one's children. Assuming the fertility outcome is not known at the time of the match, x is considered as a random draw from distribution G, where G is continuous and has a connected support $[0, \overline{x}]$. For simplicity, we assume that G is independent of (α, α', a, a'); i.e. the fertility outcome is independent of productivity variables. For ease of exposition we also assume $\overline{x} \geq \overline{a} + \overline{e}$ so that $G(\alpha) < 1$ and is strictly increasing in α for all (feasible) productivities $\alpha < \overline{a} + \overline{e}$.

Consumption within the family is a public good. Assuming agents are risk neutral, each individual's utility within the match is equal to their expected net income, less their education debts plus any happiness directly attributable to the match. Their expected utility when forming the match is therefore

$$\Pi(\alpha, \alpha', a, a', \varepsilon) = \int_0^{\overline{x}} \max[\alpha, \alpha', \alpha + \alpha' - x] dG(x) - c_0[\alpha - a] - c_0[\alpha' - a'] + \varepsilon$$

where $\max[\alpha, \alpha', \alpha + \alpha' - x]$ describes their optimal labour supply choice (either one partner works or they both do and then pay for childcare) and ε is an idiosychratic match value. Throughout we assume the payoff to being single is sufficiently small that being in a partnership is always strictly preferred.

Assuming marital consumption is a public good has two prime advantages. First it implies there is no hold-up problem; each partner would like to choose an education level which maximises joint payoff $\Pi(.)$ (though the marriage market might distort these investment incentives). As the hold-up problem is well understood in the matching literature, our public good approach allows us to focus on other issues. Second it avoids political economy type issues where the family breadwinner might have a larger say on how income is spent within the family.

An important feature of this payoff structure is that it implies utility is non-transferable between two matched partners. Thus a less able type cannot attract a more able type by committing to consume less of the marital pie. Note also that education debts play a similar role to dowries: a partner with ability a who chooses little or no education is, ex-post, less

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[4]In addition it includes other costs such as paying for cleaning and maintenance services.

productive in the workplace but brings greater starting wealth $c_0[a - \alpha]$ to the match.

We now describe the marriage market. As the focus of the paper is on equilibrium education choice and joint labour supply, the marriage market is kept relatively simple. Let F_m, F_w describe the distribution of male and female attributes (α, a) in the marriage market; i.e. $F_m(\alpha, a)$ denotes the probability a randomly chosen single male has productivity no greater than α and initial ability no greater than a. As each individual is small, assume all take these distributions as given. Note that individuals with different attributes value differently the attributes of singles on the other side of the market. As equilibrium matching with search frictions and ex-ante heterogeneous agents is complex,[5] we assume there are no search frictions and consider allocations where:

(i) each man and woman is allocated a partner;

(ii) there are no two individuals who would strictly prefer to be matched with each other than be matched with their allocated partner.

Any such allocation is termed a stable matching allocation.

We also focus on two polar cases. The first polar case assumes $\varepsilon = \varepsilon^H$ in all matches; i.e. there are no idiosyncratic match values. In this case, potential partners value only their joint net wealth position; i.e., they marry for money. We shall refer to this matching process as type $t = M$.

The second case assumes matching is instead romantic, driven by skin-deep attributes which are unrelated to productivity and debt. In particular, in the second period Nature (Cupid) randomly chooses a man and woman and assigns idiosyncratic match payoff $\varepsilon = \varepsilon^H$ should they match with each other and match value $\varepsilon = 0$ should either match with any other partner. Assuming ε^H is sufficiently high that both prefer to match with each other than match with anyone else in the market, a stable matching allocation implies they match with each other and so leave the marriage market. Nature then chooses another pair at random and repeats this process until all are matched. This process implies partnership formation is random and that all realised matches generate the same idiosyncratic match payoff ε^H. We refer to this case as romantic matching; $t = R$.

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[5] See for example Burdett and Coles (1997), Shimer and Smith (2000), Albrecht and Vroman (2001), Teuling and Gautier (2004), and Atakan (2005) for recent work in this literature.

2.1 Optimal Investment Strategies and the Definition of Equilibrium

Given a partnership (α, α', a, a'), let $\alpha^{\max} = \max[\alpha, \alpha']$ denote the productivity of the more productive partner and $\alpha^{\min} = \min[\alpha, \alpha']$. As the income value of the match is $\max[\alpha, \alpha', \alpha + \alpha' - x]$, the payoff maximising participation strategy of this pair is that the higher productivity partner participates in the workplace with probability one, the other participates if and only if the cost of childcare $x \leq \alpha^{\min}$. Hence

$$\int_0^{\overline{x}} \max[\alpha, \alpha', \alpha + \alpha' - x] dG(x) = \alpha^{\max} + \int_0^{\alpha^{\min}} [\alpha^{\min} - x] dG(x)$$

and integration by parts then implies the expected value of the match is

$$\Pi(\alpha, \alpha', a, a', \varepsilon) = \alpha^{\max} + \int_0^{\alpha^{\min}} G(x) dx - c_0[\alpha - a] - c_0[\alpha' - a'] + \varepsilon. \tag{1}$$

As both types of matching $t = M, R$ imply $\varepsilon = \varepsilon^H$ in a stable matching allocation, we simplify notation by dropping reference to ε in what follows. Conditional on the distribution of types F_m, F_w and a stable matching allocation, let $H_w(.|\alpha, a)$ denote the distribution of female types who match with a male with attributes (α, a). This clearly depends on the assumed type of matching - for example romantic matching implies $H_w = F_w$. Given H_w, an unmatched male a who invests to productivity level α obtains expected payoff

$$V_m(\alpha, a) = \int \Pi(\alpha, \alpha', a, a') dH_w(\alpha', a'|\alpha, a).$$

Restricting attention to pure strategies, let $\alpha_m^*(a)$ denote the privately optimal education choice of a man with endowed ability a; i.e.

$$\alpha_m^*(a) \in \arg\max_\alpha V_m(\alpha, a)$$

and let $\alpha_w^*(a)$ denote the privately optimal education choice of a woman. The rest of the paper identifies equilibrium as now defined.

Definition of Equilibrium:

An investment equilibrium of matching type $t = M, R$ is a sextuple of functions $\{\alpha_m^*, \alpha_w^*, F_m, F_w, H_m, H_w\}^t$ in which:

(i) the investment strategies α_m^*, α_w^* are optimal given agents expect a partner drawn from distributions H_m, H_w;

(ii) the attribute distributions F_m, F_w are consistent with the underlying distribution of abilities A and the investment strategies α_m^*, α_w^*;

(iii) the partner distributions H_m, H_w are consistent with the attribute distributions F_m, F_w and a stable allocation.

Before characterising these investment equilibria, it is insightful to describe the education and matching allocations which maximise total value in the economy when there are no idiosyncratic match values.

2.2   The First Best Matching Allocation and Investment Strategies

Suppose first we can pre-arrange a marriage; that is we know that a particular male a will match with a particular female a'. In that case, their jointly optimal education choices solve

$$\max_{\substack{\alpha \in [a, a+\overline{e}] \\ \alpha' \in [a', a'+\overline{e}]}} \Pi = \alpha^{\max} + \int_0^{\alpha^{\min}} G(x)dx - c_0[\alpha - a] - c_0[\alpha' - a']$$

where $\alpha^{\max}(.) = \max[\alpha, \alpha']$ and $\alpha^{\min}(.) = \min[\alpha, \alpha']$. Differentiating with respect to α implies

$$\partial \Pi / \partial \alpha \;=\; 1 - c_0 \text{ for } \alpha > \alpha'$$
$$=\; G(\alpha) - c_0 \text{ for } \alpha < \alpha'.$$

Notice there are increasing returns to education; i.e. $\partial \Pi / \partial \alpha$ is increasing in α and is strictly increasing for $\alpha < \alpha'$ (and α feasible). This occurs because of particpation effects. Should the worker not participate in the workplace, the ex-post realised marginal benefit to education is zero. In contrast, if the worker participates in the workplace, the ex-post realised marginal benefit to education is one (given $\alpha = a + e$). The ex-ante expected marginal benefit to education is therefore the worker's expected participation probability. The critical observation is

that participation probabilities increase with workplace productivity and so generate increasing returns to education.

As the more productive partner participates with probability one, his/her expected marginal benefit to education is one. Hence $\partial \Pi / \partial \alpha = 1 - c_0$ for $\alpha > \alpha'$. As $c_0 < 1$, it follows that the more productive partner invests to the ceiling level of education $e = \overline{e}$.

The less productive partner participates if childcare costs are low enough; i.e. participates with probability $G(.)$. Hence $\partial \Pi / \partial \alpha = G(\alpha) - c_0$ for $\alpha < \alpha'$. Note this implies strictly increasing marginal returns to education. Given constant marginal cost c_0, then optimality implies the lower productivity agent chooses either $e = 0$ or $\overline{e}$. We now characterise that choice.

Suppose that $a \geq a'$. Using the functional form for $\Pi(.)$, it is straightforward to show that it is most efficient that the more able partner, a, invests $e = \overline{e}$ and participates in the labour market with probability one. The less able partner, a', participates with probability $G < 1$ and chooses either $e = 0$ or $\overline{e}$. Their joint payoff if partner a' chooses $e = 0$ is

$$\Pi_0 = a + \overline{e} + \int_0^{a'} G(x)dx - c_0\overline{e}$$

as $\alpha^{\max}(.) = a + \overline{e}$ and $\alpha^{\min}(.) = a'$. If partner a' instead chooses $e = \overline{e}$, their payoff is

$$\Pi_1 = a + \overline{e} + \int_0^{a'+\overline{e}} G(x)dx - 2c_0\overline{e}.$$

Optimality implies partner a' chooses $e = \overline{e}$ if and only if $\Pi_1 \geq \Pi_0$ which implies the condition

$$\int_{a'}^{a'+\overline{e}} G(x)dx \geq c_0\overline{e}.$$

Reflecting there are increasing returns to education, note that the left hand side of this inequality is strictly increasing in a' for $a' \in [0, \overline{a}]$. Hence we have the following result.

Claim 1. Jointly Efficient Investment.
Given a match between a pair with abilities a, a' with $a \geq a'$, it is jointly efficient that

(i) the more able partner chooses $e = \overline{e}$;

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(ii) the less able partner chooses $e = \overline{e}$ if and only if $a' \geq \widehat{a}(c_0)$ where $\widehat{a}$ is defined by

$$\int_{\widehat{a}}^{\widehat{a}+\overline{e}} [G(x) - c_0]dx = 0. \tag{2}$$

Otherwise the less able partner chooses $e = 0$.

$\widehat{a}(c_0)$ describes a critical ability at which investment by the less able partner becomes optimal. Note such individuals only participate with probability $G(.)$. The definition of $\widehat{a}$ in (2) says that education is worthwhile if the worker's average participation probability, defined over education levels $e \in [0, \overline{e}]$, exceeds the marginal cost of education c_0. A useful insight for what follows is that individuals with ability $a \geq \widehat{a}(c_0)$ have a dominant investment strategy - regardless of their future partner's productivity, their minimal participation rate G is sufficiently high that investing $e = \overline{e}$ is always optimal. To avoid the trivial case, assume from now on that $c_0 > \underline{c}$ where $\underline{c}$ is defined by

$$\int_0^{\overline{e}} [G(x) - \underline{c}]dx = 0.$$

$c_0 > \underline{c}$ guarantees $\widehat{a}(c_0) > 0$. The optimal investment choice of a low ability type, one with $a < \widehat{a}$, is $e = 0$ if their future partner is more productive but $e = \overline{e}$ if their future partner is less productive. This implies a co-ordination problem as singles make these investment choices before meeting their partners.

Claim 1 describes the optimal investment strategies given a pre-arranged marriage. But what type of matching allocation is efficient? Note that the joint payoff function $\Pi(.)$ defined in equation (1) is submodular in (α, α'); i.e. $\alpha^L < \alpha^H$ implies

$$\Pi(\alpha^H, \alpha^H, .) + \Pi(\alpha^L, \alpha^L, .) < 2\Pi(\alpha^H, \alpha^L, .).$$

The allocation which maximises gross value is that a highly productive type - one whose productivity lies above the median value - should match with a low productivity type - one whose productivity lies below the median value. Given such matching, the socially optimal investment strategies imply the high ability types - those with ability above the median value - choose $e = \overline{e}$ and participate in the labour market with probability one, while the low ability types

choose education levels consistent with participation rate G. The above discussion establishes the following Theorem.

Theorem 1. Maximal productive efficiency implies:

(i) negative assortative matching (NAM) - those with ability above the median level match with those with ability below the median level;

(ii) investment and participation is symmetric across the sexes where

(a) those with above median ability choose $e = \overline{e}$ and participate with probability one;

(b) those with below median ability but above $\widehat{a}(c_0)$ choose $e = \overline{e}$, all others choose $e = 0$ and each participates with probability $G(\alpha)$.

Hence with socially efficient matching - negative assortative matching - the socially optimal investment strategies are symmetric across the sexes. As we shall see, equilibrium matching implies quite different education and labour market participation patterns.

3   Marrying for money

This section assumes there are no idiosyncratic match values and so each single's valuation of a potential partnership depends only on productivity and debt. Fortunately we do not need to characterise the set of stable matching allocations for all possible distributions F_m, F_w as Theorem 2 shows that an investment equilibrium necessarily implies positive assortative matching on underlying abilities.

Theorem 2 [Marrying for money]. With no idiosyncratic match values, all investment equilibria imply positive assortative matching (PAM) on underlying abilities; i.e. each man with ability a marries a woman with ability a. Further, an investment equilibrium exists where

(a) all men choose $e = \overline{e}$ and participate with probability one;

(b) women with $a \geq \widehat{a}(c_0)$ choose $e = \overline{e}$, all others choose $e = 0$ and each participates with probability $G(\alpha)$.

Proof. We establish the Theorem using an induction proof, starting with the most able types.

Suppose $\widehat{a}(c_0) < \overline{a}$ and consider the most able man with $a = \overline{a}$. Regardless of his future partner's attributes (α', a'), Claim 1 implies his joint payoff $\Pi(.)$ with that partner is maximised

by choosing $e = \overline{e}$. Now consider any woman (α', a'). The Envelope Theorem implies this woman strictly prefers to be matched with the highest ability male when he chooses $e = \overline{e}$ than with any other man. Hence the optimal choice of male $a = \overline{a}$ is to choose $e = \overline{e}$ - it maximises joint payoff given any partner and also ensures he has the first pick of all the women in the market (in a stable matching allocation). The argument also applies for the most able woman. Hence the most able man and woman both choose $e = \overline{e}$ and a stable matching allocation implies they match with each other.

Now fix an $a \in (\widehat{a}(c_0), \overline{a}]$ and suppose all with ability strictly greater than a choose $e = \overline{e}$ and match with each other. Consider then the residual market $[0, a]$. As $a > \widehat{a}(c_0)$ the same reasoning implies the most able man and woman in this residual market choose $e = \overline{e}$ - it maximises joint payoff given any partner and guarantees all those who remain in the residual market are willing to match with them. Hence they both choose $e = \overline{e}$ and a stable matching allocation implies they match with each other. Hence induction implies all those with $a \in (\widehat{a}(c_0), \overline{a}]$ choose $e = \overline{e}$ and match with each other.

Now consider the most able men and women not in $(\widehat{a}(c_0), \overline{a}]$; i.e., consider those with abilities $a \in [\widehat{a}(c_0) - \varepsilon, \widehat{a}(c_0)]$ where $\varepsilon > 0$ (small). Suppose there is a subset of men and women who match with individuals outside of this set. As an investment equilibrium implies individuals with ability $a > \widehat{a}(c_0)$ only match with each other, then these individuals must match with a lower ability partner $a' < \widehat{a}(c_0) - \varepsilon$. But this can only describe a stable matching allocation if the investment choices of agents in this subset are jointly inefficient - otherwise the Envelope Theorem implies they prefer to match with each other than with a lower ability partner. As optimality implies each chooses either $e = \overline{e}$ or 0, then Claim 1 and jointly inefficient investment requires that all in this subset choose $e = 0$ or all choose $e = \overline{e}$. Suppose then that all choose $e = 0$. But this cannot describe an investment equilibrium as by deviating to investment level $e = \overline{e}$, a stable matching allocation implies this individual now matches with a member of this subset and his/her payoff strictly increases (investments are jointly efficient with a more productive partner). Hence for any $\varepsilon > 0$, an investment equilibrium implies agents with abilities $a \in [\widehat{a}(c_0) - \varepsilon, \widehat{a}(c_0))$ must match with each other. Letting $\varepsilon \to 0$ and an induction argument now implies PAM is the unique stable matching allocation in an investment

equilibrium.

We complete the proof by showing that the investment strategies described in the Theorem imply an investment equilibrium. Note first that the investment choices defined in the Theorem are jointly efficient for partners with the same ability (see Claim 1). Given such joint efficiency, the Envelope Theorem and definition of Π implies each pair strictly prefers to match with each other than with a lower ability partner. An induction argument starting at $a = \overline{a}$ now implies PAM is the unique stable matching allocation. Given those investment strategies and PAM, consider now an individual $a \in [0, \overline{a}]$ who makes a deviating investment choice. Given the investment decisions of all the other agents and a stable matching allocation, this individual cannot match with a higher ability partner. Hence a deviating investment choice not only implies the investment is no longer jointly efficient with partner a, a stable matching allocation implies a match with a lower ability partner. Thus a deviating investment is strictly payoff reducing and so the above investment strategies and PAM describe an investment equilibrium. This completes the proof of Theorem 2.∎

Theorem 2 describes an investment equilibrium where education and labour market participation is asymmetric between the two sexes. In contrast to the socially efficient case described in Theorem 1, all men are educated and participate in the labour market with probability one. Only the highest ability women invest in education, those with ability $a \geq \widehat{a}$, but then they only participate with probability $G < 1$ reflecting childcare costs. Lower ability women, $a < \widehat{a}$, do not invest in education. Instead to attract an able partner, they choose to have no education debts (or they accumulate a 'dowry' if education costs are interpreted as foregone earnings) and anticipate focussing their energies on childcare responsibilites (with ex-post labour market participation $G(a)$).

Of course another investment equilibrium exists with the sex roles reversed - all women are educated and participate in the labour market with probability one, while men are involved in childcare. In fact there is a continuum of equilibria: equilibrium implies positive assortative matching on underlying abilities and jointly efficient investment, but at each ability level $a < \widehat{a}$ the sex of the partner who makes the education investment is not determined. It might be argued that if women are slightly better at childcare, or men slightly more productive in the

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workplace, then the outcome described in Theorem 2 is an obvious social norm. Perhaps a more compelling argument is that, as only women can give birth and breastfeed, women historically adopted the childrearing role while men took hunter/farmer roles requiring harder physical labor. It seems plausible that society might maintain that social norm as the economy gradually evolved over time.[6] But an alternative social norm is that of romantic matching - to be considered in the next section - and which might be thought of characterizing many modern Western societies.

Although negative assortative matching is socially optimal, equilibrium matching when individuals marry for money implies partners are positively assorted. A critical ingredient for this result is that utility is not transferable between partners. For example with transferable utility, a less able type might attract a more able type by committing to consume less of the marital pie. Indeed in a labour market context, submodular payoffs (and standard comparative advantage arguments) would imply a sorting equilibrium where low ability workers earn low wages and are employed in high productivity firms, while high ability workers earn high wages and are employed in low productivity firms. In that scenario, wages transfer utility from firms to workers. But there is no price mediation in the marriage market and wages are not paid within marriages. Although assuming utility is perfectly non-transferable in marriages is somewhat strong - one partner could perform more of the domestic chores, another might overindulge in expensive hobbies - it is not an unreasonable approximation. Marital contracts, where singles commit to undertake more than their fair share of domestic chores, do not exist (presumably because of verifiability issues). In the absence of such contracts, an enamoured single may be unable to transfer match utility to his/her object of desire and so may not make the match.

Although PAM is allocationally inefficient, Claim 1 implies the induced investment behaviour is constrained efficient. Indeed, the education rates implied in Theorem 2 are always no lower than those implied in the first best configuration with NAM. Such high education rates correct, at least partly, for the inefficient matching allocation relative to NAM. We shall discuss this issue further in section 5. Next we describe investment equilibria with romantic matching.

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[6]Another story might encompass political economy factors, such as those developed by John Stuart Mill (1869). The explanation of the origin of matching norms is a fascinating topic but beyond the scope of the present paper.

4 Romantic Matching

Assuming large idiosyncratic match values, the matching process as described in Section 2 implies random matching. As all realised matches obtain $\varepsilon = \varepsilon^H$, we again drop reference to ε in what follows. To demonstrate the solution method, we first characterise the (unique) symmetric equilibrium and then characterise asymmetric equilibria.

4.1 Symmetric Investment Equilibria with Romantic Matching

A symmetric investment equilibrium implies men and women of the same ability choose the same levels of education; i.e. $\alpha_m^*(.) = \alpha_w^*(.) = \alpha^*(.)$. Let $F(\alpha', a')$ describe the resulting distribution of attributes (α', a') in the marriage market. Random matching implies the expected value of being single with attributes (α, a) is

$$V(\alpha, a) \equiv \int \Pi(\alpha, \alpha', a, a') dF(\alpha', a'),$$

and equation (1) then implies

$$V(\alpha, a) = \int \left[\alpha^{\max} + \int_0^{\alpha^{\min}} G(x) dx \right] dF(\alpha', a') - c_0[\alpha - a] - c_0 E[\alpha' - a'].$$

where $\alpha^{\max} = \max[\alpha, \alpha']$, $\alpha^{\min} = \min[\alpha, \alpha']$ and the last term describes the expected education debts of a randomly selected partner. Note that $\alpha^{\max}, \alpha^{\min}$ do not depend on a'. Define $\widetilde{F}(\alpha) = F(\alpha, \overline{a})$ which is the probability that a randomly chosen partner has productivity no greater than α. Expanding the above expression implies

$$
\begin{aligned}
V(\alpha, a) &= \int_{\alpha' = \underline{\alpha}}^{\alpha} \left[\alpha + \int_0^{\alpha'} G(x) dx \right] d\widetilde{F}(\alpha') + \int_{\alpha' = \alpha}^{\overline{\alpha}} \left[\alpha' + \int_0^{\alpha} G(x) dx \right] d\widetilde{F}(\alpha') \\
&\quad - c_0[\alpha - a] - c_0 E[\alpha' - a']
\end{aligned}
$$

The first integral describes expected match payoff where the partner drawn has productivity $\alpha' \leq \alpha$ (and so $\alpha^{\max} = \alpha, \alpha^{\min} = \alpha'$) and the second when $\alpha' > \alpha$.

Assume for the moment there are no mass points in $\widetilde{F}$ (which is true in equilibrium).

Differentiating V with respect to α implies

$$\frac{\partial V}{\partial \alpha} = \left[\widetilde{F}(\alpha) + [1 - \widetilde{F}(\alpha)]G(\alpha) \right] - c_0.$$

Note, $\left[\widetilde{F}(\alpha) + [1 - \widetilde{F}(\alpha)]G(\alpha) \right]$ is the worker's expected marginal benefit to education and c_0 is marginal cost. $\left[\widetilde{F}(\alpha) + [1 - \widetilde{F}(\alpha)]G(\alpha) \right]$ is also the worker's expected participation probability; $\widetilde{F}(\alpha)$ is the probability the future partner is less productive in which case our worker participates with probability one, $[1 - \widetilde{F}(\alpha)]$ is the probability the future partner is more productive in which case our worker participates with probability $G(\alpha)$. The marginal benefit to education equals the worker's expected participation probability as the worker only realises a full return to that investment by working in the labour market. As this participation probability is increasing in α (strictly for $\alpha < \overline{\alpha}$) there are increasing returns to education.

Suppose instead $\widetilde{F}$ has a mass point $m > 0$ at α. Assuming partners with equal ability randomise fairly on who goes to work, the expected participation probability of an unmatched single with productivity α is

$$\widetilde{F}(\alpha^-) + m\frac{1 + G(\alpha)}{2} + [1 - \widetilde{F}(\alpha)]G(\alpha).$$

Hence a small increase in productivity across any mass point implies a discrete increase in the worker's participation probability. Fortunately this complication plays no important role. The critical feature is that the worker's participation probability is increasing in α which then implies there are increasing returns to education. In particular, constant marginal cost c_0 and increasing returns imply:

(a) each worker optimally chooses either $e = 0$ or $\overline{e}$;

(b) for each education level e, a more able worker must have a higher marginal benefit to education than a less able worker (as productivity $\alpha = a + e$ is greater) and so invests no less.

Increasing returns now imply a critical ability, denoted a^s, where those with ability $a \geq a^s$ optimally choose $e = \overline{e}$ while those with ability $a < a^s$ optimally choose $e = 0$. Of course a^s is identified where the education choices $e = 0$ or $\overline{e}$ yield the same expected payoff.

Consider then a symmetric equilibrium where men with ability $a > a^s$ choose $e = \overline{e}$ and

men with ability $a < a^s$ choose $e = 0$. This describes a symmetric equilibrium if and only it is then optimal for women with ability $a > a^s$ to invest $e = \overline{e}$ and those with ability $a < a^s$ to invest $e = 0$. Given these male investment strategies, consider a woman with ability $a = a^s$. If she chooses $e = 0$ her expected payoff is:

$$
\begin{aligned}
V_0 &= \int_0^{a^s} \left[a^s + \int_0^a G(x)dx \right] dA(a) + \int_{a^s}^{\overline{a}} \left[a + \overline{e} + \int_0^{a^s} G(x)dx \right] dA(a) \\
&\quad - [1 - A(a^s)]c_0\overline{e}
\end{aligned}
$$

where the first term is her expected income if her future partner has ability $a < a^s$ and therefore chose $e = 0$ (in which case $\alpha^{\max} = a^s$ and $\alpha^{\min} = a$), the second term is her expected income if her future partner has ability $a > a^s$ and therefore chose $e = \overline{e}$ (in which case $\alpha^{\max} = a + \overline{e}$ and $\alpha^{\min} = a^s$) and the last term is the expected education debts of her partner.[7]

If instead she chooses $e = \overline{e}$ her expected payoff is:

$$
\begin{aligned}
V_1 &= \int_0^{a^s} \left[a^s + \overline{e} + \int_0^a G(x)dx \right] dA(a) + \int_{a^s}^{\overline{a}} \left[a + \overline{e} + \int_0^{a^s + \overline{e}} G(x)dx \right] dA(a) \\
&\quad - c_0\overline{e} - [1 - A(a^s)]c_0\overline{e}
\end{aligned}
$$

where this payoff structure is the same as for V_0. She is indifferent between these two actions if and only if $V_1 = V_0$ and solving this condition implies (3) below. Increasing returns to education now imply the following Theorem.

Theorem 3 - Romantic Matching.

For any $c_0 \in (\underline{c}, 1)$, a symmetric investment equilibrium with romantic matching exists, is unique and $a^s \in (0, \overline{a})$ is the solution to

$$
\frac{1}{\overline{e}} \int_{a^s}^{a^s + \overline{e}} [A(a^s) + [1 - A(a^s)]G(x)]dx = c_0. \tag{3}
$$

Proof is in the Appendix.

The interpretation of (3), describing the equilibrium critical ability a^s, is the same as for $\widehat{a}$.

---

[7] We do not need to specify the education choice of men with ability a^s as A has no mass points (by assumption) and so, having zero measure, their education choice cannot affect expected female payoffs.

The integrand in (3), $[A(a^s) + [1 - A(a^s)]G(x)]$, is the expected participation probability of a worker with ability a^s who invests to productivity $x = a^s + e$ as:

(i) $A(a^s)$ is the probability this worker matches with a less able partner (who in equilibrium chooses $e = 0$) and so our worker participates with probability one, and

(ii) $[1 - A(a^s)]$ is the probability this worker matches with a more able partner (who in equilibrium chooses $e = \overline{e}$) and our worker then participates with probability $G(x)$.

Equation (3) says that worker a^s's average participation rate (defined over education levels $e \in [0, \overline{e}]$) equals the marginal cost of education, c_0. Higher ability workers have strictly higher participation probabilities and so strictly prefer $e = \overline{e}$, lower ability workers have strictly lower participation probabilities and so strictly prefer $e = 0$.

Section 5 compares equilibrium investment levels to those implied by Theorem 2 when partners marry for money. Here we complete the analysis by characterising the set of asymmetric investment equilibria where men and women use different investment strategies.

4.2 Asymmetric Investment Equilibria with Romantic Matching

The previous section showed that if men invest according to a^s (i.e. men with $a > a^s$ invest $e = \overline{e}$ and men with $a < a^s$ invest $e = 0$), where a^s satisfies (3), then the collective best response of women is that those with ability greater than a^s prefer $e = \overline{e}$ while lower ability women prefer $e = 0$. We now consider asymmetric equilibria. Suppose now men invest according to some $a^m \in [0, \overline{a}]$ where men with $a > a^m$ invest $e = \overline{e}$ and men with $a < a^m$ invest $e = 0$. We shall show that the collective best response of women implies a critical ability $a^w = a_{BR}^w(a^m)$ where women with ability greater than a^w prefer $e = \overline{e}$ while lower ability women prefer $e = 0$. We shall show that characterising asymmetric equilibria requires identifying these collective best response functions and then solving for (a^m, a^w) with $a^m = a_{BR}^m(a^w)$ and $a^w = a_{BR}^w(a^m)$. Theorem 4 below describes these equilibria.

Given an arbitrary set of female investment strategies $a_w^*(.)$ and corresponding distribution of female productivities, $\widetilde{F}_w$, a male with ability a who invests to productivity α obtains expected

payoff

$$V_m(\alpha, a) = \int_{\underline{\alpha}_w}^{\overline{\alpha}_w} \left[\alpha^{\max} + \int_0^{\alpha^{\min}} G(x) dx \right] d\widetilde{F}_w(\alpha') - c_0[\alpha - a] - c_0 E_w[\alpha' - a'].$$

The argument used in the previous section still applies. Differentiating with respect to α reveals that the marginal benefit to education is the male's expected participation probability. As this participation probability is increasing in α there are increasing returns to education; i.e. $\partial V_m / \partial \alpha$ is increasing in α. Optimality again implies:

(a) each male chooses either $e = \overline{e}$ or 0;

(b) a higher ability male invests no less than a lower ability male.

Thus for any set of female investment strategies (and corresponding $\widetilde{F}_w$) optimality implies a critical male ability, denoted a^m, where men with higher ability prefer $e = \overline{e}$ and men with lower ability prefer $e = 0$. The argument also applies to women; given any set of male investment strategies and corresponding $\widetilde{F}_m$ there is a critical female ability, denoted a^w, where women with higher ability prefer $e = \overline{e}$ and all others prefer $e = 0$. Hence identifying an investment equilibrium reduces to finding a pair $(a^m, a^w) \in [0, \overline{a}] \times [0, \overline{a}]$ where:

(i) given women invest according to a^w, then optimality implies men with ability $a \in [0, a^m)$ prefer $e = 0$, men with ability $a \in (a^m, \overline{a}]$ prefer $e = \overline{e}$;

(ii) given men invest according to a^m, then optimality implies women with ability $a \in [0, a^w)$ prefer $e = 0$, women with ability $a \in (a^w, \overline{a}]$ prefer $e = \overline{e}$.

The appendix now characterises asymmetric romantic equilibria (see the proof of Theorem 4 below) in which $a^m < a^w$; i.e. equilibria where more men are educated than women.[8] The approach uses best response arguments. If women invest according to some $a^w \in [0, \overline{a}]$ then male optimality implies a 'best response', denoted $a^m_{BR}(a^w) \in [0, \overline{a}]$, where men with ability $a < a^m_{BR}(a^w)$ prefer to invest $e = 0$ and more able men with $a > a^m_{BR}(a^w)$ prefer to invest $e = \overline{e}$. As drawn in Figures 1 and 2 below, the proof establishes that for $a^w \in [a^s, \overline{a}]$, the best response function $a^m_{BR}(.)$ exists and is a continuous and decreasing function with $a^m_{BR}(a^w) \in [0, a^s]$. Similarly if men invest according to $a^m \in [0, \overline{a}]$ then female optimality implies a 'best response', denoted $a^w_{BR}(a^m) \in [0, \overline{a}]$, where women with ability $a < a^w_{BR}(a^m)$ prefer to invest $e = 0$ and

---

[8] Of course if such an equilibrium exists, the analysis implies an equivalent equilibrium exists with $a^w < a^m$.

more able women with $a > a_{BR}^w(a^m)$ prefer to invest $e = \overline{e}$. The proof establishes that for $a^m \in [0, a^s]$, $a_{BR}^w(.)$ is also a continuous and decreasing function and implies $a_{BR}^w(a^m) \in [a^s, \overline{a}]$. As depicted in Figures 1 and 2, an investment equilibrium occurs where these best response functions intersect.

Theorem 4 - Romantic Matching.

For $c_0 \in (\underline{c}, 1)$ asymmetric investment equilibria exist if

$$\frac{A'(a^s) \int_{a^s}^{a^s+\overline{e}} [1 - G(x)]dx}{[1 - A(a^s)][G(a^s + \overline{e}) - G(a^s)]} > 1 \tag{4}$$

where a^s is given by (3). Further an interior asymmetric equilibrium, one where $a^m < a^w$ and $a^m, a^w \in (0, \overline{a})$, implies a^m, a^w are solutions to:

$$c_0\overline{e} = A(a^m)\overline{e} + \int_{a^m}^{a^w} [a^m + \overline{e} - a' + \int_{a^m}^{a'} G(x)dx]dA(a') + [1 - A(a^w)] \int_{a^m}^{a^m+\overline{e}} G(x)dx \tag{5}$$

$$c_0\overline{e} = A(a^m)\overline{e} + \int_{a^m}^{a^w} [a^w - a + \int_{a^w}^{a+\overline{e}} G(x)dx]dA(a) + [1 - A(a^w)] \int_{a^w}^{a^w+\overline{e}} G(x)dx. \tag{6}$$

Proof is in the Appendix.

The proof of Theorem 4 allows that in equilibrium, all men might prefer to invest $e = \overline{e}$ and/or all women might prefer to invest $e = 0$; i.e. a^m, a^w might take corner values 0 (all invest) or $\overline{a}$ (none invest). In an interior equilibrium, however, where $a^m, a^w \in (0, \overline{a})$, (5) and (6) are the relevant equations describing the marginal investors when $a^m < a^w$. (5) identifies the critical male a^m who is indifferent to investing $e = 0$ or $e = \overline{e}$ given women invest according to a^w. The first term on the right hand side of (5) is this man's return to education if he matches with a woman with ability $a' < a^m$ [he participates with probability one], the second term describes his return if his future partner has ability $a' \in [a^m, a^w]$ (note, she is more able but has not invested in education) and the last when $a' > a^w$ (she is not only more able but has also invested in education and so is necessarily the more productive partner). This man is indifferent between investing $e = \overline{e}$ and $e = 0$ if that return equals the investment cost $c_0\overline{e}$. Hence (5) identifies the critical male ability a^m in an interior equilibrium (given women invest according to a^w). (6) is the equivalent condition describing a^w given men invest according to

a^m. Note that these two equations are not symmetric (we are not considering a symmetric case as $a^m < a^w$).

(5) is an implicit function which describes the best response function $a^m = a^m_{BR}(a^w)$, while (6) describes $a^w = a^w_{BR}(a^m)$.[9] These best response functions pass through the symmetric equilibrium; i.e. $a^m_{BR}(a^s) = a^w_{BR}(a^s) = a^s$. An important feature however is that both of these best response functions are decreasing functions (see claims 3 and 4 in the appendix). To see why, suppose slightly fewer women choose to invest in education; i.e. consider a small increase in a^w. Given fewer women choose to be educated, the distribution of female *productivities* shifts to the left. For any given male a and education choice e, random matching implies this male is then more likely to be more productive than his future partner. As this raises his expected participation probability, it increases the return to male education and so implies more men invest in education; i.e. (5) implies a^m_{BR} is a decreasing function of a^w.

The same insight implies a^w_{BR} is a decreasing function of a^m; if more men invest in education (a fall in a^m) this implies a fall in female participation rates (women are more likely to be the less productive partner), which implies a fall in the return to education and so fewer women invest (a^w_{BR} increases).

Figure 1 depicts an example where there are no asymmetric equilibria; i.e. it is not necessarily the case that the best response functions intersect at a point $a^m < a^w$. In that case the symmetric equilibrium is the unique investment equilibrium.

Figure 1 here

Figure 2 depicts a case where asymmetric equilibria also exist. (4) in Theorem 4 implies that the slope of a^m_{BR} at the symmetric equilibrium is steeper (in absolute value) than the slope of a^w_{BR}. Using continuity arguments, the proof of Theorem 4 shows that (4) guarantees that an asymmetric equilibrium exists.[10]

Figure 2 here

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[9]Though see the proof of Theorem 4 for details when there is a corner solution.

[10]However it may imply a corner solution with $a^m = 0$ (all men invest) or $a^w = \overline{a}$ (no women invest).

In fact (4) suggests that the symmetric equilibrium is unstable. Establishing this formally requires a dynamic framework which is beyond the scope of this paper. Nevertheless consider the symmetric equilibrium $a^m = a^w = a^s$ and suppose men collectively deviate with slightly more choosing $e = \overline{e}$; i.e. suppose $a^m = a^s - \varepsilon$ with $\varepsilon > 0$ (small). If $A'/(1 - A)$ is large at $a = a^s$, this deviation implies a relatively large increase in the *number* of educated men. Random matching then implies a relatively large fall in the expected participation rates of women, and the corresponding fall in the expected return to education implies fewer women choose to invest in education. If fewer women choose $e = \overline{e}$; i.e. if $a^w = a^s + \varepsilon'$ where $\varepsilon' > 0$ (small), then $A'/(1 - A)$ large implies a relatively large decrease in the *number* of educated women. This implies a relatively large increase in the expected participation rates of men and so more men choose to invest in education. (4) suggests the symmetric equilibrium is unstable for if slightly more men choose $e = \overline{e}$ and slightly more women choose $e = 0$, then endogenous labour market participation decisions imply an even higher return to education for men and an even lower one for women.

5   Discussion and Simulations

This section compares the investment outcomes implied by Theorems 1-4. There are two main features. When the return to education is low, romantic investment equilibria can result in particularly low education rates. The difficulty for romantic investment equilibria is that future partners cannot co-ordinate their investment plans. The converse applies when people marry for money. In that case investment incentives are efficient but the resulting allocation, PAM, is inefficient. By raising average productivity in the workplace, our simulations find that romantic matching is more efficient than PAM and marrying for money.

Although many of the results described in the following two subsections (and summarized in Table 1) can be obtained analytically, we use numerical simulations to develop the appropriate insights. We consider two examples, characterized by low and high returns to education respectively. Initially assume the costs of investing in education are $c_0 = 0.95$ and thus, conditional on participation, the rate of return is $\rho = [\overline{e} - c_0\overline{e}]/[c_0\overline{e}] = 5\%$. This represents the *low returns to*

investment example discussed below. Only those who are relatively certain about participating in the labour market (probability exceeding 95%) will invest in education in this situation. For the *high returns to education* example, assume $c_0 = 3/4$, which instead implies rate of return $\rho = 33\%$. Only those who anticipate participating in the labour market with probability more than $3/4$ will make this investment.[11]

Assume the distributions of ability A and of private childcare costs G are uniform, and normalise $\overline{a} = 1$. We set $\underline{x} = 0$ and $\overline{x} = \overline{a} + \overline{e}$. The former restriction implies that the lowest ability worker with no education never works in the labour market, while the latter implies that the highest ability worker who chooses $e = \overline{e}$ always participates. We choose $\overline{e} = 1/2$ as it implies $a^s = 0.5$ in the symmetric romantic equilibrium with $c_0 = 3/4$; that is, half the population invests in education when $\rho = 33\%$. It is simple to show that, in this equilibrium, an above-median ability worker (who invests in education) has participation probability exceeding $5/6$ while a below-median ability worker (who does not invest) has participation probability below $2/3$. Note that participation rates are not continuous with ability: those who invest in education have much higher participation rates (above c_0).

5.1   The Low Returns Example, $\rho = 5\%$.

Recall that $\widehat{a}(c_0)$ is the critical ability at which investment by the less able partner becomes optimal. The above parameter values and $c_0 = 0.95$ imply $\widehat{a} > \overline{a}$.[12] Claim 1 implies it is ex-post jointly efficient that only one partner - the more able one - should invest in education. The following describes the actual investment outcomes as implied by Theorems 1-4.

(i) With no idiosyncratic match values, the first best with NAM implies men and women with above median ability ($a \geq 0.5$) choose $e = \overline{e}$ and participate with probability one, while all others choose $e = 0$ and participate with probability $G(a)$.

(ii) With no idiosyncratic match values, PAM as described in Theorem 2 implies all men get educated and participate with probability one, while no women get educated and only

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[11] This rate of return may at first seem very high but this is not so. For example, suppose a 4 year degree scheme generates a 15% increase in wages. If the cost of education is simply foregone earnings, then a working lifespan of 44 years implies $\rho = [40 \times 0.15 \times a - 4a]/[4a] = 50\%$, which is higher than in our example in the text.

[12] In general, $\widehat{a} > \overline{a}$ always occurs for c_0 large enough.

particpate with probability $G(a)$.

(iii) The symmetric romantic equilibrium implies $a^s = 0.18$; i.e. the top 18% of both sexes get educated.

(iv) The symmetric equilibrium is unstable ((4) holds) and an asymmetric equilibrium exists with $a^m = 0.68$, $a^w = 0.98$; i.e. the top 32% of men are educated as are the top 2% of women.

Investment efficiency implies it is optimal that only one partner, the more able one, invests in education. NAM and PAM (marrying for money) yield investments which are perfectly efficient. The NAM outcome describes a perfect meritocracy where the most able men and women are educated and participate in the workforce with probability one. This is summarized in Row [1] of Table 1. Equilibrium when singles marry for money also implies efficient investment, but the sex roles are perfectly differentiated: men are educated and participate in the labour market with probability one, while women are not educated and have low participation rates.[13] This is summarized in Row [2] of Table 1. Women do not choose to be educated in this equilibrium as they know their future partner will be educated, and the cost of education is too high to make it worthwhile for both to invest in education. Indeed by avoiding high education costs, each woman is able to attract a more productive male partner (a dowry effect).

When the return to education is small, the symmetric romantic investment equilibrium implies low education rates - in this case only the top 18% are educated (as opposed to education rates of 50% with NAM and PAM). It is straightforward to show in general that $a^s \to \overline{a}$ as $c_0 \to 1$; education rates collapse with romantic matching as the return to education becomes small. In contrast, PAM implies the education rates hold firm at 50% as $c_0 \to 1$. The difficulty with romantic matching is that partners cannot co-ordinate their investment decisions. When the return to education is small, the downside loss through both partners being educated is high. Hence education rates become small as the return to education becomes small. This is summarized in Row [3] of Table 1.

For these parameter values the symmetric equilibrium is unstable. An asymmetric equilibrium exists where the top 32% of men invest in education, as do the top 2% of women.

---

[13] In our discusion, for expositional ease we refer to the female as bearing the costs of childcare, but as noted in our introduction there is no need for this to be the case. All of our analysis can proceed with the gender roles reversed.

Note there are two efficiency advantages of the asymmetric equilibrium. The first is that it reduces the number of inefficient marriages where both partners are educated. In particular, the symmetric equilibrium implies both partners are educated in 3.2% of all matches, while this figure falls to 0.6% in the asymmetric equilibrium.From the educated male's perspective, the asymmetric equilibrium implies the probability his wife is also educated drops from 0.18 to 0.02.

The second efficiency advantage of the asymmetric equilibrium is that it reduces the number of matches where neither partner is educated (though this effect is small - a fall from 67.2% to 66.4%). Somewhat surprisingly it achieves this with a slightly lower average education rate of 17%, compared to 18% in the symmetric equilibrium. These predictions are summarized in Row [4] of Table 1. However it should be noted there is an efficiency loss associated with the asymmetric equilibrium. Matches occur where the woman is the more able partner but it is the man who has invested in education and participates in the workplace. This inefficiency does not occur in the symmetric equilibrium.

There are significant welfare differences between these equilibria. Not surprisingly, PAM leads to the greatest inequality as the most productive only match with each other. It is also the least efficient allocation as payoffs are submodular in productivities, and the value of high ability women's market work is lost to society. Comparing romantic equilibria, lower ability women prefer the asymmetric equilibrium. In particular, women below the top 18% do not invest in education and prefer that their future male partners invest in education and so earn a good wage. As the asymmetric equilibrium implies higher male education rates, low ability women are therefore better off in the asymmetric equilibrium. In contrast, high ability women prefer the symmetric equilibrium. A woman in the top 2% invests in education and, as $\hat{a} > \bar{a}$, high male investment rates reduces her expected payoff as she then faces the two debt problem. The converse holds for men - high ability men prefer the asymmetric equilibrium (low female education rates) and low ability men prefer the symmetric equilibrium (or even better, the asymmetric equilibrium where the top 32% of women invest in education).

Thus the matching regimes represent tradeoffs between investment and allocative inefficiency. PAM yields higher education rates and educational investment is perfectly co-ordinated

across partners. In contrast, romantic matching yields low education rates and investments that may be ex-post inefficient (for example, both partners may be uneducated, or both are educated but one has ability $a < \widehat{a}$). However romantic matching yields a more efficient allocation. Indeed, the average payoff $\overline{\Pi}$ for these parameter values is highest in the asymmetric romantic equilibrium, which is slightly higher than in the symmetric equilibrium but significantly higher than PAM. Romantic matching increases allocative efficiency, as more high ability women participate in the workplace, raising average workplace productivity. Thus with low returns to education, the allocative value of romance dominates the investment efficiency of PAM.

5.2 The High Returns Example, $\rho = 33\%$

Now suppose that all parameter values remain the same except $c_0 = 0.75$. The critical ability at which investment by the less able partner becomes optimal now becomes $\widehat{a} = 0.875 < \overline{a}$. Thus the following outcomes are implied by Theorems 1-4.

(i) With no idiosyncratic match values, the first best with NAM implies men and women with above median ability choose $e = \overline{e}$, while all others choose $e = 0$.

(ii) With no idiosyncratic match values, PAM as described in Theorem 2 implies all men get educated and participate with probability one, while women in the top 12.5% get educated.

(iii) The symmetric romantic equilibrium implies $a^s = 0.50$; i.e. the top 50% of both sexes get educated.

(iv) The symmetric equilibrium is unstable and an asymmetric equilibrium exists with $a^m = 0.42$, $a^w = 0.58$; i.e. the top 58% of men are educated as are the top 42% of women.

The much higher return to education does not affect the first best outcome with NAM - only the top 50% are educated. But PAM now yields a slight increase in the overall education rate - it has risen to 56% and remains highly skewed towards men. In the symmetric romantic equilibrium, the average education rate is a comparable 50%. The symmetric equilibrium is unstable and an asymmetric equilibrium exists where 58% of men are educated as are 42% of women.

As pointed out in the previous subsection, the romantic equilibrium implies two types of ex-post investment inefficiency: both partners may be uneducated, or both may be educated

with at least one having $a < \widehat{a} = 0.875$. The asymmetric equilibrium again reduces both of these inefficiencies. The number of uneducated couples drops (a fall from 25% to 24.4%), as does the number of partnerships where both are educated but education for one partner is ex-post inefficient (a fall from 23.4% to 22.8%). However the reductions are clearly small. Also note that, with high returns, the skewness towards male education is smaller. A low return to education implied education rates of 32% (male) and 2% (female), while the high return example implies 58% (male) and 42% (female). This suggests that male and female education rates and participation rates converge as the return to education increases. Indeed it can be shown that education rates converge to one for both men and women as $c_0 \to \underline{c}$ (recall that $\underline{c}$ is the trivial case where returns to education are so high that it pays to educate everyone).

The same welfare implications apply with high returns to education as in the low returns example. PAM yields much greater inequality between couples. With romantic matching, high ability women prefer the symmetric equilibrium while low ability women prefer the asymmetric equilibrium. The converse holds for men. The ranking in terms of average payoff is also unchanged. Although PAM leads to co-ordinated investment strategies and higher investment rates, it is allocatively inefficient. Romantic matching implies more high ability women participate in the workplace, which increases average productivity. For our chosen parameter values, the asymmetric romantic equilibrium is (very) slightly more productive than the symmetric equilibrium, which is significantly more productive than PAM. The allocative value of romance again generates the higher average payoff.

6 Conclusion

In this paper we demonstrated how customs or matching regimes affect the educational investment decisions of young singles and the subsequent joint labor supply decisions of partnered couples. We showed that labor market participation choices endogenously generate increasing returns to education and so imply a co-ordination problem. It is not optimal for each partner to invest in an intermediate level of education. As education is expensive, it may also not be optimal that they both invest in a full education, particularly if one expects to be raising

children. When making an investment choice, a single not only has to anticipate the education decision of his/her future partner but also to take into account how it affects his/her marriage prospects.

In the context of the model developed, it is shown that when people marry for money, equilibrium implies perfect positive assortative matching - men and women of the same ability match with each other. Equilibrium also implies that the investment strategies are efficient; that is, conditional on a partnership, the education choices are jointly efficient. Those investments, however, need not be symmetric across the sexes. For example, when the return to education is small, an equilibrium exists where all men invest in education and participate with probability one, while no women invest in education and are likely to spend their time childrearing. In this equilibrium, a woman maximizes her marital prospects by avoiding debt (or builds up a dowry if education costs are interpreted as foregone earnings). If the return to education is instead high, then the highest ability women also invest in education and, as positive assortative matching implies her partner will also be a high flyer, she expects to pay for private childcare.

Romantic matching, where the matching allocation is instead orthogonal to real economic variables, leads to quite different investment behaviour. It is shown that a symmetric equilibrium always exists, where the higher ability types of both sexes invest in education. But unlike the case when partners marry for money and education rates are at least 50% (because all men are always educated), education rates with romantic matching may be very low. In particular, when the return to education is low it is very costly for both partners to be educated - they have to pay for two education debts while trying to raise a family. By co-ordinating the investment plans of singles, the 'marrying for money' equilibrium avoids the two-debt problem. There is no such co-ordination with symmetric romantic matching and a low return to education then implies low education rates as singles avoid the two-debts problem.

We showed that equilibria with romantic matching may be asymmetric (more men are educated than women and have higher labour market particpation rates), and may be more efficient than those where partners marry for money. Although investment is efficient in the marrying-for-money case, the matching allocation implies there are many high ability women who do not participate in the labour market, which represents a loss to society. Romantic matching, by

generating a different allocation of partners, increases average workplace productivity and thus improves allocative efficiency.[14]

A more ambitious approach might consider when such different matching regimes arise. Modeling endogenous social institutions is an important research area - see Mailath and Postlewaite (2004) for example. A plausible scenario here might consider the role of capital market imperfections. Suppose children cannot finance their own education and it is instead paid for by their parents. To ensure efficient investment incentives, a social norm might arise which gives parents the implicit right to arrange their offspring's marriage. If parents can "punish" a child who makes an unsuitable match, say by withdrawal of financial support, and if parents do not value their child's romantic leanings, society might settle on a 'marry-for-money' equilibrium. Theorem 2 then suggests parents invest in an education (or professional training) for their sons, dowries for their daughters and the resulting match allocation is strongly hierarchical. Indeed, John Stuart Mill seems to be describing such a marriage market in 19th century Europe:

- 'Marriage being the destination appointed by society for women, the prospect they are brought up to, and the object which it is intended should be sought by all of them,...one might have supposed that everything would have been done to make this condition as eligible to them as possible, that they might have no cause to regret being denied the option of any other. Society...has preferred to attain its object by foul rather than fair means... Until a late period in European history, the father had the power to dispose of his daughter in marriage at his own will and pleasure, without any regard to hers.' JS Mill (1869), *The Subjection of Women,* p29.

If education is instead provided by the state, or capital markets are perfect and young adults finance their own college education, children become more independent from their parents and the social norm might eventually change to allow young adults a completely free choice in the marriage market. Matching may then be more romantic - and potentially more efficient.

---

[14]Various sorting regimes may have important effects on inequality, as noted in Fernandez and Rogerson (2001) and Fernandez *et al* (2005) *inter alia.*

7 Appendix

Proof of Theorem 3.

By construction (3) describes the ability a^s where a single with ability $a = a^s$ is indifferent to choosing $e = 0$ or $\overline{e}$ given singles of the opposite sex invest according to a^s. Increasing returns to education implies types with $a > a^s$ prefer $e = \overline{e}$ while those with $a < a^s$ prefer $e = 0$. Hence if a^s in (3) is well defined, this identifies a symmetric investment equilibrium. As $A(.), G(.)$ are continuous functions, the left hand side of (3) is a continuous function of a^s. Inspection establishes that the left hand side of (3) is also strictly increasing in a^s for all $a^s \in [0, \overline{a}]$. Further the left hand side equals $\underline{c}$ when $a^s = 0$ and equals one when $a^s = \overline{a}$. Hence for any $c_0 \in (\underline{c}, 1)$ a symmetric equilibrium exists, is unique and implies $a^s \in (0, \overline{a})$.

Proof of Theorem 4.

We prove this Theorem in 3 steps. We first formally define a best response function. Step 1 then identifies the best response function for men and describes its properties. Step 2 repeats the analysis for women and step 3 establishes the Theorem.

Suppose women invest according to $a^w \in [0, \overline{a}]$ and consider a male with ability $a \in [0, \overline{a}]$. Let $V_0(a, a^w)$ denote this male's expected payoff by investing $e = 0$ and $V_1(a, a^w)$ denote his expected payoff by investing $e = \overline{e}$. Let

$$S_m(a, a^w) = V_1(a, a^w) - V_0(a, a^w)$$

denote his expected surplus by investing in education. Strictly increasing returns to education implies S_m is strictly increasing in a. The male best response function, $a_{BR}^m(a^w)$, is now defined as:

$$
\begin{aligned}
a_{BR}^m(a^w) &= 0 \text{ if } S_m(a, a^w) \geq 0 \text{ for all } a \in [0, \overline{a}] \\
a_{BR}^m(a^w) &= a^m \text{ if } S_m(a^m, a^w) = 0 \text{ for some } a^m \in (0, \overline{a}) \\
a_{BR}^m(a^w) &= \overline{a} \text{ if } S_m(a, a^w) \leq 0 \text{ for all } a \in [0, \overline{a}].
\end{aligned}
$$

Note this definition implies $a_{BR}^m(a^w) \in [0, \overline{a}]$ for all $a^w \in [0, \overline{a}]$ and increasing returns implies all

men with ability $a > a_{BR}^m(a^w)$ prefer $e = \overline{e}$ while those with ability $a < a_{BR}^m(a^w)$ prefer $e = 0$. The female best response function is defined in the same way.

Step 1. Characterising the male best response function.

Fix an $a^w \in [0, \overline{a}]$ and consider a male with ability $a \in [0, \overline{a}]$. There are three cases. First suppose a satisfies $a \in (a^w - \overline{e}, a^w]$. If this male chooses $e = 0$ his expected payoff is

$$
\begin{aligned}
V_0 &= \int_{\underline{a}}^a \left[a + \int_0^{a'} G(x)dx \right] dA(a') + \int_a^{a^w} \left[a' + \int_0^a G(x)dx \right] dA(a') \\
&+ \int_{a^w}^{\overline{a}} \left[a' + \overline{e} + \int_0^a G(x)dx \right] dA(a') - c_0 E_w[\alpha' - a'].
\end{aligned}
$$

The first term describes his expected payoff if he matches with a woman with ability $a' < a \leq a^w$ (who is uneducated and so $\alpha^{\max} = a$, $\alpha^{\min} = a'$) the second if he matches with a woman $a' \in (a, a^w)$ (who is uneducated and so $\alpha^{\max} = a'$, $\alpha^{\min} = a$) the third if he matches with a woman $a' \geq a^w$ (who is educated and so $\alpha^{\max} = a' + \overline{e}$, $\alpha^{\min} = a$). The final term is the expected sunk education cost of a randomly chosen woman from distribution $\widetilde{F}_w$.

If instead this male chooses $e = \overline{e}$, his expected payoff is

$$
\begin{aligned}
V_1 &= \int_{\underline{a}}^{a^w} \left[a + \overline{e} + \int_0^{a'} G(x)dx \right] dA(a') + \int_{a^w}^{\overline{a}} \left[a' + \overline{e} + \int_0^{a+\overline{e}} G(x)dx \right] dA(a') \\
&- c_0 \overline{e} - c_0 E_w[\alpha' - a'],
\end{aligned}
$$

where the structure of payoffs is the same but our male now has productivity $a + \overline{e} > a^w$. Straightforward algebra establishes that $S_m(a, a^w) \equiv V_1 - V_0$ is given by

$$
\begin{aligned}
S_m(a, a^w) &= A(a)\overline{e} + \int_a^{a^w} [a + \overline{e} - a' + \int_a^{a'} G(x)dx] dA(a') \\
&+ [1 - A(a^w)] \int_a^{a+\overline{e}} G(x)dx - c_0 \overline{e}
\end{aligned}
\tag{7}
$$

for male abilities $a \in (a^w - \overline{e}, a^w]$. Differentiation then establishes that S_m is strictly increasing in a and a^w in this region. As depicted in figure A below, any contour $S_m(a, a^w) = k$ in the region $a \in (a^w - \overline{e}, a^w]$ is continuous and strictly downward sloping.

Now consider the second case $a \leq a^w - \overline{e}$. V_0 has the same functional form as above, but

now

$$V_1 = \int_{\underline{a}}^{a+\overline{e}} \left[a + \overline{e} + \int_0^{a'} G(x)dx \right] dA(a') + \int_{a+\overline{e}}^{a^w} \left[a' + \int_0^{a+\overline{e}} G(x)dx \right] dA(a')$$

$$+ \int_{a^w}^{\overline{a}} \left[a' + \overline{e} + \int_0^{a+\overline{e}} G(x)dx \right] dA(a') - c_0\overline{e} - c_0 E_w[\alpha' - a']$$

which is different as $a + \overline{e} \leq a^w$. Hence $S_m(a, a^w) \equiv V_1 - V_0$ is given by

$$S_m(a, a^w) = A(a)\overline{e} + \int_a^{a+\overline{e}} [a + \overline{e} - a' + \int_a^{a'} G(x)dx]dA(a')$$

$$+ [1 - A(a + \overline{e})] \int_a^{a+\overline{e}} G(x)dx - c_0\overline{e} \qquad (8)$$

for male abilities $a \leq a^w - \overline{e}$. Differentiation establishes that S_m is strictly increasing in a in this region but $\partial S_m/\partial a^w = 0$. Although the functional form for S_m changes at $a = a^w - \overline{e}$, it is easy to show that it is continuously differentiable across the join.

Given we are only characterising equilibrium with $a^m < a^w$ we do not need to describe S_m for $a > a^w$. Reflecting increasing returns to education, we need only note that S_m is strictly increasing in a for $a \in [a^w, \overline{a}]$. Claim 2 describes the essential properties of S_m for what follows.

Claim 2. S_m is continuously differentiable in a, a^w for all $a \in [0, a^w]$ and $a^w \in [0, \overline{a}]$. Further

(i) $S_m(a^s, a^s) = 0$;

(ii) S_m is strictly increasing in a for all $a \in [0, \overline{a}]$;

(iii) S_m is strictly increasing in a^w for $a \in (a^w - \overline{e}, a^w)$ and does not depend on a^w for $a \leq a^w - \overline{e}$.

Proof: (i) follows from (7) above with $a = a^w = a^s$ and Theorem 3 where a^s is given by (3). The rest of the claim follows from the previous discussion.

Figure A here.

Figure A is a contour plot $S_m(a, a^w) = k$ where S_m has properties as described in Claim 2. In the region $a \in [a^w - \overline{e}, a^w]$, S_m is given by (7) and the Implicit Function Theorem implies the contours are continuous and downward sloping. In the region $a < a^w - \overline{e}$, S_m is given by (8) and the contours are horizontal lines (as S_m does not vary with a^w). The contour of interest

is $S_m = 0$ which passes through the symmetric equilibrium (a^s, a^s), and $c_0 \in (\underline{c}, 1)$ implies $a^s \in (0, \overline{a})$. It is now straightforward to characterise the best response function $a^m_{BR}(.)$.

Claim 3. For $a^w \in [a^s, \overline{a}]$, $a^m_{BR}(a^w)$ exists and is a continuous, decreasing function satisfying $a^m_{BR}(a^w) \in [0, a^s]$.

Proof. Consider Figure A. The best response function a^m_{BR} passes through the symmetric equilibrium (a^s, a^s) and while $a^m_{BR}(a^w) \in (0, \overline{a})$ it coincides with the contour $S_m = 0$. Hence for $a^w \in [a^s, \overline{a}]$, and while $a^m_{BR}(a^w) > 0$, Claim 2 and the Implicit Function Theorem imply a^m_{BR} is a continuous and decreasing function. Should this contour imply $a^m_{BR}(a^w) = 0$ for some $a^w < \overline{a}$, then Claim 2 and the definition of a^m_{BR} implies $a^m_{BR} = 0$ for all higher values of a^w (as $S_m(0, a^w) \geq 0$ for such a^w and S_m is strictly increasing in a). This completes the proof of Claim 3.

Step 2. As the argument is the same, we quickly describe the female best response function $a^w = a^w_{BR}(a^m)$ (as described in claim 4 below).

Fix $a^m \in [0, \overline{a}]$ and consider a woman with ability $a' \in [a^m, a^m + \overline{e})$. Constructing payoffs V_0 and V_1 as before, algebra establishes that her surplus by choosing $e = \overline{e}$ is

$$
\begin{aligned}
S_w(a', a^m) &= A(a^m)\overline{e} + \int_{a^m}^{a'} [a' - a + \int_{a'}^{a+\overline{e}} G(x)dx]dA(a) \\
&\quad + [1 - A(a')] \int_{a'}^{a'+\overline{e}} G(x)dx - c_0\overline{e}
\end{aligned}
\tag{9}
$$

for $a' \in [a^m, a^m + \overline{e})$. As before, note that $S_w(a^s, a^s) = 0$; i.e. the female best response function passes through the symmetric equilibrium. It also follows that S_w is strictly increasing in own ability a' (there are increasing returns) and strictly increasing in a^m in this region. Hence any contour $S_w(a', a^m) = k$ in the region $a' \in [a^m, a^m + \overline{e})$ is downward sloping.

For $a' \geq a^m + \overline{e}$, S_w is instead given by

$$
\begin{aligned}
S_w(a', a^m) &= A(a' - \overline{e})\overline{e} + \int_{a'-\overline{e}}^{a'} [a' - a + \int_{a'}^{a+\overline{e}} G(x)dx]dA(a) \\
&\quad + [1 - A(a')] \int_{a'}^{a'+\overline{e}} G(x)dx - c_0\overline{e}
\end{aligned}
\tag{10}
$$

which is independent of a^m. Although S_w changes functional form at $a' = a^m + \overline{e}$, it is continuously differentiable across this join. These properties of S_w now imply Claim 4.

Claim 4. For $a^m \in [0, a^s]$, $a^w_{BR}(a^m)$ exists and is a continuous, decreasing function satisfying $a^m_{BR}(a^w) \in [a^s, \overline{a}]$.

Proof. The best response function a^w_{BR} passes through the symmetric equilibrium (a^s, a^s) and while $a^w_{BR}(a^m) \in (0, \overline{a})$ it coincides with the contour $S_w = 0$. A contour plot of S_w makes it clear that for $a^m \in [0, a^s]$ and while $a^w_{BR}(a^m) \in (0, \overline{a})$, a^w_{BR} is a continuous and decreasing function. Should this contour imply $a^m_{BR}(a^w) = \overline{a}$ for some $a^m \in (0, a^s)$, then the definition of a^w_{BR} implies $a^w_{BR} = \overline{a}$ for all lower values of a^m (as $S_w(\overline{a}, a^m) \leq 0$ for such a^m and S_w is strictly increasing in a'). This completes the proof of Claim 4.

Step 3. Existence of Asymmetric Equilibria.

Given the information contained in Claims 3 and 4, Figure 1 in the text depicts a case where the best response functions do not intersect at a point $a^m < a^w$ and so the symmetric equilibrium is the unique investment equilibrium. As depicted in Figure 2, a sufficient condition for the existence of an asymmetric equilibrium is that a^m_{BR} must be steeper in absolute value than a^w_{BR} at (a^s, a^s). Continuity arguments based on Claims 3 and 4 then imply the two best response functions must intersect at some value $(a^m, a^w) \in [0, a^s) \times (a^s, \overline{a}]$.

Now $a^m_{BR}(.)$ is described by $S_m(a^m_{BR}, a^w) = 0$ at $a^w = a^s$, with S_m given by (7). Hence the Implicit Function Theorem implies the slope of a^m_{BR} at (a^s, a^s) is (in absolute value)

$$slope\ a^m_{BR} = \frac{A'(a^s) \int_{a^s}^{a^s + \overline{e}} [1 - G(x)] dx}{[1 - A(a^s)][G(a^s + \overline{e}) - G(a^s)]}.$$

At (a^s, a^s) the slope of a^w_{BR} is the inverse of the slope of a^m_{BR} and so a^m_{BR} is steeper than a^w_{BR} if and only if (4) holds. Thus if (4) holds an asymmetric equilibrium with $a^m < a^s < a^w$ exists.

To complete the proof of Theorem 4, we need to show that in any interior equilibrium with $a^m < a^w$, then a^m, a^w satisfy (5) and (6). To establish this we show that an interior equilibrium with $a^m < a^w$ implies $a^m \in (a^w - \overline{e}, a^w]$; i.e. it must lie in the region where the contours are strictly downward sloping. This establishes the Theorem as an interior equilibrium requires a^m, a^w satisfy $S_m(a^m, a^w) = S_w(a^m, a^w) = 0$ and the restriction $a^m \in (a^w - \overline{e}, a^w]$ implies S_m

is given by (7) and S_w is given by (9). The conditions $S_m(a^m, a^w) = S_w(a^m, a^w) = 0$ and (7),(9) then imply equations (5),(6) as stated in the Theorem.

We show that an interior equilibrium with $a^m < a^w$ implies $a^m \in (a^w - \overline{e}, a^w)$ using a contradiction argument. Suppose not and so $a^m \leq a^w - \overline{e}$. An interior equilibrium implies $S_m(a^m, a^w) = S_w(a^w, a^m) = 0$ and $a^m \leq a^w - \overline{e}$ implies S_m is given by (8) and S_w is given by (10). Thus we have the necessary conditions:

$$c_0 \overline{e} = A(a^m)\overline{e} + \int_{a^m}^{a^m + \overline{e}} [a^m + \overline{e} - a' + \int_{a^m}^{a'} G(x)dx]dA(a') + [1 - A(a^m + \overline{e})] \int_{a^m}^{a^m + \overline{e}} G(x)dx \quad (11)$$

$$c_0 \overline{e} = A(a^w - \overline{e})\overline{e} + \int_{a^w - \overline{e}}^{a^w} [a^w - a + \int_{a^w}^{a + \overline{e}} G(x)dx]dA(a) + [1 - A(a^w)] \int_{a^w}^{a^w + \overline{e}} G(x)dx \quad (12)$$

describing a^m, a^w. Let $\widetilde{a} = a^w - \overline{e}$ and substitute out a^w in (12) to get

$$c_0 \overline{e} = A(\widetilde{a})\overline{e} + \int_{\widetilde{a}}^{\widetilde{a} + \overline{e}} [\widetilde{a} + \overline{e} - a' + \int_{\widetilde{a} + \overline{e}}^{a' + \overline{e}} G(x)dx]dA(a') + [1 - A(\widetilde{a} + \overline{e})] \int_{\widetilde{a} + \overline{e}}^{\widetilde{a} + 2\overline{e}} G(x)dx \quad (13)$$

Note that $a^m \leq a^w - \overline{e}$ implies $\widetilde{a} \geq a^m$.

(11),(13) now imply our contradiction. Differentiation establishes that the right hand side of (11) is increasing in a^m and that the right hand side of (13) is increasing in $\widetilde{a}$ (there are increasing returns). Further putting $\widetilde{a} = a^m$ implies the right hand side of (13) is strictly greater than the right hand side of (11) (as $G(x)$ is a strictly increasing function for $x \in [0, \overline{a} + \overline{e}]$ and $a^m < \overline{a}$ in an interior equilibrium). As both right hand sides must equal $c_0 \overline{e}$, this implies $\widetilde{a} < a^m$ which is the required contradiction. This completes the proof of Theorem 4.

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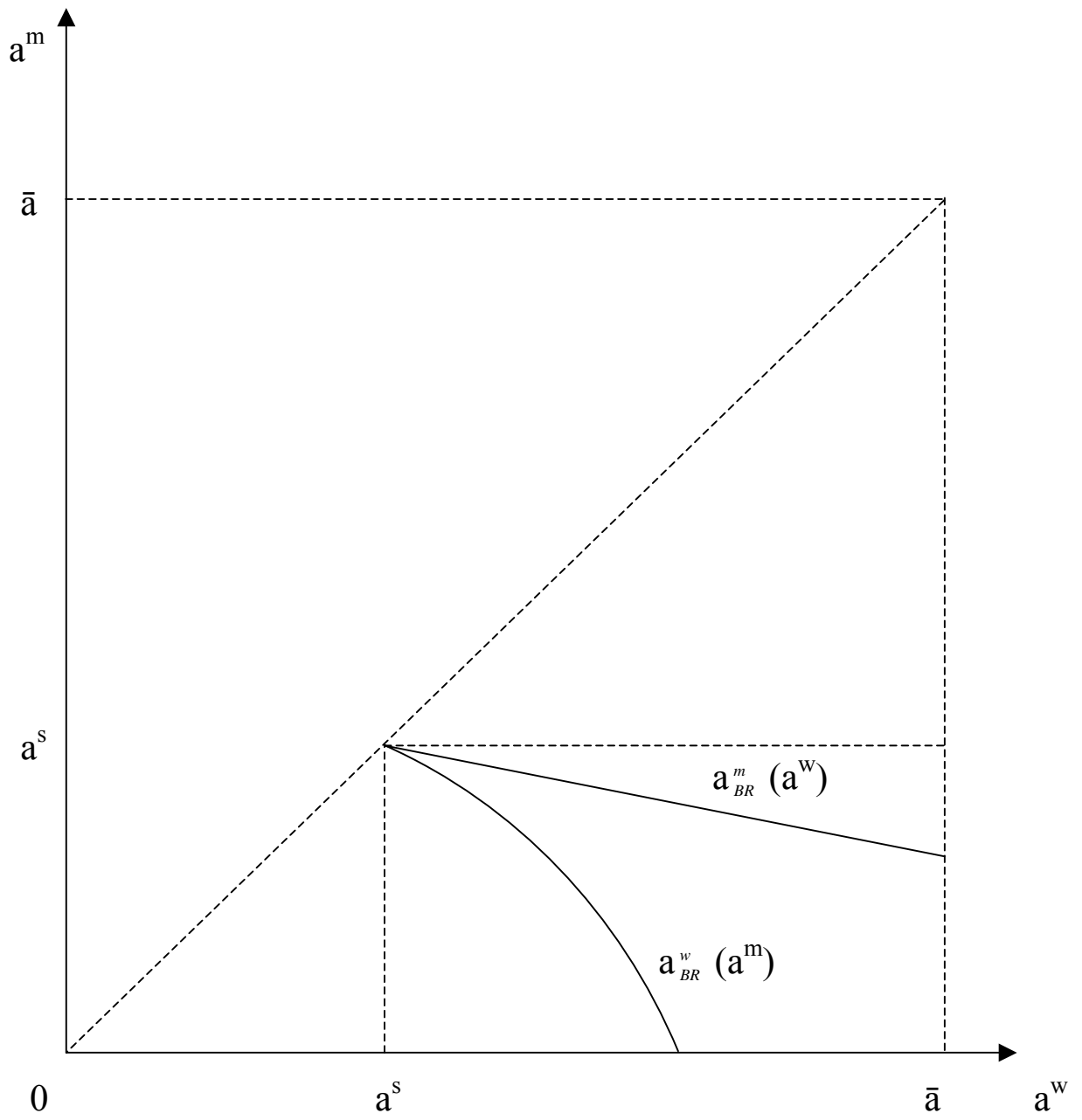
Figure 1: Unique Investment Equilibrium

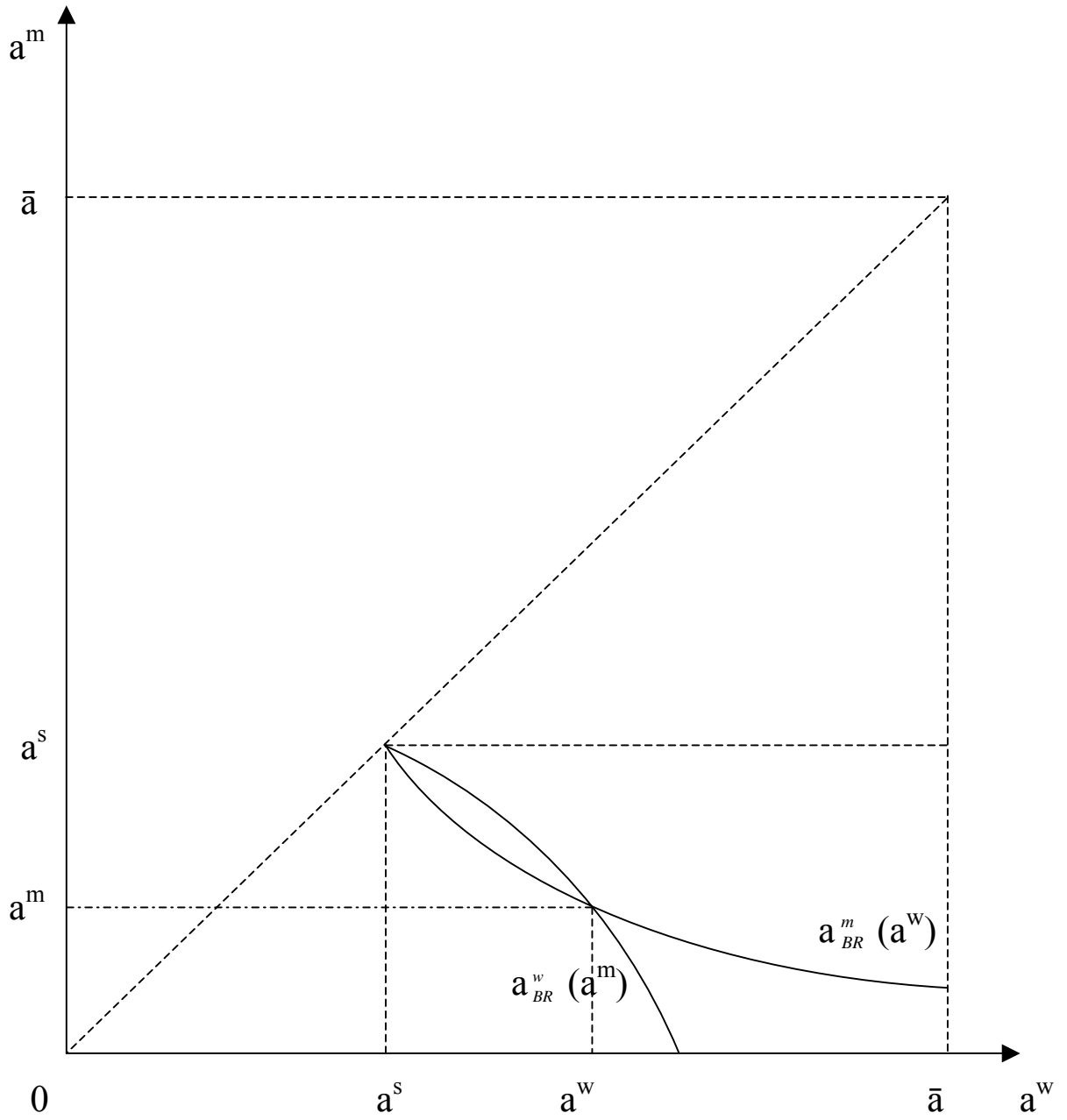# Figure 2: Existence of Asymmetric Equilibria

Figure A: Contour Plot $S_m(.)$