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Economic Growth with Bubbles

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Comments are welcome

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Abstract: This paper presents a stylized model of economic growth with bubbles. This model views asset price bubbles as a market-generated device to moderate the effects of frictions in financial markets, improving the allocation of investments and raising the capital stock and welfare. The model illustrates various channels through which asset price bubbles affect the incentives for innovation and economic reforms, and therefore, the rate of economic growth. The model also offers a new perspective on the effects of financial development on asset price bubbles and economic growth.

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Modern economies often experience large movements in asset prices that cannot be explained by changes in economic conditions or fundamentals. It is commonplace to refer to these episodes as asset price bubbles popping up and bursting. Typically, these bubbles are unpredictable and generate substantial macroeconomic effects. Consumption, investment and productivity growth all tend to surge when a bubble pops up, and then collapse or stagnate when the bubble bursts. Here, I address the following questions: What is the origin of these bubbles? Why are bubble episodes unpredictable? How do bubbles affect consumption, investment and productivity growth? In a nutshell, the goal of this paper is to develop a stylized view or model of economic growth with bubbles.[1]

The theory developed here views an asset price as the sum of two components: fundamental and bubble. The fundamental is the net present value of all future dividends, and its size depends on how productive and well managed is the asset. The bubble is a pyramid scheme, and its size depends on the market's expectation of its future size. Throughout most of the paper, I assume possible to strip any asset into these two components. As a result, any asset market can be thought of as the sum of two assets classes: productive assets or "capital" whose price is equal to the fundamental component; and unproductive assets or "bubbles" whose price equals the bubble component. I consider environments with rational, informed and risk neutral investors that follow the simple portfolio rule of holding only those assets that offer the highest expected return. The theoretical challenge is to identify situations in which these investors optimally choose to hold bubbles in their portfolios and then characterize the macroeconomic consequences of their choice.

In a seminal contribution, Jean Tirole [1985] proposed a theory of rational asset price bubbles. To satisfy their need for a store of value, economies might accumulate capital even if the investment required to keep this capital exceeds the income it produces. That is, economies might find themselves using a costly or

[1] The predominant view is that bubbles constitute a failure of markets to price assets correctly, and their main consequence is to distort incentives and lower investment efficiency. Although there is no formal model of this market failure yet, there is a widespread belief that its origin lies in some sort of irrational behavior by market participants. The view developed in this paper is almost the converse.

inefficient store of value. In this situation, a bubble with negligible maintenance costs constitutes a more efficient store of value and can favorably compete with capital. As the bubble displaces capital in the portfolios of investors, it liberates resources that are used to raise consumption and welfare. This explains the origins and effects of bubbles. Since bubbles do not have intrinsic value, their size depends on the market's expectation of their future size. In a world of rational investors, this opens the door for self-fulfilling expectations to play an important role in bubble dynamics and accounts for their unpredictability.

Although insightful, Tirole's model does not fit the facts. As Abel et al. [1989] have noted, the investment required to keep the capital stock at current levels is less than the income it produces. This raises the following question: If capital is an efficient store of value, why are some investors buying bubbles? Historical evidence also suggests that bubble episodes tend to generate increases in investment and the capital stock. This raises a second question: If bubbles displace capital in the portfolios of investors, how can they also raise aggregate investment and the capital stock? Any attempt to further develop a theory of rational asset price bubbles must provide answers to these questions.

The first contribution of this paper is to extend the theory of rational bubbles to the case of imperfect financial markets and show that this is enough to reconcile it with the facts. Assume some individuals ("entrepreneurs") are able to run their own firms, while others ("shareholders") do not have this ability and can only buy equity from publicly traded firms. Even though the technology available to both types of firms is the same, the separation between ownership and control exposes publicly traded firms to incentive problems that lower their return vis-à-vis privately held firms. Assume capital is an efficient store of value if held by privately held firms, but an inefficient one if held by publicly traded firms. This could explain why some investors ("shareholders") buy bubbles even if on average ("shareholders" plus "entrepreneurs") capital appears to be an efficient store of value. As bubbles displace capital from the portfolios of shareholders, they liberate resources that could raise both consumption *and* investment by entrepreneurs. If the latter increases enough, the extended theory

can also explain why bubble episodes raise aggregate investment and the capital stock.

Since by their very own nature bubbles must be held by publicly traded firms, the previous argument implicitly assumes that firms holding bubbles in their balance sheet have lower agency costs than firms holding capital. This seems a reasonable assumption, though. Agency costs increase with the manager's ability to influence the firm's value and decrease with the shareholders' ability to observe the actions of the manager. Since the value of productive firms depends on their capital and how it is managed, it is likely that managers could influence the firm's value through a variety of channels that are difficult to observe. Since the value of bubbly firms depends on the expectations of a rational market that knows the firm holds a bubble, it is unlikely that managers could have much of an influence on the firm's value unless the market decided to use the manager as a sunspot to coordinate expectations. Even in this case, it seems unlikely the manager could exploit this ability to his/her advantage without the shareholders knowing it.

The second contribution of this paper is to study the connection between bubbles and productivity growth. The first connection goes from productivity to bubbles. High labor productivity growth allows the bubble to grow faster and this increases the range of bubbly equilibria that are feasible. The higher is capital productivity, the faster the bubble must grow to compete with capital, and this reduces the range of bubbly equilibria that are feasible. The second connection goes from bubbles to productivity growth. The presence of a bubble raises the incentives for labor-augmenting inventions, since these inventions not only raise labor productivity but also have the additional benefit of raising the bubble. The presence of a bubble reduces instead the incentives for capital-augmenting inventions, since the increases in capital productivity must now be weighted against the additional cost of reducing the bubble. Using these insights, I construct a series of suggestive examples that illustrate the interaction between bubbles and productivity growth.

A third contribution of the paper is to analyze the effects of financial development in the presence of bubbles. This part of the paper is still incomplete, although I have written some notes in the text below that explain the main results and the sequence of ideas.

The paper is organized as follows: Section one presents the basic setup and describes the equilibrium without bubbles. Section two shows that there are additional equilibria with bubbles and formally describes them. Section three discusses the macroeconomic effects of bubbles in a stationary economy. Sections four and five introduce endogenous productivity growth and shows how bubbles and productivity growth interact with each other. Section six provides a discussion of the effects of financial development in the presence of bubbles.

1. Basic setup

Although the ultimate goal here is to study the macroeconomic effects of bubbles, it is useful to start with the benchmark case in which the only asset in positive net supply is the capital stock. I consider an economy with overlapping generations that live two periods: young and old. Each generation contains a continuum of individuals, indexed by $z \in [0,1]$. The lifetime goal of the young is to maximize the expected value of old age consumption. The young work, save all their labor income and purchase firms in the stock market. The old retire and live off their capital income. There are no bequests.

The technology is linear on labor and capital. The aggregate labor endowment is one and is uniformly distributed among the young. This unit of labor produces π^L units of a good that can be used for consumption or investment. Since labor markets are competitive, π^L is also the wage per unit of labor and the economy's labor income. A unit of capital costs one unit of the single good, and delivers π^K units of it one

generation later before depreciating. I shall refer to π^L and π^K as the labor and capital productivity.

Firms own the stock of capital. Some firms are financed and managed by an entrepreneur, and obtain a one-period gross return of π^K. Since entrepreneurs are infinitesimal, so is the size of their privately held firms: $\pi^L \cdot dz$. The rest of the firms are financed by shareholders and ran by managers. This creates an agency cost, as shareholders are forced to spend μ per unit of capital to monitor the manager and prevent him/her from embezzling all or part of the firm's capital. Since the salary of the manager is a fixed cost, publicly traded firms choose the maximum size a manager can handle, which is assumed to be non-infinitesimal. As we shall see shortly, the salary of a manager is negligible relative to the size of the firm and, as a result, the return to holding a publicly traded firm is $\frac{\pi^K}{1+\mu}$. Let ϕ be the share of monitoring costs in total capital costs, i.e. $\phi \equiv \frac{\mu}{1+\mu}$. This parameter should be interpreted as a measure of financial frictions or agency costs, and plays a crucial role in what follows.

The young differ in their ability to manage a firm. A fraction λ of the young do not know how to manage a firm and their only savings option is to become shareholders of publicly traded firms. The remaining young know how to manage a firm and have a choice between becoming entrepreneurs or managers of a publicly traded firm. If they become entrepreneurs, they work and use their labor income to finance their privately held firm. If they become managers, they receive a manager's salary and invest it in a publicly traded firm. Since there is free entry to both occupations, the salary of a manager is such that the expected utility of entrepreneurs and managers is the same.[2] Since there is a discrete number of publicly traded firms, all but a measure-zero subset of the young become entrepreneurs. This implies that the economy contains a continuum of "small" privately held firms with aggregate size

[2] This means the manager's salary is $(1-\phi)^{-1} \cdot \pi^L \cdot dz$, which is negligible as a share of the costs of a publicly held firm.

equal to $(1-\lambda)\cdot\pi^L$; and a discrete number of "large" publicly held firms with aggregate size $\lambda\cdot\pi^L$.

It is straightforward to check that this set of assumptions implies the following capital stock and consumption for the two types:

(1) $K^E_{t+1} = \pi^L$ and $K^S_{t+1} = (1-\phi)\cdot\pi^L$

(2) $C^E_{t+1} = \pi^K \cdot K^E_{t+1}$ and $C^S_{t+1} = \pi^K \cdot K^S_{t+1}$

where K^E_{t+1} (K^S_{t+1}) and C^E_{t+1} (C^S_{t+1}) are the capital stock and the consumption of entrepreneurs (shareholders) born in date t. Since there is full depreciation, K is also real investment, i.e. investment spending divided by the cost of capital. Since the lifetime objective of each individual is to maximize old age consumption, C is also welfare. Equation (1) says that the capital stock of entrepreneurs is higher than the capital stock of shareholders, since the latter have to pay monitoring costs. Equation (2) states that all individuals finance old age consumption with their capital income. Although both types have the same labor income and save all of it, entrepreneurs enjoy higher consumption since they do not have to pay monitoring costs. Finally, aggregate over all consumers to find that:

(3) $K_{t+1} = (1-\lambda\cdot\phi)\cdot\pi^L$

(4) $C_{t+1} = \pi^K \cdot K_{t+1}$

where K_{t+1} and C_{t+1} are the average stock of capital and consumption of the generation born in date t.

The model has been stripped down of all the usual choices. Since there is no disutility of work or utility of young age consumption, the labor supply and saving decisions are trivial. Throughout, I shall keep these assumptions even though it is not difficult to imagine the consequences of relaxing them. I shall focus instead on the

central problem of interest, namely the portfolio choice between capital and bubbles and its macroeconomic implications. To do this, I introduce next bubbles as an additional asset in positive net supply.

2. Equilibrium bubbles

Consider the possibility of unproductive or bubbly firms being traded at a positive price in the stock market, alongside with productive firms. Since everyone knows these firms will never deliver a dividend, their value is not the result of some sort of misperception or irrationality. Since there is no government, their value does not stem either from the need to comply with some regulation or the desire to avoid taxes. The only reason anyone would buy an unproductive firm is to resell it later at a high enough price. That is, bubbly firms are nothing but pyramid schemes.

Bubbly firms appear randomly. Each generation contains some young entrepreneurs that are born not only with the ability to manage a productive firm, but also with the fortune of creating a new bubbly firm.[3] Let B_t be the aggregate bubble or total value of the bubbly firms traded in the stock market. Since there are no costs of creating new bubbly firms, their value constitutes a pure profit or rent, R_t. If there are no old bubbly firms, these rents must equal the aggregate bubble. If there are old bubbly firms, these rents must be only a fraction of the aggregate bubble. For simplicity, let this fraction be constant and equal to ρ. Therefore, the total value of the rents obtained from the creation of bubbly firms is given by:

$$(5) \qquad R_t = \begin{cases} B_t & \text{if } B_{t-1} = 0 \\ \rho \cdot B_t & \text{if } B_{t-1} > 0 \end{cases}$$

[3] The assumption that only young entrepreneurs create bubbles is not important, and is adopted only to simplify the algebra. Nothing of substance would change if young shareholders or the old also created new bubbles.

The presence of bubbly firms means that the portfolio decision of the young is no longer trivial. Since the young are risk neutral they always choose to hold the asset or assets with the highest expected return. Remember that the expected return to capital for entrepreneurs and shareholders is π^K and $(1-\phi)\cdot\pi^K$, respectively. Since a bubbly firm contains no assets, there is nothing managers can embezzle and shareholders have no need to spend on monitoring costs.[4] The observation that bubbly firms are not subject to agency costs plays a crucial role in the analysis. It means that the return to the bubble is the same for all investors and is given by its expected price appreciation, i.e. $E_t\{B_{t+1}-R_{t+1}\}/B_t$.[5] Throughout, I focus on the case in which entrepreneurs invest all their savings on capital, but shareholders might hold a bubble. That is, I shall consider equilibria with bubbles that satisfy these conditions:

$$(6) \qquad (1-\phi)\cdot\pi^K \leq \frac{E_t\{B_{t+1}-R_{t+1}\}}{B_t} < \pi^K \qquad \text{and} \qquad B_t \leq \lambda\cdot\pi^L \quad \text{for all t.}$$

The first set of inequalities in Equation (6) says that the bubble is expected to grow fast enough to attract shareholders, but not fast enough to attract entrepreneurs. The second inequality states that the bubble must not exceed the savings of shareholders. Note that only one of the two weak inequalities in Equation (6) is strict, except for a knife-edge case that I disregard from now on.

An equilibrium bubble is a stochastic process for B_t (and the associated values for R_t) that satisfies Equations (5) and (6). As we shall see next, in some regions of the parameter space the set of equilibrium bubbles can be quite large. I shall not attempt a full analysis of this set, but instead construct a few suggestive examples. To do this, it is useful to start by characterizing the subset of deterministic bubbles, i.e. those where $E_tB_{t+1}=B_{t+1}$. It follows from Equations (5) and (6) that deterministic bubbles follow these dynamics:

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[4] The only way managers can acquire part of the value of the firm is by issuing new shares that dilute the value of old shares. But this can be easily avoided at negligible cost by shareholders.

[5] The operator E_t in front of a variable indicates its mathematical expectation conditional on all the information available to generation t. As it will become clear later, there might be some uncertainty regarding the future values of the bubble.

$$(7) \qquad B_{t+1} = B_t \cdot \begin{cases} \left[\dfrac{(1-\phi)\cdot\pi^K}{1-\rho}, \dfrac{\pi^K}{1-\rho} \right) & \text{if } B_t = \lambda \cdot \pi^L \\ \dfrac{(1-\phi)\cdot\pi^K}{1-\rho} & \text{if } B_t \leq \lambda \cdot \pi^L \end{cases}$$

The top panel of Figure 1 plots this map, assuming $1-\rho<(1-\phi)\cdot\pi^K$. Any strictly positive initial bubble would grow without bound and eventually exceed the savings of shareholders. This cannot be an equilibrium bubble. Therefore, the only equilibrium bubble is $B_t=0$, i.e. the bubbleless equilibrium of the previous section. The bottom panel plots the map above, assuming instead $(1-\phi)\cdot\pi^K<1-\rho$. Now there is also a stationary equilibrium bubble that absorbs all the savings of shareholders, i.e. $B_t=\lambda\cdot\pi^L$; and a continuum of non-stationary bubbles that vanish over time (such as the one depicted). This provides a full characterization of the set of deterministic bubbles.

Bubbles place expectations or investor sentiment at center stage.[6] In the bubbleless equilibrium all generations are pessimistic about reselling the bubble and do not buy it, validating their pessimism. In a bubbly equilibrium all generations are optimistic about reselling the bubble and buy it, validating their optimism. The model can also accommodate changes in investor sentiment. To show this, I shall construct next three stochastic and stationary equilibrium bubbles. All of them are driven by a sunspot variable that fluctuates between pessimism (s=P) and optimism (s=O), and is to be interpreted as "investor sentiment". With some probability, investor sentiment changes from one generation to the next. The different bubbles arise under different assumptions about these transition probabilities.

The first bubble is the "all-or-nothing" bubble. Let the probability of transitioning from P to O be α, while the probability of transitioning from O to P is β. Assume pessimistic generations believe the next generation of shareholders will buy the bubble with probability α, while optimistic generations believe this probability to be

[6] Expectations matter in the sense that their changes constitute a source of extrinsic uncertainty, as in Cass and Shell [1983].

1-β. Assume that $\alpha\cdot(1-\rho)<(1-\phi)\cdot\pi^K\leq(1-\beta)\cdot(1-\rho)<\pi^K$. Then, pessimistic generations of shareholders do not buy the bubble, optimistic generations do, and the probabilities assigned by both types of generation are exactly the equilibrium ones. Therefore, we have that:

$$(8)\qquad B_t=\begin{cases}0 & \text{if } s_t=P\\ \lambda\cdot\pi^L & \text{if } s_t=O\end{cases}$$

This is the "all-or-nothing" bubble because, depending on investor sentiment, it absorbs either all or none of the savings of the young shareholders.

The second bubble is the "escalating" bubble. Assume again that the probability of transitioning from P to O is α, but assume now that the probability of transitioning from O to P falls with the age of the bubble. In particular, let this probability be β_1 for the first generation that has the bubble, β_2 for the second, and so on; with $\beta_1>\beta_2>...>\beta_\infty$. As before, assume that pessimistic generations believe the next generation of shareholders will buy the bubble with probability α, while optimistic generations believe this probability to be $1-\beta_n$, where n is the age of the bubble. It is clear that, if $\alpha\cdot(1-\rho)<(1-\phi)\cdot\pi^K$, pessimistic generations of shareholders do not buy the bubble. If $(1-\phi)\cdot\pi^K<(1-\beta_\infty)\cdot(1-\rho)<\pi^K$, optimistic generations will buy the bubble. To see this, assume first that $(1-\phi)\cdot\pi^K<(1-\beta_1)\cdot(1-\rho)$. Then, $B_n=\lambda\cdot\pi^L$ for all ages and this is the "all-or-nothing" bubble. Assume instead that $(1-\beta_1)\cdot(1-\rho)<(1-\phi)\cdot\pi^K$. Then, there exists some age N such that $(1-\beta_N)\cdot(1-\rho)<(1-\phi)\cdot\pi^K<(1-\beta_{N+1})\cdot(1-\rho)$. It follows from Equation (6) that $B_n=\lambda\cdot\pi^L$ if n>N; and $(1-\beta_n)\cdot(1-\rho)\cdot B_{n+1}=(1-\phi)\cdot\pi^K\cdot B_n$ if n≤N. Using this rule recursively, we find the following bubble:

$$(9)\qquad B_t=\begin{cases}0 & \text{if } s_t=P\\ \left(\prod_{m=n}^{N}\frac{(1-\beta_m)\cdot(1-\rho)}{(1-\phi)\cdot\pi^K}\right)\cdot\lambda\cdot\pi^L & \text{if } s_t=O \text{ and } n\leq N\\ \lambda\cdot\pi^L & \text{if } s_t=O \text{ and } n>N\end{cases}$$

This is the "escalating" bubble because it is small when it pops up and, conditional on not bursting, it keeps growing continuously until it absorbs all the savings of the young entrepreneurs. The bubble is small at the beginning because young shareholders do not have much confidence on its survival, and this requires that the bubble grow fast enough to compensate them for the high probability of bursting. As the bubble becomes older, young shareholders become more confident about it and its growth rate declines steadily and eventually stabilizes.

The third bubble is the "fluctuating" bubble. Assume again that the probability of transitioning from P to O is α, but assume now the probability of transitioning from O to P fluctuates randomly between β_H and β_L, with $\beta_H>\beta_L$. I model this fluctuation with the help of a random variable that takes value high (x=H) and low (x=L) with probability θ and 1-θ, respectively. It is clear that, if $\alpha\cdot(1-\rho)<(1-\phi)\cdot\pi^K$, pessimistic generations of shareholders do not buy the bubble. If $(1-\phi)\cdot\pi^K<(1-\beta_L)\cdot(1-\rho)<\pi^K$, optimistic generations will buy the bubble. To see this, assume first that $(1-\phi)\cdot\pi^K<(1-\beta_H)\cdot(1-\rho)$. Then, $B_t=\lambda\cdot\pi^L$ if s_t=O, regardless of whether x_t=H or x_t=L. This is again the "all-or-nothing" bubble. Assume instead that $(1-\beta_H)\cdot(1-\rho)<(1-\phi)\cdot\pi^K$. It follows from Equation (6) that $B_t=\lambda\cdot\pi^L$ if s_t=O and x_t=L; and $(1-\rho)\cdot E_tB_{t+1}=(1-\phi)\cdot\pi^K\cdot B_t$ if s_t=O and x_t=H. Using this restriction, we find the following bubble:

$$(10)\qquad B_t=\begin{cases}0 & \text{if } s_t=P\\ \dfrac{(1-\theta)\cdot(1-\beta_H)\cdot(1-\rho)}{(1-\phi)\cdot\pi^K-\theta\cdot(1-\beta_H)\cdot(1-\rho)}\cdot\lambda\cdot\pi^L & \text{if } s_t=O \text{ and } x_t=H\\ \lambda\cdot\pi^L & \text{if } s_t=O \text{ and } x_t=L\end{cases}$$

This is the "fluctuating" bubble because, for as long as it does not burst, it fluctuates between high and low values that reflect the probability of bursting.

As these examples show, this model encapsulates the notion that changes in investor sentiment can be self-fulfilling and allows us to talk rigorously about the effects of a bubble popping up or bursting. We can think of these events as asset

price booms and busts created by changes in investor sentiment. The next goal is to study their effects.

3. Macroeconomic effects of bubbles

Figure 2 depicts the time series behavior of investment and consumption during a bubble episode. The top, middle and bottom panels show the “all-or-nothing”, the “escalating” and the “fluctuating” bubbles, respectively. The economy does not have a bubble until T_1, when there is a surge in investor confidence and a bubble pops up. This bubble lasts until T_2, when there is a drop in investor confidence and the bubble bursts. As the Figure shows, these movements in the bubble generate substantial effects in the capital stock and consumption. The goal of this section is to understand these movements.

To account for the effects of bubbles on the capital stock and consumption of entrepreneurs and shareholders, we generalize Equations (1) and (2) as follows:

$$(11) \qquad K^{E}_{t+1} = \pi^{L} + \frac{R_t}{1-\lambda} \qquad \text{and} \qquad K^{S}_{t+1} = (1-\phi)\cdot\left(\pi^{L} - \frac{B_t}{\lambda}\right)$$

$$(12) \qquad C^{E}_{t+1} = \pi^{K}\cdot K^{E}_{t+1} \qquad \text{and} \qquad C^{S}_{t+1} = \pi^{K}\cdot K^{S}_{t+1} + \frac{B_{t+1}-R_{t+1}}{\lambda}$$

The effect of bubbles on the capital stock depends on the type. Bubbles raise the capital owned by entrepreneurs, as the rents obtained from the creation of bubbles make them richer. But bubbles lower the capital stock owned by shareholders, as these now spend some of their savings to purchase bubbly firms. The effect of bubbles on consumption is however positive for both types. In the case of entrepreneurs, this increase in consumption is a result of the additional capital financed by rents from bubbly creation. In the case of shareholders, the increase in

consumption is because the bubble provides a better asset to invest their savings.[7] Once again, aggregating over all individuals we find the average capital stock and consumption:

$$(13) \qquad K_{t+1} = (1-\lambda\cdot\phi)\cdot\pi^{L} + R_t - (1-\phi)\cdot B_t$$

$$(14) \qquad C_{t+1} = \pi^{K}\cdot K_{t+1} + B_{t+1} - R_{t+1}$$

Equations (5) and (11)-(14), together with an equilibrium bubble determine the behavior of the economy for a given realization of the sunspot variable. For instance, the three panels of Figure 2 provide the evolution of this system assuming s_t=O for $t\in[T_1,T_2)$ and s_t=P otherwise for the bubbles in Equations (8), (9) and (10).

The first effect of the appearance of the bubble is that the economy receives a positive wealth shock or transfer from the future. This is the central feature of a pyramid scheme where the initiator claims that, by making him/her a payment now, the other party earns the right to receive a payment from a third person later. By successfully creating and selling a bubble, young entrepreneurs have assigned themselves and sold the "rights" to the savings of a generation living in the very far future or, to be more exact, living at infinity.[8] This appropriation of rights is a pure windfall or wealth gain for the lucky young entrepreneurs. Since it is the young shareholders who purchase the bubble, this gain could be as a large as their savings and this might be a substantial fraction of the economy. Since young entrepreneurs do not care about young age consumption, they save all this gain and invest it in capital so as to increase their old age consumption. This is why all the panels in Figure 2 show a spike in the capital stock and consumption of the generation that creates the bubble, i.e. the generation born in T_1.

The second effect of the appearance of a bubble is an increase in the efficiency at which the economy operates. The economy not only gains rights to the

[7] To see that expected consumption increases for shareholders, substitute Equation (11) into (12) and then use Equation (6).
[8] See Shell [1971] for the classic statement of this view.

future, but by trading them across generations also obtains the further benefit of eliminating wasteful or inefficient investment. Each generation stops making investments that are subject to agency costs and, in return, it receives a transfer from the next generation. Since the size of the transfer exceeds the income generated by inefficient investments (See Equation (6)), the economy achieves a higher level of expected consumption and welfare. If $\rho=0$, this efficiency gain is reaped in its entirety by successive generations of shareholders. If $\rho>0$, successive generations of entrepreneurs also benefit from this efficiency gain as they are able to appropriate of some of the "rights" to the future.[9] This efficiency gain explains why in all the panels of Figure 2 the bubble raises consumption even after the initial spike. In Figure 2, we also observe that, after the initial spike, the capital stock remains higher with the bubble than before the bubble. This does not happen for all parameter values, though. On the one hand, the bubble eliminates the investment of young shareholders and this reduces the capital stock by $(1-\phi)\cdot B_t$. On the other hand, the bubble gives young entrepreneurs extra income in the form of rents and this raises the capital stock by $\rho\cdot B_t$. Therefore, the bubble raises the capital stock if and only if $\rho>1-\phi$. This is the case depicted in the different panels of Figure 2.

The bursting of the bubble is akin to losing the rights to the future and therefore creates opposite wealth and efficiency gains. Suddenly, old shareholders find themselves unable to sell the bubble and experience a windfall or wealth loss. Since this was their only source of old age income, their consumption drops to zero. This explains why in all panels of Figure 2 there is a negative spike in consumption when the bubble bursts. After this, young shareholders no longer have the option of buying the bubble and go back to their inefficient investments; while young entrepreneurs lose their rents and must reduce their efficient investment. As all the panels of Figure 2 show, when the bubble bursts efficiency declines and so do the capital stock and consumption.

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[9] This is not the only mechanism through which young entrepreneurs can appropriate part of the benefits of improved efficiency. Consider, as in Ventura [2003], the possibility that the cost of capital declines with investment spending. When the bubble appears, young shareholders stop their inefficient investment, reducing the cost of capital and raising the real income of young entrepreneurs. Here this mechanism does not operate because the price of consumption and investment is fixed to one.

This simple model can be used to show that the two main critiques to the Tirole model do not apply generically to models of rational bubbles. The first one is that they crowd out capital. Although the bubble displaces the capital stock from the portfolio of shareholders, it is clear by now that this need not reduce the aggregate capital stock. To begin with, the appearance of the bubble creates wealth and part of it can be used for investment. In the specific model presented here, this wealth falls into the hands of young entrepreneurs who invest all of it. In addition, continuous bubble creation redistributes wealth from the old to the young and the latter might invest some of it. In the model, this redistribution goes from old shareholders to young entrepreneurs and the later invest all of it. It is entirely possible therefore that bubbles raise the capital stock and, in fact, this is the case depicted in Figure 2.

The second critique is that rational bubbles are not empirically relevant because capital income exceeds investment spending in industrial economies. This claim was made first by Abel et *al* [1989], who argued that investments must be efficient since they net out a surplus to the economy in all dates and, as a result, there is no demand for the bubble and it cannot therefore exist. But the model here provides a counterexample to this claim.[10] Figure 3 shows the behavior of capital income and investment spending throughout the bubble episodes depicted in Figure 2. Note that capital income exceeds investment spending in all dates. The reason is simply that observing that *on average* investments net out a surplus to the economy cannot rule out the possibility that the economy contains pockets of inefficient investors that demand the bubble. The shareholders of this model constitute one of such pockets.

The critiques to the Tirole model stem from the assumption that financial markets are frictionless and all investors face the same rate of return. Under this

[10] Take, for instance, the "all-or-nothing" bubble. Under this bubble, capital income fluctuates between $\pi^K \cdot (1-\lambda \cdot \phi) \cdot \pi^L$ without the bubble and $\pi^K \cdot [1-\lambda \cdot (1-\rho)] \cdot \pi^L$ with the bubble, while investment spending fluctuates between π^L and $[1-\lambda \cdot (1-\rho)] \cdot \pi^L$ without and with the bubble. Therefore, the condition for capital income to always exceed investment spending is $(1-\lambda \cdot \phi) \cdot \pi^K > 1$. But this condition is not incompatible with the condition required for the bubble to exist, i.e. $\alpha \cdot (1-\rho) < (1-\phi) \cdot \pi^K \leq (1-\beta) \cdot (1-\rho) < \pi^K$.

assumption, the bubble must displace capital from the portfolios of *all* investors and therefore reduce the aggregate capital stock. Under this assumption, *all* investments are equally efficient and we can rule out the existence of inefficient investments by looking only at economy-wide averages. But financial frictions create heterogeneity in rates of return. Unless one is willing to assume that financial markets are frictionless, it makes sense to think that rational asset price bubbles can raise the capital stock and cannot be ruled out with available empirical methods.

4. Economic reforms

Perhaps the most important effect of a bubble is how it affects the incentives for change. Economies continuously face opportunities to raise their productive capacity through the development and adoption of new technologies and/or the improvement of their policies and institutions. Whether economies take advantage of these opportunities or let them pass depends on some cost-benefit calculation made by those that are in a position to decide. The goal of this section is to gain some insight on how bubbles affect this calculation.

Assume the generation born in date 0 has the option of implementing a reform that would increase its labor productivity and that of future generations by γ percent. This might require incurring some implementation costs upfront that need not be specified for the argument that follows. I shall just assume that these costs are weakly positive and are drawn from some known distribution function. I assume also that no other generation has ever had the option to implement this reform before and no other generation will have it in the future.[11] The question I address here is whether the reform is more likely to be implemented with or without the bubble.

[11] In this model it does not matter whether this opportunity was anticipated by earlier generations and, as a result, I shall not take a stand on this. As one removes some of the special features of the model, whether the reform is anticipated or not matters quantitatively but not qualitatively.

Table 1 shows the effects of this reform on the expected consumption or welfare of living generations. Consider first the effects of the reform in the bubbleless equilibrium of section one, as reported in the first column of Table 1. Given the linearity of the model, the only effect of the reform is to raise the wage of the young and their labor income by $\gamma \cdot \pi^L$. Since the old are unaffected by the reform, they are not willing to contribute to its financing. All the young benefit from the reform and are willing to contribute at most a fraction γ of their labor income towards its financing. Therefore, the maximum contribution that living generations are willing to make towards the financing of the reform is equal to $\gamma \cdot \pi^L$.

With a bubble, the willingness to pay for the reform increases. This is shown in the second column of Table 1. The willingness to pay for the reform increases because now the reform not only raises the wage of the young by $\gamma \cdot \pi^L$, but also increases the value of the bubble by $\gamma \cdot \lambda \cdot \pi^L$. A fraction ρ of this wealth increase goes to young entrepreneurs and the rest to old shareholders. Therefore, the bubble raises the maximum contribution that living generations are willing to make towards the financing of the reform from $\gamma \cdot \pi^L$ to $(1+\lambda) \cdot \gamma \cdot \pi^L$.

TABLE 1: Gains from a γ percent increase in π^L

		WITHOUT BUBBLE	**WITH BUBBLE**
Entrepreneurs	Old	0	0
	Young	$\pi^K \cdot \pi^L \cdot \gamma$	$\left(1+\frac{\rho \cdot \lambda}{1-\lambda}\right) \cdot \pi^K \cdot \pi^L \cdot \gamma$
Shareholders	Old	0	$(1-\rho) \cdot \pi^L \cdot \gamma$
	Young	$(1-\phi) \cdot \pi^K \cdot \pi^L \cdot \gamma$	$(1-\rho) \cdot \pi^L \cdot \gamma$

Note: Each cell contains the change in expected consumption generated by a γ percent increase in π^L.

This observation leads to the first result about the interaction between economic reforms and asset price bubbles. Assume the reform is implemented if its cost does not exceed the willingness to pay of living generations. Then, for any given distribution of costs, the bubble increases the probability of implementing a reform that raises labor productivity. In particular, it is straightforward to construct situations in which the reform is blocked without a bubble, but implemented with a bubble.

Since I have assumed individuals are selfish, living generations do not take into consideration the effects of the reform on future generations when deciding whether to implement it or not. But it is still possible to determine the benefits of the reform on future generations and how the bubble affects them. Assume for instance that the economy has the "all-or-nothing" bubble, and let q_t^O (q_t^P) be the probability that there will be a bubble at date t, given that there is a bubble (no bubble) at date 0. It follows that $0 \leq q_t^P \leq q_t^O \leq 1$, and also that both probabilities converge as t goes to infinity. Regardless of whether there is a bubble or not, all unborn shareholders would be willing to pay a fraction γ of their labor income to finance the reform. But each generation of unborn entrepreneurs is willing to pay 'ex-ante' as much as a fraction $[1-\lambda(1-q_t^O \cdot \rho)] \cdot \gamma$ of their labor income if there is a bubble at date 0, but only a fraction $[1-\lambda \cdot (1-q_t^P \cdot \rho)] \cdot \gamma$ if there is none. Therefore, the bubble also increases the maximum contribution that unborn generations would be willing to make towards the financing of the reform. It seems reasonable therefore to conjecture that introducing a bit of intergenerational altruism in the model would make the positive interaction between the reform and the bubble even stronger.[12]

The results are the opposite however if we consider a reform that raises capital productivity, rather than labor productivity. To make this point, I assume now that the reform would increase the productivity of new capital by γ percent while leaving labor productivity unchanged. I keep all the other assumptions as in the

[12] If we introduce too much altruism however, the bubble might not be feasible. We know, for instance, that rational bubbles are not feasible when generations are linked through altruism as in Barro [1974].

previous case, and ask again whether the reform is more likely to be implemented with or without the bubble.

Table 2 shows the consumption or welfare effects of this reform on the living generations. Without the bubble, the only effect of the reform is to raise the productivity of the capital accumulated by the young by γ percent. The old are unaffected by the reform and therefore unwilling to contribute to its financing, while the young benefit and are willing to contribute a fraction γ of their labor income. This is shown in the first column of Table 2. Without the bubble, the reform that raises capital productivity is worth exactly the same as the reform that raises labor productivity for each and every member of the living generations. Both reforms would be implemented under the same range of costs.

TABLE 2: Gains from a γ percent increase in π^K

		WITHOUT BUBBLE	WITH BUBBLE	
			IF IT SURVIVES	IF IT BURSTS
Entrepreneurs	Old	0	0	0
	Young	$\pi^K \cdot \pi^L \cdot \gamma$	$\left(1+\frac{\rho \cdot \lambda}{1-\lambda}\right) \cdot \pi^K \cdot \pi^L \cdot \gamma$	$\left(\gamma-\frac{\rho \cdot \lambda}{1-\lambda}\right) \cdot \pi^K \cdot \pi^L$
Shareholders	Old	0	0	$-(1-\rho) \cdot \pi^L$
	Young	$(1-\phi) \cdot \pi^K \cdot \pi^L \cdot \gamma$	0	$[(1-\phi) \cdot \pi^K \cdot (1+\gamma)-1+\rho] \cdot \pi^L$

Note: Each cell contains the change in expected consumption generated by a γ percent increase in π^K.

With a bubble, the willingness to pay for the reform diminishes. Assume first the extreme or corner case in which the reform does not affect the size of the bubble. This happens when agency costs are so severe that, even after the reform has increased capital productivity shareholders still hold only the bubble in their portfolios. This is the case reported in the second column of Table 2. Since young shareholders do not hold capital, they do not benefit from the reform and they are not willing to pay for it. Since young entrepreneurs hold more capital with the bubble and this raises their benefits from the reform, their willingness to contribute to its financing increases. The balance of these two effects is negative and the bubble reduces the willingness to pay for the reform of living generations from $\gamma \cdot \pi^L$ to $[1-\lambda \cdot (1-\rho)] \cdot \gamma \cdot \pi^L$, making it less likely that the reform be implemented.

The likelihood of implementing the reform is further reduced if we move away from the extreme or corner case in which the reform does not affect the size of the bubble. Since the reform makes capital more attractive, in general it reduces the demand for the bubble and therefore its value. This is a negative wealth effect for the owners of bubbly firms and reduces the overall benefits of the reform. To see this, consider the other extreme case in which the increase in capital productivity eliminates the demand for the bubble and makes it burst. This case is reported in the third column of Table 2. The economy suffers a wealth loss equal to $\lambda \cdot \pi^L$; of which a fraction ρ goes to young entrepreneurs and the rest to old shareholders. Interestingly, this loss is not proportional to γ and, as a result, it could be quite large relative to the productivity gains that the reform brings about. Another effect of the bubble is to reduce the benefits of the reform for the young shareholders. The reform increases the return to their savings by a factor of $[(1-\phi) \cdot \pi^K/(1-\rho)]-1$, which is smaller than γ. Therefore, the bubble reduces the maximum contribution that living generations are willing to make for the financing of the reform from $\gamma \cdot \pi^L$ to $[\gamma-\lambda \cdot (1-\phi) \cdot \pi^K/(1-\rho)] \cdot \pi^L$, making it less likely that the reform be implemented. In fact, this willingness to pay might be negative and the reform might not be implemented even if its cost is zero.

A second result on the interaction of bubbles and economic reforms follows from this discussion: for any given distribution of costs, the bubble reduces the

probability of implementing a reform that raises labor productivity. Using the same line of reasoning we used before, it is straightforward to show that the bubble reduces the benefits of the reform not only for the living generations, but also for the unborn. Consequently, introducing a bit of intergenerational altruism into the model is likely to reinforce this result.

To sum up, bubbles impart a pro-labor and anti-capital bias to economic change. Since we have considered only an abstract change in labor and capital productivity, this result applies to a large set of events. In fact, the economic reforms studied here should be interpreted in a broad sense to include the development and adoption of new technologies, increases in education and health, advances in the design of fiscal policy and regulations, and improvements in wide range of institutions.

5. Productivity growth and bubbles

We just saw that the presence of a bubble affects the likelihood of adopting reforms that increase labor and capital productivity. But increases in productivity also affect the size of bubble. Increases in labor productivity raise the demand for the bubble and its size, while increases in capital productivity reduce the demand for the bubble and its size. These observations suggest that bubbles and productivity growth are interrelated, sometimes reinforcing each other and sometimes not. The goal of this example is to explore some of these interactions.

Let g_t^L and g_t^K be the growth rate of labor and capital productivity between t and t+1:

(15) $\pi_{t+1}^L = (1+g_t^L)\cdot\pi_t^L$ and $\pi_{t+1}^K = (1+g_t^K)\cdot\pi_t^K$

Assume the economy contains a research and development sector capable of inventing new technologies that improve factor productivity. Each labor-augmenting (capital-augmenting) improvement costs z_t^L (z_t^K) workers and raises labor productivity by γ percent, i.e. $g_t^L = \gamma$ ($g_t^K = \gamma$). These costs have strictly positive and bounded supports, i.e. $z_t^L \in [\underline{z}^L, \overline{z}^L]$ and $z_t^K \in [\underline{z}^K, \overline{z}^K]$; and are known before potential investors start inventing, i.e. $E_t\{z_t^L\} = z_t^L$ and $E_t\{z_t^K\} = z_t^K$. Only one improvement of each type can be done per generation.

All entrepreneurs have the ability to create and run research and development firms. Those who do this incur the costs of research during their youth in the hope that the government will reward them during their old age if they come up with an invention that raises the welfare of the living generations. I assume this reward just covers the cost of the invention.[13] Let R_t^L and R_t^K be these rewards. Then, we have that:

$$(16) \qquad g_t^L = \begin{cases} \gamma & \text{if } z_t^L \cdot \pi_t^L \le \dfrac{E_t\{R_{t+1}^L\}}{\pi_t^K} \\ 0 & \text{if } z_t^L \cdot \pi_t^L > \dfrac{E_t\{R_t^L\}}{\pi_t^K} \end{cases} \quad \text{and} \quad g_t^K = \begin{cases} \gamma & \text{if } z_t^K \cdot \pi_t^L \le \dfrac{E_t\{R_{t+1}^K\}}{\pi_t^K} \\ 0 & \text{if } z_t^K \cdot \pi_t^L > \dfrac{E_t\{R_t^K\}}{\pi_t^K} \end{cases}$$

That is, inventions are financed if and only if the net present value of the gains it generates exceeds the costs. The model of section two applies as the special case in which z_t^L and z_t^K are sufficiently large to discourage the invention of new technologies and, as a result, $g_t^L = g_t^K = 0$ for all t. I describe next the effects of relaxing this assumption.

[13] This assumption is typical of all models of research and development. This reward sometimes consists of granting a legal monopoly for some period of time or granting subsidies or a public contract. In many models, government policy is not optimal and the size of this reward is therefore arbitrary. Here, I assume instead that government policy is the one that would be agreed upon by all the living generations. These are willing to commit 'ex-ante' to finance any invention that raises their welfare.

Consider first the dynamics of factor productivity in the bubbleless equilibrium. In this case, only the young benefit from productivity increases. By asking how much of their labor income are the young willing to give up in order to increase their labor and capital productivity by γ percent, we find the expected gains from inventions:

(17) $$R^L_{t+1} = \frac{\gamma}{1+\gamma} \cdot \pi^L_{t+1} \qquad \text{and} \qquad R^K_{t+1} = \frac{\gamma}{1+\gamma} \cdot \pi^L_{t+1}$$

That is, both types of inventions are worth the same in terms of labor income. The next step is to use Equations (16)-(17) to study the growth process without bubbles.

An implication of Equations (16)-(17) is that capital-augmenting productivity growth must eventually stop. Capital-augmenting inventions take place only in generations where the return to these inventions exceeds the rate of return to capital, i.e. $\frac{\gamma}{z^K_t} \cdot \frac{1+g^L_t}{1+\gamma} \geq \pi^K_t$. If the return to capital is very low relative to the average return to inventions, this condition holds frequently and capital-augmenting productivity growth is fast. As the return to capital increases relative to the average return to inventions, the frequency this condition holds falls and approaches zero. As a result, capital-augmenting productivity growth declines and eventually stops, i.e. $\pi^K_t \to \pi^K_\infty$.

Equations (16)-(17) do not imply however that labor-augmenting productivity growth should ever cease. Labor-augmenting inventions take place only in generations where the return to these inventions exceeds the rate of return to capital, i.e. $\frac{\gamma}{z^L_t} \geq \pi^K_t$. Even if capital productivity has already converged to π^K_∞, it is entirely possible that labor-augmenting inventions take place with some frequency. If this is the case, labor productivity grows without bound.

Consider next the dynamics of factor productivity in equilibria with bubbles. Although the specifics of the interaction between bubbles and factor productivity depend on particular assumptions, it is possible to distill two general themes or

sources of interactions. I shall describe these next and illustrate how they can be used as building blocks to model a wide range of behavior.

The first theme is that productivity growth affects the set of bubbles that is feasible. Remember that feasible bubbles must satisfy the following conditions:

$$(18) \qquad (1-\phi)\cdot\pi_t^K \le \frac{E_t\{B_{t+1}-R_{t+1}\}}{B_t} < \pi_t^K \qquad \text{and} \qquad B_t \le \lambda\cdot\pi_t^L \quad \text{for all t.}$$

with R_t defined by Equation (4). High labor productivity growth allows the bubble to grow faster and this increases the range of bubbly equilibria that are feasible. The higher is capital productivity, the faster the bubble must grow to compete with capital, and this reduces the range of bubbly equilibria that are feasible.

The second theme is that bubbles affect the incentives for factor productivity growth. For instance, in the presence of the stationary and deterministic bubble of section two, we find that the expected gains from inventions are:

$$(19) \qquad R_{t+1}^L = \frac{\gamma}{1+\gamma}\cdot(1+\lambda)\cdot\pi_{t+1}^L \qquad \text{and} \qquad R_{t+1}^K = \frac{\gamma}{1+\gamma}\cdot[1-\lambda\cdot(1-\rho)]\cdot\pi_{t+1}^L$$

Comparing Equations (17) and (19), we find that the bubble raise the incentives for labor-augmenting innovations but lowers the incentives for capital-augmenting productivity. This was already stressed in the previous section.

Combining these two sources of interaction between bubbles and factor productivity, it is possible to construct some thought-provoking examples:

Example #1: Assume first that z_t^L and z_t^K fluctuate between two values with transition probability τ. Assume second that the initial capital productivity is high enough to rule out capital-augmenting inventions: $\pi_0^K = \pi_\infty^K$. Assume third that agency costs are high and investors coordinate to the "all-or-nothing" bubble: $(1-\phi)\cdot\pi_\infty^K < (1-\beta)\cdot(1-\rho)$. Assume fourth that changes in investor sentiment are uncorrelated with changes in

research costs. Assume fifth that research costs are such that if the cost of labor-augmenting inventions is low (high), inventions (do not) take place regardless of whether there is a bubble: $\underline{\theta}^{L} \cdot \pi_{\infty}^{K} < (1+\alpha \cdot \lambda) \cdot \gamma < [1+(1-\beta) \cdot \lambda] \cdot \gamma < \overline{\theta}^{L} \cdot \pi_{\infty}^{K}$.

In this example, the economy goes through bubble episodes driven by investor sentiment, and also through periods of rapid productivity growth driven by low research costs. An analysis of the time-series generated by this economy reveals that bubbles and productivity growth are uncorrelated. There seems however that in real economies periods of rapid productivity growth and asset price bubbles tend to come together. Of course, one could just generate this positive correlation by simply assuming that changes in investor sentiment are correlated with changes in research costs. However, the next couple of examples show that the model offers deeper reasons for this positive correlation between productivity growth and bubbles.

Example #2: Assume agency costs are intermediate and the bubble is only possible if there is high productivity growth: $(1-\beta) \cdot (1-\rho) < (1-\phi) \cdot \pi_{\infty}^{K} < (1-\beta-\tau) \cdot (1-\rho) \cdot (1+\gamma)$. The rest of assumptions remain as in Example #1. (Although the fifth assumption must be modified to say: $\underline{\theta}^{L} \cdot \pi_{\infty}^{K} < (1+\alpha \cdot \tau \cdot \lambda) \cdot \gamma < [1+(1-\beta-\tau) \cdot \lambda] \cdot \gamma < \overline{\theta}^{L} \cdot \pi_{\infty}^{K}$).

Example #3: Assume research costs are such that productivity growth is only possible if there is a bubble: $(1+\alpha \cdot \lambda) \cdot \gamma < \underline{\theta}^{L} \cdot \pi_{\infty}^{K} < [1+(1-\beta) \cdot \lambda] \cdot \gamma < \overline{\theta}^{L} \cdot \pi_{\infty}^{K}$. The rest of assumptions remain as in Example #1.

In Example #2 bubble episodes can only take place in periods of high productivity growth. In Example #3 periods of rapid productivity growth can only take place during bubble episodes. Both examples therefore feature a positive correlation between bubbles and productivity growth. Without straining the language too much, we can refer to Example #2 as an economy in which high productivity growth causes bubbles, and to Example #3 as an economy in which bubbles cause high productivity growth. The following example poses a “chicken or egg” question:

Example #4: Assume agency costs are intermediate and the bubble is only possible if there is high productivity growth: $(1-\beta)\cdot(1-\rho)<(1-\phi)\cdot\pi_{\infty}^{K}<(1-\beta-\tau)\cdot(1-\rho)\cdot(1+\gamma)$. Assume research costs are such that productivity growth is only possible if there is a bubble: $(1+\alpha\cdot\tau\cdot\lambda)\cdot\gamma<\underline{\theta}^{L}\cdot\pi_{\infty}^{K}<[1+(1-\beta-\tau)\cdot\lambda]\cdot\gamma<\overline{\theta}^{L}\cdot\pi_{\infty}^{K}$. The rest of assumptions remain as in Example #1.

In this example, bubble episodes and periods of rapid productivity growth reinforce each other. None can happen without the other. As a result, the economy will alternate between periods of high productivity growth with bubbles and periods of stagnation without bubbles.

Throughout these examples, there is no capital-augmenting productivity growth. Provided that we keep the other assumptions, assuming that $\pi_{0}^{K}<\pi_{\infty}^{K}$ does not alter much the results. At most, it introduces transitional dynamics in which there is some capital-augmenting productivity growth. But things change quite a bit if π_{∞}^{K} is high enough to eliminate the possibility of bubbles. The following example provides a dramatic example of this:

Example #5: Assume first that z_{t}^{L} and z_{t}^{K} are constant and equal to θ^{L} and θ^{K}, respectively. Assume second that $\pi_{0}^{K}<\pi_{\infty}^{K}$. Assume third agency costs are low and the bubble is only feasible if capital productivity is low and there is labor-augmenting productivity growth: $(1-\beta)\cdot(1-\rho)<(1-\phi)\cdot\pi_{0}^{K}<(1-\beta)\cdot(1-\rho)\cdot(1+\gamma)<(1-\phi)\cdot\pi_{\infty}^{K}$. Assume fourth that research costs are such that capital-augmenting productivity growth is only possible if there is no bubble and capital productivity is low: $[1-\lambda\cdot(1-\rho)]\cdot\gamma<\theta^{K}\cdot\pi_{0}^{K}<\gamma<\theta^{K}\cdot\pi_{\infty}^{K}$. Assume fifth that research costs are such that labor-augmenting productivity growth is only possible if there is a bubble and capital productivity is low: $(1+\alpha\cdot\lambda)\cdot\gamma<\theta^{L}\cdot\pi_{0}^{K}<[1+(1-\beta)\cdot\lambda]\cdot\gamma<\theta^{L}\cdot\pi_{\infty}^{K}$.

As in Example #4, this economy fluctuates between periods of high labor productivity growth with bubbles and zero labor productivity growth without bubbles. The only difference is that now there is capital-augmenting productivity growth during the periods without bubbles. Eventually, capital productivity will increase enough to

eliminate the possibility of bubbles and, with them, all labor productivity growth. The economy will therefore converge towards a steady state without growth.

This raises the following question: Should the government discourage capital-augmenting inventions? The answer is not simple. If the government could somehow sustain the maximum bubble, capital productivity would never take place and all generations would be better off. If the government cannot sustain the maximum bubble, capital productivity raises the welfare of living generations at the expense of the welfare of future generations. In this situation, whether the government should discourage capital-augmenting inventions depends on how it weights the welfare of different generations.

The overriding theme should be that economies have two different "technologies" or sources of consumption. The first one is the conventional one: the actual production of goods. The second one is less conventional: the value of rights to future production of goods. It is this insight that is likely to be quite robust to generalizations of the model. The model presented here illustrates how technological progress affects both sources of consumption. Capital-augmenting productivity growth raises actual production but lowers the value of rights to future production and might end up reducing the consumption possibilities of present and future generation. Labor-augmenting productivity growth raises both actual production and the value of rights to future production and therefore increases consumption possibilities of present and future generations beyond the actual increase in production.

6. Financial development (*Incomplete*)

Here I discuss the interactions between financial development and bubbles.

Throughout, I have assumed that financial markets are imperfect due to agency costs. A first question is: What happens if there is financial development and ϕ declines? By now the intuition should be clear. This is like an improvement in capital

productivity, but only for the shareholders. Without bubbles, financial development is always good. With bubbles, this is less clear and financial development can be blocked. In fact, keeping a poor financial sector might be a way to keep bubbles going (remember example #5 of section 5). If financial development cannot be blocked, we can tell a simple and suggestive story of its effects: We have the economy of the previous section, with episodes of high productivity growth with bubbles followed by episodes of stagnation without bubbles. Financial reform takes place in a bubble period. The impact effect is a big crisis as the bubble collapses. Afterwards, you get an impact effect on output (which can be positive or negative) followed by stagnation. Financial development has led to stagnation and a poverty trap.[14]

Throughout, I have also assumed that financial markets can strip and sell separately the fundamental and bubble components of each asset. What happens if we can only buy assets that have a fraction $\omega(>1)$ of bubble and $1-\omega$ of capital, and the converse? The model in the paper is the special case in which $\omega = 1$. As $\omega \to 1/2$, bubbles become less effective as a store of value and as a mechanism to reallocate investments from low to high productivity investors. The reason is that it is not possible to 'span' the optimal portfolios, since "shareholders" are forced to hold some capital. Bubbles no longer have negligible maintenance costs and become a costly store of value.[15]

This leads us to revise our story of financial development. What happens if this financial development consists of a decline of ϕ *and* an increase in ω? Bubbles become less needed but more powerful. Analyze the possibilities. [...]

[14] One issue: if financial markets become frictionless with probability one, the bubble cannot exist in the first place. Therefore, financial development must just be a possibility to start with.

[15] [Question: Is it possible to create a simple example in which bubbles end up distorting investments and being welfare-reducing?]

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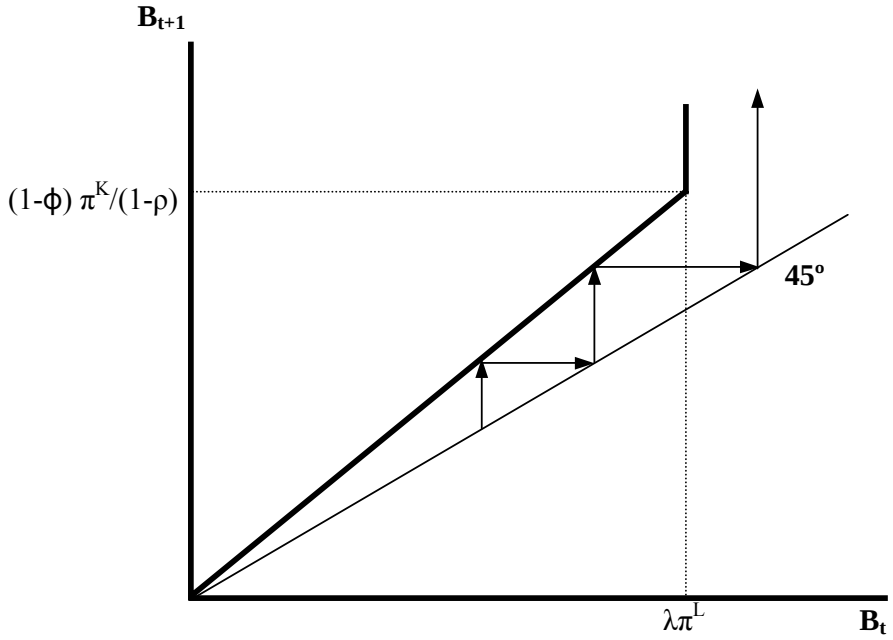
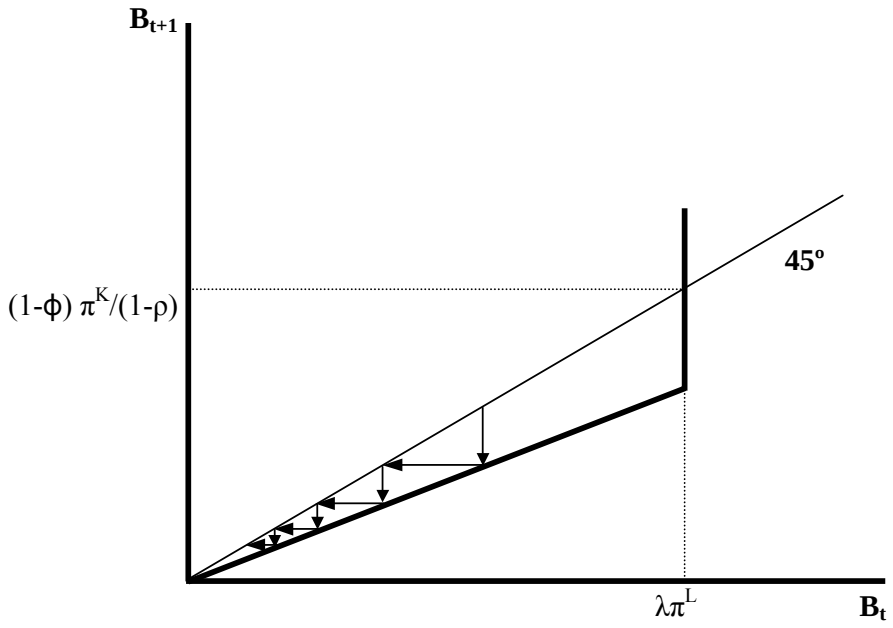
Figure 1

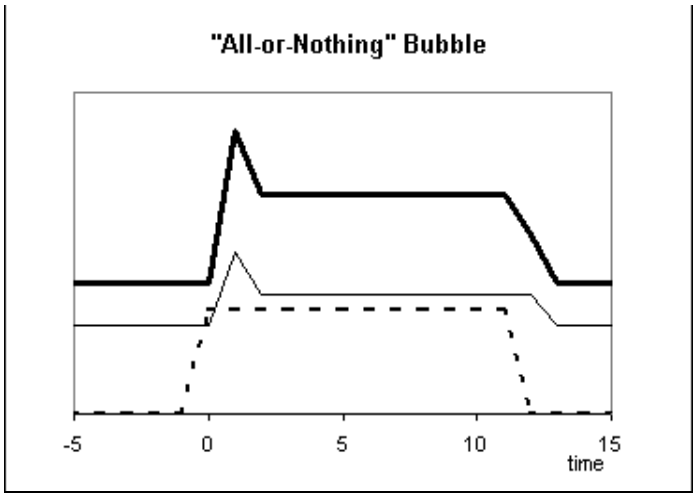


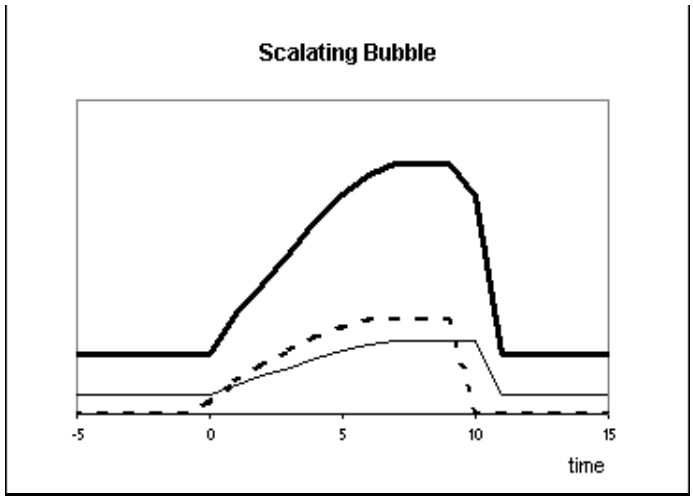
1.a $(1-\rho) < (1-\phi)\,\pi^K$. The bubble grows without bound.



1.b $(1-\phi)\,\pi^K < (1-\rho) < \pi^K$. Stationary bubble is possible.

Figure 2


"All-or-Nothing" Bubble
-5
0
5
10
15
time


Scalating Bubble
-5
0
5
10
15
time

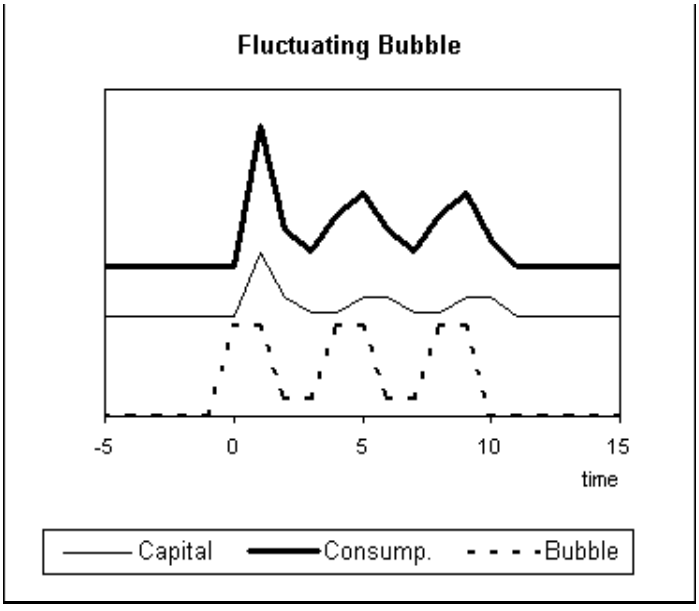
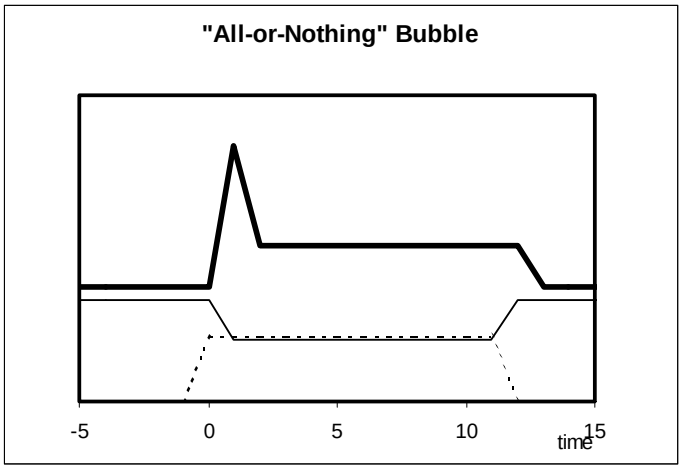
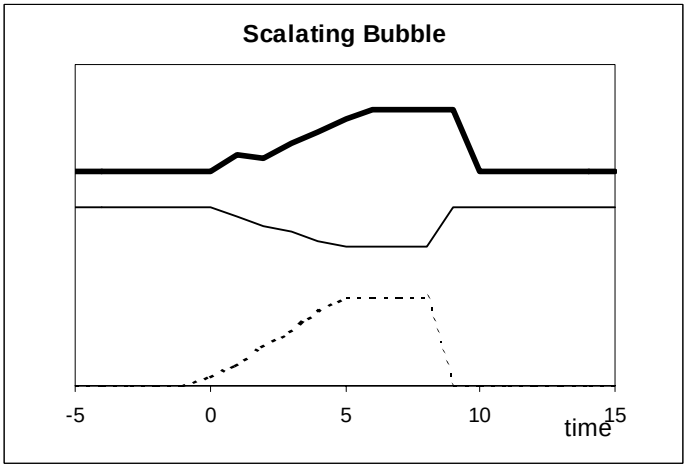
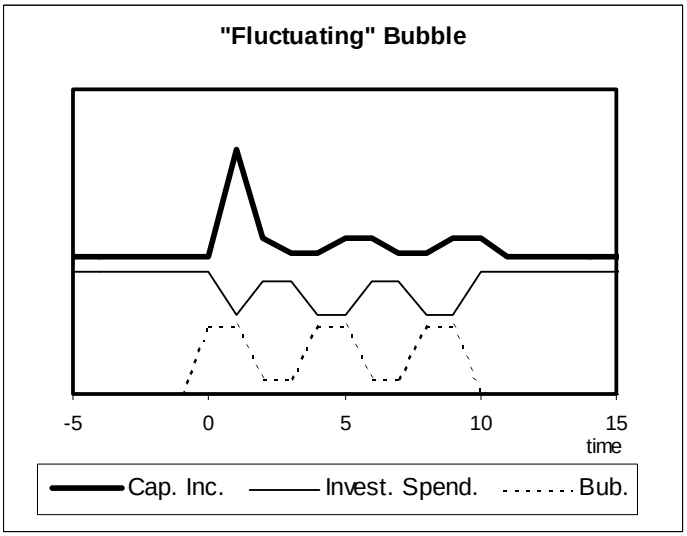

Fluctuating Bubble
-5
0
5
10
15
time
Capital
Consump.
Bubble

Figure 3


"All-or-Nothing" Bubble
-5
0
5
10
15
time


Scalating Bubble
-5
0
5
10
15
time


"Fluctuating" Bubble
-5
0
5
10
15
time
Cap. Inc.
Invest. Spend.
Bub.