

**Centre de Referència en Economia Analítica**

**Barcelona Economics Working Paper Series**

**Working Paper n° 194**

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Contemporary Pattern of Growth**

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June, 2005

# Capitalism, Unemployment and the Transition to the Contemporary Pattern of Growth

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A new model of unemployment based on an idea of Marx is presented and used to interpret the development of the British economy from the beginning of capitalism to the present. It is shown that unemployment may be created purposely by capitalists in order to weaken the bargaining position of the workers. This mechanism leads to complex temporal pattern of unemployment and can explain why wages took almost a century and a half to react to the growing capital to labour ratio that characterised early British capitalism.

*Keywords:* Capitalism, Marx, Great Britain, unemployment

**JEL classification:** E11, E24, N33, O41, O51, P1.

## 1. Introduction

The objective of this paper is to set out a new explanation of unemployment, based on an idea of Marx, and show how it provides an understanding of the changing pattern of British capitalist development since the middle of the 18th century. The paper is related to a number of papers which model, in a detailed way, the different historical phases of economic development and, in particular, the escape from the Malthusian trap. But it differs from them in that it attributes the escape, not to the slow accumulation of knowledge, but rather to the arrival of capitalism. In any case the early period of British development poses a mystery: the real wage took almost a century and a half to react to the growing capital to labour ratio. Marx, in his criticism of Malthus, had an explanation of this: he claimed that capitalists would lower the embodied labour to capital ratio, purposely create a reserve army of unemployed and thus maintain the wage at subsistence. The paper models this and shows that the economy passes through just such a period before entering into one with a growing wage and falling unemployment, similar to that of the contemporary British economy.

The Malthusian trap is that population growth forces the wage and per capita income to subsistence level because the quantity of land is fixed. The economy escapes from

this trap if per capita income and the wage enter a period of sustained growth. There are a number of papers which attempt to model this, either by postulating a switch to technologies which are immune to diminishing returns<sup>1</sup> or by assuming an increase in the rate of knowledge accumulation which compensates for the diminishing returns.<sup>2</sup> But all of these papers keep the social structure constant during the escape. It seems to me that the escape was brought about by the switch from peasant farming and artisanal production to capitalist agriculture and manufacturing. This permitted the application of capital and ideas to these activities and thus created a motive for its accumulation and their creation respectively.<sup>3</sup> Thus it is in the nature of capitalism where we should search for an explanation of the details of the escape from the Malthusian trap.

Capitalism in the United Kingdom was well established by the beginning of the 18th century, Heilbroner (1987). But if we look at the period after 1700, we are confronted with a mystery. This is detailed in Table 1. Consider the period 1700-1841. Per capita GDP grew at a rate of 0.3% and then 0.4% and reached 1.2% near the end of the period. There are no figures on the growth of capital per capita before 1771, but in the 45 years to 1816 it rose at a rate of 0.42%, and afterwards, in the 25 years to 1841 at a rate of 0.82%. Furthermore it seems likely that it was rising before 1771 since the rate of growth in the decade starting with 1771 was 0.35%. On the other hand, during the 116 years to 1816 there was no growth in the real wage at all, in the next 25 years it started to grow only slowly at a rate of 0.39% and then finally began to grow rapidly after 1841. The mystery is why, after the establishment of capitalism and the beginning of per capita income and capital growth, did the real wage wait nearly a century and a half before it started to rise?

	1700-1771	1771-1781	1781-1816	1816-1841	1841-1881
$\hat{y}$	← 0.3	→ × ← 0.4	→ × ← 1.2	→	
$\hat{k}$	← ?	→ × ← 0.42	→ × ← 0.82	→ × ← 1.38	→
$\hat{w}_{UK}$	← 0	→ × ← 0.39		→ × ← 1.34	→

*Table 1.* Historical data for the United Kingdom. Notes: A hat over a variable indicates yearly rate of growth.  $y$  is GDP per man hour from Maddison (1991). The switch from 0.4 to 1.2 occurred in 1820.  $k$  is the capital to labour ratio for Great Britain, the figures are calculated from the table in the appendix.  $w_{UK}$  is the real wage for the UK. The figure for before 1771 is from Hanson and Prescott (1999). The rest of the data is calculated from the table in the appendix. The same pattern would be observed if the data for the wage for Great Britain were used.

Malthus and Marx had different ideas about the causes of the poverty that was reflected in the constancy of the wage. Malthus' idea, and the one on which current research is based, was that the poverty was caused by an excess of population relative to the available land. Malthus thought that the only escape lay in a reduction in the rate of growth of population. As noted above, current research holds that the escape occurred because productivity increased due either to the switch to non-diminishing returns technologies or the accumulation of knowledge. Marx, on the other hand, thought that the poverty was due to the capitalist strategy of creating an excess population, or reserve army of the unemployed in order to keep the wage from rising. That is, Malthus and modern research have the problem of poverty arising from the fixed quantity of land, Marx has it arising from the nature of capitalism.

Specifically Marx thought that capitalism would pass through two stages. In the first the capitalist sector would be small, the labour supply plentiful, the wage at subsistence and the technology fixed. Employment in the capitalist sector would grow with the accumulation of capital. The second stage would start when a scarcity of labour started to raise the wage. The capitalists would react by lowering the labour to capital ratio embodied in the technology (henceforth the embodied labour to capital ratio) in order to create a reserve army of unemployed and keep the wage at subsistence. Thus in both stages the wage would be at subsistence but in the first the embodied labour to capital ratio would be constant and employment would be increasing while in the second both of these would be falling.<sup>4</sup>

The paper formally models Marx's idea that unemployment is purposely caused by capitalists to influence the wage. Because, as explained above, diminishing returns is not an issue for capitalism, the model has constant returns. The population grows exogenously and capital accumulates because of capitalist savings. The capitalists act in a coordinated way while the workers react as individuals. In each period the capitalist class, henceforth the capitalist, first chooses the embodied labour to capital ratio which immediately determines the level of unemployment, then chooses a specific group of workers sufficient to fill the jobs that have been created, and finally negotiates individually with each of these workers. The workers who are not chosen or who do not reach agreement with the capitalist are unemployed and must try to maintain themselves as best they can.

It is shown that this economy passes through three stages and approaches a fourth. These are set out in Table 2.  $k$  is the ratio of capital to the supply of workers;  $l$  is the embodied labour to capital ratio, that is the ratio of employed workers to capital;  $w$  is the wage,  $e$  is the ratio of employed to total workers or employment ratio and is equal to one minus the rate of unemployment; and finally a hat over a variable indicates a percentage change with respect to time. The first two stages correspond to the ones described by Marx, the last two approximate the pattern of contemporary growth and were not mentioned by Marx.

Stage	$k$	$l$	$w$	$e$
first	$\hat{k} > 0$	$\hat{l} = 0$	$\hat{w} = 0$	$\hat{e} > 0$
second	$\hat{k} > 0$	$\hat{l} < 0$	$\hat{w} = 0$	$\hat{e} < 0$
third	$\hat{k} > 0$	$\hat{l} < 0$	$\hat{w} > 0$	$\hat{e} > 0$
fourth	$\hat{k} > 0$	$\hat{l} < 0$	$\hat{w} > 0$	$\hat{e} = 0$

*Table 2.* The pattern of development. See text for definitions of symbols.

These stages allow one to understand the historical pattern of British development: It will be argued that the first transition between the stages took place around 1816, that the second transition occurred between 1841 and 1850 (which explains the long delay before the start of wage growth) and finally that it seems that the transition to the fourth (which is a steady state) has not yet taken place. It should be emphasised that this explanation of unemployment, with the implication that its trend should vary over time, is new in the literature.<sup>5</sup>

## 2. Model and Result

The model which gives rise to the stages described in Table 2 is set out in this section. There are two issues which are separate from the formal structure of the model which it is convenient to discuss first. They are the disagreement payment and the introduction of technical progress.

If the worker is unable to reach agreement with the capitalist or if he is not chosen, it is assumed that he must try to support himself by interacting with those workers who are employed by the capitalist. This may be by providing services like cleaning or

washing or by begging or stealing. In any case it is assumed that the total output of these activities rises both with the number of employed workers  $D$  and the number of unemployed workers  $U$ . Specifically it is assumed that total output is given by  $DU/(D+U)$ . Output per unemployed person or the disagreement payoff is then  $D/S$  where  $S=D+U$  is the total supply of workers. This is different from virtually all models of unemployment where the disagreement payment is linked to the negotiated wage via unemployment benefits, Booth (1995). There are two justifications for this variation. First, the model is mainly focused on a period in which there were no unemployment benefits and second, even today, unemployment benefits are of limited duration so that the disagreement payoff should depend in part on the productivity of activities that a formally unemployed person might undertake.

There is a problem with the introduction of technical progress in the model. On the one hand it is important to introduce it because, without it, when the model reaches steady state, there would be a counterfactually constant real wage. On the other hand it causes considerable technical difficulties during the first two stages in which the wage is equal to a subsistence wage. The solution is to introduce technical progress only after the third stage has begun and the wage has risen above subsistence. This is not ideal but I conjecture that the introduction of technical progress from the first stage on would not change the substance of the results.<sup>6</sup>

Now consider the formal description of the model. It has two state variables, the supply of workers  $S$  which grows at an exogenous rate  $\hat{S}$  and the stock of capital  $K$  which grows because of capitalist saving. There is a production function  $D K^{1-\alpha}$  where  $0 < \alpha < 1$ , and as above,  $D$  is the number of workers that the capitalist demands and actually hires. In each period the first thing that happens is that the capitalist chooses the embodied labour to capital ratio  $l=D/K$ . An alternative way to think of this is that the capital stock consists of a number of machines each operated by a single worker. The choice of  $l$  determines the amount of capital that will be embodied in each machine. Because the stock of capital and the supply of workers are fixed, this choice also determines the numbers of workers that will be employed and unemployed.

The second thing that happens is that the capitalist chooses a specific group of  $D$  workers and bargains individually with each of these workers. Because the embodied labour to capital ratio has already been embodied in the capital stock there is no connection between these negotiations. If a negotiation is successful the capitalist gets

$l^{-1} -$  where  $l^{-1}$  is the output per man and  $w$  is the wage. If the negotiation is unsuccessful it is assumed that the capitalist cannot go back to the pool of unemployed workers and so gets nothing. If the negotiation is successful the worker gets  $w$  while if not he gets the disagreement payoff  $D/S=(D/K)(K/S)=lk$  where  $k$  is the capital to supply of workers ratio, henceforth the capital to labour ratio. The Nash product for this negotiation is  $(l^{-1} - w) (w - lk)^{1-\beta}$  where  $0 < \beta < 1$  is the bargaining strength of the capitalist and  $1-\beta$  is that of the worker. The wage that would arise from this negotiation is the negotiated wage,  $w_N$ ,

$$w_N(l) = (1 - \beta)l^{-1} + \beta lk, \quad w_N(l) > 0, \quad w_N(l_*) > 0, \quad w_N(l_{**}) < 0 \quad (1)$$

where ‘ $\cdot$ ’ indicates the derivative etc. and the last two conditions are explained in a moment.<sup>7</sup> But there is, in addition, a subsistence wage  $w$  below which, for efficiency reasons, the capitalist will not pay. Thus  $w = \max(w, w_N)$ . This is illustrated in Figure 1. From the figure it is clear that  $w$  is a function of  $l$ ,

$$w(l) = w, \quad l < l_*; \quad w(l) = w_N(l), \quad l > l_*, \quad l_* < l_{**}; \quad (2)$$

that  $l_*$  and  $l_{**}$  are defined as functions of  $k$  as the larger and smaller roots of equation (3),

$$w = w_N(l) = (1 - \beta)l^{-1} + \beta lk, \quad l = l_*, l_{**}; \quad (3)$$

and that the last two conditions of (1) hold. The existence of  $l_*$  and  $l_{**}$  is dealt with below. Finally differentiating (3) and applying (1) gives

$$l_*(k) = \frac{l_*}{w_N(l_*)} = \frac{l_*}{(1 - \beta)(-1)l_*^{-2} + \beta k} < 0 \quad (4)$$

$$l_{**}(k) = -\frac{l_{**}}{w_N(l_{**})} > 0 \quad (5)$$

Thus the wage that arises from the negotiation depends on the value of  $l$  that the capitalist had chosen and indirectly on the state variable  $k$ . As  $l$  increases this wage

initially falls because the average product of labour falls but then rises because the rise in  $l$  lowers unemployment and thus raises the disagreement payoff. This effect is central to the working of the model.<sup>8</sup>

It is assumed that the capitalist chooses  $l$  to maximise profits. Thus it is necessary to look at the shape of the profit function  $\pi(l)$  for various values of  $k$ .

$$\begin{aligned}\pi(l) &= [l^{-1} - \pi(l)]lK = [l - \pi(l)l]K \\ &= [l - w_N l]K && \text{for } l_{**} < l < l_* \\ &= [l - w_N(l)l]K = [l - l^2 k]K && \text{for } l < l_{**}, l_* < l.\end{aligned}\quad (6)$$

It turns out that there are many cases. To focus on the relevant one condition (C1) is imposed,

$$k < 2/3. \quad (C1)$$

From Figure 1 it is clear that  $\pi'(l)$  has discontinuities at  $l_*$  and  $l_{**}$ . This means that  $\pi(l)$  will have discontinuities at these points as well.  $\pi'^+(l_*)$  is the left hand derivative of the profit function at  $l_*$ , that is the limit of  $\pi'(l)$  as  $l$  approaches  $l_*$  from the left, etc.. The specific expressions for these derivatives follow from the differentiation of (6):

$$\begin{aligned}\text{For } l < l_{**}, l_* < l: \\ \pi(l) &= [l^{-1} - w_N(l) - l w_N(l)]K = (l^{-1} - 2lk)K = \pi'^+(l_{**}) = \pi'^-(l_*)\end{aligned}\quad (7)$$

$$\begin{aligned}\text{For } l_{**} < l < l_*: \\ \pi(l) &= [l^{-1} - w]K = \pi'^-(l_{**}) = \pi'^+(l_*).\end{aligned}\quad (8)$$

The shapes of the profit function are given by Lemma 1.

**Lemma 1:** *Let (C1) hold. Then*

$$\pi'^+(l) > \pi'^-(l), \quad l = l_*, l_{**} \quad (9)$$

$$\pi(l) < 0 \quad \text{where defined}, \quad (10)$$

$$\lim_{k \rightarrow 0} \pi'^+(l_{**}(k)) > 0, \quad \lim_{k \rightarrow 0} \pi'^-(l_*(k)) < 0; \quad (11)$$

there are  $k_L$  and  $k_M$ ,  $k_L < k_M$ , such that

$$\pi'^+(l_*(k)) > 0 \text{ or } 0 \text{ as } k \rightarrow k_L, \quad (12)$$



$$-(l_*(k)) \text{ or } 0 \text{ as } k \text{ or } k_M. \quad (13)$$

This is illustrated in Figure 2.

*Proof of Lemma 1.* (9) follows from (7), (8), (1) and (2). (10) follows from differentiating (7) and (8). To demonstrate (11) note that the limits of  $l_{**}$  and  $l_*$  as  $k$

approaches zero are, respectively,  $\frac{w}{1-w}$  and infinity so that, from (3), the limit of

$l_*k$  is  $w$ . Substituting these values into (7) gives the result.

To demonstrate (12) note that from (8) the sign of  $+(l_*(k))$  is the same as that of  $f(l_*) = l_*^{-1} - w$ . Let

$$k_L = \left( \frac{w}{1-w} - 1 \right) \frac{1}{w} \quad (14)$$

where  $k_L > 0$  by (C1). For  $k = k_L$ , the substitution of  $l_0 = \frac{w}{1-w}$  into (3) shows that it is one of the roots of (3). Substitution of  $l_0$  into the middle term of (4) shows that the middle term is negative by (C1) and thus, because of (5), that  $l_0 = l_*(k_L)$ . (12) now follows from  $f(l_*(k_L)) = 0$ ,  $f' < 0$  and (4).

To demonstrate (13), note that from (7) the sign of  $-(l_*(k))$  is the same as the sign of  $g(k) = l_*^{-2} - 2k$ . Let

$$k_M = \frac{w}{2} \frac{1}{(1-w)^2} \quad (15)$$

For  $k = k_M$ , the substitution of  $l_{00} = \frac{w}{(1-w)^2}$  into (3) shows that it is one of the roots of (3). Substitution of  $l_{00}$  and  $k_M$  into the middle term of (4) shows that the middle term is negative because of (C1) and thus that  $l_{00} = l_*(k_M)$ . Taking account of the formula for  $l_*(k)$  in (4),  $g = (-2)l_*^{-3}l_* - 2 > 0$ . (13) follows from this and  $g(k_M) = 0$ .

Finally to demonstrate that  $k_L < k_M$  suppose the contrary. Then there is a  $k$ ,  $k_M < k < k_L$  and  $+(l_*(k)) < 0$   $-(l_*(k)) > 0$  by (12) and (13) which contradicts (9). []

It was claimed that the existence of  $l_*$  and  $l_{**}$  would be dealt with. Under (C1) the proof of lemma 1 shows that  $l_*$ , and thus also  $l_{**}$ , exist for  $0 < k < k_M$ .

The capitalist chooses  $l$  to maximise  $\pi$ . Let  $l_M(k)$  be the maximising value. Further let  $e(k) = l_M(k)k$ ,  $\tilde{w}(k)$  and  $\tilde{\pi}(k) = \pi / K = l_M(k) - \tilde{w}(k)l_M(k)$  be the employment ratio, the wage and the profit rate that arise from this choice. There are two restrictions that these variables must satisfy: the employment restriction,

$$e(k) \leq 1 \tag{R1}$$

and the Nash consistency restriction,

$$l_M^{-1}(k) - l_M(k)k > 0 \tag{R2}$$

The second restriction states that average product of a worker must be greater than the disagreement payoff for, if not, there would be nothing to bargain over. To ensure that (R1) is satisfied a second condition must be imposed,

$$w \leq \tilde{w} / (\beta + \alpha - 1). \tag{C2}$$

Lemma 2 now gives these four functions for the three ranges of the value of  $k$ .

**Lemma 2:** *Let (C1) and (C2) be satisfied, then there is a  $k_H > k_M$  such that*

$$\begin{aligned}
 \tilde{w} &= \frac{w}{\beta - 1} & \tilde{w} &= \frac{w}{\beta - 1} k \\
 l_M(k) &= l_*(k), l_M(k) < 0, & e(k) &= l_*(k), e(k) < 0 \\
 & \frac{2}{\beta} k^{\frac{1}{\beta - 2}} & & \frac{2}{\beta} \frac{1}{k^{\frac{1}{\beta - 2}}} \\
 \tilde{w}(k) &= \frac{w}{(1 - \alpha + \beta/2) \frac{2}{\beta} k^{\frac{1}{\beta - 2}}}, & \tilde{\pi}(k) &= l_*(k) - w l_*(k), \quad \tilde{\pi}(k) < 0 \\
 & & & (1 - \beta/2) \frac{2}{\beta} k^{\frac{1}{\beta - 2}}
 \end{aligned}$$

and (R1) and (R2) are satisfied for all  $0 < k < k_H$ . Here the three functions on the right hand side of each equals sign are the ones for a)  $0 < k < k_L$ , b)  $k_L < k < k_M$  and c)  $k_M < k < k_H$ , respectively.

*Proof.* For  $l_M(k)$ : from Lemma 1 (or Figure 2) for a) set  $\dot{l}(l)=0$  in (8); for b)  $l_M(k)=l_*(k)$  and, from (4),  $l_M(k) < 0$ ; and for c) set  $\dot{l}(k)=0$  in (7). For  $e(k)$ : the expressions for a), b) and c) follow from the expressions for  $l_M(k)$  and the definition of  $e(k)$ .  $e(k) = kl_*(k) + l_*(k) < 0$  where specific account is taken of the formula for  $l_*(k)$  in (4). For  $\tilde{l}(k)$ : a) and b) follow from Lemma 1 (or Figure 2) and (2); and c) follows from these two facts plus the substitution of the expression for  $l_M(k)$  into (1). For  $\dot{l}(k)$ : the expressions in a), b) and c) follow from substituting the expressions for  $\tilde{l}(k)$  and  $l_M(k)$  into the definition; for b)  $\dot{l}(k) = (l_*^{-1}(k) - w)l_*(k) < 0$  because the first factor is positive by (8) and (12) for  $k_L < k$  and the second is negative by (4). (The inequality should refer to  $k_L < k < k_M$  but this is suppressed for simplicity.)

Finally consider the restrictions. For (R2): since  $l_M^{-1}(k)$  is constant to  $k_L$  and rising afterwards and  $e(k)$  rises to  $k_L$ , falls to  $k_M$  and then rises, one only has to check the restriction at  $k_L$  and for  $k < k_M$ . Substituting the function  $l_M(k)$  for these values of  $k$  into (R2) shows it is satisfied. For (R1): noting the pattern of  $e(k)$ , once again one only has to check the restriction at  $k_L$  and for  $k < k_M$ . (C2) guarantees its satisfaction at  $k_L$ . To find  $k_H$ , consider c) and set  $e(k)=1$ . This gives

$$k_H = \frac{2}{1-\alpha} \quad (16)$$

which, by the behaviour of  $e(k)$ , is clearly greater than  $k_M$ .  $\square$

The next step is to introduce technical progress. Suppose that for all  $k < k_M$  there is technical progress in the sense that the number of efficiency units of labour that each worker represents grows at the rate  $\alpha$ . Define the following variables.

$\underline{l} = le^{-t}$ : the embodied efficiency unit of labour to capital ratio,

$\underline{k} = ke^{-t}$ : the capital to efficiency unit of labour ratio,

$\underline{w} = we^{-t}$ : the wage of an efficiency unit of labour,

$e = \frac{l}{k}$ : the employment ratio of efficiency units of labour.

When  $k > k_M$  the model can be thought of in terms of efficiency units of labour instead of units of labour. The capitalist negotiates over the wage of an efficiency unit, the disagreement payoff for an efficiency unit is  $\frac{k}{l}$  and the wage of an efficiency unit is  $\frac{1}{k}$ . Thus the model is identical with the one set out previously except that the word worker is replaced by the word efficiency unit of labour. This implies the following.

**Corollary to Lemma 2:** *Let (C1) and (C2) hold. Let there be technical progress at rate  $\lambda$  when  $k > k_M$ . Then the functions set out in Lemma 2 are valid for the efficiency unit of labour variables. That is*

$$l_M(\underline{k}) = \frac{2}{\underline{k}} \underline{k}^{-\frac{1}{2}}, \quad e(\underline{k}) = \frac{2}{\underline{k}} \underline{k}^{-\frac{1}{2}} (\underline{k})^{\frac{1-\alpha}{2}},$$

$$\hat{\underline{k}}(\underline{k}) = (1 - \alpha + \lambda/2) \frac{2}{\underline{k}} \underline{k}^{\frac{1-\alpha}{2}}, \quad \hat{e}(\underline{k}) = (1 - \alpha/2) \frac{2}{\underline{k}} \underline{k}^{-\frac{1}{2}}.$$

In addition for  $\underline{k} < \underline{k}_H = \frac{2}{\alpha} \alpha^{-\frac{1}{1-\alpha}}$ , (R1) and (R2) hold for the efficiency unit variables.

Finally consider the way  $k$ ,  $\underline{k}$ , and the other variables evolve over time. It is supposed that capital grows because the capitalist saves a proportion  $s$  of his profits. Thus

$$\hat{k} = s(k) - \hat{S}, \quad \hat{\underline{k}} = s(\underline{k}) - \hat{\underline{S}} \quad (17)$$

where  $\hat{\underline{S}} = \hat{S} + \lambda$ . Let  $\underline{k}^*$  be defined by  $0 = s(\underline{k}^*) - \hat{\underline{S}}$ . These functions as well as the first three of Lemma 2 and its Corollary are illustrated in Figure 3 for the case of  $s(k_M) - \hat{S} > 0$  and  $\underline{k}^* < \underline{k}_H$ . Figure 3 is based on Lemma 2 and its Corollary.

The description of the trajectory of the economy is given by the following proposition.

**Proposition:** *Let (C1) and (C2) be satisfied,  $k_0$  be the initial value of  $k$ ,  $0 < k_0 < k_L$ ,  $s(k_M) - \hat{S} > 0$  and  $\underline{k}^* < \underline{k}_H$ . Then equations (17) form a system of successive differential equations with the following solution:  $k(t)$ ,  $0 \leq t < t_M$ , is the solution to  $\hat{k} = s(k) - \hat{S}$ ,*

$k(0)=k_0$  , where  $t_M$  is defined by  $k(t_M)=k_M$  ;  $\hat{k}(t) > 0$ .  $\underline{k}(t)$ ,  $t_M < t$  is the solution to  $\hat{\underline{k}} = s(\underline{k}) - \hat{\underline{c}}$ ,  $\underline{k}(t_M)=k_M$ ;  $\hat{\underline{k}}(t) > 0$  and  $\underline{k}(t)$  approaches  $\underline{k}^*$ .

Define  $k(t)$ ,  $t_M < t$  as  $k(t)=\underline{k}(t)e^t$ . Then the trajectory of the economy passes through the first three stages of Table 2 and approaches the fourth. In addition (R1) and (R2) are satisfied.

*Proof:* The statements about the differential equations follow from Figure 3 as do the characteristics of the first two stages. The characteristics of the last two follow from  $l_M(t) = e^{-t} l_M(\underline{k}(t))$ ,  $e(t) = \underline{e}(\underline{k}(t))$ ,  $\dot{t} = e^{-t} \dot{\underline{k}}(\underline{k}(t))$  for  $t_M < t$  and Figure 3. (R1) and (R2) are satisfied by Lemma 2 and its Corollary.

The model affords the following description of the development of the economy after the establishment of capitalism. At first labour is plentiful and is drawn from low productivity activities by the capitalists who pay a subsistence wage and choose an embodied labour to capital ratio such that the marginal product of labour is equal to the subsistence wage. This stage comes to an end when a growing labour shortage threatens to drive the wage above subsistence. The capitalist class reacts by starting to lower the embodied labour to capital ratio. This has the benefit of creating unemployment which keeps the wage at subsistence but has the cost, which grows, of progressively increasing the gap between the marginal product of labour and the wage. The second stage comes to an end and the third begins when this cost outweighs the benefit of keeping the wage at subsistence. The capitalist class, to take advantage of the gap, begins to hire more labour by moderating the fall in the embodied labour to capital ratio so that employment, and the negotiated wage rise. This does not mean that the capitalist class has ceased to cause unemployment since it still exists and is the result of the technology they choose. The fourth state is approached as follows. Technical progress causes the number of efficiency units of labour per worker to rise. This slows the fall of the rate of profit but, in spite of this, it falls to the point where capital is growing at the same rate as efficiency units of labour. This means that capitalist demand for efficiency units grows at the same rate as their supply so that the employment ratio stabilizes. Similarly the wage of an efficiency unit stabilises so that the wage per worker grows. On the other hand, since there is progressively less labour per efficiency unit, the capital to labour ratio continues to rise and the embodied labour to capital ratio to fall.

### 3. An Interpretation of British Development

This section indicates the periods in which the transitions between the stages may have occurred. It is argued that the first transition may have occurred around 1816, the second in the period 1841-1850 and that the third may not yet have occurred.

The location of these transitions depends, in part, on movements of the rate of unemployment. Data on unemployment before 1860 are hard come by. But Feinstein (1999, p. 646) has given approximate figures. These are listed in Table 3 together with the later more reliable figures.

	1700-1816	1850	1861-1910	1951-1975
Unemployment rate	5%	8%	4.54%	1.91%

*Table 3.* Rates of unemployment for the UK. Sources: 1700-1816 and 1850: Feinstein (1999, p. 646). 1861-1910 and 1951-1975: Phillips (1958) and OECD Main Economic Indicators, Historical Statistics and Main Economic Indicators, various numbers.

Referring to Table 2, the first transition can be identified by movements of the embodied labour to capital ratio and the employment ratio. In particular the latter changes from rising to falling which implies that the rate of unemployment should first fall and then rise. From Table 3, although the rate of unemployment is only constant and does not fall up to 1816, the strong rise up to 1850 indicates tentively that the first transition occurred around 1816.

Again referring to Table 2, the second transition is characterised by movements of the wage and the employment ratio. The former changes from constant to rising while the latter implies that the unemployment rate should change from rising to falling. From Table 1 the rapid increase in the wage only started after 1841 and from Table 3 the rate of unemployment changed from rising to falling about 1850. This would indicate that the second transition occurred in the period 1841-1850.

Finally, from Table 2, the third transition is characterised by the employment ratio, it changes from rising to constant. Figure 4 plots the rate of unemployment for the United Kingdom from 1861 to 2003 as 5 year averages. The latter part of the 19th century does not seem to display a trend while the 20th century is dominated by the two shocks of the

over-valuation and depression and of the oil crises. But when one considers only the periods without shocks, one has the distinct impression of a falling unemployment rate. Specifically, suppose one discards the periods starting with 1911 and 1916 because of WWI, 1921 and 1926 because of the over-valuation of sterling, 1931 and 1936 because of the depression, 1941 and 1946 because of WWII and finally 1976 and afterwards because of the oil shocks, then one is left with the data displayed in Table 3. This shows a downward trend from 1850 until 1975. On the basis of this, it would seem that the United Kingdom had not undergone the third transition, at least not before 1975.

The placing of the first and second transitions fits nicely with the anecdotal facts. Feinstein (1999, p.651) argues that the condition of the workers only began to improve in the 1840s and notes that the period 1810 to 1840 was characterised by “the unrest and radicalism of Luddism, The Captain Swing protests, ‘collective bargaining by riot’, and Chartism. Only from the 1850s did this give way to a greater sense of harmony, safety and social stability that prevailed in the mid-Victorian ‘age of equipoise’.” It is almost too neat to suggest that especially the Luddism would be a rational response of workers if they perceived that capitalists were keeping the wage low by lowering the embodied labour to capital ratio and that these protests would have stopped once the capitalist strategy changed.

#### **4. Conclusion**

This paper has made two contributions: the first concerns the behaviour of the wage and the second the causes of unemployment. With regard to the former, it showed that the wage only started to rise nearly a century and a half after the establishment of capitalism and the beginning of the rise in income and capital per capita; and then it provided a formal explanation for this unnoticed and puzzling phenomenon. With regard to the latter, it set out a new explanation for unemployment based on an idea of Marx. This has two characteristics. First it implies a complex development of unemployment over time; and second it locates the cause of unemployment in the behaviour of capitalists who purposely create it in order to weaken the bargaining position of the workers.

The paper also raises an extremely important question. The model is a better description of 18th and 19th century economies than of the contemporary ones principally because it includes neither trade unions nor unemployment benefits. I am

the conclusion that unemployment was caused purposely by capitalists to weaken the bargaining position of workers. The important question is whether, in contemporary economies, this cause of unemployment is important empirically. If the answer was yes, it would imply a basic change in our understanding of unemployment.

## Acknowledgements

An earlier version of the paper entitled "Growth with a Marxian Reserve Army," was presented at the conference Economic Growth and Distribution in Lucca, Italy. I am indebted to the discussant, Rodolfo Signorino, for his detailed commentary, to one of the participants who, in a conversation afterward, pointed out an error, and to the participants generally for their comments. Thanks also go to the participants of the macro workshop of the IDEA program of the UAB, especially Melvyn Coles and Jordi Caballé. Comments by Valeri Sorolla were especially helpful. Finally the data provided by Charles H. Feinstein greatly improved the thrust of the paper. Financial assistance is acknowledged from grant SEC 2003-0036 of the Spanish Ministry of Science and Technology and the Barcelona Economics Program of CREA.

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## Appendix

	1771	1781	1791	1801	1811	1816	1821	1831	1841	1851	1861	1871	1881
<i>S</i>	4.01	4.32	4.66	5.01	5.69	6.05	6.41	7.37	8.49	9.38	10.52	11.66	12.95
<i>K</i>	257	281	315	364	422	459	497	609	789	1037	1271	1615	2100
<i>k</i>	62.8	65.0	67.4	71.1	74.2	75.8	77.5	82.5	93.2	110.5	120.8	138.5	162.2
	96	100	106	103	103	102	108	111	114	132	137	149	173
<i>UK</i>	97	100	105	99	98	97	102	104	107	129	139	154	183

*Table A1.* Definitions: *S*, The occupied population of Great Britain in millions; *K*, net capital stock at the beginning of the year for Great Britain at 1850-60 prices in millions of pounds. The figures for 1871 and 1881 were got by adjusting the figures for the UK, the figures for both *S* and *K* for 1816 are the averages of 1811 and 1821. and  $w_{UK}$  are wage indexes for Great Britain and for the UK. Sources: *S*: personal communication from Charles H. Feinstein; *K*, Table *viii* pp. 441-443 in Feinstein and Pollard (1988); and  $w_{UK}$ : table 5 p. 648 in Feinstein (1999).

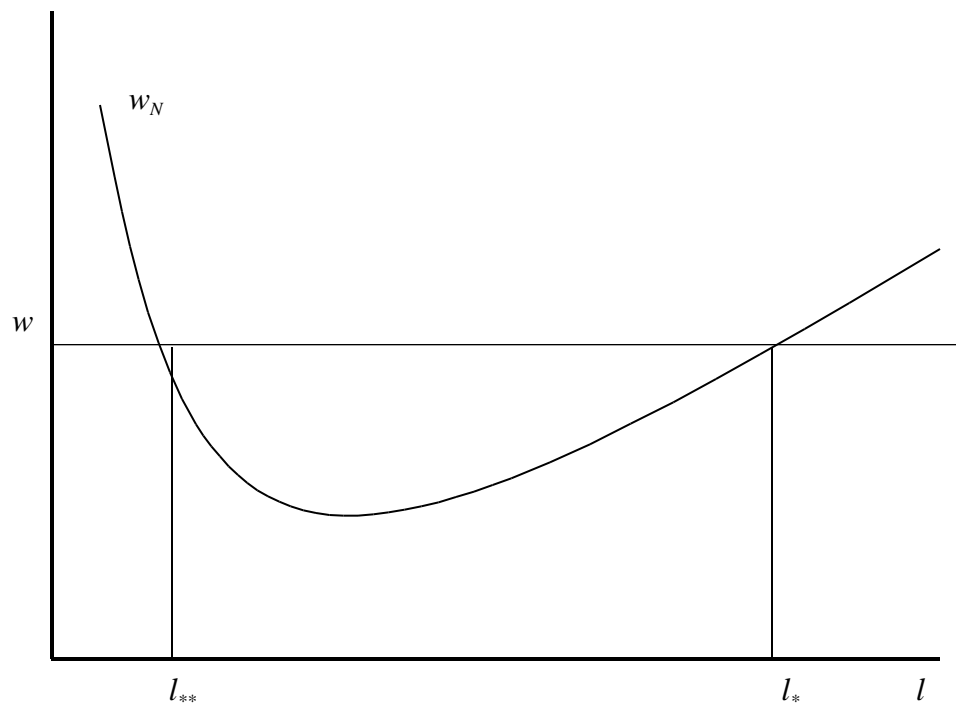


Figure 1. The Wage.

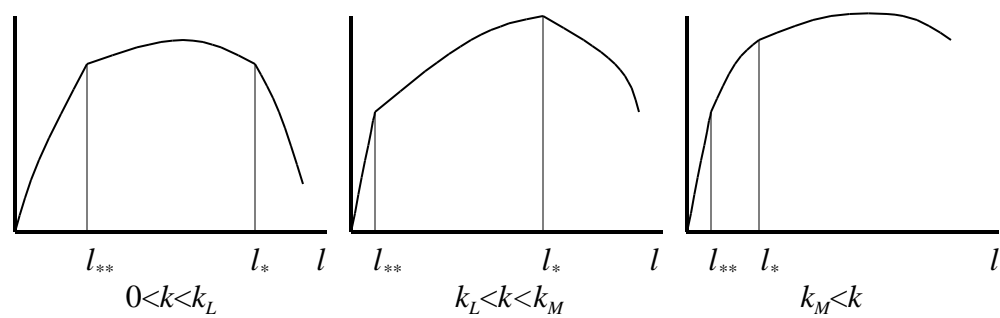


Figure 2. The shape of the profit function.

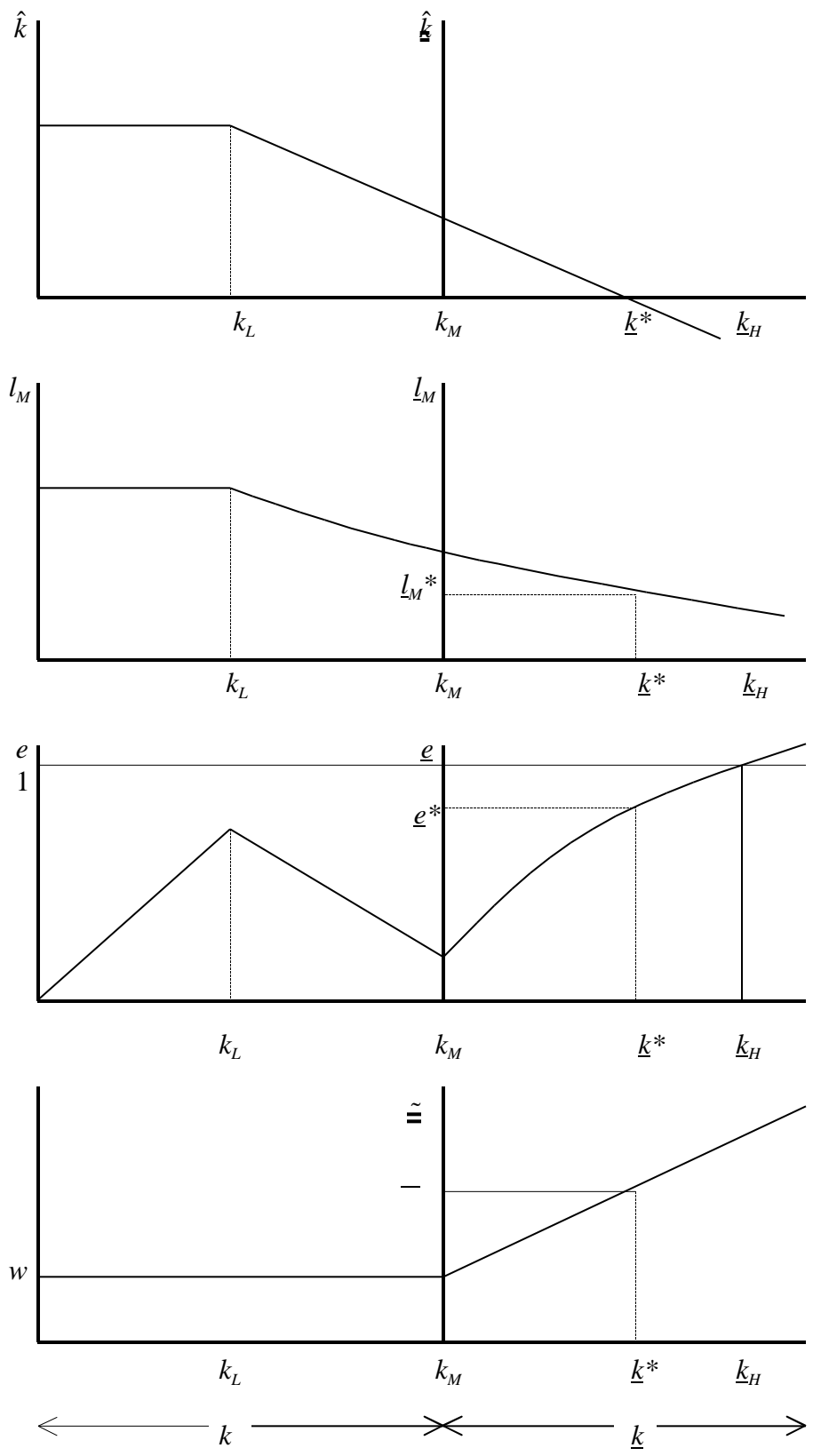


FIG. 2. Equilibrium capital stock in a two-country world



Figure 4. The UK rate of unemployment 1861-2003. The figure gives 5 year averages where each period starts with the indicated year. Sources: Phillips (1958) and OECD Main Economic Indicators, Historical Statistics and Main Economic Indicators, various numbers.

### Notes

<sup>1</sup> Goodfriend and McDermott (1995) and Hansen and Prescott (1999) have one sector models. Laitner (2000), Kögel and Prskawetz (2001) and Galor and Mountford (2000) have two sector models with manufacturing and agriculture. In most of these models the escape occurs when the technology which does not suffer from diminishing returns becomes profitable.

<sup>2</sup> These papers vary in the way knowledge is created. Jones (2001) emphasises the effect of population size, Galor and Weil (2000) have this but specifically model the way this interacts with education. Finally Galor and Moav (2001, 2002) have pro-education mutants triggering the escape.

<sup>3</sup> The explanations that depend on the existence of a technology that does not suffer decreasing returns are a bit unconvincing because, at a macro level, it is difficult to imagine a technology that uses no fixed factor. With respect to the accumulation of knowledge, Skocpol (1979, chap. 2) explains how one of the major causes of the French revolution was the inability of the French peasant agriculture to keep up with the British capitalist one. The writings of the physiocrats was a failed attempt to reform French agriculture so that it could keep up with the British. (See Eltis 2001.) If only knowledge were at issue, the French could have crossed the channel and just looked at what the British were doing. Instead they needed a revolution to change the social structure.

This point is related to the famed Brenner debate. (See Aston and Philpin 1985.) The difference is that Brenner claimed that the whole of pre-modern European economic history, not just the escape from the Malthusian trap, had to be understood in terms of class structure.

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This point has also been made in a different context by Hall and Jones (1999) who claim that differences in per capita income between countries can be explained by the extent that European cultural values, largely capitalistic, have been accepted.

<sup>4</sup> The locus classicus is Marx (1983 chap. 25). Except for the statements about the subsistence wage, this description is the generally accepted one, see Cottrell and Darity (1988). The statements about the subsistence wage are controversial, see Hollander (1984) and Baumol (1983). The question is whether the reserve army would make the wage fall to the level of physical subsistence. According to Cottrell and Darity, the issue is still to be decided. I have portrayed Marx's position as above in order to make it fit with the model. The reader should be aware that this is an oversimplification.

<sup>5</sup> Marx (1983 chap. 25) gives a number of explanations of why the reserve army is created. Some of these have been studied. For example Bowles (1985) emphasises worker discipline while Goodwin (1967) studies how labour scarcity may weaken capitalists which in turn slows capital growth and re-establishes the reserve army. There are many papers on these themes. What is claimed here is that the specific mechanism of weakening the bargaining strength has not been studied.

<sup>6</sup> The Shapiro and Stiglitz (1984) efficiency wage model suffers from a similar problem which, only after 18 years, has been fixed by Brecher, Chen and Choudhri (2002).

<sup>7</sup> The assumption that the capitalist could not go back to the pool of the unemployed may have seemed artificial. But this is compensated for by the bargaining strength parameter. For example, if it was costless to go back,  $\beta$  would be 1 and the outcome would be the disagreement payoff.

<sup>8</sup> Bean and Pissarides (1993) has just this model of bargaining where the capitalist chooses employment strategically to influence the wage. The only difference is that the disagreement payoff effect is weak, the  $w_N$  curve falls throughout so that these considerations tend to increase employment. Here, as will be seen, the effect is the reverse.