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# Money and Prices in Models of Bounded Rationality in High Inflation Economies

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**Abstract**

This paper studies the short run correlation of inflation and money growth. We study whether a model of learning can do better than a model of rational expectations, we focus our study on countries of high inflation. We take the money process as an exogenous variable, estimated from the data through a switching regime process. We find that the rational expectations model and the model of learning both offer very good explanations for the joint behavior of money and prices.

*Keywords:* Inflation and money growth, switching regimes, quasi-rationality.

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# 1 Introduction

The purpose of this paper is to explore the empirical implications of departing from rational expectations in very simple money demand models. The objective is to study the joint distribution of money and prices.

It is well known that in the long run money growth and long run inflation are highly related[1] but that in the short run this relationship is much less strong. It has been long recognized that standard rational expectations monetary models imply a correlation between money and prices in the short run that is way too high relative to the data. The reason is that velocity (the inverse of real money demand) fluctuates too little in the models relative to the data and so does the ratio of money to prices. Most attempts at reconciling this feature have explored models with price stickiness or with segmented markets.[2] Ours, is an attempt to explain the potential role of sticky expectations in driving the high frequency movements of money and prices.

Recent monetary models of bounded rationality specify processes that imply more sluggish adjustment of inflation expectations than the rational expectations versions. Thus, as long as velocity depends on expected inflation, it will also exhibit more sluggish movements than the rational expectations version. Therefore, the ratio of money and prices will also be more sluggish, so the implications for the co-movement between money and prices may be different from the rational expectations version and, hopefully, closer to the data. This paper is an attempt at clarifying this issue.

The literature on models of learning in macroeconomics has been very productive in the last two decades[3] but the applications of models of learning to empirical issues is relatively scant. Some references are Chung (1990), Arifovic, Bullard and Duffy (1997), Timmerman (1993), Sargent (1999), Evans and Honkapohja (1993) and Marcet and Nicolini (2003). Most related to our work is the paper by Saint-Paul (2001) who argues that a model of boundedly rational behavior could explain the delayed response of prices to changes in money growth. As we discuss in detail in Marcet and Nicolini

[1]This has been documented, for example, in Lucas (1980), Fitzgerald (1999) and McCandless and Weber (1995).

[2]Some examples of papers explaining the relatively low short run correlation of inflation and money growth are Rotemberg (1984), Grossman and Weiss (1983), Alvarez and Atkeson (1997) and Alvarez, Atkeson and Edmond (2001).

[3]See Sargent (1993), Marimon (1997) and Evans and Honkapohja (2001) for surveys.

(2003) (MN) the main obstacle for using models with bounded rationality to explain actual data has been that there are too many ways of being irrational, leaving room for too many degrees of freedom. A methodological device that we propose in MN to face this "free-parameters" problem is to allow only for expectation-formation mechanisms that depart from the true conditional expectation only by a small distance. We propose three different (lower) bounds to rationality and show that in a model similar to the one we use in this paper they become operative in determining the equilibrium values of the model parameters, therefore solving the free parameters problem. A key feature of the bounds, is that agents are "almost" rational in the sense that the expected value of the difference between expected or perceived inflation and the true conditional expectation of inflation is very small.[4] In this paper we follow that methodological assumption and we impose the bounds.

We show in MN that a nearly-rational model of learning can have very different implications from a rational expectations model in a setup to explain seiniorage-driven hyperinflations. The learning mechanism we use combines tracking with least squares, two of the most common mechanisms used in the literature. That mechanism works well both in stable as well as in changing environments, so it produces good forecasts both in countries with low and stable inflation and countries with high average inflation that experience, from time to time, recurrent burst of hyperinflations. We show in that paper that the equilibrium outcome can be very different from the rational expectations equilibria *only* when the government follows a high average seignorage policy, which goes along with high money growth. If the average money growth rate is low, the only equilibrium with bounded rationality is close to the rational expectations equilibrium. So, the imposition of bounds on the mistakes agents can make in equilibrium implies, in MN, that the rational expectations and bounded rationality hypothesis are almost observationally equivalent, except in countries where inflation is high on average. The reason for this result is that countries with low inflation also exhibit quite stable inflation. Thus, in these stable environments, the bounds imply that agents learn fast the true structure of the model and the outcome converges to the rational expectations outcome very quickly. However, countries with high average inflation also exhibit very unstable environments, as the econometric estimates we provide in the appendix of this paper clearly testify. In these changing environments it is much harder - although not impossible, since

[4] Rational expectations implies that difference to be exactly equal to zero.

the equilibrium must satisfy the bounds -to learn the true structure of the model with simple backward looking schemes, and therefore the equilibrium outcome can be very different from the rational expectations equilibrium for a very long transition, characterized by recurrent bursts in inflation rates.

In this paper we use this property of the model with learning in changing environments to compare the empirical implications of the bounded rationality hypothesis relative to the rational expectations version. While in our previous work we were concerned with the issue of whether or not periods of hyperinflations and high money growth could emerge endogenously, here we impose periods of high money growth by assuming a switching regime for exogenous money growth. So, we attempt to explain the potential role of sticky expectations in driving the high frequency movements of money and prices in high inflation countries, when the learning equilibria satisfies the bounds and is close to the conditional expectations. Thus, we can evaluate the empirical performance of the learning model relative to the rational expectations version, while not being subject to the free parameters criticism.

The model has a single real demand for money equation that is decreasing with expected inflation, and were the exogenous driving force is the money supply. We then fit a Markov switching regime statistical model for the money growth rate for five high inflation countries and solve both the bounded rationality and rational expectations versions of the model. Finally, we compare moments of the joint distributions of money and prices on the model and the data. We conclude that both versions of the model deliver the same quantitative implications and fit the data well. Thus, the high frequency behavior of money and prices in high inflation countries does not offer evidence either in favor nor against the bounded rationality hypothesis. Hence, this demonstrates that imposing lower bounds on the learning mechanism might not cause the model of learning to have very different behavior from the rational expectations version.

Section 2 describes the model, it explains the reasons that a model of learning can have different implications than rational expectations, it fits the Markov switching regime to the data, it solves both the rational expectations and learning versions of the model and calibrates the money demand. Section 3 presents empirical results and compares the models to the data. Section 4 concludes.

# 2 The Model

The model consists of a demand for cash balances given by

$$P_t = \frac{1}{\phi} M_t + \gamma P_{t+1}^e, \tag{1}$$

where $\phi, \gamma$ are positive parameters. $P_t, M_t$ are the nominal price level and the demand for money, and $P_{t+1}^e$ is the forecast of the price level for next period. The driving force of the model is given by the stochastic process followed by the growth rate of money. This well known money demand equation is consistent with utility maximization in general equilibrium in a context of overlapping generations model.

To complete the model one needs to specify the way agents forecast the future price level. In what follows, we compare the rational expectations version with a version in which agents use an ad-hoc algorithm that depends on past information to forecast future prices. There are several ways in which one can restrict the bounded rationality, or learning, version of the model to be "close" to rational expectations. For example, it has been common to study the conditions under which the equilibrium outcome of the learning model converges to the rational expectations model. To be more specific, consider, in the context of the money demand equation above, two alternative expectation formation mechanisms

$$\begin{aligned} P_{t+1}^e &= E_t \left[ P_{t+1} \right] \\ P_{t+1}^e &= L \left( P_{t-1}, P_{t-2}, ... \right). \end{aligned}$$

for a given function $L$[5]. We then obtain corresponding solutions $\{P_t^{RE}, P_t^L\}_{t=0}^{\infty}$ and, for the sake of the argument, assume that the rational expectations version has a unique solution. Then, letting $\rho(S_1, S_2)$ be some distance between two sequences $S_1$ and $S_2$, the distance

$$\rho(\{P_t^{RE}\}_{t=0}^{\infty}, \{P_t^L\}_{t=0}^{\infty}) \tag{2}$$

measures how different equilibria are. If the model is deterministic, convergence to rational expectations can be written as

[5]Strictly speaking, $L$ should depend on the time period, but we leave this dependence implicit.

$$\lim_{t\to\infty}(P_t^{RE} - P_t^L) = 0$$

When this occurs, the equilibrium outcome under learning "looks like" a rational expectations equilibrium in the limit. Only the transition, if long enough, is then potentially suitable to distinguish learning from rational expectations. But if the transition is not restricted in any way, we either have observational equivalence or unrestricted learning outcomes.

Near-rationality can be interpreted as imposing restrictions on the size of the systematic mistakes the agents make in equilibrium. This amounts to consider the distance

$$\rho(\{P_t^L\}_{t=0}^{\infty}, \{L\,(P_{t-1}, P_{t-2}, ...)\}_{t=0}^{\infty}) \tag{3}$$

This distance should be zero under rational expectations. For example, if we impose

$$\rho(\{P_t^L\}_{t=0}^{\infty}, \{L\,(P_{t-1}, P_{t-2}, ...)\}_{t=0}^{\infty}) = 0 \tag{4}$$

then we have (2) satisfies $\rho(\{P_t^{RE}\}_{t=0}^{\infty}, \{P_t^L\}_{t=0}^{\infty}) = 0$, by definition of rational expectations. If instead, we impose that the mistakes need not be zero, but that they have to be small, we impose no restriction on the relationship between $P_t^{RE}$ and $P_t^L$. More precisely, one can find models where even if $L$ is restricted so that (3) is small the difference with the rational expectations outcome (i.e., (2)) can be very large. If, as it is sometimes the case, $P_{t+1}^{RE}$ can be written as a function $L$ of past values, then the rational expectations equilibrium is also a learning equilibrium. But there may be other learning equilibria that are near-rational and whose equilibrium is vastly different from RE. In this case, the empirical implications of the rational expectations version and the "small mistakes" version may be different, while the learning equilibrium is still close to being rational, in the proper metric defined by (3).

One example of this behavior is MN. In that paper we impose three bounds that effectively impose different metrics $\rho$. As we mentioned in the introduction, if inflation is low, then the only learning equilibrium in MN that satisfies the bounds is the rational expectations equilibrium. However, when inflation is high, there are learning equilibrium outcomes that look different from the rational expectations one. Thus, for our empirical investigation, we focus the analysis on high inflation countries only.

## 2.1 A Regime Switching Model for Money Growth

In order to fit the process for the nominal money supply, we first look at the evidence for five Latin-American countries that experienced high average inflation and very high volatility in the last decades: Argentina, Bolivia, Brazil, Mexico and Peru. We use data for M1 and consumer prices from the International Financial Statistics published by the International Monetary Fund. To compute growth rates, we take the logarithm[6] of the time $t+1$ variable divided by the time $t$ value of the same variable.

While the high inflation years are concentrated between 1975 and 1995 for most countries, the periods do not match exactly. Thus, we chose, for each country, a sub-period that roughly corresponds to its own unstable years. Figures 1.a to 1.e plot quarterly data on nominal money growth and inflation for the relevant periods in each case.

As it can be seen from the figures, for all those countries, average inflation was high, but there are some relatively short periods of bursts - the shaded areas - in both money growth and inflation rates, followed, again, by periods of stable but high inflation rates[7]. Note also that the behavior of the money growth rate, the driving force of the model, follows a very similar pattern. Thus, we propose to fit a Markov switching process for the rate of money growth[8].

[6] This measure underestimates growth rates for high inflation countries. With this scale, it is easier to see the movements of inflation rates in the - relatively - traquile periods in all the graphs. In what follows, we make the case that the data is best described as a two regime process, so this measure biases the result against us.

[7] This feature of the data has long been recognized in case studies of hyperinflations (see, for example, Bruno, Di Tella, Dornbusch and Fisher (1988)).

[8] Actually, data suggest that during the periods of hyperinflations, both the inflation rate and the rate of growth of the money supply are increasing over time, a fact that is consistent with the model in MN. The statistical model we fit assumes, for simplicity, that during the hyperinflations both have a constant mean. We tried to fit to the data a switching regime model that allowed for growth of the growth rate in the hyperinflationary regime, but the growth element turned out not to be statistically significant. Assuming a constant mean for the growth rate of money in the hyperinflation regime biases the results against the rational expectations model since, in the learning version, agents do not take

Our empirical analysis is based on quarterly data, since we are interested in high frequency movements in money and prices. Our eyeball inspection of Figures 1.a to 1.e suggests the existence of structural breaks. This is confirmed by the breakpoint Chow Test that we present in the appendix for the five countries, so we model $\Delta\log(M_t) = \log(M_t) - \log(M_{t-1})$ as a discrete time Markov switching regime. We assume that $\Delta\log(M_t)$ is distributed $N(\mu_{s_t}, \sigma^2_{s_t})$; where $s_t \in \{0,1\}$. The state $s_t$ is assumed to follow a first order homogeneous Markov process with $\Pr(s_t = 1|s_{t-1} = 1) = q$ and $\Pr(s_t = 0|s_{t-1} = 0) = p$. The evolution of the first difference of the logarithm of the money supply can therefore be written as

$$\Delta\log(M_t) = \mu_0(1 - s_t) + \mu_1 s_t + (\sigma_0(1 - s_t) + \sigma_1 s_t)\,\varepsilon_t$$

where $\varepsilon_t$ is assumed to be iid. $N(0,1)$. All empirical results regarding the modelling of the money supply are also reported in Appendix A.

For all the countries, one state is always characterized by higher mean and higher volatility of $\Delta\log(M_t)$. Both pairs $(\mu_i,\sigma_i)$ are statistically significant, as well as the transition probabilities, $p$ and $q$, and in all cases, both states are highly persistent. The results represent very clear evidence that modelling the growth rate of money as having two states is a reasonable assumption. The rate of money growth in the high-mean/high-volatility state (henceforth, the high state) ranges from three times the rate of money growth in the low mean-low volatility state for Argentina to nine times for Bolivia while the volatility of the high state ranges from one and a half times the volatility of the low state for Mexico to eight times for Brazil. The differences across states are gigantic. The high state is always consistent with the existence of high peaks of inflation in each of the countries. These periods are represented as the shaded areas in Figures 1.a to 1.e.

These results clearly demonstrate that the economic environment can be characterized by one in which there are changes in the monetary policy regime. That is the reason why, if we want to produce a model of learning that reproduces this data and where agents are near-rational, we need to use a learning mechanism that produces reasonably low prediction errors when there is a regime switch.

into account the money supply rule, while we are providing the rational agents with a misspecified process for the money supply.

## 2.2 Rational Expectations Equilibrium

Rational expectations (RE) assumes $P_{t+1}^{e} = E_t\left(P_{t+1}\right) \equiv E_t\left(P_{t+1} \mid I_t\right)$ for all $t$, where $E_t$ is the expectation conditional on information up to time $t$, namely $I_t \equiv (M_t, P_t, M_{t-1}, P_{t-1}, ...)$. We look for non-bubble equilibria, and we conjecture that in the RE equilibrium

$$E_t\left(P_{t+1}\right) = \beta_t^{RE} P_t \tag{5}$$

with $\beta_t^{RE}$ being state dependent and

$$\beta_t^{RE} = \beta_0(1 - s_t) + \beta_1 s_t.$$

for some constants $\beta_0, \beta_1$. To solve for an equilibrium, we must find $(\beta_0, \beta_1)$.

Using the fact that $\log \frac{M_{t+1}}{M_t} \sim N\left(\mu_j, \sigma_j\right)$ we have that

$$\kappa_j \equiv E\left(\left.\frac{M_{t+1}}{M_t}\right| s_{t+1} = j, I_t\right) = \exp\left(\mu_j + \frac{1}{2}\sigma_j^2\right). \tag{6}$$

and that

$$E\left(\left.\frac{M_{t+1}}{M_t}\right| s_t = 0,\ I_t\right) = \kappa_0 p + \kappa_1(1-p) \tag{7}$$

$$E\left(\left.\frac{M_{t+1}}{M_t}\right| s_t = 1,\ I_t\right) = \kappa_0\left(1-q\right) + \kappa_1 q \tag{8}$$

Equilibrium prices are given by

*Lemma 1*

*The RE equilibrium in the model of this section is given by*

$$\begin{aligned} E\left(\left.\frac{P_{t+1}}{P_t}\right| s_t = 1,\ I_t\right) &= \beta_1 = \frac{\kappa_0\left(1-q\right) + \kappa_1 q + \kappa_0\kappa_1\gamma\left(1-q-p\right)}{1 + \gamma\kappa_0\left(1-p-q\right)} \\ E\left(\left.\frac{P_{t+1}}{P_t}\right| s_t = 0,\ I_t\right) &= \beta_0 = p\kappa_0 + \left(1-p\right)\kappa_1\frac{1-\gamma\beta_0}{1-\gamma\beta_1} \end{aligned}$$

*which gives the equilibrium price*

$$P_t = \frac{M_t}{\phi\left(1 - \gamma\beta_t^{RE}\right)}$$

*Proof*
We have

$$\begin{aligned}
E\left(P_{t+1}|s_t=0,\ I_t\right) &= pE\left(\frac{M_{t+1}}{\phi\left(1-\gamma\beta_0\right)}|s_{t+1}=0, s_t=0,\ I_t\right)+ \\
&\quad +\left(1-p\right)E\left(\frac{M_{t+1}}{\phi\left(1-\gamma\beta_1\right)}|s_{t+1}=1, s_t=0,\ I_t\right), \\
&= \frac{p}{\phi\left(1-\gamma\beta_0\right)}E\left(M_{t+1}|s_{t+1}=0, s_t=0,\ I_t\right)+ \\
&\quad +\frac{(1-p)}{\phi\left(1-\gamma\beta_1\right)}E\left(M_{t+1}|s_{t+1}=1, s_t=0,\ I_t\right), \\
&= \frac{p\kappa_0}{\phi\left(1-\gamma\beta_0\right)}M_t+\frac{\left(1-p\right)\kappa_1}{\phi\left(1-\gamma\beta_1\right)}M_t, \\
&= p\kappa_0P_t+\left(1-p\right)\kappa_1\frac{\left(1-\gamma\beta_0\right)}{\left(1-\gamma\beta_1\right)}P_t, \\
&= P_t\left(p\kappa_0+\left(1-p\right)\kappa_1\frac{\left(1-\gamma\beta_0\right)}{\left(1-\gamma\beta_1\right)}\right).
\end{aligned}$$

where the first equality arises from equation (1) and the conjecture that $P^e_{t+1}=E\left(P_{t+1}|s_t=0,\ I_t\right)=\beta_0P_t$. The third equality comes from (6), and the fourth comes from equation (1). Combining this expression with (5) we get

$$\beta_0=p\kappa_0+\left(1-p\right)\kappa_1\frac{1-\gamma\beta_0}{1-\gamma\beta_1}. \tag{9}$$

From an analogous derivation conditioning on $s_t=1$ we get

$$\beta_1=q\kappa_1+\left(1-q\right)\kappa_0\frac{1-\gamma\beta_1}{1-\gamma\beta_0}. \tag{10}$$

Solve this system of equations for the unknowns $(\beta_0,\beta_1)$ to get

$$\beta_1=\frac{\kappa_0\left(1-q\right)+\kappa_1q+\kappa_0\kappa_1\gamma\left(1-q-p\right)}{1+\gamma\kappa_0\left(1-p-q\right)}$$

and plugging this expression into (9) we obtain the solution for $\beta_0$.

Plugging this into (5) and (1) we get the equilibrium price. QED

Notice that expected inflation differs from expected money growth in state 0, due to the presence of the factor $\frac{1-\gamma\beta_0}{1-\gamma\beta_1}$ (or its inverse in state 1). This factor appears because under RE the inflation rate satisfies

$$\frac{P_t}{P_{t-1}} = \frac{1-\gamma\beta_{t-1}^{RE}}{1-\gamma\beta_t^{RE}} \frac{M_t}{M_{t-1}}.$$

so that if there is a change of regime from one period to the next, the ratio $\frac{1-\gamma\beta_{t-1}^{RE}}{1-\gamma\beta_t^{RE}}$ alters the change in inflation relative to the change in money growth. In this model, therefore, the velocity is not constant, due to the fact that expected inflation influences the relationship between money and prices. Notice that the difference between inflation and money growth is larger the larger the difference in expected inflations, and that if $p+q=1$ the two possible values of inflation are equal to the two possible values of money growth in each state.

## 2.3 Learning Equilibrium

Under learning, we use the same learning mechanism as in MN. Let

$$P_{t+1}^e = \beta_t P_t \tag{11}$$

where

$$\beta_t = \beta_{t-1} + \frac{1}{\alpha_t}\left(\frac{P_{t-1}}{P_{t-2}} - \beta_{t-1}\right), \tag{12}$$

The coefficient $\alpha_t$ is called the "gain" and it affects the sensitivity of expectations to current information. Two of the most common specifications for the gain sequence are tracking ($\alpha_t = \alpha$ for all $t$), which performs well in environments that change often, and least squares ($\alpha_t = \alpha_{t-1}+1$), that performs well in stationary environments. As the money supply process, the driving force of the model, is well approximated by a Markov Switching Regime with substantial persistence, a scheme that combines both mechanisms performs well, as we show in MN. The gain is assumed to follow

$$\begin{aligned} \alpha_t &= \alpha_{t-1}+1 \quad \text{if} \quad \left|\frac{\frac{P_{t-1}}{P_{t-2}}-\beta_{t-1}}{\beta_{t-1}}\right| \geq \nu \\ &= \overline{\alpha} \qquad\qquad\quad \text{otherwise} \end{aligned}, \tag{13}$$

where $\overline{\alpha}, \nu$ are the learning parameters. Thus, if errors are small, the gain follows a least squares rule, and as long as the regime does not switch, agents soon learn the parameters of the money supply rule. But if a large enough error is detected, the rule switches to a constant gain algorithm, so agents can learn the new parameters of the money supply rule faster.

The solution $\left\{\frac{P_t}{P_{t-1}}, \beta_t, \alpha_t\right\}$ must satisfy (12), (13) and,

$$\frac{P_t}{P_{t-1}} = \frac{1-\gamma\beta_{t-1}}{1-\gamma\beta_t}\frac{M_t}{M_{t-1}}$$

which is obtained by plugging (11) into (1).

## 2.4 Calibration: The rational expectations model

*Money demand*

We borrow the parameter values from our previous paper, in which we use observations from empirical Laffer curves to calibrate them. This is a reasonable choice: since one empirical implication of the original model is that recurrent hyperinflations characterized by two regimes occur when average inflation - driven by average seiniorage - is "high", we need to have a benchmark to discuss what high means. We use quarterly data on inflation rates and seignorage as a share of GNP for Argentina[9] from 1980 to 1990 from Ahumada, Canavese, Sanguinetti y Sosa (1993) to fit an empirical Laffer curve. While there is a lot of dispersion, the maximum observed seignorage is around 5% of GNP, and the inflation rate that maximizes seignorage is close to 60%. These figures are roughly consistent with the findings in Kiguel and Neumeyer (1992) and other studies. The parameters of the money demand $\gamma$ and $\phi$, are uniquely determined by the two numbers above. Note that the money demand function implies a stationary Laffer curve equal to

$$\frac{\pi}{1+\pi}m = \frac{\pi}{1+\pi}\phi\left(1-\gamma(1+\pi)\right) \tag{14}$$

where $m$ is the real quantity of money and $\pi$ is the inflation rate. Thus, the inflation rate that maximizes seignorage is

[9] The choice of country is arbitrary. We chose Argentina because we were more familiar with the data.

$$\pi^* = \sqrt{\frac{1}{\gamma}} - 1$$

which, setting $\pi^* = 60\%$, implies $\gamma = 0.4$. Using this figure in (14), and making the maximum revenue equal to 0.05, we obtain $\phi = 0.37$. For simplicity, we use these numbers for all the countries.

*Money supply*

For money supply, we use the estimated Markov switching models we discussed above. We fit this process to the observed behavior of money supply to each country. The results of the estimation are reported in Appendix A. While the money demand parameters are assumed the same for each country, the money supply process is estimated using data from each country.

*Learning parameters*

The parameters described above are sufficient to solve the rational expectations model. However, we still need to be specific regarding our choice of the (still free!) parameters of the learning process, $\overline{\alpha}, \nu$.

In MN, we provide an operational definition of a bound of the type described above. Intuitively, we search for values of the parameter $\overline{\alpha}$ that satisfy a rational expectations-like fixed point problem. We look for values of the parameter such that in equilibrium, agents make small systematic mistakes. Recall that the learning mechanism we propose is well suited to deal with changing environments, a result we show formally in MN. For our simulations, we use the equilibrium values we obtained in MN ($\overline{\alpha}$ between 2 and 4). For the value of $\nu$, we also follow MN, where we used a value that was roughly equal to two standard deviations of the prediction error[10].

# 3 Evaluating the models

As we already mentioned, the main goal of this paper is to investigate the ability of the simple money demand equation with exogenous money growth to replicate the short run relationship between the inflation rate and the

[10] We also solved the learning model with an alternative specification for $v$, given the Markov structure of the money growth process: we replace $v$ for $v_{st}$, where, if $s_t = 1$ is the state with the higher growth, we have $v_l = v$ and $v_1 = (\sigma_h/\sigma_l)v$. With this alternative specification we introduce the switching regime information into the learning mechanisn, but it did not make any difference in the results.

growth rate of money using both the rational expectations model and the "almost" rational version. We dub LM the equilibrium under the learning mechanism.

Following the RBC tradition, we characterize the data using empirical moments of the joint distribution of money and prices. Table 1 in Appendix B presents moments of their marginal distributions. As one should expect, average inflation is very similar to average money growth in each country (see, for example, Lucas (1980) for a discussion of this fact). There are slightly larger differences between the volatility of inflation and money growth rates in each country, without a clear pattern emerging from the table. We will focus our analysis of the joint distribution of money and prices on the crosscorrelogram. If velocity were constant, in countries where the volatility of inflation (and money growth) is much larger than the volatility of output growth, the contemporaneous correlation between money and prices ought to be close to one, and the leads and lags of the cross-correlation should be equal to the autocorrelogram of the money growth process. Figure 2.a and Figure 2.b present the leads and lags for money and prices for the five countries. It is interesting to point out that, as it has been documented before, money and prices are highly correlated contemporaneously, contrary to the case of middle and low inflation countries where the correlation is less strong[11]. In particular, the contemporaneous correlation for Mexico, the country in the sample with lower average inflation, is substantially lower than for the other countries. The results of the simulations are shown in Tables 2.a to 2.e. The columns show the moments for the inflation generated by the model under rational expectations, and under LM. Under LM we have two columns, one for each of the two possible values for $\overline{\alpha} = \{10/4, 10/3\}$.

As we can see, the three simulations (the one under RE and the two under LM) give very similar results for each of the countries. In fact, for Argentina, neither the mean nor the standard deviation of the RE model are statistically different from the ones generated by the two different versions of LM. Most importantly, none of these moments are statistically different from the actual moments of inflation. The same is true for Peru, Bolivia and Mexico. Similar results arise for Brazil, with the exception that the volatility of the inflation rate under learning overestimates the true volatility. It is interesting to point out that while the simulations for Argentina, Bolivia and

[11] See Alvarez, Atkenson and Edmond (2001) and references therein for theoretical work that aims at matching these correlations for low inflation countries.

Peru generate volatilities that underestimate the actual ones, the volatility in the model overestimates the actual volatility for Brazil and Mexico.

Figures 3.a to 3.e and 4.a to 4.e present the leads and lags of the cross-correlogram between $\log(M_t/M_{t-1})$ and the inflation rates generated by RE, LM and the actual one for each of the five countries.[12] We also include an approximation for the confidence band, $(\pm 2/\sqrt{T})$ for the cross-correlogram of the actual series (the dotted lines).

For each country, the leads and lags graphs generated with the three series are very similar. Also, with the exception of Mexico, none of the cross-correlograms generated by either model is significantly different from the actual one. Both mechanisms perform equally well in approximating the actual cross-correlogram. A noticeable fact is that in every country except Mexico the contemporaneous correlation is lower in the simulated series than in the actual ones. This is because, as we pointed out after Lemma 1, the two states of expected inflation do not match the two states of expected money growth, and velocity in the model is not equal to one; since LM looks fairly close to RE in this model, the same occurs in the learning model. The only exception is Mexico, the country with the lowest average inflation, where the simulated correlations are still close to one but the actual correlation is less than l5 (for the correlation, see the intercept in Figures 3). Furthermore, Mexico is the only country which presents significant differences between the actual and the simulated cross-correlogram. This is due to the fact that Mexico's actual inflation is not so correlated to $\log(M_t/M_{t-1})$ as the other variables are (shown in Figures 1.a and 1.b), and as the simulated inflation are highly correlated to $\log(M_t/M_{t-1})$ they perform worse for this particular case. But even in the case of Mexico the prediction of the money demand model is not very sensitive to the expectation formation mechanism.

The most important conclusion of the paper is that, although from the theoretical point of view, sticky expectations was a promising avenue to explain the short run behavior of money and prices, quantitatively, the models are empirically equivalent if the money supply is forced to behave like in the data. Both models do imply different behavior for expected inflation. Indeed, Figure 5 plots expected inflation for both models for the case of Argentina[13]. The definitions are the same as the ones stated previously in Tables 2.a-2.e,

[12] More precisely, for each model or data, the entry corresponding to $j$ in the horizontal axis of Figures 3 represents $\rho\left(\log\left(\frac{M_t}{M_{t-1}}\right), \frac{P_{t-j}}{P_{t-j-1}}\right)$.

[13] The same happens in the other five countries.

i.e., "$BETA\_LMi$", for $i = 0, 1$ corresponds to the expected inflation, $\beta$ generated by the LM labeled as $\left(\frac{10}{3}, \frac{10}{4}\right)$, respectively in Tables 2.a-2.e, while "$BETA\_RE$" stands for the expected inflation generated by the RE model. As it can be seen in the figure, expected inflation under learning exhibits more high frequency movements. Thus the real money demand also moves more under learning. However, the impact on the behavior of the cross correlogram is quantitatively very small. This suggests that for the calibrated values of the money demand elasticity, the role that high frequency fluctuations on expectations have on the short run dynamics of money and prices is negligible.

It is clear that whether or not the LM equilibrium is very different from RE will depend on the particular values for money demand that are used. For very low values of $\gamma$ the learning model can not be very different from the RE version, since such values of $\gamma$ would imply that expectations play no role in determining equilibrium outcomes. The opposite is true, obviously, if $\gamma$ is very high. Also, the variance of money growth is important, since it could dominate the movements in the variables. So what is of interest is to report values that have some empirical relevance, as we have done above.

# 4 Conclusion

The purpose of this paper is to explore the potential role of sluggish "almost-rational" expectations in explaining the high frequency movements between money and prices. The evidence has shown a sluggish response of prices to money, so sluggish expectations imply movements on velocity that could potentially explain the observed behavior of inflation. We impose the methodological restriction that the learning mechanism must produce very good forecasts within the model in a way that resembles rational expectations. We argue that the learning model we propose satisfies the methodological restriction in countries in where monetary policy exhibits frequent and substantial changes of regime and we argue that, in principle, the models are not necessarily observationally equivalent in that case. We identify five Latin-American countries and fit a Markov switching process for the exogenous driving force - the money growth rate. There is ample evidence in favor of the regime switching structure. We quantitatively solve a calibrated money demand equation under the assumptions of both rational expectations and learning with the methodological restriction. We find that for the calibrated value of the elasticity of the money demand both models generate very similar empirical implications that match the facts in almost every dimension. Thus, we conclude, the short run behavior of money and prices in high inflation countries does not offer evidence neither in favor nor against the bounded rationality hypothesis in expectations formation if agents are not allowed to make large mistakes in equilibrium, and that both models fit the data well.

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# A APPENDIX, Empirical Results

## A.1 Chow Test for Structural Breaks

The Chow Test for the corresponding sub-samples generates the following results

*Argentina (1975:01 1992:04)*

Sub-samples 1975:01-1988:04, 1989:01-1990:01, 1990:02-1992:04

| Chow Breakpoint Test: | 1989:1 1990:1 | Probability |
|---|---|---|
| F-statistic | 3.775243 | 0.002964 |
| Log likelihood ratio | 22.09987 | 0.001161 |

*Bolivia (1975:01 1995:04)*

Sub-samples 1975:01-1983:03, 1983:04-1986:04, 1987:01-1995:04

| Chow Breakpoint Test: | 1983:4 1986:4 | Probability |
|---|---|---|
| F-statistic | 4.205059 | 0.003498 |
| Log likelihood ratio | 16.47067 | 0.002448 |

*Brazil (1980:01 1995:04)*

Sub-samples 1980:01-1987:04, 1988:01-1991:01, 1991:02-1995:04

| Chow Breakpoint Test: | 1988:1 1991:1 | Probability |
|---|---|---|
| F-statistic | 4.666468 | 0.001733 |
| Log likelihood ratio | 18.12746 | 0.001165 |

*Mexico (1975:01 1995:04)*

Sub-samples 1975:01-1989:04, 1990:01-1992:03, 1992:04-1995:04

| Chow Breakpoint Test: | 1990:1 1992:3 | Probability |
|---|---|---|
| F-statistic | 4.251409 | 0.003259 |
| Log likelihood ratio | 16.63831 | 0.002272 |

*Peru (1975:01 1995:04)*

Sub-samples 1975:01-1989:04, 1990:01-1991:01, 1991:02-1995:04

| Chow Breakpoint Test: | 1990:1 1991:1 | Probability |
|---|---|---|
| F-statistic | 16.67572 | 0.000000 |
| Log likelihood ratio | 53.90424 | 0.000000 |

## A.2 Markov Switching Regime Estimation Results

In this sub-section we present the results of the Markov Switching Regimes estimation. Let $p = \Pr(s_t = 0|s_{t-1} = 0)$, $q = \Pr(s_t = 1|s_{t-1} = 1)$, and let $\mu_i$ and $\sigma_i$ be the mean and the standard deviation of the growth rate of money in state $i$. The following tables summarize the results of the estimation.

*Argentina (1975:01 1992:04)*

| | Coeff. | Std Error. | t-statistic |
|---|---|---|---|
| $\mu_0$ | 0.175533792 | 0.018297495 | 9.593323735 |
| $\mu_1$ | 0.454399941 | 0.098682095 | 4.604684802 |
| $q$ | 0.915099885 | 0.066209340 | 13.82131101 |
| $p$ | 0.919361013 | 0.074032079 | 12.41841415 |
| $\sigma_0$ | 0.072853206 | 0.011943142 | 6.100003477 |
| $\sigma_1$ | 0.303486106 | 0.079758969 | 3.805040466 |

*Bolivia (1975:01 1995:04)*

| | Coeff. | Std Error. | t-statistic |
|---|---|---|---|
| $\mu_0$ | 0.059627604 | 0.009754163 | 6.113041314 |
| $\mu_1$ | 0.536370837 | 0.420877046 | 1.274412187 |
| $q$ | 0.935126471 | 0.066531539 | 14.05538607 |
| $p$ | 0.986617102 | 0.070142749 | 14.06584602 |
| $\sigma_0$ | 0.050541181 | 0.003716448 | 13.59932350 |
| $\sigma_1$ | 0.373992612 | 0.159586583 | 2.343509119 |

*Brazil (1980:01 1995:04)*

| | Coeff. | Std Error. | t-statistic |
|---|---|---|---|
| $\mu_0$ | 0.157931616 | 0.027645929 | 5.712653628 |
| $\mu_1$ | 0.517317340 | 0.128746245 | 4.018115930 |
| $q$ | 0.926016849 | 0.125106881 | 7.401805917 |
| $p$ | 0.957205334 | 0.068582989 | 13.95689156 |
| $\sigma_0$ | 0.045445418 | 0.019317040 | 2.352607738 |
| $\sigma_1$ | 0.389689368 | 0.089648053 | 4.346880447 |

*Mexico (1975:01 1995:04)*

| | Coeff. | Std Error. | t-statistic |
|---|---|---|---|
| $\mu_0$ | 0.064677791 | 0.010181050 | 6.352762507 |
| $\mu_1$ | 0.206343338 | 0.021967776 | 9.393000814 |
| $q$ | 0.733328049 | 0.174816708 | 4.194839597 |
| $p$ | 0.951845883 | 0.060318779 | 15.780257828 |
| $\sigma_0$ | 0.047614481 | 0.011171478 | 4.262147003 |
| $\sigma_1$ | 0.061059573 | 0.025394331 | 2.404456906 |

*Peru (1975:01 1995:04)*

| | Coeff. | Std Error. | t-statistic |
|---|---|---|---|
| $\mu_0$ | 0.119329114 | 0.015084511 | 7.910632032 |
| $\mu_1$ | 0.744060948 | 0.190436648 | 3.985303268 |
| $q$ | 0.830803948 | 0.126355006 | 6.567003388 |
| $p$ | 0.971727572 | 0.048361910 | 19.86071300 |
| $\sigma_0$ | 0.082091249 | 0.008656942 | 10.06842486 |
| $\sigma_1$ | 0.383326249 | 0.134969389 | 2.847524436 |

# B Simulation Results

Table 1 shows the first and second moments of inflation and money growth for every country. Tables 2.a to 2.e show the first and second moments of the simulations.

| | | | **Table 1** | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| | | | Inflation ($\Delta \log P_t$) | | Money Growth ($\Delta \log M_t$) | |
| **Sample** | **Country** | | $\boldsymbol{\mu}$ | $\boldsymbol{\sigma}$ | $\boldsymbol{\mu}$ | $\boldsymbol{\sigma}$ |
| 1975:01-1992:04 | Argentina | | 0.146870 | 0.132087 | 0.138898 | 0.117783 |
| 1975:01-1995:04 | Bolivia | | 0.069557 | 0.138701 | 0.072275 | 0.117080 |
| 1980:01-1995:04 | Brazil | | 0.177091 | 0.131648 | 0.174454 | 0.158965 |
| 1975:01-1995:04 | Mexico | | 0.036109 | 0.029010 | 0.037837 | 0.031375 |
| 1975:01-1995:04 | Peru | | 0.104011 | 0.140325 | 0.095859 | 0.123695 |

| | | **Table 2.a** | | |
|---|---|---|---|---|
| | | **Argentina** | | |
| **Sample:** | 1975:01-1992:04 | | | |
| | | | $\mu$ | $\sigma$ |
| | $True$ | | 0.146870 | 0.132087 |
| | $\overline{\alpha} = \frac{10}{3}$ | | 0.142280 | 0.137145 |
| | $\overline{\alpha} = \frac{10}{4}$ | | 0.141884 | 0.152390 |
| | $RE$ | | 0.144074 | 0.123287 |
| | | | | |

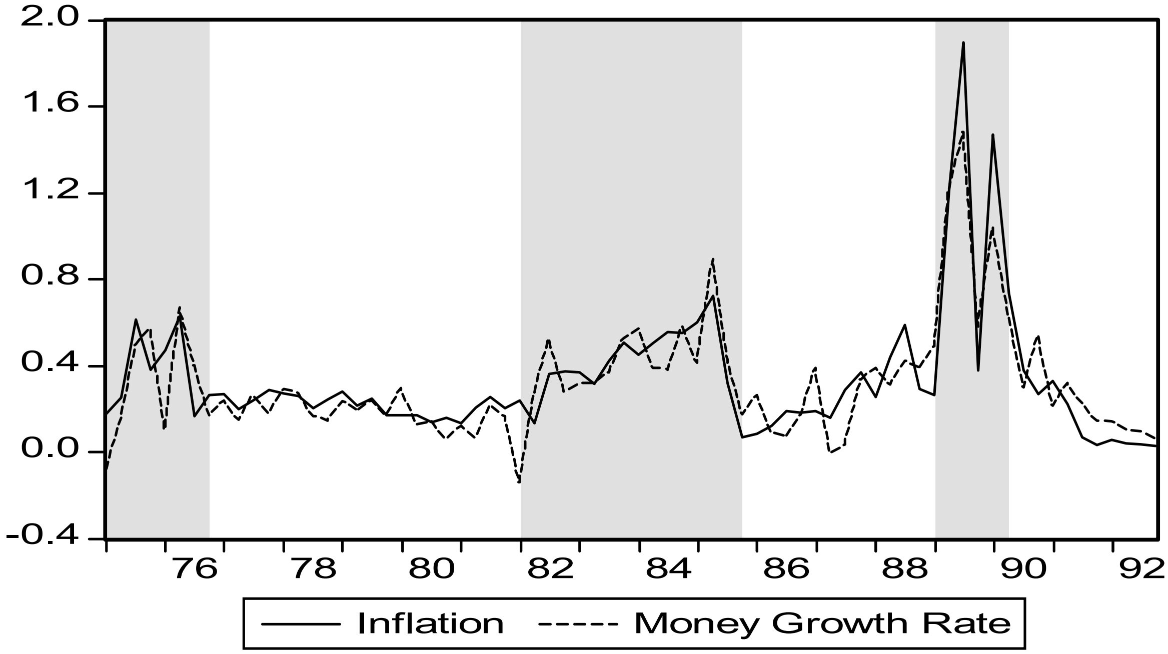
| | | **Table 2.b** | | |
|---|---|---|---|---|
| | | **Bolivia** | | |
| **Sample:** | 1975:01-1995:04 | | | |
| | | | $\mu$ | $\sigma$ |
| | $True$ | | 0.069557 | 0.138701 |
| | $\overline{\alpha} = \frac{10}{3}$ | | 0.072237 | 0.146258 |
| | $\overline{\alpha} = \frac{10}{4}$ | | 0.072189 | 0.161556 |
| | $RE$ | | 0.072755 | 0.127636 |
| | | | | |

| | | **Table 2.c** | | |
|---|---|---|---|---|
| | | **Brazil** | | |
| **Sample:** | 1980:01-1995:04 | | | |
| | | | $\mu$ | $\sigma$ |
| | $True$ | | 0.177091 | 0.131648 |
| | $\overline{\alpha} = \frac{10}{3}$ | | 0.164544 | 0.217368 |
| | $\overline{\alpha} = \frac{10}{4}$ | | 0.163226 | 0.295389 |
| | $RE$ | | 0.180857 | 0.168293 |
| | | | | |

| | | Table 2.d | | |
|---|---|---|---|---|
| | | **Mexico** | | |
| **Sample:** | 1975:01-1995:04 | | | |
| | | | $\mu$ | $\sigma$ |
| | $True$ | | 0.036109 | 0.029010 |
| | $\overline{\alpha} = \frac{10}{3}$ | | 0.037844 | 0.032996 |
| | $\overline{\alpha} = \frac{10}{4}$ | | 0.037805 | 0.033354 |
| | $RE$ | | 0.038116 | 0.033121 |
| | | | | |

| | | Table 2.e | | |
|---|---|---|---|---|
| | | **Peru** | | |
| **Sample:** | 1975:01-1995:04 | | | |
| | | | $\mu$ | $\sigma$ |
| | $True$ | | 0.104011 | 0.140325 |
| | $\overline{\alpha} = \frac{10}{3}$ | | 0.094706 | 0.146434 |
| | $\overline{\alpha} = \frac{10}{4}$ | | 0.094602 | 0.161485 |
| | $RE$ | | 0.096507 | 0.174697 |
| | | | | |

**<u>Fig 1.a</u>: Argentina**




**<u>Fig 1.b</u>: Bolivia**

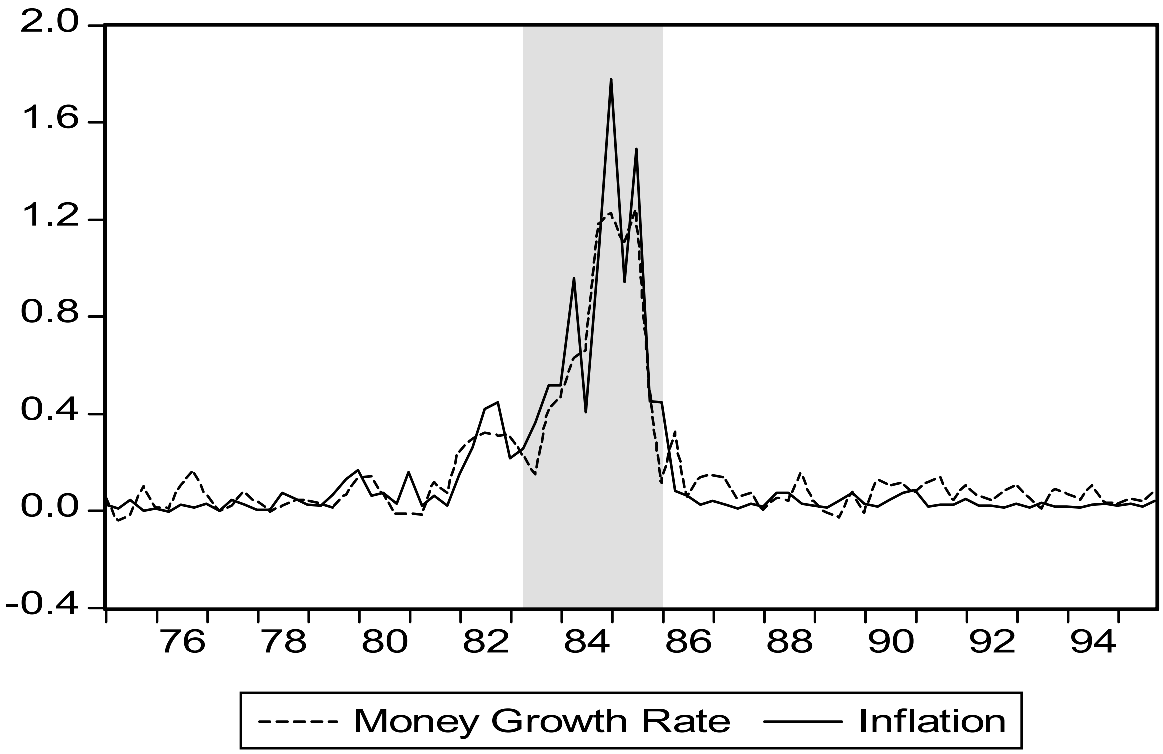


**Fig 1.c: Brazil**

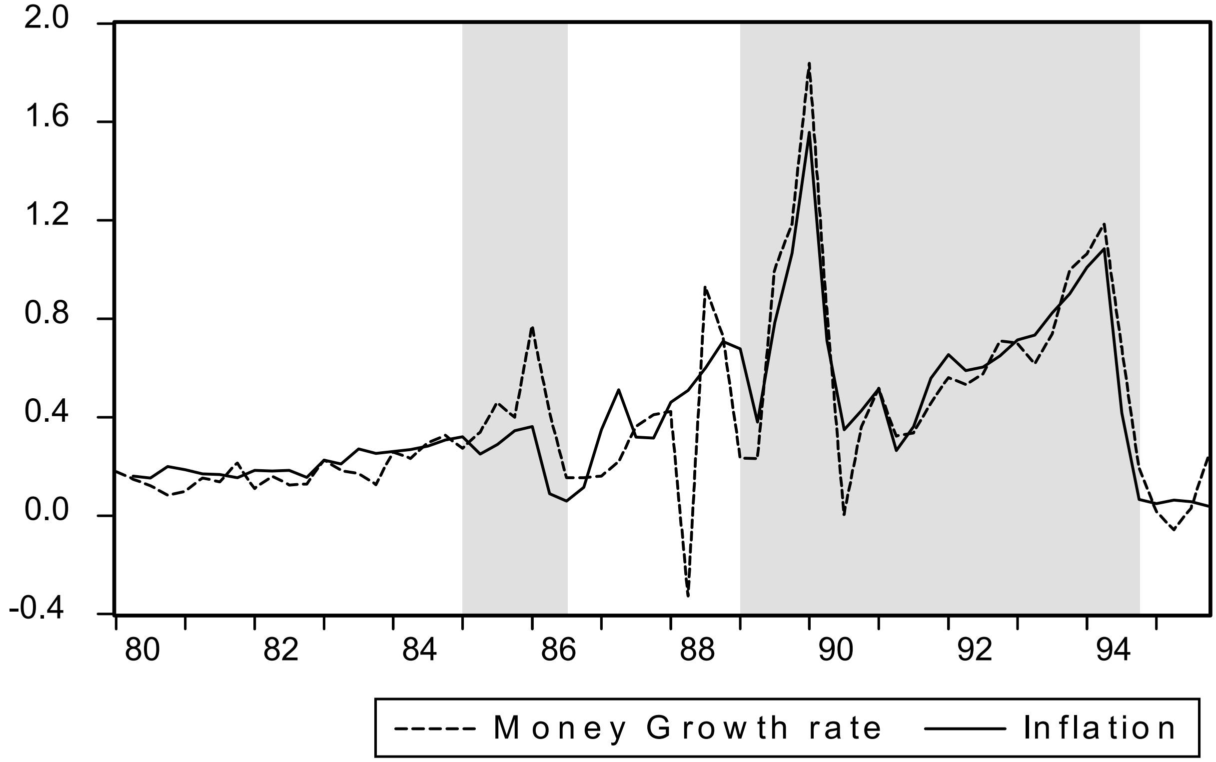
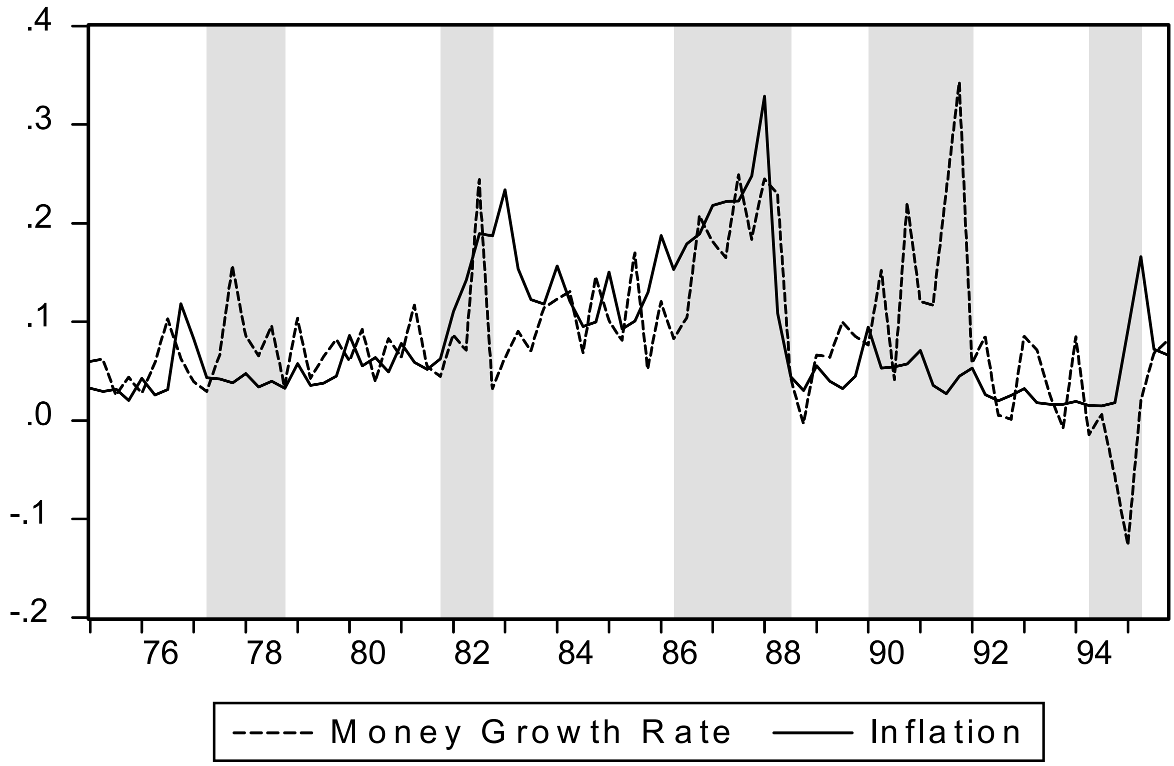



**Fig 1.d: Mexico**



**<u>Fig 1.e</u>: Peru**

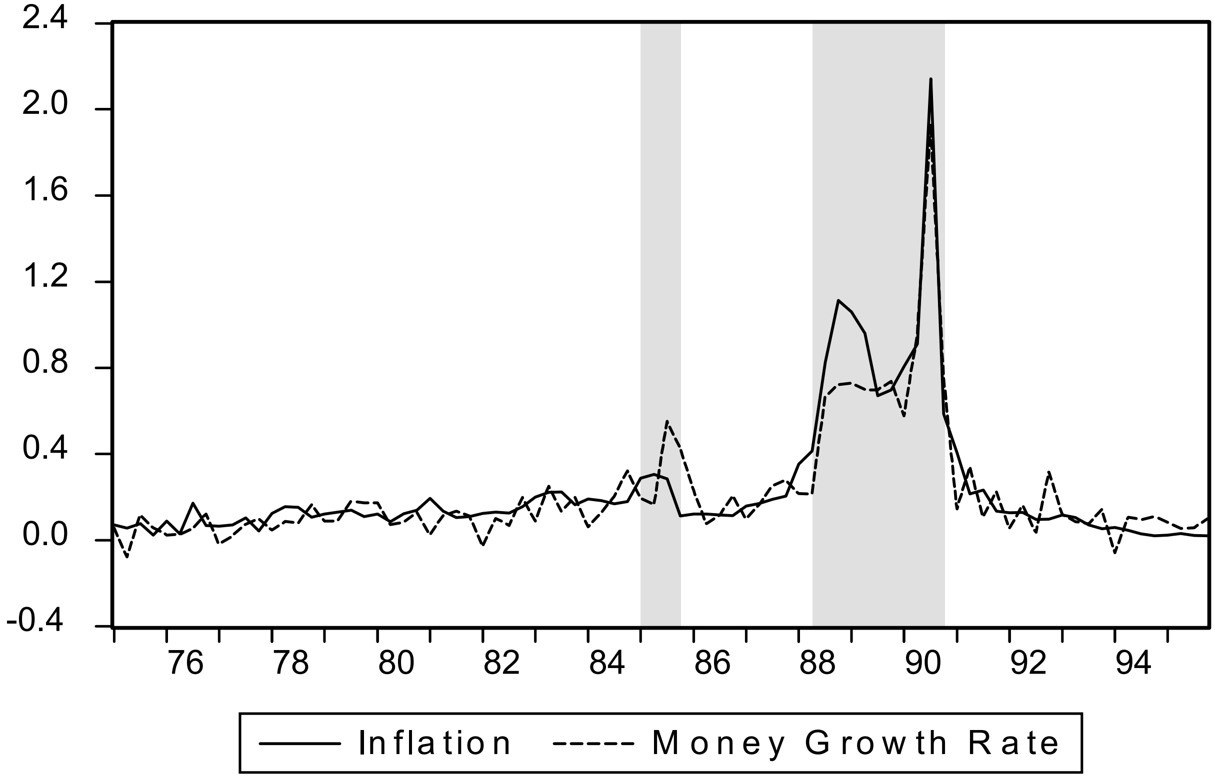


**Fig 2.a**
**Lead of all countries**

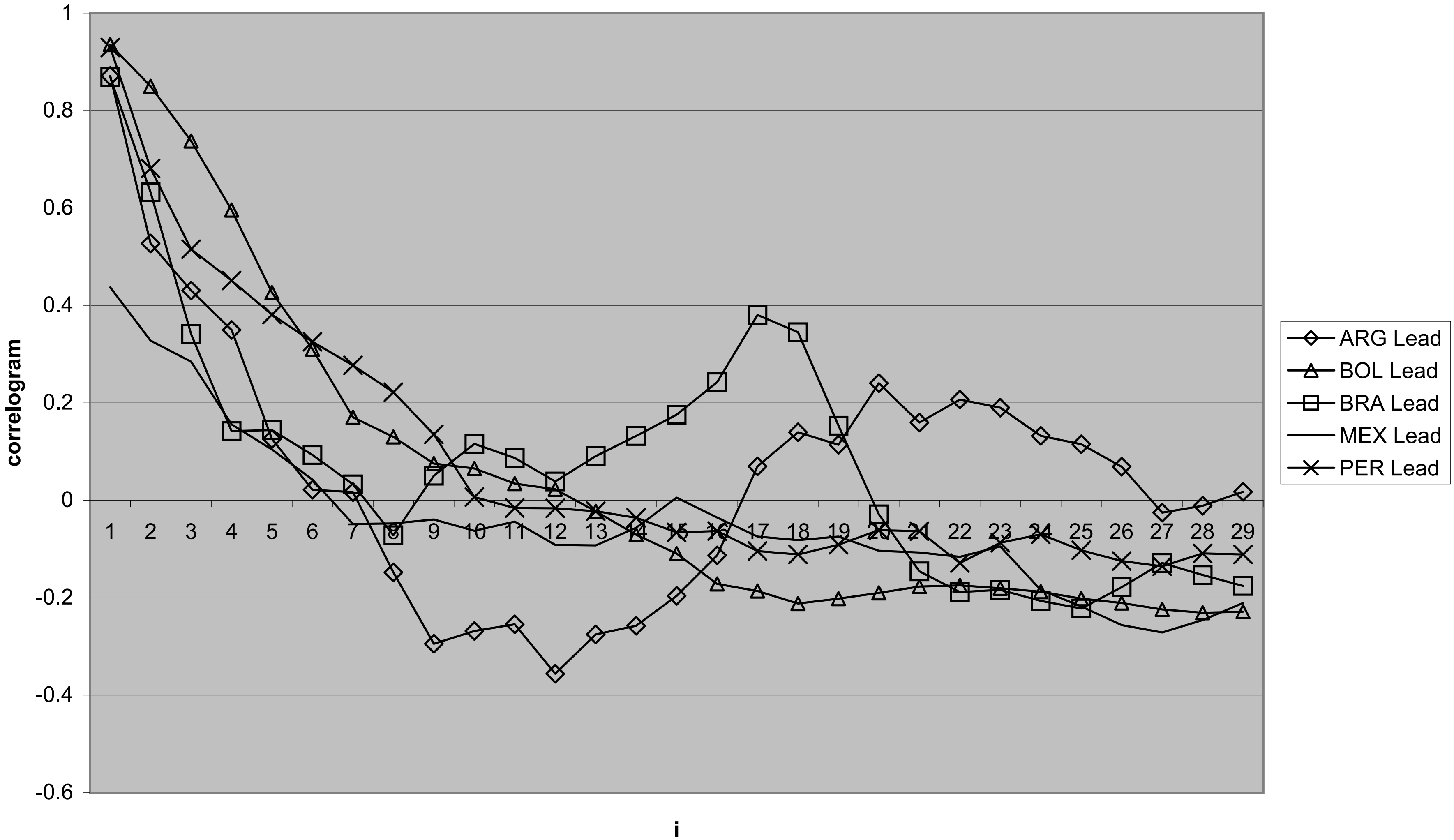


**Fig 2.b**
**Lag of all countries**

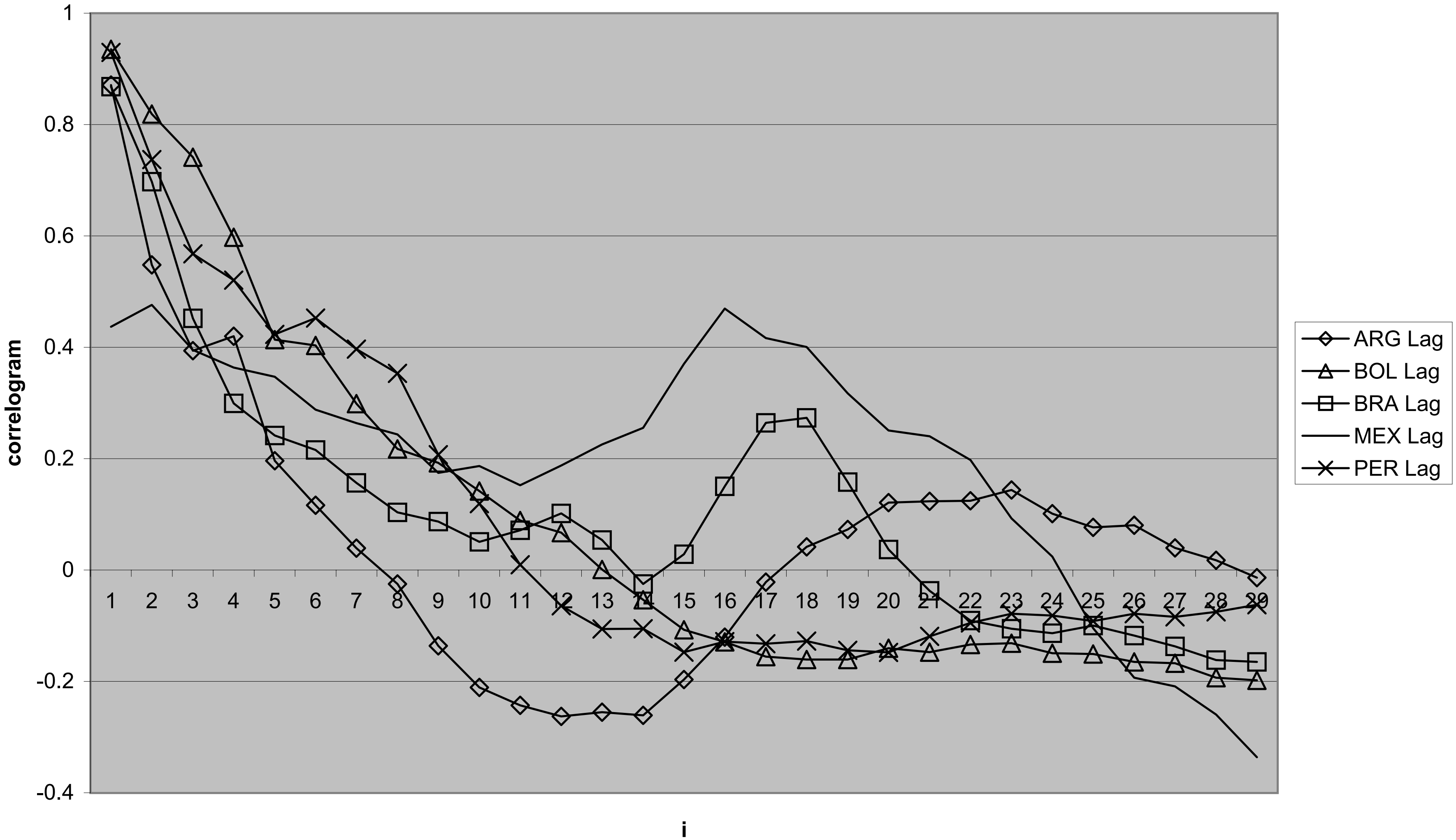


**Fig 3.a**
**Argentina: Lag with confidence band**

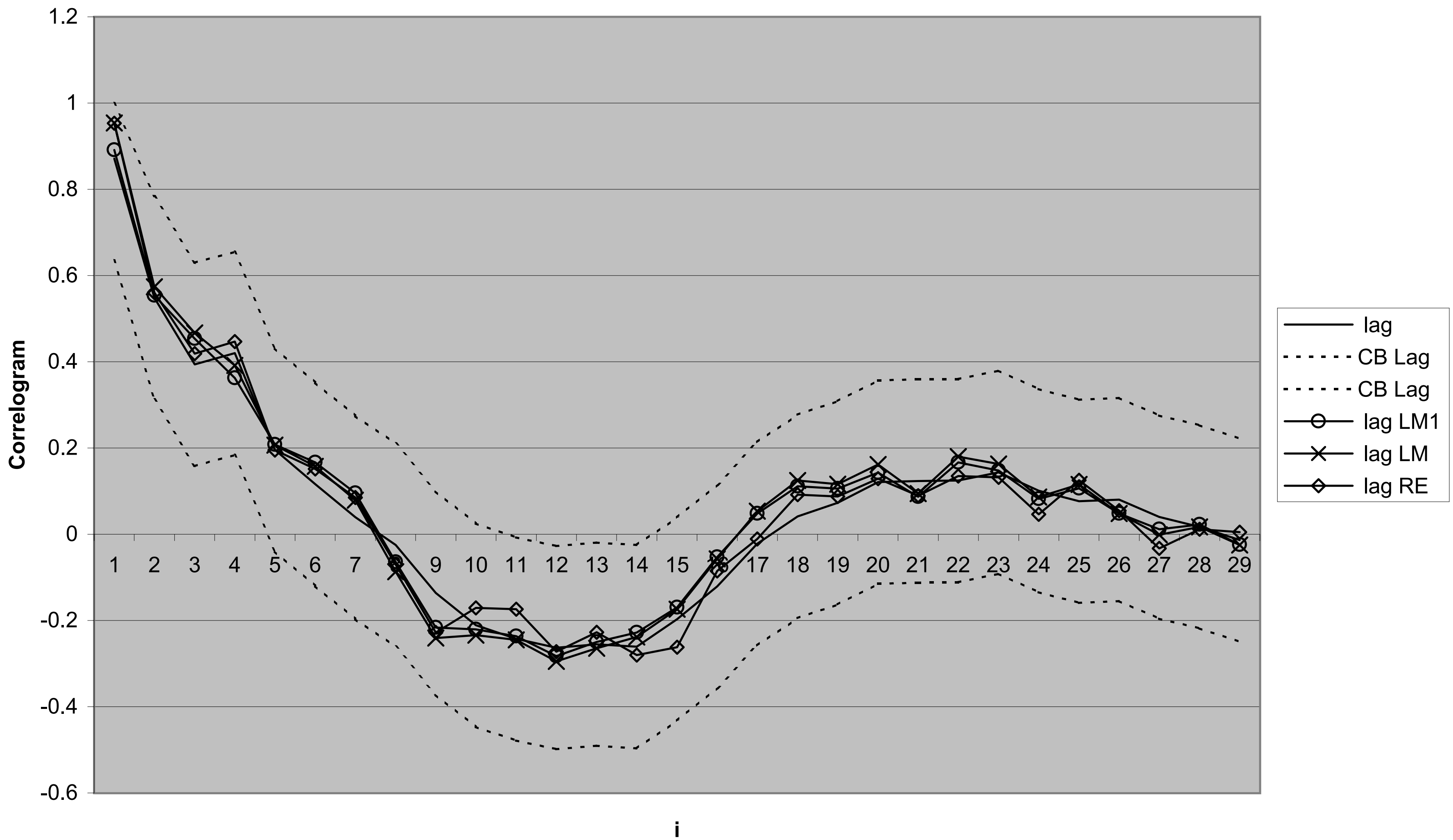


**Fig 3.b**
**Bolivia: Lag with confidence interval**

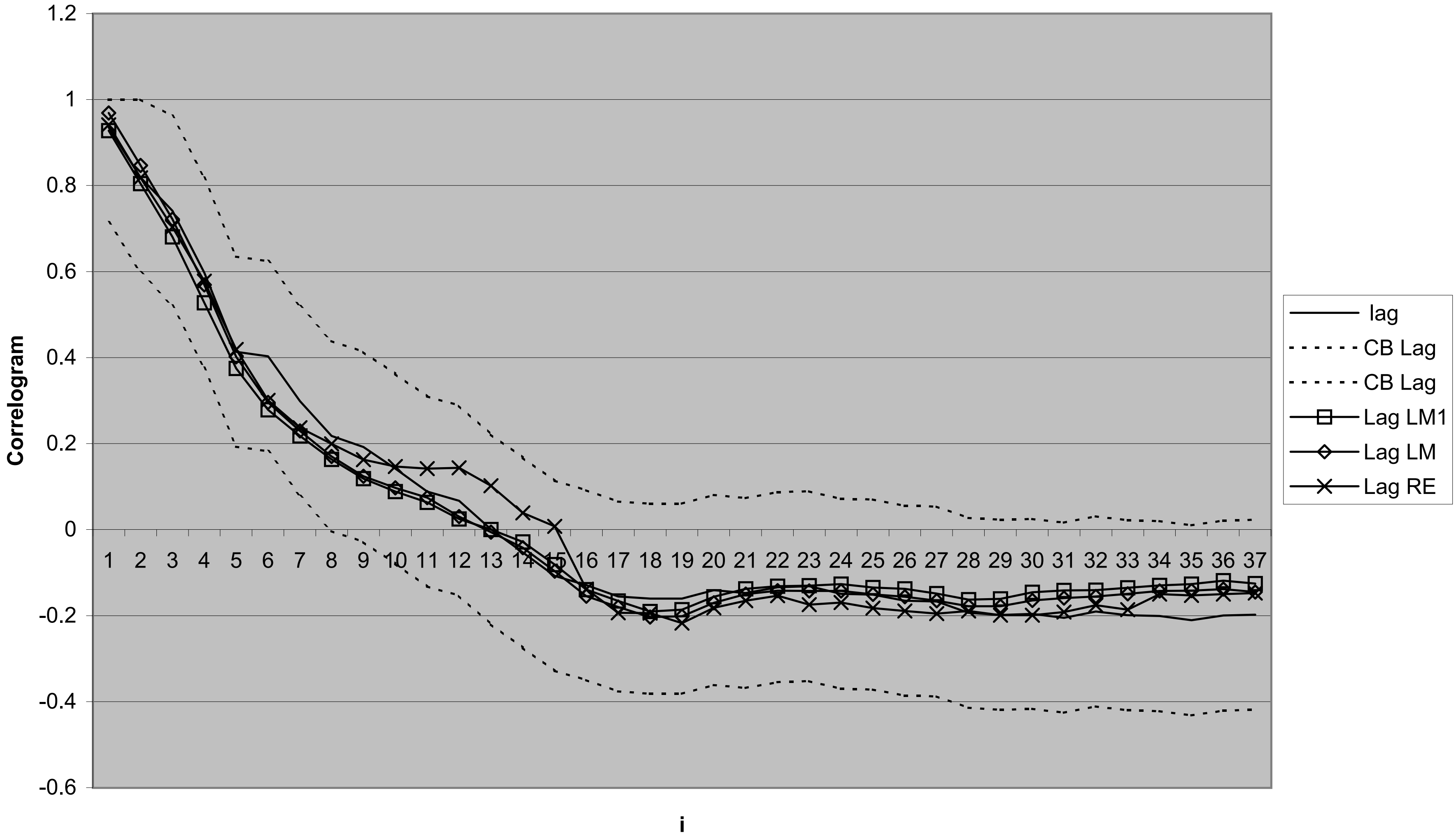


**Fig 3.c**
**Brazil: Lag with confidence interval**

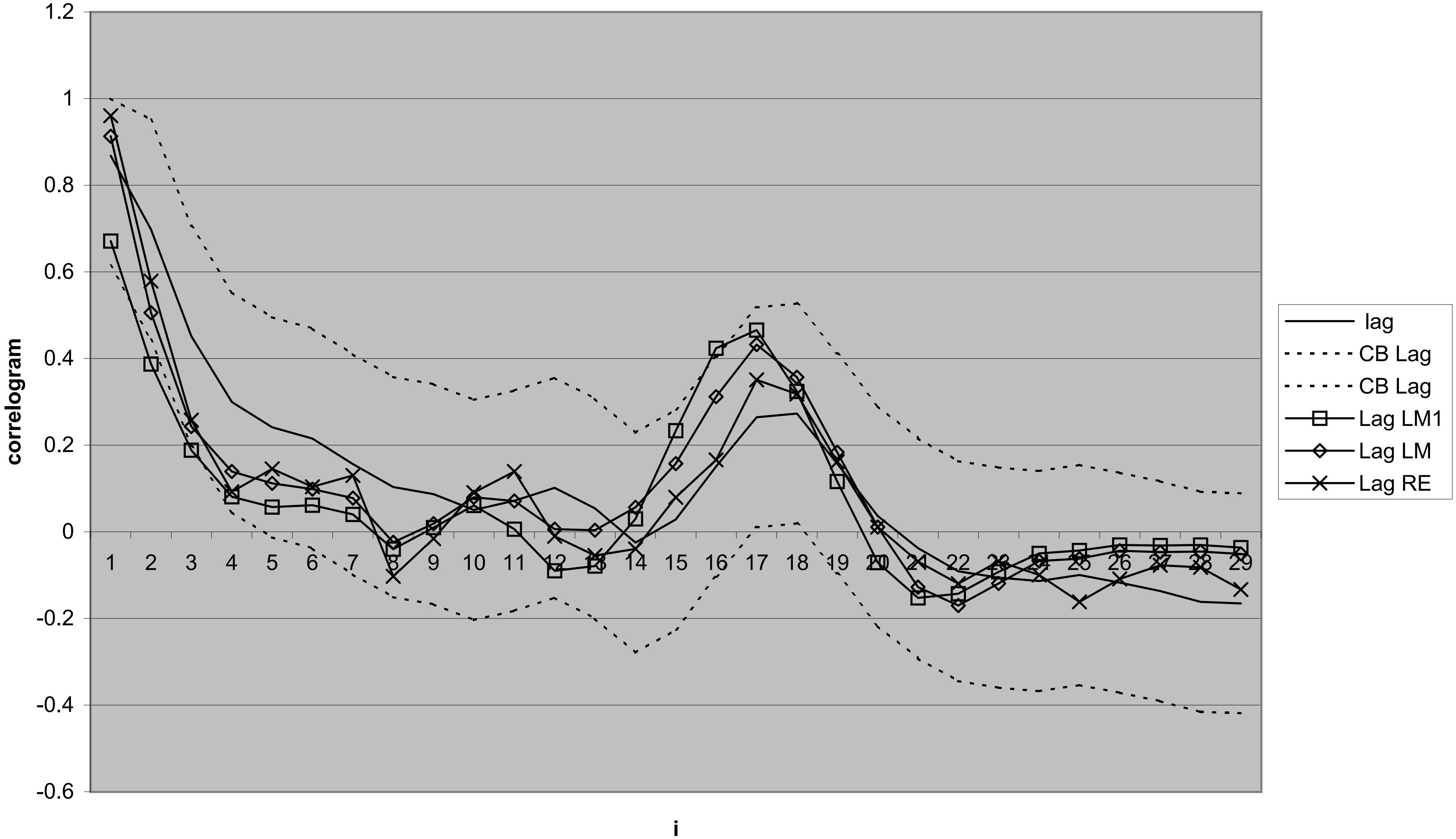


**Fig 3.d**
**Mexico: Lag with confidence interval**

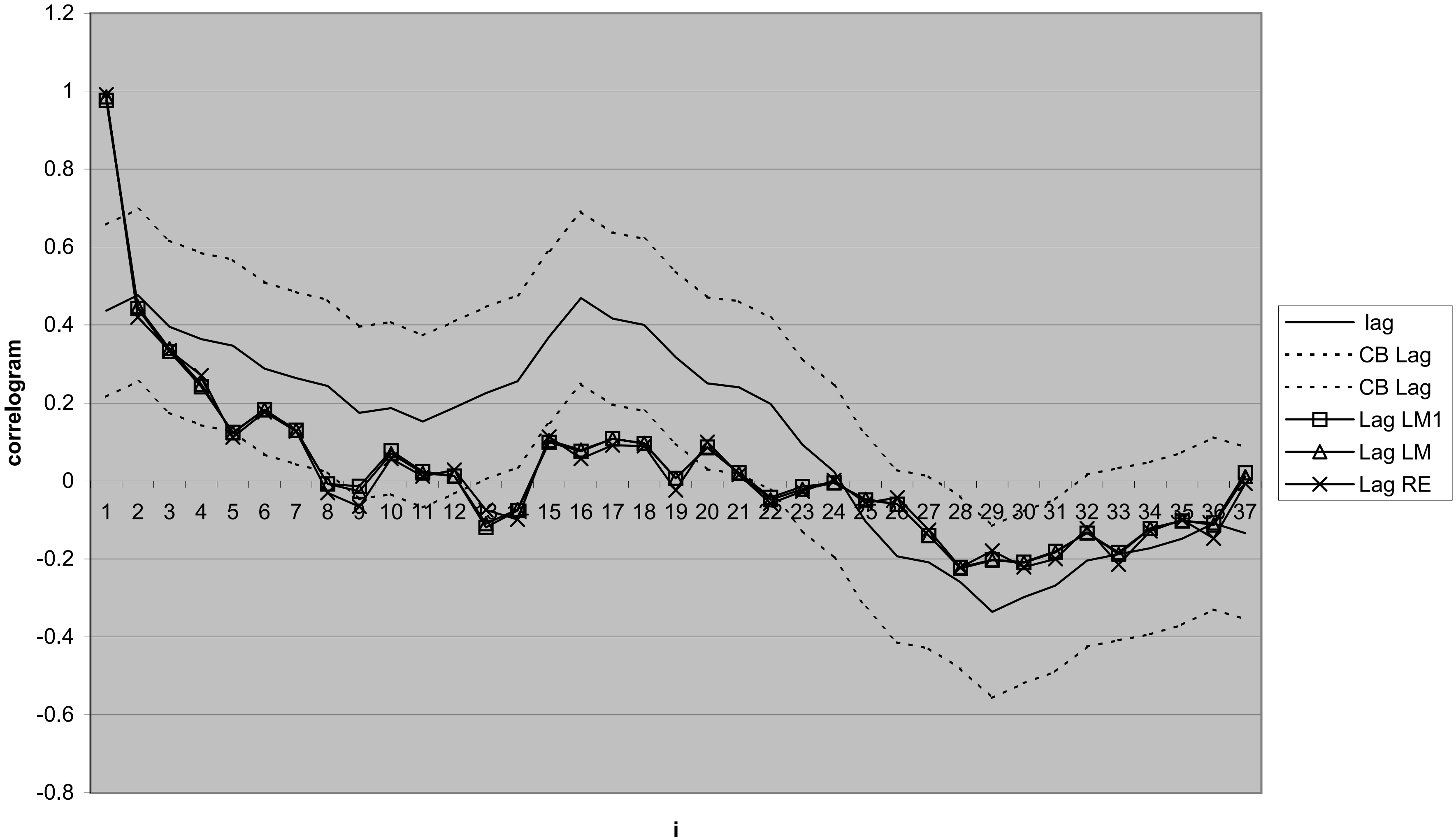


**Fig 3.e**
**Peru: Lag with confidence interval**

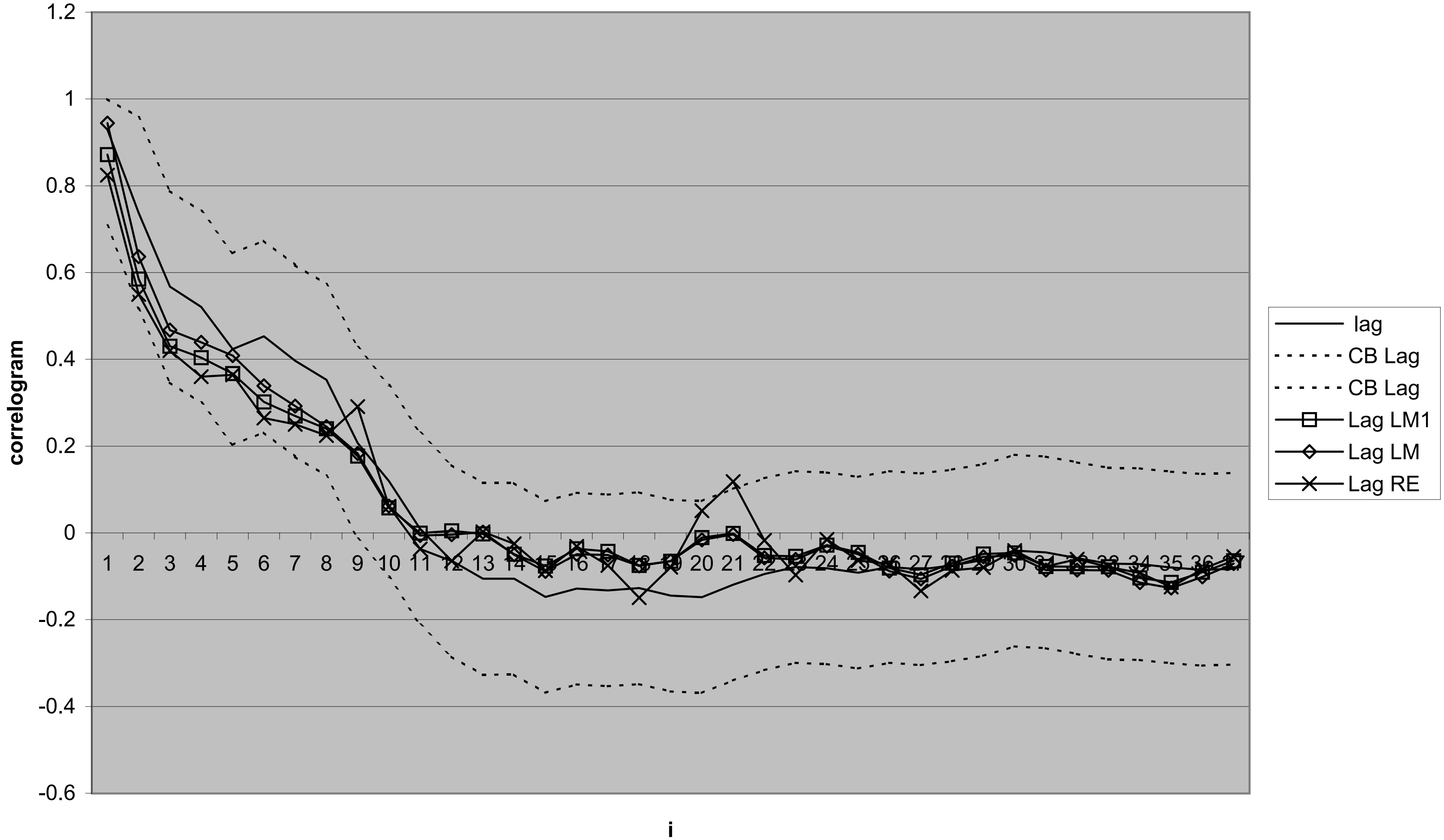


**Fig 4.a**
**Argentina: Lead with confidence interval**

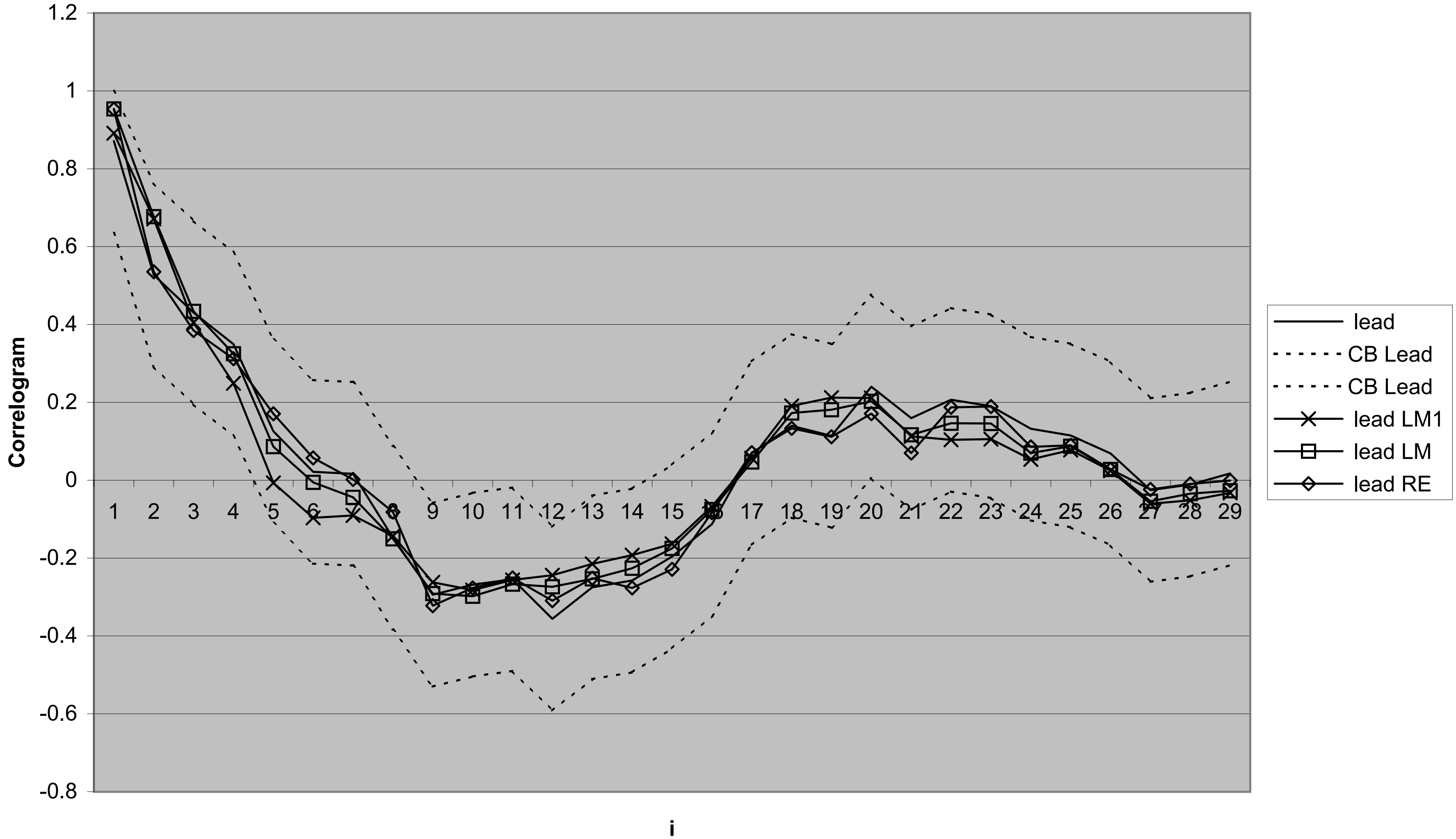


**Fig 4.b**
**Bolivia: Lead with confidence interval**

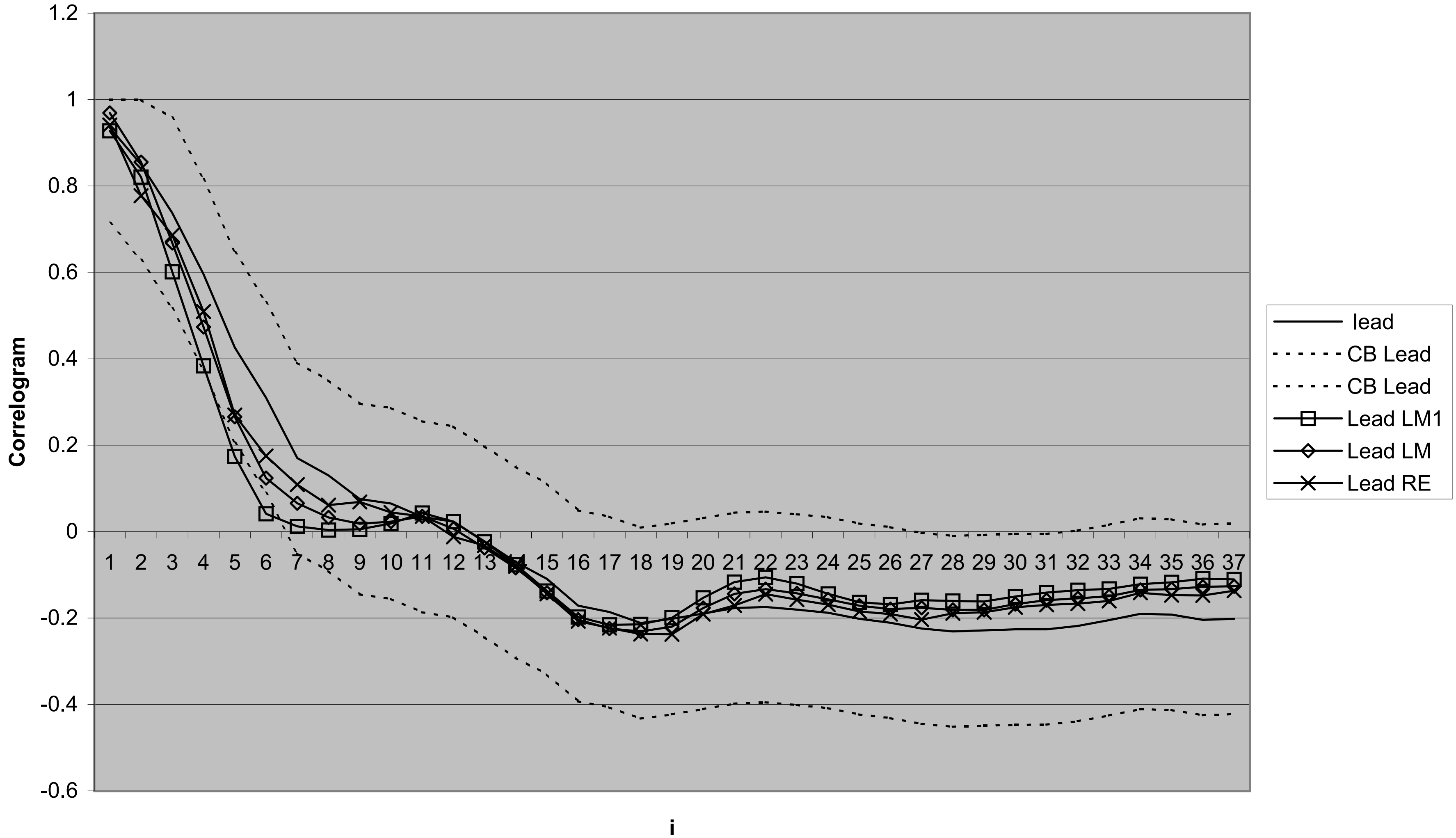


**Fig 4.c**
**Brazil: Lead with confidence interval**

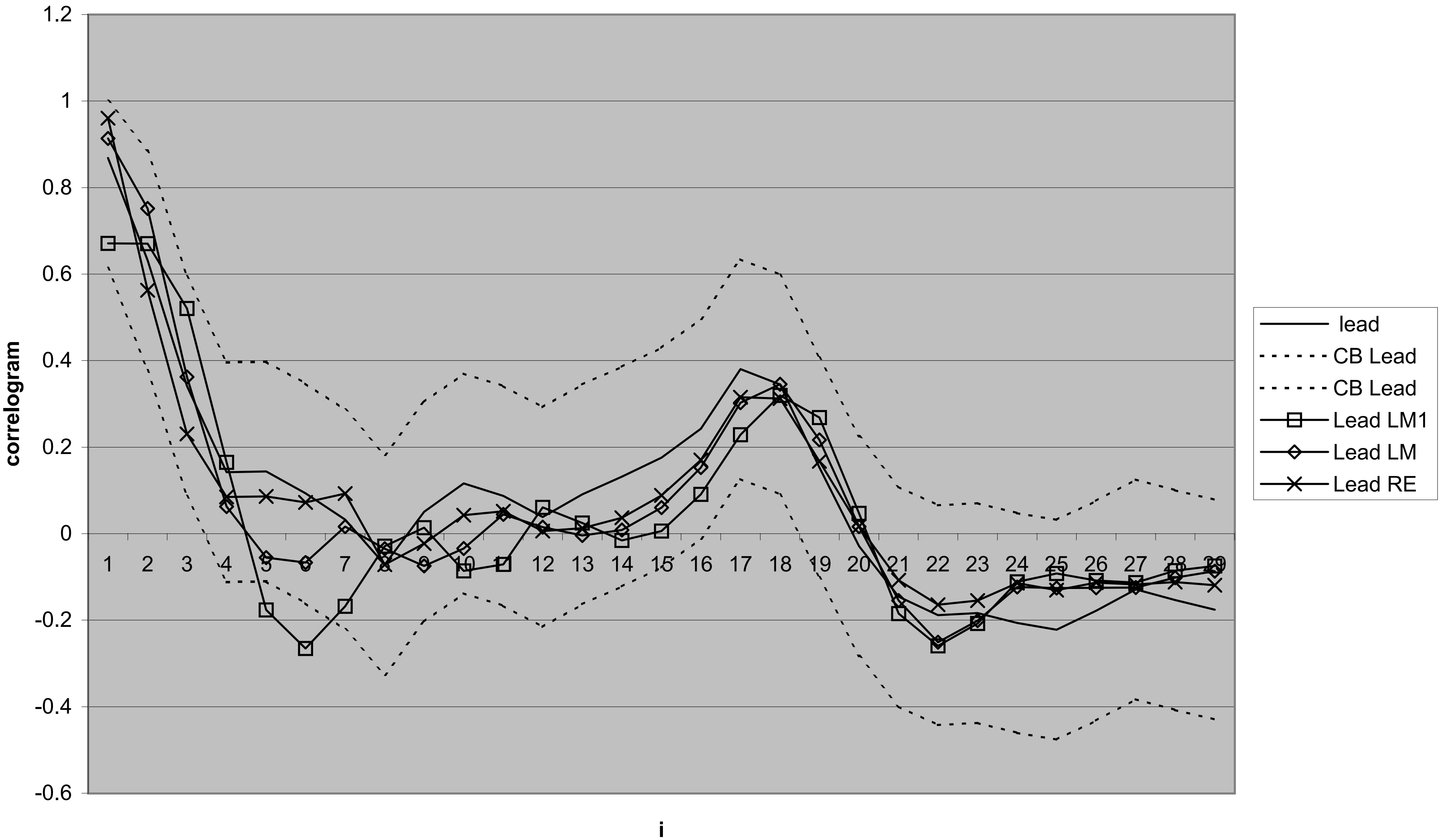


**Fig 4.d**
**Mexico: Lead with confidence interval**

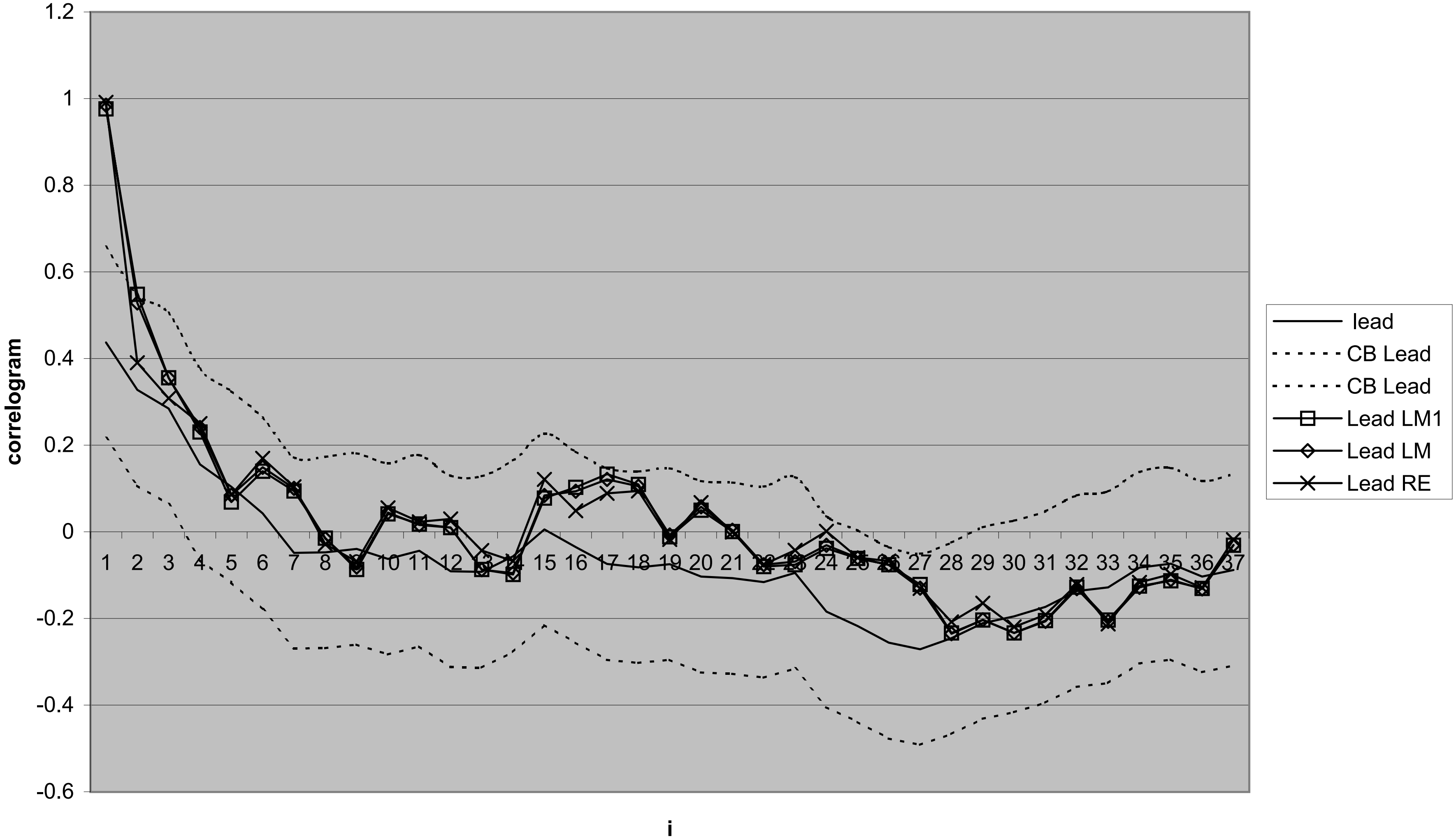


**Fig 4.e**
**Peru: Lead with Confidence interval**

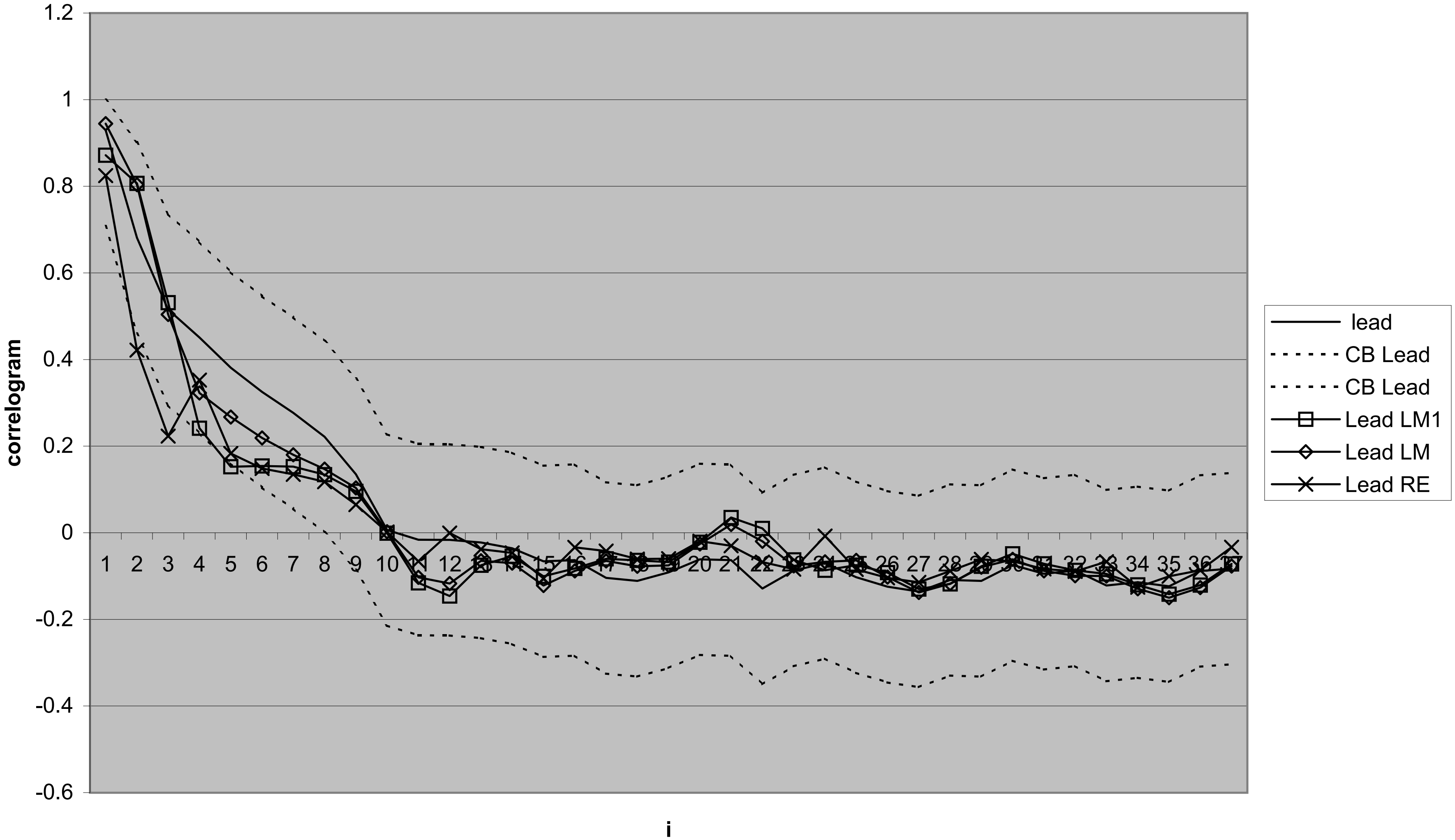


**Fig 5**
**Argentina: Inflation generated by LM, LM1 and RE**