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On Information and Competition in Private Value Auctions

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Abstract

How much information does an auctioneer want bidders to have in a private value environment? We address this question using a novel approach to ordering information structures based on the property that in private value settings more information leads to a more disperse distribution of buyers' updated expected valuations. We define the class of precision criteria following this approach and different notions of dispersion, and relate them to existing criteria of informativeness. Using *supermodular precision*, we obtain three results: (1) a more precise information structure yields a more efficient allocation; (2) the auctioneer provides less than the efficient level of information since more information increases bidder informational rents; (3) there is a strategic complementarity between information and competition, so that both the socially efficient and the auctioneer's optimal choice of precision increase with the number of bidders, and both converge as the number of bidders goes to infinity.

KEYWORDS: Auctions, Competition, Private Values, Informativeness Criteria.

JEL classification numbers: D44, D82, D83.

1 Introduction

There are numerous situations in which a seller controls the information available to potential buyers: a government agency soliciting bids to execute a public project, a company wanting to sell a subsidiary (or go public), internet auctions, etc. Such situations raise important questions, such as: should the seller make information available to bidders? How much should he make available? As much as possible? Are his incentives to provide information aligned with social ones? How does his choice depend on the number of potential buyers in the market?

We address these questions in a standard independent private values auction setting: an auctioneer wants to sell an object to n risk-neutral bidders whose valuations are independently drawn from a common and known distribution. Initially, bidders know the distribution but are uncertain about their valuations. The auctioneer, prior to the auction, can supply information (in the form of private signals) to help them obtain a more accurate estimate of their valuations. Each bidder receives a private signal which conveys information only about his private valuation and is private information to him. Effectively, the auctioneer chooses the information structure, i.e. the joint distribution of the private signals and valuations, so as to maximize his expected revenue from the auction. This choice can be interpreted either as the auctioneer producing information or as controlling access to existing information.

In order to analyze the auctioneer's problem generally, without recourse to specific families of information structures, we need a criterion of informativeness. A natural candidate is Lehmann (1988)'s criterion of effectiveness which has been used in Persico (2000) and Bergemann and Välimäki (2002) to order information structures in the related problem where bidders choose how much information to acquire. But, Lehmann's, as well as other commonly used criteria of informativeness, such as Blackwell (1955)'s, are not well suited for analyzing the auctioneer's incentives to provide information to bidders, since they do not pose specific constraints on the distribution of bidders expected valuations, which is central in our problem. We propose a new family of criteria of informativeness, that we refer to as *precision* criteria, and are defined via the effect of information on the distribu-

tion of expected valuations. Precision criteria allow us to obtain robust results on the auctioneer's incentives for the provision of information and how they change with the number of bidders in the auction.

The common characteristic of precision criteria is that they are defined by the property that in private value settings more information leads to a more spread out distribution of buyers' updated expected valuations. In our model, the auctioneer controls the informativeness of the signals but does not observe their realizations. He knows that bidders use these signals to update their expected valuations so that to him updated expected valuations are random draws from a distribution which he can influence by choosing the degree of informativeness of the signals. Further, he knows that due to the heterogeneity in bidder preferences an increase in the informativeness of the signals will have an asymmetric effect on bidder's expected valuations, raising some while reducing others. So that by increasing the informativeness of signals, the auctioneer makes the distribution of updated expected valuations more spread out.

Precision criteria are defined by this effect: an information structure will be more informative than another if its distribution of updated expected valuations is more disperse. As there are different ways in which one random variable may be more disperse than another, we will provide correspondingly different precision criteria. The main criterion used in this paper is that of *supermodular precision (sm-precision)*, which is defined using the dispersive order of Bickel and Lehmann (1976).[1] Using weaker notions of dispersion, we construct several alternative (and weaker) precision criteria, and show that commonly used informativeness criteria only imply the weakest precision criterion and correspondingly limited comparative static results.

Returning to the auctioneer's problem, we suppose the auctioneer can choose at some cost from a general class of information structures ranked in terms of their informativeness. We show that if information structures are ordered by precision criteria, we can also order the total surplus they

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[1]In addition to defining supermodular-precision, in the Appendix we identify a condition stated in terms of information structures (rather than the distribution of expected valuations) that is sufficient to order information structures in terms of supermodular-precision. This condition is the mirror image of Lehmann (1988)'s characterization of greater effectiveness.

generate and, in certain cases, the expected price. Moreover, the link between precision criteria and the outcome of the auction allows us to analyze the incentives of the auctioneer to provide information. We will now present the three main results obtained using *supermodular precision*. At the end, we introduce weaker precision criteria and discuss how they affect these results. Until then, to improve readability, we use the generic term *'precision'* in the text to also refer to *supermodular precision* whenever it does not generate ambiguity.

The first result is that more precision raises the efficiency of the allocation. We show that increasing the precision of the information structure increases the surplus of the allocation summarized by the expected valuation of the winning bidder. Hence, if granting access to information is costless, then it is efficient to give full access to all available information. If granting access is costly, then there is a trade-off between increasing the surplus from the allocation versus increasing costs.

The second result is that private and social incentives for the provision of information are not aligned. The auctioneer faces two opposing forces when deciding the precision of the signals: more precision improves the efficiency of the allocation, which increases expected revenues, while also increasing informational rents, which reduces them. Informational rents increase the cost of providing information, and the auctioneer optimally chooses an inefficient level of sm-precision.

The third result is that information and competition are complements in the following sense: total surplus and the auctioneer's expected revenue are supermodular in the number of bidders and the informativeness of the information structure in terms of sm-precision. This implies that the socially efficient and the auctioneer's optimal choice of sm-precision are increasing in the level of competition measured in terms of the number of bidders in the auction. We now provide the intuition behind this complementarity.

Total surplus is determined by the expenditure on information and the match between the characteristics of the object and bidder preferences. An extra bidder increases the marginal value of information since for any level of expenditure this extra bidder can generate additional efficiency gains from the information.

Expected revenue is total surplus minus informational rents of the winning bidder. Private

incentives to provide information increase with competition, because competition increases the benefits of information (the efficiency of the allocation) and reduces its costs (informational rents). This complementarity does not imply that providing information to the market is always valuable to the auctioneer. With two bidders, the negative effect of information (increased informational rents) overwhelms the gains from a better match, and the value of providing information is negative. As the number of bidders increases, information eventually becomes valuable. Thereafter, the precision of the information structure chosen by the auctioneer increases with competition. Finally, we show that as the number of bidders goes to infinity, informational rents disappear. Consequently total surplus and expected revenues converge and so do the optimal and the efficient levels of information.

1.1 Related Literature

The problem of information revelation by the auctioneer has been studied by Milgrom and Weber (1982) in an affiliated values setting. They establish the celebrated linkage principle. According to the linkage principle, the expected-revenue-maximizing policy for the auctioneer is to commit to fully and publicly announce all information he has. Ottaviani and Prat (2001) extend the logic of the linkage principle to a market setting. A price-discriminating monopolist would like to commit to publicly announce information that is affiliated with buyer valuations. Our results are very different since in the affiliated values setting bidders react symmetrically to information, while in the private value setting bidders react *asymmetrically* to information—valuations instead of moving together, become more disperse.

The basic idea that information in private value settings increases the differences between bidders, which generates informational rents, already appears in Lewis and Sappington (1994)'s pioneering study of information revelation by a monopolist. In Lewis and Sappington (1994), buyers' preferences over the monopolist's product are heterogenous. The monopolist chooses how much information to provide to buyers using a simple family of information structures. The information provided spreads out the distribution of consumer valuations, increasing the valuation of high-value buyers and informational rents. This leads the monopolist to select an all-or-nothing information

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strategy.

While Johnson and Myatt (forthcoming) study the problem of a monopolist's advertising and marketing decisions in a market setting, we consider their methodological approach to be the one closest to ours. As in this paper, Johnson and Myatt (forthcoming) focus on the link between the seller's supply of information and the shape of the distribution of buyers' expected valuations. In Johnson and Myatt, if consumers react asymmetrically to information (advertising) provided by the monopolist, the distribution of consumer valuations becomes more disperse, generating a rotation of the demand curve. They show that these rotations generate a convexity in the monopolist's profits which explains the optimality of the monopolist's extreme choices in Lewis and Sappington (1994). Our paper provides a general analysis of the link between information and dispersion. In particular, we present three novel informativeness criteria based on different notions of dispersion, one of which (SC-dispersion) is akin to Johnson and Myatt's notion of a rotation. SC-dispersion provides the basis for an informativeness criterion, *single-crossing precision*. We show that, as a rotation is useful in understanding the problem of costless provision of information in a market setting, *single-crossing precision* is useful in an auction setting. Moreover, we determine that *single-crossing precision* is not implied by the standard informativeness criteria, and that in order to obtain comparative statics results with costly provision of information we need a stronger precision criterion.

The findings of Lewis and Sappington (1994) have their counterpart in private value auctions. Recently, Bergemann and Pesendorfer (2003), Ganuza (2004) and Board (2005) show that the asymmetric reaction of bidders to information in private value settings may lead the auctioneer to optimally withhold information.[2,3] Our informativeness criteria could be used to extend previous analysis of the seller's information revelation decision with a much larger set of information structures. We use it to provide a general treatment of the auctioneer's problem in the symmetric

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[2]Board (2005) shows that even if bidders react symmetrically to information but with different sensitivities, the possibility that the ranking of bidders may change can lead the auctioneer to prefer not to release any information.

[3]An alternative approach, pursued in Eső and Szentes (2005), is to assume that the auctioneer can fully commit to provide any given level of information precision and charge bidders for it, prior to actually revealing any information. Thus, the auctioneer can extract all the informational rents ex-ante and the tradeoff in Lewis and Sappington (1994) vanishes.

case.

In the current paper, the auctioneer provides information symmetrically—bidders receive private signals with the same precision. In contrast, Bergemann and Pesendorfer (2003) consider the possibility of discriminating between bidders and find that some degree of discrimination may be optimal. Another difference with Bergemann and Pesendorfer (2003) is that in that paper the auctioneer designs the optimal information structure, which turns out to be representable by asymmetric partitions, that are difficult to rank in terms of informativeness. In contrast, we have the usual setup where the auctioneer chooses from a family of indexed information structures ordered in terms of informativeness, as in Bergemann and Välimäki (2006)'s survey of the role of information in mechanism design, as well as in Persico (2000) and Bergemann and Välimäki (2002).

The current paper uses the standard approach to modeling the provision of information, i.e. via private signals correlated with uncertain valuations. As Bergemann and Pesendorfer (2003) point out, this way of modeling is consistent with situations where the seller publicly reveals information about the characteristics of the object/product and buyers combine it with their preferences to determine their valuations. These valuations are private information to buyers, because they are the only ones who know how what is known about the object matches with their preferences. Ganuza (2004) follows this approach in a Salop setting where an auctioneer sells a good to bidders who are located on a circle according to a uniform distribution, and he finds similar comparative static results to ours.

The paper proceeds as follows: Section 2 formulates the model. Section 3 discusses our approach to ordering information structures and introduces the notion of supermodular-precision. Section 4 contains the main economic results concerning information revelation and competition in private value auctions. In Section 5 we study the link between information and dispersion: alternative precision criteria are presented and their relationship with existing informativeness criteria is established. This section is followed by the conclusion. All proofs are relegated to a technical appendix.

2 The Model

An auctioneer wants to sell an object he values at zero to one of n (ex-ante) identical risk-neutral bidders (indexed by $i = 1, \ldots, n$). Bidders' valuations of the object are private and uncertain. Bidder i's realized valuation after the auction is described by a random variable, V_i. For all $i = 1, \ldots, n$, V_i is independently distributed on $\mathcal{V} = [0, 1]$ according to a common distribution $H(v) = \Pr(V_i \leq v)$ with mean μ.

The utility obtained by bidder i from winning the auction is linear. If the realized valuation is v_i and he makes a monetary payment of t_i, the utility obtained is given by

$$u_i(v_i, t_i) = v_i - t_i.$$

All bidders start with identical priors, described by H, and no other information on the object. Hence, their expected valuations of the object will be the same and equal to μ.

The auctioneer can supply information prior to the auction. The production of information is costly. By paying an amount $\delta \in [0, \infty)$ the auctioneer will generate information which bidders receive in the form of private signals $(X_i)_{i=1}^n$. A higher δ generates more informative signals and δ is public information to all bidders.

Signals are independent and identically distributed random variables. We assume that these signals are drawn from the space of signals, $\mathcal{X} \subseteq \mathbf{R}$, and for each $i = 1, \ldots, n$, X_i is informative only about bidder i's own true and uncertain valuation of the object, v_i, and bidder i observes the private signal X_i and no other.

By choosing how much to spend on supplying information, the auctioneer determines the information structure, where an information structure is a joint distribution, $\mathbf{F}_\delta$, over signals, $(X_i)_{i=1}^n$ and valuations $(V_i)_{i=1}^n$, indexed by δ—we postpone until the next section, Section 3, exactly how a greater expenditure on supplying information translates into greater informativeness.

By symmetry and independence, the joint distribution can be characterized using the distribution

$F_\delta(v, x) = \Pr(V \le v, X \le x)$ as follows:

$$\mathbf{F}_\delta\left(V_1 \le v_1, \ldots, V_n \le v_n, X_1 \le x_1, \ldots, X_n \le x_n\right) = \prod_{i=1}^{n} F_\delta(v_i, x_i)$$

We leave out the i subscripts on signals and valuations whenever they are clear from the context. With a slight abuse of notation, let $F_\delta(x)$ denote the marginal distribution of X. As priors have to be consistent with the joint distribution, the marginal distribution of V is equal to the prior $H(v)$.

We will be comparing the effect of different levels of investment on information, and this requires that the signals generated by different information structures be comparable. This is done by transforming the realized signal X_i into a new random variable, Π_i, using the probability integral transformation: $\Pi_i = F_\delta(X_i)$. Then, $X_i = F_\delta^{-1}(\Pi_i)$, where F_δ^{-1} is the right-continuous inverse of the marginal distribution $F_\delta(x)$. We will assume that all random variables are non-trivial. The new signal, Π_i, has the same informational content as X_i so that we can use Π_i's instead of X_i's. More importantly, the marginal distribution of Π_i has a very useful property: it is the uniform distribution on $[0, 1]$ and independent of δ. Let $F_\delta(\pi|v)$ and $F_\delta(v|\pi)$ be the conditional distributions, where $F_\delta(v|\pi) = \Pr(V \le v | X = F_\delta^{-1}(\pi))$.

After the auctioneer has released the information, the awarding process takes place. To participate in this process, each bidder combines his knowledge of δ and the realization of the private signal, π_i, to update his expected valuation of the object $E[v_i|\pi_i, \delta]$ (also referred to as the interim valuation and denoted $W_i(\pi_i, \delta)$) using Bayes' rule. The auctioneer sells the object using a second-price sealed-bid auction. The choice of the second price auction as the awarding mechanism is done without loss of generality as long as the conditions for the revenue equivalence theorem are present. We abstract from reserve prices and assume that the object is always sold, so that the second price auction (as well as all standard auctions) is the optimal mechanism.[4] Summarizing, the model is structured as follows:

1. Everyone starts with common priors over bidders' uncertain valuations for the object.

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[4]Ganuza and Penalva (2004) study the introduction of a reserve price using a parameterized family of information structures. The possibility of using a reserve price gives the auctioneer an additional tool to control bidders informational rents. This raises his incentives to provide information and the optimal level of informativeness increases.

2. Prior to the auction, the auctioneer, knowing the number of bidders, n, chooses δ. This decision becomes public information.

3. Given δ, each bidder receives a private signal π_i over his valuation, and updates his valuation of the object.

4. The second-price sealed-bid auction takes place.

We now define and discuss what it means for the auctioneer to provide more information.

3 Supermodular Precision

Given any $\delta > 0$, signals are informative about valuations, V. We formalize this by assuming that given two (transformed) signals $\pi', \pi \in [0, 1]$, such that $\pi > \pi'$, receiving the larger signal, π, is good news in the sense of Milgrom (1981).[5] We assume that this condition holds for all pairs of signals and in every information structure throughout the paper. This implies that the agent's interim (expected) valuations, $W_i(\pi, \delta)$, are a nondecreasing function of the realization of the signal, π.

Having established that for $\delta > 0$ signals are informative about valuations we now turn to formalize how a higher δ leads to more informative signals. The primary approach to defining the informativeness of a structure, $\mathbf{F}_\delta$, is based on the pioneering work of Blackwell. Blackwell (1951) considers the decision problem of an individual who has to choose an action based on the realization of the signal. He defines a signal structure, $\mathbf{F}_\delta$, to be more informative than another, $\mathbf{F}_{\delta'}$, if every decision maker prefers $\mathbf{F}_\delta$ to $\mathbf{F}_{\delta'}$, and shows it is equivalent to the statistical notion of sufficiency. Subsequently weaker informativeness criteria (Lehmann (1988), Jewitt (1997), Athey and Levin (2001)) focus on when all decision makers in a particular class (those with supermodular payoff functions, with single-crossing incremental returns, etc.) prefer $\mathbf{F}_\delta$ to $\mathbf{F}_{\delta'}$. Our problem is different. The auctioneer will take no additional actions after choosing the information structure—the auction mechanism takes over. The auctioneer cares about how changes in δ affect the distribution

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[5]For all $\pi > \pi'$, the posterior distribution of v conditional on π, $F_\delta(v|\pi)$, dominates the posterior distribution of v conditional on π', $F_\delta(v|\pi')$, in the sense of First Order Stochastic Dominance (FOSD): $F_\delta(\cdot|\pi) \geq_{st} F_\delta(\cdot|\pi')$, (that is $F_\delta(v|\pi) \leq F_\delta(v|\pi')$ for all $v \in \mathcal{V}$).

function of updated expected valuations, which will determine price and allocations through the auction mechanism.

Let Π be a random variable uniformly distributed on $[0, 1]$. The random variable $W(\Pi, \delta)$ represents bidders' updated expected valuations after having observed a signal from an information structure indexed by δ. By choosing an information structure with more informative signals (a higher δ), the distribution of $W(\Pi, \delta)$ will become more spread out.[6] Intuitively, this is because the updated expected valuation combines the realization of the signal with the prior and assigns more weight to the signal if it is a more accurate estimate of the true valuation. Then, more informative signals lead to updated expected valuations which are more sensitive to the realization of the signal, and hence, to a more spread out distribution of $W(\Pi, \delta)$.

While all previous informativeness criteria lead to a more spread out distribution of $W(\Pi, \delta)$, our informativeness criteria are defined by requiring that this spreading out takes place in a specific way. The first of such criteria, *supermodular-precision*, orders information structures when the distribution of expected valuations generated by each of the information structures can be ordered according to Bickel-Lehmann *dispersion*:

Definition 1 (Bickel-Lehmann (1976)) *A random variable X with cumulative distribution function F is said to be more Bickel-Lehmann* disperse *than another random variable Y with cumulative distribution function G, if for all $q, p \in [0, 1]$, $q > p$*

$$F^{-1}(q) - F^{-1}(p) \geq G^{-1}(q) - G^{-1}(p).$$

Bickel-Lehmann *dispersion* defines an ordering over distribution functions which implies further orderings of the distributions of their first and second order statistics, two crucial variables in our analysis.[7,8]

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[6]Consider extreme cases: with no information the distribution of expected valuations is concentrated at the expected value, μ; if valuations were revealed perfectly, the distribution of expected valuations would be the true distribution of valuations.

[7]For an overview of results relating dispersion and order statistics see Shaked and Shantikumar (1994).

[8]Despite the importance of dispersion in statistics, Bickel-Lehmann dispersion has been used rarely in economics. An interesting application of this concept to the problem of income inequality can be found in Eckwert and Zilcha (2005).

Then, an information structure, $\mathbf{F}_\delta$, is more informative, in the sense of *supermodular-precision* (sm-precision), than another, $\mathbf{F}_{\delta'}$, if the distribution of updated expected valuations from $\mathbf{F}_\delta$ is more Bickel-Lehmann disperse than that from $\mathbf{F}_{\delta'}$. The criterion of supermodular precision owes its name to the fact that its definition can be most conveniently expressed using the familiar notion of supermodularity:

Definition 2 (SM-Precision) *The information structure $\mathbf{F}_\delta$ is more informative in terms of sm-precision than $\mathbf{F}_{\delta'}$ iff $\forall \pi, \pi' \in [0,1]$, $\pi > \pi'$,*

$$W(\pi, \delta) - W(\pi', \delta) \geq W(\pi, \delta') - W(\pi', \delta'),$$

The intuition is that more information in terms of sm-precision increases the slope of expected valuations as a function of π (as depicted in Figure 1).
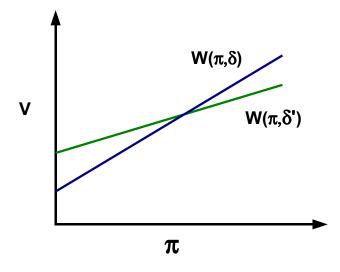

Figure 1: Precision and expected valuations

When comparing two signals from the same information structure, we said that a higher signal is better news than a low signal because it raises bidder's expected valuations. As $W(\pi, \delta)$ and $W(\pi, \delta')$ cross, we can think of increasing sm-precision as making low signals worse news and high

signals better news.[9]

Economists are more familiar with a different and yet related notion of 'spread out' distributions: Second Order Stochastic Dominance (SOSD).[10] Dispersion is a stronger condition than SOSD:

Remark 1 *Let* $\mathbf{F}_\delta$ *be more informative in terms of sm-precision than* $\mathbf{F}_{\delta'}$, *then* $W(\Pi_i, \delta)$ *is dominated by* $W(\Pi_i', \delta')$ *in the sense of SOSD.*

To illustrate our concept, we look at two families of information structures that are ordered according to sm-precision and are commonly used in the existing literature.

Example 1: Let V be normally distributed with mean μ and variance σ_v^2, and let the distribution of the signal, X, conditional on the true valuation, v, be the realization of the valuation v plus a noise term, ϵ_δ, which is normally distributed with mean zero and variance σ_δ^2. The variance of the noise term orders these information structures in the usual way: the information structure with lower variance of the noise term is more informative in terms of sm-precision.

Example 2: Let V be distributed according to the prior distribution $H(v)$ with mean μ. The signal, X, represents the bidder's valuation V, but the signal reveals the truth, i.e. $v = x$, with probability δ, and with the complementary probability the signal is false. We assume that when the signal is false it is pure noise, distributed according to the prior, $H(v)$, and independently of v. Thus, for all values of δ, the signal, X, is distributed according to $H(v)$. An increase in δ, the probability that the signal reveals the truth, leads to greater informativeness in terms of sm-precision.

In the Appendix, we characterize sm-precision further by providing a sufficient condition for greater informativeness in terms of sm-precision based on the properties of the joint distribution function of signals and valuations. In the next section, we use sm-precision to study the auctioneer's information revelation problem.

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[9]The law of iterated expectations implies $E[W(\Pi, \delta)] = E[W(\Pi, \delta')] = \mu$, so that if $W(\pi, \delta)$ and $W(\pi, \delta')$ were differentiable, they would cross at a single point (as in Figure 1). In general, $W(\pi, \delta) - W(\pi, \delta')$ is monotone increasing and changes sign once.

[10]In section 5, we use SOSD to characterize a novel informativeness criterion (integral-precision). The formal definition of SOSD is presented there.

4 Releasing Information

In this section, we will use the sm-precision criterion to study social and private incentives to release information, and how they are affected by competition.

4.1 The Efficient Release of Information

The efficient level of sm-precision is that which maximizes total surplus at the time of the information release. In our setup, total surplus is defined as the sum of the auctioneer's revenue and the expected interim utility of the bidder with the highest expected valuation at the time of the auction. As the price paid for the object is a pure transfer from the auctioneer to the winning bidder, total surplus is the expected valuation of the object by the winning bidder minus the cost of providing information. We first focus on the expected valuation of the winning bidder.

Denote the highest realization of the signal by x_1, so that $\pi_1 \equiv F_\delta(x_1)$. The winner of the auction will be the bidder receiving π_1 so that his expected valuation is: $V_1(n, \delta) = E[W(\Pi_1, \delta)]$—the notation makes explicit the number of bidders as we shall be studying the effect of changing n. Let $U_{1:n}(p)$ be the cumulative distribution function of the first order statistic of n independent uniform random variables on $[0, 1]$. Because the transformed signals Π_i are independent and uniformly distributed on $[0, 1]$, $U_{1:n}(p)$ is the cumulative distribution function of Π_1 and

$$V_1(n, \delta) = \int_0^1 W(p, \delta) \mathrm{d}U_{1:n}(p)$$

As the auctioneer increases the *precision* of the information structure this expectation increases:

Theorem 1 *The expected valuation of the winning bidder is nondecreasing in the informativeness of the information structure in terms of sm-precision, δ.*

Hence, we can compare the expected surplus generated from the allocation (the expected valuation of the winning bidder) under two different structures ordered in terms of sm-precision: the more precise information structure generates greater surplus. In Section 5 we will consider weaker precision criteria, the weakest of which (based on SOSD) is implied by all standard informativeness

criteria. Theorem 1 holds using the weakest precision criterion. The intuition behind this result is that more information improves the matching between the features of the object and the preferences of the winning bidder (and hence the expected surplus).

Theorem 1 implies that if the provision of information is costless, it is efficient to release all available information. With costly information, the trade-off faced when choosing the efficient level of precision, δ^E, is between increasing the efficiency of the allocation and increasing the costs of providing information:

$$\delta^E \in \operatorname*{argmax}_{\delta} V_1(n, \delta) - \delta \tag{1}$$

The next theorem states the relationship between the efficient level of sm-precision and the level of competition.

Theorem 2 *Total surplus is supermodular in the informativeness of the information structure in terms of sm-precision, δ, and the number of bidders, n.*

Theorem 2 states that the difference in terms of expected surplus between two information structures ordered in terms of sm-precision is larger the larger the number of bidders. So that the larger the number of bidders, the larger the social value of information. The intuition is that having more draws from the pool of bidder preferences increases the social incentives to improve the matching between the object and bidder preferences by increasing precision. Consequently, with fiercer competition it is efficient to spend more on the provision of information.

Corollary 1 *The efficient level of sm-precision, δ^E, is nondecreasing in the number of bidders.*

4.2 The Auctioneer's Optimal Information Release

Having characterized the efficient level of sm-precision, we now turn to the auctioneer's problem: choosing the level of precision so as to maximize expected revenue from the auction.

Let x_2 denote the second-highest signal, so that $\pi_2 \equiv F_\delta(x_2)$. The price in the second-price auction is determined by the bidder receiving π_2. Thus, the expected price is: $V_2(n, \delta) = E[W(\Pi_2, \delta)]$.

Let $U_{2:n}(p)$ be the cumulative distribution function of the second order statistic of n independent uniform random variables on $[0, 1]$. $U_{2:n}(p)$ is the cumulative distribution function of Π_2. The expected price in the auction is:

$$V_2(n, \delta) = \int_0^1 W(p, \delta) \mathrm{d}U_{2:n}(p),$$

Theorem 1 shows that increasing the precision of the information structure always improves efficiency. The effect of increasing precision on the expected price depends on the level of competition.

Proposition 1 *Suppose $\delta > \delta'$, (i) for $n = 2$, the expected price is nonincreasing in the informativeness of the information structure in terms of sm-precision, $V_2(2, \delta) \leq V_2(2, \delta')$. On the other hand, (ii) for all $\delta' < \delta$, there exists n' such that for all $n > n'$, the expected price is nondecreasing in the informativeness of the information structure in terms of sm-precision, $V_2(n, \delta) \geq V_2(n, \delta')$.*

Notice that if the number of bidders is small enough, increasing the precision of the information structure can reduce the expected price. Then, even if information is costless, the auctioneer prefers not to release any information. Eventually, when the number of bidders is sufficiently high, information becomes valuable to the auctioneer.[11] The intuition behind this result comes from two effects of increasing precision on the price: it increases the willingness to pay by the winning bidder, which increases the price, but it also increases informational rents, which lower the price. We proceed to demonstrate this second effect.

The expected informational rents of the winning bidder, $R_w(n, \delta)$, are the difference between the expected valuation of the winning bidder and that of the bidder with the second highest realization of the private signal:

$$R_w(n, \delta) = V_1(n, \delta) - V_2(n, \delta)$$

Proposition 2 *The expected informational rents of the winning bidder are nondecreasing in the informativeness of the information structure in terms of sm-precision, δ.*

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[11]A related result can be found in Board (2005). He shows that the auctioneer, when deciding whether or not to provide information an additional piece of information, will always choose not to reveal it if there are only two bidders. He also shows that as the number of bidders goes to infinity, the information will be revealed.

Greater sm-precision makes the distribution of expected valuations more disperse, i.e. it makes bidders more heterogeneous, which translates into higher informational rents for the winning bidder.[12]

The auctioneer's problem is to choose the level of precision, δ^A, that maximizes his expected revenue, i.e. the difference between the expected price and the cost of providing more information:

$$\delta^A \in \operatorname*{argmax}_{\delta} V_2(n, \delta) - \delta, \tag{2}$$

We find that, as in the problem of maximizing total surplus, there is a complementarity between the level of sm-precision and the number of bidders.

Theorem 3 *The auctioneer's expected profits are supermodular in the informativeness of the information structure in terms of sm-precision, δ, and the number of bidders, n.*

The difference between the expected profits generated by two information structures ordered in terms of sm-precision is larger the larger the number of bidders. Therefore, more bidders increase the incentives of the auctioneer to provide information.

Corollary 2 *The optimal level of sm-precision, δ^A, is nondecreasing in the number of bidders, n.*

4.3   Optimal vs. Efficient Provision of Information

Finally, we compare private incentives to provide information with social ones.

Theorem 4 *The optimal level of informativeness of the information structure, in terms of sm-precision, is lower than the efficient level: $\delta^A \leq \delta^E$. The difference between the efficient and the optimal level converges to 0 as the number of bidders goes to infinity.*

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[12]Our results are linked to the relationship between the Bickel-Lehmann dispersive order and order statistics. In particular, Proposition 2 can be proven using Theorem 2.B.15 in Shaked and Shantikumar (1994), which says that if one random variable, X, is more Bickel-Lehmann disperse than another, Y, the difference between the first and second order statistic from a sample of n independent draws of X stochastically dominates the same difference from a sample of n draws of Y. Our strategy for proving results on information in auctions does not make explicit use of the properties of the dispersive order over order statistics. This allows us to explore weaker dispersion criteria and to obtain results related to the relationship between Bickel-Lehmann dispersive order and the distribution of first and second order statistics, that as far as we know, cannot be found in the statistics literature.

To better understand this result, rewrite the auctioneer's problem:

$$\delta^A \in \operatorname*{argmax}_{\delta} V_1(n, \delta) - \delta - R_w(n, \delta)$$

This formulation clarifies the trade-off faced by the auctioneer when providing information to the market. On the one hand, when the auctioneer increases precision, the efficiency of the allocation goes up ($V_1(n, \delta)$ is nondecreasing in δ – Theorem 1). On the other hand, the increase in precision also raises the informational rents of the winning bidder ($R_w(n, \delta)$ is nondecreasing in δ—Proposition 2). The optimal balance of these two opposing effects leads the auctioneer to provide lower precision than would be efficient. In other words, the auctioneer will restrict the information released to the market in order to make bidders more homogeneous, with the underlying goal of intensifying competition and increasing his expected revenue.

Competition increases the positive effect of precision on expected revenues, increasing the efficiency of the allocation, while it reduces the negative effect, informational rents. The compounded effect is to increase the incentives of the auctioneer to reveal information so that as the number of bidders increases so does the optimal amount of precision. In the limit, as the number of bidders goes to infinity, informational rents disappear and with them the difference between the efficient and the optimal level of *sm-precision*.

5   Weaker Precision Criteria

In this section, we introduce weaker precision criteria, compare them with standard informativeness criteria, and discuss how previous results are affected by using informativeness criteria other than *sm-precision*.

Precision criteria establish that an information structure is more informative than another if it leads to a more disperse distribution of expected valuations. Our central criterion, sm-precision, requires that the distribution of expected valuations must be ordered in the sense of Bickel-Lehmann dispersion. To define additional informativeness criteria we consider alternative dispersive orders. We concentrate on: single-crossing dispersion (related to the idea of 'more dan-

gerous'[13] random variables), and Second Order Stochastic Dominance (SOSD).

Consider two random variables, X with cumulative distribution function F, finite mean and support on $A \subseteq \mathbf{R}$, and Y with cumulative distribution function G, finite mean and support on $B \subseteq \mathbf{R}$.

Definition 3 (SC-Dispersion) *X is dominated in terms of single-crossing dispersion by Y, if for all z and $z' \in \mathbf{R}$, $z > z'$*

$$F(z') - G(z') \leq (<) \; 0 \implies F(z) - G(z) \leq (<) \; 0$$

Definition 4 (SOSD) *X is dominated in terms of second order stochastic dominance (SOSD) by Y, if the expected value of X is the same as that of Y and for all $z \in \mathbf{R}$*

$$\int_{-\infty}^{z} F(x)\mathrm{d}x \geq \int_{-\infty}^{z} G(x)\mathrm{d}x.$$

In the same way we have used Bickel-Lehmann dispersion to define sm-precision, we can define *single-crossing precision* and *integral-precision* using single-crossing dispersion and SOSD: an information structure, $\mathbf{F}_\delta$, is more single-crossing precise (integral-precise) than another, $\mathbf{F}_{\delta'}$, if the distribution of updated expected valuations from $\mathbf{F}_\delta$ dominates according to single-crossing dispersive order (SOSD) that from $\mathbf{F}_{\delta'}$. As with sm-precision, *sc-precision* and *integral precision* can be most conveniently expressed using the expected valuation function:[14]

Definition 5 (SC-Precision) *The information structure $\mathbf{F}_\delta$ is more informative in terms of single-crossing precision than $\mathbf{F}_{\delta'}$ iff $\forall \pi, \pi' \in [0,1]$, $\pi > \pi'$,*

$$W(\pi', \delta) - W(\pi', \delta') \geq (>) \; 0 \implies W(\pi, \delta) - W(\pi, \delta') \geq (>) \; 0.$$

---

[13]The concept of dispersion based on cumulative distribution functions that cross once appears in Karlin and Novikoff (1963). The concept is very seldom used and the nomenclature comes from actuarial science (where the notion was introduced by Ohlin (1969)). The notion of a rotation introduced in Johnson and Myatt (forthcoming) is a smooth version of single-crossing cdf's. .

[14]The relationship between dispersive orders and differences in expected valuation functions is based on the the fact that if we let F be the cumulative distribution functions of $W(\Pi, \delta)$, then $F(v)$ is the inverse of $W(\pi, \delta)$, i.e. $F(v) = \pi$ iff $v = W(\pi, \delta)$. Based on this fact, one can derive the equivalence between using dispersive orders on the distributions of expected valuations and using properties of the expected valuation function.

Definition 6 (Integral-Precision) *The information structure* $\mathbf{F}_\delta$ *is more informative in terms of integral-precision than* $\mathbf{F}_{\delta'}$ *iff* $\forall \pi \in [0,1]$

$$\int_0^\pi \left(W(p, \delta) - W(p, \delta') \right) dp \leq 0$$

Just by simple inspection of the definitions of sm-precision and sc-precision, we can conclude that sm-precision is a stronger order than sc-precision. Given that $E[W(\Pi, \delta)] = E[W(\Pi, \delta')] = \mu$, we obtain $\int_0^1 \left(W(p, \delta) - W(p, \delta') \right) dp = 0$, which leads to the conclusion that sc-precision implies SOSD. Hence, our informativeness criteria are ordered: sm-precision implies sc-precision, which implies integral-precision-[15]

5.1   Precision and Standard Informativeness Criteria

As discussed above, standard informativeness criteria are based on the value of information for an individual decision maker (Blackwell (1951), Lehmann (1988), Jewitt (1997), Athey and Levin (2001)). Athey and Levin (2001) present necessary and sufficient conditions for all decision makers with different classes of payoff functions to prefer one information structure over another. If we restrict attention to the class of decision makers with nondecreasing incremental returns, the necessary and sufficient condition is denoted the MIO-ND condition.[16]   Blackwell's notion is sufficient for information to be valuable for all decision makers which makes it stronger than both Athey and Levin's and Lehmann's. Also, for two information structures to be ordered in terms of Lehmann's informativeness criterion they have to satisfy the condition:

$$\hat{\gamma}(v, \pi) = F_\delta^{-1}(F_{\delta'}(\pi|v)|v) \quad \text{is nondecreasing in } v, \text{ for each } \pi,$$

---

[15]We could obtain this result also by comparing the dispersion orders. In our setting in which we can concentrate in comparing random variables with the same mean, it can be shown that Bickel-Lehmann *dispersion* implies *single-crossing dispersion*, which implies *SOSD*.

[16]We use the following version of the MIO-ND condition in Athey and Levin (2001):

Definition 7 (MIO-ND) *The information structure* F_δ *is more informative than* $F_{\delta'}$ *according to MIO-ND if for all* $v \in \mathcal{V}$ *and* $\pi \in [0,1]$, $F_{\delta'}(v|X \leq F_{\delta'}^{-1}(\pi)) \leq F_\delta(v|X \leq F_\delta^{-1}(\pi))$

and this implies the MIO-ND condition, which makes Athey and Levin's the weakest informativeness criterion.[17]

We can now relate these notions of information with the conditions stated above.

Proposition 3 *(i) Blackwell's informativeness criterion does not imply signals are more sc-precise, and (ii) MIO-ND implies signals are more integral-precise.*

Hence, none of the standard notions of informativeness imply *sc-precision*, and all standard notions of informativeness imply *integral-precision*.

5.2 Weakening Informativeness and PV Auctions

We have assumed that the provision of information is costly. Then we know (from Milgrom and Shannon (1994)) that to ensure the quasisupermodularity of total surplus and expected revenue, we need supermodularity of the functions $V_1(n, \delta)$ and $V_2(n, \delta)$. Among our three precision criteria (sm-precision, sc-precision and integral-precision), only the strongest one, sm-precision, ensures the supermodularity of $V_1(n, \delta)$ and $V_2(n, \delta)$. However some comparative static results can be obtained using *sc-precision* and integral-precision in a framework where the provision of information is costless.

If a higher δ only implies the information structure is more integral-precise, Theorem 1 holds. Hence, in a setup with costless provision of information, the efficient policy is full information disclosure. The results of Proposition 1 are also valid: if the number of bidders is two, information has a negative value, and eventually, when there is sufficient competition, information becomes valuable. Also, Theorem 4 holds: as the efficient information revelation strategy is full disclosure, and information may lower the expected price the incentives of the auctioneer to provide information will be weakly lower than the efficient ones. Furthermore, from the second part of Proposition 1 it follows that both the efficient and the optimal level of information converge as the number of bidders goes to infinity.

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[17]The proof is in Athey and Levin (2001), in the proof of Proposition 3.

The notion of sc-precision adds on integral-precision. With sc-precision the optimal amount of information provided by the auctioneer is weakly monotonic in the number of bidders—this follows from the property that with sc-precision, for $\delta > \delta'$, $V_2(n, \delta) - V_2(n, \delta')$ is single-crossing.

6 Conclusions

This paper provides a novel approach to ordering information structures which is specially suited to problems of information revelation in private value settings. In such settings, new information will be perceived differently by different people, raising the valuations of some while reducing that of others, which leads to a more spread out distribution of expected valuations. The core idea of the paper is to use this fact to build a family of informativeness criteria: an information structure is more informative than another if it leads to a more disperse distribution of expected valuations. More stringent notions of dispersion will lead to stronger informativeness criteria.

We focus on three ordered concept of dispersion (Bickel-Lehmann *dispersion* $\succ$ *single-crossing dispersion* $\succ$ *SOSD*) and use them to characterize three ordered informativeness criteria (*Supermodular-precision* $\succ$ *Single-crossing precision* $\succ$ *Integral precision*). We then apply this family of informativeness criteria to the auctioneer's information revelation problem.

The weakest of our criteria, integral-precision, which is defined using second order stochastic dominance, has the important feature that it is implied by all standard informativeness criteria. In addition, integral-precision allows us to obtain two important comparative static results with costless provision of information: (i) if an information structure is more integral-precise than another, it will generate a greater surplus, and (ii) if there are two bidders, the auctioneer obtains a lower expected price when choosing a more integral-precise information structure, and greater integral-precision leads to a greater expected price if there is enough competition. Hence, if providing information is costless, integral-precision implies that the auctioneer's incentives to provide information are lower than the social ones since the efficient policy is full information disclosure and information may reduce the price. If there is enough competition, the incentives to provide information will be positive and, in the limit, social and private incentives will coincide.

The criterion of single-crossing precision is not implied by any of all standard informativeness criteria. This criterion adds to the results obtained with integral-precision by establishing that the incentives of the auctioneer to provide information increase with the number of bidders.

Our central notion of informativeness and the strongest one, sm-precision, allows us to obtain comparative statics results with costly provision of information. Information increases bidders' informational rents, and leads the auctioneer to optimally provide information to the market below that which is efficient. We also show that there is a strategic complementarity between information and competition, so that both the efficient level of investment in providing information to the market and the auctioneer's optimal investment increase with the number of bidders. We can also show that both converge as the number of bidders goes to infinity.

A   Appendix

A.1   A Sufficient Condition For Greater Supermodular-Precision

Supermodular-precision is based on the shape of the distribution of updated expected valuations. In some cases this condition can be verified directly but at times, such as when looking at parameterized families of bivariate distributions, one might prefer to use an informativeness condition based on some property of the joint distribution function of signals and valuations.

In this section we identify one such condition that implies *sm-precision*. This condition is a mirror image of Lehmann (1988)'s notion of effectiveness and can be found in the statistics literature on positive dependence:[18]

Definition 8 (Capéraà and Genest (1990)) *Let F_δ and $F_{\delta'}$ be two joint distribution functions with posterior distribution functions $F_\delta(v|\pi)$ and $F_{\delta'}(v|\pi)$. The distribution function F_δ is more stochastically increasing in π than $F_{\delta'}$, if the function:*

$$\gamma(v, \pi) = F_\delta^{-1}(F_{\delta'}(v|\pi)|\pi) \quad \text{is nondecreasing in } \pi. \tag{3}$$

If $F_\delta(v|\pi)$ and $F_{\delta'}(v|\pi)$ are continuous in v for all π, then $\gamma(v, \pi)$ is increasing in π and is equivalent [19] to the following single-crossing condition: for all $\pi' < \pi$, $v, v' \in \mathcal{V}$,

$$F_{\delta'}(v|\pi') \geq F_\delta(v'|\pi') \Rightarrow F_{\delta'}(v|\pi) \geq F_\delta(v'|\pi)$$

Notice that Condition (3) appears in Lehmann (1988)'s definition of effectiveness but with valuations and signals interchanged.[20]

If, in addition, $F_\delta(v|\pi)$ is differentiable in δ and v, then greater sm-precision is also equivalent

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[18]Positive dependence between two variables is the property that larger realizations of one variable are probabilistically associated with larger realizations of another.

[19]See Fang and Joe (1992).

[20]Lehmann's definition of effectiveness also includes the requirement that both F_δ and $F_{\delta'}$ have the monotone likelihood ratio property.

to the Spence-Mirrlees type condition:[21]

$$\frac{\partial F_\delta(v|\pi)/\partial \delta}{\partial F_\delta(v|\pi)/\partial v} \quad \text{is nondecreasing in } \pi. \tag{4}$$

Proposition 4 *Suppose $F_\delta(v|\pi)$ and $F_{\delta'}(v|\pi)$ are differentiable in δ and v. If F_δ is more stochastically increasing in π than $F_{\delta'}$, then $\mathbf{F}_\delta$ is more informative, in terms of sm-precision, than $\mathbf{F}_{\delta'}$.*

Proof:

If F_δ is more stochastically increasing in π than $F_{\delta'}$, then the Spence-Mirlees type condition (4) holds (Jewitt(1997), Section 5.4). Following an argument presented in LiCalzi (2005):

$$\begin{aligned}
\frac{\partial}{\partial \delta}W(\pi,\delta) &= \frac{\partial}{\partial \delta}\int_\mathcal{V}(1 - F_\delta(v|\pi))dv \\
&= -\int_\mathcal{V}\frac{\partial}{\partial \delta}F_\delta(v|\pi)dv \\
&= -\int_\mathcal{V}\frac{\partial F_\delta(v|\pi)/\partial \delta}{\partial F_\delta(v|\pi)/\partial v}dF_\delta(v|\pi)
\end{aligned}$$

As $\frac{\partial F_\delta(v|\pi)/\partial \delta}{\partial F_\delta(v|\pi)/\partial v}$ is decreasing in π, $\partial W(\pi,\delta)/\partial \delta$ is increasing in π, i.e. $W(\pi,\delta)$ is supermodular. ∎

Summarizing, condition (3) can be interpreted as a mirror image of Lehmann's effectiveness condition, and is sufficient to ensure that two information structures are ordered in terms of the sm-precision of their signals.

A.2   Definitions and Preliminary Result

We make repeated use of the following very well-known result, which we state as a lemma: if $X \geq_{st} Y$, then for all nondecreasing functions ψ, $E[\psi(X)] \geq E[\psi(Y)]$.

Lemma 1 *Let X and Y be real-valued random variables with cumulative distribution functions F and G respectively, such that $F(z) \leq G(z)$ for all $z \in \mathbf{R}$. For all bounded real-valued nondecreasing functions $\psi : \mathbf{R} \to \mathbf{R}$,*

$$\int_\mathbf{R}\psi(z)\mathrm{d}F(z) \geq \int_\mathbf{R}\psi(z)\mathrm{d}G(z)$$

---

[21] Jewitt (1997) gives a detailed comparison of different notions of informativeness and illustrates the connection between the standard Spence-Mirrlees condition and Lehmann's notion of effectiveness, which we use in the proof of Proposition 4.

We also use the following notation: $U_{i:j}(x)$ is the cumulative distribution function (cdf) of a random variable Y such that $U_{i:j}(x) = \Pr(Y \leq x)$. This random variable is the ith order statistic from a sample of j independently and identically uniform distributed random variables over $[0, 1]$, where $U_{1:j}$ refers to the cdf of the maximum of the sample, $U_{2:j}$ to the cdf of the second highest realization in the sample and so on until $U_{j:j}$, which is the cdf of the minimum realization in the sample. We will also make use of the functional form of $U_{1:n}$, $U_{1:n}(\pi) = \pi^n$, $\pi \in [0, 1]$.

A.3   Proofs

Proof of Remark 1:

Applying the law of iterated expectations: $\mu = E[E[v|X]] = E[W(\Pi, \delta)]$ and $\mu = E[E[v|X']] = E[W(\Pi', \delta')]$. If X and Y, two random variables, have the same mean and X is more disperse than Y, then X is dominated by Y in terms of SOSD (Shaked and Shantikumar (1994), Theorem 2.B.10). ∎

Proof of Theorem 1: We want to show that if $\delta > \delta'$ then $V_1(n, \delta) \geq V_1(n, \delta')$. This is equivalent to showing

$$\int_0^1 (W(\pi, \delta) - W(\pi, \delta')) \, \mathrm{d}U_{1:n}(\pi) \geq 0$$

By the law of iterated expectations, the expected valuation of the distribution of expected valuations, $E[W(\pi, \delta)] = \mu$, must not depend on the information structure. Let $U_{1:1}(\pi) = \pi$ denote the cumulative distribution function of the uniform. We can now write

$$\int_0^1 (W(\pi, \delta) - W(\pi, \delta'))\mathrm{d}\pi = \int_0^1 (W(\pi, \delta) - W(\pi, \delta'))\mathrm{d}U_{1:1}(\pi) = 0$$

Define the function $\psi(\pi) \equiv (W(\pi, \delta) - W(\pi, \delta'))$. By the definition of sm-precision, $\psi(\pi)$ is nondecreasing in π. As $U_{1:n}(\pi) = \pi^n \leq U_{1:1}(\pi)$ for all $n \geq 1$ and $\pi \in [0, 1]$, and $\psi(\pi)$ is nondecreasing, we can apply Lemma 1 and the result follows.

∎

Proof of Theorem 2: It suffices to show that $V_1(n + 1, \delta) - V_1(n, \delta) \geq V_1(n + 1, \delta') - V_1(n, \delta')$.

This is equivalent to showing

$$V_1(n + 1, \delta) - V_1(n + 1, \delta') \geq V_1(n, \delta) - V_1(n, \delta')$$

$$\Leftrightarrow \int_0^1 (W(\pi_1, \delta) - W(\pi_1, \delta')) \mathrm{d}U_{1:n+1}(\pi_1) \geq \int_0^1 (W(\pi_1, \delta) - W(\pi_1, \delta')) \mathrm{d}U_{1:n}(\pi_1)$$

As $W(\pi, \delta) - W(\pi, \delta')$ is nondecreasing in π and $U_{1:n+1}(\pi) = \pi^{n+1} \leq U_{1:n}(\pi) = \pi^n$ for all $\pi \in [0, 1]$, we can apply Lemma 1.

∎

Proof of Corollary 1: Immediate from the results of Milgrom and Shannon (1994) and Theorem 2. ∎

Proof of Proposition 1:

Part (i): We want to show that if $\delta > \delta'$ then $V_2(2, \delta) \leq V_2(2, \delta')$. With two bidders and prior to their receiving their private signals, both have the same probability $(\frac{1}{2})$ of being the bidder with the highest valuation. By the law of iterated expectations, the ex ante (before receiving the signal) expected valuation of bidders must not depend on the information structure. We can conclude that

$$\frac{V_1(2, \delta) + V_2(2, \delta)}{2} = \frac{V_1(2, \delta') + V_2(2, \delta')}{2} = \mu$$

Hence, $V_1(2, \delta) - V_1(2, \delta') = -(V_2(2, \delta) - V_2(2, \delta'))$. From Theorem 1, $V_1(2, \delta) - V_1(2, \delta') \geq 0$, which implies $V_2(2, \delta) - V_2(2, \delta') \leq 0$.

Part (ii): We want to show that there exists n' such that for $n \geq n'$, $V_2(n, \delta) - V_2(n, \delta') \geq 0$. It suffices to show first that for all n, $V_2(n, \delta) - V_2(n, \delta')$ is nondecreasing in n and then to demonstrate that $\lim_{n \to \infty} V_2(n, \delta) - V_2(n, \delta') > 0$.

To show $V_2(n, \delta) - V_2(n, \delta')$ is nondecreasing in n for all n we follow the same logic as in the proof of Theorem 2:

$$V_2(n+1, \delta) - V_2(n+1, \delta') \geq V_2(n, \delta) - V_2(n, \delta')$$
$$\Leftrightarrow \int_0^1 (W(\pi, \delta) - W(\pi, \delta')) \mathrm{d}U_{2:n+1}(\pi) \geq \int_0^1 (W(\pi, \delta) - W(\pi, \delta')) \mathrm{d}U_{2:n}(\pi)$$

As $W(\pi, \delta) - W(\pi, \delta')$ is nondecreasing in π and $U_{2:n+1}(\pi) \leq U_{2:n}(\pi)$ for all $\pi \in [0, 1]$, we can apply Lemma 1.

Finally, as n goes to infinity, $U_{2:n}$ converges to a distribution with mass only at $\pi = 1$. Thus,

$$V_2(\delta, n) - V_2(\delta', n) = \int_0^1 \psi(\pi) \mathrm{d}U_{2:n}(\pi)$$

converges to $\psi(1) > 0$. ■

Proof of Proposition 2:

We want to show that for $\delta > \delta'$, $R_w(n, \delta) \geq R_w(n, \delta')$, i.e.

$$V_1(n, \delta) - V_2(n, \delta) \geq V_1(n, \delta') - V_2(n, \delta')$$

This is equivalent to

$$V_1(n, \delta) - V_1(n, \delta') \geq V_2(n, \delta) - V_2(n, \delta')$$

i.e.,

$$\int_0^1 (W(\pi_1, \delta) - W(\pi_1, \delta')) \mathrm{d}U_{1:n}(\pi_1) \geq \int_0^1 (W(\pi_2, \delta) - W(\pi_2, \delta')) \mathrm{d}U_{2:n}(\pi_2)$$

Again, using $W(\pi, \delta) - W(\pi, \delta')$ is nondecreasing in π and the stochastic dominance of the first order statistic over the second, $U_{1:n}(\pi) \leq U_{2:n}(\pi)$ for all $\pi \in [0, 1]$, we can apply Lemma 1.

■

Proof of Theorem 3: As in the proof of Theorem 2, it suffices to show that $V_2(n+1, \delta) - V_2(n, \delta) \geq V_2(n+1, \delta') - V_2(n, \delta')$, and we have demonstrated this in the proof of Proposition 1, part (ii). ■

Proof of Corollary 2: Immediate from the results of Milgrom and Shannon (1994) and Theorem 3.

■

Proof of Theorem 4: The auctioneer's problem is

$$\delta^A \in \operatorname*{argmax}_{\delta^A} \{V_2(n, \delta) - \delta\}$$

This problem is equivalent to

$$\delta^A \in \operatorname*{argmax}_{\delta^A} \{V_1(n, \delta) - \delta - R_w(n, \delta)\}$$

where $R_w(\delta)$ is as defined in the text.

Compare the formulation of the auctioneer's problem to the formulation of the social welfare maximization problem (equation (1)). As δ^A solves the auctioneer's optimization problem

$$E[W(\Pi_1, \delta^A) - \delta^A - R_w(\delta^A)] \geq E[W(\Pi_1, \delta^E) - \delta^E - R_w(\delta^E)]$$

$R_w(\delta)$ is nondecreasing (Proposition 2) so that if $\delta^A > \delta^E$, then this last equation would imply

$$E[W(\Pi_1, \delta^A) - \delta^A] \geq E[W(\Pi_1, \delta^E) - \delta^E]$$

but this contradicts the fact that δ^E maximizes social surplus, so that $\delta^A \leq \delta^E$.

To establish the second part of the Theorem, consider the informational rents, $R_w(n, \delta)$.

$$
\begin{aligned}
R_w(n, \delta) &= V_1(n, \delta) - V_2(n, \delta) \\
&= \int_0^1 W(\pi, \delta) dU_{1:n}(\pi) - \int_0^1 W(\pi, \delta) dU_{2:n}(\pi) \\
&= \int_0^1 W(\pi, \delta) d(U_{1:n}(\pi) - U_{2:n}(\pi))
\end{aligned}
$$

We know $U_{1:n}(\pi) = \pi^n$ and $U_{2:n}(\pi) = n\pi^{n-1} - (n-1)\pi^n$.

$$
\begin{aligned}
U_{1:n}(\pi) - U_{2:n}(\pi) &= n(\pi^n - \pi^{n-1}) \\
\Rightarrow \lim_{n \to \infty} U_{1:n}(\pi) - U_{2:n}(\pi) &= 0
\end{aligned}
$$

As $W(\pi, \delta)$ is bounded and monotone in π, and $(U_{1:n}(\pi) - U_{2:n}(\pi))$ converges to zero then $R_w(n, \delta)$ also converges to zero

$$\lim_{n \to \infty} R_w(n, \delta) = \lim_{n \to \infty} \int_0^1 W(\pi, \delta) d(n(\pi^n - \pi^{n-1})) = 0$$

As information eventually becomes valuable, the objective function of the auctioneer approaches total surplus as n goes to infinity.

∎

Proof of Proposition 3:

Part (i): We proceed by constructing an example of two information structures ordered in the sense of Blackwell (1951) which are not ordered in the sense of *sc-precision*.

Let V be uniformly distributed on $[0, 1]$. Let X be equal to 0 if $v \in [0, 1/2)$ and equal to 1 if $v \in [1/2, 1]$. Similarly, let Y distributed as follows

$$Y = \begin{cases} 0 & \text{if} \quad v \in [0, 1/4) \\ 1 & \text{if} \quad v \in [1/4, 1/2) \\ 2 & \text{if} \quad v \in [1/2, 3/4) \\ 3 & \text{if} \quad v \in [3/4, 1] \end{cases}$$

Clearly Y is more informative than X in the sense of Blackwell so that the information structure for (V, Y), denoted $\mathbf{F}_\delta$, is more informative than that for (V, X), denoted $\mathbf{F}_{\delta'}$, in the sense of Blackwell. Nevertheless, $E[v|X = 0] = 1/4$ and $E[v|X = 1] = 3/4$, while

$$E[v|Y] = \begin{cases} 1/8 & \text{if} \quad Y = 0 \\ 3/8 & \text{if} \quad Y = 1 \\ 5/8 & \text{if} \quad Y = 2 \\ 7/8 & \text{if} \quad Y = 3 \end{cases}$$

So that if X is the signal from $\mathbf{F}_{\delta'}$ and Y $\mathbf{F}_\delta$, then $\psi(\pi) \equiv W(\pi, \delta) - W(\pi, \delta')$ from equals

$$\psi(\pi) = \begin{cases} -1/8 & \text{if} \quad v \in [0, 1/4) \\ 1/8 & \text{if} \quad v \in [1/4, 1/2) \\ -1/8 & \text{if} \quad v \in [1/2, 3/4) \\ 1/8 & \text{if} \quad v \in [3/4, 1] \end{cases}$$

and $\mathbf{F}_\delta$ and $\mathbf{F}_{\delta'}$ are not SC-ordered.

Part (ii): Let $\delta > \delta'$. We want to rewrite $W(\pi, \delta) - W(\pi, \delta')$ to show MIO-ND implies *integral-precision*. Using the properties of Riemann-Stieltjes integrals:

$$W(\pi, \delta) - W(\pi, \delta') = \int_{\mathcal{V}} v \, dF_\delta(v|F_\delta^{-1}(\pi)) - \int_{\mathcal{V}} v \, dF_{\delta'}\left(v|F_{\delta'}^{-1}(\pi)\right)$$

Integrating by parts

$$W(\pi, \delta) - W(\pi, \delta') = -\int_{\mathcal{V}} (F_\delta(v|F_\delta^{-1}(\pi)) - F_{\delta'}\left(v|F_{\delta'}^{-1}(\pi)\right)) dv$$

Then

$$\int_0^\pi (W(p, \delta) - W(p, \delta')) dp = -\int_0^\pi (\int_{\mathcal{V}} (F_\delta(v|F_\delta^{-1}(p)) - F_{\delta'}\left(v|F_{\delta'}^{-1}(p)\right)) dv) dp$$

Interchanging the integration limits on the RHS

$$\int_0^\pi (W(p, \delta) - W(p, \delta')) dp = -\int_{\mathcal{V}} \int_0^\pi ((F_\delta(v|F_\delta^{-1}(p)) - F_{\delta'}\left(v|F_{\delta'}^{-1}(p)\right)) dp) dv$$

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If $\delta < \delta'$ implies that the corresponding information structure is more informative according to MIO-ND (Definition 7) then:

$$\int_0^\pi \left(F_\delta \left(v | F_\delta^{-1}(p) \right) - F_{\delta'} \left(v | F_{\delta'}^{-1}(p) \right) \right) \mathrm{d}p \geq 0, \quad \forall \pi \in [0, 1] \tag{5}$$

Hence,

$$\int_0^\pi (W(p, \delta) - W(p, \delta')) \mathrm{d}p \leq 0, \quad \forall \pi \in [0, 1]$$

■

A.4 Other Results

In this section we prove several claims stated informally in the text.

A.4.1 Our two examples of precision

Example 1(normal distribution): Consider $\delta > \delta'$ and let $\Phi(x)$ be the cumulative distribution of a standard normal distribution, $\phi(x)$ the distribution function:

$$
\begin{aligned}
E[v|x] &= \mu + (x - \mu)\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\delta^2} \\
F_\delta(x) &= \pi = \Phi\left(\frac{x - \mu}{\sqrt{\sigma_v^2 + \sigma_\delta^2}} \right) \\
\Leftrightarrow F_\delta^{-1}(\pi) &= \mu + \Phi^{-1}(\pi)\left(\sqrt{\sigma_v^2 + \sigma_\delta^2} \right)
\end{aligned}
$$

So that

$$
\begin{aligned}
W(\pi, \delta) &= E[v|F_\delta^{-1}(\pi)] \\
&= \mu + \Phi^{-1}(\pi)\frac{\sigma_v^2}{\sqrt{\sigma_v^2 + \sigma_\delta^2}} \\
\Rightarrow W(\pi, \delta) - W(\pi, \delta') &= E[v|F_\delta^{-1}(\pi)] - E[v|F_{\delta'}^{-1}(\pi)] \\
&= \Phi^{-1}(\pi)\left(\frac{\sigma_v^2}{\sqrt{\sigma_v^2 + \sigma_\delta^2}} - \frac{\sigma_v^2}{\sqrt{\sigma_v^2 + \sigma_{\delta'}^2}} \right)
\end{aligned}
$$

As $\sigma_\delta^2 < \sigma_{\delta'}^2$ then

$$A(\delta, \delta') := \left(\frac{\sigma_v^2}{\sqrt{\sigma_v^2 + \sigma_\delta^2}} - \frac{\sigma_v^2}{\sqrt{\sigma_v^2 + \sigma_{\delta'}^2}} \right) > 0$$

so that

$$\frac{\partial}{\partial \pi}(W(\pi, \delta) - W(\pi, \delta')) = \frac{A(\delta, \delta')}{\phi(\pi)} > 0$$

Example 2:

Given the structure of the signal, the bidders' expected valuation is

$$
\begin{aligned}
w(x, \delta) &= E[v|x] \\
&= x_i \delta + (1 - \delta)\mu \\
\Rightarrow w(x, \delta) - w(x', \delta) &= \delta(x - x') \\
\Leftrightarrow W(\pi, \delta) - W(\pi', \delta) &= \delta\left(H^{-1}(\pi) - H^{-1}(\pi')\right)
\end{aligned}
$$

So that for $\pi > \pi'$, $\partial(W(\pi, \delta) - W(\pi', \delta))/\partial\delta > 0$.

A.4.2 Further Discussion of Section 5.2

In this section we provide a more detailed discussion of the statements made in Section 5.2.

Theorem 1 holds under weaker notions of precision, such as *integral-precision*. Using similar arguments as those used in the proof of Theorem 1,

$$V_1(n, \delta) \geq V_1(n, \delta') \Leftrightarrow \int_0^1 \psi(\pi)dU_{1:n}(\pi) \geq 0$$

where $\psi(\pi) \equiv (W(\pi, \delta) - W(\pi, \delta'))$ and let $\Psi(b) = \int_0^b \psi(\pi)d\pi$. Greater integral *precision* implies $\Psi(\pi) \leq 0, \forall \pi \in [0, 1]$.

Using $U_{1:n}(\pi) = \pi^n$ and integrating by parts, $V_1(n, \delta) \geq V_1(n, \delta')$ holds as:

$$\int_0^1 \psi(\pi)n\pi^{n-1}d\pi = -\int_0^1 \Psi(\pi)n(n-1)\pi^{n-2}d\pi \geq 0$$

where the last inequality holds as $\Psi(\pi) \leq 0, \forall \pi \in (0, 1)$.

Then, if information provision is costless, the efficient policy is to fully disclose all information—and the first part of Theorem 4 holds trivially.

Also, note that the proof of part (i) of Proposition 1 only makes use of Theorem 1. As this latter result extends to the case where information structures are ordered by integral-precision, then under this condition the value of information is also negative with only two bidders.

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As for the second part of the proposition, with integral-precision we can show that given any $\delta' < \delta$, there exists a $\hat{n}$ such that for all $n > \hat{n}$, $V_2(n, \delta) > V_2(n, \delta')$. Let $\phi(n) \equiv V_2(n, \delta) - V_2(n, \delta')$,

$$\phi(n) = n(n-1) \int_0^1 \psi(\pi)(1-\pi)\pi^{n-2}d\pi$$

Because $\mathbf{F}_\delta$ is more integral-precise than $\mathbf{F}_{\delta'}$, then

$$\forall p \in [0,1], \quad \int_0^p \psi(\pi)d\pi \leq 0$$

This and the fact that $\int_0^1 \psi(\pi)d\pi = 0$ implies $\forall p \in [0,1], \int_p^1 \psi(\pi)d\pi \geq 0$. Let $A = \{p \in [0,1] | \psi(p) < 0\}$ and $\hat{p}$ the highest p in the closure of A. Ignoring the trivial case that $\psi(\pi) \equiv W(\pi, \delta) - W(\pi, \delta')$ is equal to zero for all π, integral-precision implies that $\hat{p} \in (0,1)$ and there exists $p_1, p_2 \in (\hat{p}, 1]$ such that $\forall \pi \in [p_1, p_2], \psi(\pi) > 0$. Let $c_1 = \min_{\pi \in [0, p_1)} \psi(\pi)(1-\pi)$, and $c_2 = \min_{\pi \in [p_1, p_2]} \psi(\pi)(1-\pi)$. Notice that $c_1 < 0$ and $c_2 > 0$. Then

$$\begin{aligned}
\phi(n) &\geq n[p_1^{n-1}c_1 + (p_2^{n-1} - p_1^{n-1})c_2] \\
&= np_2^{n-1}[(p_1/p_2)^{n-1}(c_1 - c_2) + c_2]
\end{aligned}$$

Let

$$\hat{n} \equiv 1 + \frac{\ln\left(\frac{c_2}{c_2 - c_1}\right)}{\ln\left(\frac{p_1}{p_2}\right)}$$

As $p_1/p_2 < 1$, then for all $n > \hat{n}$, $(p_1/p_2)^{n-1}(c_1 - c_2) + c_2 > 0$, and $\phi(n) > 0$.

Integral-precision provides limit results but it does not give us any predictions as to the monotonicity of the optimal release of information. On the other hand, *sc-precision* does provide a sufficient condition for the optimal release of information to be monotone in the number of bidders. The reason is as follows: by *sc-precision*, $W(\pi, \delta)$ is single-crossing in (π, δ) (in the sense that for all $\delta > \delta'$, $W(\pi', \delta) - W(\pi', \delta') \geq (>)0$ implies for all $\pi > \pi'$, $W(\pi, \delta) - W(\pi, \delta') \geq (>)0$). The family of distributions $\{U_{2:n}(p)\}_n$ is ordered by n in the likelihood-ratio order. As

$$V_2(n, \delta) = \int_0^1 W(\pi, \delta)dU_{2:n}(\pi)$$

then, by the results in Athey (2002), $V_2(n, \delta)$ is single-crossing in (n, δ) and the optimal choice of δ is monotone in n.

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