Centre de Referència en Economia Analítica

Barcelona Economics Working Paper Series

Working Paper nº 97

Unemployment Insurance Design: inducing moving and retraining

John Hassler and José V. Rodríguez Mora,

December, 2003

Unemployment Insurance Design: inducing moving and retraining[¤]

John Hassler[y]   José V. Rodríguez Mora[z]

December, 2003
Barcelona Economics WP nº# 97

Abstract

Evidence suggests that unemployed individuals sometimes can a¤ect their job prospects by undertaking a costly action like deciding to move or retrain. Realistically, such an opportunity arises only for some individuals and the identity of those is unobservable. Unemployment insurance should then be designed to induce individuals to exploit existing opportunities to move or retrain without excessively diminishing the insurance value for the remaining unemployed. This problem has been neglected in previous literature on unemployment insurance design and we show that it may have important consequences. In particular, we derive closed-form solutions, showing that unemployment bene…ts should increase over the unemployment spell, having an initial period with low bene…ts and a substantial increase after this period has expired.

JEL Classi…cation: J65, J64, E24

Keywords: Unemployment bene…ts, search, moral hazard, adverse selection

1 Introduction

An important feature of the modern welfare state is the existence of an extensive unemployment insurance (UI) system. It is now well established that the design of the unemployment insurance a¤ects the incidence of unemployment by distorting the incentives of unemployed to search for a job (see, e.g., ? for a survey). This has motivated a growing literature on how the UI system should be designed to make an optimal trade-o¤ between providing good insurance on the one hand, and not distorting the incentives too much, on the other. The seminal paper by ? characterizes the optimal design of UI when search activity is unobservable. Since then, a line of papers that extend the analysis has appeared. For example, ? allow a more general set of policies; speci...cally, they assume that in addition to UI-bene...ts, also taxes paid by employed can be made contingent on the employment history of the individual. An important assumption in ? and ?, is that the insurer can fully control the individual's consumption – usually interpreted as the individual having no access to markets for saving and borrowing and no alternative sources of income. It has proven di¢cult to relax the assumption of no hidden savings, but recently important progress has been made in this respect (see ?, ? and ?). Other important extensions of the analysis, for example allowing sequential search, endogenous wage formation, job-creation and production, has also been done in recent years (see, e.g., ? , ?, ?, ? and ?).

In this paper, we will maintain most of the standard assumptions in the literature but cast the focus on an important informational problem that has been largely neglected. Speci...cally, we will consider the case when some, but not all, unemployed can increase the probability of being hired by undertaking a costly investment, e.g., by retraining or moving to a location with better employment prospects. Under the realistic assumption that the insurer is unable to observe who has this option, an incentive problem arises and a failure to take this into account may lead to sub-optimal UI-design. One concievable way to mitigate the problem would be to o¤er subsidies to moving or retraining. However, as we will argue below, full cost-compensation is not feasible in the realistic case when the insurer can not fully distinguish volontary and unvolontary job-separations.

Although an empirical investigation is outside the scope of this paper, we argue that the consequences of not providing reasonable incentives for people to move or retrain may be of substantial quantitative importance. For instance, ? documents that the proportion of geographical mobility in the U.S. caused by the decision to change jobs is one-half of all

migration decisions for young workers and one third of all migration decisions for workers above the age of 45. Furthermore, geographical mobility is substantially lower in continental Europe, and ? document in panel-data a negative correlation between geographical mobility and UI-generosity as well as between mobility and aggregate unemployment rates. Other empirical documentations of the link between unemployment and geographical mobility are ?, ? and ?.

In, ?, a constant UI-bene...t is assumed and, of course, the higher this is, the weaker are the incentives to move. Following the tradition in the optimal UI-design literature, we will investigate if non-constant bene...t rates can strengthen the incentives to move without reducing the insurance value of UI. Since we believe that also the standard moral hazard problem of providing incentives for a continuous job-search are important, we will include this in the analysis.

There is empirical evidence indicating that precautionary saving is used in order to self-insure against unemployment risk. Using PSID, ? ...nds that, in absence of UI, consumption falls by 22% when an individual become unemployed, showing that individuals are able to smooth consumption also when there is no UI. Similarly, ? show that UI crowds out ...nancial savings, indicating that households use ...nancial markets to self-insure against unemployment risk.[1] The assumption that the insurer can perfectly control individual consumption is thus not entirely realistic. Building on the emerging tradition in the recent papers cited above, we will therefore allow the individual to make her own consumption decisions, allowing access to a market for saving and borrowing.

To facilitate understanding of the results, we will make assumptions that allow analytical characterizations and, speci...cally, graphical and closed form solutions for optimal bene...ts as well as for observables like the changes in individual consumptions levels associated with a change of job status. Our model also easily lends itself to allowing multiple incentive problems, e.g., adding a moral hazard problem in job-retention e¤ort like in ?.

Two important assumptions are key to analytical tractability; First, we assume constant absolute risk-aversion implying that search incentives are independent of asset holdings. Second; Individuals have access to a perfect market for borrowing and lending. As shown in ?, a constant bene...t scheme is optimal under these assumptions when search e¢ciency is constant. This allows us to focus on simple bene...t schemes with a limited number of bene...t

---

[1]Also if access to the formal capital market is limited, alternative means to smooth consumption may exist, see e.g., ?.

3

levels. Neither of the key assumptions is perfectly realistic, and the model does therefore not directly lend itself to quantitative policy recommendations. Our purpose is instead to illustrate a mechanism not previously explored in the literature, thereby providing guidence for future quantitative work.

The paper is structured in the following way. The model is presented in section 2, where in subsection 2.1 we derive the relevant value functions, in subsection 2.2 incentive compatibility constraints are derived. In section 3 and subsection 3.1 the main results are derived and discussed and section 4 concludes. Some proofs can be found in the text, others in the appendix and the remaining are available upon request from the authors.

2 The model

Consider an economy in continuous time where individuals can be employed or unemployed. They have access to a market for safe saving and borrowing with an exogenous return r; equal to the subjective discount rate (possibly including a positive probability of dying). Unemployed individuals can a¤ect their chances of …nding a job. As noted in the introduction, we will focus on the case where some, but not necessarily all, individuals can make a costly investment increasing their chances of becoming employed. Allowing unobservable heterogeneity in this respect creates and informational problem similar to an adverse selection problem and makes full insurance infeasible.[2] In addition, we will allow a more standard moral hazard problem where search activity entails a ‡ow cost.

Speci…cally, we assume that employed individuals loose their jobs at rate q. A share $p \in [0; 1]$ of those who loose their job can undertake a costly investment. We will interpret this as representing a cost of moving, denoted $m > 0$ (for example between geographical locations or between occupations that require some retraining). For simplicity, we assume that if the unemployed pays this cost ("moves"), she is immediately rehired. Unemployed who cannot, or decide not to move and who search for a job …nd one at rate h. Searching has a cost of $s \geq 0$ per unit of time. We may consider this cost as representing the opportunity cost of searching, arising from, for example, some alternative economic activity. Whether the agent actually searches or not and whether she has the opportunity to move are assumed

---

[2]There are few papers on UI deal with adverse selection. One recent paper is ?, where individuals have di¤erent hiring rates are separated by being o¤ered di¤erent menues of bene…ts.

to be her own private information. To make the problem interesting, we assume that it is optimal to induce individuals to search and move (if they have the opportunity). It is easy to show that under this assumption, agents who have the option to move should be induced to do so immediately. Therefore, in the optimal solution, no mass of agents should be unemployed while having the opportunity to move.

An employed individual is said to be in state 1, receiving an exogenous gross wage w. An individual who looses her job and do not move enters into state 2 and is then called short-term unemployed, receiving bene...ts denoted b_2. To analyze the issue of whether unemployment bene...ts should be increasing or decreasing, we allow two bene...t levels, b_2 and b_3, the latter being given to individuals in state 3, who are denoted long-term unemployed. To facilitate a simple presentation of the results, we assume that an individual in state 2 enters state 3 with a constant instantaneous probability f.[3] Since state 3 is an administrative state associated with long unemployment duration, we assume individuals who search to have the same hiring rates, h, in the two unemployment states.[4]. Motivated by practical considerations, and in contrast to, e.g., ?, we assume that bene...t levels can be given conditional only on current unemployment status (2 or 3), not not on employment history or asset holdings.

Individuals maximize their intertemporal utility, given by

$$E \int_0^1 e^{-rt} U(c_t) \, dt,$$

where c_t is consumption at time t and r the subjective discount rate. In order to facilitate analytical solutions when individuals have access to markets for saving and borrowing, we choose the CARA utility function

$$U(c_t) \, \acute{} \, \text{¡} \, e^{-^\circ c_t},$$

where $^\circ$ is the coe¢cient of absolute risk aversion.[5] All individuals are born (enter the

---

[3]This assumption implies that seach incentives remain constant as long as the individual remain in state 2. An alternative would be to use discrete time and assume that short-term UI bene...ts are paid for one period only as done by e.g., ?. Assuming that UI bene...ts change after some ...xed period of time would make search incentives depend on the remaining time of current bene...ts and considerably complicate the analysis with little gain.

[4]This assumption could, however, easily be relaxed.

[5]Given this, assets will not a¤ect individual decisions. For other utility functions, the decision to move and to search for a job would depend on the individual asset level. Then, asset dependent bene...ts would be required to satisfy the incentive constraints exactly. We believe that some asset dependence, like a means-tested UI-system might be reasonable, but we leave such schemes for future research.

labor market) as employed without assets and are identical at that point.

The purpose of this paper is to discuss how an unemployment insurance system should be constructed when there are incentive problems. To his end, we want to remove other motives for unemployment bene...ts than providing insurance. In particular, we are in this paper not interested in motives to use the UI system to create non-actuarial transfers between individuals with di¤erent characteristics. Therefore, we assume that individuals face an actuarially fair insurance. This means that when an individual enters the labor force, the expected present discounted value of the bene...ts she will receive during her life-time exactly balances the expected present discounted value of her contributions. An alternative interpretation of actuarial fairness is that in a decentralized equilibrium, where individuals can sign binding insurance contracts with competitive insurance companies when entering their ...rst job, actuarial fairness is identical to a break-even condition for the insurance companies, which would be satis...ed under perfect competition.[6]

Without loss of generality, we let individuals pay lump-sum taxes, denoted ι, implying that

$$\dot{A}_t = rA_t + \omega - c_t - \iota; \tag{1}$$

except at the points in time when the cost of moving is paid, and where $\omega \in \{w; b_1 - s; b_2 - s\}$, depending on the employment state. We de...ne the average discounted probabilities (ADP's) of being in state 2 and 3, respectively, by

$$\Pi_2 \equiv r \int_0^1 e^{-rt} \pi_{2;t} dt;$$
$$\Pi_3 \equiv r \int_0^1 e^{-rt} \pi_{3;t} dt:$$

where $\pi_{2;t}$ and $\pi_{3;t}$ are the probabilities of being short term and long term unemployed at time t, respectively, conditional on being employed at time zero. Solving for the ADP's assuming that individuals who can move do so and that unemployed search for a job yields

$$\Pi_2 \equiv \tilde{q} \frac{h + r}{(r + h + \tilde{q})(r + h + f)}; \tag{2}$$
$$\Pi_3 \equiv \Pi_2 \frac{f}{h + r};$$

where $\tilde{q} \equiv q(1 - p)$ equals the rate of ‡ow into unemployment.

---
[6]Since we use the CARA speci...cation, individual assets do not a¤ect preference over insurance so older employed agents with non-zero asset holdings would not want to renegotiate their contract.

The actuarial fairness requirement of the UI system can then be written

$$\iota = \tau_2 b_2 + \tau_3 b_3. \tag{3}$$

2.1 Value functions and consumption

It is well known that the value functions for the three states can be written as

$$V_j(A_t) = -\frac{1}{r}e^{-rA_t}e^{-\mu_j}, \quad j \in \{1,2,3\}, \tag{4}$$

where μ_j are state-dependent constants and where the state dependent consumption functions are

$$c_j(A_t) = rA_t + \mu_j, \quad j \in \{1,2,3\}.$$

Note that individuals optimally consume the permanent income from their asset holdings (rA_t) plus a state dependent constant μ_j. The difference in consumption between individuals in different states but with identical assets holdings thus satisfy

$$c_1(A_t) - c_2(A_t) = \mu_1 - \mu_2 \equiv \Phi_2 \text{ and} \tag{5}$$

$$c_1(A_t) - c_2(A_t) = \mu_1 - \mu_3 \equiv \Phi_3.$$

It is straightforward to check that the Bellman equation for the individuals who search and move is satisfied if the constants μ_j, satisfy

$$\mu_1 = w - \iota - q\frac{pe^{-rm} + (1-p)e^{-(\Phi_2)} - 1}{-r}, \tag{6}$$

$$\mu_2 = b_2 - s - \iota + h\frac{1 - e^{-(\Phi_2)}}{-r} - f\frac{e^{-(\Phi_3 - \Phi_2)} - 1}{-r},$$

$$\mu_3 = b_3 - s - \iota + h\frac{1 - e^{-(\Phi_3)}}{-r},$$

in which case individual intertemporal utility is maximized under (1) and a No-Ponzi condition.

Our objective is to maximize welfare of an individual entering the labor market with no assets, $V_1(0)$, subject to incentive constraints and actuarial fairness. From (4), we note that i) this is equivalent to maximizing μ_1, subject to the constraints, and ii) the solution will maximize welfare of all employed, regardless of their asset holdings and previous employment history.

Our procedure will be done in two steps. First, we will maximize μ_1 over the consumption differences subject to incentive constraints (to be defined shortly) and actuarial

7

fairness, thus ...nding the optimal c_2 and c_3. Second, we characterize the unique combination of bene...ts b_2 and b_3 that implements the optimal allocation. To do this and to express actuarial fairness (3) in terms of c_2 and c_3, we subtract the second and third line of (6), respectively, from the ...rst, yielding

$$c_2 = w - (b_2 - s) - q\frac{pe^{-rm} + (1-p)\,e^{-c_2} - 1}{-r} - h\frac{1 - e^{-c_2}}{-r} + f\frac{e^{-(c_3-c_2)} - 1}{-r}; \qquad (7)$$

$$c_3 = w - (b_3 - s) - q\frac{pe^{-rm} + (1-p)\,e^{-c_2} - 1}{-r} - h\frac{1 - e^{-c_3}}{-r}. $$

Notice that (7) establishes a one-to-one relationship between $\{c_2, c_3\}$ and $\{b_2, b_3\}$. If we subtract the two equations in (7) it is easy to see that $(c_2 - c_3)$ is a monotonously increasing function of $b_3 - b_2$ that crosses the origin. Furthermore, whenever c_2 is larger than c_3; bene...ts are necessarily larger for long run than for short run unemployed.

2.2 Incentive constraints

2.2.1 Incentives to move

Now, consider a person who has lost her job and has the ability to move. She should be induced to do so voluntarily. If her assets at separation were A_t, her value immediately after moving is

$$V_1(A_t - m) = -\frac{1}{r}e^{-r(A_t - m)}e^{-\mu_1}.$$

We compare this to the value of a one-period deviation, i.e., the value if she does not move during this unemployment spell, given by

$$V_2(A_t) = -\frac{1}{r}e^{-rA_t}e^{-\mu_2}.$$

To induce moving we need $V_1(A_t - m) \geq V_2(A_t)$: It follows immediately that this can be written

$$c_2 \geq rm: \qquad (8)$$

We label (8) the ICM-condition. Note that ICM-condition is independent of assets, implying that it can never be individually rational to wait in the short-term unemployment state and move later, while still in state 2.[7]

---

[7]Of course, when $b_2 > b_3$, it could be individually rational not to move in state 2 but move when state 3 is entered.

Note that the ICM is independent of $¢_3$: This does not mean that the incentives to move are independent of long-run bene...ts. On the contrary, as seen in (7), $¢_2$ depends on $¢_3$, which, in turn, depends on b_3: However, an important advantage of focusing on the incentives $¢_2$ and $¢_3$, is that incentive constraints in state j can be expressed in terms of $¢_j$ only. As we will see, this orthogonality will hold also for the remaining incentive constraints, discussed in the next subsection, and will make the analysis simple.

2.2.2 Incentives to search

Let us now consider the incentives for searching during unemployment. A long-term unemployed who does not search will remain unemployed for ever, consuming her permanent income, given by $b_3 ¡ ¿ + rA_t$: This yields an intertemporal utility of $¡\frac{1}{r}e^{¡°rA_t}e^{¡°(b_3 ¡ ¿)}$. The long-term unemployed will search if this is less than her intertemporal utility when searching, i.e., if $V_3 (A_t) ¸ ¡\frac{1}{r}e^{¡°rA_t}e^{¡°(b_3 ¡ ¿)}$. This, again, is independent of assets, and can be written as

$$¾_3 ¸ b_3 ¡ ¿: \tag{9}$$

Using this in the individual dynamic budget constraint yields,

$$A_t = rA_t + b_3 ¡ ¿ ¡ s ¡ ¾_3 ¡ rA_t$$
$$· rA_t + b_3 ¡ ¿ ¡ s ¡ (b_3 ¡ ¿) ¡ rA_t = ¡ s$$

As we see, (9) requires that consumption for a long term unemployed ($c_3 (A_t) = ¾_3 + rA_t$) must be larger than or equal to her income net of taxes ($b_3 ¡ ¿ + rA_t$). This means that incentives have to be at least large enough to make the individual willing to borrow to ...nance her search cost. This, in turn, means that consumption necessarily falls as long as the individual remains long-term unemployed. Using (6), (9) can be written as

$$¢_3 ¸ ¡\frac{\ln\left(1 ¡ \frac{°rs}{h}\right)}{°} · ¢\left(\frac{rs}{h}; °\right): \tag{10}$$

which we label the IC3-condition. As we see, the increase in consumption a long-term unemployed achieves by ...nding a job needs to be larger than $\hat{¢}\left(\frac{rs}{h}; °\right)$ where $\hat{¢}$ is a strictly increasing function of $rs=h$ and, in addition, only dependent on $°$: Since the gain from searching comes in the future, it is intuitive that the potential reward, $¢_3$; has to be larger as discounting increases. It is also intuitive that increased search cost and reduced search e¤ectiveness requires a larger reward for individuals to want to search. The sign of

the derivative, $@\hat{\mathbb{C}}\left(\frac{rs}{h};°\right)=@°$ is on the other hand non-monotonic, being positive for low values of $°$ and becoming negative as $°$ approach h=rs:

Note that while the search incentive in general depends on the extent to which the value function increases when employment is gained, in this case, the incentive constraint can be written as only depending on the extent to which consumption increases at re-employment. This is due to the stationary risk-environment we impose in combination with the CARA utility.

Now, we turn to search incentives for short-run unemployed. For the short term unemployed, we compute the value associated with a one-period deviation, i.e., no search in the current employment state, conditional on searching in future states. In the appendix, we show that this value is $\frac{ei^{°rA}ei^{°c_{2;n}}}{r}$ where $\frac{3}{4}_{2;n}$ satis...es

$$\frac{3}{4}_{2;n} = b_2 - ¿ + \frac{f\left(1 - ei^{°(\frac{3}{4}_{3}i\frac{3}{4}_{2;n})}\right)}{°r}:$$

The IC2 constraint is $\frac{3}{4}_2 \, \frac{3}{4}_{2;n};$ which can be written as

$$\mathbb{C}_2 \, \hat{\mathbb{C}}\left(\frac{rs}{h};°\right); \tag{11}$$

which we label the IC2-condition.

As noted in the previous subsection, the incentive constraints for the two state, IC2 and IC3, are orthogonal, only depending on the relevant incentive ($\mathbb{C}_2$ or $\mathbb{C}_3$) and exogenous variables. To repeat, this does, of course, not mean that only b_2 (b_3) matters for search incentives of the short-term (long-term) unemployed. On the contrary, both b_2 and b_3 a¤ect consumption in all states, as seen in (6). However, individual optimization and access to markets for saving and borrowing imply the value function to be a monotonous transformation of consumption. Thus, the wedge between consumption in the current state and during employment is a su¢cient statistic to determine if search incentives are su¢ciently strong.

Furthermore, note that the RHS of IC2 and IC3 are identical. In other words, given that the hiring probability and search costs are the same for short-term unemployed and long-term unemployed, individuals in these states need the same reward in terms of consumption increases after a successful job-search to be willing to search. Allowing di¤erent search costs and/or hiring probabilities, would simply change the argument of the $\hat{\mathbb{C}}(:)$ function, while maintaining orthogonality between the two constraints.

The optimal insurance contract should then be chosen to maximize $\frac{3}{4}_1 = w_i \cdot _i$ $q\frac{pe^{\circ rm}+(1_i p)e^{\circ \Phi_2}_i 1}{\circ r}$, over Φ_2 and Φ_3; subject to the incentive constraints (8), (10) and (11), and the actuarial fairness constraint (de...ned by (3) and (7)).

3 Characterization of the preferred UI-scheme

Since the focus of this paper is the incentive problems associated with moving, we start the analysis with the assumption that search costs are zero, while moving costs are strictly positive. Speci...cally, we ...rst assume that only the ICM condition binds, i.e., that $\Phi_2 =$ $rm > 0$: In addition, we require $\Phi_3 \, \, \Phi(0; \circ) = 0$: If this constraint were violated, long-term unemployed would strictly prefer to remain unemployed.

To provide an understanding of our analytical results below, we start by deriving a graphical representation of the problem. By substituting for $\cdot$ in the objective function, solving (7) for bene...ts and substituting into (3), and dividing by $\frac{1}{2}$ the problem can be written

$$\max_{\Phi_2;\Phi_3} \left\{ K + \Phi_2 + \frac{f}{h+r}\Phi_3_i \frac{1}{\circ r}\left[(r+h+f)e^{\circ \Phi_2} + he^{i \circ \Phi_2} + fe^{\circ(\Phi_3_i \Phi_2)} + \frac{fh}{h+r}e^{i \circ \Phi_3}\right]\right\}$$

(12)

$$s{:}t{:} \quad \Phi_2 \, \, rm; \Phi_3 \, \, 0{:}$$

where K is a constant.[8]

For $\Phi_3; \Phi_2 \, \, 0$ and $f\Phi_3; \Phi_2 g \, f0; 0g$; the indi¤erence curve for this problem has a slope given by

$$\frac{d\Phi_2}{d\Phi_3}_{\frac{3}{4}_1 \text{ constant}} = _i \frac{\frac{f}{h+r}_i \frac{1}{r}\left[fe^{\circ(\Phi_3_i \Phi_2)}_i \frac{fh}{h+r}e^{i \circ \Phi_3}\right]}{1_i \frac{1}{r}\left[(r+h+f)e^{\circ \Phi_2}_i he^{i \circ \Phi_2}_i fe^{\circ(\Phi_3_i \Phi_2)}\right]}$$

(13)

In ...gure 1, we make a graphical representation of the problem. The bliss point is at full insurance, when $f\Phi_3; \Phi_2 g = f0; 0g$. The indi¤erence curves have elliptical shapes around this point, of which we are only interested in the segment in the positive quadrant since incentive compatibility certainly requires $\Phi_3; \Phi_2 \, \, 0$: Speci...cally, the slope of an

---

[8]The constant is given by

$$K \; \acute{} \; w\frac{(1_i \, _2_i \, _3)}{_2}_i \frac{(_2 + _3)}{_2}s$$
$$_i \frac{1_i \, _2_i \, _3}{_2}g\frac{pe^{\circ rm}_i 1}{r}_{\circ} + \frac{1}{\circ r}\left(h + f\right)\left(1 + \frac{h}{h+r}\right)$$

indi¤erence curve is; i) negative at $z_3 = z_2$, and ii) positive at $z_3 = 0$ and $z_2 > 0$: Since the ICM condition is horizontal, the ...rst fact implies that bene...ts at the tangency must satisfy $z_3 < z_2$, $b_3 > b_2$. The second fact implies that at the tangency, $z_3 > 0$, implying a strictly positive search incentive also for the long-term unemployed. To conclude, the tangent to the ICM constraint (z_2, rm) must be at a point where $z_2 > z_3 > 0$; implying $b_2 ¡ s < b_3 ¡ s < w$. To understand these results, note that when $z_3 = 0$ while $z_2 = rm$, long-term unemployed are as well of as the employed (given assets) and their expected marginal utility is relatively low. A reallocation from long-term to short-term bene...ts therefore increases the value of the insurance so the tax-cost of providing a given insurance value can be reduced. Thus, indi¤erence curves have positive slopes at $z_3 = 0$; $z_2 > 0$. On the other hand, when $z_2 = z_3$ (i.e., when $b_2 = b_3$) the opposite happens. The expected marginal utility of a long-run unemployed is larger than for a short term unemployed, as assets are depleted during the unemployment spell (see ? for more on this). A reallocation from long-term to short-term bene...ts therefore increases the overall value of the insurance.
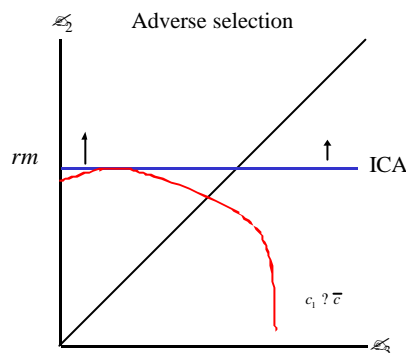

Figure 1: Indi¤erence curve (constant c_1) and Incentive Constraint for Moving

The economic reason for our results can now be phrased in the following way; To separate individuals who have the option to move from those who have not, a positive z_2 is required. However, this does not call for an ine¢ cient structure of the bene...t schedule. Speci...cally, starting from a ‡at bene...t schedule, the welfare in all states can be increased, while maintaining the necessary wedge $z_2 = rm$, by increasing bene...ts for long-term unemployed and reducing bene...ts for short-term unemployed. The reason for this is that

12

expected marginal utility is higher for individuals who have been unemployed for a long time. The optimum is, however, reached before bene...ts to long-term unemployed are high enough to make the latter indi¤erent between having a job and staying unemployed.

Now, let us derive closed-form solutions to our problem. Using the binding ICM condition $c_2 = rm$ to substitute for c_2, the objective function, $\frac{3}{4}_1$, can be rewritten as $w_i \grave{\iota}_i q \frac{e^{\circ rm_i} 1}{\circ r}$ where everything except ι is exogenous. In other words, the problem is to minimize taxes over c_3, respecting actuarial fairness and that bene...ts must be consistent with the chosen c_3 and $c_2 = rm$. After removing constants from the objective function, the problem can then be written

$$\max_{c_3 \in R^+} \left(\rfloor_3 \left(c_{3i} h \frac{e^{i \circ c_3}}{\circ r} \right) \rfloor \rfloor_2 f \frac{e^{\circ(c_{3i} rm)}}{\circ r} \right) : \qquad (14)$$

These terms have straightforward interpretations; the ...rst term is due to the bene...t of reducing the tax-cost of long-term bene...ts. This term is increasing in c_3 since higher c_3 is achieved by lower bene...ts for long-term unemployed, which reduces taxes in proportion to the ADP of long term unemployment $\rfloor_3$: Note that this tax reduction comes from two sources; there is a direct e¤ect that is proportional to c_3 but there is also a indirect e¤ect, captured by the second term inside the parenthesis. Long-term unemployed ...nd jobs at a positive rate h. The prospect of ...nding a job keeps up consumption so that it falls less than proportionally to the reduction in bene...ts. Conversely, given an increase in c_3, bene...ts can be reduced more than proportionally.

The second term in (14) is due to the bene...t of reducing tax cost of short-term bene...ts. It is decreasing in c_3 since less consumption for long-term unemployed has a negative impact on consumption also of the short-term unemployed, proportional to f: As c_3 increases, bene...ts to the short-term unemployed must therefore increase to keep $c_2 = rm$: This has a tax-cost proportional to the ADP of short-run unemployment $\rfloor_2$:

The second derivative of (14) with respect to c_3 is strictly negative, the ...rst derivative is strictly positive when $c_3 = 0$; and strictly negative for $c_3 = rm$: Thus, the unique solution to the problem is obtained by the solution to the ...rst-order condition, given by

$$c_3^\pi = \iota \frac{\ln \left(q \left[\frac{\rfloor_2}{\frac{r}{2h}} + e^{i \circ rm} \left(\frac{h+r}{h} \right) \iota \frac{r}{2h} \right] \right)}{\circ} > 0;$$

which implies (from 7), that

$$b_3^\pi \iota b_2^\pi = rm \iota c_3^\pi + \left[f + he^{i \circ c_3} \right] \frac{1 \iota e^{i \circ (rm_i c_3^\pi)}}{\circ r} > 0; \qquad (15)$$

13

where stars denote optimal values.

Notice also that since the solutions for c_3 and b_3 are independent of f, we see that b_2 falls monotonically in f: That is, as the duration of the short-term unemployment spell falls, the difference $b_3 - b_2$ should increase.[9]

As is clear from the analysis above, a reduction in m reduces c_2 and allows more generous unemployment insurance. Such a reduction could be achieved by subsidies to moving or retraining. However, full compensation is unlikely to be optimal in reality. Suppose, realistically, that individuals with a job sometimes experience a preference or productivity shock, making another job or a job in another location more attractive than the current. Suppose also, that these shocks are not large enough to induce voluntary separation and moving if the individual has to pay the moving cost herself. Clearly, such moves are then not socially optimal.

The insurer would now like to fully subsidize the moving cost of individuals who involuntarily are separated from their job, but not subsidize it for individuals who voluntary separate in order to claim the subsidy. This, however, is infeasible if the insurer cannot distinguish voluntary and involuntary separations. Therefore, we argue that although partial subsidies may be feasible and, in fact, optimal, full subsidization is not. More specifically, it seems clear that subsidies should be as large as possible, without inducing inefficient voluntary separation. We could thus interpret m as the cost of moving or retraining net of the optimal subsidy. Although this point could be formalized by introducing preference shocks along the lines just described, we abstain in order not to complicate the analysis unnecessarily.

3.1 Search costs

We can now easily analyze the conditions such that IC3 and IC2 are satisfied despite positive search costs. Graphically, the constraints are simply horizontal and vertical lines. If search

---

[9]It can be shown that the derivative of the objective function with respect to f is always positive. Low values of f is an inefficient way of inducing separation between those who can move and those who cannot, as agents expect to spend a longer stochastic time suffering the low short-run benefits. Without showing this formally, we conjecture that if lump-sum benefits were allowed, the best policy would be to punish unemployment by a lump-sum unemployment tax when an individual becomes unemployed. In reality, however, it may be politically difficult or even infeasible to implement a lump-sum punishment on those who loose their jobs. Similarly, a lower bound on b_2 might be imposed for political reasons, in which case this would pin down f from (15).

costs are sufficiently small, specifically, if

$$c_3^\pi \geq \hat{c}\left(\frac{rs}{h};\circ\right)^3 \tag{16}$$

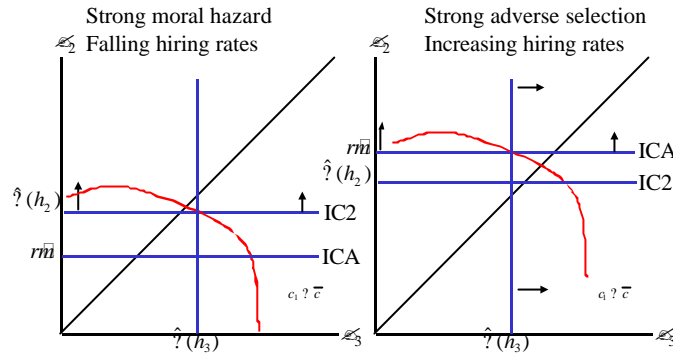none of the search constraints bind, as shown in figure 2.

Figure 2: Low search costs.

Increasing search costs shift out IC2 and IC3 and eventually, (16) is no longer satisfied. This situation is depicted in figure 3, where we see that c_3 remains smaller than c_2 implying $b_2 < b_3$: Specifically, since IC3 and ICM are orthogonal, they will both bind and c_2 should be set equal to rm and equal c_3 to $\hat{c}\left(\frac{rs}{h};\circ\right)$: This means that individuals will be indifferent in the choice of moving and that long-term unemployed are indifferent to searching, while the short-term unemployed strictly prefer to search.

A further increase in search costs will eventually call a situation like in graph 4. Here both search constraints bind, while the moving constraint is slack. Benefits are constant over time since $c_2 = c_2 = \hat{c}\left(\frac{rs}{h};\circ\right)$:[10]

4 Conclusion

In this paper, we have argued that there are reasons to believe that an important moral hazard problem associated with unemployment insurance has been neglected in the previous literature. This problem stems from the fact that unemployed individuals sometimes have

---

[10]This is special case of the result in Werning (2002) who shows that constant benefits are optimal under CARA utility in a general class of UI-schemes.

Transitions in absence of adverse
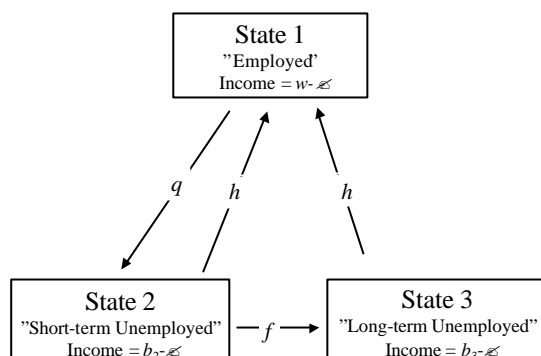selection and moral hazard

Figure 3: Moderate search costs.

the option to make an up-front investment that could increase their chances of ...nding a job. Examples of such investments are retraining and moving to another location. Since it is reasonable to assume that it is di¢cult or impossible to observe who has these options, the UI system should give incentives for people to take advantage of any reasonable option to increase their labor market prospect. By deriving analytical closed-form solutions for the optimal two-tier system, we have shown that such incentives can be provided without reducing the value of the unemployment insurance excessively. This requires an initial period of relatively low bene...ts. The intuition here is straightforward, by setting initial bene...ts at a low level, individuals with good opportunities to get new jobs are induced to exploit these. On the other hand, individuals with worse opportunities value insurance against long-term unemployment more than insurance against short-term unemployment. The value of the UI system can therefore be maintained by providing generous bene...ts after the initial period.

We have assumed that individuals can self-insure via unobservable savings, i.e., that individual consumption is unobservable or uncontractable. If the insurer has control over the consumption of the individual, it is well known that there would be a tendency to provide a downward sloping path of consumption (and bene...ts, if the individual has no other income) to provide good search incentives. Nevertheless, the point of this paper, that a period of low initial UI bene...ts is an e¢cient way to separate individuals who can move from those who cannot would still be true. Which of the two e¤ects dominates would
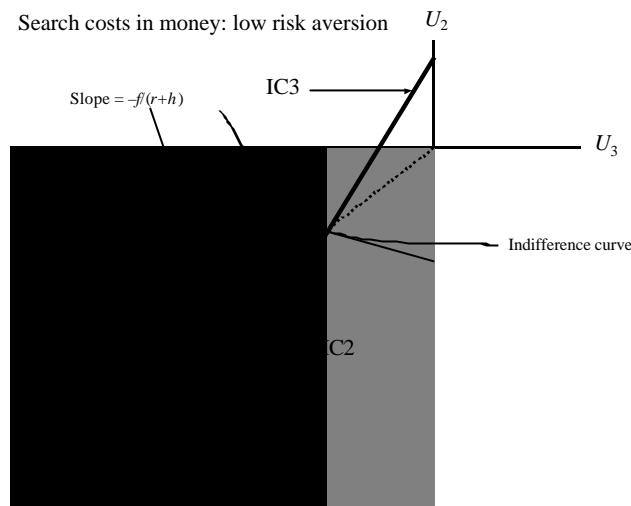
Figure 4: High search costs.

depend on how important the two di¤erent incentive constraints are. In a working paper version of this paper we provide a model in which both e¤ects cancel, so that constant bene…ts are optimal.

We also assume constant absolute risk-aversion in this paper. This representation of individual preferences is not necessarily the most realistic. Let us therefore speculate on the consequences of allowing constant relative risk-aversion. In such a case, the analysis is greatly complicated by the fact that, in general, search incentives would depend on asset holdings. Therefore, incentive compatibility would not in general be consistent with a …nite number of bene…ts that are independent of individual asset holdings. However, the intuition for the results in this paper appear not to be related to such e¤ects. In our model the preference for increasing bene…ts arises from the need to separate between the two types of workers and the fact that individual assets are depleted during unemployment, (which is true for general speci…cations of utility, in particular for CRRA, as shown in e.g., ?). Both mechanisms are likely to be present also under more general preference speci…cations. However, since search incentives in general depend on asset holdings and the duration of unemployment is likely to be correlated with the individual's asset holdings, unobservability of the latter may have consequences for optimal bene…t time pro…les. For example, if the search incentives are reinforced as wealth decumulates and individuals with long unemployment spells are likely to have less wealth, this might call for increasing bene…ts. The analysis of optimal UI design with hidden savings when individual behavior

depends on asset holdings is likely to demand numerical models. We leave this for future research.

5 Appendix

5.1 The IC2 condition

The IC2 constraint is given by

$$\sigma_2 - \sigma_{2;n} \geq 0:$$

Furthermore,

$$
\begin{aligned}
\sigma_2 - \sigma_{2;n} &= -s + \frac{h\left(1 - e^{-\rho\phi_2}\right)}{\rho r} - \frac{f\left(e^{\rho(\phi_3 - \phi_2)} - e^{-\rho(\sigma_3 - \sigma_{2;n})}\right)}{\rho r} \\
&= -s + \frac{h}{\rho r}\left(1 - e^{-\rho\phi_2}\right) - \frac{f}{\rho r}e^{\rho(\phi_3 - \phi_2)}\left(1 - e^{-\rho(\sigma_2 - \sigma_{2;n})}\right) \\
&\stackrel{\cdot}{=} R\left(\sigma_2 - \sigma_{2;n}\right)
\end{aligned}
\tag{17}
$$

Clearly, R is a monotonously decreasing function that has an horizontal asymptote at $-s + \frac{h}{\rho r}\left(1 - e^{-\rho\phi_2}\right) - \frac{f}{\rho r}e^{\rho(\phi_3 - \phi_2)}$ (achieved as $\sigma_2 - \sigma_{2;n}$ approaches infinity), approaches infinity as $\sigma_2 - \sigma_{2;n}$ approaches minus infinity and $R(0) = -s + \frac{h}{\rho r}\left(1 - e^{-\rho\phi_2}\right)$. The solution to (17) is the unique fixed-point of R. This value is non-negative if and only if $-s + \frac{h}{\rho r}\left(1 - e^{-\rho\phi_2}\right) \geq 0$. So

$$
\sigma_2 \geq \sigma_{2;n} , \quad \phi_2 \geq -\frac{\ln\left(1 - \frac{\rho rs}{h}\right)}{\rho} = \hat{\phi}(h)
$$

is true

QED

5.2 More than 3 states

Suppose we have n, states, then the consumption constants are

$$\tfrac{3}{4}_1 = w - \gamma - q\frac{pe^{\circ rm} + (1-p)\,e^{\circ(c_2)} - 1}{\circ r};\tag{18}$$

$$\tfrac{3}{4}_2 = b_2 - s - \gamma + h\frac{1 - e^{-\circ(c_2)}}{\circ r} - f\frac{e^{\circ(c_3 - c_2)} - 1}{\circ r};$$

$$\tfrac{3}{4}_3 = b_3 - s - \gamma + h\frac{1 - e^{-\circ(c_3)}}{\circ r} - f_3\frac{e^{\circ(c_4 - c_3)} - 1}{\circ r}\tag{19}$$

$$\vdots\tag{20}$$

$$\tfrac{3}{4}_{n-1} = b_{n-1} - s - \gamma + h\frac{1 - e^{-\circ(c_{n-1})}}{\circ r} - f_{n-1}\frac{e^{\circ(c_n - c_{n-1})} - 1}{\circ r}\tag{21}$$

$$\tfrac{3}{4}_n = b_n - s - \gamma + h\frac{1 - e^{-\circ(c_n)}}{\circ r};$$

Now, $\gamma = \sum_{s=2}^{n} b_s \vert s$, and the ICM constraing is binding, so $c_2 = rm$, implying that we should minimize taxes. Using the above, and $c_2 = rm$ we have

$$c_2 = w - b_2 + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ rm}}{\circ r} + f\frac{e^{\circ(c_3 - c_2)} - 1}{\circ r}$$

$$c_3 = w - b_3 + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ c_3}}{\circ r} + f_3\frac{e^{\circ(c_4 - c_3)} - 1}{\circ r}$$

$$\vdots$$

$$c_{n-1} = w - b_{n-1} + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ c_{n-1}}}{\circ r} + f_{n-1}\frac{e^{\circ(c_n - c_{n-1})} - 1}{\circ r}$$

$$c_n = w - b_n + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ(c_n)}}{\circ r}$$

or

$$b_2 = w - c_2 + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ rm}}{\circ r} + f_2\frac{e^{\circ(c_3 - c_2)} - 1}{\circ r}$$

$$b_3 = w - c_3 + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ c_3}}{\circ r} + f_3\frac{e^{\circ(c_4 - c_3)} - 1}{\circ r}$$

$$\vdots$$

$$b_{n-1} = w - c_{n-1} + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ c_{n-1}}}{\circ r} + f_{n-1}\frac{e^{\circ(c_n - c_{n-1})} - 1}{\circ r}$$

$$b_n = w - c_n + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ(c_n)}}{\circ r}$$

$$\gamma = \vert_2\left(w - rm + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ rm}}{\circ r} + f\frac{e^{\circ(c_3 - rm)} - 1}{\circ r}\right)$$

$$+ \sum_{i=3}^{\mathbf{X}^{1}}\vert_3\left(w - c_i + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ c_i}}{\circ r} + f_s\frac{e^{\circ(c_{i+1} - c_i)} - 1}{\circ r}\right)$$

$$+ \vert_n\left(w - c_n + s - q\frac{e^{\circ rm} - 1}{\circ r} - h\frac{1 - e^{-\circ c_n}}{\circ r}\right)$$

Removing constants,

$$\ddot{\imath} = \text{constant} + \beta_2 \left(f \frac{e^{-(c_3 - rm)}}{\sigma r} \right)$$

$$+ \sum_{i=3}^{\infty} \beta_i \left[\mu c_i + h \frac{e^{-\sigma c_i}}{\sigma r} + f_s \frac{e^{-(c_{i+1} - c_i)}}{\sigma r} \right]$$

$$+ \beta_n \left[\mu c_n + h \frac{e^{-\sigma c_n}}{\sigma r} \right]$$

First order conditions are

$$c_{i2f3;n-1g}; \quad \beta_{i-1} \frac{f_{i-1}}{r} e^{-(c_i - c_{i-1})} \mu - \beta_i \left[1 + \frac{h}{r} e^{-\sigma c_{i-1}} + \frac{f_i}{r} e^{-(c_i - c_{i-1})} \right] = 0$$

$$c_n; \quad \beta_{n-1} \frac{f_{n-1}}{r} e^{-(c_n - c_{n-1})} \mu - \beta_n \left[1 + \frac{h}{r} e^{-\sigma c_n} \right] = 0;$$

where $c_2 = rm$:

Suppose this is satis...ed for $c_3 = c_4 = \ldots c_n = c$:Then,

$$e^{-(c - rm)} = \frac{r\beta_3}{f_2\beta_2} \mu \left[1 + \frac{h}{r} e^{-\sigma c} + \frac{f_3}{r} \right]$$

$$\frac{f_{i-1}}{r} = \frac{\beta_i}{\beta_{i-1}} \mu \left[1 + \frac{h}{r} e^{-\sigma c} + \frac{f_i}{r} \right]$$

$$\frac{f_{n-1}}{r} = \frac{\beta_n}{\beta_{n-1}} \mu \left[1 + \frac{h}{r} e^{-\sigma c} \right]$$

or

$$e^{-(c - rm)} = \frac{r\beta_3}{f_2\beta_2} \mu \left[1 + \frac{h}{r} e^{-\sigma c} + \frac{\beta_4}{\beta_3} \mu \left(1 + \frac{h}{r} e^{-\sigma c} + \frac{f_4}{r} \right) \right]$$

$$= \frac{r\beta_3}{f_2\beta_2} \mu \left[1 + \frac{h}{r} e^{-\sigma c} + \frac{\beta_4}{\beta_3} \mu \left(1 + \frac{h}{r} e^{-\sigma c} + \frac{\beta_5}{\beta_4} \mu \left(1 + \frac{h}{r} e^{-\sigma c} + \frac{f_5}{r} \right) \right) \right]$$

$$= \frac{r}{f_2} \mu \left(1 + \frac{h}{r} e^{-\sigma c} \right) \left[\frac{\beta_3}{\beta_2} + \frac{\beta_4}{\beta_3} + \ldots + \frac{\beta_{n-1}}{\beta_{n-2}} + \frac{\beta_n}{\beta_2} \right]$$

$$= \frac{r}{f_2} \mu \left(1 + \frac{h}{r} e^{-\sigma c} \right) \left[\sum_{i=3}^{\infty} \frac{\beta_i}{\beta_{i-1}} + \frac{\beta_n}{\beta_2} \right]$$

Clearly, there exists a c^π such that this is satis...ed, consequently $c_i = c^\pi 8i \; 2 \; f3; 4; \ldots; ng$ satis...es all the ...rst-order conditions. This allocation is implemented by a b_2^π and a constant bene...t sequence $b_3^\pi = b_4^\pi = \ldots b_n^\pi$ thereafter. Finally, we note that since individuals face identical conditions in states $3; \ldots n;$the allocation would not change if the number of states is reduced as long as $n > 3$: Thus, the optimal value of b_2 is independen of n if $n > 3$: Consequently, the optimal bene...t schedule is to have $b_2 = b_2^\pi$ and a constant bene...t level $b_3 = b_3^\pi$ thereafter.

6 Proofs not intended for publication

6.1 Finding value functions

Guessing that the value function is $-e^{-\rho(rA_t+\xi_j)}$ for $j \in \{1,2,3\}$, the Bellman equation for the employed is,

$$-\frac{1}{r}e^{-\rho(rA_t+\xi_1)} = \max_\xi -e^{-\rho(rA_t+\xi)}dt$$
$$-(1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\rho(rA_{t+dt}+\xi_1)} + qdt\frac{1}{r}e^{-\rho(rA_{t+dt}+\xi_2)}\right].$$

Using first-order linear approximations and dividing by $e^{-\rho rA_t}$, this becomes

$$-\frac{1}{r}e^{-\rho\xi_1} = \max_\xi -e^{-\rho\xi}dt$$
$$-(1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\rho\xi_1}(1-\rho r(w-\zeta-\xi)dt) + qdt\frac{1}{r}e^{-\rho\xi_2}(1-\rho r(w-\zeta-\xi)dt)\right].$$

Adding $\frac{1}{r}e^{-\rho\xi_1}$ to both sides, dividing by dt and letting dt approach zero, yields

$$0 = \max_\xi \left\{ -re^{-\rho(\xi-\xi_1)} + r + \rho r(w-\zeta-\xi) + q\left(1-e^{-\rho(\xi_2-\xi_1)}\right) \right\}. \qquad (22)$$

Similarly, for the short-term and long-run unemployed, we obtain

$$0 = \max_\xi \left\{ -re^{-\rho(\xi-\xi_2)} + r + \rho r(b_2-s-\zeta-\xi) + h + f - he^{-\rho(\xi_1-\xi_2)} - fe^{-\rho(\xi_3-\xi_2)} \right\};$$
$$(23)$$
$$0 = \max_\xi \left\{ -re^{-\rho(\xi-\xi_3)} + r + \rho r(b_3-s-\zeta-\xi) + h\left(1-e^{-\rho(\xi_1-\xi_3)}\right) \right\}.$$

Equations (22) and (23) are maximized at $\xi = \xi_j$, implying that for the Bellman equation to be satisfied, the constants ξ_j, must satisfy

$$\xi_1 = w-\zeta - \frac{q\left(e^{\rho\Phi_2}-1\right)}{\rho r}$$

$$\xi_2 = b_2 - s - \zeta + \frac{h\left(1-e^{-\rho\Phi_2}\right)}{\rho r} - \frac{f\left(e^{\rho(\Phi_3-\Phi_2)}-1\right)}{\rho r}$$

$$\xi_3 = b_3 - s - \zeta + \frac{h\left(1-e^{-\rho\Phi_3}\right)}{\rho r}.$$

6.2 Derivation of 12

Doing the substitution in the text and collecting endogenous terms, we have

$$\xi_1 = w - \zeta_2\left(w+s - q\frac{pe^{\rho rm}-1}{\rho r}\right) - (h+f)\frac{1}{r} - \zeta_3\left(w+s-q\frac{pe^{\rho rm}-1}{\rho r} - \frac{h}{\rho r} - q\frac{pe^{\rho rm}-1}{\rho r}\right)$$
$$-\zeta_2\left(-\Phi_2 - q\frac{(1-p)e^{\rho\Phi_2}}{\rho r} + h\frac{e^{-\rho\Phi_2}}{\rho r} + f\frac{e^{\rho(\Phi_3-\Phi_2)}}{\rho r}\right)$$
$$-\zeta_3\left(-\Phi_3 - q\frac{(1-p)e^{\rho\Phi_2}}{\rho r} + h\frac{e^{-\rho\Phi_3}}{\rho r}\right) - q\frac{(1-p)e^{\rho\Phi_2}}{\rho r}$$

Dividing by κ_2, and de…ning

$$K \; \prime \; \frac{w \, \text{¡} \, \kappa_2 \left(w + s \, \text{¡} \, q \frac{pe^{\circ rm} \text{¡} 1}{\circ r} \text{¡} (h+f) \frac{1}{\circ r} \right)}{\kappa_2}$$

$$\text{¡} \; \frac{\kappa_3 \left(w + s \, \text{¡} \, q \frac{pe^{\circ rm} \text{¡} 1}{\circ r} \text{¡} \frac{h}{\circ r} \right) + q \frac{pe^{\circ rm} \text{¡} 1}{\circ r}}{\kappa_2}$$

$$= \; w \frac{(1 \, \text{¡} \, \kappa_2 \, \text{¡} \, \kappa_3)}{\kappa_2} \; \text{¡} \; \frac{(\kappa_2 + \kappa_3)}{\kappa_2} s$$

$$\text{¡} \; \frac{1 \, \text{¡} \, \kappa_2 \, \text{¡} \, \kappa_3}{\kappa_2} \frac{q}{r} \left(pe^{\circ rm} \text{¡} 1 \right) + \frac{1}{\circ r} \left(h + f \right) \left(1 + \frac{h}{h+r} \right)$$

we get

$$\frac{\tfrac{3}{4}_1}{\kappa_2} \; = \; K + \mathbb{C}_2 + \frac{f}{h+r} \mathbb{C}_3 + q \frac{(1 \, \text{¡} \, p) e^{\circ \mathbb{C}_2}}{\circ r} \left(1 \, \text{¡} \, \frac{1}{\kappa_2} + \frac{f}{h+r} \right)$$

$$\text{¡} \; h \frac{e^{i \, \circ \mathbb{C}_2}}{\circ r} \, \text{¡} \, f \frac{e^{\circ (\mathbb{C}_3 \text{¡} \mathbb{C}_2)}}{\circ r} \, \text{¡} \, \frac{f}{h+r} h \frac{e^{i \, \circ \mathbb{C}_3}}{\circ r}$$

$$= \; K + \mathbb{C}_2 + \frac{f}{h+r} \mathbb{C}_3 \, \text{¡} \, q \frac{(1 \, \text{¡} \, p) e^{\circ \mathbb{C}_2}}{\circ r} \frac{r + h + f}{(1 \, \text{¡} \, p) q}$$

$$\text{¡} \; h \frac{e^{i \, \circ \mathbb{C}_2}}{\circ r} \, \text{¡} \, f \frac{e^{\circ (\mathbb{C}_3 \text{¡} \mathbb{C}_2)}}{\circ r} \, \text{¡} \, \frac{f}{h+r} h \frac{e^{i \, \circ \mathbb{C}_3}}{\circ r}$$

In the following …gure, we plot this, for $r = 0.05; f = 1; \circ = 1; h = 1$ against $\mathbb{C}_2; \mathbb{C}_3$ viewing it from above an cutting all values above -100.2. As we see, the isoquant has an elliptical form.