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Preferred Suppliers and Vertical Integration in Auction Markets

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Abstract

We consider a model of preference in an asymmetric procurement auction with two suppliers. The buyer can award the contract to a preferred supplier at the bid of a competing supplier. As such, the preferred supplier has a *right-of-first-refusal*. The preferred supplier may be an independent firm who has paid for the preference or may be a subsidiary of the buyer. Preference creates an allocative distortion that is qualitatively different than the distortion that arises in an asymmetric first-price auction. For a family of power distributions on the costs, we examine the effects of preference on the expected price paid by the buyer. If the buyer is not compensated for the preference, the expected price in the preference auction will be higher than either an efficient auction or a first-price auction. However, if the buyer can sell preference in a pre-auction, the net expected price will always be lower than an efficient auction, or even the first-price auction. The stronger supplier with the more favorable cost distribution has a greater willingness to pay for preference, and will outbid the weaker supplier to acquire the preference in the pre-auction. Similarly, the buyer can lower the net expected price by acquiring the stronger supplier as a preferred subsidiary.

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1. Introduction

In this paper, we examine preference in a procurement auction. Corporations and governments frequently have a preference for particular suppliers of various goods. Preference may arise in a variety of settings. First, preference for one supplier may arise from successful contractual relationships in the past between the buyer and this supplier. Second, preference for one supplier may also arise from the special capabilities possessed only by this supplier. Third, preference may arise from bribery of a buyer's procurement officer by the supplier. In contrast to these explanations, the model of this paper will assume that preference is explicitly sold by the buyer to one of the suppliers. In particular, a buyer may auction preference to one supplier or may acquire one of the suppliers as a preferred internal subsidiary.

In the basic model, preference occurs when the buyer creates a *right-of-first-refusal* for the preferred supplier, allowing him to accept the contract at the price bid by the competing supplier. The preferred supplier will clearly accept the contract whenever his cost is below the bid of the competing supplier. We first examine how preference affects the bidding strategy of the competing supplier. We then identify the effects of preference on the allocation of contracts, the expected price paid by the buyer, and the expected profits of the suppliers. With the expected profits, we examine the incentives of the suppliers to acquire preference. In particular, we examine an open auction to sell preference to one of the suppliers. Finally, we examine the incentive of the buyer to create preference by acquiring one of the suppliers.

Bikhchandani, Lippman, and Ryan (2003) have recently examined the *right-of-first-refusal* in symmetric sealed-bid second-price auctions. They focus on the gains and losses to a seller, the buyer with the *right-of-first-refusal*, and the other buyers for private-value, common-value, and affiliated-value settings. In the private-value setting, they find that the buyer with the *right-of-first-refusal* gains at the expense of the other buyers and the seller. With their second-price auction, this gain is equivalent to the loss of the seller. Thus, they conclude that there is no incentive for the seller to award a *right-of-first-refusal* to one of the buyers. In contrast, our paper examines an asymmetric (sealed-bid first-price) procurement auction with two suppliers in a private-value setting. The buyer is obviously harmed by granting a *right-of-first-refusal* to one of the suppliers without some payment. However, this supplier gains more than the buyer losses, resulting in gains from trade for the *right-of-first-refusal*. In addition, we allow the buyer to

auction the *right-of-first-refusal* to one of the suppliers, and find that the buyer can always gain by holding an open auction to sell the *right-of-first-refusal*. Moreover, we find that the stronger supplier will always outbid the weaker supplier for the *right-of-first-refusal*, and may earn higher expected profits than in an efficient auction.

In Section 2, we construct a procurement auction with preference for one of two potential suppliers. One competing supplier bids for the contract, while the other preferred supplier has a *right-of-first-refusal* to accept or reject the contract at that bid. We characterize the equilibrium bidding function for the competing supplier. After doing so, we briefly discuss the impact of preference on the bidding function for general cost distributions. We then focus on a family of power distributions for the costs of the suppliers. This family allows us to obtain closed-form solutions for the bidding function of the competing supplier.

In Section 3, we examine the expected price in the preference auction. We demonstrate analytically that this price exceeds the expected price in an efficient auction. In addition, we demonstrate computationally that this price also exceeds the expected price in a first-price auction. The increase in the expected price from the preference auction is significant and is larger when the suppliers are more symmetric.

In Section 4, we examine the allocative distortion in the preference auction. Preference creates a distortion because the contract is awarded to the preferred supplier in cases where his cost is below the bid of the competing supplier, but above the cost of the competing supplier. We compare this allocative distortion to that which arises in an asymmetric first-price auction. We also compute the expected distortion for the preference auction and show that it is significant relative to the expected price in an efficient auction.

In Section 5, we allow the buyer to sell preference to one of the suppliers prior to the procurement auction. With the family of power distributions on the costs, we can unambiguously define the stronger supplier as the one with a higher probability of lower costs. We find that the stronger supplier has a greater willingness to pay for preference. Thus, if the buyer holds a pre-auction for preference, the stronger supplier will outbid the weaker supplier.

In Section 6, we examine the net expected price paid by the buyer with a preference auction and after holding a pre-auction for preference. We find that this net expected price would always be lower than the expected price from an efficient auction. The price of preference paid by the stronger supplier more than compensates for the higher expected price

after preference. In addition, we also find that the net expected price paid by the buyer after the pre-auction for preference would even be lower than the expected price from a first-price auction. As such, the buyer would benefit from a preference auction in which preference is sold in a pre-auction to the stronger supplier.

In Section 7, we examine the expected profits of the two suppliers after the stronger supplier acquires preference. The weaker supplier will clearly be less profitable than he would have been with an efficient auction. However, the stronger preferred supplier may be more profitable than he would have been with an efficient auction. This can occur if the stronger supplier is sufficiently stronger than the weaker supplier. If so, the price of preference does not extract all the gains from being preferred in the procurement auction for the stronger supplier.

In Section 8, we examine vertical integration in which the buyer acquires one of the suppliers. Vertical integration is an alternative to auctioning preference. The acquired supplier becomes an internal subsidiary of the buyer, and the buyer optimally prefers the subsidiary during the procurement auction. Using a natural specification for the acquisition price, we find that it is profitable for the buyer to acquire either supplier, but that it is more profitable to acquire the stronger supplier. The net expected price paid by the buyer is lower than before the acquisition. Moreover, the net expected price is lower than it would have been with a preference auction after holding a pre-auction for preference.

In Section 9, we conclude with the policy implications of these results. Appendix 1 discusses the distinctions between this preference auction and the other models of exclusive dealing and vertical integration. Appendix 6 compares the pre-auction for preference with a uniform entry fee. We find net expected price paid by the buyer can be lower with a pre-auction for preference than with a uniform entry fee charged to both suppliers.

2.   The Model of a Preference Auction

The buyer has a value v for a good with a fixed quantity and quality. There are two suppliers, and their cost distributions for producing the good may differ. The buyer could employ an efficient auction (EA) such as a second-price auction (SPA) or an open auction. If so, the contract would be awarded to the supplier with the lowest cost at a price equal to the second lowest cost. The buyer could also employ a sealed-bid first-price auction (FPA). If so, the

contract would be awarded to the supplier with the lowest bid at a price equal to that bid. The winning supplier need not have the lowest cost, so this FPA need not be efficient. As an alternative to either an efficient auction or a first-price auction, we allow the buyer to employ a preference auction in which one supplier is the preferred supplier (PS) and the other is the competing supplier (CS). In the preference auction, the CS will bid for the contract, but the PS will then be offered the contract at a price equal to the bid of the CS. The contract will be accepted by the PS if his cost is below the bid of the CS, and rejected otherwise. Thus, preference means that the PS has a *right-of-first-refusal* at the bid of the CS. The PS would not bid against the CS because under-bidding the CS would only lower the expected price he would receive.[1]

An important feature of our model is that the suppliers can be asymmetric with different distributions on the cost of producing the good. We assume that each supplier draws his cost of production c_i, where $i = d$ (PS) or h (CS), from a distribution $G_i(c)$ with a common support $[0,1]$, and a positive density $g_i(c)$ over this support.[2] The cost c_i is private information for each supplier, but the distribution functions are common knowledge. For simplicity, we also assume that the value of the buyer exceeds the highest possible cost realization ($v > 1$). Finally, we assume that the costs of the suppliers are independently distributed. Thus, we will examine preference in an asymmetric independent private value auction. With preference, the PS but does not bid against the CS. Instead, the PS awaits the bid of the CS. The CS realizes his cost and then bids for the contract, knowing the cost distribution of the PS, and also knowing that preference represents a *right-of-first-refusal* for the PS. The buyer then offers the contract to the PS at a price equal to the bid of the CS.[3]

Let c_d be the cost realization of the PS and b_h be the bid of the CS. If $b_h < c_d$, the contract is awarded to the CS at a price equal to his bid. The PS does not want the contract at

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[1] If the PS won, his bid must have been below the bid of the CS. However, the bid of the CS is the price at which the PS would be offered the *right-of-first-refusal*. Of course, this assumes that any payment for the *right-of-first-refusal* is independent of any bid that the PS might make. If so, the optimal bid of the PS is unity, the highest possible cost realization. In effect, the PS submits the maximum possible bid so that he would never win the contract outright. See Burguet and Perry (2002).

[2] The subscripts h and d correspond to *honest* and *dishonest*, respectively, from our paper Burguet and Perry (2002). We employ the same subscripts in this paper so that the reader can easily reference the relevant results from that earlier paper.

the price b_h because he would incur a loss. On the other hand, if $b_h > c_d$, the contract is awarded to the PS at a price equal to the bid b_h of the CS. In this case, there is a *surplus* $(b_h - c_d)$ which is retained by the PS. If preference is awarded, and not sold, the PS would retain the expected surplus. However, if the buyer can sell the preference to one supplier, the buyer may be able to appropriate part or all of this expected surplus.

For general cost distributions, we can now characterize the optimal bid of the CS. Assuming that the PS will not reject contracts at a price above his cost, the CS has a dominant bidding strategy. Indeed, the CS is effectively bidding against the cost of the PS because the PS will obtain the contract by preference whenever his costs are below the bid of the CS. The CS calculates his bid by solving the following problem:

(1) $$\max_b \Pi_h[b;c] = (b-c)\cdot[1 - G_d(b)] .$$

The first-order condition for this problem is

(2) $$[1 - G_d(b)] - (b-c)\cdot g_d(b) = 0 .$$

The optimal bidding function for the CS, $b_h(c)$, is implicitly defined by (2).[4] This bidding function is equivalent to the best take-it-or-leave-it offer that a supplier with cost c can make to a buyer with a random reserve price in the interval $[0,1]$ given by the distribution function G_d.[5] Even though the preference auction is dominance solvable, the first-order condition (2) is still cumbersome. For the remainder of the paper, we will employ a family of power distributions on the costs of the suppliers. This family of distributions allows us to obtain a closed-form solution for the bidding function of the CS while maintaining asymmetry between the suppliers.[6]

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[3] We assume that the bid of the CS is verifiable in that the buyer can show the PS a signed document with the bid of the CS.

[4] The second order condition requires $b - [(1 - G_d(b))/g_d(b)]$ to be increasing in b. This condition will be satisfied by the power family of distributions that we define in this section.

[5] With only one CS, all oral and sealed-bid auctions are equivalent. With more than one competing supplier, this would only be true if the competing suppliers were symmetric.

[6] In general, the equilibrium bidding function of the CS may differ from what it would have been with a FPA. However, the equilibrium bidding function of the CS is not uniformly more or less aggressive. For example, one cannot conclude that the CS bids more aggressively (lower bids at any cost realization) because he knows that his bid must be below the cost of the PS. See Burguet and Perry (2002) for a general comparison of the first-order conditions in a preference auction and a FPA auction, and a symmetric example of $G_h(c) = G_d(c) = c^2$ in which the bidding function of the CS is more aggressive for low cost realizations but less aggressive for high cost realizations.

Define the one-parameter family of distribution functions $G(c;t) = 1 - [1-c]^t$ over the support [0,1] where c is the cost and $t > 0$ is a parameter which can differ for the two suppliers.[7] The corresponding density function is $g(c;t) = t \cdot [1-c]^{t-1}$. Without loss of generality, we assume that $t_1 \geq t_2$. Thus, the first supplier has a more favorable cost distribution in that there is a higher probability of obtaining a cost below any given c: $G(c;t_1) > G(c;t_2)$ for all c in the support. In other words, the cost distribution of the first supplier stochastically dominates that of the second supplier. For this reason, we will call the first supplier the stronger supplier, and refer to t_1 and t_2 as the capacities of the suppliers.[8]

The buyer will decide which supplier, if either, is preferred. So let t_d be the capacity of the PS and t_h be the capacity of the CS, and denote $G_d(c) = G(c;t_d)$ and $G_h(c) = G(c;t_h)$. The equilibrium bidding function of the CS from (2) has the following linear form:

$$(3) \qquad\qquad b_h(c) = \frac{1}{1+t_d} + \frac{t_d}{1+t_d} \cdot c \ .$$

The intercept and slope of this bidding function are defined by the capacity t_d of the PS. The CS clearly bids more aggressively when the PS is stronger. If preference were awarded to the stronger supplier, he would no longer bid for the contract even though he had the higher probability of obtaining low costs and thus making low bids. However, the weaker supplier would bid more aggressively as the CS than would the stronger supplier. These tradeoffs will be relevant in understanding the effects of preference on the expected prices.

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[7] Waehrer and Perry (2003) show that this distribution function is not as restrictive as it might seem. Distributions of this form follow directly from natural properties, particularly a property corresponding to constant returns to scale. Note that power distributions of the form c^s are not members of this family, except for $s=1$. Instead, these distributions would be members of a larger family of power distributions: $G(c;s,t) = 1 - [1-c^s]^t$.

[8] The parameter t is a continuous representation of the number of draws that a supplier would have from a uniform distribution on costs. For example, t would correspond to the number of equal size plants owned by a supplier, where each plant has a uniform distribution on the cost of producing the good, and the supplier uses the lowest cost plant to produce the good for the contract. As such, t can be loosely interpreted as capacity, and the stronger supplier has a larger capacity than the weaker supplier.

It is useful to compare this equilibrium bidding function to the FPA for the symmetric case in which $t = t_h = t_d$. The general symmetric first-order condition for the bidding function in a FPA is

$$(4) \qquad \left[1 - G(b^{-1}(b))\right] - (b-c) \cdot g(b^{-1}(b)) \cdot \frac{db^{-1}(b)}{db} = 0 .$$

With this family of cost distributions $G(c;t)$, the solution to (4) takes the linear form:

$$(5) \qquad b(c) = \frac{1}{1+t} + \frac{t}{1+t} \cdot c .$$

Thus, when the suppliers are symmetric, the bidding function of the CS is the same for both the preference auction and the FPA.[9] This fact makes it easy to compare the preference auction with other auctions in the symmetric case. In addition, it provides a general reference point for the asymmetric cases where we will analytically compare the preference auction to an efficient auction (EA).

3. The Expected Price in the Preference Auction

If the buyer awards preference to the PS without any compensation, then the expected price paid will increase from what it would have been in either an EA or a FPA. In asymmetric cases, the CS may bid more aggressively after preference is awarded to the PS. However, this will not offset the fact that the PS no longer bids to win the contract. Let $Ep(t_d, t_h)$ be the expected price in the preference auction. This expected price has a very simple expression:

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[9] This is a consequence of the family of cost distributions. With this family, we find that

$$\frac{1 - G(b)}{g(b)} = \frac{1 - G(b^{-1}(b))}{g(b^{-1}(b)) \cdot db^{-1}(b)/db} ,$$ where $b^{-1}(b)$ represents the inverse of the symmetric bidding function

without preference. That is, the hazard rate of the distribution of bids is the same as the hazard rate of the costs generating those bids. When this is true, the tradeoff facing the CS is the same with and without preference.

$$(6) \qquad Ep(t_d, t_h) \;=\; \frac{1 + t_d + t_h}{(1 + t_d)(1 + t_h)} \;=\; \frac{1 + t_1 + t_2}{(1 + t_1)(1 + t_2)} \; .$$

Note that the expected price paid by the buyer in the preference auction is independent of which supplier is preferred. Thus, without confusion, we can switch the indices in the expression and refer to this price as $Ep(t_1, t_2)$. The expected price is solely determined by the cost distribution and bidding function of the CS. The cost distribution obviously depends solely on the capacity of the CS, but the bidding function in (3) depends solely on the capacity of the PS. When the CS is the weaker supplier, he has a lower probability of obtaining a low cost, but he bids more aggressively. On the other hand, when the CS is the stronger supplier, he has a higher probability of obtaining a low cost, but he bids less aggressively. With this family of cost distributions, these two forces exactly offset each other and the buyer is indifferent between which supplier is preferred.[10]

We can now compare the expected price in the preference auction to the expected price that would arise with an efficient auction (EA), defined as $Ep_{EA}(t_1, t_2)$.[11]

$$(7) \qquad Ep_{EA}(t_1, t_2) \;=\; \frac{1 + t_1 + t_2}{(1 + t_1)(1 + t_2)} \;-\; \frac{t_1 t_2}{(1 + t_1)(1 + t_2)(1 + t_1 + t_2)} \; .$$

Efficient auctions would include an open descending-price auction or a sealed-bid second-price auction. Comparing expressions (6) and (7), we obtain the following result:

Proposition 1(a): For all (t_1, t_2), the expected price in a preference auction is higher than the expected price in an efficient auction: $Ep(t_1, t_2) > Ep_{EA}(t_1, t_2)$.

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[10] In order to see that this result is not general, consider the case in which $G_1(c) = c$ and $G_2(c) = c^2$. The second distribution does not belong to this family of cost distributions, but the first supplier is clearly the stronger supplier. In this case, the expected price for the preference auction would be 0.7747 when the first supplier is the CS and 0.8333 when the second supplier is the CS. Thus, it is more important for the stronger supplier to be the competing supplier.

[11] Without preference, the allocation of the contract virtually determines the total surplus that can be divided between the buyer and the suppliers, and how it is divided. This is an implication of the Revenue Equivalence Theorem. See Myerson (1981). Since an efficient auction allocates the contract to the lowest cost supplier, it is a natural reference point for evaluating other auctions.

The second term in (7) is the difference between the two expected prices and is the loss to the buyer from granting preference without a payment.[12] This increase in the expected price from awarding preference without compensation is often substantial. Moreover, the percentage increase is larger when the combined capacity T is larger, and when the suppliers are more symmetric. If we define $t_1 = \lambda \cdot T$ and $t_2 = (1-\lambda) \cdot T$ where $\lambda \geq 0.5$, the percentage increase in the expected price can be expressed as $\{\lambda(1-\lambda)T\}/\{(1+T)^2 - \lambda(1-\lambda)T^2\}$. When $\lambda = 0.5$, the percentage increase rises from 12.5% when $T = 2$, to 33.3% as $T \rightarrow \infty$. Alternatively, when $T = 2$, the percentage increase rises from 4.2% when $\lambda = 0.9$ to 12.5% when $\lambda = 0.5$. In sum, the buyer will not award preference without some compensation to offset the resulting higher expected price.

In order to provide the intuition for Proposition 1(a), consider an EA in which payments are made only to the winner of the auction. For any such EA, an "implicit bidding function" for the ith supplier can be defined as the expected price paid by the buyer to ith supplier when he wins with a cost c_i, where the expected price is equal to the expected cost of the jth supplier. For this family of cost distributions, the implicit bidding function in such an EA is:

$$(8) \qquad b_{EA,i}(c_i) = E\left[c_j \mid c_j \geq c_i\right] = \frac{\int_{c_i}^1 x \cdot dG(x;t_j)}{1 - G(c_i;t_j)} = \frac{1}{1+t_j} + \frac{t_j}{1+t_j} \cdot c_i.$$

The bidding function of the CS in the preference auction is identical to this implicit bidding function in an EA. In particular, expression (8) is equivalent to expression (3) when $i = h$ and $j = d$. Thus, preference for the PS does not induce the CS to bid more or less aggressively than such an EA.[13] However, since the PS is no longer bidding to win the contract, the expected price paid by the buyer must increase.

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[12] Interpreted in this procurement setting, Bikhchandani, Lippmann, Ryan (2003) find the same result for the symmetric case with general cost distributions.

[13] This finding is a consequence of the family of power distributions on the costs, and is not necessarily true for other cost distributions. Consider the larger family of power distributions: $G(c;s,t) = 1 - [1-c^s]^t$, and two symmetric suppliers with the same capacity. When $s = 2$, the CS would bid more aggressively than the implicit bidding function of this EA or the bidding function of the symmetric FPA. On the other hand, when $s = 0.5$, the CS would bid less aggressively.

Proposition 1(a) compares the preference auction to an asymmetric EA which has no allocative distortions. However, it is also interesting to compare the preference auction to an asymmetric FPA. Let $Ep_{FPA}(t_1, t_2)$ be the expected price paid by the buyer in a FPA. In the symmetric case, the expected price in the FPA is equivalent to the expected price in an EA, and thus, both are lower than expected price in the preference auction: $Ep_{FPA}(t,t) = Ep_{EA}(t,t) < Ep(t,t)$. For the asymmetric case, the comparison is more difficult because the asymmetric FPA does not have closed-form solutions for the bidding functions with this family of cost distributions. However, we can state the following computational proposition.

Proposition 1(b): For a fine grid of (t_1, t_2), the expected price in a preference auction is always higher than the expected price in a first-price auction: $Ep(t_1, t_2) > Ep_{FPA}(t_1, t_2)$.

Proposition 1(b) is obtained using the Bidcomp[2] program of Li and Riley (1999). With repeated Bidcomp[2] calculations for the expected price in the asymmetric FPA, we found no examples of the capacity parameters (t_1, t_2) for which the expected price is lower with a preference auction. Proposition 1(b) is virtually a corollary of Proposition 1(a) because the computations from Bidcomp[2] also reveal that the expected price in an EA is greater than the expected price in a FPA: $Ep_{EA}(t_1, t_2) > Ep_{FPA}(t_1, t_2)$.

Propositions 1(a) and 1(b) demonstrate that awarding preference to one supplier without compensation will result in a higher expected price paid by the buyer. The findings are not surprising in that the PS has no incentive to bid against the CS. The goal of the remainder of the paper is to examine the potential benefits to the buyer if preference is auctioned to one of the suppliers. But before doing so, we briefly examine the allocative distortion introduced by the preference auction.

4.   The Allocative Distortion in the Preference Auction

In this section, we examine how preference affects the efficiency of a procurement auction. By definition, an EA generates no allocative distortion because the contract is awarded

to the lowest cost supplier. An asymmetric FPA generates an allocative distortion because the weaker supplier who bids more aggressively will be awarded the contract in some situations where his cost is higher than the stronger supplier who bids less aggressively. The preference auction generates an allocative distortion in that the PS who does not bid will be awarded the contract in some situations where his cost is higher than the CS.

The allocative distortion in a preference auction is qualitatively different from the allocative distortion in the FPA. If the weaker supplier is the PS, the allocative distortion favoring the weaker supplier in the FPA is replaced with the new allocative distortion from the preference for the weaker supplier. However, if the stronger supplier is the PS, the allocative distortion favoring the weaker supplier is eliminated and the new allocative distortion is created from the preference for the stronger supplier. Since we cannot obtain closed-form solutions for the bidding functions in the asymmetric FPA, we cannot analytically compare these two allocative distortions. However, we can illustrate them.

Consider first the allocative distortion in preference auction. There are three regions of the cost space, two in which the lower cost supplier obtains the contract, and one in which the higher cost PS obtains the contract. The latter region gives rise to the allocative distortion. Consider a cost realization c_h of the CS. The PS will be awarded the contract whenever $c_d \leq b_h(c_h)$. In the region where $c_d < c_h$, there is no allocative distortion because the PS has a lower cost and preference efficiently awards him the contract. However, in the second region where $c_h < c_d \leq b_h(c_h)$, the preference auction introduces an allocative distortion because the PS has a higher cost but obtains the contract. Finally, in the third region where $b_h(c_h) < c_d$, there is no allocative distortion because the PS has a cost higher than the bid of the CS and does not want the contract. Figure 1 illustrates a representative bidding function for the CS, and these three regions.

Now consider the allocative distortions in the asymmetric FPA. Let $b_1(c)$ and $b_2(c)$ be the equilibrium bidding functions for the two suppliers. The weaker supplier with the smaller capacity t_2 bids more aggressively than the stronger supplier with larger capacity t_1. Thus, the weaker supplier inefficiently wins contracts when $b_1^{-1}(b_2(c_2)) < c_1 \leq c_2$. A preference auction with preference for the stronger supplier ($t_1 = t_d$) would eliminate this allocative distortion, but

would create the new allocative distortion when $c_2 < c_1 \le b_h(c_2)$ discussed previously. Figure 2 illustrates the bidding functions of the two suppliers in a FPA, and the region of allocative distortion in favor of the weaker supplier.

For the preference auction we can solve for the allocative distortion. Assuming that the stronger supplier is the preferred supplier, the distortion can be expressed as:

$$
(9) \qquad D(t_1, t_2) = \frac{t_2}{(1+t_1)(1+t_1+t_2)} \cdot \left(1 - \left(\frac{t_1}{1+t_1} \right)^{t_1} + \left(\frac{t_1}{1+t_1} \right)^{t_1+1} \right)
$$

This allocative distortion is typically substantial. Table 1 summarizes the relative magnitude, $D(t_1,t_2) / Ep_{EA}(t_1,t_2)$, of the allocative distortion for symmetric and asymmetric cases (for T and $t_1 = \lambda \cdot T$ where $\lambda \ge 0.5$). For example, when $t_1 = t_2 = 1$, the distortion represents 16.7% of the expected price in an EA. The distortion D becomes more significant when industry capacity increases. Thus when $t_1 = t_2 = 2$, the distortion D is 25.6% of the expected price in an EA.

5.   The Pre-Auction for Preference

In this section, we allow the buyer to sell preference to one of the suppliers. We first examine the incentives of each supplier to acquire preference from the buyer. We then model the sale of preference as a pre-auction in which one supplier will outbid the other and pay the buyer for the *right-of-first-refusal* in the subsequent procurement auction. In particular, we assume that this pre-auction will be an ascending-price open auction in which each supplier bids to become the PS before knowing his cost realization for producing the good.[14] The willingness to pay for preference is the difference between the expected profits as the PS and as the CS. The winning bid will then depend on which supplier has the highest willingness to pay for preference.

In order to calculate the willingness to pay, we first solve for the expected profits of the PS and the CS:

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[14] With common knowledge of the capacities t_1 and t_2, the pre-auction could also be a sealed-bid auction.

$$(10) \quad \text{PS:} \quad \Pi_d\left(t_d, t_h\right) = \frac{t_d}{1+t_d} \cdot \left\{ \frac{1}{1+t_h} + \frac{t_h}{\left(1+t_d\right) \cdot \left(1+t_d+t_h\right)} \cdot \left(\frac{t_d}{1+t_d}\right)^{t_d} \right\},$$

$$(11) \quad \text{CS:} \quad \Pi_h(t_d, t_h) = \frac{t_h}{\left(1+t_d\right) \cdot \left(1+t_d+t_h\right)} \cdot \left(\frac{t_d}{1+t_d}\right)^{t_d}.$$

In order to understand the pre-auction to become the PS, it is useful to summarize several relationships between the expected profits of the suppliers. We continue to assume that $t_1 \geq t_2$ without loss of generality. We can now state the following proposition.

Proposition 2: (a) The expected profits of a stronger PS are greater than the expected profits of a weaker PS: $\Pi_d\left(t_1, t_2\right) > \Pi_d\left(t_2, t_1\right)$ for all $t_1 > t_2$;

(b) The expected profits of a stronger CS are greater than the expected profits of a weaker CS: $\Pi_h\left(t_2, t_1\right) > \Pi_h\left(t_1, t_2\right)$ for $t_1 > t_2$;

(c) If t_1 is sufficiently larger than t_2, the expected profits of a stronger CS are greater than the expected profits of a weaker PS.

The proof of Proposition 2 is contained in Appendix 2. Propositions 2(a) and 2(b) make it clear that having a more favorable cost distribution will benefit either the PS or the CS. Proposition 2(c) states that a more favorable cost distribution can even overcome the disadvantage from being the CS, rather than the PS.

We can now examine which supplier will outbid the other to become the PS. Define V_i as the willingness to pay by the ith supplier for preference. V_i is the expected profit from becoming the PS minus the expected profit from becoming the CS. Part of the willingness to pay by each supplier is to obtain the advantage of preference, but another part is to avoid the disadvantage of competing against preference for the other supplier. The willingness to pay V_i for each supplier can be expressed as follows:

(12a) $\qquad V_1(t_1,t_2) = \Pi_d(t_1,t_2) \; - \; \Pi_h(t_2,t_1) \, .$

(12b) $\qquad V_2(t_1,t_2) = \Pi_d(t_2,t_1) \; - \; \Pi_h(t_1,t_2) \, .$

The question is whether the stronger supplier has a higher willingness to pay for preference, that is, whether $V_1 > V_2$ when $t_1 > t_2$. The answer is not transparent because there are two forces at work. First, the supplier with the lower cost realization has a higher willingness to pay for preference. Since the stronger supplier is more likely to have a lower cost, this suggests that he might have a higher willingness to pay for preference. However, there is a second force. For any two cost realizations, the weaker supplier with the lower cost would gain more from being preferred than the stronger supplier with the lower cost. This occurs because the CS bids less aggressively against a weak PS, so the difference between his bid and the cost of the preferred supplier is higher. This suggests that the weaker supplier might have a higher willingness to pay for preference. This second force is illustrated in Figures 3(a) and 3(b). The vertical two-headed arrows measure how much the two suppliers gain from being preferred for two values of their costs. In Figure 3(a) where $c_1 > c_2$, the willingness to pay by the weaker supplier with c_2 [$b_{\{h=1\}}(c_1) - c_2$] is much larger than the willingness to pay by the stronger supplier with c_1 [$b_{\{h=2\}}(c_2) - c_1$]. However, in Figure 3(b) where $c_2 > c_1$, the willingness to pay by the stronger supplier with c_1 is only slightly larger than the willingness to pay by the weaker supplier with c_2. The following proposition demonstrates that the first effect dominates the second effect.

Proposition 3: The stronger supplier always has a higher willingness to pay for preference than the weaker supplier: $V_1 > V_2$ whenever $t_1 > t_2$. As a result, if the buyer held an ascending-price open auction for preference, the stronger supplier would outbid the weaker supplier and pay the buyer at least the willingness to pay V_2 of the weaker supplier.

The proof of Proposition 3 is contained in Appendix 3. If the buyer runs a pre-auction for preference the weaker supplier would exit the bidding at a price of V_2.

The willingness to pay by the weaker supplier V_2 is the equilibrium price of preference. We can summarize certain features of the function $V_2(t_1,t_2)$. For the symmetric case in which $t_1 = t_2 = t = T/2$, $V_2 = V_1$ can then be expressed as:

(13) $$V_2(t,t) = \frac{t}{(1+t)^2} \cdot \left(1 - \frac{1}{1+2t} \cdot \left(\frac{t}{1+t}\right)^t\right).$$

This function is increasing for small t and then decreasing for larger t. The maximum occurs at approximately $t = 1.314$ at which it attains a value of approximately 0.213. When $t = 1.314$, the expected price $Ep(t,t)$ is approximately 0.71, so that V_2 is approximately 30% of the expected price.

The function $V_2(t_1,t_2)$ need not be monotonically increasing or decreasing in t_1 or t_2. There are two forces which can be working in the opposite direction. For a given cost realization c_2, and conditional on receiving the contract, the difference in expected profits of the weaker supplier when preferred versus when not preferred decreases as the weaker supplier becomes stronger. This first force implies that the willingness to pay for preference V_2 could decline as t_2 increases toward t_1. However, the difference in the probability of the weaker supplier receiving the contract when preferred versus when not preferred increases as the weaker supplier becomes stronger. This second force implies that the willingness to pay for preference V_2 could increase as t_2 increases toward t_1. Appendix 4 contains the mathematical expressions for this intuition.

When t_1 is relatively small ($t_1 < 2$), calculations demonstrate that $V_2(t_1,t_2)$ increases as t_2 increases from zero to t_1. For these cases, the stronger supplier would pay a higher price for preference as the weaker supplier becomes stronger, and the symmetric value $V_2(t,t)$ in (13) would be the highest price for preference. However, when t_1 is larger ($t_1 > 2$), calculations demonstrate that $V_2(t_1,t_2)$ first increases and then decreases as t_2 increases from zero to t_1. Thus, as the weaker supplier initially becomes stronger, the stronger supplier would have to pay more to acquire preference, but the maximum price for preference would occur at some value $t_2{}^* < t_1$.

We can also examine the behavior of $V_2(t_1,t_2)$ when the industry capacity is fixed such that $t_1 + t_2 = T$. When T is relatively small ($T < 5$), calculations demonstrate that $V_2(t_1,T\text{-}t_1)$ uniformly declines from the value in (13) as the suppliers become more asymmetric (t_1 larger

and t_2 smaller). Thus, the maximum price of preference V_2 for a given industry capacity occurs when the suppliers each have half the industry capacity. However, for larger T, $V_2(t_1, T-t_1)$ can exhibit a variety of non-monotonic behaviors. One type of behavior is that price of preference V_2 attains a maximum value when t_1 is close to T and t_2 is close to zero.

The previous intuition and findings indicate that the behavior of $V_2(t_1, t_2)$ is very sensitive to the capacity of the weaker supplier t_2, particularly when $t_2 < 1$. Thus, we also examined the behavior of $V_2(t_1, t_2)$ for a given $t_2 \geq 1$, letting the capacity of the stronger supplier t_1 increase above t_2. For these cases, calculations demonstrate that the price of preference uniformly declines as the stronger supplier becomes even stronger. Thus, for $t_2 = t \geq 1$, the maximum price of preference is $V_2(t,t)$ from (13) and declines thereafter as the stronger supplier $t_1 > t$ becomes yet stronger. These cases correspond to what we expected might happen, that the stronger supplier would have to pay less for preference when he becomes yet stronger than the weaker supplier.

6.   The Buyer's Net Expected Price After the Pre-Auction for Preference

As we demonstrated by Propositions 1(a) and 1(b), preference for either supplier increases the expected price paid by the buyer, but the buyer now receives a payment of V_2 from the stronger PS.   Thus, the question is whether this payment for preference lowers the *net expected price*, $NEp(t_1, t_2)$, paid by the buyer:

(14) $NEp(t_1, t_2) = Ep(t_1, t_2) - V_2(t_1, t_2)$.[15]

The following proposition demonstrates that the net expected price after auctioning preference is always lower than the expected price in an EA.

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[15]   The net expected price would be the same if the buyer could sell preference to the weaker supplier at his willingness to pay V_2 for the preference. However, the buyer cannot implement this outcome with a pre-auction for preference. Instead, the buyer would have to know the cost distributions of the suppliers in order to calculate V_2 and make the weaker supplier a take-it-or-leave-it offer.

Proposition 4(a): For all (t_1, t_2), the net expected price paid by the buyer in the preference auction when the stronger supplier acquires preference in ascending-price open pre-auction is lower than the expected price with an efficient auction: $NEp(t_1, t_2) < Ep_{EA}(t_1, t_2)$.

The proof of Proposition 4(a) is contained in Appendix 5. Figure 4 illustrates the relationship between Ep, Ep_{EA}, and NEp when $t_1 + t_2 = T = 2$. When the suppliers are symmetric, $Ep(1,1) = 3/4$, $Ep_{EA}(1,1) = 2/3$, and $NEp(1,1) = 13/24$. From expression (6), we see that $Ep(t_1, t_2)$ is minimized at $t_1 = t_2 = T/2$, and increasing to the upper bound on costs when $t_1 = T$ and $t_2 = 0$. Similarly, from expression (7), $Ep_{EA}(t_1, t_2)$ has the same behavior, but is everywhere below $Ep(t_1, t_2)$ (Proposition 1(a)). Proposition 4(a) now demonstrates that $NEp(t_1, t_2)$ is everywhere below $Ep_{EA}(t_1, t_2)$, the exact amount depending on the behavior of V_2 when $t_1 > t_2$.[16]

The percentage price reduction relative to an EA can be defined as $\Delta_{EA}(t_1, t_2) = \{ V_2 - [Ep - Ep_{EA}] \} / Ep_{EA}$. For the symmetric case, the percentage price reduction is 18.75% when $T = 2$, and approaches 33.3% as $T \to \infty$. Table 2 provides the percentage price reductions for other symmetric and asymmetric cases of the preference auction (for T and $t_1 = \lambda \cdot T$ where $\lambda \geq 0.5$). Except for very small T, the percentage reductions are substantial, typically above 10% for $T > 2$, typically above 20% for $T > 5$, and above 30% as $T \to \infty$. For a given T, the percentage price reductions decline only slightly as the first supplier becomes increasingly stronger than the second supplier (larger λ).

Proposition 4(a) demonstrates that the price of preference V_2 dominates the increase in the expected prices $[Ep - Ep_{EA}]$, resulting in a lower net expected price after preference is auctioned. The increase in the expected prices is largest when the suppliers are symmetric, but the price of preference V_2 is also largest in the symmetric case. The price of preference is often more than twice as large as the increase in the expected price for cases of symmetry and modest degrees of asymmetry. For example, when $t_1 = t_2 = 1$, the increase in expected price is 12.5%, but the reduction in the net expected price after auctioning preference is 18.75%. Thus, the V_2 is two and half times the magnitude of $[Ep - Ep_{EA}]$.

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[16] Note that the slope of $NEp(t_1, t_2)$ is not zero at $t_1 = t_2 = T/2$. The reason is that the derivative of V_2 is not zero at $t_1 = t_2$, but instead continues to decline.

We can also make some comparisons of the net expected price in a preference auction with the expected price in a FPA. When the suppliers are symmetric ($t_1 = t_2 = t = T/2$), the expected price in an EA is equivalent to the expected price in a FPA. Thus, Proposition 4(a) implies that the net expected price in the preference auction is also less than the expected price in a FPA: $NEp(t,t) < Ep_{EA}(t,t) = Ep_{FPA}(t,t)$. The more difficult and interesting cases occur when the suppliers are asymmetric. We can now state the following proposition based on computations using the Bidcomp[2] program of Li and Riley (1999).

Proposition 4(b): For a fine grid of (t_1,t_2), the net expected price paid by the buyer in the preference auction when the stronger supplier acquires preference in ascending-price open pre-auction is lower than the expected price with a first-price auction: $NEp(t_1,t_2) < Ep_{FPA}(t_1,t_2)$.

Table 3 illustrates both Propositions 4(a) and 4(b) by providing examples of expected prices for these three auctions: $NEp(t_1,t_2)$, $Ep_{FPA}(t_1,t_2)$, and $Ep_{EA}(t_1,t_2)$. As with the other tables, these expected prices are calculated for a given industry capacity, $t_1 + t_2 = T$, and a given degree of asymmetry, $t_1 = \lambda \cdot T$ where $\lambda \geq 0.5$. The expected prices $NEp(t_1,t_2)$ and $Ep_{EA}(t_1,t_2)$ have closed-form solutions, but $Ep_{FPA}(t_1,t_2)$ must be computed using Bidcomp[2]. Table 2 also provides the percentage prices reductions from the FPA auction: $\Delta_{FPA}(t_1,t_2) = \{ V_2 - [Ep - Ep_{FPA}] \} / Ep_{FPA}$. The price reductions from auctioning preference remain substantial relative to the expected price in the FPA. For cases in which the stronger supplier is only moderately stronger than the weaker supplier, the percentage price reductions are only modestly smaller than those relative to an EA.

7. The Effect of Preference on the Profits of the Suppliers

Propositions 4(a) and 4(b) make it clear that the buyer would benefit from selling preference to one of the suppliers. In this section, we examine the effect of preference on the

profitability of the suppliers. It is clear that the CS cannot have higher expected profits after preference is sold to the PS. However, the PS could have higher expected profits after paying V_2 for preference.

If the suppliers are asymmetric, we cannot make direct comparisons of the expected profits between the preference auction and the FPA. However, we can compare the expected profits between the preference auction and an EA, and this will allow us to make some conclusions about the comparison with the FPA. The expected profits of the ith supplier in an EA can be obtained by integrating the profit margin over the cost distributions, conditional on the ith supplier winning the contract. The profit margin is simply the implicit bidding function in (8), net of the cost: $b_{EA,i}(c_i) - c_i$. The resulting expected profits can be easily expressed as:

$$(15) \qquad \Pi_{EA,i}(t_i, t_j) = \frac{t_i}{(1+t_j) \cdot (1+T)} \quad .$$

If the suppliers are symmetric, expression (15) also represents the expected profits in a FPA.

After preference is sold to the stronger supplier, the expected profits of the weaker CS would be lower. This is obvious from comparing the expected profits of the CS using expression (11) [where $t_d = t_1$ and $t_h = t_2$] and expression (15) [where $t_j = t_1$ and $t_i = t_2$]

$$(16) \qquad \Pi_h(t_1, t_2) - \Pi_{EA,2}(t_2, t_1) = \frac{t_2}{(1+t_1)(1+T)} \cdot \left\{ \left(\frac{t_1}{1+t_1} \right)^{t_1} - 1 \right\} < 0 \quad .$$

Now consider the stronger PS. The stronger supplier would clearly benefit if preference were granted to him without a payment: $\Pi_d(t_1, t_2) > \Pi_{EA,1}(t_1, t_2)$. This gain from preference can be decomposed into two positive components. The first component is equal to the loss of the buyer defined by the second term of (7), and the second component is a fraction $[t_1/(1+t_1)]$ of the expected profits of the CS.[17] Thus, there are gains from trade for the buyer and the preference

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[17] Interpreted in this procurement setting, Bikhchandani, Lippmann, and Ryan (2003) find that the gains to the PS would be equal to the losses to the buyer when the competing suppliers bid in a second-price auction.

supplier. However, this is not the relevant question. Instead, we need to examine the expected profits of the stronger supplier after he has paid the buyer for preference.

Proposition 3 implies that the stronger supplier earns higher expected profits after acquiring preference in the pre-auction rather than allowing the weaker supplier to acquire the preference. Using the definition for V_1 from expression (12a), we quickly see that $V_1 > V_2 \Leftrightarrow$ $\Pi_d(t_1,t_2) - V_2 > \Pi_h(t_2,t_1)$. The more difficult question is whether the stronger PS has higher expected profits, after paying the buyer for preference, than he would have earned in an EA. One part of the willingness to pay V_1 of the stronger supplier arises from the advantage of being preferred, but the other part arises from avoiding the disadvantage of not being preferred. Thus, depending on the price of preference V_2, the stronger PS would be partially paying to avoid the negative consequences from not being preferred.

The expected profits of this stronger PS are equal to $\Pi_1 = \Pi_d(t_1,t_2) - [\Pi_d(t_2,t_1) - \Pi_h(t_2,t_1)]$. On the other hand, the expected profits of this stronger supplier in an EA would be $\Pi_{EA} = t_1/[(1+t_2)(1+t_1+t_2)]$. Thus, if we examine the difference $\Pi_1 - \Pi_{EA}$. Substituting from (10) and (11), we find that

$$(17) \quad \Pi_1 - \Pi_{EA} = \frac{t_2}{(1+t_1)(1+T)} \cdot \left\{ -1 + \frac{t_1}{(1+t_1)} \cdot \left(\frac{t_1}{1+t_1}\right)^{t_1} + \frac{t_1(1+t_1)}{t_2(1+t_2)^2} \cdot \left(\frac{t_2}{1+t_2}\right)^{t_2} \right\}.$$

In the symmetric case when $t_1 = t_2 = t$, the term in braces is $\{ -1 + [t/(1+t)]^t \} < 0$. Thus, the PS earns lower expected profits than an EA. Since either supplier would pay the buyer his full willingness to pay in order to acquire preference, the expected profits of either supplier would be lower after the acquisition of preference. However, the stronger PS can earn higher expected profits when its capacity t_1 is sufficiently larger that t_2. For example, if $t_2 = 1$, the stronger PS would earn higher expected profits when $t_1 \geq 2$.[18] In these cases, the gains from having

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[18] As compared to an EA, the FPA typically induces both a lower total surplus and a lower expected price. Thus, the expected profits of the suppliers are necessarily lower. Moreover, the FPA generates a lower expected price for the buyer by distorting the allocation in favor of the weaker supplier, thereby reducing the informational rents of the stronger supplier. For these reasons, the stronger supplier earns a lower expected profit in a FPA than he would in

preference in the procurement auction exceed the price of purchasing preference. In sum, when the stronger supplier is sufficiently stronger than the weaker supplier, both the buyer and the stronger supplier can mutually benefit from the auction of preference.

Since the stronger supplier often decreases his expected profits after acquiring preference, one might ask whether the stronger supplier could refuse to participate in the pre-auction for preference. This would only make sense if he knew that the buyer and the weaker supplier could not reach a mutually beneficial agreement for preference. The combined expected profits of the buyer and the weaker supplier with the preference auction are $[v - Ep(t_1, t_2)] +$ $\Pi_d(t_2, t_1)$. Alternatively, their combined expected profits in an EA are $[v - Ep_{EA}(t_1, t_2)] +$ $\Pi_{EA,2}(t_2, t_1)$. Substituting from the relevant expressions (6), (7), (10), and (15), we see that the exchange of preference will generate larger combined profits for the buyer and the weaker supplier, at the expense of the stronger supplier. As a result, the buyer and the weaker supplier can always negotiate a mutually beneficial price for preference, relative to an EA. Thus, the stronger supplier cannot afford to bypass the pre-auction for preference, even though his expected profits may decline relative to the EA.

8.   Vertical Integration

Instead of selling preference to one supplier, the buyer could acquire one of the suppliers, making it an internal subsidiary. If so, the buyer would optimally choose to give its subsidiary a *right-of-first-refusal* to supply the good. If the subsidiary had a cost below the bid of the independent supplier, the buyer would employ the subsidiary. Otherwise, the buyer would award the contract to the independent supplier. Thus, the independent supplier would bid just as a competing supplier in the preference auction, and the internal subsidiary would be the preferred supplier. As such, the procurement auction would remain unchanged. However, there could be a difference between the price of preference and the acquisition cost of one of the suppliers.

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an EA.   Thus, the preference auction would also generate higher expected profits for the stronger PS relative to the expected profits in an FPA when t_1 is sufficiently larger than t_2.

Consider the acquisition price of either supplier if the buyer announces his intention to acquire one of the two suppliers. The expected price paid for the good by the buyer, $Ep(t_2,t_2)$ from (6), is independent of which supplier is acquired. Thus, the expected profits for the buyer (value v – net expected price NEp) depends on the difference between the expected profits of the preferred subsidiary and the acquisition cost of that subsidiary. From Proposition 2(a), $\Pi_d(t_1,t_2) > \Pi_d(t_2,t_1)$ for all $t_1 > t_2$. Thus, the buyer would experience a larger increase in expected profits from acquiring the stronger supplier. However, the acquisition cost of the stronger supplier may also be higher. If the announcement to acquire one of the suppliers is credible, each supplier can only guarantee itself the expected profits of the CS after the other supplier is acquired. From Proposition 2(b), $\Pi_h(t_2,t_1) > \Pi_h(t_1,t_2)$ for $t_1 > t_2$. Thus, the stronger supplier would command a higher acquisition price.

Proposition 3 states that $V_1 > V_2$, and this finding immediately implies that the increase in expected profits of the buyer would be greater after acquiring the stronger supplier. From the definitions of V_1 and V_2 in (12a) and (12b), V_1 is the net increase in expected profits from acquiring the stronger supplier, and V_2 is the lower net increase in expected profits from acquiring the weaker supplier. In addition, the acquisition of the stronger supplier would be more profitable than auctioning preference to the stronger supplier. The preference auction would result in the stronger supplier purchasing preference, but the price received by the buyer would be only V_2, not V_1. The following proposition extends this finding to any acquisition price that arises from a Nash bargaining solution.

Proposition 6: If the weaker supplier can be acquired at his reservation price $\Pi_h(t_1,t_2)$, then any Nash bargaining solution between the buyer and the stronger supplier over the acquisition price of the stronger supplier would result in higher expected profits (lower net expected price) for the buyer than holding an ascending-price open pre-auction for preference.

The proof is straight-forward. If the weaker supplier can be acquired at his reservation price $\Pi_h(t_1,t_2)$, then the buyer can guarantee himself V_2, the same that he receives from the stronger supplier in a pre-auction for preference. On the other hand, the stronger supplier can only guarantee himself $\Pi_h(t_2,t_1)$ if the weaker supplier is acquired. Since $V_1 > V_2$, the acquisition

of the stronger supplier would generate an expected surplus $\Pi_d(t_1, t_2)$ which exceeds the sum of these threat points for the buyer and the stronger supplier $\Pi_d(t_1, t_2) > V_2 + \Pi_h(t_2, t_1)$. Thus, any Nash bargaining solution over the acquisition price would generate an increase in the expected profits for the buyer which exceeds his threat point of V_2. Alternatively, the buyer pays a lower net expected price for the good.

9.   Conclusions and Policy Implications

The sale of preference clearly empowers the buyer. In particular, the buyer can extract rents from the suppliers because part of the willingness to pay for preference arises from the desire to avoid the reduced expected profits if the other supplier is preferred. Propositions 4(a) and 4(b) indicate that the buyer will always pay a lower net expected price for the good after the sale of preference. If the buyer is competing in some subsequent final good market, this could benefit the ultimate consumers if the savings to the buyer on his inputs are passed on to these consumers through some competitive process in the final market. Despite this benefit to the buyer and his consumers, the creation and sale of preference raises several concerns for efficiency and future competition.

The first concern for competition policy is that one cannot infer that preference arises from efficiency considerations. The standard justification for exclusive dealing is that the interests of the buyer and the seller are better aligned, and various externalities in their decisions to produce and distribute the good are internalized. On the contrary, preference arises in this model even though it generates no efficiency gains, and actually creates an allocative distortion in the award of the procurement contract. The incentive of the buyer to extract rents from the suppliers naturally gives rise to the sale of preference, defined as a *right-of-first-refusal*. In particular, the buyer can take advantage of the fact that preference distorts the competition between the suppliers such that the expected profits from being preferred are much larger than the expected profits from remaining the competing supplier. This distortion can then be used to extract rents from the suppliers by holding a pre-auction for the preference. In effect, preference creates an artificial asymmetry in the procurement auction which inflates the willingness to pay for preference by both suppliers.

The second concern for competition policy is that the pre-auction for preference distorts the auction in favor of the stronger supplier. The stronger supplier always outbids the weaker supplier for preference. Moreover, if the stronger supplier is sufficiently stronger, he may actually earn higher expected profits after paying for preference. This preference in favor of the stronger supplier is clearly contrary to prescription of an optimal auction mechanism. But in addition, it accentuates the underlying asymmetry from the differing capacities of the suppliers. In this model, the effects on the three players are fully examined. However, there could be other

anti-competitive effects in an extended model with investment by suppliers and competition between multiple buyers in a final market.

Consider the question of investments by suppliers. In our related paper Burguet and Perry (2000), we examined the incentives of suppliers to make investments when they can bribe the auctioneer for favoritism, which corresponds to preference in this model. Although favoritism enhanced the investment incentives of the favored supplier, it severely undermined the investment incentives of the disfavored supplier. Moreover, aggregate investment in capacity was lower with favoritism and was less than the social optimum.

Now consider the question of competition among multiple buyers of the input in a final market, and more suppliers than the buyers. In the current model with one buyer, preference to one supplier does not withdraw his capacity from the input market. However, with multiple buyers, the capacity available to bid on contracts by one buyer diminishes as each buyer auctions preference to another one of the suppliers. Thus, competition among the suppliers is bifurcated and there might be a negative impact on wholesale prices and final prices.

In sum, there is more work to be done on the effects of preference in auction markets and the analogies to exclusive dealing in non-auction markets.

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Appendix 1: Related Literature

Awarding a *right-of-first-refusal* to one supplier is similar to exclusive dealing with the buyer or vertical integration between one supplier and the buyer. A number of authors have examined models in which two manufacturers compete to foreclose each other by executing an exclusive dealing contract with a downstream distributor who has a monopoly in one or all of the markets.   These papers employ a model of consumer demand with either homogeneous or differentiated products, and focus on complete foreclosure. This literature includes papers by Mathewson and Winter (1987), Bernheim and Whinston (1986, 1998), Besanko and Perry (1994), Martimort (1996), and O'Brien and Shaffer (1997). In our model, the final market is suppressed in order to focus on the wholesale market. Preference does not result in complete foreclosure because the buyer retains the opportunity to buy from the competing suppliers instead of his preferred supplier.

This paper builds on the model of Burguet and Perry (2000). That earlier paper examines various models of bribery in a model with one auctioneer and two suppliers competing to provide an input for the ultimate buyer. The auctioneer represents the buyer in selecting the supplier to receive the contract, but the auctioneer can be bribed by one of the suppliers. In the basic model of bribery, the auctioneer gives one supplier a *right-of-first-refusal* at the price bid by the other supplier in return for a bribe equal to a share of the profits on the contract. The paper then examines the implications of bribery for the expected price, the allocative distortion, expected profits of the suppliers, and the *ex ante* investments by the suppliers. This paper uses the same auction model between the suppliers, but eliminates the auctioneer and allows the buyer to hold a pre-auction for the *right-of-first-refusal*.

Another related paper is Aghion and Bolton (1987). An incumbent seller enters into a contract with a monopoly buyer in which there is a payment for the good when an entrant seller with a lower cost of producing the good does not arise. However, if a lower cost entrant arises, the buyer makes a lower payment to seller to breach the contract and then purchase the good from the entrant.   The incumbent seller and buyer can mutually benefit from this contract by extracting rents from the entrant seller.   In our model, the buyer auctions the *right-of-first-refusal* to one of the suppliers, and purchases from the other supplier only if his bid is below the

cost of the preferred supplier. In a similar manner, the buyer and possibly the preferred supplier can increase their expected profits at the expense of the competing supplier.

Appendix 2: Proof of Proposition 2

Proof of Proposition 2(a): The difference in the expected profits of the stronger supplier can be rearranged into the following expression:

$$\Pi_d(t_1, t_2) - \Pi_d(t_2, t_1) \ =$$

$$\frac{t_1 t_2}{(1 + t_1 + t_2)} \cdot \left\{ \left(\frac{1}{t_2(1+t_2)} - \frac{1}{(1+t_2)^2} \left(\frac{t_2}{1+t_2} \right)^{t_2} \right) - \left(\frac{1}{t_1(1+t_1)} - \frac{1}{(1+t_1)^2} \left(\frac{t_1}{1+t_1} \right)^{t_1} \right) \right\}.$$

Since $\Pi_d(t_1, t_2) - \Pi_d(t_2, t_1) = 0$ when $t_1 = t_2$, we examine $\Pi_d(t_1, t_2) - \Pi_d(t_2, t_1) > 0$ for $t_1 > t_2$. The term in braces is the difference between two expressions having the form:

$$\left(\frac{1}{t(1+t)} - \frac{1}{(1+t)^2} \left(\frac{t}{1+t} \right)^t \right) \ .$$

Thus, the term in braces is positive if this expression is decreasing in t. Taking the derivative of this expression, we find that

$$\frac{d(-)}{dt} \ = \ -\frac{1}{(1+t)^2} \cdot \left[\frac{1}{t^2} + \left(\frac{1}{t} - \frac{1}{(1+t)} \left(\frac{t}{1+t} \right)^t \right) + \left(\frac{1}{t} + \ln \left(\frac{t}{(1+t)} \right) \left(\frac{t}{1+t} \right)^t \right) \right] < 0 \ .$$

All three terms in the brackets are positive for all t. The first and second terms are obviously positive. The third term is also positive because $[1/t + \ln(t/(1+t))] > 0$. This follows from the fact that $[1/t + \ln[t/(1+t)]$ approaches zero as $t \to \infty$ and is decreasing for all t. Since this derivative is negative, $\Pi_d(t_1, t_2) - \Pi_d(t_2, t_1) > 0$ when $t_1 > t_2$. Q.E.D.

Proof of Proposition 2(b): The difference in the expected profits of the stronger supplier can be rearranged into the following expression:

$$\Pi_h(t_2,t_1) - \Pi_h(t_1,t_2) = \frac{t_1 \cdot t_2}{(1+t_1+t_2)} \cdot \left\{ \frac{1}{t_2 \cdot (1+t_2)} \cdot \left(\frac{t_2}{1+t_2} \right)^{t_2} - \frac{1}{t_1 \cdot (1+t_1)} \cdot \left(\frac{t_1}{1+t_1} \right)^{t_1} \right\}$$

Since $\Pi_h(t_2,t_1) - \Pi_h(t_1,t_2) = 0$ when $t_1 = t_2$, we examine $\Pi_h(t_2,t_1) - \Pi_h(t_1,t_2) > 0$ for $t_1 > t_2$. The term in braces is the difference between two expressions having the form

$$\frac{1}{t \cdot (1+t)} \cdot \left(\frac{t}{1+t} \right)^{t}$$

Thus, the term in braces is positive if this expression is decreasing in t. Taking the derivative of this expression, we find that

$$\frac{d(-)}{dt} = \frac{1}{t \cdot (1+t)} \cdot \left(\frac{t}{1+t} \right)^{t} \left[-\frac{1}{t} + \ln\left(\frac{t}{1+t} \right) \right] < 0 \ .$$

The term in brackets is obviously negative for all t. Thus, $\Pi_h(t_2,t_1) - \Pi_h(t_1,t_2) > 0$ when $t_1 > t_2$. Q.E.D.

Proof of Proposition 2(c): The difference in the expected profits from being a stronger CS rather than a weaker PS is

$$\Pi_h(t_2,t_1) - \Pi_d(t_2,t_1)$$

$$= \frac{t_1}{(1+t_2)(1+t_1+t_2)} \cdot \left(\frac{t_2}{1+t_2} \right)^{t_2} - \frac{t_2}{(1+t_1)(1+t_2)} - \frac{t_1 \cdot t_2}{(1+t_2)^2(1+t_1+t_2)} \cdot \left(\frac{t_2}{1+t_2} \right)^{t_2}$$

$$= \frac{t_1}{(1+t_2)^2(1+t_1+t_2)} \cdot \left(\frac{t_2}{1+t_2} \right)^{t_2} - \frac{t_2}{(1+t_1)(1+t_2)}$$

When $t_1 = t_2$, the second term dominates the first term, so that $\Pi_h(t_2,t_1) < \Pi_d(t_2,t_1)$. This is no surprise. However, as the CS becomes stronger, the first term eventually dominates. In particular, the second term approaches zero as $t_1 \to \infty$, but the first term converges to a strictly positive number in terms of t_2: $\frac{1}{(1+t_2)^2} \cdot \left(\frac{t_2}{1+t_2} \right)^{t}$ Thus, there exists some $t_1(t_2) > t_2$ such that $\Pi_h(t_2,t_1) > \Pi_d(t_2,t_1)$ for all $t_1 > t_1(t_2)$.

Appendix 3: Proof of Proposition 3

Proof of Proposition 3:   The difference in the willingness to pay of the stronger and weaker suppliers can be rearranged as follows:

$$V_1 - V_2 = \frac{t_1 t_2}{(1+t_1+t_2)} \cdot \left\{ \left(\frac{1}{t_2(1+t_2)} - \frac{(1+2t_2)}{t_2 \cdot (1+t_2)^2} \left(\frac{t_2}{1+t_2} \right)^{t_2} \right) - \left(\frac{1}{t_1(1+t_1)} - \frac{(1+2t_1)}{t_1 \cdot (1+t_1)^2} \left(\frac{t_1}{1+t_1} \right)^{t_1} \right) \right\}.$$

Since $V_1 - V_2 = 0$ when $t_1 = t_2$, we examine $V_1 - V_2 > 0$ for $t_1 > t_2$. The term in braces is the difference between two expressions having the form:

$$\left(\frac{1}{t(1+t)} - \frac{(1+2t)}{t \cdot (1+t)^2} \cdot \left(\frac{t}{1+t} \right)^t \right) .$$

Thus, the term in braces is positive if this expression is decreasing in t. Taking the derivative of this expression, we find that:

$$\frac{d(-)}{dt} = -\frac{(1+2t)}{t(1+t)^2} \cdot \left[\frac{1}{t} + \ln\left(\frac{t}{1+t} \right)\left(\frac{t}{1+t} \right)^t + \frac{(1+2t+2t^2)}{t(1+t)(1+2t)} \cdot \left(\frac{t}{1+t} \right)^t \right] < 0 .$$

The first and third terms are obviously positive, but the second term is negative. However, the first term dominates the second term because $1/t + \ln[t/(1+t)] > 0$. This follows from the fact that $[1/t + \ln(t/(1+t))]$ approaches zero as $t \to \infty$ and is decreasing for all t. Since this derivative is negative, $V_1 > V_2$ whenever $t_1 > t_2$. Q.E.D.

Appendix 4: Mathematical Intuition for $V_2(t_1,t_2)$

Given c_2, and conditional on receiving the contract, the expected profit of the weaker supplier with preference is:

$$\pi_d(t_2,t_1;c_2) = \frac{1+t_1+t_2}{(1+t_1)(1+t_2)} - c_2 \quad \text{if } c_2 < \frac{1}{1+t_2}$$

$$= \frac{1-c_2}{1+t_1} \quad \text{otherwise}$$

Similarly, the expected profit of the weaker supplier without preference is:

$$\pi_h(t_1,t_2;c_2) = \frac{1-c_2}{1+t_1} .$$

The difference between these expected profits is:

$$\pi_d(t_2, t_1; c_2) - \pi_h(t_1, t_2; c_2) = \frac{t_1}{1 + t_1} \cdot \left(\frac{1}{1 + t_2} - c_2 \right) \quad \text{if } c_2 < \frac{1}{1 + t_2} \quad .$$
$$= 0 \quad \text{otherwise}$$

Clearly, this difference is decreasing in t_2.

Now consider the probability of receiving the contract for the weaker supplier. If the weaker supplier is preferred, the probability of receiving the contract is

$$\text{Pr}_h = 1 - \frac{t_1}{t_1 + t_2} \cdot \left(\frac{t_2}{1 + t_2} \right)^{t_2} \quad .$$

If the weaker supplier is not preferred, the probability of receiving the contract is:

$$\text{Pr}_d = \frac{t_2}{t_1 + t_2} \cdot \left(\frac{t_1}{1 + t_1} \right)^{t_1} \quad .$$

Consider the derivative of the difference in these probabilities with respect to t_2 :

$$\frac{d[\text{Pr}_h - \text{Pr}_d]}{dt_2} = \frac{t_1}{(t_1 + t_2)^2} \cdot \left\{ \left(\frac{t_2}{1 + t_2} \right)^{t_2} - \left(\frac{t_1}{1 + t_1} \right)^{t_1} - (t_1 + t_2) \cdot d \left(\frac{t_2}{1 + t_2} \right)^{t} / dt_2 \right\}$$

Since $[t/(1+t)]^t$ is decreasing in t, the positive first term dominates the negative second term when $t_1 > t_2$, and the third term is also positive. Thus, the difference is increasing in t_2. Q.E.D.

Appendix 5: Proof of Proposition 4(a)

Proof of Proposition 4(a): The proof is very straight-forward. Substituting for the expressions in $NEp(t_1, t_2)$ from the prior expressions, we find that $NEp(t_1, t_2) < Ep_{EA}(t_1, t_2)$ if and only if:
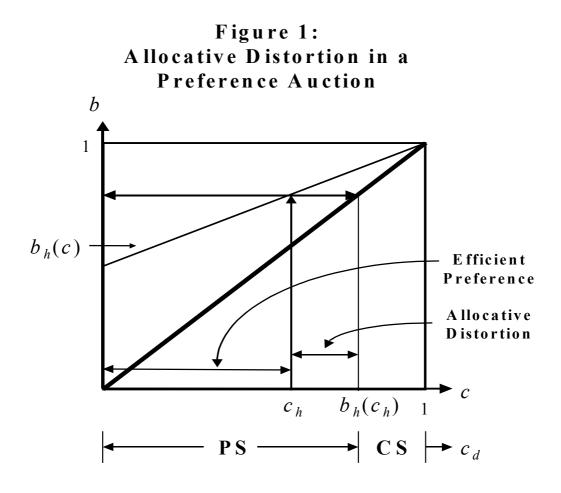
$$\frac{t_2}{(1 + t_1)(1 + t_1 + t_2)} \cdot \left\{ -1 + \left(\frac{t_1}{1 + t_1} \right)^{t_1 + 1} - \frac{t_1(1 + t_1)}{t_2(1 + t_2)^2} \left(\frac{t_2}{1 + t_2} \right)^{t_2} \right\} < 0 \quad .$$
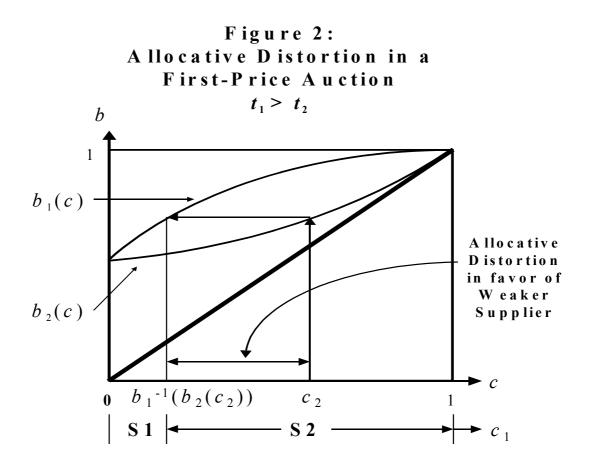
Since the second term in the braces is less than one for all t_1, this condition is satisfied. Q.E.D.
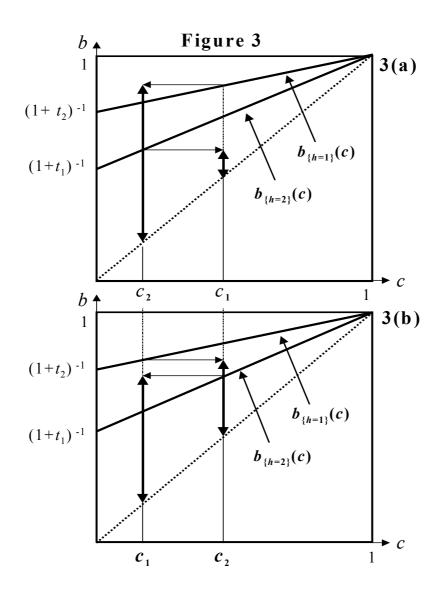
Appendix 6: Preference versus a Uniform Entry Fee

If the buyer can charge the suppliers entry fees to participate in the auction prior to the suppliers learning their actual cost, he can extract all the surplus from suppliers in a number of cases. First, if the buyer can set discriminatory entry fees, he can extract all of the surplus from the suppliers even if they are asymmetric. Second, if the buyer can only set uniform entry fees that are the same for any supplier, then he can extract all of the surplus from the suppliers if they are symmetric. In either of these cases, the buyer would pay a lower net expected price than he would from auctioning preference to the stronger supplier. However, if the suppliers are asymmetric and the buyer is constrained to set a uniform entry fee, preference might generate a lower net expected price. This case would apply is the buyer cannot identify the suppliers. In this appendix, we compare preference with an asymmetric efficient auction when the buyer can charge a uniform entry fee. The buyer can extract all the expected profits of the weaker supplier, but not from the stronger supplier.

Consider an EA with uniform entry fees. The uniform entry fee is equal to the expected profits of the weaker supplier $\Pi_{EA,2}$. Thus, the net expected price paid by the buyer would be $NEp_{EA}(t_1, t_2) \;=\; Ep_{EA}(t_1, t_2) \;-\; 2 \cdot \Pi_{EA,2}(t_2, t_1)$. Upon comparing this net expected price with the net expected price under preference from (14), we find that preference can result in lower net expected price. However, the cases in which preference generates a lower net expected price involve a significant degree of asymmetry between the suppliers. For example, when the capacity of the weaker supplier is $t_2 = 0.5$, the stronger supplier must have a capacity $t_1 > 3.66$. Similarly, when $t_2 = 1$, the stronger supplier must have a capacity $t_1 > 5.59$.

**Figure 1:
Allocative Distortion in a
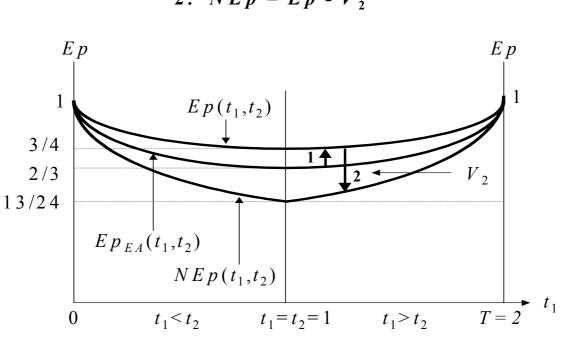Preference Auction**

F i g u r e   2 :
A l l o c a t i v e   D i s t o r t i o n   i n   a
F i r s t - P r i c e   A u c t i o n
$t_1 > t_2$

Figure 3

Figure 4: Expected Prices
1: $Ep > Ep_{EA}$
2: $NEp = Ep - V_2$

Table 1: Allocative Distortion From Preference (% of EA)

Industry Capacity **Capacity of Largest Supplier**

Auction	EA/FPA	EA	EA	EA	EA
T	$.5T$	$.6T$	$.7T$	$.8T$	$.9T$
0,25	1,7%	1,8%	1,7%	1,5%	1,0%
0,50	4,0%	4,2%	4,0%	3,4%	2,2%
0,75	6,0%	6,3%	6,1%	5,2%	3,3%
1,00	7,7%	8,2%	7,9%	6,7%	4,2%
2,00	12,5%	13,5%	13,1%	10,9%	6,4%
3,00	15,4%	16,7%	16,3%	13,5%	7,6%
4,00	17,3%	18,8%	18,4%	15,1%	8,4%
5,00	18,6%	20,4%	20,0%	16,3%	8,8%
10,00	21,9%	24,2%	23,7%	19,0%	9,9%
20,00	24,0%	26,6%	26,1%	20,8%	10,5%
Limit	27,0%	30,0%	29,5%	23,0%	11,5%

Table 2: Price Reduction From Auctioning Preference (% of EA and FPA)

Industry Capacity		Capacity of Largest Supplier				
Auction		EA/FPA	EA	FPA	EA	FPA
EA	FPA	EA	FPA			
T		.5T	.6T	.6T	.7T	.7T
0,25		2,95%	2,67%		2,23%	
0,5		6,64%	5,97%		5,02%	
0,75		9,76%	8,77%		7,42%	
1		12,30%	11,04%		9,43%	
2		18,75%	16,88%		14,81%	
3		22,20%	20,06%	19,50%	17,93%	
4		24,34%	22,05%	21,50%	19,97%	17,50%
5		25,79%	23,41%	22,80%	21,42%	18,50%
10		29,16%	26,63%	25,70%	25,01%	20,80%
20		31,13%	28,55%	27,50%	27,28%	22,40%
Limit		33,33%	30,73%		30,02%	

Table 3: Expected Prices

Industry Capacity		Capacity of Largest Supplier												
Auction	NEP	FPA/EA	NEP	FPA	EA	NEP	FPA	EA	NEP	FPA	EA	NEP	FPA	EA
1	.57	.57	.67	.67	.67	.77	.77	.77	.87	.87	.87	.97	.97	.97
0,25	0,9489	0,978	0,9525	NA	0,979	0,9594	NA	0,981	0,9694	NA	0,986	0,9829	NA	0,992
0,50	0,8713	0,933	0,8700	NA	0,936	0,8962	NA	0,944	0,9206	NA	0,957	0,9543	NA	0,975
0,75	0,7969	0,883	0,8097	NA	0,888	0,8338	NA	0,901	0,8707	NA	0,923	0,9236	NA	0,956
1,00	0,7308	0,833	0,7466	NA	0,839	0,7766	NA	0,868	0,8236	NA	0,889	0,8933	NA	0,936
2,00	0,5417	0,667	0,5625	NA	0,677	0,6036	NA	0,708	0,6719	NA	0,766	0,7868	NA	0,857
3,00	0,4279	0,550	0,4490	0,558	0,562	0,4915	NA	0,599	0,5661	NA	0,669	0,7007	NA	0,790
4,00	0,3531	0,467	0,3732	0,475	0,479	0,4143	0,502	0,518	0,4889	NA	0,594	0,6325	NA	0,732
5,00	0,3004	0,405	0,3191	0,413	0,417	0,3580	0,439	0,456	0,4308	0,496	0,533	0,5766	NA	0,682
6,00	0,2613	0,357	0,2787	0,365	0,369	0,3151	0,390	0,407	0,3843	0,444	0,484	0,5300	NA	0,638
7,00	0,2312	0,319	0,2473	0,327	0,331	0,2814	0,351	0,367	0,3472	0,403	0,443	0,4905	NA	0,600
8,00	0,2073	0,289	0,2223	0,296	0,299	0,2542	0,319	0,335	0,3167	0,368	0,409	0,4566	NA	0,566
9,00	0,1878	0,264	0,2119	0,270	0,274	0,2318	0,292	0,307	0,2911	0,339	0,379	0,4271	NA	0,536
10,00	0,1717	0,242	0,1849	0,249	0,252	0,2131	0,269	0,284	0,2694	0,315	0,354	0,4012	0,427	0,509
11,00	0,1582	0,225	0,1705	0,230	0,233	0,1971	0,250	0,264	0,2507	0,293	0,331	0,3784	0,402	0,486
12,00	0,1466	0,209	0,1582	0,215	0,217	0,1834	0,233	0,247	0,2344	0,275	0,312	0,3580	0,380	0,462
13,00	0,1366	0,196	0,1476	0,201	0,204	0,1714	0,218	0,232	0,2201	0,258	0,294	0,3398	0,360	0,442
14,00	0,1279	0,184	0,1383	0,189	0,191	0,1609	0,205	0,218	0,2075	0,244	0,279	0,3233	0,343	0,424
15,00	0,1202	0,173	0,1301	0,178	0,180	0,1517	0,194	0,206	0,1962	0,231	0,264	0,3084	0,327	0,407
20,00	0,0924	0,134	0,1003	0,138	0,140	0,1177	0,152	0,162	0,1543	0,182	0,211	0,2506	0,266	0,338