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Matching Model of Commodity Money**

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## **Abstract**

This paper generalizes the original random matching model of money by Kiyotaki and Wright (1989) (KW) in two aspects: first, the economy is characterized by an arbitrary distribution of agents who specialize in producing a particular consumption good; and second, these agents have preferences such that they want to consume any good with some probability. The results depend crucially on the size of the fraction of producers of each good and the probability with which different agents want to consume each good. KW and other related models are shown to be parameterizations of this more general one.

## 1. Introduction

This paper generalizes the original random matching model of money by Kiyotaki and Wright (1989) (KW) in two aspects: first, the economy is characterized by an arbitrary distribution of agents who specialize in producing a particular consumption good; and second, these agents have preferences such that they want to consume any good with some probability. In contrast, KW assumes a constant distribution of agents who are specialized in producing and consuming in a very particular way. We focus on a simple three goods-and-agents version of the model. Our goal is to concentrate on the role of the patterns of production and consumption of agents in the determination of which object emerges as commodity money. In order to do this, storage costs, arguably the driving force of some of the main results in the seminal contribution by KW, are assumed away and all goods have identical intrinsic properties. As in KW and other related papers, we are basically interested in characterizing the circumstances under which a particular commodity emerges as medium of exchange. Our results depend crucially on the size of the fraction of producers of each good and the probability with which different agents want to consume each good.

There are two previous papers that are related to this. Wright (1995) generalizes the KW model introducing arbitrary distributions of specialized producers and consumers. Cuadras-Morató and Wright (1997) adds homogeneous generalists consumers who, with some common probability, consume any good. We take a step further and generalize consumer preferences, so that now producers of each type want to consume goods with idiosyncratic probability distributions.

We think that the model is interesting at least for three reasons. First, this is a general model, in the sense that the previously mentioned models are particular parameterizations of it. Thus, this model can be used as a general framework to analyze known results from previous contributions to the literature. Second, we abstract completely from storage costs and other intrinsic properties of goods. Our goal is to study the effects of the patterns of specialization in production and consumption in the emergence of commodity money. The results here depend only on the relative number of producers of each goods and the probability with which they want to consume the goods. Third, although the model can be solved analytically only for some particular cases, it can be solved numerically in general. The equilibrium set is considerably larger and gives us new insights into previous results derived from more restrictive models and, thus, into the general issue of the nature of commodity money.

A quick snapshot of the results of the paper is as follows. We first show that storage costs are crucial for the existence of monetary equilibria in KW. Second, we argue that it is still possible to have monetary equilibria in a very parsimonious way, just by assuming a more general distribution of specialized consumers and producers as in Wright (1995). Third, we argue that if the structure of preferences is identical for all consumers, then the model is equivalent to a representative agent model in which all individuals use identical trade strategies, as in Cuadras-Morató and Wright (1997). Fourth, we show that a commodity that nobody consumes cannot be commodity money. Finally, we present economies with equilibria in which agents accept as commodity money goods they will never consume in exchange for goods they would eventually have consumed.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses the results and Section 4 concludes. All proofs are relegated to the Appendix.

## 2. The model

The economy is populated by a  $[0,1]$  continuum of infinite-lived agents who are specialized in producing one of three different consumption goods, each of which will be called *good  $i$*  ( $i = 1,2,3$ ). Production costs,  $D$ , are common across producers. The fraction of agents specialized in producing good  $i$  (type  $i$  agents) is  $\sigma_i$  ( $\sum_i \sigma_i = 1$ ). Agents are generalists in consumption. At every date,  $t=0,1,2\dots$  each agent gets a taste shock that determines the good she desires that period. More specifically,  $\delta_{ij}$  ( $\sum_j \delta_{ij} = 1$ ) represents the probability that an agent of type  $i$  wants to consume good  $j$  ( $j=1,2,3$ ) at any particular period of time.<sup>1</sup> This is independent across inventory holdings and time. If a trader desires good  $j$ , she gets utility  $U$  from consuming good  $j$  and zero utility from consuming any other good different from  $j$ . After consuming a good, traders immediately produce one unit of their own production commodity. Agents cannot produce unless they have previously consumed. The net utility of the joint action of consuming plus producing is denoted by  $u=U-D$ . All goods are indivisible and have identical properties (durability, portability, storability, etc.).

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<sup>1</sup> This is a generalization of the assumption in Cuadras-Morató and Wright (1997), where  $\delta_{1j} = \delta_{2j} = \delta_{3j} = \delta_j$ .

Agents are initially endowed with one unit of the good they produce. Given the setup of the model, every agent will be always holding one unit of an object. The sequence of events will be the following: every period of time, agents start with some object in inventory. Then, they get a taste shock. If they are holding the good they want to consume, then they consume it, produce a new good and wait for next period. Otherwise, they enter a trading process (*market*) in which they are randomly matched with other traders in their same situation. Once matched, the agents have to decide whether they want to trade or not. If they want to trade, then they swap inventories one-for-one. Whenever an agent gets the good she desires, she consumes it and immediately produces a new good; otherwise, she keeps the object she obtained and waits for next period. If they do not want to trade, then they part company and wait for the next period.

The strategic decision of traders is the following. At a given period of time, an agent in the market is holding some good she does not want to consume (obviously, she would never go to the market holding the good she wants to eat). Then, she is paired with another agent who offers her one of the two following alternatives: either the good she wants to consume today (which would be accepted and consumed immediately) or another object which she does not want to eat today either. The only trade decision to be made is a choice between two objects that are not desired for immediate consumption. To analyze it, let  $V_{ij}$  be the value function for a type- $i$  individual, at the end of a period holding good  $j$  other than the one currently desired for consumption.  $V_{ij}$  can be interpreted as the value of good  $j$  as asset. The structure of taste shocks that we have assumed guarantees that  $V_{ij}$  does not depend on the good that is desired in the current period. The strategic problem of agents can be formalized in the following way: an agent of type  $i$  will accept good  $h$  in exchange for good  $j$  iff  $V_{ih} > V_{ij}$  (that is, if the value



of good  $h$  as asset is larger than the value of good  $j$ ). Iff  $V_{ij} \geq V_{ih}$ , then agents will not accept good  $h$  in exchange for good  $j$ . For most of the paper, we focus on pure strategies and assume that agents simply do not want to trade when they are indifferent between two goods (it would be enough to assume a positive arbitrarily small transaction cost to get this as a result of the model). The behavior of agents of type  $i$  can be characterized by a ranking of the three value functions corresponding to the three goods of the economy. This can be represented by a strategy vector of three elements  $s^i = (\dots, s_{hj}^i, \dots)$  where  $s_{hj}^i$  is defined as follows:

$$s_{hj}^i \in \begin{cases} \{1\} & \text{iff } V_{ij} > V_{ih} \\ \{0\} & \text{iff } V_{ij} \leq V_{ih} \end{cases}$$

where  $j$  is the good held by agent  $i$  and  $h$  is the good that is being offered in exchange. The behavior of all agents can be summarized by  $s = (\dots s^i \dots)$ .

As it is customary in this literature, commodity money in this economy is any object that is accepted by an agent not to be consumed immediately, but because of its relative high value as asset. In equilibrium, each agent ranks all objects from more acceptable to less acceptable, reflecting their relative liquidity in the economy.

In order to continue the analysis, we need to specify some notation. Let  $p_{ij}$  denote the measure of type  $i$  agents with good  $j$  at the start of a period,  $(\sum_j p_{ij} = \sigma_i)$ . Let  $\mathbf{p} = (\dots p_{ij}, \dots)$ . The total number of agents with good  $j$  who go to the market is  $\sum_i (1 - \delta_{ij}) p_{ij}$  and the total number of agents who go to the market is  $N = \sum_j \sum_i [(1 - \delta_{ij}) p_{ij}]$ . Given  $s$ , the distribution  $\mathbf{p}$  evolves according to some law of motion  $\mathbf{p}' = f(\mathbf{p}; s)$ . A steady state is a solution to  $\mathbf{p} = f(\mathbf{p}; s)$ .

Given all this, it is straightforward to write the flow value functions in the following form  $rV_{ij} = (u + V_{ii} - V_{ij})(\delta_{ij} + \sum_{k \neq j} \delta_{ik} \varepsilon_{kj}) + \sum_{k \neq j} (V_{ik} - V_{ij}) \delta_{ih} \varepsilon_{kj} s_{jk}^i$   $h \neq j, k$ ,

where  $r > 0$  is the rate of time preference and  $\varepsilon_{hj} = \frac{1}{N} \sum_i p_{ih} (\delta_{ij} + \delta_{ik} s_{hj}^i)$ ,  $k \neq h, j$ , is the probability of meeting in the market an agent who holds good  $h$  and is willing to accept good  $j$  in exchange. From this definition of the payoffs functions, the following incentive compatibility conditions can be derived.

$$\begin{aligned} s_{12}^1 = 1 & \text{ iff } V_{11} - V_{12} = [\delta_{11}(1 - \varepsilon_{12}) + \delta_{12}(\varepsilon_{21} - 1) + \delta_{13}(\varepsilon_{31} - \varepsilon_{32})] (r + \delta_{13} + \delta_{11}\varepsilon_{13} + \delta_{12}\varepsilon_{23} + \delta_{12}\varepsilon_{13}s_{31}^1 + \delta_{11}\varepsilon_{23}s_{32}^1) + \\ & [\delta_{11}(1 - \varepsilon_{13}) + \delta_{12}(\varepsilon_{21} - \varepsilon_{23}) + \delta_{13}(\varepsilon_{31} - 1)] \delta_{11}\varepsilon_{32}s_{23}^1 + [\delta_{11}(\varepsilon_{13} - \varepsilon_{12}) + \delta_{12}(\varepsilon_{23} - 1) + \delta_{13}(1 - \varepsilon_{32})] \delta_{12}\varepsilon_{31}s_{13}^1 > 0 \\ s_{12}^1 = 0 & \text{ iff } V_{11} - V_{12} \leq 0 \end{aligned} \quad (\text{IC})$$

Similarly for  $s_{23}^1$  and  $s_{31}^1$  and also for agents of type 2 and 3.

We are now ready to define the equilibrium of the model.

DEFINITION. A steady state, pure-strategy equilibrium is a vector  $s$  of strategies  $s = (\dots s^i \dots)$ ,  $i \in \{1, 2, 3\}$  and a steady state distribution  $p$  such that: a) given  $p$ , the incentive compatibility conditions (IC) hold; and b) given  $s$ ,  $p = f(p; s)$ .

The following result will be helpful to solve some versions of the model:

LEMMA. If  $\delta_{ij} = 1$ , then in equilibrium  $s_{jh}^i = 0$  and  $s_{hj}^i = 1$ ,  $h \neq j$ ,  $h, j \in \{1, 2, 3\}$ .

Proof. It is immediate from the examination of the incentive compatibility conditions (IC).

### 3. Results

The procedure to solve the model (both numerically and analytically) and present some results involves the following steps:

1. Define an *economy*  $(\delta, \sigma)$
2. Make hypothesis about the *equilibrium* strategies  $(s)$
3. Compute  $p$ 's and  $\varepsilon$ 's (the Appendix includes the system of nonlinear equations which characterizes the steady state distribution  $\boldsymbol{p}$ )
4. Check that the incentive compatibility conditions (IC) hold.

Despite the simplicity of the setting up of the model, its analysis is not straightforward at least for two reasons: first, the number of potential equilibria is very large; and second, the computation of the inventory distribution involves solving a system of nonlinear equations which does not allow to get general analytical results. This will only be possible for particular cases of the model. Nevertheless, the model can always be solved numerically.

The following lines present the results for some simple economies, comparing them with previous literature.

Let us define *Economy 1* as  $\delta_1=(0,0,1)$ ,  $\delta_2=(1,0,0)$ ,  $\delta_3=(0,1,0)$ , and  $\sigma=\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ . This is the case of agents uniformly specialized in production and consumption without any double coincidence of wants analyzed in KW. We can prove the following:<sup>2</sup>

*Proposition 1. In Economy 1 there does not exist any pure strategy equilibrium.*

It should not come as a surprise that the pure strategy equilibria of the KW model, which crucially depend on storage costs, disappear once we abstract from those. Intuitively, without storage costs, all goods in this economy are perfectly symmetric. This means that the only possible equilibria ought to keep this same symmetric structure (see below). Despite this non-existence result, the model is still capable of supporting equilibria in which commodity money is accepted in a very parsimonious way.

To see this, let us now define *Economy 2* as  $\delta_1=(0,0,1)$ ,  $\delta_2=(1,0,0)$ ,  $\delta_3=(0,1,0)$ , and  $\sigma=(\sigma_1,\sigma_2,\sigma_3)$ . This is identical to *Economy 1*, but for general distributions of production types. This is the case analyzed in Wright (1995). The following result holds.

*Proposition 2. In Economy 2 there are three pure strategy equilibria. If  $\sigma_1>1/2$ , then  $s^1=(1,1,0)$ ,  $s^2=(0,0,1)$ , and  $s^3=(1,0,0)$  is an equilibrium. (Relabeling the goods accordingly we obtain the other two equilibria).*

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<sup>2</sup> The proofs involve mainly very simple algebra, the details of which are relegated to the Appendix.

If  $\sigma_1 > 1/2$ , then good 2 emerges as the only commodity money. Since all goods are identical, to have one of them acting as the unique commodity money, we need some kind of asymmetry in the economy that characterizes this particular good and makes it different from the rest. The only source of such an asymmetry in *Economy 2* is the distribution of production types. In this particular case, producers of good 1 (who are also consumers of good 3) are relatively numerous. This means that the best strategy for agents who produce good 3 is to stick with it. Since they are consumers of good 2, agents of type 1 will find it useful to accept and hold good 2 in order to trade it for their consumption good. This means that agents who produce good 2 (and consume 1) will be able to get their consumption good by holding their production good (in fact, they are perfectly indifferent between holding goods 2 and 3, so they will never accept good 3). Summing up, good 2 is the only good that is accepted as commodity money by agents of type 1. As it has already been noted, in this economy in which storage costs are zero, all goods have identical properties. This means that any existing equilibrium has to be accompanied by other two, which are mere products of renaming the goods of the economy.

Figure 1 represents the space of the parameters for which the three different equilibria exist. Note that there is an area of the parameter space for which there does not exist any pure strategy equilibrium. Our hypothesis is that there exist mixed strategy equilibria for these values of the parameters. For instance, for the particular case  $\sigma = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$  (this is *Economy 1*), it can be shown that  $s_{12}^1 = s_{23}^2 = s_{31}^3 = \frac{2}{3}$  is a mixed strategy equilibrium.

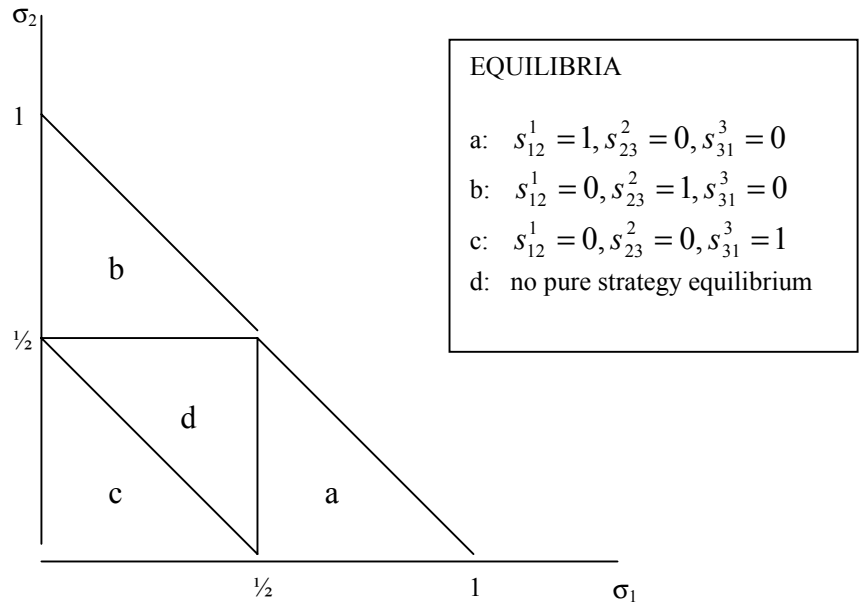


Figure 1. Equilibria with  $\delta_1=(0,0,1)$ ,  $\delta_2=(1,0,0)$ ,  $\delta_3=(0,1,0)$

Let us define *Economy 3* as  $\delta_1 = \delta_2 = \delta_3 = (\delta_1^*, \delta_2^*, \delta_3^*)$  and  $\sigma=(\sigma_1, \sigma_2, \sigma_3)$ . This is the economy analyzed in Cuadras-Morató and Wright (1997).

*Proposition 3.* [Proposition 1 in Cuadras-Morató and Wright (1997)]. All agents in *Economy 3* use identical trading strategies,  $s^1=s^2=s^3=(s_{12},s_{23},s_{31})$ .

Homogeneous preferences for goods mean that this is equivalent to a representative agent economy in which everybody uses identical trading strategies. Agents are heterogeneous in this economy because they produce different goods. Nevertheless, since they draw their taste shocks from the same distribution, this type of heterogeneity does not affect their trading strategies at all.

*Economy 4* is defined as  $\delta_1=(0, \delta_{12}, 1-\delta_{12})$ ,  $\delta_2=(0, \delta_{22}, 1-\delta_{22})$ , and  $\delta_3=(0, \delta_{32}, 1-\delta_{32})$ . This is an economy characterized by the fact that there is a good (good 1 in this particular case) that nobody ever consumes.

*Proposition 4. In Economy 4, equilibria are always of the form  $s^i=(1, s_{23}^i, 0)$ .*

Proposition 4 says that all equilibria in Economy 4 are such that good 1 is never accepted in exchange for another good. It is worth going into the details of this. In KW, a good that nobody ever consumes, fiat money, is proved to be acceptable as medium of exchange under some circumstances. Here good 1, a good nobody consumes, is never going to be accepted as media of exchange. The reason is that good 1 is not the same as fiat money in KW because is produced by agents of type 1. Suppose, for instance, that good 1 was accepted by some agent (obviously not to consume it). Agents of type 1 would get goods they could eventually consume. After consuming them, they would produce good 1, but then goods of type 1 would become so abundant that, in the end, would make no sense to accept them in exchange for something that can eventually be consumed. Note that this result generalizes some of the results in Proposition 3 in Cuadras-Morató and Wright (1997).<sup>3</sup>

Notice that, so far, we have not proved that an agent may prefer a good she can never consume (obviously only to use it as money) to another good that she may

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<sup>3</sup> Although the result is similar, the logic underlying it is somehow different in Cuadras-Morató and Wright (1997). In that model, everybody has identical consumption preferences. Suppose that there is some good that everybody wants to consume with very high probability and suppose someone accepted the good that is never consumed (good 1) as commodity money because she thinks that this will make it easier to get the most consumed good. Notice though, that the latter is at least as acceptable by everyone as good 1 and, besides, can be consumed. This means that the holder of that good will not be willing to accept good 1 and, so, nobody else will.

eventually consume. This is partly due to the particular assumptions about consumption preferences in the economies we have studied above. For instance, in Economies 1 and 2 agents are completely specialized in consumption. This means that accepting the own consumption good is always the best strategy. There might be commodity money, but it would never be preferred to the only good one wants to consume. One could imagine, however, economies in which agents, who do not consume at all a particular good, still accept it as commodity money and rank it above goods they eventually consume. This can be seen as the next proposition.

Let us first define *Economy 5* as  $\delta_1=(0,\delta,1-\delta)$ ,  $\delta_2=(0,\delta,1-\delta)$ ,  $\delta_3=(1,0,0)$ , and  $\sigma=(\sigma,\sigma,1-2\sigma)$  ( $\sigma<1/2$ ). The following proposition holds.

*Proposition 5.* *If  $\delta < \frac{1}{2}$  and  $\sigma < \frac{1-2\delta}{2-\delta}$ , then  $s^1=(0,1,0)$ ,  $s^2=(0,1,0)$ , and*

*$s^3=(0,1,1)$  is an equilibrium.*

Notice that in this particular equilibrium, good 1, which agents of type 1 and 2 never consume, will be preferred by both types to good 2, which they consume with positive (but not too big) probability. This is because type 3 agents, who are relatively numerous, have good 3 (they produce it) and are willing to consume good 1. What makes this equilibrium interesting is the fact that a relatively large number of agents willing to consume a good may induce other agents, for whom this good does not have any consumption value, to prefer it and use it as commodity money, accepting it in exchange for goods that they eventually consume.



## 4. Summary and conclusions

We have presented a general equilibrium model that further generalizes the consumer preference structure of the existing random matching models of commodity money. We analyze the equilibrium conditions and focus on what commodities appear as money. The results depend on the size of the fraction of producers of each good and the probability with which different agents want to consume each good. Previous models that address similar questions are shown to be particular parameterizations of this one.

## Appendix

### Steady-state distribution of inventories

In order to find the steady state distribution of inventories, the following system of equations ought to be solved (for  $i,j=1,2,3$ ):

$$p_{ii} \sum_{h \neq i} \delta_{ih} \varepsilon_{ki} s_{ik}^i = \sum_{h \neq i} p_{ih} [\delta_{ih} + \delta_{ii} \varepsilon_{ih} + \delta_{ik} (\varepsilon_{kj} + \varepsilon_{ij} s_{ji}^i)] \quad k \neq h, i \quad (A1)$$

$$p_{ij} [\delta_{ij} + \delta_{ii} (\varepsilon_{ij} + \varepsilon_{kj} s_{jk}^i) + \delta_{ik} (\varepsilon_{kj} + \varepsilon_{ij} s_{ji}^i)] = \sum_{h \neq j} p_{ih} \delta_{im} \varepsilon_{jh} s_{hj}^i \quad k \neq j, i \quad m \neq h \quad (A2)$$

$$\sigma_i = \sum_j p_{ij} \quad (A3)$$

## Proofs of results

Proofs of the results of the paper imply very simple and repetitive algebra. In what follows we sketch the proofs of the propositions, but do not go over all the details of them.

### *Proof of Proposition 1*

By Lemma, the equilibria in *Economy 1* will be of the form  $s^1 = (s_{12}^1, 1, 0)$ ,  $s^2 = (0, s_{23}^2, 1)$  and  $s^3 = (1, 0, s_{31}^3)$ . This means that there are eight candidate strategy sets to be equilibrium. We need to check whether the incentive compatibility conditions (IC) hold for each of them.

We first conjecture the following equilibrium strategy set:  $s^1 = (1, 1, 0)$ ,  $s^2 = (0, 0, 1)$ ,  $s^3 = (1, 0, 0)$ . From (A1)-(A3),  $p_{31}=p_{32}=p_{21}=p_{23}=p_{13}=0$ ,  $p_{33}=p_{22}=1/3$ , and  $p_{11}=p_{12}=1/6$ . Consequently,  $\epsilon_{31}=0$ ,  $\epsilon_{21}=\epsilon_{32}=1/3$ , and  $\epsilon_{12}=\epsilon_{13}=\epsilon_{23}=1/6$ . From (IC), it is easy to check that  $V_{31} > V_{33}$ , so  $s_{31}^3 = 0$  cannot be part of the equilibrium. Hence, the above strategy set is not equilibrium. By symmetry,  $s^1 = (0, 1, 0)$ ,  $s^2 = (0, 1, 1)$ ,  $s^3 = (1, 0, 0)$  and  $s^1 = (0, 1, 0)$ ,  $s^2 = (0, 0, 1)$ ,  $s^3 = (1, 0, 1)$  are not equilibrium either. Similar arguments apply to the rest of candidate equilibria.

### *Proof of Proposition 2*

By Lemma, we have again the same candidate equilibria as in Proposition 1. We proceed in an identical manner. We conjecture the following equilibrium strategy set:

$s^1 = (1,1,0), s^2 = (0,0,1), s^3 = (1,0,0)$ . From (A1)-(A3),  $p_{31}=p_{32}=p_{21}=p_{23}=0$ ,  $p_{33}=\sigma_3$ ,  $p_{22}=\sigma_2$ ,  $p_{11}=\frac{\sigma_1\sigma_3}{\sigma_2+\sigma_3}$ , and  $p_{12}=\frac{\sigma_1\sigma_2}{\sigma_2+\sigma_3}$ . Consequently,  $\varepsilon_{31}=0$ ,  $\varepsilon_{21}=\sigma_2$ ,  $\varepsilon_{32}=\sigma_3$ , and  $\varepsilon_{12}=\varepsilon_{13}=\frac{\sigma_1\sigma_3}{\sigma_2+\sigma_3}$ , and  $\varepsilon_{23}=\frac{\sigma_1\sigma_2}{\sigma_2+\sigma_3}$ . It is easy to check that  $V_{12}>V_{11}$  and  $V_{22}=V_{23}$ , so  $s_{12}^1 = 1$  and  $s_{23}^2 = 0$  are optimal strategies. Also,  $V_{33}>V_{31}$  iff  $\sigma_1>1/2$ . Applying a symmetry argument, we have that  $s^1 = (0,1,0), s^2 = (0,1,1), s^3 = (1,0,0)$  is equilibrium iff  $\sigma_2>1/2$  and  $s^1 = (0,1,0), s^2 = (0,0,1), s^3 = (1,0,1)$  iff  $\sigma_3>1/2$ . Following the same steps is easy to check that the incentive compatibility conditions (IC) do not hold for the rest of the candidate equilibria.

### *Proof of Proposition 3*

We remit the readers to Proposition 1 in Cuadras-Morató and Wright (1997).

### *Proof of Proposition 4*

The logic underlying this result would be the following. Suppose that there was an equilibrium in which, for some  $i$ ,  $s_{12}^i \neq 1$  and  $s_{31}^i \neq 0$ . In such equilibrium, good 1 would be used as medium of exchange, but it would be never consumed. Every time producers of good 1 got to consume some good, however, they would produce afterwards a unit of good 1. This means that the amount of agents carrying good 1 would grow. Eventually good 1 would be ubiquitous and, hence, valueless (there will be too much of it) and agents would refuse to accept in exchange for a good they may want to consume eventually. By backward induction, good 1 is unacceptable, so  $s_{12}^i = 1$  and  $s_{31}^i = 0$ .

*Proof of Proposition 5*

Given the strategies that we hypothesize, we have that  $\varepsilon_{23} = \varepsilon_{21} = \frac{p_{22}(1-\delta)}{N}$ ,

$$\varepsilon_{32} = \frac{(p_{13} + p_{23})\delta}{N}, \quad \varepsilon_{31} = \frac{p_{33}}{N} = \frac{1-2\sigma}{N}, \quad \varepsilon_{12} = \frac{(p_{11} + p_{21})\delta}{N}, \quad \text{and} \quad \varepsilon_{13} = \frac{p_{11} + p_{21}}{N}.$$

By Lemma,  $s_{12}^3 = 0$  and  $s_{31}^3 = 1$ . Also, from the examination of the incentive compatibility conditions (IC), it is immediate to conclude that  $s_{23}^3 = 1$ , and  $s_{31}^1 = s_{31}^2 = 0$ . Moreover,  $s_{23}^1 = s_{23}^2 = 1$  if the following inequality holds:

$$[\delta(1-\varepsilon_{23}) + (1-\delta)(\varepsilon_{32} - 1)](r + \delta\varepsilon_{21} + \varepsilon_{31}) + [\delta(\varepsilon_{21} - \varepsilon_{23}) + (1-\delta)(\varepsilon_{31} - 1)](1-\delta)\varepsilon_{12} < 0$$

Since now  $\varepsilon_{23} = \varepsilon_{21}$ , it will be sufficient to prove that  $\delta(1-\varepsilon_{23}) + (1-\delta)(\varepsilon_{32} - 1) < 0$ .

Substituting for the values of  $\varepsilon$ 's and after some straightforward algebra, it is easy to

prove that  $\delta < \frac{1}{2}$  and  $\sigma < \frac{1-2\delta}{2(1-\delta)^2}$  are sufficient conditions that guarantee that the

inequality holds. Also,  $s_{12}^1 = s_{12}^2 = 0$  if the following inequality holds:

$$[\delta(\varepsilon_{21} - 1) + (1-\delta)(\varepsilon_{31} - \varepsilon_{32})](r + 1 - \delta + \delta\varepsilon_{23}) + [\delta(\varepsilon_{23} - 1) + (1-\delta)(1 - \varepsilon_{32})]\delta\varepsilon_{31} > 0$$

Notice that we have already provided sufficient conditions for the second part of the inequality to be positive. We need to evaluate the sign of  $\delta(\varepsilon_{21} - 1) + (1-\delta)(\varepsilon_{31} - \varepsilon_{32})$ .

Substituting for the values of  $\varepsilon$ 's, and after some algebra, we prove that  $\sigma < \frac{1-2\delta}{2-\delta}$  is a

sufficient condition that guarantees that the inequality holds.

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