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Giffen Goods and Market Making

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Giffen Goods and Market Making ⋆

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Summary. This paper shows that information effects *per se* are not responsible for the Giffen goods anomaly affecting traders' demands in multi asset noisy, rational expectations equilibrium markets. The *role* that information plays in traders' strategies also matters. In a market with risk averse, uninformed traders, informed agents have a dual trading motive: *speculation* and *market making*. The former entails using prices to assess the effect of error terms; the latter requires employing them to disentangle noise traders' demands within aggregate orders. In a correlated environment this complicates the signal extraction problem and may generate upward sloping demand curves. Assuming *(i)* that competitive, risk neutral market makers price the assets or that *(ii)* uninformed traders' risk tolerance coefficient grows unboundedly, removes the market making component from informed traders' demands rendering them well behaved in prices.

Keywords and Phrases: Giffen Goods, Financial Economics, Asset Pricing, Information and Market Efficiency.

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1 Introduction

In a well known paper [1] showed that multi asset noisy rational expectations equilibrium (NREE) markets display a number of *anomalies*. In particular, owing to correlation effects, traders' demand functions could be upward sloping in prices. This "Giffen good" anomaly was attributed to the contemporaneous workings of an *information* and a *substitution* effect generated by prices in an economy with asymmetric information. Indeed, a price increase in a NREE could either signal an increase in the value of the asset pay-off or be the effect of a demand pressure from noise traders. For some parameter configurations, traders could then interpret such a price increment as *good news* about the asset's fundamental and increase their (long) position in the asset. Recently, Giffen goods anomalies have been related to market behavior around crashes (see [8] and [2]) and to *un*stable equilibria (see [4]). Upward sloping demand curves make traders shy away from assets whose price plummets and increase their long position in assets whose price rockets, eventually amplifying market movements. In this perspective, understanding the extent to which information effects *per se* determine such anomalies is therefore relevant.

This paper shows that information effects *alone* are not responsible for Giffen goods anomalies: the *role* that information plays in traders' strategies also matters. Intuitively, owing to their superior knowledge, privately informed traders should be able to better disentangle noise from information and this should lead them to choose their positions by comparing prices with their private signals. On the contrary, traders that only observe (endogenous) public information (i.e. equilibrium prices) should rely on correlation effects in order to disentangle the informative content of a price movement. Building on this insight, I show that in a market with risk averse uninformed traders, informed agents have a dual motive for trading: *speculation* and *market making*. They *speculate* on the difference between their private signals and equilibrium prices; they *accommodate traders' total demand* in each asset by comparing (common) prior information to equilibrium prices. While speculation entails assessing the effect of private signal biases, market making requires disentangling noise traders' effects from fundamental information within the observed aggregate orders. This complicates the signal extraction problem and (may) generate upward sloping demand curves. I therefore attribute Admati's "Giffen" good anomaly to the market making component of informed traders' demands.

Based on this intuition, I then give sufficient conditions under which the Giffen goods phenomenon disappears from informed traders' strategies. Intuitively, this occurs whenever informed traders find it unprofitable to accommodate liquidity shocks. Thus, either assuming that competitive, risk neutral market makers price the assets or letting the risk tolerance parameter of uninformed traders grow unboundedly, allows to remove the anomaly from the demand of informed agents but not from that of uninformed agents.

The paper is organized as follows: in the next section, I outline the model's assumptions, define notation and recall the equilibrium result of [1]. I then show by means of examples that the market making component of an informed trader's demand is responsible for the Giffen good anomaly. In section 3, I introduce risk neutral competitive market makers in the model and show that this removes the anomaly from informed demands. In section 4, I generalize the model considering a market where informed and uninformed traders interact. This allows to show that the result of section 3 can be obtained as a limit result when uninformed traders' risk tolerance grows unboundedly.

2 The Model

Consider a market where two classes of agents exchange a vector of $K > 1$ risky assets with random liquidation value $\boldsymbol{v} \sim N(\bar{\boldsymbol{v}}, \boldsymbol{\Pi}_{\boldsymbol{v}}^{-1})$ and a risk less one with return given by $R \geq 1$: a continuum of risk-averse informed traders distributed in the interval $[0, 1]$ and noise traders, trading for liquidity purposes. Prior to trading, each informed agent i receives a K-dimensional vector of private signals $\boldsymbol{s}_i = \boldsymbol{v} + \boldsymbol{\epsilon}_i$ where $\boldsymbol{\epsilon}_i \sim N(\boldsymbol{0}, \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}^{-1})$, $\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}^{-1} \neq \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_j}^{-1}$, and $\boldsymbol{\epsilon}_i$, $\boldsymbol{\epsilon}_j$ are independent for $i \neq j$. Assume that his preferences are represented by a CARA utility $U(W_{1i}) = -\exp\{-W_{1i}/\gamma_i\}$, where $\gamma_i > 0$ is agent i's coefficient of constant absolute risk tolerance, $W_{1i} = W_{oi}R + \boldsymbol{x}_i'(\boldsymbol{v} - R\boldsymbol{p})$ indicates his final wealth that comes from buying $\boldsymbol{x}_i' = (x_{i1}, x_{i2}, \ldots, x_{iK})$ units of each asset at price $\boldsymbol{p}$, and $W_{oi} \geq 0$ denotes his initial wealth. Let noise traders submit a K-dimensional vector of (price-inelastic) random demands $\boldsymbol{u} \sim N(\bar{\boldsymbol{u}}, \boldsymbol{\Pi}_{\boldsymbol{u}}^{-1})$. Assume that the random vectors $\boldsymbol{v}, \boldsymbol{u}$, and $\boldsymbol{\epsilon}_i$ are mutually independent $\forall i$ and that the Strong Law of Large Numbers holds (i.e. $\int_0^1 \boldsymbol{\epsilon}_i di = \boldsymbol{0}$, almost surely). Finally, let each of $\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1}, \boldsymbol{\Pi}_{\boldsymbol{u}}^{-1}$, and $\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}^{-1}$ be positive definite and suppose that the distributional assumptions are common knowledge among the agents in the economy.

2.1 The Equilibrium

Suppose that in the above market each trader submits a vector of demand functions indicating the position desired in each asset at every price, contingent on his private information. Owing to market clearing, the resulting equilibrium price vector will then reflect all traders' information. This, in turn, will provide each agent with an additional signal *beyond* the one he privately observes, that he can exploit in forming his optimal demand. Therefore, in a rational expectations equilibrium, prices perform two functions: they *clear* all markets and they *convey information* to traders. In turn, traders' beliefs are *endogenous* and their demand functions are defined *only* for equilibrium prices. [1] The following definition formally describes the rational expectations equilibrium concept for the above market:

Definition 1 *A rational expectations equilibrium for the above market is a price vector $\boldsymbol{p}$ and demand functions $\{\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p})\}_{i\in[0,1]}$ such that (i) $\boldsymbol{p}$ is $(\boldsymbol{v}, \boldsymbol{u})$ measurable; (ii) $\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) \in \arg\max_{\boldsymbol{x}_i} E[U(W_{1i})|\boldsymbol{s}_i, \boldsymbol{p}]$; and (iii) $\int_0^1 \mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p})di + \boldsymbol{u} = 0$, almost surely.*

[1] For noisy rational expectations equilibrium models with a single risky asset see [11], [7] and [9].

The first condition requires prices *not* to depend on single signals' realizations. Indeed, in the large market as each informed agent is small and private signals are independently distributed, equilibrium prices should only vary *either* because of changes in the value of the pay-off vector *or* because of noise traders' demand realizations. The second condition requires traders to choose optimal equilibrium demand functions *given* the equilibrium price *and* their private information. Finally, the last condition requires the price vector to clear all the markets.

To apply definition 1 to the current context, assume each informed trader i submits a vector of demand functions $\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p})$ and restrict attention to *linear* equilibria where, thus, the price is a linear function of informed traders' aggregate signals and noise traders' demands. Owing to CARA utility and the normality assumption, an informed agent's equilibrium demand is linear in his private signal $\boldsymbol{s}_i$ and in the equilibrium price vector $\boldsymbol{p}$: $\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \mathbf{A}_i\boldsymbol{s}_i + \phi_i(\boldsymbol{p})$, where $\mathbf{A}_i$ and $\phi_i(\boldsymbol{p})$ denote respectively the matrix of trading aggressiveness and a linear function of the price to be determined in equilibrium. The market clearing equation thus reads as $\int_0^1 \mathbf{A}_i\boldsymbol{s}_i + \phi_i(\boldsymbol{p})di + \boldsymbol{u} = \mathbf{0}$, and the following result holds:

Proposition 1 *In the market outlined above there exists a unique linear equilibrium. Agents' strategies are given by*

$$\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \tag{1}$$
$$\mathbf{A}_i(\boldsymbol{s}_i - R\boldsymbol{p}) + (\gamma_i/\bar{\gamma})\left(\boldsymbol{\Lambda}^{-1} - \bar{\mathbf{A}}\right)(\bar{\boldsymbol{v}} - R\boldsymbol{p}) - \gamma_i\left(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\right)^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}},$$

and the vector of equilibrium prices is given by

$$\boldsymbol{p} = (1/R)\boldsymbol{\Lambda}\boldsymbol{z} + (1/R)(I - \boldsymbol{\Lambda}\bar{\mathbf{A}})\bar{\boldsymbol{v}} - (\bar{\gamma}/R)(\bar{\gamma}\boldsymbol{\Pi} + \bar{\mathbf{A}})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}},$$

where $\mathbf{A}_i = \gamma_i\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$, $\bar{\gamma} = \int_0^1 \gamma_i di$, $\bar{\mathbf{A}} = \int_0^1 \gamma_i\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}di$, $\boldsymbol{z} = \bar{\mathbf{A}}\boldsymbol{v} + \boldsymbol{u}$, $\boldsymbol{\Lambda} = (\bar{\gamma}\boldsymbol{\Pi} + \bar{\mathbf{A}})^{-1}(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})$, *and* $\boldsymbol{\Pi} = (\mathrm{Var}[\boldsymbol{v}|\boldsymbol{z}])^{-1} = \boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}}$.

Proof See the appendix.

The vector $\boldsymbol{z}$ denotes the intercept of the total net demand due to traders' private information and noise traders' supply shocks. Insofar as it conveys a signal about the "true" value of the asset payoffs, it captures the "informational content" of the order flows. The matrix $\boldsymbol{\Lambda}^{-1}$ maps equilibrium prices into the traders' total net demand: for $R = 1$ and for a unitary price vector $\boldsymbol{p}' = (1, 1, \ldots, 1)$, $\boldsymbol{\Lambda}^{-1}$ measures the size of the traders' aggregate demand intercept in each asset that is either due to private information or to a liquidity shock.

According to (1) an agent's demand function has two components. The first component ($\mathbf{A}_i(\boldsymbol{s}_i - R\boldsymbol{p})$) reflects the agent's "speculative" position based on private information. The second component ($(\gamma_i/\bar{\gamma})(\boldsymbol{\Lambda}^{-1} - \bar{\mathbf{A}})(\bar{\boldsymbol{v}} - R\boldsymbol{p}) + \gamma_i(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}}$) denotes his position in (potentially) accommodating both the *expected* ($\gamma_i(I+\bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}}$), and the *unexpected* ($(\gamma_i/\bar{\gamma})(\boldsymbol{\Lambda}^{-1}-\bar{\mathbf{A}})(\bar{\boldsymbol{v}}-R\boldsymbol{p})$) net demand in each asset k.

Traders' speculative aggressiveness is given by the conditional precision matrix of their private signals weighted by their risk tolerance coefficient: $\mathbf{A}_i = \gamma_i\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$. As $\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$ is positive definite, $\gamma_i > 0$, and $R \geq 1$ the speculative component of a trader's demand in an asset k is *decreasing* in its own price for every asset k. The aggressiveness of their unexpected "market making" component is captured by the difference between traders' total net demand and informed agents' speculative aggressiveness in each asset for a unitary price vector (and for $R = 1$), weighted by their relative risk-tolerance vis-à-vis the whole market. This matrix has no particular structure and thus nothing can be said a priori about the sign of its diagonal elements. Indeed, given R, $\boldsymbol{p}$ differs from $\bar{\boldsymbol{v}}$ either because of noise traders' liquidity shocks, or because of informed traders' demands; thus, an informed agent attempts to establish whether the order he faces is due to the former or to the latter. If $(\boldsymbol{\Lambda}^{-1})_{kk} - \bar{\mathbf{A}}_{kk} > 0$, then he attributes it to a supply shock and thus accommodates it. [2] This corresponds to the "normal goods" case of consumer theory in which the cheaper is an asset, the more of it a trader wants to buy. If, however, $(\boldsymbol{\Lambda}^{-1})_{kk} - \bar{\mathbf{A}}_{kk} \leq 0$, then the trader attributes the total net demand he faces to informed trading, refrains from taking the other side of the trade and a Giffen good may arise.

Notice that Giffen goods in the present context have a different interpretation from the one they have in consumer theory. Indeed, in the latter setting prices are *exogenous* to traders' demands whereas in the former prices are *endogenous* equilibrium prices and demand functions are *equilibrium* demands. Furthermore, a Giffen good in consumer theory is due to the presence of a strong income effect that offsets the substitution effect and leads to an increase (decrease) in the trader's demand when the good's price increases (decreases). However, in the current setting, owing to the assumed exponential utility function and the presence of a risk-less asset, income effects do not exist. As the following examples show, an asset here can be a Giffen good as a result of the information extraction problem that informed traders face when forming the market making component of their demand functions.

Example 1 Suppose $K = 2$ and indicate with τ_{x_k} and ρ_x, respectively the precision of the random variable x_k and the correlation coefficient of the random variables x_1, x_2. Suppose that $R = 1$, $\bar{\boldsymbol{u}} = \mathbf{0}$, $\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i} = \boldsymbol{\Pi}_{\boldsymbol{\epsilon}}$, and $\gamma_i = \gamma$, for every agent $i \in [0,1]$. Then, if $\rho_v = \rho_\epsilon = \rho_u = 0$,

$$\mathbf{A}_i = \mathbf{A} = \gamma \begin{pmatrix} \tau_{\epsilon_1} & 0 \\ 0 & \tau_{\epsilon_2} \end{pmatrix}, \qquad \boldsymbol{\Lambda} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},$$

where $\lambda_k = (1+\gamma a_k \tau_{u_k})/(a_k+\gamma\tau_k)$, $a_k = \mathbf{A}_{kk} = \gamma\tau_{\epsilon_k}$, and $\tau_k = (\mathrm{Var}[v_k|\boldsymbol{z}])^{-1} = \tau_{v_k} + a_k^2\tau_{u_k}$ indicate, respectively, the reciprocal of market depth, the trader private signal aggressiveness, and the public precision associated with market $k = 1, 2$. Hence, a trader i's strategy in asset k is given by $X_{ik}(\boldsymbol{s}_i, \boldsymbol{p}) = a_k(s_{ik} - p_k) + (1+\gamma\tau_{u_k} a_k)^{-1}\gamma\tau_{v_k}(\bar{v}_k - p_k)$. As explained above, informed traders have two trading motives: they speculate on private information, and they absorb the

[2] To be sure: $\boldsymbol{\Lambda}^{-1}$ and $\bar{\mathbf{A}}$ respectively measure the size of total traders' demand intercept and the average speculative component of informed trader's demand for a unitary price vector ($\boldsymbol{p}' = (1, 1, \ldots, 1)$) and risk less asset return ($R = 1$). Therefore if $(\boldsymbol{\Lambda}^{-1})_{kk} - \bar{\mathbf{A}}_{kk} > 0$, $(\boldsymbol{\Lambda}^{-1})_{kk} - \bar{\mathbf{A}}_{kk}$ captures the part of the total net demand for asset k that for a unitary price vector, in the trader's opinion, is *not* due to informed agents' superior information about asset k.

supply shock taking the counterpart of each limit order book and clearing markets (i.e. buying when the price declines and selling when it increases w.r.t. its expected value). While speculation is due to private information, "market making" is the result of the price discount (premium) informed traders receive on each transaction because of risk aversion. To see this, rewrite prices and strategies as follows:

$$p_k = E\left[v_k|\boldsymbol{z}\right] - \frac{\tau_{v_k}}{a_k\tau_{u_k}(a_k+\gamma\tau_k)}\left(\bar{v}_k - E\left[v_k|\boldsymbol{z}\right]\right),$$
$$X_{ik}(\boldsymbol{s}_i,\boldsymbol{p}) = a_k(s_{ik}-p_k) + \frac{\gamma\tau_k\tau_{v_k}}{a_k\tau_{u_k}(a_k+\gamma\tau_k)}\left(\bar{v}_k - E\left[v_k|\boldsymbol{z}\right]\right).$$

Whenever the traders in the market for asset k believe that on average asset k's value is lower than its unconditional expectation (i.e. $\bar{v}_k > E[v_k|\boldsymbol{z}]$) an informed trader buys the asset at a discount (i.e. $p_k - E[v_k|\boldsymbol{z}]$) to be compensated for the risk that $v_k < E[v_k|\boldsymbol{z}]$. The opposite occurs when traders on average believe the asset value to be higher than its unconditional expected value: in this case a trader sells the asset at a premium. Clearly, $a_k = \gamma\tau_{\epsilon_k} > 0$ and $(\tau_{v_k}/(1+\gamma\tau_{u_k}a_k)) > 0$. Thus, no Giffen good appears in this case.

Example 2 Keeping the assumptions of the previous example, suppose now, as in [1], that $\mathbf{A} = I$ and

$$\boldsymbol{\Pi}_{\boldsymbol{v}}^{-1} = \begin{pmatrix} 1 & 5 \\ 5 & 26 \end{pmatrix}, \qquad \boldsymbol{\Pi}_{\boldsymbol{u}}^{-1} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}.$$

Then

$$\boldsymbol{\Lambda} = \begin{pmatrix} -0.02 & 0.19 \\ -0.45 & 1.08 \end{pmatrix}, \qquad \boldsymbol{\Lambda}^{-1} - \mathbf{A} = \begin{pmatrix} 15.6 & -3 \\ 7 & -1.3 \end{pmatrix},$$

and a trader's strategies are given by: $X_{i1}(\boldsymbol{s}_i,\boldsymbol{p}) = s_{i1} - p_1 + 15.6(\bar{v}_1 - p_1) - 3(\bar{v}_2 - p_2)$, $X_{i2}(\boldsymbol{s}_i,\boldsymbol{p}) = s_{i2} - p_2 - 1.3(\bar{v}_2 - p_2) + 7(\bar{v}_1 - p_1)$. Notice that asset 2 is a Giffen good: an increase (decrease) in its price leads the trader to increase (decrease) his position in the asset. Furthermore, notice that the Giffen good "anomaly" is *entirely* due to the *market making* component of the trader's demand. In particular, whenever $\bar{v}_2 > p_2$, the trader is no longer willing to accommodate the supply shock (as in example 1). Rather, for any given value of the speculative component of his demand, in the presence of a price decrease he reduces his position in the asset. This is so because of the information he extracts from the *two* order flows. Indeed, suppose $\bar{v}_1 > p_1$ and $\bar{v}_2 > p_2$. Should the trader attribute this price realization to informed trading or to a supply shock? The positive correlation across pay-offs makes a contemporaneous value reduction in both assets likely. However, as the distribution of asset 1 is more concentrated than the one of asset 2, $\bar{v}_1 > p_1$ is probably due to a selling pressure from noise traders; on the contrary $\bar{v}_2 > p_2$ may be the result of "bad news." Such inference is reinforced by the higher dispersion of noise traders' demand in asset 1 (w.r.t. asset 2) and by the fact that noise traders' demands are positively correlated. Hence, informed traders align their behavior to the rest of the market in asset 2 and "lean against the wind" in asset 1.

Expressing the equilibrium price and traders' demands as done in example 1, sheds further light on the agents' demand market making component. Indeed, rearranging the equilibrium price vector in proposition 1 gives:

$$\boldsymbol{p} = (1/R)\left(E[\boldsymbol{v}|\boldsymbol{z}] - \left(\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}(\bar{\mathbf{A}} + \bar{\gamma}\mathbf{\Pi})\right)^{-1}\mathbf{\Pi}_{\boldsymbol{v}}((\bar{\boldsymbol{v}} - E[\boldsymbol{v}|\boldsymbol{z}]) + \bar{\boldsymbol{u}})\right).$$

Hence,

$$\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \mathbf{A}_i(\boldsymbol{s}_i - R\boldsymbol{p}) + \gamma_i\mathbf{\Pi}\left(\left(\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}(\bar{\mathbf{A}} + \bar{\gamma}\mathbf{\Pi})\right)^{-1}(\mathbf{\Pi}_{\boldsymbol{v}}(\bar{\boldsymbol{v}} - E[\boldsymbol{v}|\boldsymbol{z}]) + \bar{\boldsymbol{u}})\right).$$

Using the above parameter values: $p_1 = E[v_1|\boldsymbol{z}] - 9.7(\bar{v}_1 - E[v_1|\boldsymbol{z}]) + 1.85(\bar{v}_2 - E[v_2|\boldsymbol{z}])$, $p_2 = E[v_2|\boldsymbol{z}] - 20.8(\bar{v}_1 - E[v_1|\boldsymbol{z}]) + 3.98(\bar{v}_2 - E[v_2|\boldsymbol{z}])$, and, $X_{i1}(\boldsymbol{s}_i, \boldsymbol{p}) = s_{i1} - p_1 + 105.3(\bar{v}_1 - E[v_1|\boldsymbol{z}]) - 20.14(\bar{v}_2 - E[v_2|\boldsymbol{z}])$, $X_{i2}(\boldsymbol{s}_i, \boldsymbol{p}) = s_{i2} - p_2 + 47.19(\bar{v}_1 - E[v_1|\boldsymbol{z}]) - 9.02(\bar{v}_2 - E[v_2|\boldsymbol{z}])$. Notice that differently from example 1, a trader is *not* willing to accommodate the total net demand in asset 2. Whenever the traders in the market for asset 2 believe that on average asset 2's value is lower than its unconditional expectation ($\bar{v}_2 > E[v_2|\boldsymbol{z}]$), an informed trader *sells* the asset at a *premium* (instead of *buying* it at a *discount*) to be compensated for the risk that $v_2 > E[v_2|\boldsymbol{z}]$. [3]

Summarizing, when all traders in the market are risk averse, the demand of an informed agent can be decomposed into a speculative and a market making component. Owing to correlation effects, the market making component may make informed agents willing to increase (decrease) their position in a given asset when its price increases (decreases). Thus, intuitively, if an informed agent were to find it unprofitable to accommodate the unexpected net demand, the market making component should disappear rendering his demand well behaved in prices. The next section shows that this intuition is indeed correct. [4]

3 The Market with Risk Neutral Market Makers

In this section I keep the same information structure of section 2 and introduce competitive *risk neutral* market makers as in [13] and [5]. Market makers can be seen as *uninformed* agents that aggregate all traders' orders and set a single market clearing price vector. As a result of risk neutrality, prices *do not* incorporate a risk premium and informed traders find it *unprofitable* to accommodate traders' total orders. Hence, they only trade to speculate on private information and their

[3] Strictly speaking, the trader *decumulates* his long position if $s_{i2} - p_2 > 0$ and *accumulates* it if the reverse occurs.

[4] It is important to emphasize, though, that such a decomposition is based on the trader's private information. As traders' information is diverse, what a trader thinks of being a non-information-driven trade may be perceived as information-driven by another trader (see [10] for a discussion of this issue in the context of a one-asset, dynamic, noisy rational expectations equilibrium model).

demand functions are well behaved. However, insofar as market makers clear all trades, they use the equilibrium price to disentangle noise from information and correlation effects can induce the Giffen phenomenon in their demand functions.

More formally, let each informed trader i submit a vector of demand functions $\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p})$, indicating the position desired in each asset k at every price vector $\boldsymbol{p}$, contingent on his private information. Denote with $W_{I1i} = W_{Ioi} + \boldsymbol{x}'_{Ii}(\boldsymbol{v} - R\boldsymbol{p})$, the informed trader final wealth, and with γ_i his risk tolerance coefficient. Noise traders' demand $\boldsymbol{u}$ is price inelastic and random. Risk neutral market makers observe the aggregate order flow $\mathbf{L}(\boldsymbol{p}) = \int_0^1 \mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p})di + \boldsymbol{u}$ and set prices *efficiently*: $\boldsymbol{p} = (1/R)E[\boldsymbol{v}|\boldsymbol{p}]$. [5] Restricting attention to linear equilibria, the following result holds:

Proposition 2 *In the market with competitive, risk neutral market makers, there exists a unique linear equilibrium. Informed traders (I) and market makers (MM) trade according to the following strategies:*

$$\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \mathbf{A}_i(\boldsymbol{s}_i - R\boldsymbol{p}), \tag{2}$$

$$\mathbf{X}_{MM}(\boldsymbol{p}) = (\boldsymbol{\Lambda}_{RN}^{-1} - \bar{\mathbf{A}})(\bar{\boldsymbol{v}} - R\boldsymbol{p}) - \bar{\boldsymbol{u}}, \tag{3}$$

and the vector of equilibrium prices is given by

$$\boldsymbol{p} = (1/R)E[\boldsymbol{v}|\boldsymbol{z}] = (1/R)(\boldsymbol{\Lambda}_{RN}(\boldsymbol{z} - \bar{\boldsymbol{u}}) + (I - \boldsymbol{\Lambda}_{RN}\bar{\mathbf{A}})\bar{\boldsymbol{v}}),$$

where $\mathbf{A}_i = \gamma_i \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$, $\bar{\gamma} = \int_0^1 \gamma_i di$, $\bar{\mathbf{A}} = \int_0^1 \gamma_i \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i} di$, $\boldsymbol{z} = \bar{\mathbf{A}}\boldsymbol{v} + \boldsymbol{u}$, $\boldsymbol{\Lambda}_{RN} = \boldsymbol{\Pi}^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}$, *and* $\boldsymbol{\Pi} = (\mathrm{Var}[\boldsymbol{v}|\boldsymbol{z}])^{-1} = \boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}}$.

Proof See the appendix.

Remark 1 Notice that as the matrix of traders' average speculative aggressiveness ($\bar{\mathbf{A}}$) coincides in propositions 1 and 2, the informational content of the order flows ($\boldsymbol{z}$) does not change in the two equilibria. As a consequence, the inference traders can make by observing equilibrium prices in the two markets is the *same*.

Informed traders' behavior has now changed. Owing to market makers' risk neutrality, the risk premia incorporated into asset prices disappear and market making becomes *unprofitable* to risk averse, informed traders. Therefore, as $\mathbf{A}_i$ is positive definite (and $R \geq 1$), no Giffen good appears in their demand functions. On the contrary, market makers' demand may still display the anomaly as the following example shows.

Example 3 Keeping the data of example 2, $\bar{\mathbf{A}} = I$ and

$$\boldsymbol{\Lambda}_{RN} = \begin{pmatrix} -0.11 & 0.21 \\ -0.65 & 1.11 \end{pmatrix}, \qquad \boldsymbol{\Lambda}_{RN}^{-1} - \bar{\mathbf{A}} = \begin{pmatrix} 115 & -22 \\ 68 & -13 \end{pmatrix}.$$

[5] As will become clear in the proof of proposition 2, in equilibrium $\boldsymbol{p}$ is observationally equivalent to $\boldsymbol{z}$. Therefore, $\boldsymbol{p} = (1/R)E[\boldsymbol{v}|\boldsymbol{z}] = (1/R)E[\boldsymbol{v}|\boldsymbol{p}]$. Efficient pricing can be seen as the result of Bertrand competition among risk neutral market makers for *each* asset order flow (see [14]).

Hence, $X_{Ii,1}(\boldsymbol{s}_i,\boldsymbol{p}) = s_{i1} - p_1$, $X_{Ii,2}(\boldsymbol{s}_i,\boldsymbol{p}) = s_{i2} - p_2$, $X_{MM,1}(\boldsymbol{p}) = 115(\bar{v}_1 - p_1) - 22(\bar{v}_2 - p_2)$, and $X_{MM,2}(\boldsymbol{p}) = -13(\bar{v}_2 - p_2) + 68(\bar{v}_1 - p_1)$. Asset 2 is the Giffen good and an intuition along the lines given in example 2 applies here too.

Therefore, combining the intuition drawn from examples 2, and 3 with proposition 2, and remark 1, information effects *per-se* cannot be responsible for the Giffen goods anomaly. [6] For, if this was the case, they should also affect the strategy of an informed trader displayed in proposition 2. Rather, the *role* that prices have in agents' strategies *also matters*.

With *no* risk neutral market makers, prices have two roles: (1) they allow to disentangle error terms from information in traders' private signals; (2) they allow to separate noise from information in the observed order flow realizations. The first role is related to the *speculative* component of the trader's demand; the second role is related to the *market making* component. Indeed, assuming for simplicity that $\bar{\boldsymbol{u}} = \boldsymbol{0}$ and $R = 1$, for ρ_v, ρ_u, $\rho_\epsilon \neq 0$, according to proposition 1 when $K = 2$, an informed strategy is given by

$$X_{ik}(\boldsymbol{s}_i,\boldsymbol{p}) = \frac{\gamma_i \tau_{\epsilon_{ik}}}{1-\rho^2_{\epsilon_i}}(s_{ik} - p_k) - \frac{\gamma_i \rho_{\epsilon_i}\sqrt{\tau_{\epsilon_{ik}}\tau_{\epsilon_{ih}}}}{1-\rho^2_{\epsilon_i}}(s_{ih} - p_h) + \left(\frac{\gamma_i}{\bar{\gamma}}\right)\sum_{l=1}^{2}\left(\boldsymbol{\Lambda}^{-1} - \bar{\mathbf{A}}\right)_{kl}(\bar{\boldsymbol{v}} - \boldsymbol{p})_{kl}.$$

To see how prices perform the *first* role, assume that $\rho_{\epsilon_i} > 0$ and that trader i receives two signals s_{ik}, s_{ih} such that $s_{ik} > p_k$ and $s_{ih} > p_h$. This can happen for two reasons: either both assets are worth more than what the market thinks (i.e. asset prices are biased downward e.g. by noise traders' selling pressure); or both signals are biased upward. The existence of positive correlation across signal-error terms strengthens the hypothesis of a contemporaneous, upward bias into the trader's signals. [7] Given this, he reinforces his belief that the good news he received is due to the effect of error biases and reduces his demand for both assets.
As far as the *second* role, example 2 provided an intuition for it. As soon as risk neutral market makers are introduced in the model, informed traders find no longer profitable to absorb the liquidity shock and prices cease to perform the second role for them. However, since market makers take the counterpart of the order books and clear markets, such a second role is *relevant* to their objectives. Hence, the Giffen good anomaly only characterizes risk neutral market makers' demand functions.

Remark 2 The above conclusion also clarifies the effect of assuming infinitely dispersed noise traders' demands (see [1], p. 647). Formally, letting $\boldsymbol{\Pi}_{\boldsymbol{u}} \to \boldsymbol{0}$ in proposition 1 (in any norm on matrices) implies that $\boldsymbol{\Lambda} \to \boldsymbol{\Lambda}^* = (\bar{\gamma}\boldsymbol{\Pi}_{\boldsymbol{v}} +$

[6] See [1], p. 645.
[7] This is the case because an error that biases upward the information contained in s_{ik} is more likely to happen together with an error biasing upwards the information about asset h as well.

$\bar{\mathbf{A}})^{-1}$. Thus, $\boldsymbol{p}$ and $\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p})$, converge respectively (and almost surely) to $\boldsymbol{p}^* = (1/R)(\boldsymbol{\Lambda}^*\boldsymbol{z} + (I - \boldsymbol{\Lambda}^*\bar{\mathbf{A}})\bar{\boldsymbol{v}})$ and $\mathbf{X}_i^*(\boldsymbol{s}_i, \boldsymbol{p}^*) = \mathbf{A}_i(\boldsymbol{s}_i - R\boldsymbol{p}^*) + \gamma_i \boldsymbol{\Pi}_{\boldsymbol{v}}(\bar{\boldsymbol{v}} - R\boldsymbol{p}^*)$. As $\boldsymbol{\Pi}_{\boldsymbol{v}}$ is positive definite and $\gamma_i > 0$, the market making component of an informed trader's demand is well behaved, and Giffen goods disappear. Indeed, as noise traders' demand dispersion in *every* asset increases without bound, informed traders cannot use prices to disentangle noise from information in the observed order flows' realization. Furthermore, the risk of trading with an informed agent vanishes and risk averse traders are always willing to accommodate the total net demand they face. Notice, however, that prices *do* aggregate information (i.e. reflect the value of $\boldsymbol{z}$) allowing informed traders to use them to disentangle the error terms affecting their signals. Therefore, in this Walrasian equilibrium, prices perform the *first* role but not the *second* role. [8]

Remark 3 The result that Giffen goods only characterize market makers' demand functions, is likely to depend on the competitive assumption about informed traders' behavior. Indeed, a strategic insider could exploit such anomalous market makers' behavior and induce a price increase to speculate on it. This possibility leads to conjecture that in the presence of a *non* atomistic trader, the Giffen good anomaly should disappear also from market makers' strategies. Indeed, [3] in a multi-asset generalization of [12], find that in equilibrium the matrix mapping order flows into prices must be positive definite, ruling out the existence of Giffen goods. [9]

4 The Market with Uninformed Traders

In this section I generalize the model studied in section 2 adding a sector of risk-averse uninformed traders. This allows to obtain a model where the equilibrium of proposition 2 arises as a limit result when the risk-bearing capacity of uninformed traders grows without bound.

Formally, assume that a continuum of uninformed traders distributed in the interval $[0, 1]$ is added to the market of section 2. Every uninformed trader j's preferences are represented by a CARA utility $U(W_{U1j}) = -\exp\{-W_{U1j}/\gamma_{Uj}\}$, where $\gamma_{Uj} > 0$ indicates the agent's coefficient of constant absolute risk tolerance, $W_{U1j} = W_{Uoj}R + \boldsymbol{x}'_{Uj}(\boldsymbol{v} - R\boldsymbol{p})$ denotes his final wealth that comes from buying $\boldsymbol{x}'_{Uj} = (x_{Uj,1}, x_{Uj,2}, \ldots, x_{Uj,K})$ units of each asset at price $\boldsymbol{p}$, and $W_{Uoj} \geq 0$

[8] It is interesting to contrast this equilibrium with its counterpart in the market with risk neutral market makers. As shown in proposition 2, the equilibrium price there is given by $\boldsymbol{p} = (1/R)E[\boldsymbol{v}|\boldsymbol{z}]$. However, as noise traders' demand is infinitely dispersed, market makers *cannot* extract *any* information from $\boldsymbol{z}$ to estimate $\boldsymbol{v}$. As a consequence, $\boldsymbol{p} = (\bar{\boldsymbol{v}}/R)$, informed traders cannot use the information conveyed by $\boldsymbol{z}$ to disentangle the error terms in their private signals, and $\mathbf{X}_{Ii}^* = \mathbf{A}(\boldsymbol{s}_i - (\bar{\boldsymbol{v}}/R))$. Thus, differently from the case analyzed above, the presence of competitive, risk neutral market makers prevents the equilibrium price from aggregating *any* information about the asset payoffs.

[9] Being a generalization of [12], the insider in Caballé and Krishnan's model submits non price-contingent orders to competitive, risk neutral market makers differently from the informed traders of the present context.

designates his initial wealth. Assume that every uninformed trader submits a vector of demand functions $\mathbf{X}_{Uj}(\boldsymbol{p})$ indicating the desired position in each asset k at every price vector $\boldsymbol{p}$. Finally, indicate with γ_{Ii} the risk tolerance coefficient of an informed trader i. Restricting attention to linear equilibria, the following result applies:

Proposition 3 *In the market with a sector of (CARA) uninformed traders (U), there exists a unique linear equilibrium. Agents' strategies are given by*

$$\begin{aligned}\mathbf{X}_{Ii}(\boldsymbol{s}_i,\boldsymbol{p}) &= \mathbf{A}_i(\boldsymbol{s}_i - R\boldsymbol{p}) \\ &\quad + (\gamma_{Ii}/\bar{\gamma})\left(\boldsymbol{\Lambda}_U^{-1} - \bar{\mathbf{A}}\right)(\bar{\boldsymbol{v}} - R\boldsymbol{p}) - \gamma_{Ii}(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}}, \\ \mathbf{X}_{Uj}(\boldsymbol{p}) &= (\gamma_{Uj}/\bar{\gamma})\left(\boldsymbol{\Lambda}_U^{-1} - \bar{\mathbf{A}}\right)(\bar{\boldsymbol{v}} - R\boldsymbol{p}) \\ &\quad - \gamma_{Uj}(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}},\end{aligned} \tag{4}$$

and the vector of equilibrium prices is given by

$$\boldsymbol{p} = (1/R)(\boldsymbol{\Lambda}_U\boldsymbol{z} + (I - \boldsymbol{\Lambda}_U\mathbf{A})\bar{\boldsymbol{v}}) - (\bar{\gamma}/R)(\bar{\mathbf{A}} + \bar{\gamma}\boldsymbol{\Pi})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}},$$

where $\mathbf{A}_i = \gamma_{Ii}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$, $\boldsymbol{z} = \bar{\mathbf{A}}\boldsymbol{v} + \boldsymbol{u}$, $\bar{\mathbf{A}} = \int_0^1 \mathbf{A}_i di$, $\boldsymbol{\Lambda}_U = (\bar{\mathbf{A}} + \bar{\gamma}\boldsymbol{\Pi})^{-1}(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})$, $\boldsymbol{\Pi} = (\mathrm{Var}[\boldsymbol{v}|\boldsymbol{z}])^{-1} = \boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}}$, *and* $\bar{\gamma} = \int_0^1 \gamma_{Ii} di + \int_0^1 \gamma_{Uj} dj$.

Proof See the appendix.

Notice that informed traders speculate on private information (as in proposition 1) and, together with uninformed traders, accommodate the total net demand. Also, as in proposition 1, the Giffen good anomaly (potentially) comes from the market making component of a trader's demand.

Corollary 1 *Assume that* $\gamma_{Uj} = \gamma_U, \forall j \in [0,1]$. *Then, if* $\gamma_U \to \infty$ *the equilibrium of proposition 3 converges (almost surely) to the one of proposition 2.*

Proof It follows immediately from the fact that $\bar{\mathbf{A}}$ does not depend on γ_U and as $\gamma_U \to \infty$, $\boldsymbol{\Lambda}_U \to \boldsymbol{\Lambda}_{RN}$. Thus, $\boldsymbol{p}$ converges (almost surely) to $(1/R)E[\boldsymbol{v}|\boldsymbol{z}]$.

Therefore, when uninformed agents display homogeneous risk attitude, if their risk-bearing capacity grows unboundedly, *(i)* the risk premia incorporated into equilibrium prices disappear, *(ii)* informed traders find no longer profitable to accommodate the total net demand they face, and *(iii)* their demand function becomes "well behaved" in prices.

5 Conclusions

Recent work in finance theory has highlighted the role played by Giffen goods in affecting stock market behavior around "unusual" events. Giffen goods characterize both unstable equilibria and episodes of market crashes. Indeed, when faced with the problem of extracting a signal about the asset fundamentals, traders with a "backward bending" demand curve shy away from assets whose price plummets and increase their position in assets whose price rockets. Depending on the specific model, this either *de*stabilizes the market (as in [4]) or introduces discontinuities in the function mapping the asset supply into its equilibrium price (as in [8] and [2]). These contributions testify the importance of understanding the conditions under which Giffen goods arise in markets with asymmetric information.

Building on [1], in this paper I have shown that contrary to previous intuitions in a market where informed and noise traders exchange vectors of assets, information effects *per se* are *not* responsible for the existence of Giffen goods. The *role* that prices play in informed traders strategies also matters. In particular, I have demonstrated that whenever all agents in a market are risk averse, an informed trader has two trading motives: *speculation* and *market making*. Insofar as the trader uses equilibrium prices to separate informed from noise traders' orders, the presence of correlation effects can lead him to attribute the total net demand he faces to informed trading. As a consequence, he may thus refrain from taking the other side of the trade, giving rise to the Giffen good anomaly. I have then given sufficient conditions that allow to remove the anomaly from informed traders demands.

While the results are robust to general model specifications, the analysis clearly relies on the competitive assumption about informed traders. Indeed, as conjectured in the paper, the presence of a non-atomistic trader, should rule out the Giffen phenomenon from *all* traders' strategies. In particular, it would be interesting to study a model where imperfectly competitive insiders submit multidimensional demand functions to risk neutral market makers. In this setup, one could analyze the behavior of the equilibrium price mapping as the number of insiders grows large, gauging what is the *degree* of competition beyond which a Giffen good appears.

Appendix

The following lemma, which is useful to compute conditional expected values, adapts a standard result from normal theory to the present context (see e.g. [6], Theorem 1, section 9.9).

Lemma 1 *Suppose that* $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ *is a random sample from a multivariate normal distribution with unknown mean vector* $\mathbf{M}$ *and specified precision matrices* $\mathbf{\Sigma}_i$, $i = 1, 2, \ldots, n$. *Suppose also that the prior distribution of* $\mathbf{M}$ *is multivariate normal with mean vector* $\boldsymbol{\mu}$ *and precision matrix* $\mathbf{\Sigma}$ *such that* $\boldsymbol{\mu} \in \mathbb{R}^K$ *and* $\mathbf{\Sigma}$ *is a symmetric positive definite matrix. Then the posterior distribution of* $\mathbf{M}$ *when* $\mathbf{X}_i = \mathrm{x}_i$ $(i = 1, 2, \ldots, n)$ *is a multivariate normal with mean vector* $\bar{\boldsymbol{\mu}}$ *and precision matrix* $\bar{\mathbf{\Sigma}} = \mathbf{\Sigma} + \sum_{i=1}^n \mathbf{\Sigma}_i$, *where* $\bar{\boldsymbol{\mu}} = \bar{\mathbf{\Sigma}}^{-1}(\mathbf{\Sigma}\boldsymbol{\mu} + (\sum_{i=1}^n \mathbf{\Sigma}_i)\,\widetilde{\boldsymbol{x}})$ *and* $\widetilde{\boldsymbol{x}} = (\sum_{i=1}^n \mathbf{\Sigma}_i)^{-1}(\sum_{i=1}^n \mathbf{\Sigma}_i \boldsymbol{x}_i)$.

Proof For $\mathbf{M} = \boldsymbol{m}$ and $\mathbf{X}_i = \boldsymbol{x}_i$ $(i = 1, 2, \ldots, n)$, the likelihood function $f_n(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n | \boldsymbol{m})$ satisfies the following relation:

$$f_n(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n | \boldsymbol{m}) \propto \exp\left\{-(1/2)\sum_{i=1}^n (\boldsymbol{x}_i - \boldsymbol{m})'\mathbf{\Sigma}_i(\boldsymbol{x}_i - \boldsymbol{m})\right\}. \quad (5)$$

However, $\sum_{i=1}^n (\boldsymbol{x}_i - \boldsymbol{m})'\mathbf{\Sigma}_i(\boldsymbol{x}_i - \boldsymbol{m}) = (\boldsymbol{m} - \widetilde{\boldsymbol{x}})'(\sum_{i=1}^n \mathbf{\Sigma}_i)(\boldsymbol{m} - \widetilde{\boldsymbol{x}}) + \sum_{i=1}^n (\boldsymbol{x}_i - \widetilde{\boldsymbol{x}})'\mathbf{\Sigma}_i(\boldsymbol{x}_i - \widetilde{\boldsymbol{x}})$, and since the last term in the previous equation does not involve $\boldsymbol{m}$, we can rewrite (5) as follows:

$$f_n(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n | \boldsymbol{m}) \propto \exp\left\{-(1/2)(\boldsymbol{m} - \widetilde{\boldsymbol{x}})'\left(\sum_{i=1}^n \mathbf{\Sigma}_i\right)(\boldsymbol{m} - \widetilde{\boldsymbol{x}})\right\}. \quad (6)$$

The prior p.d.f. of $\mathbf{M}$ satisfies

$$\varphi(\boldsymbol{m}) \propto \exp\left\{-(1/2)(\boldsymbol{m} - \boldsymbol{\mu})'\mathbf{\Sigma}(\boldsymbol{m} - \boldsymbol{\mu})\right\}, \quad (7)$$

and the posterior p.d.f. $g(\cdot | \boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$ of $\mathbf{M}$ will be proportional to the product of the functions specified by (6) and (7). However, $(\boldsymbol{m} - \boldsymbol{\mu})'\mathbf{\Sigma}(\boldsymbol{m} - \boldsymbol{\mu}) + (\boldsymbol{m} - \widetilde{\boldsymbol{x}})'(\sum_{i=1}^n \mathbf{\Sigma}_i)(\boldsymbol{m} - \widetilde{\boldsymbol{x}}) = (\boldsymbol{m} - \bar{\boldsymbol{\mu}})'\bar{\mathbf{\Sigma}}(\boldsymbol{m} - \bar{\boldsymbol{\mu}}) +$ terms not involving $\boldsymbol{m}$. Hence, we can write $g(\boldsymbol{m} | \boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n) \propto \exp\{-(1/2)(\boldsymbol{m} - \bar{\boldsymbol{\mu}})'\bar{\mathbf{\Sigma}}(\boldsymbol{m} - \bar{\boldsymbol{\mu}})\}$. This is the p.d.f. of a multivariate normal distribution for which the mean vector and the precision matrix are as specified in the statement of the lemma. □

Proof of proposition 1

As is well known the assumption of a CARA utility function and multivariate normality imply that $E[-\exp\{\gamma_i^{-1} W_{1i}\} | \boldsymbol{s}_i, \boldsymbol{p}] = -\exp\{-\gamma_i^{-1}(E[W_{1i} | \boldsymbol{s}_i, \boldsymbol{p}] - (1/2\gamma_i)\mathrm{Var}[W_{1i} | \boldsymbol{s}_i, \boldsymbol{p}])\}$. Therefore, the agent's demand is given by $\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \gamma_i(\mathrm{Var}[\boldsymbol{v} | \boldsymbol{s}_i, \boldsymbol{p}])^{-1}(E[\boldsymbol{v} | \boldsymbol{s}_i, \boldsymbol{p}] - R\boldsymbol{p})$.

Consider a candidate linear equilibrium $\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \mathbf{A}_i \boldsymbol{s}_i - \mathbf{B}_i \boldsymbol{p} + \mathbf{C}_i$, where $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ denote matrices of parameters to be determined in equilibrium. The market clearing equation reads as follows: $\int_0^1 \mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p})di + \boldsymbol{u} = \boldsymbol{z} - \bar{\mathbf{B}}\boldsymbol{p} + \bar{\mathbf{C}} = \mathbf{0}$, where $\boldsymbol{z} = \bar{\mathbf{A}}\boldsymbol{v} + \boldsymbol{u}$, $\bar{\mathbf{A}} = \int_0^1 \mathbf{A}_i di$, $\bar{\mathbf{B}} = \int_0^1 \mathbf{B}_i di$, and $\bar{\mathbf{C}} = \int_0^1 \mathbf{C}_i di$. Given the market clearing equation, and assuming that $\bar{\mathbf{B}}$ is non singular, $\boldsymbol{p}$ and $\boldsymbol{z}$ are observationally equivalent, and traders condition indifferently on $\boldsymbol{p}$ or $\boldsymbol{z}$ when determining their positions. To compute equilibrium strategies, assume that the matrix $\bar{\mathbf{A}}$ is invertible. Then, $\bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})|\boldsymbol{v} \sim N(\boldsymbol{v}, \bar{\mathbf{A}}^{-1}\mathbf{\Pi}_{\boldsymbol{u}}^{-1}(\bar{\mathbf{A}}')^{-1})$. Also, $\boldsymbol{s}_i|\boldsymbol{v} \sim N(\boldsymbol{v}, \mathbf{\Pi}_{\boldsymbol{\epsilon}_i}^{-1})$. Therefore, applying lemma 1, with $n = 2$, $\boldsymbol{x}_1 = \boldsymbol{s}_i$, $\boldsymbol{x}_2 = \bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})$, and $\boldsymbol{m} = \boldsymbol{v}$, $E[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] = (\mathbf{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}_i})^{-1} (\mathbf{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}(\boldsymbol{z} - \bar{\boldsymbol{u}}) + \mathbf{\Pi}_{\boldsymbol{\epsilon}_i}\boldsymbol{s}_i)$, and $\text{Var}[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] = (\mathbf{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}} + \mathbf{\Pi}_{\boldsymbol{\epsilon}_i})^{-1}$. Substituting these expressions into the agent strategy and simplifying

$$\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \gamma_i \mathbf{\Pi}_{\boldsymbol{\epsilon}_i}(\boldsymbol{s}_i - R\boldsymbol{p}) + \gamma_i(\mathbf{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}(\boldsymbol{z} - \bar{\boldsymbol{u}}) - R\mathbf{\Pi}\boldsymbol{p}), \qquad (8)$$

where $\mathbf{\Pi} = \mathbf{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}\bar{\mathbf{A}}$. Hence, $\mathbf{A}_i = \gamma_i \mathbf{\Pi}_{\boldsymbol{\epsilon}_i}$ and the matrix $\bar{\mathbf{A}}$ is symmetric and positive definite. Substituting (8) into the market clearing equation and solving for the equilibrium price, $\boldsymbol{p} = (1/R)(\mathbf{\Lambda}\boldsymbol{z} + (I - \mathbf{\Lambda}\bar{\mathbf{A}})\bar{\boldsymbol{v}}) - (\bar{\gamma}/R)(\bar{\mathbf{A}} + \bar{\gamma}\mathbf{\Pi})^{-1}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}}$, where $\mathbf{\Lambda} = (\bar{\mathbf{A}} + \bar{\gamma}\mathbf{\Pi})^{-1}(I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})$, and $\bar{\gamma} = \int_0^1 \gamma_i di$. Thus, $\bar{\mathbf{B}} = (1/R)\mathbf{\Lambda}$, and given our assumptions this matrix is non singular. Solving for $\boldsymbol{z}$ in the equilibrium price, and substituting it into (8), gives

$$\begin{aligned}
&\mathbf{X}_i(\boldsymbol{s}_i, \boldsymbol{p}) = \\
&\quad \mathbf{A}_i(\boldsymbol{s}_i - \boldsymbol{p}) + \gamma_i \big((I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}(\mathbf{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} - \bar{\boldsymbol{u}}) \\
&\qquad +(\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}(I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}} - (I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\mathbf{\Pi}_{\boldsymbol{v}} \\
&\qquad\qquad -(I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}})R\boldsymbol{p}\big) .
\end{aligned}$$

Notice that $\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}(I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}} = (I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}}$, hence using the definition of $\mathbf{\Lambda}$ and simplifying the previous expression gives the traders equilibrium demand functions displayed in proposition 1. □

Proof of proposition 2

Consider a candidate linear equilibrium $\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \mathbf{A}_i\boldsymbol{s}_i + \phi_i(\boldsymbol{p})$, where $\mathbf{A}_i$ and $\phi_i(\cdot)$ denote respectively the matrix of trading intensities and a linear function of current prices. Owing to linear strategies, the aggregate order flow is given by $\mathbf{L}(\boldsymbol{p}) = \int_0^1 \mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p})di + \boldsymbol{u} = \boldsymbol{z} + \phi(\boldsymbol{p})$, where $\boldsymbol{z} = \bar{\mathbf{A}}\boldsymbol{v} + \boldsymbol{u}$, $\bar{\mathbf{A}} = \int_0^1 \mathbf{A}_i di$, and $\phi(\boldsymbol{p}) = \int_0^1 \phi_i(\boldsymbol{p})di$. Because of competition for each order flow and risk neutrality, $\boldsymbol{p} = (1/R)E[\boldsymbol{v}|\boldsymbol{z}]$. Assume that $\bar{\mathbf{A}}$ is invertible and notice that $\bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})|\boldsymbol{v} \sim N(\boldsymbol{v}, \bar{\mathbf{A}}^{-1}\mathbf{\Pi}_{\boldsymbol{u}}^{-1}(\bar{\mathbf{A}}')^{-1})$, hence we can apply lemma 1 with $n = 1$, $\boldsymbol{m} = \boldsymbol{v}$ and $\boldsymbol{x}_1 = \bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})$ to obtain: $\boldsymbol{p} = (1/R)\mathbf{\Pi}^{-1}(\mathbf{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}(\boldsymbol{z} - \bar{\boldsymbol{u}})) = (1/R)(\mathbf{\Lambda}_{RN}(\boldsymbol{z} - \bar{\boldsymbol{u}}) + (I - \mathbf{\Lambda}_{RN}\bar{\mathbf{A}})\bar{\boldsymbol{v}})$, where $\mathbf{\Pi} = (\text{Var}[\boldsymbol{v}|\boldsymbol{z}])^{-1} = \mathbf{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}}$ and $\mathbf{\Lambda}_{RN} = \mathbf{\Pi}^{-1}\bar{\mathbf{A}}'\mathbf{\Pi}_{\boldsymbol{u}}$. Let us now turn attention to traders' strategies. As seen in the previous proof, the assumptions of a CARA utility function and multivariate normality imply that the agent's demand is given by $\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \gamma_i(\text{Var}[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}])^{-1}(E[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] - R\boldsymbol{p})$. As $\boldsymbol{s}_i|\boldsymbol{v} \sim N(\boldsymbol{v}, \mathbf{\Pi}_{\boldsymbol{\epsilon}_i}^{-1})$, and $\boldsymbol{p}$ is in equilibrium observationally equivalent to $\bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})$, we can again apply lemma 1 with $n = 2$, $\boldsymbol{x}_1 = \boldsymbol{s}_i$, $\boldsymbol{x}_2 = \bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})$, and $\boldsymbol{m} = \boldsymbol{v}$.

This gives $E[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] = \boldsymbol{\Pi}_i^{-1}(\boldsymbol{\Pi} R\boldsymbol{p} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}\boldsymbol{s}_i)$, where $\boldsymbol{\Pi}_i = (\text{Var}[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}])^{-1} = \boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$. Plugging these expressions into the equilibrium strategy and simplifying,

$$\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \gamma_i \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}(\boldsymbol{s}_i - R\boldsymbol{p}).$$

Thus, $\mathbf{A}_i = \gamma_i \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$, and $\phi_i(\boldsymbol{p}) = -\mathbf{A}_i R\boldsymbol{p}$. As $\mathbf{A}_i = \mathbf{A}_i' = \gamma_i\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$, the assumption that $\bar{\mathbf{A}}$ is nonsingular is correct in equilibrium. To determine market makers' demand, consider the market clearing condition

$$\boldsymbol{z} - \bar{\mathbf{A}}R\boldsymbol{p} + \mathbf{X}_{MM}(\boldsymbol{p}) = \mathbf{0}. \tag{9}$$

Solving for $\boldsymbol{z}$ from the equilibrium price, $\boldsymbol{z} = (\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})^{-1}(\boldsymbol{\Pi} R\boldsymbol{p} - \boldsymbol{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}})$. Substituting the latter expression in (9), and isolating $\mathbf{X}_{MM}(\boldsymbol{p})$ gives: $\mathbf{X}_{MM}(\boldsymbol{p}) = (\boldsymbol{\Lambda}_{RN}^{-1} - \bar{\mathbf{A}})(\bar{\boldsymbol{v}} - R\boldsymbol{p}) - \bar{\boldsymbol{u}}$. □

Proof of proposition 3

Along the same lines of the previous proofs, CARA utility functions and multivariate normality imply $\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \gamma_{Ii}(\text{Var}\,[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}])^{-1}(E\,[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] - R\boldsymbol{p})$, and $\mathbf{X}_{Uj}(\boldsymbol{p}) = \gamma_{Uj}(\text{Var}\,[\boldsymbol{v}|\boldsymbol{p}])^{-1}(E\,[\boldsymbol{v}|\boldsymbol{p}] - R\boldsymbol{p})$. Consider a candidate linear equilibrium $\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \mathbf{A}_i\boldsymbol{s}_i - \mathbf{B}_{Ii}\boldsymbol{p} + \mathbf{C}_{Ii}$, $\mathbf{X}_{Uj}(\boldsymbol{p}) = \mathbf{C}_{Uj} - \mathbf{B}_{Uj}\boldsymbol{p}$, where $\mathbf{A}_i$, $\mathbf{B}_{Ii}$, $\mathbf{B}_{Uj}$, $\mathbf{C}_{Ii}$, and $\mathbf{C}_{Uj}$ denote matrices of parameters to be determined in equilibrium. The market clearing equation reads as follows: $\int_0^1 \mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p})di + \int_0^1 \mathbf{X}_{Uj}(\boldsymbol{p})dj + \boldsymbol{u} = \boldsymbol{z} - \bar{\mathbf{B}}\boldsymbol{p} + \bar{\mathbf{C}} = \mathbf{0}$, where $\boldsymbol{z} = \bar{\mathbf{A}}\boldsymbol{v} + \boldsymbol{u}$, $\bar{\mathbf{A}} = \int_0^1 \mathbf{A}_i di$, $\bar{\mathbf{B}} = \int_0^1 \mathbf{B}_{Ii}di + \int_0^1 \mathbf{B}_{Uj}dj$, and $\bar{\mathbf{C}} = \int_0^1 \mathbf{C}_{Ii}di + \int_0^1 \mathbf{C}_{Uj}dj$. Given the market clearing equation, and assuming that $\bar{\mathbf{B}}$ is non singular, $\boldsymbol{p}$ and $\boldsymbol{z}$ are observationally equivalent, and traders condition indifferently on $\boldsymbol{p}$ or $\boldsymbol{z}$ when determining their positions. To compute equilibrium strategies, assume that the matrix $\bar{\mathbf{A}}$ is invertible. Then, $\bar{\mathbf{A}}^{-1}(\boldsymbol{z} - \bar{\boldsymbol{u}})|\boldsymbol{v} \sim N(\boldsymbol{v}, \bar{\mathbf{A}}^{-1}\boldsymbol{\Pi}_{\boldsymbol{u}}^{-1}(\bar{\mathbf{A}}')^{-1})$. Also, $\boldsymbol{s}_i|\boldsymbol{v} \sim N(\boldsymbol{v}, \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}^{-1})$. Therefore, applying lemma 1, $E[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] = (\boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i})^{-1}\,(\boldsymbol{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}'\boldsymbol{\Pi}_{\boldsymbol{u}}(\boldsymbol{z} - \bar{\boldsymbol{u}}) + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}\boldsymbol{s}_i)$, and $\text{Var}[\boldsymbol{v}|\boldsymbol{s}_i, \boldsymbol{p}] = (\boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}} + \boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i})^{-1}$. Substituting these expressions into the informed agent strategy and simplifying

$$\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \gamma_{Ii}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}(\boldsymbol{s}_i - R\boldsymbol{p}) + \gamma_{Ii}(\boldsymbol{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}'\boldsymbol{\Pi}_{\boldsymbol{u}}(\boldsymbol{z} - \bar{\boldsymbol{u}}) - R\boldsymbol{\Pi}\boldsymbol{p}), \tag{10}$$

where $\boldsymbol{\Pi} = \boldsymbol{\Pi}_{\boldsymbol{v}} + \bar{\mathbf{A}}'\boldsymbol{\Pi}\bar{\mathbf{A}}$. Hence, $\mathbf{A}_i = \gamma_{Ii}\boldsymbol{\Pi}_{\boldsymbol{\epsilon}_i}$ and the matrix $\bar{\mathbf{A}}$ is symmetric and positive definite. Similarly,

$$X_{Uj}(\boldsymbol{p}) = \gamma_{Uj}(\boldsymbol{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} + \bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}(\boldsymbol{z} - \bar{\boldsymbol{u}}) - R\boldsymbol{\Pi}\boldsymbol{p}). \tag{11}$$

Substituting (10) and (11) into the market clearing equation and solving for the equilibrium price, $\boldsymbol{p} = (1/R)(\boldsymbol{\Lambda}_U\boldsymbol{z} + (I - \boldsymbol{\Lambda}_U\bar{\mathbf{A}})\bar{\boldsymbol{v}}) - (\bar{\gamma}/R)(\bar{\mathbf{A}} + \bar{\gamma}\boldsymbol{\Pi})^{-1}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}}\bar{\boldsymbol{u}}$, where $\boldsymbol{\Lambda}_U = (\bar{\mathbf{A}} + \bar{\gamma}\boldsymbol{\Pi})^{-1}(I + \bar{\gamma}\bar{\mathbf{A}}\boldsymbol{\Pi}_{\boldsymbol{u}})$, and $\bar{\gamma} = \int_0^1 \gamma_{Ii}di + \int_0^1 \gamma_{Uj}dj$. Thus,

$\bar{\mathbf{B}} = (1/R)\mathbf{\Lambda}_U$, and given our assumptions this matrix is non singular. Solving for $\boldsymbol{z}$ in the equilibrium price, and substituting it into (10), gives

$$
\begin{aligned}
&\mathbf{X}_{Ii}(\boldsymbol{s}_i, \boldsymbol{p}) = \\
&\quad \mathbf{A}_i(\boldsymbol{s}_i - \boldsymbol{p}) + \gamma_{Ii}\big((I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}(\mathbf{\Pi}_{\boldsymbol{v}}\bar{\boldsymbol{v}} - \bar{\boldsymbol{u}}) \\
&\qquad + (\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}(I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}} - (I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\mathbf{\Pi}_{\boldsymbol{v}} \\
&\qquad\qquad - (I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}})R\boldsymbol{p}\big) .
\end{aligned}
$$

Notice that $\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}(I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}} = (I + \bar{\gamma}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}})^{-1}\bar{\mathbf{A}}\mathbf{\Pi}_{\boldsymbol{u}}\bar{\mathbf{A}}$, hence using the definition of $\mathbf{\Lambda}_U$ and simplifying the previous expression gives the informed traders equilibrium demand functions displayed in proposition 3. Along the same lines, one obtains the second equation of (4). □

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