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**Testing Financing Constraints on Firm  
Investment using Variable Capital**

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# Testing Financing Constraints on Firm Investment using Variable Capital\*

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## Abstract

A recent literature has criticised the sensitivity of a firm's investment to its own cash flow as an adequate measure of financing constraints. In this paper we develop a new method to detect the presence of financing constraints at firm level. We consider a structural dynamic model of investment with financing imperfections and with both fixed and variable capital. We solve the model and simulate an industry with many firms. We show that the irreversibility of fixed capital is the main reason why the sensitivity of fixed investment to cash flow is not a good measure of financing constraints. Using the fact that variable capital is reversible, we develop a new test of financing constraints based on a reduced form variable capital investment equation. Simulation results show that our test correctly identifies financially constrained firms also when the estimation of firms' investment opportunities is very noisy. Moreover our test is valid regardless of the type of adjustment costs of fixed capital. We confirm empirically the validity of this method on a sample of US companies.

JEL classification: D21, G31

Keywords: Financing Constraints, Investment

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# I Introduction

In order to explain the aggregate behaviour of investment and production, it is necessary to understand the factors that affect investment at the firm level. Financing imperfections may prevent firms to access external finance and make them unable to invest unless internal finance is available. It is therefore important to study to what extent financing constraints matter for the investment decisions of firms. This is useful also for other areas of research, such as the literature on the role of internal capital markets and banks as well as the macro literature on the financial accelerator.

The seminal paper by Fazzari, Hubbard and Petersen (1988) develops a financing constraints test based on two considerations. First, if adjustment costs of fixed capital are quadratic and there are no financing imperfections, firm investment is a linear function of Tobin's marginal  $Q$ . Second, if a firm is unable to raise external financing then it only invests when internally generated funds become available. Fazzari, Hubbard and Petersen (1988) propose to detect the presence of financing constraints by estimating an augmented  $Q$  model where cash flow is included as an explanatory variable. Several studies support the validity of this approach.<sup>1</sup> They estimate the  $Q$  model for several groups of firms and find that the positive correlation between investment and cash flow, conditional on Tobin's  $Q$ , is most important for firms more likely to face capital-market imperfections.<sup>2</sup>

The motivation of this paper is that recent studies have criticized the validity of these results. Erickson and Whited (2000) and Bond *et al* (2004) show that errors in measuring the expected profitability of firms explain most of the observed positive correlation between fixed investment and cash flow. Gomes (2001), Pratap (2003) and Moyen (2005) take another approach. They develop structural models of firm investment with financing imperfections, and simulate artificial economies where a fraction of the firms face a binding financing constraint. The simulated data show that the investment-cash flow

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<sup>1</sup>See Hubbard (1998) for a review of this literature.

<sup>2</sup>However Kaplan and Zingales (1997 and 2000) and Cleary (1999) show that this result is not robust across the criteria for selecting the groups of firms, and find that the investment-cash flow correlation is stronger for firms which are financially very wealthy and surely not financially constrained.

correlation is not generally increasing in the intensity of financing constraints.<sup>3</sup>

However the theoretical models of Gomes (2001), Pratap (2003) and Moyen (2005) depart from the standard assumption of quadratic adjustment costs of fixed capital. This assumption is necessary in order to obtain a linear relationship between investment and marginal  $Q$  and therefore it is essential for the empirical literature cited before. As a consequence, it is still an open question to what extent observational errors or misspecification problems are responsible for the failure of the financing constraints tests adopted in the previous literature.

The objective of this paper is to answer this question, and to use such answer to develop a new financing constraints test that is robust to the above criticisms. We consider a structural model of firm investment with financing imperfections, adjustment costs of capital and a multifactor production technology. We simulate an artificial industry, and we study the effects of adjustment costs and measurement errors on the relationship between internal finance and investment. We prove that the correct specification of adjustment costs is key to obtain a consistent estimate of the financing constraints of the firms. We demonstrate that when fixed capital is irreversible then the augmented  $Q$  model considered by the previous literature is misspecified, and it fails to detect financing constraints even when marginal  $Q$  is perfectly measured. We show that instead a consistent test of financing constraints can be based on the correlation between internal finance and the investment in a reversible factor of production. This new test is based on a simple linear investment equation, but is not subject to the criticisms mentioned before, because of two main reasons: i) the test is robust to different types of adjustment costs of capital; ii) simulation results show that the test is very efficient in detecting financing constraints also when the firm's expected profitability is very noisily estimated.

The model considers the optimal investment decisions of a risk neutral firm which generates output using two complementary factors of production, fixed and variable capital. Fixed capital is subject to adjustment costs, while variable capital can be adjusted

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<sup>3</sup>Alti (2003) and Abel and Eberly (2003) and (2004) develop theoretical frameworks in which positive investment-cash flow correlations arise in the absence of financial markets imperfections.

without frictions. Because of an enforceability problem, the firm can obtain external financing only if it secures it with collateral. The assets of the firm can only be partially collateralisable and some downpayment is needed to finance investment. We solve the model and simulate several artificial industries where firms are subject to different types of adjustment costs of fixed capital, and we estimate the  $Q$  model of investment both with and without observational errors. Because we allow firms to accumulate financial assets, the model predicts that the effect of financing constraints on firm investment is identified by including cash stock, rather than cash flow, as a regressor in the investment equation.<sup>4</sup>

Simulation results show that the correct specification of adjustment costs, rather than the absence of measurement errors, is key to obtain a consistent financing constraints test. If adjustment costs of fixed capital are quadratic, then the  $Q$  model of investment is well specified. We show that in this case the correlation between fixed investment and cash stock is a useful indicator of the intensity of financing constraints. This is true even when marginal  $Q$  is very noisily estimated. On the contrary if fixed capital is irreversible then the fixed investment of financially unconstrained firms can be more sensitive to internal finance than that of financially constrained firms, even if there is no noise in the estimation of marginal  $Q$ . This happens because the firms use a reversible factor of production (variable capital) in conjunction with an irreversible one (fixed capital). The consequence is that when the irreversibility constraint is currently binding or it was binding in the previous period, then only variable capital is sensitive to internal finance for financially constrained firms. This finding is important, because it is a well documented fact that fixed investment at the firm level is subject to a significant degree of irreversibility.<sup>5</sup>

The main contribution of this paper is to develop a new financing constraints test that is robust both to measurement errors and to different types of adjustment costs

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<sup>4</sup>The same result applies to structural models of investment with financing imperfections where the marginal cost of external finance increases with the leverage of the firm (Gilchrist and Himmelberg, 1998; Caggese 2005).

<sup>5</sup>See for example Caballero, Engel and Haltiwanger (1995), Eberly (1997), Cooper and Haltiwanger (2000) and Chirinko and Shaller (2004).

of capital. Simulation results show that such a test can be based on the sensitivity of variable capital investment to financial wealth. This sensitivity is estimated using a reduced form variable capital equation, and it is shown to be a good indicator of the intensity of financing constraints, even in presence of: i) different types of adjustment costs of fixed capital; ii) large, persistent and positively correlated observational errors in the regressors. Importantly, the simulations show that this financing constraints test is reliable also when variable capital is not perfectly flexible, as long as it is a reversible factor of production.

Finally, another advantage of this test is that it does not require the estimation of marginal  $Q$ , but only of the expected productivity shock of the firm. This means that it is easier to apply to datasets of small firms not quoted on the stock market, for which the information about the market value of the assets is not normally available.

We verify empirically the validity of this new test of financing constraints on a balanced panel of US companies, drawn from the Worldscope database, with 8 years of balance sheet data (1996-2003). We first estimate the  $Q$  model augmented with cash flow, separately for groups of firms a priori considered with different likelihood of facing financing constraints, and we confirm the finding that the fixed investment-cash flow correlation is not a good indicator of financing constraints.

We then estimate our new test of financing constraints. For many firms in this sample we do not have the information about the most flexible input factors such as the cost of materials. Therefore we approximate variable capital using the labour input (number of employees). Adjustment costs are likely to affect labour dynamics at high frequencies, as a large body of literature shows.<sup>6</sup> However, we believe that in the context of our dataset labour is a factor of production flexible enough for the purpose of being used in our financing constraints test. There are two reasons for this: i) we use yearly data aggregated at the company rather than at the plant level. The empirical evidence shows that for US data adjustment costs mostly affect labor dynamics at monthly and quarterly

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<sup>6</sup>See Hamermesh and Pfann (1996) for a review of this literature.

frequency<sup>7</sup>. As Hall (2004) says, “*Many economists believe that adjustment costs for labour are unimportant from one year to the next...*”. ii) Our simulations show that the new financing constraints test still works in the presence of convex adjustment costs in variable capital.

Estimation results show that the correlation between labour input and net financial wealth is positive and higher for firms more likely to be financially constrained than for the other firms. The difference is strongly statistically significant, and the correlation increases for the groups of firms with the highest probability to be financially constrained. Furthermore we show that the results are robust to several different specifications and inclusion of additional variables.

This paper contributes to both the theoretical and empirical literature on financing constraints and firm investment. The theoretical section of this paper is related to Gomes (2001), Pratap (2003) and Moyen (2005). With respect to these papers we clarify the relationship between measurement errors, adjustment costs and the investment-internal finance relationship.

Because of its emphasis on the importance of adjustment costs to understand the investment decisions of firms, this paper is related to Barnett and Sakellaris (1998) and to Abel and Eberly (2002), who analyze the implications of different types of adjustment costs on the relationship between marginal  $Q$  and investment at the firm and at the aggregate level. Moreover it is related to Whited (2004) who shows that, in the presence of fixed costs of investment, constrained firms are less likely to undertake a new large investment project than unconstrained firms, after controlling for expected productivity and for the time lapsed since the last large investment project.

The empirical section of this paper, in the spirit of the seminal paper by Fazzari, Hubbard and Petersen (1988), uses the structural model of firm investment to derive a financing constraints test that is based on a simple reduced form linear investment equation. A similar motivation is behind the recent papers by Hennessy, Levy and Whited

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<sup>7</sup>See footnote n.6

(2005) and Almeida and Campiello (2005). Hennessy, Levy and Whited (2005) derive an enhanced version of the  $Q$  model that allows for the presence of financing frictions and debt overhang. Almeida and Campiello (2005) test the hypothesis that, for a financially constrained firm, the sensitivity of investment to cash flow is increasing in the degree of liquidity of the assets of the firm. In contrast with these two papers, which both maintain the standard assumption of quadratic adjustment costs of fixed capital, and focus on the  $Q$  model, our paper shows that the presence of irreversibility of fixed capital causes an important bias in the empirical relationship between fixed investment, Tobin's  $Q$  and internal finance. Furthermore, our paper demonstrates the advantage of adopting a financing constraints test based on a reversible factor of production, which is robust regardless of the type of adjustment costs of fixed capital.

Our method to test for financing constraints on firm investment can be applied using any reversible factor of production. Caggese (2005) uses the cost of materials as a proxy for variable capital, and shows that the test is able to identify the presence of financing constraints on a sample of small Italian manufacturing firms. Nevertheless, in this paper we propose a specific application that uses labour input. Therefore our paper is also related to earlier works that show how financial factors affect labour demand of firms, like Nickell and Wadhvani (1991) and Sharpe (1994).

This paper is organized as follows. Section II describes the model. Section III defines the new financing constraints test. Section IV illustrates the simulation results. Section V verifies the validity of the new financing constraints test using a balanced panel of US firms. Finally, section VI summarizes the conclusions.

## II The model

The aim of this section is to develop a structural model of investment with financing constraints and with adjustment costs of fixed capital. We consider a risk neutral firm which has the objective to maximize the discounted sum of future expected dividends. The discount factor is equal to  $1/R$ , where  $R = 1 + r$ , and  $r$  is the lending/borrowing risk



free interest rate.

The firm operates with two inputs,  $k_t$  and  $l_t$ , that are respectively fixed and variable capital. New capital installed at time  $t$  generates output at time  $t + 1$ . The production function is strictly concave in both factors. We assume a Cobb-Douglas functional form:

$$y_t = \theta_t k_t^\alpha l_t^\beta \text{ with } \alpha + \beta < 1 \quad (1)$$

All prices are constant and normalized to 1.<sup>8</sup>  $\theta_t$  is a productivity shock that follows a stationary stochastic process. For simplicity we assume that variable capital is nondurable and fixed capital is durable:

$$1 = \delta_l > \delta_k \quad (2)$$

$\delta_l$  and  $\delta_k$  are the depreciation factors of variable capital and fixed capital respectively. Moreover variable capital investment is not subject to adjustment costs, while fixed capital investment is subject to the adjustment cost function  $\mu(i_t)$ , where  $i_t$  is gross fixed investment:

$$i_t = k_{t+1} - (1 - \delta_k) k_t \quad (3)$$

$$\mu(i_t = 0) = 0; \mu(i_t \neq 0) \geq 0 \quad (4)$$

Assumption (4) allows for both concave and convex adjustment costs. Financial imperfections are introduced by assuming that new shares issues and risky debt are not available. At time  $t$  the firm can borrow from (and lend to) the banks one period debt, with face value  $b_{t+1}$ , at the market riskless interest rate  $r$ . A positive (negative)  $b_{t+1}$  indicates that the firm is a net borrower (lender). Banks only lend secured debt, and the only collateral they accept is physical capital. Therefore at time  $t$  the borrowing capacity of the firm is limited by the following constraint:

$$b_{t+1} \leq \tau_k k_{t+1} + \tau_l l_{t+1} \quad (5)$$

$$0 < \tau_k \leq 1 - \delta_k; \tau_l = 1 - \delta_l = 0 \quad (6)$$

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<sup>8</sup>This simplifying assumption will be relaxed in the empirical section of the paper.

$\tau_k$  and  $\tau_l$  are the shares of variable capital and fixed capital that can be used as collateral. One possible justification for constraint (5) is that the firm can hide the revenues from the production. Being unable to observe such revenues the banks can only claim the residual value of the firm's physical assets as repayment of the debt (Hart and Moore, 1998).<sup>9</sup> If  $\tau_k = 1 - \delta_k$  then all the residual value of fixed capital is accepted as collateral. This is possible because we assume that adjustment costs do not apply when the firm is liquidated and all its assets are sold.<sup>10</sup>

The timing of the model is the following: at the beginning of period  $t$  the firm's technology becomes useless with an exogenous probability  $1 - \gamma$ . In this case the assets of the firm are sold and the revenues are distributed as dividends. Instead with probability  $\gamma$  the firm continues activity. It inherits from time  $t - 1$  the stock of fixed and variable capital  $k_t$  and  $l_t$ . Then  $\theta_t$  is realized,  $y_t$  is produced and  $b_t$  repaid.  $w_t$ , the net financial wealth, is the following:

$$w_t = y_t + (1 - \delta_k)k_t - b_t \quad (7)$$

After producing, the firm allocates  $w_t$  plus the new borrowing between dividends, fixed capital and variable capital, according to the following budget constraint:

$$d_t + l_{t+1} + k_{t+1} + \mu(i_t) = w_t + b_{t+1}/R \quad (8)$$

Let's denote the value at time 0 of the firm, conditional on not liquidating the activity in period 0, and after  $\theta_0$  is realized, by  $V_0(w_0, \theta_0, k_0)$  :

$$V_0(w_0, \theta_0, k_0) = \underset{(k_{t+1}, l_{t+1}, b_{t+1})_{t=0,1,\dots,\infty}}{MAX} d_0 + E_0 \left\{ \sum_{t=1}^{\infty} \left( \frac{\gamma}{R} \right)^t \left[ d_t + \frac{1 - \gamma}{\gamma} w_t \right] \right\} \quad (9)$$

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<sup>9</sup>Some authors argue that variable capital has an higher collateral value than fixed capital (Berger et al, 1996). Nevertheless the results derived in this section are consistent with alternative specifications that allow for a positive value of variable capital as collateral.

<sup>10</sup>In theory, the interactions between financing constraints and adjustment costs of fixed capital may imply that in some cases the firm is forced to liquidate the activity to repay the debt, even if it would be profitable to continue. In order to simplify the analysis, we focus in this paper on the set of parameters for which this outcome never happens in equilibrium.

The firm maximizes (9) subject to (5), (8) and the non negativity constraint on dividends:

$$d_t \geq 0 \quad (10)$$

For a large class of adjustment cost functions  $\mu(\cdot)$  these constraints define a compact and convex feasibility set for  $l_{t+1}$ ,  $k_{t+1}$ ,  $b_{t+1}$  and  $d_t$ , and the law of motion of  $w_{t+1}$  conditional on  $w_t$ ,  $k_t$  and  $\theta_t$  is continuous. Therefore, given the assumptions on  $\theta_t$  and the concavity of the production function, a unique solution to the problem exists. In order to describe the optimality conditions of the model, we use equation (8) to substitute  $d_t$  in the value function (9). Moreover in the following analysis we assume that the marginal adjustment cost function  $\mu'(i_t)$  is continuous and smooth in  $i_t$ . In appendix 3, equations (60)-(63) illustrate the optimality conditions of the problem when fixed capital is irreversible and this assumption is not satisfied.

Let  $\lambda_t$  and  $\phi_t$  be the Lagrangian multipliers associated respectively with constraints (5) and (10). The solution of the problem is defined by the equations (11)-(14):

$$\phi_t = R\lambda_t + \gamma E_t(\phi_{t+1}) \quad (11)$$

$$E_t\left(\frac{\partial y_{t+1}}{\partial k_{t+1}}\right) = UK + R\mu'(i_t) - \Phi_t E_t[\mu'(i_{t+1})] + E_t(\Psi_{t+1}^k) \quad (12)$$

$$E_t\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}\right) = UL + E_t(\Psi_{t+1}^l) \quad (13)$$

$$D_k k_{t+1} + D_l l_{t+1} + \mu(i_t) \leq w_t - d_t \quad (14)$$

Where:

$$D_k = 1 - \frac{\tau_k}{R}; \quad D_l = 1 - \frac{\tau_l}{R}; \quad UK = R - (1 - \delta_k); \quad UL = R \quad (15)$$

$$\Phi_t = \frac{\gamma(1 - \delta_k) [1 + E_t(\phi_{t+1})]}{1 + \gamma E_t(\phi_{t+1})} \quad (16)$$

$$E_t(\Psi_{t+1}^k) = \frac{R[R + R\mu'(i_t) - \tau_k] \lambda_t - \gamma [(1 - \delta_k) cov[\phi_{t+1}, \mu'(i_{t+1})] - cov(\phi_{t+1}, \frac{\partial y_{t+1}}{\partial k_{t+1}})]}{1 + \gamma E_t(\phi_{t+1})} \quad (17)$$

$$E_t \left( \Psi_{t+1}^l \right) = \frac{R(R - \tau_l) \lambda_t - \gamma \text{cov} \left( \phi_{t+1}, \frac{\partial y_{t+1}}{\partial l_{t+1}} \right)}{1 + \gamma E_t \left( \phi_{t+1} \right)} \quad (18)$$

Equations (11), (12) and (13) are the first order conditions of  $b_{t+1}$ ,  $l_{t+1}$  and  $k_{t+1}$  respectively.  $D_z$  for  $z \in \{k, l\}$ , is the downpayment required to purchase one additional unit of capital. Equation (14) combines together the budget constraint (8) and the collateral constraint (5) and implies that the downpayment necessary to buy  $k_{t+1}$  and  $l_{t+1}$  must be lower than the residual net worth after paying the dividends. By iterating forward equation (11) we obtain:

$$\phi_t = R \sum_{j=0}^{\infty} E_t (\lambda_{t+j}) \quad (19)$$

Equation (19) implies that as long as there are some current or future expected financing constraints, then  $\phi_t > 0$  and the firm does not distribute dividends:  $d_t = 0$ .<sup>11</sup> In this case the solution of the investment problem depends on whether or not the collateral constraint is binding. If constraint (14) is not binding then equations (12) and (13) evaluated at  $\lambda_t = 0$  can be solved to determine the optimal unconstrained capital levels  $k_{t+1}^* (k_t, \theta_t)$  and  $l_{t+1}^* (k_t, \theta_t)$ . The collateral constraint is instead binding when financial wealth is not sufficient as a downpayment for  $k_{t+1}^*$  and  $l_{t+1}^*$ , even if  $d_t = 0$ :

$$D_k k_{t+1}^* + D_l l_{t+1}^* + \mu \left( k_{t+1}^* - (1 - \delta) k_t \right) > w_t \quad (20)$$

In this case the constrained levels of capital  $k_{t+1}^c (k_t, \theta_t, w_t)$  and  $l_{t+1}^c (k_t, \theta_t, w_t)$  are such that:

$$D_k k_{t+1}^c + D_l l_{t+1}^c + \mu \left( k_{t+1}^c - (1 - \delta) k_t \right) = w_t \quad (21)$$

And the solution is determined by the values  $k_{t+1}^c$ ,  $l_{t+1}^c$  and  $\lambda_t$  that satisfy equations (12), (13) and (21).

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<sup>11</sup>It is possible to allow financing constraints and positive dividends to coexist in equilibrium by assuming that the discount factor of the firm is smaller than  $\frac{1}{R}$ . This means that some dividends are distributed even when there is some probability of being financially constrained. This alternative assumption would complicate the analysis in the model, but would not affect the results derived in the paper.

### III A new test of financing constraints based on variable capital

Equation (12) shows that the expected marginal productivity of fixed capital depends on current and future expected marginal adjustment costs and on the term  $E_t(\Psi_{t+1}^k)$ . Instead equation (13) shows that the expected marginal productivity of variable capital only depends on  $UL$  ad on the term  $E_t(\Psi_{t+1}^l)$ . Importantly, the term  $E_t(\Psi_{t+1}^l)$  summarizes the effect of financing constraints on variable capital investment. This is illustrated in proposition 1:

**Proposition 1** *if  $\lambda_t = 0$ , then  $\frac{\partial E_t(\Psi_{t+1}^l | \theta_t)}{\partial w_t} = 0$ . If  $\lambda_t > 0$ , then  $\frac{\partial E_t(\Psi_{t+1}^l | \theta_t)}{\partial w_t} < 0$*

**Proof:** see appendix 3.

Proposition 1 states that if the financing constraint is binding then  $E_t(\Psi_{t+1}^l)$  is monotonously decreasing in  $w_t$ , conditional on the productivity shock. Moreover simulation results show that the relationship between  $E_t(\Psi_{t+1}^l | \theta_t)$  and  $w_t$  is convex, due to the decreasing marginal productivity of both factors. Our new financing constraint test is based on proposition 1 and on equation (13). When the financing constraint is binding, and for a given productivity shock  $\theta_t$  and fixed capital stock  $k_{t+1}$ , variable capital investment  $l_{t+1}$  is a monotonously increasing function of  $w_t$ .<sup>12</sup> Instead when the financing constraint is not binding then variable capital investment  $l_{t+1}$  is not sensitive to  $w_t$ .<sup>13</sup>

Proposition 1 is ignored, to our knowledge, by the previous literature, which almost exclusively focuses on the first order condition of fixed capital. In the case of fixed capital, proposition 1 does not apply because the presence of adjustment costs excludes a

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<sup>12</sup>In the model we assume that the user cost of capital is constant, and therefore it does not affect investment decisions. In the empirical section of the paper we will discuss the consequences of relaxing this assumption.

<sup>13</sup>Actually when the financing constraint is not binding in period  $t$  then  $\frac{\partial E_t(\Psi_{t+1}^l | \theta_t)}{\partial w_t}$  can be positive if  $cov(\phi_{t+1}, \frac{\partial y_{t+1}}{\partial l_{t+1}})$  is positive and  $\frac{\partial cov(\phi_{t+1}, \frac{\partial y_{t+1}}{\partial l_{t+1}})}{\partial w_t}$  is negative (see equation 13). This happens when a positive productivity shock at time  $t + 1$  increases at the same time  $\frac{\partial y_{t+1}}{\partial l_{t+1}}$  and  $\phi_{t+1}$  because the financing constraint becomes binding in period  $t + 1$ . This implies that  $l_{t+1}$  can actually be negatively related to  $w_t$  for a financially unconstrained firm. This effect would in theory reinforce our financing constraints test. However, the results of the simulations show it to be always negligible for realistic parameter values.

direct mapping from financial wealth to the term  $E_t(\Psi_{t+1}^k)$  (see equation 17), and from  $E_t(\Psi_{t+1}^k)$  to the marginal productivity of fixed capital (see equation 12). The standard approach of the literature is instead to assume that adjustment costs are quadratic:

$$\mu(i_t) = \frac{1}{2}bi_t^2 \quad (22)$$

$$\mu'(i_t) = bi_t \quad (23)$$

Using (23) in (12) and substituting recursively forward, we obtain the following investment equation:

$$i_t = \frac{1}{Rb} \sum_{j=0}^{\infty} \Phi_t \frac{\gamma^j(1-\delta_k)^j}{R^j} \left[ E_t \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) - UK - E_t(\Psi_{t+1}^k) \right] \quad (24)$$

With

$$\Phi_t = \prod_{i=1}^j \frac{1 + E_t(\phi_{t+j})}{1 + \gamma E_t(\phi_{t+j})} \quad (25)$$

If there are no current and future financing constraints, then  $\Phi_t = 1$  and  $E_t(\Psi_{t+1+j}^k) = 0$  for any  $t$ . In this case equation (24) is the standard  $q$  model of investment:

$$i_t = \frac{1}{Rb} \sum_{j=0}^{\infty} \frac{\gamma^j(1-\delta_k)^j}{R^j} \left[ E_t \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) - UK \right] \equiv \frac{1}{Rb} (q_t - 1) \quad (26)$$

Where  $q_t$  is Tobin's marginal  $Q$ . With financing constraints, since  $\gamma$  is close to one we can approximate  $\Phi_t \approx 1$  in order to obtain an augmented version of the  $Q$ -model:

$$i_t \approx -\frac{1}{Rb} + \frac{1}{Rb}q_t - \sum_{j=0}^{\infty} \frac{\gamma^j(1-\delta_k)^j}{R^j} E_t(\Psi_{t+1+j}^k) \quad (27)$$

The last term  $\sum_{j=0}^{\infty} \frac{\gamma^j(1-\delta_k)^j}{R^j} E_t(\Psi_{t+j+1}^k)$  is positive. Moreover it is possible to show that if adjustment costs are quadratic then conditional on  $q_t$  an increase in internal funds ( $w_t$ ) relaxes the financing constraints, reducing  $\sum_{j=0}^{\infty} \frac{\gamma^j(1-\delta_k)^j}{R^j} E_t(\Psi_{t+j+1}^k)$  and increasing  $i_t$ . Hence we expect a positive relationship between the stock of financial wealth  $w_t$  and  $i_t$ .

In explaining the failure of the financing constraints tests based on the  $Q$  model, Erickson and Whited (2000) and Bond *et al* (2004) argue that it is very difficult to accurately estimate  $q_t$ . Therefore a positive correlation between investment and internal

finance could arise because internal finance captures the effect of the unobservable productivity shock. In the next section we solve the model and simulate the production and investment activity of many firms, and we demonstrate that the major problem of this approach is instead the fact that equation (27) is misspecified if adjustment costs are not quadratic. In this case the correlation between  $w_t$  and  $i_t$ , conditional on  $q_t$ , can be higher for financially unconstrained than for financially constrained firms, even when  $q_t$  is perfectly estimated.

Therefore we develop a new test of financing constraints which uses variable capital and is justified by proposition 1. Our approach still has the problem of estimating the productivity shock  $\theta_t$ . However the task of estimating  $\theta_t$  is easier than estimating  $q_t$ . The latter is a forward looking variable that depends on the current and the future expected productivity shocks. The previous literature has usually substituted marginal  $Q$  with average  $Q$ , which can be estimated as the ratio of the market value of a firms with the replacement value of its assets. But this approach also presents problems. Average  $Q$  and marginal  $Q$  coincide only under restrictive assumptions (Hayashi, 1982). Furthermore Bond and Cummings (2001) show that the ratio of the market value and the book value of a company is a poor measurement of average  $Q$ , because the stock market valuation of a company often, and for long periods of time, diverges from the fundamentals and overestimates or underestimates the true value of the firm.

The advantage of our approach is that  $\theta_t$  only depends on the current productivity shock and can be estimated using balance sheet data (for example as a Solow residual of the production function). Importantly, our simulation results show that our new method to identify financing constraints is valid even when  $\theta_t$  is very noisily estimated.

In order to derive a reduced form variable capital equation, we take logs of both sides of equation (13). By noting that  $E_t \left( \frac{\partial y_{t+1}}{\partial l_{t+1}} \right) = \beta E_t (\theta_{t+1}) k_{t+1}^\alpha l_{t+1}^{\beta-1}$ , we obtain the following:

$$\ln \beta + \ln E_t (\theta_{t+1}) + \alpha \ln k_{t+1} + (\beta - 1) \ln l_{t+1} = \ln \left[ UL + E_t \left( \Psi_{t+1}^l \right) \right] \quad (28)$$

We take the linear approximation of  $\ln [UL + E_t(\Psi_{t+1}^l)]$  around  $\ln [UL + \bar{\Psi}^l]$  and we solve for  $\ln l_{t+1}$ :

$$\ln l_{t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{t+1}) + \pi_2 \ln k_{t+1} - \pi_3 \frac{E_t(\Psi_{t+1}^l)}{UL + \bar{\Psi}^l} \quad (29)$$

$$\pi_0 = \frac{\ln \beta - \ln [UL + \bar{\Psi}^l] + \frac{\bar{\Psi}^l}{UL + \bar{\Psi}^l}}{1 - \beta}; \quad \pi_1 = \frac{1}{1 - \beta}; \quad \pi_2 = \frac{\alpha}{1 - \beta}; \quad \pi_3 = \frac{1}{(1 - \beta)}$$

Equation (29) shows that financing constraints directly affect variable capital investment choices through the term  $E_t(\Psi_{t+1}^l)$ . Since  $E_t(\Psi_{t+1}^l)$  is a convex and decreasing function of  $w_t$ , we substitute  $-\frac{E_t(\Psi_{t+1}^l)}{UL + \bar{\Psi}^l}$  with a concave transformation of  $w_t$ :

$$\ln l_{t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{t+1}) + \pi_2 \ln k_{t+1} + \pi_3 f(w_t) \quad (30)$$

$$f'(\cdot) > 0; \quad f''(\cdot) < 0$$

Equation (30) is the variable capital investment equation that we estimate for our financing constraints test. The financing constraints hypothesis predicts that  $\pi_3$  is positive if the firm is subject to financing constraints. Instead if the firm is financially unconstrained then  $\pi_3$  is equal to zero. Proposition 1 ensures that this financing constraints test is valid regardless of the type of fixed capital adjustment costs.

## IV Simulation results

In this section we use the solution of the model to simulate the activity of many firms, all ex ante identical and all subject to an idiosyncratic productivity shock that is uncorrelated across firms and autocorrelated for each firm. The simulated data is used to test our new method to detect financing constraints on firm investment, and to compare it with the  $Q$  model augmented with internal finance. We use the same methodology commonly used in empirical applications since the seminal paper of Fazzari, Hubbard and Petersen (1988). We use a priori information to select a subsample of firms more likely to face financing



imperfections, and then we verify if the sensitivity of investment to internal finance is higher for this group than for the other firms.

We simulate two different industries. In both industries firms become financially constrained when the borrowing constraint (5) is binding, and their internal finance is not sufficient to finance all profitable investment opportunities. Moreover, in one industry fixed capital is subject to irreversibility and in the other it is subject to the quadratic adjustment costs function (22). Prices and interest rate are constant. As our objective is to analyze the effects of financing constraints at firm level, the partial equilibrium nature of this exercise does not restrict the analysis in any important way. In the remainder of the paper we include the subscript  $i$  to indicate the  $i$ -th firm. The irreversibility constraint is the following:

$$k_{i,t+1} \geq (1 - \delta) k_{i,t} \quad (31)$$

The idiosyncratic shock  $\theta_{i,t}$  is modeled as follows:

$$\theta_{i,t} = \theta_{i,t}^I \theta_{i,t}^P \quad (32)$$

$\theta_{i,t}^I$  is an idiosyncratic shock that has value 1 with probability  $1 - \xi$  and  $\xi$  with probability  $\xi$ , where  $0 < \xi < 1$ . This shock represents a small probability  $\xi$  to have an unexpected drop in output, and it increases the volatility of cash flow and the chance of experiencing negative net income. If  $\xi = 0$  then simulated firms would almost never generate negative net income, which instead is observed for 24.6% of all firm year observations in the sample used for the empirical analysis in the next section.  $\theta_{i,t}^P$  is a persistent idiosyncratic shock:

$$\ln \theta_{i,t+1}^P = \rho \ln \theta_{i,t}^P + \varepsilon_{i,t} \quad (33)$$

$$0 < \rho < 1$$

$$\varepsilon_{i,t} \sim iid(0, \sigma_\varepsilon^2) \text{ for all } i \quad (34)$$

The dynamic investment problem is solved using a numerical method (see appendix 4 for details). The model is parametrized assuming that the time period is one year. Table I summarizes the parameters choices.  $r = 6.5\%$  (long term average risk free rate). The

sum of  $\alpha$  and  $\beta$  matches returns to scale equal to 0.97. This is consistent with studies on disaggregated data that find returns to scale to be just below 1 (Burnside, 1996).  $\beta$  is set to match the ratio of fixed capital over variable capital. In the model variable capital fully depreciates in one period, and therefore we consider as variable capital the sum of materials cost and wages, and we consider as fixed capital land, buildings, plant and equipment. Using yearly data about manufacturing plants from the NBER-CES database (which includes the information about the cost of materials), we calculate a ratio of fixed capital to variable capital between 0.5 and 0.7 in the 1980-1996 period.  $\delta_k$  and  $\delta_l$ , the annual depreciation rates of fixed capital and variable capital, are set respectively equal to 0.12 and 1.  $b, \rho$  and  $\sigma_\varepsilon$  match the average, standard deviation, and autocorrelation of the fixed investment rate of the US Compustat database, as reported in Gomes (2001). The quadratic adjustment cost parameter  $b$  is very small because adjustment costs are a proportion of gross investment  $i_{i,t}$  instead of the investment rate  $\frac{i_{i,t}}{k_{i,t}}$ .  $\xi$  is equal to 0.1, and matches the standard deviation of the cash flow as a fraction of fixed capital. Regarding the collateral value of the assets, we follow the imperfect enforceability argument used to justify constraint (5) and set  $\tau_l = 0$ .<sup>14</sup>  $\tau_k$  is set to match the average debt/assets ratio of US corporations.  $\gamma$  is equal to 0.94, implying that in each period 6% of firms exit and 6% of firms are newborn. This value is consistent with the empirical evidence about firms turnover in the US. The second part of table I reports the matched moments. The simulated industries do not match perfectly the empirical moments, given the presence of nonlinearities in the mapping from the parameters to the moments, but they get sufficiently close for our purpose. We simulate 50000 firm-year observations for both industries. From equation (27) we derive the reduced form fixed investment equation according to the  $q$  –  $model$ :

$$i_{i,t} = \alpha_0 + \alpha_1 q_{i,t} + \alpha_2 w_{i,t} + \epsilon_{1,i,t} \quad (35)$$

$q_{i,t}$  is marginal  $Q$ .  $w_{i,t}$  captures the effect of current and future expected financing

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<sup>14</sup>We also tried alternative parametrisations in which both  $\tau_l$  and  $\tau_k$  are greater than zero and these do not change the results.

constraints, summarized by the term  $-\sum_{j=0}^{\infty} \frac{\gamma^j(1-\delta_k)^j}{R^j} E_t(\Psi_{t+1+j}^k)$  in equation (27).  $w_{i,t}$  is inserted linearly to preserve the analogy with the investment - cash flow literature. A concave transformation of  $w_{i,t}$  would improve the fit of the model but would not change the results. Our new financing constraints test is instead based on the following variable capital model, derived from equation (30):

$$\ln l_{i,t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}) + \pi_2 \ln k_{i,t+1} + \pi_3 \ln w_{i,t} + \epsilon_{2,i,t+1} \quad (36)$$

We instrument  $\ln k_{i,t+1}$  with its lagged value  $\ln k_{i,t}$ .  $w_{i,t}$  is included in the estimation as a concave transformation. Since a wide range of transformations yield similar qualitative results, we report the results obtained using the logarithmic transformation.<sup>15</sup> The financing constraint test is based, in both models, on the coefficient of financial wealth,  $\alpha_2$  in the case of the  $Q$  model (35) and  $\pi_3$  in the case of the variable capital model (36). These models are useful in detecting financing constraints if the estimated coefficients of financial wealth are significantly higher for firms with an higher intensity of financing constraints than for the other firms. Tables II and III report the estimated parameters for equations (35) and (36) respectively. In these tables, as well as in the other tables presented in this section, we do not report the standard deviations of the estimated coefficients, because all coefficients are strongly significant. Both equations are estimated for financially constrained and unconstrained firms and for the two industries with quadratic adjustment costs and irreversibility. In these simulations we are able to perfectly sort financially constrained firm year observations, because the intensity of financing constraints is measured by the value of the Lagrangian multiplier  $\lambda_{i,t}$ . However, in estimations that use real data, firms are usually sorted according to criteria that are imperfectly correlated with the likelihood to face capital markets imperfections. Therefore we also want to verify the ability of the two tests to identify financially constrained firms when the sorting criterion is imprecise. This is reported in the bottom part of tables II and III, where we

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<sup>15</sup>Regarding equation (36), the Box - Cox transformation  $w_{i,t}^{tr} = \frac{w_{i,t}^\nu - 1}{\nu}$  that achieves the best fit is the one with  $\nu = -0.2$  for the economy with irreversibility and  $\nu = -0.02$  for the economy with quadratic adjustment costs.

compare the estimates of  $\alpha_2$  and  $\pi_3$  for groups that include only the most constrained firms (measured by the magnitude of  $\lambda_{i,t}$ ) and the complementary groups. In these tables, as well as in the other tables of this section, we do not report test statistics of the difference between the coefficients because such difference is always significant. The first column of table II shows that in the case of quadratic adjustment costs, the  $q$  model is useful in identifying financially constrained firms, because the coefficient  $\alpha_2$  is consistently higher for constrained firms than for the complementary sample, even when the criterion to sort firms is not precise. Importantly table VI in the next section shows that in the case of quadratic adjustment costs, the test based on the  $q$  model still works when we include a large observational error in  $q_t$  and  $w_t$ . In other words, if the reduced form equation is well specified, then measurement errors are not relevant and the investment - internal finance correlation is a good measure of the intensity of financing constraints.

On the contrary, if adjustment costs are in the form of irreversibility of fixed capital then the investment-internal finance correlation is not a useful indicator of the intensity of financing constraints. The second column of table II shows that in this case fixed capital investment is sensitive to internal finance for all firms, and more so for less constrained than for more constrained firms for some of the sorting criteria. This happens even though marginal  $Q$  is perfectly measured, because of two main reasons. First, the coefficient of  $w_{i,t}$  is positive for unconstrained firms because the model is misspecified. Investment responds in a nonlinear way to changes in marginal  $Q$ , and a positive wealth coefficient captures this misspecification. Second,  $i_{i,t}$  of a financially constrained firm may not respond to a change in  $w_{i,t}$  after a productivity shock. This is obvious if the irreversibility constraint is binding, and gross investment in fixed capital is constrained to be zero. But also when the productivity shock is positive and the irreversibility constraint is not binding,  $i_{i,t}$  may not be sensitive to changes in  $w_{i,t}$ . This happens when financing problems have forced the firm to cut variable capital investment in the past. Such firm, after a positive shock that increases wealth, will initially only invest in variable capital, and only afterwards will it start investing again in fixed capital. This is illustrated in table IV. The second column

shows that the most constrained firms (those with higher shadow cost of money) are more likely to have both constraints binding. The third column shows that, if we only include observations for which the irreversibility constraint is not binding in the previous and current period, then the investment sensitivity to internal finance is higher for financially constrained than for unconstrained firms. The fourth column considers firms that had a binding irreversibility constraint in the previous period but not in the current period. In this case  $i_{i,t}$  is almost always less sensitive to  $w_{i,t}$  for financially constrained than for unconstrained firms. Taken together tables II, IV and VI suggest that the failure of the augmented  $Q$  model to identifying financially constrained firms, documented in the recent empirical literature, depends on the irreversibility of fixed capital investment rather than on the presence of measurement errors in  $Q$ .

In Table III we report the estimation of the variable capital investment equation (36).  $\pi_3$  is shown to be a good indicator of financing constraints, no matter what type of adjustment costs affect fixed capital. The estimations show that a higher sensitivity of variable capital investment to internal finance (coefficient  $\pi_3$ ) always correctly signals an higher intensity of financing constraints no matter how we split the sample.

## A Measurement errors

In this section we report the estimates of the two models in the presence of measurement errors. In table (V) we report the estimation of the variable capital equation (36), where all the explanatory variables are observed with error:

$$\ln E_t(\theta_{t+1})^* = \ln E_t(\theta_{t+1}) + \eta_{1,t+1} \quad (37)$$

$$\ln k_{t+1}^* = \ln k_{t+1} + \eta_{2,t+1}$$

$$\ln w_t^* = \ln w_t + \eta_{3,t} \quad (38)$$

$\eta_1, \eta_2$  and  $\eta_3$  are normally distributed and first order autocorrelated errors with zero mean and with the AR(1) coefficient equal to 0.5. We consider 8 different cases. The first four columns of table (V) refer to the case in which the errors are not correlated with

each other. For the other four columns the errors are correlated according to the following covariance matrix:

$$\begin{array}{c|ccc} & \eta_1 & \eta_2 & \eta_3 \\ \hline \eta_1 & 1 & & \\ \eta_2 & 0.5 & 1 & \\ \eta_3 & 0.5 & 0.5 & 1 \end{array}$$

We make a further distinction. In the “signal/noise=4” columns the standard deviation of the error is 25% of the standard deviation of the variable. In the “signal/noise=1” columns the standard deviation of the error is 100% of the standard deviation of the variable. The estimation results show that despite the large observational errors the test is still able to identify financially constrained investment. The estimate of  $\pi_3$  is always positive for more financially constrained firms, and always higher than the complementary group. An exception are the last two columns, where the errors are very large and are positively correlated. In this case the  $\pi_3$  coefficient may be negative for constrained firms, but in any case the monotonicity property ( $\pi_3$  is higher for more constrained firms than for the complementary sample) is always preserved. Table VI estimates the  $Q$  model when  $Q_t$  and  $w_t$  are estimated with noise. If adjustment costs are quadratic, then the errors do not affect the outcome of the test, which is almost always able to identify more financially constrained firms. Conversely if fixed capital is irreversible, large observational errors (signal to noise equal to one) imply that the test always gives the wrong answer.

## B Misspecification

The analysis has so far maintained the assumption that the variable capital reduced form equation (36) is well specified. This may not happen in reality, among other reasons because we may apply the test on factors of production that are not perfectly flexible. For the sake of simplicity, instead of deriving an extended model where both fixed and variable capital are subject to adjustment costs, we assume that the dependent variable in equation (36) is fixed capital  $\ln k_{t+1}$ . By combining the fixed capital first order condition (12) and the variable capital first order condition (13) we can derive the following reduced

form fixed capital equation:

$$\ln k_{t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{t+1}) + \pi_3 f\left\{\mu'(i_t), E_t[\mu'(i_{t+1})], \lambda_t, E_t(\phi_{t+1})\right\} + \epsilon_{3i,t+1} \quad (39)$$

Where  $\pi_1 = \frac{1}{1-\alpha-\beta}$  and  $f(\cdot)$  is a nonlinear function of current and future expected marginal adjustment costs of fixed capital and of current and future expected financing constraints. We then introduce  $\ln w_t$  in order to proxy for the effect of  $\lambda_t$  and  $E_t(\phi_{t+1})$ , while we omit  $\mu'(i_t)$  and  $E_{t-1}[\mu'(i_{t+1})]$  from the estimation:

$$\ln k_{t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{t+1}) + \pi_2 \ln w_{i,t} + \epsilon_{4i,t+1} \quad (40)$$

Equation (40) is as a version of equation (36) where the adjustment costs of the dependent variable introduce a misspecification problem. The fact that a factor of production is included as a regressor in equation (36) but not in equation (40) does not affect the results of this exercise. If we extended the model to allow for another factor of production that is subject to adjustment costs, then equation (40) would include such factor as a right hand side variable. The analysis would therefore be conditional on such factor, and the only misspecification problem in equation (40) would still be the presence of adjustment costs in the dependent variable. Estimation results with and without noisy regressors are reported in table VII. If the adjustment costs of the dependent variable are quadratic, estimation results show that the test maintains its ability to identify financially constrained firms. Instead if the dependent variable is subject to the irreversibility constraint then the test almost always gives the wrong answer. Therefore table VII shows that equation (36) can be a valid financing constraints test even if variable capital is not perfectly frictionless, as long as it is a reversible factor of production.

Summing up, the simulation results illustrated in tables III-VII suggest that the new test of financing constraints based on the variable capital equation (36) should work also when applied on empirical data, because they support the following conclusion: the finding that  $\pi_3$  is higher for firms more likely to face capital market imperfections than for the other firms indicates that the investment of the former group of firms is financially constrained. This result is robust to i) different types of adjustment costs of fixed capital

ii) large, persistent and positively correlated observational errors in the regressors iii) misspecification of the model due to convex adjustment costs of variable capital.

## V Empirical evidence

In this section we verify the validity of our new test of financing constraints on a sample of US companies. In order to allow for a comparison with the findings of the previous literature we select, from the Worldscope database, a balanced panel of 1357 US firms that have complete financial information for 8 years (the 1996-2003 period).<sup>16</sup> This sample is constructed with the same criteria as the sample used by Cleary (1999) in his estimation of the augmented  $Q$  model. We will show that our sample, which refers to a different time period, confirms Cleary’s finding that the fixed investment-cash flow correlation is not a good indicator of financing constraints. Then we will use the sample to perform our new test of financing constraints.

As in Cleary (1999), our main criterion to select firm more likely to face capital markets imperfections is the dividend policy. We create a binary variable that has the value of one for all the observations in which the firms increase the dividends per share (2105 observations) and zero for all the observations in which firms reduce the dividends per share (432 observations). On the one hand firms usually increase dividends if they can sustain the increase in the long term. Hence this action is likely to signal that the cost of obtaining external finance is not higher than the opportunity cost of internal finance, and these firms are likely to be not financially constrained. On the other hand a reduction in dividends is a strongly negative signal for the markets. Firms will do it only if the cost of distributing dividends is very high, and hence they presumably face an high cost of obtaining external finance. These firms are considered “likely financially constrained”.

We use these two groups to run a discriminant analysis based on several regressors:

$$Z = \beta_1 Current + \beta_2 FCCov + \beta_3 W/K + \beta_4 NI\% + \beta_5 GrSales + \beta_6 Debt + \beta_7 W \quad (41)$$

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<sup>16</sup>We start by selecting all US firms in the Worldscope database with complete information from 1996 to 2003. Then we delete from the sample banks, insurance companies, other financial companies and utility companies. The complete list of all the companies included in the sample is available upon request.



*Current* = current ratio; *FCCOV* = fixed charge coverage; *NI%* = net income margin; *GrSales* = sales growth; *Debt* = debt ratio. *W* = real value of the financial slack. Details about these variables are in appendix 1. This discriminant analysis correctly predicts which firms will cut or raise dividends 76% of the times, a result analogous to the estimation by Cleary (1999). We then calculate the discriminant score *Z* for all firms that did not increase nor decrease dividends, and hence were excluded from the discriminant analysis, and then following Cleary (1999) we create three equally sized groups of firm years observations, one with low score (likely financially constrained), one with medium score, and one with high score. Table VIII reports summary statistics about these groups. The firms in the likely financially constrained group are relatively smaller, more leveraged and less profitable than the firms in the other groups. However the variances are high and the differences are always not statistically significant. Table IX illustrates the estimations of the augmented *Q* model of investment. The top part reports the estimates obtained by Cleary (1999), and shows that the investment-cash flow sensitivities are decreasing rather than increasing in the intensity of financing constraints. The mid part of the table shows that in our sample we obtain the same qualitative result.<sup>17</sup> In both cases the cash flow coefficient is positive mainly because it is positively correlated with the unobserved productivity shock, which is only partly captured by *Q*. In the bottom part of the table we report consistent estimates of the same coefficients obtained using a GMM estimation method. The significance of the cash flow coefficient is greatly reduced, and still not related to the intensity of financing constraints. This is consistent with the results obtained by Erickson and Whited (2000) and Bond *et al* (2004).

We now proceed to estimate our alternative method to detect financing constraints. With respect to equation (36) we initially assume that fixed capital becomes productive one period after it is installed, while newly hired workers are immediately productive.

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<sup>17</sup>The magnitude and significance of the cash-flow coefficients are lower than in Cleary (1999). This may be due to the different treatment of outliers in the two samples. We eliminate outliers by cutting out all values of the variables beyond a certain threshold. Cleary (1999) keeps them in the sample by assigning a value equal to the threshold itself.

Therefore the equation we estimate is the following:

$$\ln l_{i,t+1} = a_i + d_t + \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}) + \pi_2 \ln k_{i,t} + \pi_3 \ln w_{i,t} + \epsilon_{i,t+1} \quad (42)$$

We introduce time and firm specific dummies to take into account, among other things, of the fact that the user cost of capital is non constant.  $k_{i,t}$  is the real value of net fixed assets (property, plant and equipment) at the end of period  $t$ .  $E_t(\theta_{i,t+1})$ , the expected productivity shock, is estimated from the total factor productivity of the firm.  $w_{i,t}$  is the real value of the financial slack available at the end of period  $t$ .  $l_{i,t+1}$  is number of employees reported by the company at the end of period  $t + 1$ . More detailed information about the variables used is reported in appendix 1.

Ideally we would have preferred to use a more flexible factor of production, such as the cost of materials, as the dependent variable in equation (42). Unfortunately this information is not available for most of the firms in the sample. Therefore we use the number of employees instead. A large literature finds evidence of adjustment costs in high frequency labor dynamics. However, we believe that in our dataset labor is a flexible enough factor to be used as dependent variable in our test. This is because of the following reasons: i) our data is yearly, and aggregated at company rather than at the plant level. Temporal aggregation ensures that most of the adjustment to shocks takes place within the period. For example Mairesse and Brigitte (1985) examine a yearly panel data of US firms and show that 5/6 of the labour adjustment to shocks is completed in one year. Studies on aggregate data also show a fast speed of adjustment of labour, and infer small labour adjustment costs (see Hamermesh and Pfann (1996) for a review). ii) Our simulations show that our method to detect financing constraints is robust to the presence of misspecification due to convex adjustment costs in the dependent variable (see table VII).

We estimate equation (42) using a System-GMM estimation technique (Blundell and Bond, 1998) applied to first differences.<sup>18</sup> We eliminate as outliers all the observations smaller than the 1% percentile and larger than the 99% percentile of every variable. The

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<sup>18</sup>We use the command `xtabond2` on the software package STATA.

details of the specification tests run to determine the appropriate estimation method, and the *partial*  $R^2$  test of the validity of the instruments, are reported in appendix 2. Tables from X to XVIII show the estimation results. In all the tables we provide the Hansen test of overidentifying restrictions, which is robust to heteroskedasticity and autocorrelation of unknown form. In the main estimations in tables (X)-(XIII) the orthogonality of the instruments is never rejected at the 10% significant level, with only one exception.

The four columns of table X report the estimates of equation (42) for the three groups of firm-year observations selected according to the Z score and for the sample complementary to the low Z group. The results confirm the financing constraints hypothesis, because the value of the coefficient of  $\ln w_{i,t}$  is significantly higher for the “likely financially constrained” (low Z) group than for the other groups.

In the mid part of table X we repeat the same estimation adding a variable that measures the cost of capital.  $ccap_{i,t+1}$  is the ratio of the interest payment of debt during period  $t + 1$  over total debt at the end of period  $t$ . In the theoretical model the user cost of capital is constant and included in  $\pi_0$ . Therefore the variable  $ccap_{i,t+1}$  could capture changes in the user cost of capital that are not already captured by firm specific effects and time dummies. This is potentially important because the positive estimated value of  $\pi_3$  for likely constrained firms could have the following explanation not related to financing constraints: firms that experience an increase in  $w_{i,t}$  reduce their probability of bankruptcy and banks are willing to finance them at a lower interest rate. This reduces the cost of capital and increases investment. The estimates in the second part of table X reject this hypothesis. Even after the inclusion of the cost of capital the estimate of  $\pi_3$  is still significantly higher for the likely financially constrained firms than for the other firms.

The last column shows that  $\pi_3$  is also significantly higher than zero for the high Z group. Table VIII shows that these observations on average belong to the most productive and fast growing firms, and a positive  $\pi_3$  could be capturing the unobservable productivity shock. Since the coefficient of  $\ln E_t(\theta_{i,t+1})$  has the expected sign but is not significant,

we add two additional variables that are correlated with the productivity of the firm:  $\left(\frac{\text{profits}}{\text{sales}}\right)_{i,t+1}$ , the ratio of earnings before interest and taxes to net sales during period  $t+1$ ;  $Q_{i,t+1}$ , the ratio of average market value during period  $t+1$  to book value at the end of period  $t$ . The inclusion of these two variables slightly reduces the  $\ln w_{i,t}$  coefficient for the high Z group and increases the statistical significance of the difference in this coefficient between likely constrained and likely unconstrained firms . After each regression in table X we report the p-value of a test of equality of the  $\ln w_{i,t}$  coefficient between the group of likely financially constrained observations and the other groups. In order to ensure that the test is robust, we adopt a bootstrap procedure. We randomly select subsamples of firm year observations that have same size of the groups considered in table X. We perform the estimation of the model on these subsamples, and we compute the differences in the coefficient of  $\ln w_{i,t}$ . We repeat this procedure 1000 times. The p-value reported in the table is the ratio of the number of times the difference in the random coefficients is greater than the difference between the likely financially constrained group and the other groups. Therefore it represents the p-value of a one sided test of the equality of the coefficients. The results show that the  $\ln w_{i,t}$  coefficient is significantly higher for the likely financially constrained group than for the complementary sample at more than the 0.1% significance level. The same result is obtained when we compare likely financially constrained and medium Z groups. Instead the  $\ln w_{i,t}$  coefficient is higher for the likely financially constrained group than for the high Z group at the 7.9%, 6% and 2.5% significance levels in the base model, the augmented model 1 and the augmented model 2 respectively. These results show that our method successfully identifies more financially constrained firms on a sample where the investment-cash flow approach fails. In the remainder of this paper we illustrate the estimations of several alternative specifications that prove the robustness of this result.

## A Box Cox transformation

Our theory predicts that a concave transformation of  $w_{i,t}$  is an explanatory variable in equation (42), but the degree of concavity depends on several factors, such as the type of adjustment costs of fixed capital and the type of financing imperfections and constraints on external financing. Therefore we perform a grid search on different values of  $\nu$ , the coefficient of the following Box-Cox transformation:

$$\frac{w_{i,t}^\nu - 1}{\nu} \quad (43)$$

The transformation which best fits the data for the overall sample is the one with  $\nu = 0.082$ . In table XI we report the corresponding estimated coefficients. This transformation confirms the previous results and does not improve the significance of the estimated parameters. Therefore we choose to keep the log transformation for the remainder of this paper.

## B Alternative selection of groups

We have followed so far the group selection criterion of Cleary (1999), which divides all the observations in three equally size groups. This criterion implicitly assumes that at least 33% of the firm-year observations are likely financially constrained, in the sense that, for a given level of expected productivity, they would invest and hire more if they had more cash available. This seems a very high percentage for an economy with well developed financial markets. Therefore we estimate equation (42) for a different selection of groups. In table XII we report the estimation results for 5 groups of firm-year observations selected according to the five quintiles of the Z score. The first quintile, which represents the 20% observations most likely to be financially constrained, has the highest sensitivity of employment to financial wealth. Moreover this sensitivity increases as we focus on the 10% and then the 5% observations most likely to be financially constrained, and it is always significantly higher than the sensitivity of the other groups of observations.

## C Alternative selection criteria

We have so far always used the same criterion, the  $Z$  score from the discriminant analysis, to select observations in likely financially constrained and unconstrained groups. This leaves us with the doubt that the results could depend on some specific feature of this selection criterion which is not related to financing constraints. Therefore in this subsection we sort firms according to two alternative criteria to sort financially constrained firms, size and zero dividend policy.<sup>19</sup> Small firms are more likely to face capital market imperfections and therefore should have a bigger wealth coefficient in equation (42). Firms that do not distribute dividends in any of the sample years should on average have a premium in the cost of external finance with respect to the firms that distribute dividends. This criterion was originally used by Fazzari Hubbard and Petersen (1988), and is consistent with the predictions of our theoretical model. Table XIII shows the estimation results for the size criterion. smaller firms have a significantly higher  $\Delta \ln \mathbf{w}_{i,t}$  coefficient than larger firms. Moreover conditional on a certain size class we also show that low  $Z$  observations have an higher  $\Delta \ln \mathbf{w}_{i,t}$  coefficient (except in the case of the largest class of firms with more than 10000 employees) than the complementary group. Importantly, for the medium to large size firms (250 to 10000 employees) the financing constraints test works and at the same time the estimated coefficient of the productivity shock  $\Delta \ln E_t(\theta_{i,t+1})$  is significantly greater than zero. This is consistent with the simulations results presented in the previous section, which showed that the amount of noise in the estimation of the productivity shock does not affect the power of the financing constraints test.

Table XIV shows the estimation results for the zero dividends criterion, and it confirms the results obtained before. The coefficient of  $\Delta \ln \mathbf{w}_{i,t}$  is positive and strongly significant for firms that do not distribute dividends in any sample year, while it is not significantly different from zero for the firms that distribute dividends.

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<sup>19</sup>Another commonly used selection criterion is whether or not firms issue bonds. Unfortunately we do not have access to this information for the firms in our sample.

## D Contemporaneous capital

In the previous estimations we maintained the assumption that capital is only productive one year after it is installed. In reality the time to install capital is likely to be less than one year, and therefore part of period  $t$  investment in fixed capital is likely to contribute to period  $t$  output. The omission of this component may bias the results of the estimations. In order to verify this, we repeat the analysis including contemporaneous rather than lagged fixed capital in the estimation of equation (42). The results are reported in table XVIII. The coefficient of contemporaneous fixed capital is much larger and significant than the coefficient of lagged fixed capital in the previous tables, even though it is still smaller than the value implied by the structural parameters.<sup>20</sup> Importantly, the coefficient of wealth is still able to identify financially constrained firms. The coefficient is significantly positive only for the likely financially constrained groups, and it increases as we consider the groups of 20%, 10% and 5% observations most likely to be financially constrained.

## E The Q model

Table IX shows that the investment-cash flow correlation is not a good proxy for the intensity of financing constraints. However our theoretical model with financing imperfections and quadratic adjustment costs of capital suggests that cash stock rather than cash flow should be included as a regressor in the  $Q$  model. Therefore in the upper part of table XVI we estimate the  $Q$  model adding the ratio of wealth over fixed capital among the regressors. Since the model does not predict whether financial wealth should enter in the  $Q$  model linearly or as a concave transformation, we perform a grid search over the full sample and find that the transformation with the Box-Cox coefficient equal to 0.21 achieves the best fit. The results confirm that the  $Q$  model is not useful to identify

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<sup>20</sup>The coefficient of  $\Delta \ln k_{i,t+1}$  is equal to  $\frac{\alpha}{1-\beta}$ , where  $\alpha$  and  $\beta$  are respectively the elasticities of capital and labour to output. Consistent estimates of  $\alpha$  and  $\beta$  for the balanced sample are found to be respectively 0.35 and 0.58, which implies that the coefficient of  $\Delta \ln k_{i,t+1}$  should be around 0.83.

financing constraints. In the first regression the wealth coefficient is marginally higher for the likely financially constrained group, but it is not significantly different from zero. In the second regression the wealth coefficient is not increasing in the intensity of financing constraints.

## **F Irreversibility of fixed capital**

The regression results presented in table XVI show that the correlation between fixed investment and internal finance (cash stock) is not a good measure of the intensity of financing constraints. This confirms the findings of Bond *et al* (2004), on a sample of large UK companies. Our theoretical model predicts that this happens because fixed capital is subject to a certain degree of irreversibility. This assumption is supported by a large empirical literature.<sup>21</sup> In this section we provide some simple anecdotal evidence about fixed capital irreversibility in our sample. We calculate the net investment rates of fixed capital and labour for every firm year observation. We eliminate fixed effects by subtracting from the net investment rates the firm specific averages and we normalize by dividing them for the firm specific standard deviations. As pointed out by Caballero, Engel and Haltiwanger (1995) if fixed capital is irreversible then the response of fixed capital investment to the productivity shocks is asymmetrical, and the distribution of the standardized investment rates is skewed to the left even though the distribution of productivity shocks is symmetrical. In the upper part of table XVII we report the summary statistics for the standardized net investment rates of fixed capital, labour, and the productivity shock for the subset of financially unconstrained observations (medium Z and high Z groups). We exclude the low Z group because the above analysis applies in the absence of financing constraints. The distribution of fixed capital net investment rates presents positive skewness despite the distribution of the productivity shocks has a very mild negative skewness. The Kurtosis of all series is very close to 3. This is consistent with the empirical evidence produced by Doms and Dunne (1998), who show that the investment rates at the plant

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<sup>21</sup>See footnote n.5.



level present both positive skewness and excess kurtosis. The latter indicates lumpy investment. The same authors show that investment rates aggregated at the firm level are smoother (lower kurtosis), but still present positive skewness.

In the lower part of table XVII we report the statistics for the financially unconstrained observations in the simulated economy with irreversibility of fixed capital. Both skewness and kurtosis are much higher than for the empirical data, because the data are “disaggregated at the productive unit level”. Simulated fixed capital is skewed because of the irreversibility constraint. Simulated variable capital is skewed despite is a completely frictionless factor of production, because of its complementarity with fixed capital. We observe a similar pattern also in the empirical data. Labour net investment rates are positively skewed, but less so than fixed capital net investment rates. This finding is consistent with the prediction of the model that the sensitivity of labour to internal finance is a useful indicator of financing constraints because labour is not subject to the same degree of irreversibility than fixed capital.

## **G Alternative dependent variable**

We have so far estimated equation (42) always using labour as the dependent variable. We claim that, in our dataset, labor is a flexible enough factor to be used as dependent variable in our test. This claim is supported by the statistics in table XVII. As an additional robustness check, we now consider alternative dependent variables for the estimation of equation (42). We first consider fixed capital. We expect that in this case the coefficient of internal finance should not be a good indicator of financing constraints. This is confirmed by the regression results in table XVIII, which show that the coefficient of  $\Delta \ln \mathbf{w}_{i,t}$  is almost identical for low Z firms and for the complementary firms. The same coefficient is higher for no dividend firms than for the other firms, but it is very imprecisely estimated and the null hypothesis of no significance cannot be rejected for both groups of firms. Finally, the coefficient is not significant for groups of firms selected according to size, with the exception of the firms between 250 and 2500 employees.

We then consider an alternative type of variable input, the R&D expenditure. We argue that R&D investment requires initial large fixed costs to establish a research unit in a company, but that conditional on these fixed costs R&D expenditure is more flexible than fixed capital expenditure. This is confirmed by the fact that the distribution of the standardized net percentage changes in R&D expenditures (calculated only for firms that present positive R&D expenditures in all the sample years) is much less skewed than the distribution of the standardized investment rates of both fixed capital and labour (see table XIX). Therefore we consider an extended production function that includes also R&D expenditures, called  $r_t$ :

$$y_t = \theta_t r_t^\eta k_t^\alpha l_t^\beta \text{ with } \alpha + \beta + \eta < 1 \quad (44)$$

In this case the reduced form variable capital equation is the following:

$$\ln r_{i,t+1} = a_i + d_t + \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}) + \pi_2 \ln k_{i,t} + \pi_3 \ln l_{i,t+1} + \pi_4 \ln w_{i,t} + \epsilon_{i,t+1} \quad (45)$$

The estimation results are in table XX. The sensitivity of *R&D* investment to internal finance is generally positive, and much higher for firms likely to be financially constrained (low *Z* firms, no dividends firms and smaller firms) than for the other firms, the difference being strongly statistically significant. The sensitivity of *R&D* investment to internal finance is the strongest for the group of firms smaller than 250 employees, while is not significantly different from zero at the 5% significance level for the group of firms larger than 10.000 employees.

## VI Conclusions

In this paper we develop a new test to detect financing constraints on firm investment. We develop a structural model of firm investment with financing imperfections and with fixed and variable capital. We solve the model using a numerical method and we simulate two industries, one with quadratic adjustment costs, the other with the irreversibility of fixed capital. Both industries are calibrated to match the US industry. The results of

the simulations show that if fixed capital is irreversible then the correlation between fixed investment and internal finance cannot detect financially constrained firms, even if firms investment opportunities are perfectly observable. This is because changes in internal finance mainly affect variable capital investment for financially constrained firms.

The same simulations show that the correlation between variable capital investment and internal finance is a useful indicator of the intensity of financing constraints. This is true even when firm investment opportunities are very noisily estimated, and regardless of the type of adjustment costs of fixed capital.

We confirm empirically the predictions of the model with a balanced panel data of US companies (1996 to 2003). The sample, drawn from the *Worldscope Database*, is analogous to the one used by Cleary (1999), and therefore has the appealing feature of allowing a comparison with the results of the previous literature.

Our analysis confirms Cleary's (1999) finding that fixed investment is most sensitive to cash flow for the firms less likely to be financially constrained. Importantly, we show that our new test of financing constraints instead yields consistent results: the sensitivity of variable capital investment to internal finance is always significantly higher for firms more likely to be subject to capital market imperfections. This is confirmed no matter whether we select more constrained firms using dividend policy, or size, or whether we use a combination of these two criteria. Finally, one additional advantage of our new test of financing constraints is that it is more easily applied to smaller firms, because it does not need the estimation of Tobin's marginal  $Q$ . It only needs the estimation of the expected marginal productivity of capital, and it works even if the latter is very noisy.

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## Appendix 1

We describe the variables used in the empirical section of the paper. The discriminant analysis employs the same variables used by Cleary (1999), with the exception of the fixed charge coverage ratio, which is not corrected for taxation:

$Current$  = current assets/current liabilities;

$Debt$  = current portion of long term debt/total assets;

$FCCOV$  = earnings before interest and taxes/(interest expense + preferred dividends payments);

$NI\%$  = (net income before extraordinary items  $\pm$  extraordinary items and discontinued operations)/net sales;

$GrSales$  = (net sales<sub>t</sub> - net sales<sub>t-1</sub>)/net sales<sub>t-1</sub>;

$W = w_{i,t}$  (see below).

The variables used for the financing constraints tests are listed below:

$i_{i,t}$  = real value of gross capital expenditure during year t;

$k_{i,t}$  = book value, in real terms, of fixed assets at the beginning of year t;

$c_{i,t}$  = real value of: net income + depreciation and amortization expenses + change in deferred taxes during year t;

$l_{i,t}$  = number of employees at the end of year t;

$\left(\frac{\text{profits}}{\text{sales}}\right)_{i,t}$  =earnings before interest and taxes/net sales during period  $t$ ;

$Q_{i,t}$  =average market value during period  $t$  divided by book value at the beginning of period  $t$ ;

$w_t$  =real value of :cash+short term financial assets-short term loans+0.5\*inventories+0.7\*accounts receivable. all stocks are measured at the beginning of period  $t$ ;

$r_t$  =real value of R&D expenditure during year  $t$ ;

$E_{t-1}(\theta_{i,t})$ =expected productivity at time  $t$  conditional on the information set in period  $t - 1$ . In order to compute it we estimate the following production function:

$$\ln y_{i,t} = a_i + d_t + \alpha \ln k_{i,t} + \beta \ln l_{i,t} + \varepsilon_{i,t} \quad (46)$$

$y_{i,t}$  is the real value of net sales during period  $t$ . We estimate  $\hat{\alpha}$  and  $\hat{\beta}$  from equation (46) using a SYSTEM GMM estimation method (Blundell and Bond, 1998 and 2000), separately for 6 groups of firms selected according to homogeneous types of activity. We use the estimates  $\hat{\alpha}$ ,  $\hat{\beta}$  and  $\hat{d}_t$  to calculate the total factor productivity.  $\ln E_{t-1}(\theta_i)$  is the firm specific component of the total factor productivity in period  $t - 1$  (aggregate and sector specific components are eliminated using dummy variables).

## Appendix 2

We describe the specification tests performed to determine the estimation method of equation (42). We expect that some or all the right hand side variables may be endogenous and correlated to  $\varepsilon_{i,t+1}$ , because the term  $\ln E_t(\theta_{i,t+1})$  does not capture entirely the unobservable productivity shock. Moreover they are also most likely correlated with the firm specific effect  $a_i$ . In this case a suitable estimation strategy is to first difference equation (42) to eliminate the unobservable firm specific effect  $a_i$ , and then estimate it with a GMM estimation technique, using the available lagged levels of the explanatory variables as instruments for their first differences. In this case the set of instruments is different for each year, and equation (42) is estimated as a system of cross sectional equations, each one corresponding to a different period  $t$  (Arellano and Bond, 1991). More recent lags are likely to be better instruments, but they may be correlated with the error term if this is itself autocorrelated. The test of overidentifying restrictions can be used to assess the orthogonality of the instruments with the error term. Moreover, under the assumption that  $E(\Delta z_{i,t-j}, a_i) = 0$ , with  $z = \{\ln E_t(\theta_{i,t+1}), \ln k_{i,t}, \ln w_{i,t}\}$ ,  $\Delta z_{i,t-j}$  are valid instrument for equation (42) estimated in levels. Blundell and Bond (1998) propose a SYSTEM GMM estimation technique that uses both the equation in level (instrumented using lagged first differences), and the equation in first differences (instrumented using lagged levels).

In table XXI we test the validity of the instruments for the estimation of equation (42). From the first stage regression we report, for each regressor, the *partial R<sup>2</sup>* measure proposed by Shea (1997). The higher is the *partial R<sup>2</sup>*, the more the instruments are correlated with the instrumented regressor. Moreover we test the exogeneity of the instruments with the Hansen test of overidentifying restrictions, which is robust to heteroskedasticity and serial correlation of unknown form. We consider both the overall sample and the subsamples of firms selected according to the Z score.

$t - 1$  to  $t - 3$  lagged first differences are reasonably good instruments for the equation in levels for both  $\ln E_t(\theta_{i,t+1})$  and  $\ln w_{i,t+1}$ , but their exogeneity is rejected by the orthogonality test.  $t - 2$  to  $t - 3$  lagged levels are less good instruments for the equation in first differences. This was expected given that the series in levels are very persistent. In any case also these instruments are rejected by the orthogonality test. We also report the orthogonality test for longer lags of the instruments, and we find that in most cases also -5 and -6 lags are rejected. The presence of different growth path across firms, which implies the presence of firm specific effects in the first difference equation, can explain these findings. Consider for example the case in which the error  $\epsilon_{i,t+1}$  includes an unobservable productivity trend:

$$\epsilon_{i,t+1} = \kappa_i t + v_{i,t} \quad (47)$$

Since we impose that the production function has decreasing returns to scale, equation (47) implies that firms are allowed to expand at different growth rates  $\kappa_i$ . In this case first differencing equation (42) yields:

$$\Delta \ln l_{i,t+1} = \Delta d_t + \pi_1 \Delta \ln E_t(\theta_{i,t+1}) + \pi_2 \Delta \ln k_{i,t} + \pi_3 \Delta \ln w_{i,t+1} + \kappa_i + \Delta v_{i,t} \quad (48)$$

If  $\kappa_i$  in equation (48) is correlated with  $a_i$  in equation (42) then lagged levels are not good instruments for equation (48) and lagged first differences are not good instruments for equation (42). The assumption that equation (48) contains a firm specific effect is consistent with the previous literature on firm investment. Reduced form investment equations, where the dependent variable is the fixed capital gross investment rate, usually present a firm specific effect which is correlated with the other regressors (see Bond, 2002, for an example). In equation (48) the dependent variable is an approximation of the net investment rate in labour input. In order to eliminate the firm specific effect  $\kappa_i$  we first differentiate equation (48). Table XXII shows that the lagged second differences of  $\ln E_t(\theta_{i,t+1})$ ,  $\ln k_{i,t}$  and  $\ln w_{i,t+1}$  are valid instruments for the equation in first differences, and that the lagged first differences are valid instruments for the equation in second differences. Therefore we decide to use both sets of restrictions and to estimate equation (48) with the System GMM estimation technique.

## Appendix 3

We provide both an intuitive and a more formal proof of proposition 1. For simplicity we assume that  $\theta_t$  follows a two state stationary stochastic process without persistency, even though the proof can be generalized also for persistent stationary stochastic processes:

$$\theta_t \in \{\theta_L, \theta_H\} \quad (49)$$

$$\theta_H > \theta_L > 0 \quad (50)$$

$$pr(\theta_t = \theta_L) = 0.5; pr(\theta_t = \theta_H) = 0.5 \quad (51)$$

In this case  $\theta_t$  is not a state variable of the problem. If equation (14) is not binding with equality, the solution is given by the capital levels  $l_{t+1}^*(k_t)$  and  $k_{t+1}^*(k_t)$ . The functional form that relates  $l_{t+1}^*$  and  $k_{t+1}^*$  to  $k_t$  depends on the type of adjustment costs.



We defined  $\underline{w}_t^*(k_t)$  as the minimum level of wealth that allows to finance  $l_{t+1}^*(k_t)$  and  $k_{t+1}^*(k_t)$ . Therefore the financing constraint is binding when  $w_t < \underline{w}_t^*(k_t)$ . In this case  $\lambda_t$  is positive and it is jointly determined with  $l_{t+1}^c$  and  $k_{t+1}^c$  by the three following equations:

$$E_t \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) - UK = \Omega_t + \Gamma_t \lambda_t \quad (52)$$

$$E_t \left( \frac{\partial y_{t+1}}{\partial l_{t+1}} \right) = UL + \frac{R(R - \tau_l) \lambda_t - \gamma \text{cov}(\phi_{t+1}, y_{t+1}^l)}{1 + \gamma E_t(\phi_{t+1})} \quad (53)$$

$$D_k k_{t+1} + D_l l_{t+1} + \mu(i_t) = w_t \quad (54)$$

$$\Gamma_t \equiv \frac{R[R + R\mu'(i_t) - \tau_k]}{1 + \gamma E_t(\phi_{t+1})}$$

$$\Omega_t \equiv R\mu'(i_t) + \frac{\gamma(1 - \delta_k) [1 + E_t(\phi_{t+1})] E_t[\mu'(i_{t+1})] - \{ \gamma [(1 - \delta_k) \text{cov}[\phi_{t+1}, \mu'(i_{t+1})] + \text{cov}(\phi_{t+1}, y_{t+1}^k)] \}}{1 + \gamma E_t(\phi_{t+1})}$$

Before providing a sketched proof of proposition 1, we illustrate an intuitive argument of why  $l_{t+1}(w_t | k_t, \theta_t)$  is positively related to  $w_t$ , conditional on a binding financing constraint ( $w_t < \underline{w}_t^*$  and  $\lambda_t > 0$ ). We illustrate the argument for the different types of adjustment costs of fixed capital.

**i) Irreversibility constraint.** If the irreversibility constraint is not binding, then a reduction in  $w_t$  causes a fall in both  $l_{t+1}$  and  $k_{t+1}$ . If the irreversibility constraint binds, then  $k_{t+1} = (1 - \delta)k_t$ , but still a reduction in wealth causes a reduction in  $l_{t+1}$ . An analogous argument applies when  $w_t$  increases.

**ii) Convex adjustment costs.** If optimal investment choices require an increase in fixed capital, then  $i_t^c > 0$ . Equation (52) implies that in this case the excess productivity of capital,  $E_t \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) - UK$ , is compensated by the net marginal adjustment costs ( $\Omega_t$ ) and the financing constraints costs ( $\lambda_t$ ). As  $w_t$  decreases, The firm will find convenient to reduce  $i_t^c$ . This will reduce fixed capital adjustment costs, and the firm will be able to compensate the reduction in wealth with a smaller reduction in  $i_{t+1}$  and  $l_{t+1}$ . In equation (52) the increase in  $\lambda_t$  is partly offset by a reduction in  $\Omega_t$ , and partly by an increase in  $E_t \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) - UK$ , but still there is a positive relationship between  $l_{t+1}$  and  $w_t$ .

If optimal investment choices require a decrease in fixed capital, then  $i_t^c < 0$ . In this case both  $E_t \left( \frac{\partial y_{t+1}}{\partial k_{t+1}} \right) - UK$  and  $\Omega_t$  are negative. When  $w_t$  decreases then the firm must decrease fixed investment, but this increases adjustment costs. Therefore most of the reduction in wealth will be absorbed by a reduction in variable capital.

**iii) fixed adjustment costs.** We assume fixed symmetrical adjustment costs:

$$\mu(i_t = 0) = 0; \quad \mu(i_t \neq 0) = F$$

We do not provide a formal treatment of this case, but the existing literature on the subject (see Caballero, 1997, for a review) shows that in this case optimal investment

choices imply “investment bunching”. Optimal fixed investment  $i_t$  is zero if the loss in the value function from not investing is smaller than  $F$ . Otherwise investment is different from zero and equal to internal solution of the firm maximization problem. Therefore there are three distinct cases:

**positive action:** suppose that optimal fixed investment is positive ( $i_t^* > 0$ ) conditional on  $w_t \geq \underline{w}_t^*$  ( $\lambda_t = 0$ ). The reduction in wealth below  $\underline{w}_t^*$  initially reduces fixed and variable capital investment and increases  $\lambda_t$ . At some point the gain from investing will become too small to justify the fixed cost  $F$ . In this case  $i_t^c$  falls to zero, and also  $l_{t+1}$  falls down because the two factors are complementary. Then further decreases in wealth only affect variable capital. If the fixed cost  $F$  is small relative to  $w_t$ , at some point it may be optimal to pay it to reduce fixed capital together with variable capital. But if  $F$  is large then this may not be possible, and eventually the firm may be forced to exit from activity. However in all the previous cases the reduction in  $w_t$  always causes a reduction in  $l_{t+1}$ . Therefore the monotonous relationship between  $l_{t+1}(w_t | k_t, \theta_t)$  and  $w_t$  also applies in the case of fixed costs. But in this case  $\frac{\partial l_{t+1}(w_t | k_t, \theta_t)}{\partial w_t}$  is not continuous in  $w_t$ , and this could induce biases in the estimation of the variable capital equation. Therefore if fixed costs are expected to be important, the estimation should yield better results if it is performed conditional on positive or zero fixed capital investment, whose probabilities can themselves be estimated as an hazard function.

**Inaction,  $i_t^* = 0$ , or negative action,  $i_t^* < 0$ .** Essentially the above analysis applies.

The above discussion clarifies that, for a given productivity shock, variable capital investment is directly related to financial wealth when the financing constraints is binding. For a more formal proof of proposition 1, let’s consider the solution of the model for the quadratic adjustment costs and irreversibility cases, which are those used in the simulation section of the paper.

**Quadratic adjustment costs.** In this case marginal adjustment costs are smooth and continuous in  $i_t$ . We differentiate  $E_t(\Psi_{t+1}^l)$ , as defined by equation (13), by  $w_t$  :

$$\frac{\partial E_t(\Psi_{t+1}^l)}{\partial w_t} = \frac{R^2 D_l (\partial \lambda_t / \partial w_t) - \gamma [\partial cov(\frac{\partial y_{t+1}}{\partial l_{t+1}}, \phi_{t+1}) / \partial w_t]}{1 + \gamma E_t(\phi_{t+1})} - \frac{\gamma E_t(\Psi_{t+1}^l) (\partial E_t(\phi_{t+1}) / \partial w_t)}{[1 + \gamma E_t(\phi_{t+1})]^2} \quad (55)$$

since the production function has decreasing returns to scale it follows that as wealth and investment decrease the total factor productivity must increase, and with it also the shadow cost of a binding financing constraint  $\lambda_t$ . therefore:

$$\left( \frac{\partial \lambda_t}{\partial w_t} \mid w_t \leq \underline{w}_t^* \right) < 0 \quad (56)$$

Moreover, the lower is  $w_t$ , the higher the probability that also future wealth will be lower, and this increases future expected financing constraints:

$$\left( \frac{\partial E_t(\phi_{t+1})}{\partial w_t} \mid w_t \leq \underline{w}_t^* \right) = \left( \frac{\partial R \sum_{j=0}^{\infty} E_t(\lambda_{t+1+j})}{\partial w_t} \mid w_t \leq \underline{w}_t^* \right) \leq 0 \quad (57)$$

Finally, let's consider the covariance term  $cov\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}, \phi_{t+1}\right)$ . Conditional on  $\theta_{t+1} = \theta_H$  we have that  $\frac{\partial y_{t+1}}{\partial l_{t+1}} > E_t\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}\right)$ . Moreover the positive productivity shock increases wealth and reduces future expected financing constraints:  $\phi_{t+1} < E_t\left(\phi_{t+1}\right)$ . Using the symmetric argument for  $\theta_{t+1} = \theta_L$ , it is easy to show that  $cov\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}, \phi_{t+1}\right) < 0$ . Moreover the lower is the wealth, the more expected financing constraints are sensitive to a change in wealth, and therefore such covariance becomes more negative as  $w_t$  decreases:

$$\left(\frac{\partial \left|cov\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}, \phi_{t+1}\right)\right|}{\partial w_t} \mid w_t \leq \underline{w}_t^*\right) > 0 \quad (58)$$

Now we can turn to the sign of  $\frac{\partial E_t(\Psi_{t+1}^l)}{\partial w_t}$ . According to the discussion above, the first term on the right hand side of equation (55) is negative, while the second is positive. The numerical solution of the problem shows that the sum of the two terms is always negative. We are not able to provide an analytical proof of it, but the intuition is as follows: changes in wealth affect current financing constraints more than future expected financing constraints, because the productivity shock is stationary. Therefore  $|\partial \lambda_t / \partial w_t|$  is large relative to  $|\partial E_t(\phi_{t+1}) / \partial w_t|$ . Moreover  $E_t(\Psi_{t+1}^l)$  is typically much smaller than one, and therefore the last term at the right hand side of (55) is always smaller than the first. Therefore:

$$\frac{\partial E_t(\Psi_{t+1}^l \mid \theta_t)}{\partial w_t} < 0$$

Which proves proposition 1.

**Irreversibility of fixed capital.** In this case the assumption of smooth marginal adjustment costs is substituted by the irreversibility constraint (31), and the budget constraint becomes the following:

$$d_t + l_{t+1} + k_{t+1} = w_t + b_{t+1}/R \quad (59)$$

The firm maximizes the value function (9) subject to (5), (10), (31) and (59). Let  $\lambda_t$ ,  $\phi_t$  and  $\mu_t$  be the Lagrangian multipliers associated respectively with constraints (5), (10) and (31). In this case (11) and (13) are still the first order conditions of the problem. The only change in the solution regards (12) and (14), which become the following:

$$E_t\left(\frac{\partial y_{t+1}}{\partial k_{t+1}}\right) = UK + R\mu_t - \Phi_t E_t(\mu_{t+1}) + E_t(\Psi_{t+1}^k) \quad (60)$$

$$D_k k_{t+1} + D_l l_{t+1} \leq w_t - d_t \quad (61)$$

Where:

$$\Phi_t = \frac{\gamma(1 - \delta_k) [1 + E_t(\phi_{t+1})]}{1 + \gamma E_t(\phi_{t+1})} \quad (62)$$

$$E_t(\Psi_{t+1}^k) = \frac{R[R + R\mu_t - \tau_k] \lambda_t + \gamma cov\left(\phi_{t+1}, \frac{\partial y_{t+1}}{\partial k_{t+1}}\right)}{1 + \gamma E_t(\phi_{t+1})} \quad (63)$$

Since  $E_t(\Psi_{t+1}^l)$  is still defined by equation (13), the proof of proposition 1 is the same as for the case of quadratic adjustment costs.

## Appendix 4

We briefly describe the method we use to solve the dynamic maximization problem of the firm. We write the value function (9) in recursive form, after  $\theta_t$  is realized and before knowing if the firm is liquidated at time  $t$  :

$$V_t(w_t, \theta_t, k_t) = \gamma w_t + (1 - \gamma) \left\{ \underset{k_{t+1}, l_{t+1}, b_{t+1}}{MAX} d_t + \frac{1}{R} E_t [V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})] \right\} \quad (64)$$

We discretise the state space of  $w_t$ ,  $\theta_t$  and  $k_t$ , then we guess the value of  $E_t [V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})]$ , and based on this guess we find the policy functions  $k_{t+1}(w_t, \theta_t, k_t)$ ,  $l_{t+1}(w_t, \theta_t, k_t)$  and  $b_{t+1}(w_t, \theta_t, k_t)$  that maximize  $V_t(w_t, \theta_t, k_t)$ .

We use the maximized value function to reformulate a guess of  $E_t [V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})]$ , and we repeat this procedure until convergence is achieved.

# Tables

Table I: Calibrated parameters and matched moments

	Parameter values		Empirical restriction	Matched moments		
	Q. adj. costs	Irrev.		Data	Q.a.c	Irr.
$r$	0.065	0.065	Real interest rate	0.065	0.065	0.065
$\alpha$	0.105	0.08	Returns to scale	0.97	0.97	0.97
$\beta$	0.865	0.89	Fixed capital/variable capital	0.5-0.7	0.68	0.51
$\delta_k$	0.12	0.12	Depr. of fixed capital	0.12	0.12	0.12
$\delta_l$	1	1	Depr. of variab. capital	1	1	1
$b$	0.00004	0	Average(I/K)	0.145	0.20	0.17
$\rho$	0.68	0.83	Std.(I/K)	0.139	0.26	0.16
$\sigma_\varepsilon$	0.063	0.031	Autocorr.(I/K)	0.239	0.15	0.15
$\xi$	0.1	0.1	Std.(cash flow)	0.292	0.34	0.28
$\tau$	$1-\delta_k$	$0.75(1-\delta_k)$	Debt/assets ratio	0.2	0.17	0.15
$\gamma$	0.94	0.94	6% firms exit each year	6%	6%	6%

Table II: The  $q$ -model with financial wealth . No measurement errors

<b>regression:</b> $i_{i,t} = \alpha_0 + \alpha_1 q_{i,t} + \alpha_2 w_{i,t} + \epsilon_{1,i,t}$		
	Quad. adjust. costs	Irreversibility
unconstrained firms		
<i>constant</i>	-26427	-69815
$q_{i,t}$	25762	68634
$w_{i,t}$	0.008	0.063
$R^2$	0.96	0.52
all constrained firms		
<i>constant</i>	-23026	-5616.53
$q_{i,t}$	21817	5665
$w_{i,t}$	0.232	0.065
$R^2$	0.81	0.64
$\hat{\alpha}_2$ : financially constr./compl. sample	0.232/0.008	0.065/0.063
$\hat{\alpha}_2$ : 80% most constr./compl. sample	0.248/0.014	0.059/0.064
$\hat{\alpha}_2$ : 60% most constr./compl. sample	0.328/0.019	0.065/0.055
$\hat{\alpha}_2$ : 40% most constr./compl. sample	0.381/0.025	0.064/0.050
$\hat{\alpha}_2$ : 20% most constr./compl. sample	0.464/0.029	0.033/0.048

OLS estimates on simulated data. All the estimated coefficients are statistically significant. The lower part of the table compares the estimates of  $\alpha_2$  for different subsamples selected according to the intensity of financing constraints. All the differences across coefficients are statistically significant.

Table III: The variable capital model with financial wealth. No measurement errors

<b>regression:</b> $\ln l_{i,t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}) + \pi_2 \ln k_{i,t+1} + \pi_3 \ln w_{i,t} + \epsilon_{2i,t+1}$		
	Quad. adjust. costs	Irreversibility
Unconstrained firms		
<i>constant</i>	-1.54	-1.63
$\ln E_{i,t}(\theta_{t+1})$	7.41	9.07
$\ln k_{i,t+1}$	0.78	0.73
$\ln w_{i,t}$	-0.00003	0.0003
$R^2$	1	1
Constrained firms		
<i>constant</i>	-0.073	-1.44
$\ln E_{i,t}(\theta_{t+1})$	2.26	6.88
$\ln k_{i,t+1}$	0.078	-0.22
$\ln w_{i,t}$	0.78	0.94
$R^2$	0.99	0.97
$\hat{\pi}_3$ : financially constr./compl. sample	0.78/0.00	0.94/0.00
$\hat{\pi}_3$ : 80% most constr./compl. sample	0.78/0.00	1.04/0.04
$\hat{\pi}_3$ : 60% most constr./compl. sample	0.88/0.04	1.05/0.08
$\hat{\pi}_3$ : 40% most constr./compl. sample	0.91/0.07	1.06/0.13
$\hat{\pi}_3$ : 20% most constr./compl. sample	0.95/0.09	1.10/0.21

TOLS estimates on simulated data.  $\ln k_t$  is instrumented by  $\ln k_{t-1}$ . All the estimated coefficients are statistically significant. The lower part of the table compares the estimates of  $\pi_3$  for different subsamples selected according to the intensity of financing constraints. All the differences across coefficients are statistically significant.

Table IV: Fixed investment-net worth correlation conditional on the irreversibility constraint

<b>regression:</b> $i_{i,t} = \alpha_0 + \alpha_1 q_{i,t} + \alpha_2 w_{i,t} + \epsilon_{1,i,t}$				
	Intensity of financing constraint ( $\bar{\lambda}$ )	% of firms with both constraints binding at times $t$ or $t-1$	$\hat{\alpha}_2 \mid (i_{i,t} > 0 \text{ and } i_{i,t-1} > 0)$	$\hat{\alpha}_2 \mid (i_{i,t} > 0 \text{ and } i_{i,t-1} = 0)$
Unconstrained firms	0	0	0.070	0.087
1 <sup>st</sup> quintile of $\lambda$	0.0043	16.2	0.088	0.027
2 <sup>nd</sup> quintile of $\lambda$	0.0130	24.8	0.080	0.022
3 <sup>rd</sup> quintile of $\lambda$	0.0197	26.8	0.171	0.020
4 <sup>th</sup> quintile of $\lambda$	0.0312	43.4	0.212	0.088
5 <sup>th</sup> quintile of $\lambda$	0.0677	67.0	0.139	0.050

OLS estimates on simulated data. All the estimated coefficients are statistically significant.

Table V: The variable capital model with financial wealth. Measurement errors

<b>regression:</b> $\ln l_{i,t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}^*) + \pi_2 \ln k_{i,t+1}^* + \pi_3 \ln w_{i,t}^* + \epsilon_{2i,t+1}$								
	Uncorrelated errors				Correlated errors			
	sign./noise=4		sign./noise=1		sign./noise=4		sign./noise=1	
	Q.a.c.	Irr.	Q.a.c.	Irr.	Q.a.c.	Irr.	Q.a.c.	Irr.
unconstrained firms								
<i>const</i>	-1.61	-1.66	0.66	0.86	-0.34	1.43	3.11	5.28
$\ln E_t(\theta_{i,t+1}^*)$	8.62	9.61	6.91	6.91	1.61	-2.34	-14.84	-16.12
$\ln k_{i,t+1}^*$	0.67	0.59	0.43	0.46	1.12	1.39	2.04	1.95
$\ln w_{i,t}^*$	0.04	0.09	0.07	0.11	-0.11	-0.29	-0.39	-0.45
all constrained firms								
<i>const</i>	0.02	-1.18	2.24	3.55	0.46	-0.84	3.03	4.49
$\ln E_{i,t}(\theta_{t+1}^*)$	2.46	6.38	3.17	4.52	0.40	5.38	-6.75	-1.17
$\ln k_{i,t+1}^*$	0.33	-0.06	0.32	0.16	0.60	-.02	1.37	0.60
$\ln w_{i,t}^*$	0.52	0.80	0.18	0.21	0.34	0.78	-0.26	0.018
coefficient of $\ln w_{i,t}^*$								
All const./c. sample	.52/.04	.80/.09	.18/.07	.21/.11	.34/-.11	.78/-.29	-0.26/-0.39	.02/-.45
80% m.c./c.sample	.57/.04	.77/.11	.19/.07	.16/.10	.44/-.11	.76/-.33	-0.21/-0.39	.09/-.49
60% m.c./c.sample	.70/.06	.76/.13	.20/.08	.15/.11	.65/-.11	.77/-.30	0.03/-.40	.14/-.53
40% m.c./c.sample	.76/.08	.78/.16	.21/.09	.16/.13	.78/-.10	.79/-.24	.27/-.40	.16/-.53
20% m.c./c.sample	.69/.10	.75/.21	.14/.10	.14/.13	.72/-.10	.76/-.10	.22/-.39	.10/-.49

TOLS estimates on simulated data.  $\ln k_t$  is instrumented by  $\ln k_{t-1}$ . All the estimated coefficients are statistically significant. The lower part of the table compares the estimates of  $\pi_3$  for different subsamples selected according to the intensity of financing constraints. All the differences across coefficients are statistically significant. Q.a.c.: industry with quadratic adjustment costs of fixed capital. Irr.: industry with irreversibility of fixed capital

Table VI: The  $q$  model with financial wealth- measurement errors

<b>regression:</b> $i_{i,t} = \alpha_0 + \alpha_1 q_{i,t}^* + \alpha_2 w_{i,t}^* + \epsilon_{1,i,t}$								
	Uncorrelated errors				Correlated errors			
	sign./noise=4		sign./noise=1		sign./noise=4		sign./noise=1	
	Q.a.c.	Irr.	Q.a.c.	Irr.	Q.a.c.	Irr.	Q.a.c.	Irr.
coefficient of $w_{i,t}^*$								
All const./c. sample	.213/.009	.059/.054	.088/.008	.027/.029	.210/.006	.052/.057	.078/-.001	.022/.030
80% m.c./c.sample	.228/.013	.052/.052	.096/.008	.018/.026	.224/.012	.051/.049	.082/.000	.010/.026
60% m.c./c.sample	.298/.018	.057/.048	.111/.008	.019/.027	.282/.016	.056/.046	.073/.001	.008/.026
40% m.c./c.sample	.342/.023	.056/.045	.121/.009	.019/.026	.323/.021	.056/.043	.066/.003	.008/.026
20% m.c./c.sample	.391/.027	.029/.044	.112/.011	.010/.027	.350/.026	.027/.043	.000/.005	.002/.026

OLS estimates on simulated data. The table compares the estimates of  $\alpha_2$  for different subsamples selected according to the intensity of financing constraints. All the differences across coefficients are statistically significant.

Table VII: The variable capital model with financial wealth - misspecification

<b>regression:</b> $\ln k_{i,t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}) + \pi_2 \ln w_{i,t} + \epsilon_{4i,t+1}$				
	Quadratic adj. costs		Irreversibility	
	No noise	Corr. errors (s/n)=1	No noise	Corr. errors (s/n)=1
coefficient of $\ln w_{i,t}$				
All const./c.sample	.73/.23	.63/.32	.56/.45	.46/.57
80% m.c./c.sample	.74/.26	.63/.33	.55/.56	.27/.51
60% m.c./c.sample	.73/.30	.59/.35	.54/.57	.25/.53
40% m.c./c.sample	.72/.34	.56/.37	.52/.57	.26/.53
20% m.c./c.sample	.68/.41	.46/.40	.58/.61	.29/.54

TSLS estimates on simulated data.  $\ln k_t$  is instrumented by  $\ln k_{t-1}$ . The table compares the estimates of  $\pi_3$  for different subsamples selected according to the intensity of financing constraints. All the differences across coefficients are statistically significant.

Table VIII: Summary statistics

	Low $Z_{FC}$ (likely financially constrained)	Medium $Z_{FC}$	High $Z_{FC}$
Mean net fixed assets	670117 (4365691)	983064 (3763386)	956132 (4151552)
Median net fixed assets	36108	130400	81568
Median number of employees	1476	3200	1900
Current ratio <sup>1</sup>	2.43(1.99)	2.16(1.41)	2.73(2.03)
Debt ratio <sup>1</sup>	0.35(0.22)	0.26(0.15)	0.14(0.14)
Fixed charge coverage ratio <sup>2</sup>	-5.9(25)	9.6(40)	44.4(89)
Net income margin (%) <sup>2</sup>	-0.17(0.41)	0.035(0.044)	0.063(0.19)
Market-to-book ratio <sup>2</sup>	1.34(1.58)	1.32(1.11)	2.19(1.71)
Sales growth <sup>2</sup>	-0.024(0.23)	0.079(0.15)	0.23(0.29)
Slack/ $K^2$	2.16(3.55)	1.64(2.98)	2.79(4.61)
Cash Flow/ $K^2$	-0.31(1.45)	0.50(0.76)	0.81(1.22)
Investment/ $K^1$	0.22(0.22)	0.23(0.20)	0.33(0.27)
Discriminant score <sup>2</sup>	-2.27	-0.29	0.90

Standard deviations in parenthesis. 1) Largest 1% excluded from the computation; 2) largest 1% in absolute value excluded from the computation.



Table IX: The Q model with cash flow added as an explanatory variable

dependent variable: $\left(\frac{i}{k}\right)_{i,t}$	$Q_{i,t}$	$\left(\frac{cf}{k}\right)_{i,t}$	$Adj. R^2$	N. obs.
1988-1994 sample, fixed effects estimates (from Cleary, 1999)				
All firms	0.024 (12.3)	0.096 (29.7)	0.1176	9219
Low $Z$ score (financially constr.)	0.020 (5.8)	0.064 (14.0)	0.0778	3073
Medium $Z$ score	0.028 (7.7)	0.090 (14.1)	0.0928	3073
High $Z$ score	0.018 (5.8)	0.153 (23.5)	0.1824	3073
1997-2003 sample, fixed effects estimates				
All firms	0.049 (22.18)	0.032 (15.12)	0.1547	10191
Low $Z$ score (financially constr.)	0.043 (10.06)	0.006 (1.76)	0.0973	3149
Medium $Z$ score	0.036 (7.02)	0.027 (4.37)	0.1107	3280
High $Z$ score	0.030 (7.42)	0.103 (13.30)	0.2146	3169
1997-2003 sample, GMM estimates				
	$Q_{i,t}$	$\frac{\text{Cash Flow}}{\text{Net fixed assets}}$	Overid. t.	N. obs.
All firms	0.025 (1.39)	-0.036 (-0.8)	0.057	8996
Low $Z$ score (financially constr.)	0.044 (2.80)	0.007 (0.4)	0.282	2848
Medium $Z$ score	0.085 (3.39)	-0.007 (-0.3)	0.102	3146
High $Z$ score	0.024 (2.09)	0.063 (2.6)	0.120	2959

The outliers of the 1988-1994 sample are winsorised, see Cleary (1999) for details. The 1997-2003 sample is cleaned of outliers before estimation. We consider as outliers in each variable all observations above the 99% percentile and below the 1% percentile.

$\left(\frac{i}{k}\right)_{i,t}$  = gross fixed capital expenditure during period  $t$  divided by net fixed assets at the beginning of period  $t$ .  $Q_{i,t}$  = market value divided by the book value of firm  $i$  at the beginning of period  $t$ .  $\left(\frac{cf}{k}\right)_{i,t}$  = cash flow during period  $t$  divided by net fixed assets at the beginning of period  $t$ .

Both fixed effects and GMM estimates include time dummies. The GMM estimates are computed by first differencing and then using  $t-3$  and  $t-4$  lagged levels as instruments of the dependent variables. The Overidentification test reports the p-value of the Hansen test of overidentifying restrictions, which is robust to autocorrelation and heteroskedasticity of unknown form.

Table X: The new test of financing constraints based on variable capital

Dependent variable: $\Delta \ln l_{i,t+1}$				
	Base Model			
	Likely fin. constrained (low Z)	Compl. sample	Medium Z	High Z
$\Delta \ln k_{i,t}$	.062(2.4)	.059(3.1)	.026(1.1)	.067(2.7)
$\Delta \ln w_{i,t}$	<b>.049(4.0)</b>	<b>.018(2.3)</b>	<b>.004(0.5)</b>	<b>.028(2.2)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	.042(1.4)	.019(0.8)	.025(0.8)	.022(0.6)
Hansen test (p. val)	0.231	0.558	0.652	0.213
AR(1) in residuals	0.000	0.000	0.000	0.000
AR(2) in residuals	0.481	0.089	0.044	0.916
test of equality of $\Delta \ln w_{i,t+1}$ coeff.		0.000	0.000	0.070
	Augmented model 1			
$\Delta \ln k_{i,t}$	0.64(2.5)	0.063(3.2)	0.029(1.3)	0.071(2.8)
$\Delta \ln w_{i,t}$	<b>0.049(4.0)</b>	<b>0.018(2.3)</b>	<b>0.004(0.4)</b>	<b>0.027(2.2)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	0.040(1.4)	0.020(0.8)	0.027(0.8)	0.019(0.6)
$\Delta ccap_{i,t+1}$	-0.0009(-0.4)	-0.00004(-4.2)	0.0026(0.1)	-0.00002(-2.7)
Hansen test (p. val)	0.377	0.650	0.525	0.331
AR(1) in residuals	0.000	0.000	0.000	0.000
AR(2) in residuals	0.482	0.088	0.044	0.905
test of equality of $\Delta \ln w_{i,t+1}$ coeff.		0.000	0.000	0.060
	Augmented model 2			
$\Delta \ln k_{i,t}$	0.062(2.4)	.067(3.4)	.042(1.9)	.068(2.6)
$\Delta \ln w_{i,t}$	<b>0.052(4.3)</b>	<b>.016(2.0)</b>	<b>.003(0.3)</b>	<b>.025(2.0)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	.022(0.8)	.015(0.6)	.022(0.7)	.025(0.7)
$\Delta ccap_{i,t+1}$	-.001(-.4)	-.00004(-5.4)	.01(.24)	-.00002(-1.4)
$\Delta \left(\frac{profits}{sales}\right)_{i,t+1}$	.043(1.1)	-.022(-0.4)	.12(1.5)	-.076(-1.2)
$\Delta Q_{i,t+1}$	-.002(-1.1)	-.03(-3.1)	-.02(-2.2)	-.02(-1.4)
Hansen test (p. val)	0.380	0.134	0.070	0.189
AR(1) in residuals	0.000	0.000	0.000	0.000
AR(2) in residuals	0.494	0.119	0.049	0.777
test of equality of $\Delta \ln w_{i,t+1}$ coeff.		0.000	0.000	0.025
number of observations	2363	5443	2762	2681

One step robust System GMM estimator. Time dummies included as strictly exogenous regressors. Other instruments are lags -1 to -3 of the first differences of the regressors for the equation in levels and lags -2 and -3 of the levels for the equation in first differences.  $t$  statistics in parenthesis.

$l_{i,t+1}$ =number of employees at the end of period  $t + 1$ .  $k_{i,t}$  = real value of net fixed assets at the end of period  $t$ .  $w_{i,t}$  = real value of financial slack at the end of period  $t$ .  $E_t(\theta_{i,t+1})$ =expected productivity shock based on period  $t$  information set.

$ccap_{i,t+1}$  = interest payment on debt during period  $t + 1$  divided by total debt at the beginning of period  $t + 1$ .  $\left(\frac{profits}{sales}\right)_{i,t+1}$  = earnings before interest and taxes divided by total sales during period  $t$ .  $Q_{i,t+1}$  = market value during period  $t + 1$  divided by the book value at the beginning of period  $t + 1$ .

The p-value of the Hansen test of overidentifying restrictions is reported. This test is robust to autocorrelation and heteroskedasticity of unknown form. AR(1) and AR(2) report the p-value of the Arellano-Bond (1991) test of autocorrelation in the residuals of the first differences. We eliminate as outliers of each variable all observations above the 99% percentile and below the 1% percentile.

Table XI: The new test of financing constraints based on variable capital. Box - Cox transformation

Dependent variable: $\Delta \ln l_{i,t+1}$				
	Likely financially constr. (low Z)	Complementary sample	Medium Z	High Z
$\Delta \ln k_{i,t}$	.062(2.4)	.067(3.4)	.034(1.5)	.072(2.7)
$\Delta \frac{w_{i,t}^{0.082}-1}{0.082}$	<b>.034(4.3)</b>	<b>.008(1.7)</b>	<b>.0003(0.1)</b>	<b>.015(2.0)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	.027(0.9)	.014(0.6)	.023(0.7)	.022(0.7)
$\Delta ccap_{i,t+1}$	-0.001(-.4)	-0.00004(-4.3)	.005(.22)	-0.00002(-2.4)
$\Delta \left(\frac{profits}{sales}\right)_{i,t+1}$	.04(1.0)	-0.019(-0.4)	.112(1.4)	-0.072(-1.2)
$\Delta Q_{i,t+1}$	-0.002(-1.2)	-0.029(-3.1)	-0.02(-2.4)	-0.016(-1.4)
n.obs.	2370	5438	2759	2679
Hansen test (p. val)	0.254	0.289	0.123	0.219
Test of AR(1) in res.	0.000	0.000	0.000	0.000
Test of AR(2) in res	0.651	0.141	0.059	0.712
Test of equality of $\Delta \ln w_{i,t+1}$ coefficient with low Z group (p-value)				
		0.000	0.000	0.025

One step robust System GMM estimator. See footnote to table X for details.

Table XII: The new test of financing constraints based on variable capital. Alternative selection of groups (quintiles of the Z score)

Dependent variable: $\Delta \ln l_{i,t+1}$							
	1 <sup>st</sup> quintile	2 <sup>nd</sup> quint.	3 <sup>rd</sup> quint.	4 <sup>th</sup> quint.	5 <sup>th</sup> quint.	10% lowest Z	5% lowest Z
$\Delta \ln k_{i,t}$	.067(2.0)	.023(0.8)	.055(1.9)	.095(1.7)	.053(1.7)	.036(.9)	-.033(-.7)
$\Delta \ln w_{i,t}$	.057(4.0)	.031(2.6)	-.004(-.4)	.027(1.9)	.001(0.5)	.068(3.3)	.086(3.3)
$\Delta \ln E_t(\theta_{i,t+1})$	.036(1.1)	.015(0.4)	.019(0.4)	.040(1.0)	.042(.97)	.072(1.9)	.072(1.7)
$\Delta ccap_{i,t+1}$	.001(.5)	-.01(-1.6)	-.002(-.1)	-.0001(-5.6)	-.042(-.8)	-.001(-1.4)	-.01(-1.2)
$\Delta \left(\frac{profits}{sales}\right)_{i,t+1}$	.063(1.5)	.049(0.7)	-.1(-.15)	.071(0.8)	-.074(-1.2)	.052(1.3)	.009(.3)
$\Delta Q_{i,t+1}$	-.002(-.9)	-.01(-1.1)	-.01(-2.1)	-.06(-4.1)	-.004(-0.6)	-.002(-1)	-.02(-3.7)
n.obs.	1563	1578	1576	1560	1546	780	389
Hansen test	0.650	0.275	0.496	0.428	0.680	0.438	0.536
AR(1) in res.	0.000	0.000	0.004	0.000	0.000	0.000	0.003
AR(2) in res.	0.092	0.199	0.111	0.881	0.727	0.615	0.900
Tests of equality of $\Delta \ln w_{i,t+1}$ coefficients (p-value)							
			compl. sample	2 <sup>nd</sup> quint.	3 <sup>rd</sup> quint.	4 <sup>th</sup> quint.	5 <sup>th</sup> quint.
1 <sup>st</sup> quintile			0.000	0.079	0.000	0.052	0.000
10% lowest Z			0.000	0.062	0.002	0.039	0.002
5% lowest Z			0.017	0.038	0.000	0.028	0.000

One step robust System GMM estimator. See footnote to table X for details.

Table XIII: The new test of financing constraints based on variable capital. Firms selected according to size

Dependent variable: $\Delta \ln l_{i,t+1}$				
	Size classes (average number of employees in the sample period)			
	<250	>250 & <2500	>2500 & <10000	>10000
$\Delta \ln k_{i,t}$	.103(3.0)	.059(2.7)	.125(3.5)	.061(2.2)
$\Delta \ln w_{i,t}$	<b>.071(3.8)</b>	<b>.040(3.1)</b>	<b>.026(2.0)</b>	<b>-.001(-.1)</b>
$\Delta \ln w_{i,t+1} \mid Low Z$	<b>.095(3.3)</b>	<b>.049(2.4)</b>	<b>.043(2.1)</b>	<b>-.009(-.7)</b>
$\Delta \ln w_{i,t+1} \mid No low Z$	<b>.036(2.2)</b>	<b>.021(1.5)</b>	<b>-.002(-0.2)</b>	<b>-.004(-.2)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	-.006(-.2)	.052(1.8)	.059(1.65)	-.002(-.1)
$\Delta ccap_{i,t+1}$	.030(.7)	-.0004(-.3)	.001(.02)	-.00006(-7.0)
$\Delta \left( \frac{profits}{sales} \right)_{i,t+1}$	.014(.5)	.084(1.3)	.098(.8)	.065(.9)
$\Delta Q_{i,t+1}$	-.009(-2.4)	-.028(-2.6)	-.009(-1.3)	-.0007(-1.6)
n.observations	1059	3124	2355	1811
% with low Z	39%	32%	23%	21%
Hansen test	0.352	0.464	0.246	0.105
AR(1) in res.	0.000	0.000	0.000	0.000
AR(2) in res.	0.127	0.783	0.195	0.059

One step Robust System GMM estimator. See footnote to table X for details. The coefficients of  $(\Delta \ln w_{i,t+1} \mid low Z)$  and  $(\Delta \ln w_{i,t+1} \mid no low Z)$  are obtained from the unrestricted model (all coefficients are allowed to be different for each size group and Z-score group).

Table XIV: The new test of financing constraints based on variable capital: sample selected according to the zero dividends criterion

Dependent variable: $\Delta \ln l_{i,t+1}$		
	Zero dividends firms	Positive dividends firms
$\Delta \ln k_{i,t}$	0.089 (4.4)	0.063 (3.1)
$\Delta \ln w_{i,t}$	<b>0.051 (4.9)</b>	<b>0.010 (1.1)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	0.033 (1.4)	0.004 (0.2)
$\Delta ccap_{i,t+1}$	-0.00004 (-4)	0.002 (1.1)
$\Delta \left( \frac{profits}{sales} \right)_{i,t+1}$	0.28 (0.7)	0.02 (0.5)
$\Delta Q_{i,t+1}$	-0.14 (0.1)	-0.001 (-1.1)
n.obs.	4184	3988
Hansen test (p. val)	0.110	0.055
Test of AR(1) in res.	0.000	0.000
Test of AR(2) in res	0.697	0.035

One step robust System GMM estimator. See footnote to table X for details.

Table XV: A new test of financing constraints based on variable capital. Contemporaneous fixed capital

Dependent variable: $\Delta \ln l_{i,t+1}$							
	Constr. (low Z)	Compl. sample	Med. Z	High Z	20% low. Z	10% low. Z	5% low. Z
$\Delta \ln k_{i,t+1}$	.452(8.8)	.495(7.3)	.472(6.7)	.491(7.7)	.383(7.7)	.404(7.8)	.357(7)
$\Delta \ln w_{i,t}$	<b>.026(2.6)</b>	<b>.004(0.5)</b>	<b>-.003(-.4)</b>	<b>.007(.6)</b>	<b>.032(2.8)</b>	<b>.034(2.3)</b>	<b>.05(2.7)</b>
$\Delta \ln E_t(\theta_{i,t+1})$	-.003(-.1)	.028(1.1)	.019(0.7)	.041(1.3)	.016(0.5)	.038(1.0)	.031(1)
$\Delta ccap_{i,t+1}$	.009(3.1)	-.0001(-5)	.016(.9)	-.0001(-2)	.009(5.2)	-.001(-.0)	.002(.5)
$\Delta \left(\frac{profits}{sales}\right)_{i,t+1}$	.034(.98)	.047(0.8)	.168(2.3)	.001(0.1)	.059(1.6)	.054(1.6)	0.08(.3)
$\Delta Q_{i,t+1}$	.00(-0.5)	-.01(-1.7)	-.006(-1.2)	.006(0.1)	-.001(-1)	-.001(-1)	-.013(-3)
n.observations	2362	5460	2760	2683	1559	777	387
Hansen test	0.375	0.315	0.638	0.161	0.741	0.437	0.696
AR(1) in res.	0.000	0.000	0.000	0.000	0.000	0.001	0.014
AR(2) in res.	0.250	0.128	0.099	0.985	0.588	0.718	0.264
One sided tests of equality of $\Delta \ln w_{i,t+1}$ coefficients							
	Complementary sample		Medium Z score		High Score		
Low Z	0.008		0.002		0.047		
1st quintile of Z	0.008		0.002		0.018		
10% lowest Z	0.043		0.021		0.067		
5% lowest Z	0.008		0.006		0.023		

One step robust System GMM estimator. See footnote to table X for details.

Table XVI: The Q model augmented with cash stock

Dependent variable: $\Delta \left(\frac{i}{k}\right)_{i,t+1}$				
	Base specification			
	Low Z	Compl. sample	Med. Z	High Z
$\Delta \left(\frac{c}{k}\right)_{i,t+1}$	-0.035(-0.6)	.028(1.0)	-0.008(-0.3)	.067(2.0)
$\Delta \left(\frac{w}{k}\right)_{i,t+1}$	<b>.009(1.2)</b>	<b>.002(1.5)</b>	<b>.001(0.1)</b>	<b>.002(2.8)</b>
$\Delta Q_{i,t+1}$	.015(0.9)	-0.001(-.1)	-0.016(-1)	-0.008(-0.8)
Hansen test (p. val)	0.129	0.470	0.774	0.403
Test of AR(1) in res.	0.000	0.000	0.000	0.000
Test of AR(2) in res.	0.808	0.154	0.978	0.167
	Box Cox transformation			
$\Delta \left(\frac{c}{k}\right)_{i,t+1}$	.019(0.4)	.054(2.2)	.029(1.0)	.069(2.0)
$\Delta \frac{(w/k)_{i,t+1}^{0.21} - 1}{0.21}$	<b>.094(2.7)</b>	<b>.101(2.9)</b>	<b>.060(1.6)</b>	<b>.141(3.3)</b>
$\Delta Q_{i,t+1}$	.036(2.3)	-0.011(-.9)	-0.011(-1)	-0.016(-1.3)
Hansen test (p. val)	0.140	0.556	0.688	0.412
Test of AR(1) in res.	0.000	0.000	0.000	0.000
Test of AR(2) in res.	0.852	0.000	0.221	0.010
n. of observations	2624	5847	2989	2842

One step robust System GMM estimator. Time dummies included as strictly exogenous regressors. Other instruments are lags -2 to -4 of the first differences of the regressors for the equation in levels and lags -3 and -4 of the levels for the equation in first differences. t statistics are reported in parenthesis. The

sample is cleaned of outliers before estimation.

We considered as outliers in each variable all observations above the 99% percentile and below the 1% percentile.  $\left(\frac{i}{k}\right)_{i,t}$  = gross fixed capital expenditure during period t dividend by net fixed assets at the beginning of period t.  $Q_{i,t+1}$  = market value during period t + 1 divided by the book value at the beginning of period t + 1.  $\left(\frac{cf}{k}\right)_{i,t}$  =cash flow during period t dividend by net fixed assets at the beginning of period t.

The P-value of the Hansen test of overidentifying restrictions is reported. This test is robust to autocorrelation or heteroskedasticity of unknown form. AR(1) and AR(2) report the p-value of the Arellano-Bond (1991) test of autocorrelation in the residuals of the first differences.

Table XVII: Distribution of the standardised net investment rates of  $k_{i,t}$  and  $l_{i,t}$ 

Empirical data, financially unconstrained firms (low Z excluded)		
	Skewness	Kurtosis
$\frac{\Delta k_{i,t}}{k_{i,t-1}}$	0.800	3.50
$\frac{\Delta l_{i,t}}{l_{i,t-1}}$	0.587	3.52
$\Delta \ln \theta_{i,t}$	-0.128	2.73
Simulated data, financially unconstrained firms ( $\lambda_t = 0$ )		
$\frac{\Delta k_{i,t}}{k_{i,t-1}}$	1.777	7.480
$\frac{\Delta l_{i,t}}{l_{i,t-1}}$	1.491	6.352
$\Delta \ln \theta_{i,t}$	-0.034	3.21

Empirical data:  $l_{i,t}$  = number of employees at the end of period  $t$ ;  
 $k_{i,t}$  = real value of total assets at the end of period  $t$ .

Table XVIII: A new test of financing constraints based on variable capital. fixed capital as dependent variable

Dependent variable: $\Delta \ln k_{i,t+1}$								
	Low Z	Comp. sample	Size 1	Size 2	Size 3	Size 4	No div.	Comp. sample
$\Delta \ln l_{i,t+1}$	.78(7)	.66(7)	.74(6)	.44(5)	.64(8)	.82(9)	.85(9)	.70(8)
$\Delta \ln \mathbf{w}_{i,t}$	.014(1.2)	.013(1.6)	.028(1.4)	.034(2.4)	.002(.2)	.012(1.2)	.019(1.5)	.004(.5)
$\Delta \ln E_t(\theta_{i,t+1})$	.03(.8)	-.03(-1.1)	.003(.1)	-.01(-.4)	-.01(-.2)	.03(.7)	-.03(-1)	.05(1.5)
$\Delta ccap_{i,t+1}$	-.02(-.6)	.000(.1)	.002(.1)	-.01(-1)	-.02(-.5)	.000(.2)	.000(1.4)	-.02(-6)
$\Delta \left( \frac{prof.}{sales} \right)_{i,t+1}$	-.04(-.8)	-.13(-2)	-.01(-.3)	-.08(-1.3)	-.01(-.01)	-.06(-1)	-.1(-1.5)	-.04(-1.2)
$\Delta Q_{i,t+1}$	-.01(-1)	-.03(-1)	-.01(-1.9)	-.03(-2.8)	-.003(-.5)	-.00(-1.4)	-.01(-1.3)	-.001(-.3)
n.observations	2362	5985	1057	2123	2355	1812	4182	4165
Hansen test	.028	.527	.492	.186	.210	.409	.075	.067
AR(1) in res.	.000	.000	.000	.000	.000	.000	.000	.000
AR(2) in res.	.042	.142	.563	.069	.209	.115	.924	.031

One step robust System GMM estimator. See footnote to table X for details. Size 1: firms smaller than 250 employees. Size 2: firms between 250 and 2500 employees. Size 3: firms between 2500 and 10000 employees. Size 4: firms larger than 10000 employees.

Table XIX: Distribution of the standardised net percentage changes of the expenditure in Research and Development,

Empirical data, financially unconstrained firms (low Z excluded)		
	Skewness	Kurtosis
$\frac{\Delta r_{i,t}}{r_{i,t-1}}$	0.488	3.28

Table XX: A new test of financing constraints based on variable capital. expenditure in Research and Development as dependent variable

Dependent variable: $\Delta \ln r_{i,t+1}$								
	Low Z	Comp. sample	Size 1	Size 2	Size 3	Size 4	No div.	Comp. sample
$\Delta \ln k_{i,t}$	.12(2.9)	.10(2.8)	.11(2.3)	.06(1.3)	.14(2.5)	.14(2.9)	.12(3.3)	.15(3.6)
$\Delta \ln l_{i,t+1}$	.44(4.9)	.36(4.9)	.41(3.6)	.31(2.9)	.53(8.5)	.42(3.1)	.39(5.9)	.29(2.6)
$\Delta \ln \mathbf{w}_{i,t}$	.129(5.3)	.045(2.9)	.133(4.1)	.067(2.6)	.077(3.3)	.033(1.7)	.106(5.3)	.048(3.2)
$\Delta \ln E_t(\theta_{i,t+1})$	-.11(-2.8)	.008(.2)	-.06(-1.2)	-.09(-2.3)	-.07(-1)	-.14(-1.7)	-.06(-1.8)	.004(.11)
$\Delta ccap_{i,t+1}$	.01(1.6)	.01(.7)	-.02(-.6)	.006(1.4)	.04(1.6)	.12(1.4)	.009(1.7)	.012(.93)
$\Delta \left(\frac{prof.}{sales}\right)_{i,t+1}$	-.01(-.3)	-.20(-2.2)	-.02(-.5)	-.20(-2.6)	-.21(-2.0)	-.17(-2.1)	-.048(-1)	-.18(-1.6)
$\Delta Q_{i,t+1}$	-.005(-1)	-.02(-2.1)	-.001(-.1)	-.09(-2.3)	-.02(-1.9)	-.01(-1.3)	-.01(-1.3)	-.03(-3.2)
n.observations	862	2065	432	1317	940	778	1464	1528
Hansen test	.038	.115	1.000	.144	.970	1.000	.153	.304
AR(1) in res.	.000	.000	.000	.000	.000	.000	.000	.000
AR(2) in res.	.841	.665	.646	.253	.077	.065	.265	.707

One step robust System GMM estimator. See footnote to table X for details. Size 1: smaller than 250 employees. Size 2: between 250 and 2500 employees. Size 3: between 2500 and 10000 employees. Size 4: larger than 10000 employees.



Table XXI: Specification tests - model in levels

$\ln l_{i,t+1} = \pi_0 + \pi_1 \ln E_t(\theta_{i,t+1}) + \pi_2 \ln k_{i,t} + \pi_3 \ln w_{i,t} + \epsilon_{i,t+1}$				
$t - 1, t - 2$ and $t - 3$ first diff. as instr. of the equation in levels				
	All sample	Low Z	Medium Z	High Z
Shea's partial $R^2$				
$\ln E_t(\theta_{i,t+1})$	0.39	0.45	0.37	0.12
$\ln k_{i,t}$	0.06	0.09	0.06	0.02
$\ln w_{i,t+1}$	0.15	0.18	0.14	0.17
Hansen test of overid. restrictions (p-value)				
<i>lags</i> 1,2,3	0.000	0.006	0.012	0.000
<i>lags</i> 2,3,4	0.000	0.027	0.030	0.001
<i>lags</i> 3,4,5	0.000	0.021	0.035	0.029
<i>lags</i> 4,5,6	0.000	0.010	0.029	0.091
$t - 2$ and $t - 3$ levels as instr. of the equation in first differences				
	All sample	Low Z	Medium Z	High Z
Shea's partial $R^2$				
$\ln E_t(\theta_{i,t+1})$	0.11	0.14	0.12	0.11
$\ln k_{i,t}$	0.04	0.04	0.05	0.05
$\ln w_{i,t+1}$	0.04	0.04	0.06	0.05
Hansen test of overid. restrictions (p-value)				
<i>lags</i> 2,3	0.175	0.001	0.540	0.000
<i>lags</i> 3,4	0.067	0.029	0.621	0.000
<i>lags</i> 4,5	0.177	0.000	0.568	0.000
<i>lags</i> 5,6	0.188	0.003	0.581	0.000

Table XXII: Specification tests. Model in first differences

$\Delta \ln l_{i,t+1} = \pi_0 + \pi_1 \Delta \ln E_t(\theta_{i,t+1}) + \pi_2 \Delta \ln k_{i,t} + \pi_3 \Delta \ln w_{i,t} + \epsilon_{i,t+1}$				
$t - 1, t - 2$ and $t - 3$ first diff. as instr. of the equation in levels				
	All sample	Low Z	Medium Z	High Z
Shea's partial $R^2$				
$\ln E_t(\theta_{i,t+1})$	0.80	0.81	0.77	0.81
$\ln k_{i,t}$	0.60	0.60	0.58	0.61
$\ln w_{i,t+1}$	0.82	0.81	0.83	0.80
Hansen test of overid. restrictions (p-value)				
<i>lags</i> 1,2,3	0.560	0.505	0.922	0.581
$t - 2$ and $t - 3$ levels as instr. of the equation in first differences				
	All sample	Low Z	Medium Z	High Z
Shea's partial $R$				
$\ln E_t(\theta_{i,t+1})$	0.62	0.66	0.62	0.58
$\ln k_{i,t}$	0.45	0.50	0.46	0.41
$\ln w_{i,t+1}$	0.62	0.65	0.65	0.59
Hansen test of overid. restrictions (p-value)				
<i>lags</i> 2,3	0.629	0.452	0.286	0.547