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**Fiat Money in a Search Theoretical Model
with Generalist Consumers**

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Abstract

There are two general ways in which the role of fiat money has been introduced in the standard monetary search-theoretical model. The first is to bring in the model a fiat object with different intrinsic properties. The second is to introduce a centralized institution that favors the use of fiat money through specific transaction policies. We carry out a similar exercise for a modified version of the model in which agents have a different structure of preferences. We characterize the conditions for which there exist equilibria with circulating fiat money and evaluate the main differences with the results derived from the standard model.

1. Introduction

There are two general ways in which the role of fiat money has been introduced in monetary search-theoretical models. The first [see Kiyotaki and Wright (1991) and Aiyagari and Wallace (1992)] is to bring in the model a fiat object with different intrinsic properties, such as storability. The second [see Aiyagari and Wallace (1997) and Li and Wright (1998)] is to introduce a centralized institution (e.g. the government) that favors the use of fiat money through its specific transaction policies. Using these two approaches, the aforementioned papers provide the conditions for existence of equilibria in which fiat money circulates as medium of exchange. They do it, however, in the context of a particular version of the search theoretical model of money, in which agents are specialized in production and consumption in a very specific way. In the current paper we carry out a similar exercise for a different version of the search-theoretical model in which the structure of preferences of agents implies that they are generalists in consumption. Thus, we start from a commodity money model without any role for fiat money [based on Cuadras-Morató and Wright (1997)], introduce fiat money using both approaches mentioned above and characterize the new conditions for existence of equilibria with valued fiat money. This is itself a first contribution of the paper. Secondly, we compare the main results of this exercise with the conclusions obtained in previous literature. One of the main findings is that some of the conclusions reached with the standard model do not hold when the environment is subject to the described changes. In particular, with agents less specialized in consumption, it becomes more difficult to get fiat money to play a role as universal medium of exchange.

The model from which we start our analysis [Cuadras-Morató and Wright (1997)] is a version of the search theoretical model of money in which goods differ by the number of their potential producers (“supply”) and the probability with which agents want to consume them (“demand”). The main difference from the standard specification of the search theoretical model [Kiyotaki and Wright (1989)] is the way consumption preferences are defined. Kiyotaki and Wright (1989) has agents who derive utility from consuming always the same good, while in the current setting agents have tastes that change over time according to some common distribution that determines their preference for the different commodities. Notice that both settings are particular extreme cases of a more general specification of the consumer preferences in which a Markov process determines the good a consumer likes at any point of time. Kiyotaki and Wright (1989) is a special case of a degenerative process in which the conditional probability of liking the same good as in the past period of time is unity. The model presented here is a special case of serial independence.

Modeling agents as generalists in consumption is interesting for at least two reasons. First, it seems a very natural assumption from an intuitive point of view. Second, one may want to assess what is the importance of specific assumptions on consumer preferences to determine the role that fiat money plays in monetary search models. Agents who are very specialized in both consumption and production need to rely a lot on trade and the use of some medium of exchange such as fiat money to consume at all. Once we alter this assumption, making consumers less specialized, this may be no longer true. It is important to see how these changes in the environment modify some of the conclusions from previous literature.

The rest of the paper is organized as follows. Section 2 describes a simple economy with fiat money and storage costs. Section 3 introduces the role of government transaction policy. Section 4 concludes with a summary of the arguments and offers suggestions for future research.

2. An Economy with Fiat Money and Storage Costs

This section describes a simple version with two consumption goods of the model of Cuadras-Morató and Wright (1997), including fiat money and storage costs.¹ The economy is populated by a $[0,1]$ continuum of infinite-lived agents. There are three different objects: two consumption goods (goods 1 and 2), and fiat money, a valueless piece of paper that cannot be consumed and has no use in production processes. All objects are indivisible and identically durable, homogeneous, portable, etc. Storability is the only difference. Consumption goods can be stored at a cost c_I , in terms of disutility, and fiat money is storable at cost c_0 . There are two different types of agents according to the commodity that they are specialized in producing. Agents of type 1(2) produce good 1(2) at cost D . We shall denote by σ ($1-\sigma$) the fraction of agents who are of type 1(2). Agents are generalists in consumption. Thus, at every date, $t=0,1,2,\dots$, each agent gets a taste shock that determines the good she desires that period. We shall denote by δ ($1-\delta$) the probability that an agent desires good 1(2) at any particular period of time. This is independent across types of agents, inventory holdings, and time. If a trader desires good

¹ Since we present results only for an example, we will not give the details of a more general formulation.

The interested reader can find the description of the general setup in Cuadras-Morató (2000).

1(2), she gets utility U from consuming good 1(2) and zero utility from consuming any other good. After consuming a good, traders immediately produce one unit of their own production commodity. We shall denote by $u=U-D$, the net utility of the joint action of consuming plus producing.

There is a fixed stock of indivisible units of fiat money in the economy, m (m is also the proportion of all agents in the economy who hold one unit of fiat money). At the initial date every agent is endowed with a single unit of an object. In particular, we shall assume that a proportion $1-m$ of agents is endowed with a unit of the good they produce, while the rest, m , are endowed with fiat money. All agents have exactly the same probability of getting fiat money, independently of their production type. Following Aiyagari and Wallace (1992) and Kiyotaki and Wright (1993), we assume that agents cannot produce unless they have consumed. Thus, every agent will be always holding one unit of an object and the initial stock of money is kept invariant.

The sequence of events will be as follows: every period of time, agents start with some object in inventory (a consumption good or fiat money) and pay the corresponding storage cost (c_1 or c_0). Then, they get a taste shock. If they happen to store the good they want to consume, then they consume it, produce a new good and wait for next period. Otherwise, they enter a trading process (*market*) in which they will be randomly matched with other traders in their same situation. Once matched, the agents will have to decide whether they want to trade or not. If they want to trade, then they swap inventories one-for-one. Whenever an agent gets the good she desires, she consumes it and immediately produces a new good; otherwise, she keeps the object she obtained and

waits for next period. If they do not want to trade, then they part company and wait for the next period.

The strategic decision of traders is very simple in the economy described so far. At a given period of time, agents in the market must be holding one of two things: fiat money or some good they do not want to consume today (they would obviously never go to the market holding the good they want to eat). Then, they will be paired with another agent who will offer them one of the three following possibilities. First, the good they already have (which will be not accepted). Second, the good they want to consume today (which they would like to accept and consume immediately).² And third, fiat money. Clearly, the only trade decision to be taken by agents in this environment is a choice between the good they hold in inventory in the market and fiat money.

In order to analyze the strategic decision of agents in this economy, let V_{ij} be the value function for a type- i individual ($i \in \{1, 2\}$), at the end of a period holding good j other than the one currently desired for consumption ($j \in \{0, 1, 2\}$; good 0 here is simply the notation used for fiat money). V_{ij} can be interpreted as the value of good j as asset. The structure of taste shocks that we have assumed guarantees that V_{ij} does not depend on the good that is desired in the current period. The strategic problem of agents can be formalized in the following way: an agent of type i wants to accept good j in exchange for good h iff $V_{ij} > V_{ih}$ (that is, if the value of good j as asset is larger than the value of

² We shall make explicit below which are the restrictions on the parameters of this economy that guarantee that money holders will not refuse to accept and consume their desired goods. This would be optimal if, for instance, u was sufficiently small and holding money allowed them to save on storage costs.

good h). Iff $V_{ih} \geq V_{ij}$, then agents do not want to accept good j in exchange for good h .

We do not consider mixed strategies and assume that agents do not want to trade when they are indifferent between two goods.³ We can characterize the behavior of agents of type i simply by ranking the three value functions V_{i0} , V_{i1} , and V_{i2} . This can be represented by a strategy vector of three elements $s(i)=[s(i)_{12}, s(i)_{01}, s(i)_{02}]$ where $s(i)_{hj}$ is defined as follows:

$$s(i)_{hj} \in \begin{cases} \{1\} & \text{iff } V_{ij} > V_{ih} \\ \{0\} & \text{iff } V_{ij} \leq V_{ih} \end{cases}$$

and h is the good in inventory and j what is being offered in exchange.

The structure of this economy is such that it is optimal for all types of agents to use the same trading strategies. This is equivalent to saying that all agents will rank all objects in exactly the same way, independently of their production type. The reason for this is simply that, independently on the good they produce, all agents draw their taste shocks from a common distribution and, in order to evaluate the value of holding a particular object, what matters is simply the chance of consuming in future periods. In this particular sense, this is a representative agent model, where all agents are identical from the point of view of which trading strategies are optimal. In other words, the heterogeneity of agents in this model derives from the fact that they produce different sorts of commodities, but this feature does not affect the nature of the strategic problem they face. In order to avoid unnecessary repetitions, we do not provide the formal proof

³ This seems the natural assumption to make. Considering the existence of an arbitrarily small transaction cost would be enough to derive it as a result of the model.

of this result here and refer the reader to Proposition 1 in Cuadras-Morató and Wright (1997), where this result has been proved in the context of a model with three commodity goods and no fiat money, but otherwise identical. This is an important feature of the model and allows us to summarize the strategic behavior of all agents by a single vector, s .

Notice that commodity money is ruled out in this setup [i.e., type 1(2) agents never holds good 2(1) in inventory].⁴ In order to see this, suppose an agent of type 1 who eventually produces good 1. Then, she might want to consume it with probability δ , producing good 1 again afterwards, or she might want to consume good 2 with probability $1-\delta$ and then go to the market. Once in the market, someone who wants to consume good 1 might offer her good 2, which means that trade will take place. She could also be offered good 1, which she will refuse. Finally, she could be offered fiat money. If she accepts fiat money, she will not trade it unless it is for the good that she wants to eat. To see this, suppose that this agent of type 1 has accepted fiat money and meets someone who holds good 2. If she wants to eat good 2, she will be willing to accept it. If she wants to eat good 1, then she and her matching partner will face the same decision: either holding fiat money or good 2. Since both use the same trading strategy, no mutually beneficial agreement can be struck. Consequently, type 1 agents will never end up holding good 2 in storage. Hence, the strategic decision in this economy is reduced to a choice between one's production good and fiat money. We will sum up the behavior of all agents as a vector $s=(s_{01},s_{02})$.

⁴ Since our focus here is fiat money, the fact that commodity money is ruled out will not be a major concern [see Cuadras-Morató and Wright (1997) for a detailed examination of commodity money in a similar context].

In order to continue the analysis, we need to specify some notation. Let p_{ij} denote the measure of type i agents with good j at the start of a period ($i \in \{1, 2\}$ and $j \in \{0, 1, 2\}$). Let $\mathbf{p} = (p_{10}, p_{11}, p_{20}, p_{22})$ be the distribution of inventories of the economy. The measure of agents who hold good j is $P_j = \sum_i p_{ij}$.⁵ The total number of agents with good 1(2) who go to the market is $(1-\delta)P_1$ (δP_2) and the total number of agents who go to the market is $N = (1-\delta)P_1 + \delta P_2 + P_0$. Let $\pi_{j1} = P_j \delta / N$ ($\pi_{j2} = P_j (1-\delta) / N$) be the probability of meeting someone in the market who is holding good j and wants to consume good 1(2). Given \mathbf{s} , the distribution \mathbf{p} evolves according to some law of motion $\mathbf{p}' = f(\mathbf{p}; \mathbf{s})$. A steady-state is a solution to $\mathbf{p} = f(\mathbf{p}; \mathbf{s})$. Once we know \mathbf{p} , we can determine the steady-state value of $\boldsymbol{\pi} = (\pi_{12}, \pi_{21}, \pi_{01}, \pi_{02})$ as a function of \mathbf{s} (given the economy described by the parameters m , δ and σ).

We are now in a position to make explicit the value functions for the agents of this economy. Let r be the rate of time preference. In flow terms we have

$$rV_{11} = -c_1 + [\delta + (1-\delta)\pi_{21}]u + (1-\delta)\pi_{01}(1-s_{01})(V_{10} - V_{11}) \quad (1)$$

$$rV_{10} = -c_0 + [\delta\pi_{12}(1-s_{01}) + (1-\delta)\pi_{21}(1-s_{02})](u + V_{11} - V_{10}) \quad (2)$$

⁵ Note that $P_0 = m$.

Expression (1) represents the value function in flow terms for a type 1 agent holding good 1 at the end of the period. Any agent who ends up holding a consumption good at the end of a period must pay storage cost, c_1 , at the beginning of the next period. In that period the following events may occur. With probability δ , the agent will consume the good he is holding in storage. With probability $(1-\delta)\pi_{21}$, the agent does not want to consume good 1 and goes to the market where she will find an agent holding good 2 and desiring to consume good 1. In this case, trade takes place and the agent consumes good 2. Finally, with probability $(1-\delta)\pi_{01}(1-s_{01})$, the agent goes to the market, meets an agent who wants to consume good 1 and holds fiat money and they trade. Any other meeting in the market would finish without trade agreement. A similar line of reasoning applies to expression (2). In order to simplify the analysis, we shall adopt the following notation: $\gamma_i = \delta + (1-\delta)\pi_{2i}$ (similarly for γ_2) and $\gamma_0 = \delta\pi_{12}(1-s_{01}) + (1-\delta)\pi_{21}(1-s_{02})$. Intuitively, γ_i is the probability of consumption next period conditional on holding good i .

We are now ready to define the equilibrium of the model.

DEFINITION. A symmetric, steady-state, pure-strategy equilibrium is a vector s of strategies $s=(s_{01} s_{02})$ and a steady state distribution \mathbf{p} such that: a) given \mathbf{p} , for $i=1,2$ $s_{0i}=1$ iff $V_{i0} \leq V_{i1}$ and $s_{0i}=0$ otherwise; and b) given s , $\mathbf{p}=f(\mathbf{p};s)$.

The following lemma will be useful to characterize equilibria in the model.

LEMMA 1. For $i=1,2$, $V_{ii}-V_{i0} > 0 \Leftrightarrow (\gamma_i - \gamma_0) > (c_1 - c_0)/u$

Proof. Only solve the system of linear equations (1) and (2) for type 1 agents (and the equivalent expressions for agents of type 2).

The intuition of Lemma 1 is straightforward: good i will be preferred to fiat money if and only if the expected utility of consuming when holding good i minus the storage cost ($\gamma_i u - c_i$) is larger than the expected utility of consuming when holding fiat money minus the corresponding storage cost ($\gamma_0 u - c_0$).

There are three different types of equilibria in this model:

a) Fiat money equilibrium (M), $s=(0,0)$, in which all agents accept fiat money.

b) There are two different equilibria in which fiat money is partly accepted, $s=(1,0)$ and $s=(0,1)$ (we shall refer to them respectively by P and P'). Agents holding good 1(2) do not accept fiat money in equilibrium P (P'). Both cases are perfectly symmetric.

c) Barter equilibrium (B), $s=(1,1)$. No agent accepts fiat money. This means that agents always keep their production good until they can consume it or exchange it for something they will consume immediately.

To carry on the analysis we need to solve for the steady-state distribution of inventories of the different agents, \mathbf{p} . This will be determined by our meeting technology

plus the particular trading strategy followed by the agents. In general, we need to solve the following system of equations.

$$p_{11}(1-\delta)(1-p_{11}-p_{22})\delta(1-s_{01}) = (\sigma-p_{11})\delta(1-\delta)[p_{11}(1-s_{01})+p_{22}(1-s_{02})]$$

$$p_{22}(1-\delta)(1-p_{11}-p_{22})\delta(1-s_{02}) = (1-\sigma-p_{22})\delta(1-\delta)[p_{11}(1-s_{01})+p_{22}(1-s_{02})]$$

For the different equilibria the results are the following:

Equilibrium B. The steady-state distribution which corresponds to equilibrium B (that is, trade strategies $\mathbf{s}=(1,1)$) is the following: $p_{11}=\sigma(1-m)=P_1$, $p_{22}=(1-\sigma)(1-m)=P_2$, $p_{10}=\sigma m$, and $p_{20}=(1-\sigma)m$.⁶ Consequently, $\pi_{12}=\sigma(1-m)(1-\delta)/N$, $\pi_{21}=(1-\sigma)(1-m)\delta/N$, $\pi_{01}=m\delta/N$, and $\pi_{02}=m(1-\delta)/N$.

Equilibrium M. The steady-state distribution is exactly the same as in equilibrium B (although the previous note does not apply now).

Equilibrium P. If $1-\sigma \geq m$, the steady-state distribution is $p_{11}=\sigma=P_1$, $p_{22}=1-\sigma-m=P_2$, $p_{10}=0$, and $p_{20}=m$. Consequently, $\pi_{12}=\sigma(1-\delta)/N$, $\pi_{21}=(1-\sigma-m)\delta/N$, $\pi_{01}=m\delta/N$, and $\pi_{02}=m(1-\delta)/N$. If $1-\sigma \leq m$, then the distribution is $p_{11}=1-m=P_1$, $p_{22}=0=P_2$, $p_{10}=m-(1-\sigma)$,

⁶ This result depends on the assumption we made about the initial distribution of fiat money among agents. This is because since nobody accepts fiat money in equilibrium B, the agents who were given it in the first place get stuck with it forever. We have assumed that a proportion m of agents of each type was given fiat money in the initial period.

and $p_{20}=1-\sigma$.⁷ Consequently, $\pi_{12}=(1-m)(1-\delta)/N$, $\pi_{21}=0$, $\pi_{01}=m\delta/N$, and $\pi_{02}=m(1-\delta)/N$ (the results for equilibrium P' are symmetric).

Before characterizing the equilibria of the model, the following lemma makes explicit the restrictions on the parameters that ensure that $u+V_{ii}>V_{i0}$ or, in words, that money holders are willing to accept and consume immediately their desired goods.

LEMMA 2. a) In Equilibrium M, $u+V_{11}>V_{10}$ iff

$$r + \delta \frac{1 - (1 - m)\delta\sigma + m(1 - \delta)}{1 - (1 - m)[\delta\sigma + (1 - \delta)(1 - \sigma)]} > \frac{c_1 - c_0}{u}$$

(similarly for $u+V_{22}>V_{20}$).

b) In Equilibrium P, $u+V_{22}>V_{20}$ iff

$$r + (1 - \delta) \left[1 + \frac{\delta}{m + (1 - \delta)(1 - m)} \right] > \frac{c_1 - c_0}{u} \text{ if } 1 - \sigma \leq m, \text{ and}$$

$$r + (1 - \delta) \frac{m + (1 - \delta)\sigma + \delta}{m + (1 - \delta)\sigma + \delta(1 - \sigma - m)} > \frac{c_1 - c_0}{u} \text{ if } 1 - \sigma \geq m$$

(symmetrically for $u+V_{11}>V_{10}$ in Equilibrium P').

Proof. Solving the system of equations (1) and (2) we get that $u+V_{11}>V_{10}$ iff $r + \gamma_1 + (1 - \delta)\pi_{01}(1 - s_{01}) > (c_1 - c_0)/u$ (and similarly for $u+V_{22}>V_{20}$). Substituting for the values of equilibrium strategies and steady state distributions we get the expressions almost immediately.

⁷ If $\sigma=1-m$, then the steady-state distribution is obviously $p_{11}=\sigma$, $p_{20}=1-\sigma$, and $p_{10}=p_{22}=0$.

Notice that there always exist values of the parameters r , u , c_0 , and c_1 such that these restrictions do not bind.

Given these results and Lemma 1, we can state now the following Proposition 1.

PROPOSITION 1. *a) Equilibrium B exists iff*

$$\frac{c_1 - c_0}{u} < \delta \left[1 + \frac{(1 - \delta)(1 - \sigma)(1 - m)}{1 - (1 - m)[\delta\sigma + (1 - \delta)(1 - \sigma)]} \right] \quad \text{and}$$

$$\frac{c_1 - c_0}{u} < (1 - \delta) \left[1 + \frac{\delta\sigma(1 - m)}{1 - (1 - m)[\delta\sigma + (1 - \delta)(1 - \sigma)]} \right]$$

b) Equilibrium M exists iff

$$\frac{c_1 - c_0}{u} > \delta \left[1 - \frac{(1 - \delta)\sigma(1 - m)}{1 - (1 - m)[\delta\sigma + (1 - \delta)(1 - \sigma)]} \right] > 0 \quad \text{and}$$

$$\frac{c_1 - c_0}{u} > (1 - \delta) \left[1 - \frac{\delta(1 - \sigma)(1 - m)}{1 - (1 - m)[\delta\sigma + (1 - \delta)(1 - \sigma)]} \right] > 0$$

c) Equilibrium P exists iff

$$\delta > \frac{c_1 - c_0}{u} > (1 - \delta) \left[1 + \frac{\delta(2\sigma + m - 1)}{1 - \delta\sigma - (1 - \delta)(1 - \sigma - m)} \right] > 0 \quad \text{for } 1 - \sigma \geq m, \text{ and}$$

$$\delta > \frac{c_1 - c_0}{u} > (1 - \delta) \left[1 + \frac{\delta(1 - m)}{1 - \delta(1 - m)} \right] > 0 \text{ for } 1 - \sigma \leq m$$

(symmetrically for P').

Proof. The proof is a straightforward application of Lemma 1, given the values for the steady-state distribution of inventories.

Proposition 1 allows us to fully characterize the conditions under which fiat money will circulate in this economy. It states that a necessary and sufficient condition under which fiat money will circulate in equilibrium is that $c_1 - c_0$ is positive and large enough. This means that fiat money must have strictly better intrinsic properties than the rest of goods to be valued or, in other words, that $c_1 - c_0 \leq 0$ is sufficient to rule out the circulation of fiat money. This characterization of the monetary equilibria is complementary, and obviously not contradictory, with Proposition 3 in Cuadras-Morató and Wright (1997), which says that, in a model with no storage costs (or, alternatively, with $c_1 - c_0 = 0$) fiat money does not take on value.

In the context of a similar model with two commodities and fiat money, Aiyagari and Wallace (1992) provide a necessary condition for existence of equilibrium with fiat money. Only if its storage cost is smaller than that of goods ($c_1 - c_0 > 0$) will fiat money be accepted by all agents in the economy (see their Proposition 3.1.). This result is similar to part b) in Proposition 1, although we require that $c_1 - c_0$ be strictly larger than some positive cutoff point which depends on the values of the parameters.

<INSERT FIGURE 1, 2, AND 3 HERE>

Figures 1, 2, and 3 represent the points in the parameter space (σ, δ) for which the different equilibria of the model exist [given $m=0.1$ and $(c_1-c_0)/u=0.4$]. Notice that equilibria M and B do not appear for extreme values of δ (e.g. $\delta \rightarrow 1$), for which the only existing equilibrium is P (we do not represent equilibrium P' in the figures because is perfectly symmetric to P), independently of the values of σ . The intuition for this is straightforward: if $\delta \rightarrow 1$, good 1 is too valuable to be exchanged for fiat money, so equilibrium M cannot exist; also, good 2 is worth very little as asset and it is costly to hold as well, so agents will accept fiat money to save on storage costs, and equilibrium B will not exist. Thus, the only equilibrium is P.

Summing up, there are differences worth mentioning between the standard search model with agents specialized in consumption and production and those in which agents are not specialized in consumption. In general terms, with agents less specialized in consumption in a rather natural way, it becomes more difficult to get fiat money to play a dominant role in exchange. First, we want to emphasize that the necessary condition for monetary equilibrium in Aiyagari and Wallace (1992) model of two commodities is strengthened in our model. While the necessary condition in their model was that the difference between storage costs of commodities and that of fiat money was positive, the characterization of the equilibria in our model shows that this difference should be larger than a positive number given by the values of the parameters of the model. Second, in models with more than two goods, Kiyotaki and Wright (1991) and Aiyagari and Wallace (1992) show that there exist equilibria in which fiat money is valued even if it has worse intrinsic properties than the rest of goods. In contrast with

this, fiat money can only be valued in our environment if its storability properties are strictly better than those of consumption goods (see Proposition 3 in Cuadras-Morató and Wright (1997)). The reason for all this should be intuitively clear: if all goods are equally storable, fiat money will never be preferred to the most valued consumption good, because, at best, they are equally liquid and consumption goods have additional consumption value (they can eventually be consumed). Hence, no agent will be able to get the most valued consumption good by holding fiat money, which means that fiat money will necessarily be the least valued object of all and will never be accepted in trade (equilibrium B).

3. Government Transaction Policy and Fiat Money

As we have argued in the Introduction, a second approach to introduce fiat money in the model is to study the way the government gets involved in the process of exchange. The government can be regarded as an important agent whose behavior could influence other agents' trading strategies. We now want to examine the role of government transaction policy into the problem of the determination of the medium of exchange.⁸ In order to do this, we need to formalize the existence of a new set of agents, which will be named *government agents* [our modeling of the government follows

⁸ In order to keep the analysis simple, we suppress storage costs (which are not central to our analysis now), so that all commodities are storable at no cost. As we shall see below, the results of the following sections hold as long as storage costs associated to fiat money are not too low compared to the rest of commodities.

Aiyagari and Wallace (1997); see also Li and Wright (1998) and Ritter (1995)]. Agents in the economy will be of two different types: a proportion G of agents will be government agents, while the rest $1-G$ will be private agents.⁹ Private agents have exactly the same characteristics of the agents we have described in the previous section: they produce according to their respective specialization and have a stochastic structure of preferences for all goods. The new ingredient now is the government sector. These agents do not produce anything and they do not consume either. They only go to the market, meet other agents and trade (we can think about them as if they were vending machines) according to some predetermined rule, which we shall call the government transaction policy. In this economy, the government adopts the following transaction policy: always accept fiat money, always give up fiat money, and exchange a consumption good for another consumption good with probability θ .¹⁰ The parameter θ is measuring the degree with which the government is favoring the emergence of fiat money as the medium of exchange. The lower the value of θ , the more favored is the emergence of fiat money as medium of exchange by the government policy. If $\theta=1$, then holding fiat money does not make easier to get a consumption good held by a government agent when compared to holding any other commodity (that is, it reduces the exchange value of fiat money).

Introducing government agents and government transaction policy are the only new elements in the environment of the economy described in Section 2. Even though the strategic decisions to be taken by agents do not change at all, there are a few aspects

⁹ We assume $G > m$.

¹⁰ See Li and Wright (1998) for a detailed analysis of more general transaction policies.

of the model that are different. First, notice that now it is possible that an agent of type 1(2) is holding good 2(1) at some point. The reason for this is that now she could get it from a government agent, who is always willing to exchange it for fiat money.¹¹

Notation has to be extended to include inventories held by government agents. Let p_{Gj} be the measure of government agents who happen to have in inventory good j . The distribution vector is $\mathbf{p}=(p_{10}, p_{11}, p_{12}, p_{20}, p_{21}, p_{22}, p_{G0}, p_{G1}, p_{G2})$. Moreover, $\sum_j p_{Gj}=G$, $\sum_j p_{1j}=\sigma(1-G)$ and $\sum_j p_{2j}=(1-\sigma)(1-G)$. Let $P_j=\sum_i p_{ij}$ and $\sum_j P_j=1-G$. The measure of agents in the economy who hold fiat money is now $m=P_0+p_{G0}$. Every period of time, a proportion $N [N=1-\delta P_1-(1-\delta)P_2]$ of agents go to the market and are randomly matched with other agents who are in the same situation. As before, the probability of meeting in the market an agent holding good j and wanting to consume good 1(2) is $\pi_{j1}(\pi_{j2})$. Similarly, any agent going to the market will meet with probability $\alpha_j=p_{Gj}/N$ a government agent holding good j . It is easy to see that $\sum_j \alpha_j + \sum_i \sum_j \pi_{ji}=1$.

Let us be more explicit about the derivation of the value functions in the new version of the model. The payoff functions, in flow terms, for a trader of type 1 (similarly for type 2) can be written as follows:

$$rV_{11} = \delta u + (1-\delta)[(\pi_{21} + \alpha_2 \theta)u + (\pi_{01} + \alpha_0)(1-s_{01})(V_{10} - V_{11})] \quad (3)$$

¹¹ Note that now trade is possible trade when agent 1(2) wants to exchange her fiat money for good 2(1). This is because government agents are always willing to take fiat money (as it is defined in their transaction policy). We have explained above why this would not possible in the previous model.

$$rV_{12} = (1 - \delta)(u + V_{11} - V_{12}) + \delta[(\pi_{12} + \alpha_1\theta)(u + V_{11} - V_{12}) + \pi_{02}(1 - s_{02})(V_{10} - V_{12})] \quad (4)$$

$$rV_{10} = \delta\{[\pi_{12}(1 - s_{01}) + \alpha_1](u + V_{11} - V_{10}) + \alpha_2 s_{02}(V_{12} - V_{10})\} + (1 - \delta)\{[\pi_{21}(1 - s_{02}) + \alpha_2](u + V_{11} - V_{10}) + \alpha_1 s_{01}(V_{11} - V_{10})\} \quad (5)$$

Expressions (3)-(5) are very similar to expressions (1) and (2). There are a few obvious differences worth commenting. First, storage costs have disappeared from the picture now. Second, we have the expression (4) for V_{12} , which was not present before. Third, traders can now meet government agents who are holding good j with probability α_j .

The definition of equilibrium and the equilibrium types do not change. In spite of the similarities between this version of the model and the previous one, this is now more difficult to solve. The difficult bit is, above all, computing the steady-state distribution of inventories, which has now become more complicated with the presence of government agents. Due to this, it will be impossible to get the type of general equilibrium characterization obtained in Proposition 1. The next proposition states the main result of this section.

PROPOSITION 2. *Equilibrium M does not exist in this economy.*

Proof. Substituting for the values of $s_{0i}=0$ in (3), (4) and (5) and solving the system of equations, we obtain that $V_{i0} > V_{i1}$ iff $\gamma_0 > \gamma_1$, where $\gamma_0 = \delta(\pi_{12} + \alpha_1) + (1 - \delta)(\pi_{21} + \alpha_2)$ and $\gamma_1 = \delta + (1 - \delta)(\pi_{21} + \alpha_2\theta)$ (similarly for good 2). Arbitrarily, we will suppose that $\delta \geq 0.5$. Then,

$$\gamma_1 = \delta + (1 - \delta)(\pi_{21} + \alpha_2\theta) > \delta$$

$$\gamma_0 = \delta(\pi_{12} + \alpha_1) + (1 - \delta)(\pi_{21} + \alpha_2) < \delta$$

so $\gamma_0 > \gamma_1$ cannot be an equilibrium M does not exist. If we suppose that $\delta \leq 0.5$ it is easy to show that $\gamma_0 > \gamma_2$ cannot be.

The intuition behind this result is simple. The value of holding any commodity (in terms of the probability of consumption) can be decomposed in two different elements. First, there is the consumption value, as any commodity, except fiat money, might eventually be consumed. Second, there is the liquidity value, as commodities might be exchanged in the market for other commodities that might be consumed. Fiat money has obviously zero consumption value. Its liquidity value is equal to $\delta(\pi_{21} + \alpha_1) + (1 - \delta)(\pi_{12} + \alpha_2)$, given that $s_{0i} = 0$. Good 1 has consumption value equal to δ and liquidity value $(1 - \delta)(\pi_{12} + \alpha_2\theta)$. Similarly, good 2 has consumption value $1 - \delta$ and liquidity value $\delta(\pi_{21} + \alpha_1\theta)$. Notice that although fiat money always has larger liquidity value than consumption goods, Proposition 2 proves that it cannot have larger total value than all consumption goods. This is because always at least one good has consumption value large enough to compensate for its lower liquidity value relative to fiat money. In other words, Proposition 2 shows that fiat money cannot be ranked as the

best asset in the economy and, then, equilibrium M does not exist.¹² This is in contrast with the result in Cuadras-Morató and Wright (1997) (see Proposition 3 in that paper). There, without government agents, a fiat object without any consumption value would always be the least valued asset and, consequently, will never appear as medium of exchange.

Proposition 2 is substantially different from the results in previous models that analyze government transaction policy. Thus, Aiyagari and Wallace (1997) (see Proposition 1 in that paper) and Li and Wright (1998) (see Proposition 1 in that paper) both identify general conditions under which there exists a monetary equilibrium (unique or not) in which all agents in the economy use fiat money as medium of exchange. This is precisely what we cannot get in our present set up. There might be equilibria in which fiat money is used quite generally, but never by all agents. In particular, agents holding the best-valued consumption good will never accept fiat money.

Does the nonexistence result in Proposition 2 hold when $n > 2$? There are two effects that may cast doubt on its robustness. On the one hand, the more goods there exist, the lower could be the consumption value of each single one and, hence, it would be possible to have fiat money better accepted than the rest of goods. On the other hand, with more than two goods, a problem of double coincidence of wants arises, so that fiat money might now be valued as medium of exchange (higher liquidity value). These two

¹² Proposition 2 states that fiat money cannot be the most valued object, although there might be many instances in which fiat money will still be used by some agents (this would be the case of equilibrium P and P').

factors mean that fiat money might be more valued the larger is n . Notice, however, that the logic behind Proposition 2 applies trivially to a different model in which the number of consumption good is $n > 2$. In particular, there will be always at least one good (the one that is more often wanted for consumption) that would not be accepted in exchange for fiat money. So, an equilibrium in which fiat money is universally accepted would not exist.

4. Summary and Concluding Remarks

We have presented in this paper counterexamples for some of the results on fiat money derived from the standard search theory [Kiyotaki and Wright (1989) and related literature]. As we have argued, those particular results hinged in limiting cases of a more general specification of the model. Consequently, we should be very cautious about the interpretation of some of the outcomes of the model. In particular, we have presented results from a version of the search theoretical model of money based on Cuadras-Morató and Wright (1997) in which agents no longer derive utility only from consuming always the same good, but every period of time get a taste shock that determines the good they want to consume, according to a common probability distribution.

There are several extensions of this model worthwhile mentioning here. First, the introduction of bargaining over the terms of trade which would allow us to make prices endogenous and discuss more explicitly about topics such as inflation. A monetary equilibrium in such a model would consist of a set of prices and trading strategies for which all or some agents accept fiat money. Obviously, the most desired goods would

have higher prices, in terms of money, than the least consumed and this could make fiat money acceptable by all agents in the economy, even those holding the most valued consumption goods. Second, the introduction of several currencies with different properties will give rise to the issues of currency substitution and rates of exchange. Finally, we have studied only a very limited type of government policy. One could think about different policies and the way they affect the conclusions we have drawn in this paper.

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Figure 1. Equilibrium B with $(c_1-c_0)/u=0.4$ and $m=0.1$

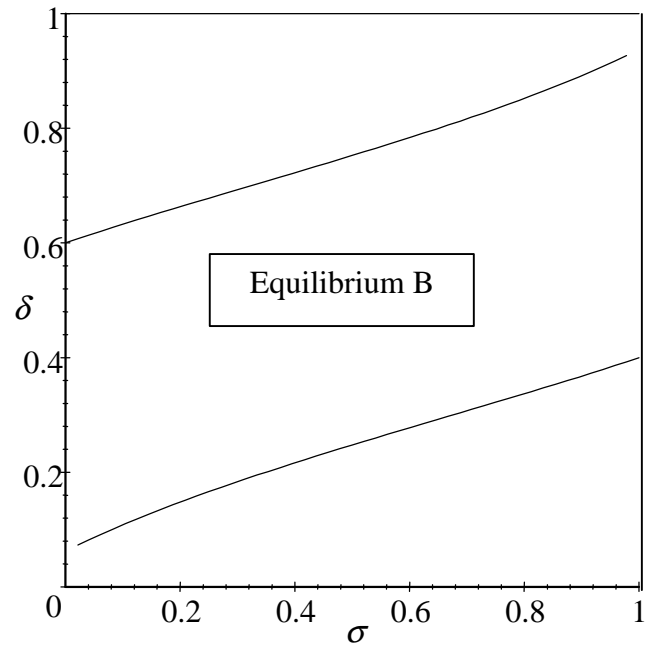


Figure 2. Equilibrium M with $(c_1-c_0)/u=0.4$ and $m=0.1$

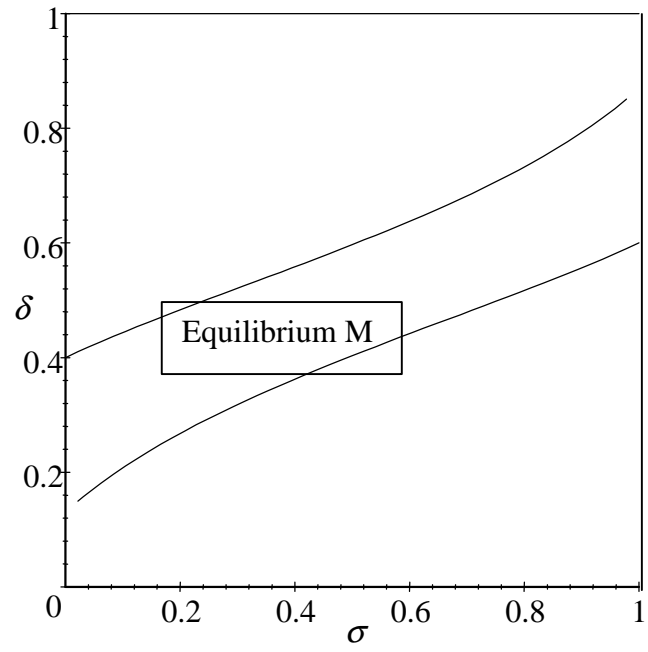


Figure 3. Equilibrium P with $(c_1-c_0)/u=0.4$ and $m=0.1$

