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Inequality and Public Resource Allocation

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Abstract

We set up a signaling game where individuals differ in ability and wealth. Higher ability means larger benefit if supported by the government. Costly signals are used to transmit information regarding own deservingness. However, capital market imperfections may perturb the signals by limiting the capacity of poor people to send the appropriate signal. We examine the cost in efficiency produced by the existing inequality in the distribution of wealth.

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1 Introduction

1.1 This Paper

In this paper we propose a particular channel through which wealth inequality contaminates the resource allocation process. Our analysis rests on four premises:

[1] Governments play a role in the development process. They can facilitate (or hinder) economic activity in certain geographical regions or in certain sectors by the use of subsidies, tax breaks, infrastructural allocation, preferential credit treatment, and permissions or licenses.

[2] Governments lack information — just as private agents do – regarding which sectors are worth pushing in the interests of rapid economic development.

[3] Agents lobby the government for preferential treatment. Moreover, lobbying sometimes but certainly does not always entail bribery of corrupt government bureaucrats. It involves industrial confederations, processions, demonstrations, signature campaigns, media manipulation and a host of other visible means to demonstrate that preferential treatment to some group will ultimately benefit "society" at large.

[4] A government – even if it honestly seeks to maximize economic efficiency — may be confounded by the possibility that *both* high wealth and true economic desirability create loud lobbies.

Of these four premises, it is immediately necessary to defend the premise of "efficiency maximization" in the last one. We do not believe that every single public decision-maker is truly honest. Nor do we believe that all honest governments will seek to maximize economic efficiency before all else. We merely use this assumption as a device to understand the signal-jamming created by the interplay of wealth and true profitability. In doing so, we develop a theory of the interaction between economic inequality and bad resource allocation. We believe that this particular interaction is important in our understanding of the development process.

Our point is that in a world of imperfect information, where lobbying plays the role of providing information to policy-makers, wealth inequality may distort the signals transmitted by economic agents. It is true that profitable sectors have more of an incentive to lobby intensively. At the same time, sectors dominated by wealthy interest groups find it *easier* to lobby more intensively. Consequently, honest governments can make bad resource-allocation decisions. This point has already been raised by Mork at al [1998] concerning what they term as the "Canadian disease": high inequality in inheritance confers strong lobbying power to individuals whose interests are tied to traditional production sectors. Public policies supporting these low productivity sectors slow down growth. Mork et al [1998] find empirical evidence supporting that countries with high inheritance inequality have lower growth rates.

Corruption may not make things that much worse. There is an abundant literature arguing that high inequality —specially in developing countries— is negative for growth because this facilitates the bribery and corruption of politicians and triggers intense rent-seeking. We shall summarize this literature in the next section. The point we wish to make here is that one can explain (at least partially) the observation that poor countries with high inequality appear to manage resources rather poorly without having to appeal to the *deus ex machina* of money-pocketing politicians.

The model we choose to make these points is an extension of Esteban and Ray [2000] to the case of an unequal distribution of wealth. In spite of its simplicity, a detailed analysis proves to be embarrassingly complicated. We conceive of a government, or a social planner, as an agency that publicly provides essential goods (or permissions) to carry out productive activities. We shall call these goods *permissions*.

There is a continuum of economic agents (we might think of them as individuals, production groups, or sectoral/regional interests). They receive productivity shocks which are independently distributed. These shocks endow individuals with "high" or "low" productivity. The type of productivity shock received is private to each individual.

A permission granted to an agent permits unrestrained economic activity on the part of that agent. Moreover, production can be carried out only by individuals endowed with a permission. The government has a limited number of permissions and wishes to allocate them so as to maximize economic efficiency. However, because the productivity shock is private information, the government does not know who the productive agents are.

Agents can send costly signals conveying implicit information regarding the gains expected from operating one's own technology. We assume that at the lobbying stage, agents are wealth-constrained. [In the main body of the paper, we discuss this assumption in more detail.] Moreover, wealth is also private information. The intensity of the signal emitted is therefore conditioned by wealth as well as the productivity level. We characterize the signaling equilibria of this model, and relate the resulting efficiency of resource allocation to the level of wealth inequality.[1][2]

Before we embark on the analysis, we discuss relevant research and set our exercise in the context of this literature. In Section 2 we describe the model. This section contains the basic setup, a discussion of possible embeddings into a growth framework, and a precise definition of equilibrium. In Section **??**, we obtain a general restriction on equilibrium outcomes that applies, in our opinion, to a much wider class of "allocation models" in which a principal must allocate some fixed supply of a good to a group of agents. Nevertheless, a wide range of equilibrium outcomes are still possible. We

[1]In Esteban and Ray [2000], where there is no inequality, we could examine only the types of equilibria depending on the benefits from permissions relative to the common wealth level.

[2]Signalling games under wealth constraints have also been developed in models of educational attainement such as Fernandez and Gal and Bedard [2001].

refine these in Section **??**, using the well-known intuitive criterion and a principle of "belief-action parity", introduced in this paper. In Section 3 we completely characterize those equilibria which survive these refinements. In Section **??** we provide a preliminary analysis of equilibrium losses.

1.2 The Literature

The relationship between economic inequality and related aspects of economic development is a recurrent matter of concern to economists and political scientists.[3] Recent literature examines the connections between inequality and national output (or growth of output) from a new starting point. Broadly put, countries with high inequality experience output (or growth) losses because high inequality has some functional impact on aggregate "capabilities" via the resource allocation process. This paper will squarely follow that tradition.

This recent literature on inequality and development can be structured around two distinct, but complementary explanations for the efficiency costs produced by inequality. The first type of explanation focusses on the role played by capital market imperfections, which impose a borrowing constraint on low wealth individuals (as seen in the earlier work of McKinnon [1973], Loury [1981] and others, and more recently in Aghion and Bolton [1997], Banerjee and Newman [1993], Galor and Zeira [1993], Lee and Roemer [1998], Lundqvist [1993], Mani [1998], Piketty [1997] and Ray and Streufert [1993]). Unable to borrow, poor individuals are constrained to make choices which do not duly develop their abilities, so that aggregate output is below its potential level. It follows that the extent of the efficiency loss critically depends on the number of individuals for which this constraint is binding. Moreover, given the fragmentation in the credit market, and one of a variety of possible nonconvexities (present in the majority of the cited papers), economic development will be severely constrained by initial inequalities.

The second type of linkage between inequality and growth adopts a broad political-economy approach (see, for instance, Alesina and Rodrik [1993], Benabou [1998], Benhabib and Rustichini [1996], Chang [1998], Perotti [1993, 1994, 1996], Persson and Tabellini [1994] and Saint Paul and Verdier [1997]). We can divide this literature, in turn, into two parts. The first (exemplified by the work of Alesina and Rodrik [1993] and Persson and Tabellini [1994]) considers redistribution induced by the democratic process. In situations of high inequality, poor voters will use their political power to redistribute wealth. If such redistributions are constrained — presumably by political or informational considerations — to act "on the margin" (rather than assuming intramarginal forms such as a one-off land reform), they will bite into investment incentives and

[3] The early work of economists such as Kaldor [1955], Pasinetti [1962] and especially Kuznets [1955, 1966] is well-known and needs no special elaboration here.

reduce growth.[4]

A second line of research in this broad area studies the relationship between social conflict and growth (see Benabou [1996], Benhabib and Rustichini [1996], Chang [1998] and Tornell and Velasco [1992]). While several of these papers do not explictly consider inequality as a driving force of possible social conflict, some explictly attempt to make the connection. For instance, high inequality may more easily serve to rupture an implicit social pact between various economic groups to not engage in costly resource-grabbing for current consumption (see Benabou's [1996] exposition of and variation on the Benhabib-Rustichini theme). Or see Olson's [1965] argument that government corruption is more likely in highly unequal societies.[5]

The literature on political instability and growth also falls into this broad category. Alesina, Özler, Roubini and Swagel [1996] demonstrate statistically significant associations between low growth, social polarization and political instability (see also Mauro [1995], Perotti [1996] and Svensson [1998]). But the connection between political instability, conflict and growth is a bit of a black box.[6]

These two main approaches — the one based on missing markets, the other on political or social struggle — have been developed quite independently from each other. There is much to be gained in marrying the two. Models of imperfect capital markets, while insightful in themselves, would be enriched by taking on board the political process. The implication for the second strand of literature — at least, the part that focusses on democratic redistribution — is more damaging. As Perotti [1994, 1996] and others have noted, initial inequalities may be related to slower growth, but evidently not through the channels proposed in this part of the theoretical literature. Unequal societies tend to under- rather than over-redistribute. Thus unequal societies may have inimical effects on growth because they *stay* unequal (and therefore suffer from one or more of the woes in the missing-markets story), and not because of some incentive-sapping drive towards equality.

Rodríguez [1997] and Benabou [1998] address this issue explictly. These authors follow a line that possibly goes back as far as Plato: even in a democratic society, *effective* political power is correlated with wealth. One can model this in a reduced-form way — as Benabou does – by assuming that the political weight of a voter depends on his rank in the wealth distribution, and then examining the implications of such a postulate.

[4] Higher taxes might also have negative effects on investment because they would induce capital flight to less redistributive countries (see, for example, Tornell and Velasco [1992] and Saint Paul and Verdier [1997]).

[5] Mauro [1995] finds evidence of a statistically significant negative relationship between corruption and growth.

[6] Alesina and Perotti [1997] propose the effects of political instability on investment as the possible channel linking it to growth. In particular, the intuititon is that in highly polarized societies it is more likely that we observe radical changes in policies. It is this policy uncertainty that restrains investment.

One way to view our paper is that it provides a foundation to the assumption of increasing political power (in wealth), albeit in special form. But there are other ingredients that are present.

First, we draw on the idea that the development process is fraught with informational gaps. This is especially so in societies that are undergoing rapid transformation. It may be very hard for a planner to understand and foresee the correct directions in which the economy must go.[7] In this sense, lobbying serves as a generator of possibly useful information, in contrast to the black-box models of rent-seeking analyzed in profusion in the literature.[8]

Second, the possibility (in our model) that wealth inequality can jam information is based squarely on the assumption that capital markets are imperfect at the lobbying stage — one cannot borrow to finance such activities. This draws upon the extensive literature on inequality and development based on credit-market imperfections, alluded to earlier.

Third, we maintain as a working hypothesis the assumption that the social planner is honest. This contrasts sharply with almost the entire literature on lobbying or rent-seeking in developing societies, which explictly or implictly assumes that corruption is at the heart of the problem.[9] In this sense, we are partly in line with Banerjee [1997], who seeks to understand bureaucratic red tape as the outcome between a welfare-maximizing government and a money-grabbing bureaucrat. As he shows, an analysis based on the premise of a fully corrupt government can be problematic in some respects. We don't go in this direction — in fact, we dispense with the nasty bureaucrat altogether — but like Banerjee, we are interested in understanding aspects of an economic system that are not based on corruption alone.

Fourth, even in the most conspicuous democracies there is only a limited number of issues which are decided by majority voting. Referenda are exceptional. The population elects a government, who most of the time must make decisions on the many matters left unspecified in their electoral platform. The point is that in a democracy there is ample room left for discretionary governmental decisions, which can be influenced by the

[7]On the possibility of degraded information in the course of development, and its implications for market functioning, see Ghosh and Ray [1996], Ray [1998, Chapters 13 and 14], and the World Development Report [1998/99].

[8]In this respect, our model shares features in common with literature that views lobbying as information communication; see, for example, Austen-Smith and Wright (1992), Bennedsen and Feldman (2002), Lohman (1994) and Rasmussen (1993).

[9]On rent-seeking and lobbying, see, for instance, Mohtadi and Roe [1998], Rama and Tabellini [1998], Shleifer and Vishny [1993] and Verdier and Ades [1996]. Both in rent-seeking and in lobbying players use resources in order to induce government decision most favorable to their interests. Whether these resources are wasted away or used to bribe government officers does not seem to be essential to the story. Indeed, in abstract models it is difficult to distinguish between a politician that is honestly impressed by the amount of lobbying by an agent and one that gets bribed by the agent who pays the most.

citizens. It is at this stage where wealth does play a role. To be sure, it is the existence of this discretionary space that explains the development of rent-seeking, lobbying, and even corruption.

The idea that wealthy agents may confound the resource allocation process because of their greater ability to corner resources is not new. The point has been made in development contexts time and again (see Bhagwati and Desai [1956] and Bardhan [1984] for insightful analyses along these lines). But the point has usually been made in the context of corrupt bureaucracies — scarce permissions or infrastructure may be bought by wealthy agents by simply bribing corrupt officials. In contrast, we emphasize the information-jamming aspects of wealth, something that can occur even when governments and bureaucrats are perfectly honest.

2 Information Transmission through Lobbying

2.1 Basic Data

A government must allocate a publicly provided input to facilitate production among economic agents. We will think of this input as *permissions*. The important restriction is that the number of permissions α is limited, that is $\alpha \in (0, 1)$.

Agents are distributed on $[0, 1]$. Assume that only two types of agents exist, indexed by the profitability in the event of being granted a permission. Denote the two levels of profitability by a and b, with $0 < a < b$. The high type b is bestowed on the agents with iid probability β.

An agent of type λ ($\lambda = a, b$) and wealth w who has expended resources r in lobbying and will be awarded a permission with probability p has an expected return of

$$p\lambda + (w - r). \tag{1}$$

Suppose that the distribution of wealth w over economic agents is given by some cumulative distribution function $F(w)$. We shall use the expression $rich_x$ to denote an individual with wealth $w \geq x$. We assume that this distribution is independent of the distribution of productivity types, so that conditional on being a high or a low type, the distribution is exactly the same. Formally, we assume

[F.1] The cumulative distribution function of wealth, F , is the same for both types of agents.

[F.2] F is continuous with full support on an interval $[0, M]$, where $M > a$.

We will assume that an agent with wealth w cannot spend more than w in lobbying. The assumption of an imperfect capital market is critical to this model. It is obvious that with wealth being distributed over $[0, M]$ some individuals will be wealth constrained in their feasible announcements. Yet, to make the problem interesting it has to be that not all the individuals are wealth constrained. Notice that a is the maximum announcement a low type will be willing to make if grated a permission for sure. Moreover, as we will soon show, equilibrium announcements by all, high and low-types, never exceed a. Therefore, $M > a$ is necessary to have that some (sufficiently rich) individuals are not wealth constrained. All the individuals with wealth $w < a$ will be effectively wealth constrained for some probability function p. There will be $F(a)$ such individuals. We shall use $F(a)$ as a measure of inequality.

2.2 Equilibrium

The government cares about efficiency alone: it would like to single out the high types and give them permissions to produce. If there are less permissions than high types, this is all he would like to do. If there are more permissions than high types, he would not mind giving the remaining permissions to the less productive (but still productive) low types. Individuals on their side would like to be identified as high types and thus qualify for a permission. To this end, they engage in lobbying in an attempt to persuade the government that they value a permission very highly. Thus, we view lobbying as a potential device to solve the informational problem.

We proceed to a formal definition of equilibrium. Given some distribution function F (as well as the other parameters that we have already described) an *equilibrium* consists of three objects $\{\theta, \mu, p\}$, where

[1] θ maps wealth-productivity pairs into lobbying expenditure, which we denote by r. Moreover, θ is a best response in that for each such pair (w, λ), the announcement $r = \theta(w, \lambda)$ is optimal given the probability function p (see (3) below).

[2] μ maps all lobby expenditures r (not just the equilibrium ones) to posterior beliefs held by the planner regarding the proportion of high types at r. For announcements in the support of θ we require that μ must be obtainable using Bayes' Rule, applied to the prior belief that the proportion of high types is β, and then using information from the shape of the function θ. Off the equilibrium support the concept of a sequential equilibrium does not impose any restriction on μ.

[3] p maps all lobbying expenditures r to the probability $p(r)$ that an announcement at r will receive a permission from the planner. Given the posterior beliefs μ held by the planner, we require that the probability function p must be chosen in order to maximize the expected number of permissions that accrue to the high types.

Notice that our definition posits a *reactive* government which cannot commit to a particular line of action during the lobbying process. Thus, the signalling game we describe is a simultanous move game.[10] We take this approach because we believe that the no precommitment case is a better description of reality when lobbying is involved. Numerous government officials are often involved in the allocation decisions, so that the reputational concerns that might underpin a commitment model (with screening) are attenuated. In addition, when the objects to be distributed are costly to the government —think of choosing the location of infrastructures, for instance— and because of budgetary constraints, the amount of objects to be allocated is given before hand.

Our equilibrium notion fails to put any restrictions on off-equilibrium beliefs. This generates an alarming variety of implausible equilibria. We rule these out in a completely standard way by applying the *intuitive criterion* of Cho and Kreps [1987], which we denote by IC. In addition, we want to discard the class of equilibria in which the planner discriminates between two announcements with the exactly same posterior beliefs over the share of high b types by assigning licenses with different probabilities. Formally, we introduce the following criterion:

BELIEF-ACTION PARITY. If the planner believes that the proportion of high types is the same across two announcements, then he should allocate permissions with equal probability to individuals making the two announcements.

In the rest of our analysis, we impose the belief-action parity criterion (henceforth christened BAP), as well as the IC.

3 Equilibrium Outcomes

Our analysis of the equilibria of this signalling game starts with the introduction of a fundamental result concerning the maximum number of announcements in an equilibrium. We then introduce a number of properties shared by all the equilibria satisfying IC and BAP. This will greatly simplify our subsequent characterization of equilibria presented at the end of this section.

3.1 The Posterior Principle

We start by showing that under very mild restrictions, there can be no more than *three* equilibrium announcements of r. This is no mere technical point; it greatly assists us in the search for equilibria of this model. Moreover, the observations in this section appear to be quite general and go beyond the confines of the particular model studied here.

The first step towards this outcome is a general restriction on the beliefs of the planner We prove a general result regarding the *number* of different posterior beliefs

[10]This assumption on the government can be contrasted with a *mechansim design* approach in which the government commits first an allocation rule and agents then react. See Banerjee [1997] for a model along these lines.

that a planner can possess over the equilibrium set. As a matter of fact, we shall estate this result for the case of an arbitrary number of types $k \geq 2$. The returns of a licence to the different types will be denoted by a_j, $j = 1, ...k$. Without loss of generality we assume that $a_j < a_{j+1}$.

The government posterior associated to a given announcement r, $\mu(r)$, is an k-dimensional vector of the probability of finding an individual of type m at this announcement level.

The government is interested in the expected return of a license at every equilibrium announcement using $\mu(r)$ and will award licences accordingly with a probability function $p(r)$ such that the aggregate return is maximized.

Say that an equilibrium *exhibits n revelation levels* if there are n distinct vectors $\mu(r)$ in the set of equilibrium announcements. [Notice that we are only referring to the equilibrium announcements and not to the behavior of beliefs off the equilibrium path.] To be sure, $n = \infty$ is also permitted. Observe, by the way, that many different levels of revelation is not necessarily "better" than, say, just two — for instance, a separating equilibrium that fully identifies the highest types has just two revelation levels ($\mu(r) = 1$ for some values of r and 0 for other values of r). Nevertheless, the notion of revelation levels is related to how complex an equilibrium can get.

Call an equilibrium announcement *pivotal* if, without it, one level of revelation is removed. In other words, a pivotal announcement carries a posterior belief that is not present in any other equilibrium announcement.

Proposition 1 *No equilibrium can exhibit more than three revelation levels. Moreover, if there are exactly three levels of revelation, then the zero announcement must be pivotal.*

Proof. Suppose that either one (or both) of the two statements in the proposition is violated. Then there must exist three *strictly positive* equilibrium announcements r_1, r_2 and r_3 such that $\mu(r_h) \neq \mu(r_j)$, for $h \neq j$, $h, j = 1, 2, 3$. Without loss of generality suppose that $r_1 < r_2 < r_3$ is the largest announcement. Then notice that the probability of permission grant $p(r_j)$ must be strictly less than $p(r_3)$ for both $j = 1$ and $j = 2$. [If this were not the case no one would ever announce r_3.] In particular, we may conclude that $p(r_j) < 1$ for $j = 1, 2$. By the same argument we deduce that $p(r_1) < p(r_2)$.

Note, moreoever, that $p(r_j) > 0$ for $j = 1, 2$, otherwise no one would announce r_j: 0 would be a better announcement (here we use the fact that all the r_j's are strictly positive). We therefore have the following observations:

$$0 < p(r_1) < p(r_2) < p(r_3) \leq 1$$

and

$$\mu(r_1) \neq \mu(r_2).$$

It must thus be that the expected return to a license is increasing in the announcement r.

But these two observations together contradict the equilibrium requirements for the function p. Permissions (equivalently, probability) can be transferred from the announcement r_1 to the higher announcement r_2 and increase the aggregate expected return of licenses. ∎

This result, which we might dub the *posterior principle*, is extremely general, in the sense that it must hold for many situations in which a fixed number of permissions or permits are being allocated. We may generalize by allowing for all sorts of planner preferences (including "corrupt preferences" in which agent wealth levels enter), and for general agent preferences (as long as permissions are desirable and announcements are costly). Notice too that defining permissions as a (0,1) decision is not critical either. The fact that permissions are awarded with given probabilities renders the expected valuation of a permission a continuous variable. What is crucial is that a planner services all announcements for which his posterior probability puts high weight on his "preferred type" *before* going on to service another announcement (with lower posterior).

Now it is easy to see that there cannot be three or more *distinctly costly* announcements with different posteriors attached to them. For in that case, the announcement with the poorest posterior must be serviced (with positive probability), otherwise no agent would make that announcement. But this means that the other two announcements must be fully serviced — that is, the probability of allocation at those announcements must *both* be unity. But now we have a contradiction to agent optimality — no one will wish to make the costlier of those two announcements.

The following proposition is immediate, given the posterior principle.

Proposition 2 *Under the principle of belief-action parity, there can be no more than three equilibrium announcements. Moreover, if there are exactly three equilibrium announcements, one of them must be at zero.*

Proof. By Proposition 1, there can be no more than three levels of revelation, and if there are exactly three, the zero announcement is pivotal. Therefore, if either of the statements in the proposition is violated, there must exist two distinct announcements of positive values of r, say r_1 and r_2, such that $\mu(r_1) = \mu(r_2)$. But then, by belief-action parity, $p(r_1) = p(r_2)$. It follows that no agent will wish to make the higher announcement, a contradiction. ∎

The interested reader can find in Esteban and Ray [2001] a detailed discussion of the role of BAP in refining the equilibrium set.

3.2 Some Properties of Equilibria

In Section 2 we have defined an equilibrium as consisting of three objects: a probability function p, the posterior beliefs by the planner and the announcements by the players. The following three Lemmas provide some properties of each of the three ingredients of an equilibrium.

Lemma 3 *In any equilibrium:*

[1] If r is an equilibrium announcement and $r' < r$ is any other announcement, then $p(r) > p(r')$.

[2] If r and r' are equilibrium announcements, if $\mu(r) > \mu(r')$, and if $p(r') > 0$, then $p(r) = 1$.

[3] Let R be the maximal equilibrium announcement. Then, at this maximal announcement, either $\mu(R) = 1$ or $p(R) = 1$.

Proof. The first two parts are pretty obvious. For part [1], if $p(r') \geq p(r)$, then no one should bid the higher value r . For part [2], since both are equilibrium announcements, and $\mu(r) > \mu(r')$, all permissions should be given to the r announcers before any are handed out to the r' announcers. So if $p(r') > 0$, this must mean that $p(r) = 1$.

As for part [3], suppose that the statement is false. Then both $\mu(R) < 1$ and $p(R) < 1$. In particular, low types are involved in making the maximal announcement. It follows that the highest possible equilibrium return to the low type (net of wealth) is

$$p(R)a - R, \tag{2}$$

and the highest possible equilibrium return to the high type is

$$p(R)b - R. \tag{3}$$

Define an announcement r by the condition that the low type is indifferent between this announcement and his highest equilibrium return, given by (2), *even if* the low type is granted the permission with probability one:

$$a - r \equiv p(R)a - R.$$

Notice that by [F.2], such announcements are feasible for wealthy enough individuals of either type.

Every announcement that even slightly exceeds this value r must be equilibrium-dominated for the low type. But it is easy enough to see, examining (3), that the high type would benefit from making some such announcement, as long as $p(R) < 1$.

It follows from the Intuitive Criterion that such announcements must receive posterior probability $\mu(r) = 1$ and consequently (since a zero measure of individuals is making the announcement), a probability of permission-receipt equal to one. This contradicts the supposition that we have an equilibrium in the first place. ∎

Lemma 4 *Assuming BAP, consider an equilibrium with at least two distinct equilibrium announcements. Then*

[1] If R is the highest equilibrium announcement, $\mu(R) > \beta$.

[2] If r' is the lowest equilibrium announcement, $\mu(r') < \beta$.

Proof. To prove part [1], notice that if the assertion is false, then $\mu(R) \leq \beta$. Because β must be a weighted average of the $\mu(z)$'s over all equilibrium announcements z, there must exist an equilibrium announcement $r < R$ with $\mu(r) \geq \beta$. Using the optimality of the planner's reaction as well as the BAP, it follows that $p(R) \leq p(r)$. This contradicts Lemma 3 [1].

To prove part [2], suppose on the contrary that $\mu(r') \geq \beta$. Then there exists an equilibrium announcement $r > r'$ with $\mu(r) \leq \beta$. Using the optimality of the planner's reaction as well as the BAP, it follows that $p(r) \leq p(r')$. This again contradicts Lemma 3 [1]. ∎

Lemma 5 *Under BAP and IC, consider any equilibrium with at least two equilibrium announcements. Let R be the maximal announcement, then*

[1] if not all low $rich_R$ types bid R, for every equilibrium announcement r

$$p(r)a - r = p(R)a - R, \tag{4}$$

[2] all high $rich_R$ types must announce R,

[3] $R \leq a$, and

[4] the lowest announcement must be zero.

Proof. We start with part [1]. By the assumption of the lemma, it must be the case that some low rich_R type bids $r < R$. Therefore

$$p(r)a - r \geq p(R)a - R. \tag{5}$$

If some low rich_R types are also bidding R, then the opposite weak inequality holds as well, and we are done.

Otherwise, no low rich$_R$ types are bidding R. Then $\mu(R) = 1$. Now, if strict inequality holds in (5), then there exists $\epsilon > 0$ such that

$$p(r)a - r > p(R)a - (R - \epsilon).$$

Now consider a deviation to the announcement $R - \epsilon$. By BAP, the *best* probability that the planner can attach to permission-granting under this announcement is $p(R)$ (because there are only high types announcing at R). But even then, the above inequality shows that it would not pay for any low type to announce such a deviation. In other words, the deviation to $R - \epsilon$ is equilibrium-dominated for the low type (but obviously not for the high types at R). We can conclude that the planner must believe that the deviation is by a high type, and (again by BAP) must use $p(R - \epsilon) = p(R)$. But then this is a profitable deviation for a high rich$_R$ type (previously announcing R), contradicting the presumption that we have an equilibrium to begin with. [This completes the proof in case there are only two equilibrium announcements.]

Suppose, now, that there is a third announcement r'. Then, of course,

$$p(r')a - r' \leq p(r)a - r$$

otherwise no low rich$_R$ type would bid r, as presumed. If strict inequality holds in the expression above, consider two cases. If $r' > r$, then no low person — poor or rich — would ever bid r'. This means that $\mu(r') = 1$, so that $p(r') \geq p(R)$ (by BAP), a contradiction.

If, on the other hand, $r' < r$ (and strict inequality holds) it follows that no low rich$_r$ person would ever bid r'. But *every* poor$_r$ person, high or low, must bid r'. It follows that $\mu(r') \geq \beta$, which contradicts part [2] of Lemma 4.

We now prove part [2] by contradiction. Suppose that the statement in [2] were false. Then some high rich$_R$ person weakly prefers a lower equilibrium announcement r. That is,

$$p(r)b - r \geq p(R)b - R. \tag{6}$$

By Lemma 3 [1], it must be that $p(R) > p(r)$. Using this information along with (6), we may conclude that

$$p(r)a - r > p(R)a - R,$$

but this contradicts (4) of part [1] of this Lemma that be have just proven.

We establish part [3] by a simple application of the intuitive criterion. Suppose that the lemma is false. Then notice that by the definition of a, only high types can announce R. Now consider an observed deviation to $R - \epsilon > a$. Then this announcement must be equilibrium-dominated for the low type, because $p(R - \epsilon)a - (R - \epsilon) \leq a - (R - \epsilon) < 0$. But it is not equilibrium-dominated for the high type announcing R . Therefore the planner must believe that the type at this announcement is high. He must therefore use the same probability at $R - \epsilon$ of granting a permission as he did at R. But then this is a profitable deviation for a high rich_R type, a contradiction.

Finally, part [4] follows immediately from [F.2]. There are players with wealth (and hence maximum announcement) arbitrarily close to zero. ∎

3.3 Characterization of the Set of Equilibria

There are two types of equilibria —separating and partially pooling— and within each type there are restrictions on the equilibrium configuration that give the model a certain degree of cutting power. In a word, while lots of things can happen, *everything* cannot happen. In Section 4 we show that there is even more cutting power if we simply restrict ourselves to equilibrium losses — both in terms of pure signaling waste as well as allocative losses due to the presence of inequality. In other words, several equilibria look the same in terms of the generated losses.

In our analysis of equilibria, it will be useful to mentally picture an equilibrium *correspondence* as α varies from low values to high. Later, we will attempt to relate this variation to changes in the inequality of wealth distribution, as captured by the cumulative distribution function F. Because an equilibrium is characterized by bids, allocation probabilities and beliefs, it is somewhat harder to visualize the range of this correspondence. While we keep this aspect vague for now, we will be able to generate a more "quantitative" range once we have our estimate of the losses along each equilibrium.

In the separating equilibrium there will be two announcements while in the partially pooling there can be either two or three announcements. None of these types of equilibria exists throughout the full range as α varies between 0 and 1. However, we can say quite a bit about the range of existence, as well as the various losses (in terms of lobbying expenditure and allocation) that they entail.

In what follows two particular values of α will turn out to be critical: α_1 and α_2. We define α_1 as

$$\alpha_1 \equiv \frac{\beta}{a} \max_{0 \le R \le a} R\,[1 - F(R)] \tag{7}$$

and α_2 as

$$\alpha_2 \equiv \beta[1 - F(a)]. \tag{8}$$

From the definition of α_1 it immediately follows that $\alpha_2 \le \alpha_1 < \beta$.

The following Proposition presents the complete description of the three types of equilibria possible which we denote by Type I, II and III, respectively. The proof is relegated to Appendix 2.

Proposition 6 *The equilibrium set consists of the following three types of equilibria:*

[1] Separating Equilibria [Type I]. *For every $\langle \alpha, \beta, a, F \rangle$ such that $\alpha \le \alpha_1$ there exists a separating equilibrium (θ, μ, p) in which only high rich_R types are serviced at R,*

and no one else receives a permission. This equilibrium achieves full separation —hence $\mu(R) = 1$— and the only waste comes from signaling. In this equilibrium, the low bid is zero with a zero probability of being serviced, while the high bid R solves

$$R\left[1 - F(R)\right]] = \frac{\alpha a}{\beta}, \tag{9}$$

and the probability of receving a license there is $p(R) = \frac{R}{a}$. Further, there might be more than one equilibrium R for given $\langle \alpha, \beta, a, F \rangle$.

[2] Partially Pooling Equilibria with Two Signals [Type II]. *For every $\langle \alpha, \beta, a, F \rangle$ such that $\alpha \geq \alpha_2$ there exists an equilibrium (θ, μ, p) in which the high rich$_R$ types plus a fraction $\lambda \in [0, 1)$ of the low rich$_R$ types are serviced at a bid R and in addition the remaining permissions δ —$\delta = \alpha - (\beta + \lambda(1 - \beta))\left[1 - F(R)\right]$— are distributed to those that bid nothing. Every pair (λ, R) solving*

$$\frac{(1 - \alpha)a}{(a - R)\left[1 - F(R)\right]} = \beta + (1 - \beta)\,\lambda$$

is an equilibrium, with

$$p(0) = \frac{\alpha - \left[\beta + (1 - \beta)\,\lambda\right]\left[1 - F(R)\right]}{1 - \left[\beta + (1 - \beta)\,\lambda\right]\left[1 - F(R)\right]}.$$

For every λ there is a unique equilibrium R. Clearly, $\mu(R) = \frac{\beta}{\beta + (1-\beta)\lambda} > \beta$. Under type 2 equilibrium there are losses both in terms of the efficacy of public allocation and because of resources expended in signaling.

[3] Partially Pooling Equilibria with Three Signals [Type III]. *For every $\langle \alpha, \beta, a, F \rangle$ such that $\alpha \in (\alpha_2, 1 - F(a)]$ there exists an equilibrium (θ, μ, p) with three bids $(R, r, 0)$ in which the high rich$_R$ types plus a fraction $\lambda \in [0, 1)$ the low rich$_R$ types are fully serviced at a bid $R = a$, all these constitute the high bidders which we denote by $B(a)$. The remaining permissions δ —$\delta = \alpha - B(a)$— are distributed to those who bid r denoted by $B(r)$, i.e. all the high rich$_r$ types with $r \leq w \leq R$ and a fraction π of the set of potential low type bidders composed of the low rich$_r$ types who have not bid a. There might also be equilibria for some $\alpha > 1 - F(a)$ but there exists $\alpha_3 < 1$ such that no equilibrium exists for $\alpha \geq \alpha_3$. These equilibria are characterized by a tuple (λ, π, r) solving*

$$\begin{aligned} B(a) &= \left[\beta + (1 - \beta)\,\lambda\right]\left[1 - F(a)\right], \\ B(r) &= \beta\left[F(a) - F(r)\right] + \pi(1 - \beta)\left[(1 - \lambda)(1 - F(a)) + F(a) - F(r)\right], \textit{and} \\ \alpha a &= aB(a) + rB(r). \end{aligned}$$

The equilibrium probability for the bid r is $p(r) = \frac{r}{a}$.

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Figure 1: A Schematic View of Equilibrium

The equilibrium posterior beliefs are

$$\mu(a) = \frac{\beta\,[1-F(a)]}{\alpha-\delta} > \mu(r) = \frac{\beta\,[F(a)-F(r)]}{\delta a} > \mu(0) = \frac{\beta r - (\alpha-\delta)\,\mu\,(a)\,r - \delta a \mu\,(r)}{r-(\alpha-\delta)\,r-\delta a}.$$

Under this type of equilibrium too there are losses both in terms of the efficacy of public allocation, and because of resources expended in signaling.

For mental organization, Figure 1 describes the "equilibrium correspondence" as α ranges from 0 to 1. The phrase is in quotes because (for now) we put no variable in the range of this correspondence. The picture merely serves as a schematic description of equilibrium. Because an equilibrium is characterized by bids, allocation probabilities and beliefs, it is somewhat harder to visualize the range of this correspondence. While we keep this aspect vague for now, in the next section we will be able to generate a more "quantitative" range once we have our estimate of the losses along each equilibrium.

4 Lobbying and Waste: Conflictual and Allocative

The equilibria described so far generally result in wasted resources. That in itself is no surprise. It is well known that signaling private information is a costly enterprise. High productivity types need to convince the government that they are indeed high, and this cannot be done free of charge.

In sharp contrast with the separation property that is available in signaling models without wealth constraints, the loss manifests itself in two distinct flavors. First, there is what we might call the *conflictual loss*. We measure this simply by the total resources consumed in the signaling process. This is the standard waste that most models of rent-seeking concentrate on. Second, there is what might be called the *allocative loss*. A planner, honest though she may be, may be unable to separate the high and low types completely. Permissions may go to the low types when they were actually meant to go to the high types. We measure this loss by the expected measure of wrong types receiving the permission, multipled by the profit differential between a typical high type and a typical low type. We denote by T the total cost and by C and A its conflictual and allocational components, respectively.

Suppose for a moment that the government were to act without trying to learn about individual characteristics. This would result in a random allocation of premissions across the population. Instead, by being responsive to lobbying, the government induces private individuals to incur into *conflictual* costs. However, in exchange, the government may acquire information permitting a more efficient allocation of resources and hence a reduction in the allocation costs as compared with the random allocation. Does the improvement in resource allocation compensate for the conflictual costs incurred? Whether lobbying is an efficient way of producing information has to be ascertained by comparing the conflictual costs incurred with the eventual allocative benefits relative to the random allocation.

In order to compute the allocative losses incurred by a random allocation of licenses, T^R, we need to bear in mind that when $\alpha > \beta$ some licenses have to go to the low types any way. We thus compute T^R as the difference between the potential benefits of a perfectly informed allocation relative to the random allocation. Specifically, we shall have that

$$T^R = (b - a) \left[\min(\alpha, \beta) - \alpha\beta \right]. \tag{10}$$

Taking into account that for equilibria of Types II and III the costs can take on any value between minimum and maximum costs, we shall say that *lobbying is strongly efficient (inefficient) if* $T_{\max} \leq T^R$ $(T_{\min} \geq T^R)$.

In this section we separately examine the conflictual and allocative losses that correspond to the different equilibria as parametrized by α[11] and then address the issue of efficiency, that is, of whether conflictual costs compensate for the allocational benefits. In the next section we will discuss how inequality affects these costs.

4.1 Type I Equilibria

We know that separating equilibria can exist for all $\alpha \leq \alpha_1$. Moreover, when $\alpha \leq \alpha_2 \leq \alpha_1$ the separating equilibria are the only equilibrria possible. Because it is separating, this equilibrium awards permissions *only* to the high types. So there is no allocative loss. The conflictual loss is given by the value of the maximal announcement R times the measure of those who announce R:

$$C^I = R\beta \left[1 - F(R) \right] = a\alpha, \tag{11}$$

where we make use of (9). Despite the possible multiplicity of equilibria in this equilibrium type, the conflictual loss is uniquely defined (for each value of α).

We summarize this easy case in a picture (see Figure 2), the first component of a more complicated diagram for all three equilibrium types that we construct piece by piece.

Let us now examine whether separating equilibria are efficient relative to the random allocation. Indeed, separating equilibria are the polar case of a random allocation. We have conflictual losses but no allocational loss.

[11] The calculations need detailed but straightforward work. Readers interested in the detailed calculations are addressed to Esteban and Ray (2001).

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Figure 2: LOSSES FOR TYPE 1 EQUILIBRIA.

Substracting (11) from (10) we obtain

$$T^R - T^I = (b-a)\min(\alpha,\beta) - [\beta b + (1-\beta)a]\,\alpha. \tag{12}$$

It follows that lobbying will be efficient if and only if

$$b \;\geq\; \frac{2-\beta}{1-\beta}a \text{ when } \alpha \leq \beta, \text{ and} \tag{13}$$

$$b \geq \frac{(1-\alpha)\beta+\alpha}{(1-\alpha)\beta}a \text{ when } \alpha \;\geq\; \beta.$$

Notice that

$$\frac{(1-\alpha)\beta+\alpha}{(1-\alpha)\beta} \geq \frac{2-\beta}{1-\beta} \text{ for } \alpha \geq \beta,$$

and that the LHS $\to \infty$. as $\alpha \to 1$.

These restrictions tell us that, as far as separating equilibria are concerned, it does not pay to the government looking for information when the differential benefits from identifying the high types are not large enough. Furthermore, when the number of licenses exceeds the number of high types the restriction on the differential benefits becomes unbounded as $\alpha \to 1$. In any case, whenever $b < 2a$ the information revealed through a separating equilibrium will never be worth the conflictual costs incurred, irrespective of the number of licenses α.

4.2 Type II Equilibria

This equilibrium class involves *all* high rich$_R$ types and some low rich$_R$ types announcing at R, receiving permissions for sure. An amount δ of permissions are left over and are distributed equiprobably to the remainder of the population, who announce zero. Clearly, there will be allocative as well as conflictual losses.

Notice now, that for every $\langle \alpha, \beta, a, F \rangle$ there can be multiple equilibria involving different shares δ of the available permissions being spilled over the second announcement and different proportions of the low types at the low levels of announcement polluting the signals. Therefore, we shall proceed by computing the maximal and minimal losses possible of both types, conflictual and allocative which we will denote by $C_{\max}, C_{\min}, A_{\max}, A_{\min}$, respectively. The maximal losses[12] are recorded when δ is set to its lowest value possible, while the minimal losses are obtained when δ is set at its highest value.

[12]The maximal loss often is not attained and should thus be viewed as a supremum.

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Figure 3: CONFLICTUAL LOSSES FOR TYPE 2 EQUILIBRIUM.

We need first to define $\bar{R}$ and $\underline{R}$ as the respective solutions to the equations

$$\bar{R}F(\bar{R}) = a(1-\alpha) \tag{14}$$

and

$$\beta\underline{R}F(\underline{R}) + (1-\beta)\underline{R} = a(1-\alpha). \tag{15}$$

Notice that by substracting $aF(a)$ on both sides of (14) we obtain

$$\bar{R}F(\bar{R}) - aF(a) = a\left[1 - F(a) - \alpha\right].$$

Since $xF(x)$ is strictly increasing in x, we have that

$$\bar{R} \gtreqless a \quad \text{as} \quad \alpha \lesseqgtr 1 - F(a). \tag{16}$$

Notice now that $\underline{R}$ is strictly decreasing in $\alpha \in [\alpha_2, 1)$ and that the value of $\underline{R}$ that corresponds to α_2 is exactly a. It follows that

$$\underline{R} < a \text{ for all } \alpha \in (\alpha_2, 1). \tag{17}$$

We can now compute the maximal losses as,

$$C^{II}_{\max} = \alpha a + \min[0, \bar{R} - a] \text{ and } A^{II}_{\max} = (b-a)\left\{\min(\alpha,\beta) - \beta\max\left[\alpha, 1 - F(a)\right]\right\}. \tag{18}$$

As for the minimal losses, we have

$$C^{II}_{\min} = \alpha a - (a - \underline{R}) \text{ and } A^{II}_{\min} = (b-a)\left\{\min(\alpha,\beta) - \alpha + (1-\beta)\frac{a-\underline{R}}{a}\right\}. \tag{19}$$

Notice that the maximum conflict losses under this type of equilibria might be lower than under the fully separating equilibria of Type 1. Notice that this will be the case only when $a > \bar{R}$ and this can happen only if the number of licences is sufficiently large, ie. $\alpha > 1 - F(a)$. Yet, then the allocative losses will be high because a large number of licenses will end up in the hands of low types.

We summarize the losses under Type II equilibrium, by means of two diagrams, Figures 3 and 4. Figure 3 reports maximal and minimal conflictual losses, while Figure 4 does the same for the allocative losses.

In studying these diagrams keep in mind the following points.

First, the (maximal) conflictual loss bears no ready relationship to the allocative loss. This is because the allocative loss (obviously) depends on the gap in profitabilities

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Figure 4: Allocative Losses for Type 2 Equilibrium.

between the two types, while the conflictual loss only depends on the profitability of the low type. The reason this is so is broadly similar to the reason why the second-highest bidder's valuation determines equilibrium (English) auction prices. We should note that this property is special to the particular model we have chosen. Nevertheless, the point remains that in general, there will be no obvious comparison of the two losses.

Second, both conflictual and allocative losses are generally inverted-U-shaped as α increases, provided that α is bigger than $\beta[1-F(a)]$ to begin with. [We have actually drawn Figure 4 for the case in which β is less than $1-F(a)$, so that the maximum allocative loss exhibits a flat here (see (18)). In the opposite case, $A_{\max}$ strictly increases, then falls.]

Third, the maximum and minimum losses are fully "spanned" by losses in between. Corresponding to each point bounded by the maximum and minimum loss, there is an equilibrium of Type II which yields that loss.

Let us now turn to the efficency of Type II equilibria. We have just computed the maximum and minimum losses for each value of the parameter α. We shall now check whether the maximum costs of a signalling equilibrium of Type II can ever be lower and the minimum costs larger than the allocation costs incurred in a random allocation. Starting with the maximum we can easily obtain from (18) and (10) the following expression:

$$T^R - T^{II}_{\max} = (b-a)\beta\left[\max(\alpha, 1-F(a)) - \alpha\right] - \left[a\alpha + \min(0, \bar{R}-a)\right]. \qquad (20)$$

That is, bearing in mind that $a \lesseqgtr \bar{R}$ as $\alpha \lesseqgtr 1-F(a)$,

$$\begin{aligned} T^R - T^{II}_{\max} &= (b-a)\beta\left[1-F(a)-\alpha\right] - a\alpha, \text{ for } \alpha \leq 1-F(a)\ , \text{ and} \qquad (21)\\ T^R - T^{II}_{\max} &= (1-\alpha)a - \bar{R} = -\bar{R}[1-F(\bar{R})], \text{ for } \alpha \geq 1-F(a). \end{aligned}$$

It is immediate that there are no parameter values warranting strong efficiency in general. Clearly, we cannot have it for $\alpha \geq 1-F(a)$, nor for α smaller than this threshold, but sufficiently close to it.

For

$$b \geq \frac{2-\beta}{1-\beta}a \qquad (22)$$

we can have strong efficiency for sufficiently small values of α, close to α_2. Yet, when (22)is not satisfied $T^R < T^{II}_{\max}$ for all α.

Let us now check whether it can be that the minimum costs $T^{II}_{\min}$ exceed the random costs. From (19) and (10) and operating we have

$$T^R - T^{II}_{\min} = \left[(1-\beta)\frac{b-a}{a} - 1\right][\underline{R} - (1-\alpha)a]. \tag{23}$$

We start by noticing that $\underline{R}$ in the second squared bracket does not depend on b. Further, using (15) we can write

$$\underline{R} - (1-\alpha)a = \beta\underline{R}\,[1 - F(\underline{R})] > 0.$$

Therefore, $T^R \gtrless T^{II}_{\min}$ as $b \gtrless \frac{2-\beta}{1-\beta}a$. Small differential gains by the high typres relative to low types make lobbying strongly inefficient.

4.3 Type III Equilibria

Type III equilibria exist in an interval of the form (α_2, α_3), where $\alpha_3 < 1$. A typical equilibrium of this type must consist of three announcements: a, r, and 0. At a, all the high rich$_a$ types and (possibly) some of the low rich$_a$ make an announcement, and all of them are granted permissions. A measure δ of permissions spills over and is given to all who announce r (these contain all the high rich$_r$ types who did not announce a and some — but not all — the low rich$_r$ types). Finally, the rest announce 0 and get nothing.

Here again we shall have to compute the the maximum and the minimum equilibrium costs possible.[13] This depends on the number of high types being serviced at the different equilibria and this depends on δ. As it turns out, the conflict costs are the same for all the equilibria, conditional on the value of α, as in the equilibria of Type I. The key point thus is how the allocative costs vary across equilibria

The maximal losses are,

$$C^{III}_{\max} = \alpha a \text{ and } A^{III}_{\max} = (b-a)\left\{\min[\alpha,\beta] - \beta\max[\alpha, 1-F(a)]\right\}. \tag{24}$$

Note that the maximum allocative losses coincide with the ones recorded for equilibria of Type II.

In order to present the minimum allocative costs we need first to introduce $\hat{\alpha}$, which is defined by

$$\hat{\alpha} \equiv \alpha_2 + \frac{\beta}{a}\max_{0\le r\le a}[r(F(a) - F(r))]. \tag{25}$$

One can show that $\hat{\alpha} < \beta$.[14]

[13] As with equilibria Type II the maximum cost has to be seen as a supremum.

[14] As long as $F(r)$ has full support we have that $\max_{0\le r\le a}[r(F(a) - F(r))] < aF(a)$. The result follows by using this inequality in (25) and using the definition of α_2 in (8).

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Figure 5: Conflictual Losses for Type 3 Equilibrium.

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Figure 6: Allocative Losses for Type 3 Equilibrium.

The minimal losses thus are

$$C_{\min}^{III} = \alpha a \text{ and } A_{\min}^{III} = (b-a)\left\{\min(\alpha,\beta) - \min(\alpha,\hat{\alpha})\right\}. \tag{26}$$

We summarize the losses under Type III equilibrium, by means of two diagrams, Figures 5 and 6. Figure 5 simply observes that conflictual losses are unique for each α and are linear in α, while Figure 6 sketches maximal and minimal allocative losses.

Much the same comments apply to these losses as they did to Type II equilibria, so we only point out some salient features.

First, notice that Type III equilibria do not exist once α becomes close to unity, and the diagram reflects this by abruptly terminating the graph.

Second, notice that unlike Type II equilibria, conflictual loss is uniquely defined no matter which Type III equilibrium we pick: it has the same functional form as conflictual loss under Type I equilibrium, and as maximal conflictual loss under Type II equilibrium.

Third, the *maximal* allocative loss is exactly the same as that under Type II equilibrium.

Finally, the *minimum* allocative loss for Type III equilibria is different from its Type III counterpart. Indeed, a Type III equilibrium can actually achieve perfect separation provided that α is not too much larger than $\beta[1-F(a)]$, something that no Type II equilibrium can achieve. However, after some threshold, even these minimum allocative losses rise linearly in α (flattening out after α crosses the value β).

We now turn to the efficiency of Type III equilibria. We know that the maximum costs under equilibria of Type III are identical to the ones for Type II for $\alpha \leq 1-F(a)$ and that for $\alpha > 1-F(a)$ the conflict costs in Type III are larger than under Type II. It thus follows that in Type III equilibria we can have strong efficiency in exactly the same circumstances than in Type II equilibria: when (22) is satisfied and in addition α is close enough to α_2.

Let us now examine the possibility of strong inefficiencies. The minimum cost differential is given by

$$T^R - T_m^{III} = (b-a)\left[\min(\alpha,\hat{\alpha}) - \alpha\beta\right] - a\alpha, \tag{27}$$

where $\hat{\alpha}$ is as defined in (25). We now wish to verify whether (27) can ever be negative so that this type of partially pooling equilibrium performs in all circumstances worse than the random allocation.

For $\alpha \leq \hat{\alpha}$ (27) becomes,

$$T^R - T_m^{III} = \alpha \left[(b-a)(1-\beta) - a \right].$$

Therefore,

$$T^R \gtrless T_m^{III} \text{ as } b \gtrless \frac{2-\beta}{1-\beta} a \text{ if } \alpha \leq \hat{\alpha}. \tag{28}$$

When $\hat{\alpha} \geq \alpha$ (27) becomes,

$$T^R - T_m^{III} = (b-a) \left[\alpha_2 + \frac{\beta}{a} \max_{r \leq a} r \left[F(a) - F(r) \right] - \alpha\beta \right] - a\alpha. \tag{29}$$

Let $\tilde{r} \leq a$ be the solution to this constrained maximization problem. Then,

$$\begin{aligned} \frac{\beta}{a} \tilde{r} \left[F(a) - F(\tilde{r}) \right] &= \frac{\beta}{a} \tilde{r} \left[1 - F(\tilde{r}) \right] - \frac{\beta}{a} \tilde{r} \left[1 - F(a) \right] \leq \\ &\leq \alpha_1 - \frac{\tilde{r}}{a} \alpha_2. \end{aligned}$$

Using this result in (29), we obtain

$$\begin{aligned} T^R - T_m^{III} &\leq (b-a) \left[\frac{a-\tilde{r}}{a} \alpha_2 + \alpha_1 - \alpha\beta \right] - a\alpha \leq (b-a) \left[\alpha_1 - \alpha\beta \right] - a\alpha \leq \\ &\leq \alpha \left[(b-a)(1-\beta) - a \right]. \end{aligned} \tag{30}$$

Therefore, a sufficient condition for $T^R < T_m^{III}$ is that

$$b < \frac{2-\beta}{1-\beta} a \text{ if } \alpha \geq \hat{\alpha}.$$

We can conclude that a sufficient condition for the strong inefficiency of Type III equilibria for all admissible α is the same as for equilibria of Type II and it is that

$$b < \frac{2-\beta}{1-\beta} a. \tag{31}$$

4.4 Summary Proposition on Lobbying Costs and Efficiency

Putting together the previous observations, we have the following Propositions.

Proposition 7 *The lobbying costs for the different types of equilibria are:*
(i) for Type I equilibria

$$C^I = a\alpha \text{ and } A^I = 0,$$

(ii) for Type II equilibria

$$\begin{aligned} C^{II}_{\max} &= \alpha a + \min[0, \bar{R} - a] \text{ and } A^{II}_{\max} = (b-a)\left\{\min(\alpha,\beta) - \beta \max[\alpha, 1-F(a)]\right\} \\ C^{II}_{\min} &= \alpha a - (a - \underline{R}) \text{ and } A^{II}_{\min} = (b-a)\left\{\min(\alpha,\beta) - \alpha + (1-\beta)\frac{a-\underline{R}}{a}\right\}. \end{aligned}$$

(iii) for Type III equilibria

$$\begin{aligned} C^{III}_{\max} &= \alpha a \text{ and } A^{III}_{\max} = (b-a)\left\{\min(\alpha,\beta) - \beta \max[\alpha, 1-F(a)]\right\} \\ C^{III}_{\min} &= \alpha a \text{ and } A^{III}_{\min} = (b-a)\left\{\min(\alpha,\beta) - \min(\alpha,\hat{\alpha})\right\}. \end{aligned}$$

As for these costs relative to random allocation, we have the following proposition.

Proposition 8 *Lobbying equilibria have the following costs and efficiency properties:*
(i) Equilibria of Type I are strongly efficient if $b > \frac{2-\beta}{1-\beta}a$ *and strongly inefficient if* $b < \frac{2-\beta}{1-\beta}a$.
(ii) Equilibria of Types II and III are strongly efficient if $b > \frac{2-\beta}{1-\beta}a$ *and* α *(*$< 1 - F(a)$*) is sufficiently close to* α_2.
(iii) Equilibria of Types II and III are strongly inefficient for all α *if* $b < \frac{2-\beta}{1-\beta}a$.

5 Inequality, Conflict of Interests and Resource Allocation

Given the multiplicity of equilibria, it would be asking for too much to pin equilibrium losses down completely, but nonetheless the model thus far generates a definite pattern of losses as we vary the number of available permissions, α, over $[0, 1]$. The purpose of this section is to demonstrate that we can also say quite a bit about the direction of change in the "loss correspondence" as the inequality of wealth changes for any given α.

We have seen that individual announcements do not depend on the wealth level unless the desider signal exceeds the personal wealth endowment. Further, we know that in any equilibrium the unconstrained best response announcements never exceed a. Therefore, a change in the distribution of wealth can have an effect on the equilibrium outcome only in as much as it hits the distribution of wealth in the relevant range, i.e.$w \leq a$. We shall thus focus on changes in F in $[0, a]$.

Now we make precise what we mean by "a change in the inequality of wealth distribution". A general rule would be second-order stochastic dominance, but given that we have placed very few assumptions on the shape of F, the effects of general changes would be quite varied. So we use the following sharper form:

By *higher inequality*, we refer to any form of Lorenz-worsening of F that increases (or at least does not decrease) $F(R)$ for all $R \leq a$.

As justification for this restriction, consider some arbitrary Lorenz-worsening of F. Then, if the productivity of the low type is low enough (so that a is close to zero), this is *also* likely to satisfy our stronger definition. Our stronger definition essentially rules out the consideration of Lorenz-worsening wealth transfers *among* the "poor", and focuses on broader transfers from "poor" to "rich", i.e. from constrained to unconstrained individuals.

Now we study each of the equilibrium types in turn.

5.1 Type I Equilibrium

Recall that under Type I equilibrium there are no allocative losses, while the conflictual losses are reproduced here:

$$C = \alpha a. \tag{32}$$

So there is little one need say about this class of equilibrium: a change in inequality has *no* effect on Type I losses as long as this type of equilibrium exists. It should be noted, however, that while this result is immediate *given* (32), it isn't obvious *a priori.* After all, there may be many equilibria of Type I, and in general, the equilibrium signals under this type are sensitive to the distribution of wealth — see (35).

While the costs are insensitive, an increase in inequality does affect the range for which Type I equilibrium exists. In the first place, — see (8) — the range of parameter values for which separating is the only type of equilibrium shrinks as $F(a)$ increases. Therefore, in this sense, an increase in inequality renders the lobbying mechanism less informative.

Let us now examine the upper limit on α for which a separating equilibrium can exist. This is given by α_1 as defined by (7). We shall now show that α_1 does diminish too following an increase in inequality.

Let R^* be the solution to $\max_{0 \le R \le a} R\,[1 - F(R)]$ and $\tilde{R}$ the solution when we use $\tilde{F}$ instead, with $\tilde{F}(x) > F(x)$ for all $x \le a$. Using (7) we can write the following inequalities

$$\tilde{\alpha}_1 = \frac{\beta}{a}\tilde{R}\left[1 - \tilde{F}(\tilde{R})\right] < \frac{\beta}{a}\tilde{R}\left[1 - F(\tilde{R})\right] \le \frac{\beta}{a}R^*\left[1 - F(R^*)\right] = \alpha_1^*. \tag{33}$$

We can thus conclude that an increase in inequality, while leaving the (conflictual) costs unchanged does shrink the set of parameter values for which a separating equilibrium exists as well as the set for which this is the only type of equilibrium possible.

5.2 Type II Equilibrium

As we have seen, under Type II equilibrium, there is an entire range of conflictual and allocative losses. Luckily, this range can be simply parametrized by the maximal and minimal losses, because all losses in between are achievable as the outcome of *some* Type II equilibrium.

Let us start with the maximum costs. In view of (18) it is immediate that the conflictual costs will depend on inequality —via $\bar{R}$— only when $\bar{R} \leq a$ (that is, when $\alpha \geq 1 - F(a)$) and will be insensitive to it otherwise. It is easy to see from (**??**) that $\bar{R}$ decreases as $F(a)$ increases. It follows that for $\alpha \geq 1 - F(a)$ an increase in inequality *decreases* the conflictual costs. Turning now to the allocative costs (18), an increase in inequality can have an effect only if $\alpha < 1 - F(a)$. In this case, it is straightforward that an increase in $F(a)$ will *increase* the allocative costs.

Putting this information together, we can now evaluate the effect of an increase in inequality over the total costs: when there is little inequality —$F(a) < 1 - \alpha$— an increase in $F(a)$ increases the total maximum costs, while when inequality is beyond this threshold a further increase lowers the total maximum costs.

We examine now how minimum costs react to an increase in inequality. In view of (19), the eventual effects are to be through the induced change in $\underline{R}$. We know from (17) that $\underline{R} < a$. and hence an increase in inequality will raise $F(\underline{R})$. Therefore, we can deduce from (15) that an increase in $F(\underline{R})$ will reduce $\underline{R}$. This decrease in $\underline{R}$ will clearly decrease the minimum conflictual losses and increase the allocative losses for all parameter values.

What will be the net effect on total costs? This will depend on the value of b relative to a. It can be readily obtained that if $b < \frac{2-\beta}{1-\beta}a$ an increase in inequality will *reduce* total minimum costs, while total costs will be increased otherwise.

5.3 Type III Equilibrium

For Type III equilibria we have been able to pin down the maximum and minimum losses. As a matter of fact, conflictual losses are equal to the losses under Type I equilibria (i.e. αa) while maximum allocative losses are identical than under Type II equilibria.

We start with the maximum losses as given in (24). Conflictual losses are insensitive to inequality. The allocative losses —and hence total maximum losses— will increase only if $F(a) < 1 - \alpha$, remaining constant otherwise.

We know from (26) that minimum conflictual losses are equal to αa and hence will be unaffected by a change in inequality. Minimum allocative losses will depend on inequality via its effect on $\hat{\alpha}$ as defined in (25) and this only when $\alpha > \hat{\alpha}$,otherwise they will also be insensitive to inequality changes.

From the definition of $\hat{\alpha}$ it is immediate that whether the term $\max_{0 \leq r \leq a} \left[r \left(F(a) - F(r) \right) \right]$ will increase or decrease with a raise in inequality critically depends on how the increase in F is distributed over the range $[0, a]$. We can identify the sign of the change induced by increased inequality if we introduce an additional restriction on the shift of the distribution function from F to $\tilde{F}$. Specifically, if $\tilde{F}(r) - F(r) \geq \tilde{F}(a) - F(a)$ for all $r \leq a$, it is easy to see that $\hat{\alpha}$ will strictly decrease with an increase in inequality.[15] Thus it follows that under this added restriction the allocative (and the total) minimum costs remain constant for $\alpha < \hat{\alpha}$ and will increase when $\alpha > \hat{\alpha}$.

5.4 Inequality and Resource Allocation

The complexity of the structure of signalling equilibria has made us to evaluate the impact of inequality on lobbying costs type by type of equilibria. We wish now to put together our results on the effects of inequality on the costs of lobbying and furnish a broad view of their implications.

We formally present the results obtained in this section in the form of a Proposition.

Proposition 9 *An increase of inequality (an increase in $F(a)$) will produce the following changes in the different costs:*

(i) under separating equilibria, conflictual, allocational and thus total costs are unaffected by inequality;

(ii) for the two types of partially pooling equilibria the share of the allocative costs over total costs —maximum as well as minimum— will systematically increase with inequality for all parameter values.

(iii) for the two types of partially pooling equilibria maximum total costs will increase with inequality if $F(a) \leq 1-\alpha$ and decrease otherwise.

(iv) for Type II equilibria total minimum costs will increase or decrease as $b \gtrless \frac{2-\beta}{1-\beta}a$, respectively, while for Type III total minimum costs will remain constant when $\alpha < \hat{\alpha}$ or may increase if $\alpha > \hat{\alpha}$ (and $\tilde{F}(r) - F(r) \geq \tilde{F}(a) - F(a)$ for all $r \leq a$).

What are the patterns emerging from our analysis? What is the rationale of htis result?

We start by noting that the wealth level a plays a crucial role in the analysis. In all the partially pooling equilibria, the individuals making the highest announcement get the license for sure and a is the maximum amount that a low type would be ready to pay to obtain a license with probability one. Moreover, because of the intuitive criterion IC, the high types never need bidding more than that. Hence, individuals with wealth below a

[15] Let $\hat{r}$ and $\tilde{r}$ be the solutions to $\max_{0 \leq r \leq a} [r(F(a) - F(r))]$ under the distributions F and $\tilde{F}$, respectively, with $\tilde{F}(r) \geq F(r)$ for $r \leq a$. Since $\hat{r}$ and $\tilde{r}$ the maximizers, it has to be that

$$\begin{aligned} \hat{r}[F(a) - F(\hat{r})] &\geq \tilde{r}[F(a) - F(\tilde{r})], \text{ and} \\ & \tilde{r}\left[\tilde{F}(a) - \tilde{F}(\tilde{r})\right] \geq \hat{r}\left[\tilde{F}(a) - \tilde{F}(\hat{r})\right]. \end{aligned}$$

Therefore, if $F(a) - \tilde{F}(a) \geq F(\tilde{r}) - \tilde{F}(\tilde{r})$ we shall have that $\hat{r}[F(a) - F(\hat{r})] \geq \hat{r}\left[\tilde{F}(a) - \tilde{F}(\hat{r})\right]$. It follows that $\hat{\alpha}$ will fall following a shift from F to $\tilde{F}$.

are wealth constrained in their signals. This is why the signalling costs —conflictual and allocative— critically depend on the share of the population that is wealth constrained, $F(a)$.

When there is little inequality, $F(a) \leq 1 - \frac{\alpha}{\beta}$, there are enough individuals with sufficient wealth to signal their type. Although there will be individuals that would like to signal, but could not because they are poor, on the aggregate these individual constraints will not emerge.

As inequality goes beyond this point, there will be an insufficient number of high types rich enough to separate out from the low types: we will be entering the partially pooling zone where only imperfect information being revealed. Thus, allocative losses will be produced as some licenses start being awarded to low types. Yet, as long as $F(a) \leq 1 - \alpha$ —that is, $1 - F(a) \geq \alpha$ and hence there are more rich people than licenses— increases in inequality will not reduce the conflictual costs. This is so because the number of "rich" individuals who can afford the worth of a sure licence to a low type, $1 - F(a)$, exceeds the number of available licenses. Thus, in the worse scenario possible (yielding maximum costs) individual wealth constraints will not be sufficiently widespread among the population to reduce the (constrained) aggregate expenditures in lobbying. In the limit all rich_a individuals —high and low— bid a and get awarded a license with a probability that approaches unity as $F(a) \uparrow 1 - \alpha$. Here society incurs into maximum allocation costs as licences are distributed with equal probability on both types.

As inequality exceeds $1 - \alpha$ —we continue with equilibria of Type II— there will be even fewer individuals who can afford the high bid. Continuing with the case of maximum costs, at such high levels of inequality the efficiency in resource allocation cannot improve, but the aggregate resources expended in lobbying will come down as fewer individuals can afford to separate out to be spotted as deserving individuals.

6 Conclusions

In this paper we have examined the effect of wealth inequality on lobbying and public decision making. We have focused in the case in which lobbying is a mechanism of information transmission about individual characteristics. An efficiency-seeking government observes these signals and tries to derive the information they might convey. With the information available the government allocates the limited resources (licenses) in view of maximizing expected aggregate output.

In an economy with imperfect capital markets, the distribution of wealth affects the allocation of resources on two counts. In the first place, the number of wealth constrained individuals limits the number of those who can afford lobbying and the size of the signals they can transmit. Thus, the aggregate resources employed in the production of information depends on the degree of inequality. Secondly, whenever individuals are effectively wealth constrained, the intensity of the signals produced will turn out to be positively correlated to wealth. Yet, since individual deservingness of a public license is uncorrelated to wealth, this bias corrupts the informational content of the signals and hence limits the efficiency of the public decisions based on this information.

Our results show that wealth inequality has opposite effects on the two counts. The larger is the number of individuals that cannot send the level of signal they would have wished to send the smaller will be the size of aggregate resources expended in competitive signalling. By the very same reason, as the signals become increasingly corrupted by high wealth inequality, the larger will be the social cost incurred because of the induced misallocation of resources. As long as the number of individuals that can afford the highest signal possible —$1 - F(a)$— exceeds the number of available licenses α, inequality has no effect on the aggregate resources expended in lobbying. When inequality is beyond this threshold a further increase in inequality will reduce the (maximum) resources expended in conflictual signalling. As for the allocative costs, when inequality is low lobbying fully reveals the relevant information and public decisions can be fully efficient. If the proportion of unconstrained individuals is smaller than $\frac{\alpha}{\beta}$ governments may end up by allocating resources imperfectly. The (maximum) allocational costs increase with inequality up until the number of unconstrained individuals is equal to the number of licenses available. At this point allocational costs reach a maximum. Further increases of inequality do not have any additional effect on the allocational costs.

Which of these two effects of opposite sign will be stronger? Focusing on the case of miximum costs, we see that there are two critical values for wealth inequality. These are when the number of unconstrained individuals equals the available licenses and when it equals the ratio of licenses to high types, i.e. $1 - F(a) = \alpha$ and $1 - F(a) = \frac{\alpha}{\beta}$. When the number of wealth constrained individuals is low —and hence $1 - F(a) > \frac{\alpha}{\beta}$— increases in inequality leave total costs unchanged. For higher numbers of wealth constained individuals —that is $1 - F(a) \in (\alpha, \frac{\alpha}{\beta})$— the increases in the allocational costs dominate and thus total costs go up with higher inequality. Beyond that threshold the misallocation of resources cannot go any worse but signalling costs will come down with increased inequality. Therefore, total maximum costs will be inverted U-shaped with respect to inequality.

These results suggest that an efficiency-seeking government might do better by giving up on trying to acquire information and hence triggering a lobbying process: a random allocation may all in all produce smaller aggregate costs. In these circumstances, discretionary decisions —hence uncorrelated with deservingness— might be a desirable course of action from the efficiency point of view.[16] Thus, it might well be that the existence

[16] Notice that a direct allocation of licenses on the basis of wealth —either with a bias for the rich or for the poor— will be a random mechanism since wealth is uncorrelated with productivity type.

of informative lobbying activities or the extend of arbitrariness in public decisions reflect the degree of inequality in the distribution of wealth rather than the honesty or corruption of a particular government.

When an efficient government should allocate licenses using a seemingly arbitrary mechanism? Our results imply that when the differential benefit of a license between the two types is not large enough —i.e.$b < \frac{2-\beta}{1-\beta}a$— it always pays to take discretionary decisions regardless of the degree of inequality. Thus, we should observe discretion —and allocational inefficiencies— in small matters and active information gathering and higher efficiency in larger matters. Both types of behavior, we insist, are consistent with the assumption of an honest government that simply tries to maximize efficiency. When the differential benefit is sufficiently large, arbitrariness will be strongly dominated (i.e. the maximum costs of lobbying being smaller than the allocative costs induced by a random allocation) by informative lobbying only if inequality is sufficiently small. For higher levels of wealth inequality total lobbying costs may be larger or smaller depending on the particular equilibrium the economy settles in. Therefore, for "important matters" with $b > \frac{2-\beta}{1-\beta}a$, we should see maximum efficiency in countries with low inequality and the frequency of seemingly arbitrary, inefficient behavior to increase for higher degrees of inequality.

Let us consider a given distribution and proceed to an rescaling of wealth levels by a factor $\lambda > 1$. It is immediate that the number of individuals with wealth not exceeding a under the scaled distribution will be smaller than under the original distribution. Hence, larger or smaller values of $F(a)$ can also be interpreted as corresponding to richer or poorer countries, keeping the underlying basic distribution unchanged as well as the rest of the parameters of the model. Bearing this interpretation in mind, we can rephrase our previous observation in terms of levels of development of the corresponding economies: inefficiency and seemingly discretionary behavior should be inversely correlated with the country's wealth level.

So far we have examined the effects of inequality on the efficiency of public decision making. Let us now briefly discuss the impact of such public decisions on the outcoming inequality in the distribution of wealth. Let us start with the case in which the government can perfectly separate high from low types and can thus allocate licenses with full efficiency. Even if being low or high is uncorrelated with wealth, only the rich high types (with $w \geq a$) will be able to send the appropriate signal. Therefore, as a result of the imperfections of the capital market, the outcome will look as if the government had a bias in favor of the rich. The outcoming distribution will display higher inequality that the original one since only people who could make big benefits have been supported and in addition this has been made with a bias towards the rich. Therefore, in countries with low initial inequality efficiency-guided government decisions will tend to increase inequality.

Suppose now that we are in a society with higher inequality with a share δ of licenses spilled over the zero bidders. The outcome will clearly be not so elite prone. From the distributional point of view, partially pooling equilibria will not increase inequality as much as under full separation on two counts: all bidding rich individuals —high and low— will benefit from a license and all poor individuals will have a chance of receiving a license and increase their wealth. Thus, we shall have that the rich bidders will be able to increase the money distance from the low wealth group as they all will be awarded a public license, but with a lower increase in within-group inequality as all, high and low, will benefit from the license. At the lower end of the distribution we shall have that some randomly selected individuals will be able to make extra profits and increase their wealth. This will increase the spread of the distribution of wealth at the lower end. But the spread will be completely uncolrrelated with initial wealth, unlike what we shall partially have at the upper end of the distribution. It follows that as the initial inequality is larger and larger and the share that goes to the poorer becomes larger too, the impact of the government licenses on the resulting inequality will be smaller.

Summing up, licenses *per se* will increase inequality by permiting the beneficiaries to obtain extra benefits. The higher is the efficiency in this allocation the larger will be the inequality-increasing effects of the licenses. Since this efficiency turns out to be negatively related to inequality, higher inequality will produce lower inequality increases. In addition, the degree of efficiency critically depends on having a sufficient number of individuals rich enough to separate out from the rest by means of costly signals. Therefore, the higher the degree of efficiency, the larger will be the pro-rich bias in the allocation of licenses. Hence, high inequality will lead to low inequality increasing effects of the allocation of licenses.

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8 Appendix 1: Proof of Proposition 3

8.1 Type I Equilibria: Two Signals With Full Separation

Choose $R \leq a$ such that

$$\beta[1 - F(R)] \geq \alpha \tag{34}$$

and

$$R = \frac{\alpha a}{\beta[1 - F(R)]} \tag{35}$$

Now construct an equilibrium as follows: all the *high* rich_R types announce R, and are given a permission with probability

$$p(R) = \frac{\alpha}{\beta[1 - F(R)]}.$$

All others (including the low rich_R types and all the poor_R types) bid zero and get nothing.

The planner considers any announcement less than R to come from a low type and sets $p(r) = 0$ for such announcements. He considers any announcement greater than or equal to R to come from a high type and sets $p(r) = p(R)$.

To check that this is indeed an equilibrium, consider deviations. The only serious ones we have to consider are deviations by low rich_R to announcing R, and by high rich_R types to announcing zero.

The low rich_R are getting 0. If they announce R, they will get

$$p(R)a - R = \frac{\alpha}{\beta[1 - F(R)]}a - R = 0,$$

where we use (35) here. So there is no incentive to deviate. Moreover, it is easy to see that the high rich_R will be *strictly* worse off by going down to 0.

Finally, we need to check that the planner's beliefs do not violate the IC. First, we show that positive announcements of $r < R$ are *not* equilibrium dominated for the low type. In the best case, if the planner believes that such an announcement is coming from a high type, he will give such an announcement the permission with probability of $p(R)$,[17] and using (35) the low type would be better off than by announcing zero. So such an announcement may very well be made by a low type and there is no contradiction here.

What about announcements in excess of R? Well, these are equilibrium dominated for *both* types so the IC places no restriction — we are free to believe that such announcements are coming from high types (or low types, or a mixture of the two, it would not matter).

[17] We are using the second role of BAP here but it is not really cutting any extra grass in any case. Just notice that we are restricting the best case scenario by not allowing the planner to award permissions with probability exceeding $p(R)$, because this is all that he is doing for the high types residing at R.

Equality (35) suggests that this class of equilibrium may not always exist. We need to assure ourselves that there exists $R \leq a$ such that (34) holds and such that the above equality is satisfied. To do this, rewrite (35) as

$$R[1 - F(R)] = \frac{\alpha a}{\beta}, \tag{36}$$

where we use the definition of a. Notice that as long as (36) holds for some $R \leq a$, (34) will automatically be satisfied. So the condition (36) (for some $R \leq a$) is the only one that needs to be met.

Notice that if α is reduced parametrically these conditions must hold if they were holding before (simply reduce R and use the continuity of F together with the observation that $z[1 - F(z)] \to 0$ as $z \downarrow 0$). This shows that this equilibrium class exists for α in some interval.

This "existence interval" — call it $[0, \alpha_1]$ — clearly includes the range $[0, \alpha_2]$ (recall the definition of α_2 from (8) and examine (36)). But it must also be contained in the range $[0, \beta]$ (examine (36) again).

8.2 Type II Equilibria: Two Signals With Partial Pooling

For analytical convenience we shall work with the size of the licenses spilled over the second announcement δ instead of the share λ of the low rich$_R$ polluting the high bid R.

Choose $R \leq a$ and $\delta \geq 0$ such that

$$R = \frac{1 - \alpha}{1 - \alpha + \delta} a. \tag{37}$$

Impose on δ the following additional restriction:

$$\alpha - [1 - F(R)] < \delta \leq \alpha - \beta[1 - F(R)] \tag{38}$$

noting that the first inequality is redundant if $\alpha < 1 - F(R)$.

An equilibrium in this class is described as follows: all high rich$_R$ and some low rich$_R$ may bid at R, but a nonnegative measure of permissions — δ — is left over. These will be given to all those that announce zero. On the equilibrium set, the planner uses $\mu(R) = \frac{\beta[1-F(R)]}{\alpha-\delta}$, $p(R) = 1$ and $\mu(0) = \frac{\beta F(R)}{1-\alpha+\delta}$, $p(0) = \frac{\delta}{1-\alpha+\delta}$. Off the equilibrium set, the planner uses $\mu(r) = p(r) = 0$ for all $r \in (0, R)$, and $\mu(r) = 1$, $p(r) = p(R)$ for all $r > R$.

The on-equilibrium beliefs satisfy Bayes' rule. Off the equilibrium path, we note that announcements strictly between 0 and R are not equilibrium-dominated for either type,[18] so the IC places no restrictions on beliefs, while announcements in excess of R

[18]For the low type, notice that when $0 < r < R$, $p(r)a - r = a - r > a - R = \frac{\delta}{1-\alpha+\delta} a = p(0)a$, where the second-last equality uses (37). For the high type, $p(r)b - r = b - r > b - R = Rb - R$. So r is not equilibrium dominated for either type.

are equilibrium-dominated for both types,[19] so once again no restrictions are implied by IC.

Finally, using (37), it is very easy to check that no agent wishes to switch to a signal other than the one specified in equilibrium.

The additional restrictions on δ will be explained later, when we show that these are the only equilibria possible in this class.

When do equilibria of type 2 exist? Clearly, these equilibria cannot exist as long as $\alpha < \alpha_2 = \beta[1 - F(a)]$. That is because $R \leq a$, which means that $\alpha < \beta[1 - F(R)]$. But then, by (38), δ cannot be nonnegative.

But these equilibria do exist everywhere else; that is, for $\alpha \geq \alpha_2$. To see this, notice that by combining (37) and (38) we obtain the following restriction on R: $R \leq a$, and

$$\frac{1-\alpha}{F(R)} a > R \geq \frac{1-\alpha}{1-\beta[1-F(R)]} a. \tag{39}$$

Meeting these restrictions will ensure that (37) and (38) will hold for some (R, δ).

Note that there always exist values of R that satisfy these restrictions, and that they lie in an interval. To see this, observe that the second inequality in (39) can always be satisfied by taking $R = a$. By lowering R, we can assure ourselves that the second inequality will hold with an equality. At this point the first (strict) inequality must be satisfied.[20] This gives us the lower bound of the interval, call it $R_*(\alpha)$. To find the upper bound, simply increase R (the second inequality will continue to hold)[21] until one of two things happen:

(a) $R = a$, or

(b)$R \leq a$, but the first strict inequality becomes an equality.

In case (a), set the upper bound $R^*(\alpha) = a$. The interval of R's for which a type 2 equilibrium exists is then given by the closed interval $[R_*(\alpha), R^*(\alpha)]$. In case (b), set $R^*(\alpha)$ equal to the appropriate threshold for which the first inequality holds with equality but do not include it. Notice that in this case, $R^*(\alpha) < a$ (except at one value of α — see below). The interval of R's for which a type 2 equilibrium exists is then given by the *half-open* interval $[R_*(\alpha), R^*(\alpha))$.

The following observations regarding type 2 equilibria are of interest:

[1] The existence interval is always nondegenerate as long as $\alpha > \alpha_2$, but shrinks to the singleton set $\{a\}$ as $\alpha \downarrow \alpha_2$. Moreover, δ must shrink to zero. Thus our entire equilibrium set of type 2 patches into one of the equilibrium branches of type 1 (coming in from the left as $\alpha \uparrow \alpha_2$).

[19] Simply note that when $r > R$, the two inequalities in the previous footnote are reversed.

[20] This is simply because the first term in (39) is always strictly greater than the third term. For $F(R) < 1 - \beta[1 - F(R)]$.

[21] This is because $\frac{1-\alpha}{1-\beta[1-F(R)]} a$ is decreasing in R.

[2] Next, allow α to rise to its maximum value, which is 1. In that case notice that the restriction (39) must squeeze the equilibrium set of R-values to a singleton yet again. In the limit as $\alpha \uparrow 1$, $[R_*(\alpha), R^*(\alpha)) \to \{0\}$. All conflict disappears as permissions are available for all, which is exactly as it should be.

[3] Finally, note that the type 2 equilibrium interval is closed over some range and half-open over the rest. In fact, it is easy to check that the equilibrium set is of the form $[R_*(\alpha), a]$ until α reaches the value $1-F(a)$. Thereafter, it is of the form $[R_*(\alpha), R^*(\alpha))$. The reason for this seeming anomaly is the belief-action parity principle.

Are There Other Equilibria With Two Signals?

We pause in our description of equilibrium to note that type 1 and type 2 equilibria collectively exhaust the entire class of equilibria with two signals.[22] To see this, note that by [F.2] (full support of F), an equilibrium with two signals must have one signal at zero. We therefore divide up various possibilities using the maximal announcement R in any two-signal equilibrium.

Possibility 1. $R > a$. This possibility is ruled out by Lemma 5 [3].

Possibility 2. $R \leq a$, but not all the high rich$_R$ get to bid the value R. This is ruled out by Lemma 5 [2].

Combining these arguments, we must have, in any equilibrium, $R \leq a$ *and* all the high rich$_R$ types announcing R. Now continue with two subcases.

Possibility 3. The measure of permissions is no greater than the measure of high rich$_R$ types: $\alpha \leq \beta[1-F(R)]$. [This is the subcase into which type 1 falls.] This means that no positive measure of low types can announce R as well, for then $\mu(R) < 1$ *and* $p(R) < 1$, contradicting Lemma 3 [3]. Therefore everybody else except for the high$_R$ types must announce 0 (because there are no permissions to give them).

We are almost done with this subcase. It only remains to observe that equation (35) *must* hold in this situation. This is an immediate consequence of Lemma 5 [1].

Possibility 4. The measure of permissions is greater than the measure of high rich$_R$ types: $\alpha > \beta[1 - F(R)]$. [This is the subcase into which type 2 falls.] In this case, all high rich$_R$ and some low rich$_R$ may bid at R, but there must be a nonnegative measure of permissions — δ — left over.[23] These will be given to all those that announce zero. By Lemma 5 [1], it must be the case that

$$p(0)a = p(R)a - R.$$

[22] To be sure, there may be other off-equilibrium *beliefs* — other than the ones we have described — that support equilibria in types 1 and 2. We do not count these as separate equilibria, though.

[23] If this were not the case, then once again we have $p(R) < 1$ and $\mu(R) < 1$, which contradicts Lemma 3.

Recall that $p(R) = 1$ in this class. Moreover, $p(0) = \frac{\delta}{1-\alpha+\delta}$, because δ is the measure of permissions left over and $1-(\alpha-\delta)$ is the measure of the population left over after $\alpha-\delta$ permissions have already been handed out at R. Using these pieces of information, the above equality can be rewritten as

$$\frac{\delta}{1-\alpha+\delta}a = a - R,$$

or as

$$R = \frac{1-\alpha}{1-\alpha+\delta}a, \tag{40}$$

which is precisely one of the restrictions — see (37) — for type 2 equilibrium.

It remains to derive the restrictions on δ. Simple adding-up demands that δ satisfy the constraint

$$0 \le \delta \le \alpha - \beta[1 - F(R)],$$

so to establish our second restriction — see (38) — it only remains to show that

$$\delta > \alpha - [1 - F(R)].$$

[As observed, this is only relevant in case $\alpha \ge 1-F(R)$.] To see why this is true, suppose that the inequality fails. Then it is easy to see that $\mu(R) \le \beta$, which contradicts part [1] of Lemma 4. Intuition: if δ is very low, then too many low types are also bidding at R, more than their numerical ratio $1-\beta : \beta$ warrants. This is unacceptable as $\mu(R)$ must have the highest value. Moreover, even having exactly the same ratio of low types bid at R is ruled out, this time by the belief-action parity principle.

These arguments prove that two-signal equilibria must be either of Type I or Type II, together with their associated restrictions.

8.3 Type III Equilibria: Three Signals With Partial Pooling

It will be pedagogically simpler to arrive at our third type of equilibrium through a process of eliminating other three-signal equilibria. Thus we both describe our equilibria and eliminate others in the same breath.

We know by Lemma 5 [3] that the maximal announcement R at any equilibrium cannot exceed a, so let us begin with some $R \le a$. If there are three signals, then at least one other signal must stand a chance of receiving permissions. Moreover, by Lemma 5 [3], all high rich$_R$ types must bid R. It follows that at any three-signal equilibrium,

$$\beta[1 - F(R)] < \alpha. \tag{41}$$

Let δ be the measure of permissions that spill over from R. Then, by exactly the same arguments that established (38), we know that $\delta \ge 0$ and

$$\alpha - [1 - F(R)] < \delta \le \alpha - \beta[1 - F(R)], \tag{42}$$

(where, as before, the first inequality is only relevant in case $\alpha \geq 1 - F(R)$).

Let there be two other bids. By Proposition 2 one of them must be zero. It follows from Lemma 5 [1] that $p(0) = 0$ and that all equilibrium returns to the low types must be zero; that is,

$$p(r)a - r = p(R)a - R = 0. \tag{43}$$

Using $p(R) = 1$ (which is the same as $\delta \geq 0$), (43) implies in particular that

$$R = a \tag{44}$$

That is, three-point equilibria in this class must all have their maximal bid set at the choke price a.

It remains to construct the exact set of equilibria in this class. To this end, allow all the high rich$_R$ types to announce $R = a$, and allow some low rich$_a$ types to do so as well, but making sure that there is a nonnegative measure δ of permissions left over. Because $p(0) = 0$, it must be that all the δ permissions will be distributed to those who announce r. But for this to satisfy equilibrium conditions, certain restrictions must be met. To these we now turn.

Let $B(r)$ be the measure of *all* agents — high or low — who announce r. Then it must be that $p(r) = \delta/B(r)$, and using this information in (43), we conclude that

$$\frac{\delta}{B(r)}a - r = 0,$$

or that

$$B(r) = \frac{\delta a}{r}. \tag{45}$$

The restriction on our equilibrium set comes from the observation that $1 = p(a) > p(r) > p(0) = 0$, so that (by BAP) we must have

$$\mu(a) > \mu(r) > \mu(0).$$

Let $\bar{\mu}$ be the proportion of high types over all the population that *does not* announce a. Then $\bar{\mu}$ must be a weighted average of $\mu(r)$ and $\mu(0)$, so that the above set of inequalities is equivalent to

$$\mu(a) > \mu(r) > \bar{\mu}. \tag{46}$$

Recall that $\mu(a) = \frac{\beta[1-F(a)]}{\alpha-\delta}$. Moreover, by simple adding-up, β is just the average of $\mu(a)$ and $\bar{\mu}$ weighted by population; that is,

$$\beta = (\alpha - \delta)\mu(a) + (1 - \alpha + \delta)\bar{\mu} = \beta[1 - F(a)] + (1 - \alpha + \delta)\bar{\mu},$$

which implies that

$$\bar{\mu} = \frac{\beta F(a)}{1 - \alpha + \delta}. \tag{47}$$

It remains to calculate $\mu(r)$. Remember that P is the total population announcing r. Also, note that *every* high rich$_r$ person who has *not* announced $R = a$ must announce r. For by (43), it must be that

$$p(r)b - r > 0$$

and 0 is the return to announcing zero. It follows (using (45)) that

$$\mu(r) = \frac{\beta[F(a) - F(r)]}{B(r)} = \frac{\beta[F(a) - F(r)]r}{\delta a}. \tag{48}$$

Substituting the values of $\bar{\mu}$ and $\mu(r)$ (from 47) and (48) respectively) into (46) and removing the common term β, we can write our first main restriction for type 3 equilibria as

$$\frac{1 - F(a)}{\alpha - \delta} > \frac{[F(a) - F(r)]r}{\delta a} > \frac{F(a)}{1 - \alpha + \delta} \tag{49}$$

Recalling that $R = a$ and using this in (42), we obtain the second main restriction for Type III equilibria:

$$\alpha - [1 - F(a)] < \delta \leq \alpha - \beta[1 - F(a)], \tag{50}$$

Notice that these arguments tell us what the planner's on-equilibrium beliefs must be:

$$\mu(a) = \frac{\beta[1 - F(a)]}{\alpha - \delta},$$
$$\mu(r) = \frac{\beta[F(a) - F(r)]r}{\delta a},$$

while $\mu(0)$ is backed out from the adding-up constraint:

$$\beta = (\alpha - \delta)\mu(a) + \frac{\delta a}{r}\mu(r) + \left[1 - (\alpha - \delta) - \frac{\delta a}{r}\right]\mu(0),$$

and can be seen from (46) to be smaller than $\mu(r)$, which in turn is smaller than $\mu(a)$.

The on-equilibrium allocation probabilities are

$$p(a) = 1,$$
$$p(r) = \frac{r}{a},$$
$$p(0) = 0.$$

As usual, a variety of off-equilibrium beliefs are admissible (all of which satisfy BAP and IC); for instance:

$$\mu(r') = 0, \quad p(r') = 0 \text{ for } r' \in (0, R), r' \neq r$$
$$\mu(r') = \mu(a), \quad p(r') = 1 \text{ for } r' > R$$

[To see that IC is satisfied, note that the former set of off-equilibrium announcements are not equilibrium-dominated for either type, while the latter set are equilibrium-dominated for both types.]

It is easy to see that both agents and planner are choosing optimal strategies given these beliefs.

Having established that Type III describes the only possible class of three-signal equilibria, we turn to an analysis of existence. Equivalently, we examine whether the restrictions (49) and (50) are both satisfied.

For Type II equilibria to exist it is necessary that δ be strictly positive. In view of (50), this requires that $\alpha > \alpha_2 = \beta[1 - F(a)]$.

The following points concerning type 3 equilibria are to be noted:

[1] Existence occurs over some range of α that includes all $\alpha \in [\alpha_2, 1 - F(a))$. In this case, (50) reduces to the following restriction on δ

$$0 \leq \delta \leq \alpha - \beta[1 - F(a)]. \tag{51}$$

To see existence in this case, note that the first term in (49) always exceeds the third term[24] So let us choose δ very small, satisfying (51), so that (for instance)

$$\frac{[F(a) - F(a/2)](a/2)}{\delta a} > \frac{F(a)}{1 - \alpha + \delta}$$

and then adjust r by bringing it close enough to a (or close enough to zero, either will do) such that

$$\frac{[F(a) - F(r)]r}{\delta a} > \frac{F(a)}{1 - \alpha + \delta}$$

but by a tiny amount. Then the first inequality in (49) will automatically hold.

[2] Existence occurs over an *interval* of α's. For suppose that (49) and (50) hold for some α. Reduce α to α', where $\alpha' > \alpha_2$. Notice that (49) continues to hold for the same values of r and δ. The only difficulty is that the second inequality in (50) may now fail. In that case, lower δ just enough so that (50) continues to be satisfied, and then adjust r in the same way as in [1] above.

[3] Type 3 equilibria must fail to exist once α is close enough to unity. To see this, consider the second inequality in (49).[25] Cross-multiplying, the second inequality is equivalent to

$$F(a)\,[r(1 - \alpha) - \delta(a - r)] > F(r)r[(1 - \alpha) + \delta],$$

[24] $\frac{[1-F(a)]}{\alpha-\delta} > \frac{F(a)}{1-\alpha+\delta}$ if $\frac{[1-F(a)]}{\alpha} > \frac{F(a)}{1-\alpha}$, and this is always true given that $1 - F(a) > \alpha$ in this part of the discussion.

[25] It can be shown that the first inequality can always be made to hold for suitable choices of δ satisfying (50) and $0 < r < a$.

and a *necessary* condition for this to hold can be found by substituting in the lower bound on δ in (50). This gives us the requirement:

$$F(a)\left[r(1-\alpha)-\alpha-(1-F(a))\,(a-r)\right] > F(r)rF(a)$$

Simplifying this inequality, we need the following to hold:

$$r\left[F(a)-F(r)\right] > a\{\alpha-[1-F(a)]\}. \tag{52}$$

It should be obvious that (52) *must fail to hold* at some threshold — call it α_3 — strictly less than unity.

[4] It is of interest to know that — along with type 2 equilibrium — these equilibria all patch into a type 1 equilibrium branch as $\alpha \downarrow \beta[1-F(a)]$; indeed, the same branch that patches into type 2 equilibria. For as $\alpha \downarrow \beta[1-F(a)]$, the restriction on δ (see (50) becomes tighter and tighter, making it ever smaller. Therefore r must slide closer to a or closer to zero, and a very small number of permissions must be handed out at r. In the limit this intermediate announcement completely disappears, completing the required patch.