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Doh-Shin Jeon

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[*] Department of Economics and Business, Universitat Pompeu Fabra, Ramon Trias Fargas, 25-27, 08005 Barcelona, Spain. Tel: (34) 93 542 1657. Fax: (34) 93 542 1746. E-mail address: doh-shin.jeon@upf.edu

Abstract

In this paper, we study mechanism design under collusion and uniform transfers focusing on the transaction costs in coalition formation created by asymmetric information among agents. We are particularly interested in the interaction between incompleteness of contracts and collusion: in our setting, the regulator (the principal) is constrained to use uniform transfers and this gives rise to room for collusion between the regulated firms. Collusion takes place under adverse selection and moral hazard since each firm's cost, observable to the regulator, is determined by its efficiency parameter and by its effort, which are the firm's private information. We first show that when the gains from collusion are smaller than a threshold, the firms fail to realize the gains because of the transaction costs created by asymmetric information. When the gains are larger than the threshold, we characterize the optimal collusion-proof mechanism which fully exploits the transaction costs. Finally, we show that when the regulator is constrained to use uniform transfers, the collusion-proofness principle does not hold.

Key words: Collusion, Asymmetric information, Transaction costs, Uniform Transfers.

JEL Classification: D8, L2

1 Introduction

The revelation principle supposes that agents behave in a non-cooperative way. This can be justified if the principal has a complete control over communication between the agents or if the cost of communication between the agents is very large. However, in reality, agents can often engage in communication at low cost and this might open room for collusive behavior to promote their joint interests.[1] In this case, the principal needs to take into account collusion when she designs her incentive schemes.[2]

In this paper, we study mechanism design under collusion focusing on the transaction costs in coalition formation created by asymmetric information among the agents. In particular, we want to capture a situation in which incompleteness of contracts gives rise to room for collusion. By the incompleteness, we mean that for some reasons outside of the model, a restriction is imposed on the set of the contracts available to the principal. Although the interaction between incompleteness of contracts and collusion is a general theme, we study it in a regulation setting. Regulators in the real world are constrained by federal (or national) codes of regulation, administrative procedures or laws.[3] In our model, we assume that the regulator is constrained to use uniform transfers. As Kahn (1988) argues, regulation is inescapably involved with the political process and the inherent defects of the political process render regulation quite imperfect.[4] Thus, the incompleteness of the regulatory contract in our setting can be seen as a consequence of this involvement with the political process.[5] In this paper, we do not model the political process, take the incompleteness of the regulatory contract as given and study the interaction between the incompleteness and collusion.

Starting from the optimal mechanism without collusion, we first show that when gains from collusion are smaller than a threshold, the agents fail to realize the gains because of the transaction costs created by asymmetric information. When the gains are larger than the threshold, we characterize the optimal collusion-proof mechanism which fully exploits the transaction costs. Finally, we show that when the regulator is constrained to use uniform transfers, the collusion-proofness principle does not hold.

We study a regulation setting a la Laffont and Tirole (1986). The principal

[1]Numerous examples of collusion can be found in the studies done by sociologists and political economists (Crozier(1967), Dalton (1959), Rose-Ackerman (1978)).

[2]In what follows, we use 'she' to represent the principal or the regulator and 'he' to represent an agent or the third-party.

[3]For a brief survey on administrative and political constraints faced by regulators, see Laffont and Tirole (1993, pp.4 - 6).

[4]See pp. 325-7 in Kahn (1988).

[5]For instance, politicians may insist on fairness or simplicity and this might induce the regulator to use her instruments in an inflexible or uniform way as is assumed in our model.

(regulator) proposes a regulatory contract (mechanism) to two firms (agents) producing complementary inputs. The principal can only observe the costs of the firms and a firm's cost is determined both by its cost parameter (type) and by the level of effort exerted by its manager. A regulatory contract specifies for each firm its cost target and the transfer that it receives from the regulator. We assume that the principal is constrained to use uniform transfers in the sense that given a state of nature, both firms receive the same transfer.[6] A firm can have either a low-cost or a high-cost type and the types are independently distributed. A firm has private information about its type and the level of effort exerted by its manager. Hence, collusion between the firms takes place under adverse selection and moral hazard. In our model, this double asymmetric information is the source of the transaction costs in coalition formation since otherwise (i.e., if the agents know each other's type or if they can perfectly monitor each other's effort), collusion is efficient.

We first characterize the optimal mechanism without side-contracting (when there is no collusion). It turns out that the fact that the principal is constrained to use uniform transfers does not generate any loss. Under the optimal mechanism, an agent's effort level depends only on his own type and a low-cost type's level is higher than a high-cost type's one.[7] The principal has a residual degree of freedom in transfers such that she can satisfy the incentive constraints in dominant strategies without loss.

After characterizing the optimal mechanism without side-contracting, we identify the condition under which gains from joint manipulations of reports exist in the absence of any transaction cost in coalition formation. Since the regulatory mechanism specifies a cost target for each agent, any manipulation of reports should be accompanied by a coordination of efforts to meet the cost targets. When the principal offers the optimal dominant-strategy mechanism without side-contracting, the optimal transfer is decreasing in the sum of the agents' cost parameters. This can create gains for downward manipulations of reports[8] such that if a certain condition holds, the agents, regardless of their types, have the incentive to announce that each of them has a low-cost type in order to receive the largest transfer.

After identifying the potential gains from collusion, we study collusion under asymmetric information. Following Laffont and Martimort (1997, 2000), we

[6]The uniform transfers assumption is a way to capture imperfect real-world regulations in our highly stylized regulation model. We note that Laffon and Martimort (1997) also make the same assumption and justify it by supposing that the agents make an ex ante agreement to share equally any asymmetric monetary transfers they receive from the principal.

[7]Indeed, the optimal level of effort is equal to the optimal level in a one-agent setting: See Laffont and Tirole (1993, Chapter 1) for the analysis of the one-agent setting.

[8]A downward manipulation of report occurs when an agent manipulates his report from a high-cost type to a low-cost type.

model collusion under asymmetric information by a side-contract offered by a benevolent and *uninformed* third-party. The third-party maximizes the sum of the agents' payoffs subject to incentive, participation and budget balance constraints. We characterize the coalition incentive constraints under asymmetric information: they are written in terms of *virtual disutilities* of effort instead of real disutilities when the incentive constraint in the side-contract binds.

As a main result, we show that when the gains from collusion are smaller than a threshold, the agents fail to realize the gains because of the transaction costs created by asymmetric information. The intuition can be given as follows. Suppose that the third-party asks each agent to always announce to the principal that he has a low-cost type. After the manipulation, a high-cost type has to exert more effort than before since he has to meet the low-cost type's cost target. Since the rent that a low-cost type obtains by pretending to have a high-cost type is increasing in the level of effort exerted by the high-cost type,[9] the manipulation increases a low-cost type's incentive to pretend to have a high-cost type to the third-party. Therefore, in order to induce truth-telling, the third-party has to concede to a low-cost type a rent larger than the one that he could obtain in the absence of collusion. We define this increase in the rent as the transaction costs created by asymmetric information and show that the transaction costs are larger than the gains from collusion.

In the case in which the optimal mechanism without side-contracting is not collusion-proof, we characterize the optimal collusion-proof mechanism. In the mechanism, the individual incentive constraint is binding for an upward manipulation while the coalition incentive constraints are binding for downward manipulations. Hence, collusion creates countervailing incentives and this makes the optimal collusion-proof effort schedule exhibit an upward distortion at the top and a downward distortion at the bottom with respect to the optimal effort schedule without side-contracting.[10]

Finally, we examine how uniform transfers affect the collusion-proofness principle. This principle states that any outcome that is obtained by letting collusion occur can be equivalently achieved by offering a collusion-proof mechanism. If the principal is not constrained to use uniform transfers, the principle holds in our setting. However, when she is constrained, we show that the principle does not hold. Since the third-party is not constrained to use uniform transfers, to implement a given cost schedule, the principal might prefer letting collusion occur to offering collusion-proof mechanisms. This result is in line with Tirole (1990)'s

[9] This is a standard result from the regulation model a la Laffont and Tirole (1986).

[10] In the literature on countervailing incentives, countervailing incentives arise because the value of an agent's outside-option is correlated with his type: see Lewis and Sappington (1989), Maggi and Rodriguez (1995) and Jullien (1999). In our setting, collusion generates countervailing incentives even though the agents have the same reservation utility regardless of their types.

conjecture that if the principal's mechanisms are incomplete, letting collusion occur could be desirable.

The theory of collusion under asymmetric information is mainly developed in the auction literature. In the literature,[11] bidders collude without knowing how much other bidders are ready to pay for the object in auction and usually a benevolent and uninformed third-party is introduced as the organizer of the collusion. However, this literature has a weakness in the sense that the principal optimizes in a very restricted set of mechanisms[12] and uses few means - very often a reservation price only - against collusion. In this regard, Laffont and Martimort (1997, 2000) take a broader perspective as they characterize the set of collusion-proof mechanisms and optimize in this set.

Our paper is closely related to their first paper (Laffont and Martimort, 1997). They study collusion between two regulated firms producing complementary goods in a setting a la Baron and Myerson (1982). In their model, the regulator is constrained to use uniform transfers[13] and this creates room for collusion as in our setting.[14] Although our setting is similar to theirs, our results are distinct from theirs.[15] First, contrary to our findings, in their paper, the optimal mechanism without side-contracting is never collusion-proof and asymmetric information does not generate any transaction cost in the optimal collusion-proof mechanism. Second, in their optimal collusion-proof mechanism, there exists no countervailing incentive since the coalition incentive constraints are binding for upward manipulations. Last, although they assert that the collusion-proofness principle holds when the principal is constrained to use uniform transfers, we prove that the principle does not hold.

In Laffont and Martimort (2000)'s recent paper, they study collusion in an environment with correlated types. Since, without collusion, the principal can fully extract the rents, room for collusion exists even though the principal can use complete contracts. They show that the first-best outcome can not be achieved in the presence of collusion. They also find that asymmetric information does not generate any transaction cost in the weak positive correlation case while it generates transaction costs in the polar case of almost perfect correlation. The result that asymmetric information generates transaction costs in coalition formation is also obtained by Jeon (2001) in a setting in which the agents have

[11] See, for instances, Caillaud and Jehiel (1998), Graham and Marshall (1987), Mailath and Zemsky (1991) and McAfee and McMillan (1992).

[12] Usually, they restrict their attention to the first-price or second-price auction.

[13] In their paper, they use term "anonymous transfers" instead of "uniform transfers".

[14] They also consider a two-type setting in which the types are independently distributed. Both in their setting and in our setting, in the absence of the uniformity constraint on transfers, the second-best outcome can be implemented in dominant strategies and in a collusion-proof way.

[15] See also Remark 3 at the end of Section 5 for a comparison between their paper and this paper.

correlated types and limited liability. However, both papers consider complete contracts and double asymmetric information inside coalitions is not considered in any of the papers.

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes as a benchmark the optimal mechanism without side-contracting and identifies the condition under which there exist gains from manipulation of reports. Section 4 characterizes the set of collusion-proof mechanisms. Our main results are presented in Sections 5-7. Section 5 shows that when the gains from collusion are less than a threshold, the optimal mechanism without side-contracting is collusion-proof. Section 6 characterizes the optimal collusion-proof mechanism when the optimal mechanism without side-contracting is not collusion-proof. Section 7 shows that the collusion-proofness principle does not hold when the principal is constrained to use uniform transfers. Section 8 briefly presents the extension to an n-agent case. Concluding remarks are gathered in Section 9.

2 The Model

2.1 The General Setting

We consider two regulated firms (agents) who participate in a public project. The firms produce perfectly complementary goods which are indispensable for the realization of the project: firm 1 produces an intermediary good which is used by firm 2 who produces the final good. We consider an indivisible public project which has a social value equal to V_s, which is assumed to be large enough to employ both firms for any realization of the efficiency parameters introduced below.[16]

The cost of firm i is given by

$$C_i = \beta_i - e_i, \quad i = 1 \text{ and } 2,$$

where β_i is an efficiency parameter and e_i is the effort exerted by the manager of firm i. Neither the regulator (principal) nor the other firm j ($\neq i$) knows the true value of β_i and observes i's effort. Hence, β_i is an adverse selection parameter and e_i is a moral hazard variable, both of which are known only by firm i. The parameter β_i, for $i = 1$ and 2, is drawn independently from the same commonly known distribution with support $\left\{\underline{\beta}, \overline{\beta}\right\} \equiv \Theta$ where $\Delta\beta \equiv \overline{\beta} - \underline{\beta} > 0$. We assume that with probability ν (respectively, $1-\nu$) the firm i has a low-cost type (respectively, a high-cost type) and β_i takes the value $\underline{\beta}$ (respectively, $\overline{\beta}$).

[16] It is a simplifying assumption and our main results hold for the case in which V_s is not large enough: see Remark 2 at the end of Section 5.

If (the manager of) firm i exerts effort e_i, it decreases the (monetary) cost by e_i and incurs a disutility (in monetary units) of $\psi_i(e_i)$. We assume that the disutility function is the same for each firm, $\psi(\cdot) = \psi_1(\cdot) = \psi_2(\cdot)$. It increases with effort $\psi' > 0$ for $e_i > 0$ at an increasing rate $\psi'' > 0$ and satisfies $\psi(0) = 0$ and $\psi''' \geq 0$.[17] For expositional ease, we assume that efforts remain strictly positive over the relevant range of equilibrium efforts.

The regulator can observe each firm's cost C_i. We take the accounting convention that costs are reimbursed to the firms by the regulator. Firm i is compensated by a net monetary transfer t_i in addition to the reimbursement of the cost. Let U_i be firm i's utility level:

$$U_i = t_i - \psi(e_i).$$

Let λ (> 0) denote the shadow cost of public funds. For a utilitarian regulator, ex post social welfare is

$$V_s - (1+\lambda)[\sum_{i=1}^{2}(t_i + C_i)] + \sum_{i=1}^{2}(U_i).$$

A regulatory contract M, or a grand-mechanism between the regulator and the firms, specifies for each firm a cost target and a transfer and takes the form of $\left\{t_i(\widehat{\beta}_1, \widehat{\beta}_2), C_i(\widehat{\beta}_1, \widehat{\beta}_2)\right\}$ where $\widehat{\beta}_i$ is firm i's report about its efficiency parameter.[18] Alternatively, a grand-mechanism can be written as $\left\{t_i(\widehat{\beta}_1, \widehat{\beta}_2), e_i(\widehat{\beta}_1, \widehat{\beta}_2)\right\}$ since the regulator can indirectly control the effort level e_i by choosing the cost target C_i: given a report $(\widehat{\beta}_1, \widehat{\beta}_2)$, to request a cost level $C_i(\widehat{\beta}_1, \widehat{\beta}_2)$ is equivalent to request an effort level $e_i(\widehat{\beta}_1, \widehat{\beta}_2) = \widehat{\beta}_i - C_i(\widehat{\beta}_1, \widehat{\beta}_2)$.

2.2 Collusion

We model collusion between the two regulated firms by a side-contract offered by a benevolent and uninformed third-party as in Laffont and Martimort (1997, 2000).[19] The benevolent third-party can be viewed as a fictitious modeling device

[17] The assumption $\psi''' \geq 0$ makes $\Phi(e) = \psi(e) - \psi(e - \Delta\beta)$, introduced in Proposition 1, convex, which ensures that the regulator's objective function is concave. In particular, the assumption makes it not worthwhile to consider stochastic mechanisms (See Laffont and Tirole (1993) pp.119-120).

[18] It is implicitly assumed that if a firm misses the cost target on which he agreed, the regulator can and will punish the firm and therefore the firm has no incentive to miss the target.

[19] The readers might wonder why we do not use a bargaining between the agents to describe collusion. However, any outcome of a bargaining under asymmetric information between the agents can be achieved by a side-contract designed by the third-party. Hence, the modelling strategy of using the third-party as a side-contract designer is a shortcut which allows us to characterize the highest bound of what can be achieved by collusion under asymmetric information.

which maximizes the sum of the agents' payoffs subject to incentive, acceptance and budget balance constraints.

A side-contract S takes the following form

$$\left\{\phi(\widetilde{\beta}_1,\widetilde{\beta}_2), y_1(\widetilde{\beta}_1,\widetilde{\beta}_2), y_2(\widetilde{\beta}_1,\widetilde{\beta}_2)\right\},$$

where $\widetilde{\beta}_i$ is firm i's report about its efficiency parameter to the third-party. $\phi(\cdot)$ is the report manipulation function which maps any pair of reports made by the agents to the third-party, i.e., $(\widetilde{\beta}_1,\widetilde{\beta}_2)$, to a pair of reports made to the seller. We assume that $\phi(\cdot)$ can specify stochastic manipulations as this convexifies the third-party's feasible set. More precisely, let $\widetilde{\phi}\in\Theta^2$ denote an outcome of $\phi(\cdot)$. Then, $\phi(\cdot)$ specifies $P^{\phi}(\widetilde{\beta}_1,\widetilde{\beta}_2,\widetilde{\phi})$ the probability that the third party, after receiving reports $(\widetilde{\beta}_1,\widetilde{\beta}_2)$, requires the agents to report $\widetilde{\phi}$ to the principal. $y_1(\cdot)$ and $y_2(\cdot)$ are two monetary transfers from the third-party to each agent respectively. Once the agents manipulate their reports into the grand-mechanism, they should realize the cost levels which are consistent with their reports $C_1(\phi(\cdot))$ and $C_2(\phi(\cdot))$. The third-party observes the realized costs but cannot observe the level of effort exerted by each agent. Since the third-party is not a source of money in our model, we assume that the ex post budget balance constraint must be satisfied:

$$\sum_{i=1,2} y_i(\beta_1,\beta_2)=0,\quad \forall(\beta_1,\beta_2)\in\Theta^2.$$

There is no loss of generality in restricting the set of feasible side-contracts to direct revelation mechanisms. Indeed, the revelation principle applies at this stage of the game. To be accepted along an equilibrium path, the side-contract must guarantee to agent i with $i=1,2$ an interim utility level $U_i(\beta_i)$ greater than the utility level which he expects to get from playing non-cooperatively the grand-mechanism M, denoted by $U_i^M(\beta_i)$. We also assume that the side-contract is enforceable even though the secrecy of this contract implies that there is no court of justice available to enforce it.

2.3 Uniform transfers and Timing

As we said in the introduction, we introduce an incompleteness of regulatory contracts by assuming that the regulator can use only uniform transfers: $t_1(\beta_1,\beta_2)=t_2(\beta_1,\beta_2)$. Since the two agents are perfectly symmetric, in the absence of any restriction on the set of mechanisms, there is no loss of generality in restricting our attention to the set of symmetric mechanisms: $t_1(\beta_i,\beta_j)=t_2(\beta_j,\beta_i)$ and $C_1(\beta_i,\beta_j)=C_2(\beta_j,\beta_i)$ for any $(\beta_i,\beta_j)\in\Theta^2$. As $t_1(\beta_i,\beta_j)=t_2(\beta_i,\beta_j)$ holds for $\beta_i=\beta_j$ in any symmetric mechanism, assuming uniform transfers implies that the principal loses one degree of freedom in transfers such that $t_1(\beta_i,\beta_j)=t_2(\beta_j,\beta_i)$

for $\beta_i \neq \beta_j$. In what follows, we assume that the principal offers a symmetric grand-mechanism with uniform transfers given as follows:

$$\begin{aligned} t_i(\beta_1, \beta_2) &= t(\beta_1, \beta_2) \text{ for } i = 1, 2 \text{ and } (\beta_1, \beta_2) \in \Theta^2; \\ C_i(\underline{\beta}, \underline{\beta}) &= C(\underline{\beta}, \underline{\beta}), C_1(\underline{\beta}, \overline{\beta}) = C_2(\overline{\beta}, \underline{\beta}), \\ C_1(\overline{\beta}, \underline{\beta}) &= C_2(\underline{\beta}, \overline{\beta}), C_i(\overline{\beta}, \overline{\beta}) = C(\overline{\beta}, \overline{\beta}), i = 1, 2. \end{aligned}$$

$\left\{e(\underline{\beta}, \underline{\beta}), e_1(\underline{\beta}, \overline{\beta}), e_1(\overline{\beta}, \underline{\beta}), e(\overline{\beta}, \overline{\beta})\right\}$ is defined similarly to the cost target $C_i(\cdot)$.

The timing of the overall game of contract offers and coalition formation is given as follows:

1. Nature draws the value of each firm's efficiency parameter β_i, for $i \in \{1, 2\}$. Each firm learns only its own parameter.

2. The regulator proposes a grand-mechanism M.

3. Each firm accepts or refuses M. If at least one firm refuses, each firm gets a reservation utility normalized to 0 and the following stages do not occur.

4. The third party offers a side-contract S to the agents.

5. Each firm accepts or refuses S. If at least one firm refuses, M is played non-cooperatively. In this case, reports are directly made in the grand-mechanism M and the next two stages of the game do not occur.

6. If S has been accepted, reports in S are made. Each agent reports non-cooperatively his type to the third party.

7. The corresponding side-transfers and the reports in M requested by the manipulation of report function are made.

8. The efforts requested by S and the monetary transfers requested by M are enforced.

3 The optimal grand-mechanism without side-contracting

In this section, we study, as a benchmark, the optimal grand-mechanism without side-contracting and then find the condition under which the mechanism exhibits room for collusion.

3.1 Characterization of the optimal grand-mechanism

To induce truth-telling, a grand-mechanism should satisfy the following incentive compatibility constraint for each type:

$$
\begin{aligned}
(BIC_L) \quad & U(\underline{\beta}) \equiv \nu\left[t(\underline{\beta},\underline{\beta}) - \psi(\underline{\beta} - C(\underline{\beta},\underline{\beta}))\right] + (1-\nu)\left[t(\underline{\beta},\overline{\beta}) - \psi(\underline{\beta} - C_1(\underline{\beta},\overline{\beta}))\right] \\
& \geq \nu\left[t(\overline{\beta},\underline{\beta}) - \psi(\underline{\beta} - C_1(\overline{\beta},\underline{\beta}))\right] + (1-\nu)\left[t(\overline{\beta},\overline{\beta}) - \psi(\underline{\beta} - C(\overline{\beta},\overline{\beta}))\right]; \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
(BIC_H) \quad & U(\overline{\beta}) \equiv \nu\left[t(\overline{\beta},\underline{\beta}) - \psi(\overline{\beta} - C_1(\overline{\beta},\underline{\beta}))\right] + (1-\nu)\left[t(\overline{\beta},\overline{\beta}) - \psi(\overline{\beta} - C(\overline{\beta},\overline{\beta}))\right] \\
& \geq \nu\left[t(\underline{\beta},\underline{\beta}) - \psi(\overline{\beta} - C(\underline{\beta},\underline{\beta}))\right] + (1-\nu)\left[t(\underline{\beta},\overline{\beta}) - \psi(\overline{\beta} - C_1(\underline{\beta},\overline{\beta}))\right]. \qquad (2)
\end{aligned}
$$

To be accepted, a grand-mechanism should satisfy the following individual rationality constraint for each type:

$$(BIR_L) \quad U(\underline{\beta}) \geq 0; \qquad (3)$$

$$(BIR_H) \quad U(\overline{\beta}) \geq 0. \qquad (4)$$

Let $p(\beta_1, \beta_2)$ denote the probability of having $(\beta_1, \beta_2) \in \Theta^2$. The benevolent regulator maximizes social welfare, defined below, subject to the constraints (1) to (4):

$$
\begin{aligned}
SW \equiv V_s - \sum_{(\beta_1,\beta_2)\in\Theta^2} \sum_{i=1,2} p(\beta_1,\beta_2) \{ & (1+\lambda)\left[t_i(\beta_1,\beta_2) + C_i(\beta_1,\beta_2)\right] \\
& - \left[t_i(\beta_1,\beta_2) - \psi(\beta_i - C_i(\beta_1,\beta_2))\right]\},
\end{aligned}
$$

In the next proposition, we characterize the optimal grand-mechanism without side-contracting.

Proposition 1 *The optimal grand-mechanism without side-contracting is characterized by:*

1. *The effort schedule is given by:*

$e^*(\underline{\beta},\underline{\beta}) = e_1^*(\underline{\beta},\overline{\beta}) = \underline{e}^* > e_1^*(\overline{\beta},\underline{\beta}) = e^*(\overline{\beta},\overline{\beta}) = \overline{e}^*$, *where* $\underline{e}^*$ *and* $\overline{e}^*$ *are defined by:*

$$
\begin{aligned}
\psi'(\underline{e}^*) &= 1, \\
\psi'(\overline{e}^*) &= 1 - \frac{\lambda}{1+\lambda}\frac{\nu}{1-\nu}\Phi'(\overline{e}^*),
\end{aligned}
$$

with $\Phi(e) \equiv \psi(e) - \psi(e - \Delta\beta)$.

2. *Only* (BIC_L) *and* (BIR_H) *are binding.*

Proof. The proof is standard and is omitted.

As usual, only the low-cost type's incentive constraint and the high-cost type's participation constraint are binding. The effort level requested for an agent depends only on his own type and is equal to the optimal level in a one-agent setting.[20] Let e represent the effort level exerted by a high-cost type. Then, the information rent that a low-cost type can obtain by pretending to be a high-cost type is given by $\psi(e) - \psi(e - \Delta\beta) \equiv \Phi(e)$ where $\Phi(e)$ is increasing in e. Because of the well-known trade-off between efficiency and rent extraction, it is optimal to introduce a downward distortion in the level of effort exerted by a high-cost type compared to the first-best level while a low-cost type's effort is equal to the first-best level.

Since the principal can choose three different transfers and there are only two binding constraints, the uniformity constraint on transfers does not generate any loss to the principal. Furthermore she can use the residual degree of freedom to satisfy the incentive constraints in dominant strategies given as follows:

$$t(\beta_i,\beta_j) - \psi\left(\beta_i - C_1(\beta_i,\beta_j)\right) \geq t(\widehat{\beta}_i,\beta_j) - \psi\left(\beta_i - C_1(\widehat{\beta}_i,\beta_j)\right), \forall\ (\beta_i,\beta_j) \in \Theta^2, \forall \widehat{\beta}_i \in \Theta, \tag{5}$$

where $i, j = 1, 2$ and $i \neq j$. If (5) is satisfied, truth-telling is a dominant strategy. Let $\{t^*(\beta_1,\beta_2)\}$ denote the transfer schedule that implements the optimal effort schedule in dominant strategies. From the binding low-cost type's incentive constraints[21], we have:

$$t^*(\underline{\beta},\underline{\beta}) - t^*(\underline{\beta},\overline{\beta}) = t^*(\underline{\beta},\overline{\beta}) - t^*(\overline{\beta},\overline{\beta}) = \psi(\underline{e}^*) - \psi(\overline{e}^* - \Delta\beta) > 0. \tag{6}$$

Let M^D represent the grand-mechanism $\{t^*(\beta_1,\beta_2), e_i^*(\beta_1,\beta_2)\}$ or equivalently $\{t^*(\beta_1,\beta_2), C_i^*(\beta_1,\beta_2)\}$ where $C_i^*(\beta_1,\beta_2) = \beta_i - e_i^*(\beta_1,\beta_2)$.

3.2 Room for collusion

In this section, we find the condition under which M^D exhibits room for collusion. Define the *null side-contract* S^0 as the side-contract in which there is no manipulation of report and no side-transfer $\{\phi(\cdot) = I_d(\cdot), y_1(\cdot) = y_2(\cdot) = 0\}$. Then, we state that room for collusion exists if there exists any side-contract which gives greater utility to each agent over S^0 when the agents collude under complete information.

[20] In fact, if the effort level is different from the optimal one in a one-agent setting, one can easily find a contradiction since starting from the optimal mechanism in a two-agent setting, one can construct an optimal mechanism for the one-agent setting.

[21] They are given by: $t(\underline{\beta},\underline{\beta}) - \psi\left(e^*(\underline{\beta},\underline{\beta})\right) \geq t(\overline{\beta},\underline{\beta}) - \psi\left(e_1^*(\overline{\beta},\underline{\beta}) - \Delta\beta\right)$ and $t(\underline{\beta},\overline{\beta}) - \psi\left(e_1^*(\underline{\beta},\overline{\beta})\right) \geq t(\overline{\beta},\overline{\beta}) - \psi\left(e^*(\overline{\beta},\overline{\beta}) - \Delta\beta\right)$.

It is easy to check that no room for collusion exists for any upward manipulation of reports while it can exist for downward manipulations. For instance, in the case of an LH-coalition (the coalition made of one low-cost type and one high-cost type), it has the incentive to announce $(\underline{\beta}, \underline{\beta})$ instead of $(\underline{\beta}, \overline{\beta})$ if the following inequality is satisfied:

$$2t^*(\underline{\beta}, \overline{\beta}) - \psi(\underline{e}^*) - \psi(\overline{e}^*) < 2t^*(\underline{\beta}, \underline{\beta}) - \psi(\underline{e}^*) - \psi(\underline{e}^* + \Delta\beta). \tag{7}$$

More generally, consider either an LH-coalition or an HH-coalition (the coalition made of two high-cost types) and suppose that a high-cost type manipulates his report from $\overline{\beta}$ to $\underline{\beta}$. Then, first, the transfer received by each agent increases by $\psi(\underline{e}^*) - \psi(\overline{e}^* - \Delta\beta)$. Second, the effort level of the agent who manipulated his report increases by $\psi(\underline{e}^* + \Delta\beta) - \psi(\overline{e}^*)$ while that of the other agent remains unchanged. Let GC denote the change in the agents' total payoffs induced by a downward manipulation of report:

$$GC \equiv 2\left[\psi(\underline{e}^*) - \psi(\overline{e}^* - \Delta\beta)\right] - \left[\psi(\underline{e}^* + \Delta\beta) - \psi(\overline{e}^*)\right].$$

GC represents the gains from collusion per downward manipulation of report. Therefore, if $GC > 0$, an HH-coalition maximizes its total payoff by reporting $(\underline{\beta}, \underline{\beta})$ to the principal. This implies that when $GC > 0$, all the coalitions have the incentive to report $(\underline{\beta}, \underline{\beta})$. Summarizing, we have:

Proposition 2 *Suppose that collusion takes place under complete information between the agents and the principal offers M^D.*

a. Room for collusion can exist only for downward manipulations of reports.

b. It exists if and only if $GC > 0$. In this case, both an LH-coalition and an HH-coalition have the incentive to manipulate their reports to $(\underline{\beta}, \underline{\beta})$.

Example 1 *Let $\psi(e) = \frac{e^2}{2}$. Then, $GC > 0$ if*

$$\frac{2}{k+3} > \Delta\beta, \text{ where } k = \frac{\lambda\nu}{(1+\lambda)(1-\nu)}.$$

4 Characterization of collusion-proof grand-mechanisms

In this section, we define collusion-proof grand-mechanism and characterize the set of collusion-proof grand-mechanisms.

4.1 Collusion-proof grand-mechanism

In order to define collusion-proof grand-mechanism, we first introduce a definition concerning side-contracts. Let $p(\beta_i)$ denote the probability of having $\beta_i \in \Theta$.

Definition 1 *A side-contract $S^* = \{\phi^*(\cdot), y_i^*(\cdot)\}$ is coalition-interim-efficient with respect to a grand-mechanism $M = \{t(\cdot), C_i(\cdot)\}$ providing the reservation utilities $U_i^M(\beta_i)$, $i = 1, 2$, if and only if it is the solution of the following third-party's program (T):*

$$\max_{\phi(\cdot), y^i(\cdot)} \sum_{(\beta_1,\beta_2)\in\Theta^2} p(\beta_1, \beta_2)[U_1(\beta_1) + U_2(\beta_2)]$$

subject to

$$\begin{array}{l} U_i(\beta_i) \equiv \sum_{\beta_j\in\Theta} p(\beta_j) \left\{ \sum_{\widetilde{\phi}\in\Theta^2} P^{\phi}(\beta_i, \beta_j, \widetilde{\phi}) \left[t(\widetilde{\phi}) + y_i(\beta_i, \beta_j) - \psi(\beta_i - C_i(\widetilde{\phi})) \right] \right\}, \\ \qquad \forall \beta_i \in \Theta,\ i = 1, 2,\ j = 1, 2,\ i \neq j, \\ (BIC^S)\ U_i(\beta_i) \geq \sum_{\beta_j\in\Theta} p(\beta_j) \left\{ \sum_{\widetilde{\phi}\in\Theta^2} P^{\phi}(\widetilde{\beta}_i, \beta_j, \widetilde{\phi}) \left[t(\widetilde{\phi}) + y_i(\widetilde{\beta}_i, \beta_j) - \psi(\beta_i - C_i(\widetilde{\phi})) \right] \right\} \\ \qquad \forall \beta_i \in \Theta\ , \forall \widetilde{\beta}_i \in \Theta,\ i = 1, 2,\ j = 1, 2,\ i \neq j, \\ (BIR^S)\ U_i(\beta_i) \geq U_i^M(\beta_i), \forall \beta_i \in \Theta,\ i = 1, 2, \\ (BB) \quad \sum_{i=1,2} y_i(\beta_1, \beta_2) = 0, \forall (\beta_1, \beta_2) \in \Theta^2. \end{array}$$

We now define a collusion-proof grand-mechanism.

Definition 2 *A grand-mechanism $M = \{t(\cdot), C_i(\cdot)\}$ providing the reservation utilities $U_i^M(\beta_i)$, $i = 1, 2$, is collusion-proof when the null side-contract is coalition-interim-efficient with respect to this mechanism.*

In words, a grand-mechanism is collusion-proof if the third-party finds it optimal not to manipulate the reports and not to use any side-transfer.

4.2 Characterization of collusion-proof grand-mechanisms

We here characterize the symmetric collusion-proof grand-mechanisms with uniform transfers. For this purpose, we introduce the following notation:

$$\begin{aligned} V^{\epsilon}(\widehat{\beta}_1, \widehat{\beta}_2; \beta_1, \beta_2) \quad \equiv \quad & 2t(\widehat{\beta}_1, \widehat{\beta}_2) - \psi(\beta_1 - C_1(\widehat{\beta}_1, \widehat{\beta}_2)) - \psi(\beta_2 - C_2(\widehat{\beta}_1, \widehat{\beta}_2)) \\ & - \epsilon \frac{\nu}{1-\nu} \left[\Phi(\beta_1 - C_1(\widehat{\beta}_1, \widehat{\beta}_2)) \mathbf{1}_{[\beta_1 = \overline{\beta}]} + \Phi(\beta_2 - C_2(\widehat{\beta}_1, \widehat{\beta}_2)) \mathbf{1}_{[\beta_2 = \overline{\beta}]} \right]. \end{aligned}$$

As it becomes clear later on, $V^{\epsilon}(\cdot)$ represents the total *virtual* payoff that a coalition with types (β_1, β_2) obtains by reporting $(\widehat{\beta}_1, \widehat{\beta}_2)$ to the principal. In the next proposition, we focus on the subset of collusion-proof mechanisms where the high-cost type's incentive constraint is not binding.[22]

[22]We will check later that the high-cost type's incentive constraint is slack in the optimal collusion-proof grand-mechanism.

Proposition 3 *A Bayesian incentive compatible mechanism $M = \{t(\cdot), C_i(\cdot)\}$ such that the high-cost type's incentive constraint is not binding is collusion-proof if and only if there exists ϵ $(1 > \epsilon \geq 0)$ such that*

(a) the following coalition incentive constraints hold[23]*: for an LL-coalition,*

$$(CIC_{LL,LH}) \quad V^{\epsilon}(\underline{\beta},\underline{\beta};\underline{\beta},\underline{\beta}) \geq V^{\epsilon}(\underline{\beta},\overline{\beta};\underline{\beta},\underline{\beta}), \tag{8}$$

$$(CIC_{LL,HH}) \quad V^{\epsilon}(\underline{\beta},\underline{\beta};\underline{\beta},\underline{\beta}) \geq V^{\epsilon}(\overline{\beta},\overline{\beta};\underline{\beta},\underline{\beta}), \tag{9}$$

for an LH-coalition,

$$(CIC_{LH,LL}) \quad V^{\epsilon}(\underline{\beta},\overline{\beta};\underline{\beta},\overline{\beta}) \geq V^{\epsilon}(\underline{\beta},\underline{\beta};\underline{\beta},\overline{\beta}), \tag{10}$$

$$(CIC_{LH,HH}) \quad V^{\epsilon}(\underline{\beta},\overline{\beta};\underline{\beta},\overline{\beta}) \geq V^{\epsilon}(\overline{\beta},\overline{\beta};\underline{\beta},\overline{\beta}), \tag{11}$$

$$(CIC_{LH,HL}) \quad V^{\epsilon}(\underline{\beta},\overline{\beta};\underline{\beta},\overline{\beta}) \geq V^{\epsilon}(\overline{\beta},\underline{\beta};\underline{\beta},\overline{\beta}), \tag{12}$$

and for an HH-coalition,

$$(CIC_{HH,LL}) \quad V^{\epsilon}(\overline{\beta},\overline{\beta};\overline{\beta},\overline{\beta}) \geq V^{\epsilon}(\underline{\beta},\underline{\beta};\overline{\beta},\overline{\beta}), \tag{13}$$

$$(CIC_{HH,LH}) \quad V^{\epsilon}(\overline{\beta},\overline{\beta};\overline{\beta},\overline{\beta}) \geq V^{\epsilon}(\underline{\beta},\overline{\beta};\overline{\beta},\overline{\beta}). \tag{14}$$

(b) If $\epsilon > 0$, the low-cost type's incentive compatibility constraint is binding in the side-contract. If it is slack, $\epsilon = 0$.

Proof. See Appendix 1.

If all the coalition incentive constraints are satisfied, the third-party has no incentive to manipulate the agents' reports and therefore the grand-mechanism is collusion-proof. For instance, if $(CIC_{LL,LH})$ is satisfied, an LL-coalition prefers truthtelling to reporting $\left(\underline{\beta},\overline{\beta}\right)$. When there is complete information between the agents, any potential gain from joint manipulations of reports can be realized. The coalition incentive constraints under complete information are obtained by taking $\epsilon = 0$ in the constraints characterized in Proposition 3.

The coalition incentive constraints under asymmetric information are written by replacing the real disutility of effort with the *virtual disutility*. A low-cost type's virtual disutility is equal to the real one while a high-cost type's virtual disutility is larger than the real one and is given by $\psi(e) + \epsilon\frac{\nu}{1-\nu}\Phi(e)$. ϵ can be positive if the low-cost type's incentive compatibility constraint is binding

[23]From the definition, $V^{\epsilon}(\widehat{\beta}_1,\widehat{\beta}_2;\underline{\beta},\underline{\beta})$ does not depend on ϵ. Since M is symmetric, $V^{\epsilon}(\underline{\beta},\overline{\beta};\underline{\beta},\underline{\beta}) = V^{\epsilon}(\overline{\beta},\underline{\beta};\underline{\beta},\underline{\beta})$ and $V^{\epsilon}(\underline{\beta},\overline{\beta};\overline{\beta},\overline{\beta}) = V^{\epsilon}(\overline{\beta},\underline{\beta};\overline{\beta},\overline{\beta})$.

in third-party's program while ϵ is zero if the constraint is slack. Because of the tension between the incentive and the participation constraints in the side-contract, giving a rent to a low-cost type can be costly to the third-party. ϵ is a parameter which measures how costly the rent is.[24] We note that the principal has some flexibility in choosing ϵ because the null-side contract S^0 satisfies the necessary and sufficient conditions for optimality in the third-party's problem for any $\epsilon \in [0, 1)$.

Remark 1 (more general mechanisms): One might argue that the principal might ask the agents for the information that they may have learned during the course of coalition formation. But then the third-party could react by inducing further manipulations of those reports of the learned information. These reactions and counter-reactions lead naturally to a problem of infinite regress. By restricting the principal to use grand-mechanisms only contingent on the agents' types, we cut arbitrarily this process in favor of colluding agents. This fits our desire to give collusive behavior its best chance.

5 Transaction costs from asymmetric information and collusion-proofness

In this section, we show that even though M^D exhibits room for collusion, it can be collusion-proof because of the transaction costs created by asymmetric information.

From Proposition 3, we know that the coalition incentive constraints under asymmetric information are written by replacing the high-cost type's disutility of effort with his virtual disutility $\psi(e)+\epsilon\frac{\nu}{1-\nu}\Phi(e)$. For instance, when the principal offers M^D, $(CIC_{LH,LL})$ is written as follows:

$$\begin{aligned} & 2t^*(\underline{\beta}, \overline{\beta}) - \psi(\underline{e}^*) - \psi(\overline{e}^*) - \epsilon\frac{\nu}{1-\nu}\Phi(\overline{e}^*) \\ \geq \; & 2t^*(\underline{\beta}, \underline{\beta}) - \psi(\underline{e}^*) - \psi(\underline{e}^* + \Delta\beta) - \epsilon\frac{\nu}{1-\nu}\Phi(\underline{e}^* + \Delta\beta), \end{aligned} \tag{15}$$

which is equivalent to

$$\epsilon\frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right] \geq GC, \tag{16}$$

where ϵ belongs to $[0, 1)$. Since we maximize the principal's payoff, we focus on the Sup of her payoff by allowing her to choose $\epsilon = 1$.[25] Hence, if GC is smaller

[24] Precisely, $\epsilon = \frac{\delta}{a+\delta}$ where $a > 0$ and δ is a Lagrange multiplier which is positive only if the low-cost type's incentive constraint is binding in the third-party's program: see Appendix 1.

[25] Although the principal cannot choose $\epsilon = 1$, there exists a mechanism with $\epsilon \in [0, 1)$ which allows the principal to achieve a payoff arbitrarily close to the one that she obtains when $\epsilon = 1$.

than a threshold $\frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right]$, the agents fail to realize the gain from the manipulation of the reports. It is easy to check that M^D satisfies $(CIC_{HH,LL})$ and $(CIC_{HH,LH})$ if GC is smaller than the threshold. Therefore, we have:

Proposition 4 *In the presence of asymmetric information between the agents, M^D is collusion-proof if the following inequality holds;*

$$GC < \frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right].$$

The intuition of why the agents fail to realize the gains from collusion can be provided as follows. Note first that under M^D, a high-cost type has to exert a higher level of effort when he reports $\underline{\beta}$ than when he reports $\overline{\beta}$: $\underline{e}^* + \Delta\beta > \overline{e}^*$. Therefore, when a low-cost type pretends to have a high-cost type to the third-party, the information rent he obtains is higher after a downward manipulation than in its absence: $\Phi(\underline{e}^* + \Delta\beta) > \Phi(\overline{e}^*)$. This in turn implies that, in order to induce a low-cost type's truth-telling, the third-party has to give him a rent larger than the one he can obtain in the absence of the manipulation. We define this increase in the rent as the transaction costs created by asymmetric information: $TC \equiv \Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)$. When the third-party requests the high-cost type to always report $\underline{\beta}$, the expected gain is equal to $2(1-\nu)GC$ while the expected increase in the low-cost type's rent which is necessary to implement this manipulation is equal to $2\nu TC$. Therefore, if GC is smaller than $\frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right]$, the agents fail to realize the gains from collusion.

In Proposition 4, the principal used the optimal dominant-strategy mechanism without side-contracting M^D. Thus, we can ask whether the principal can do better by using an optimal Bayesian mechanism in order to implement the optimal effort schedule $\{e_i^*(\beta_1, \beta_2)\}$ in a collusion-proof way. It turns out that M^D is better than any other mechanism. To see this, notice first that under M^D, $t^*(\underline{\beta}, \underline{\beta}) - t^*(\underline{\beta}, \overline{\beta}) = t^*(\underline{\beta}, \overline{\beta}) - t^*(\overline{\beta}, \overline{\beta})$. Therefore, whenever $t(\underline{\beta}, \overline{\beta}) \neq t^*(\underline{\beta}, \overline{\beta})$, the binding (BIR_H) and (BIC_L) imply that either $t(\underline{\beta}, \underline{\beta}) - t(\underline{\beta}, \overline{\beta}) > t^*(\underline{\beta}, \underline{\beta}) - t^*(\underline{\beta}, \overline{\beta})$ or $t(\underline{\beta}, \overline{\beta}) - t(\overline{\beta}, \overline{\beta}) > t^*(\underline{\beta}, \overline{\beta}) - t^*(\overline{\beta}, \overline{\beta})$ holds. Hence, whenever the principal uses transfers different from those in M^D, either $(CIC_{LH,LL})$ or $(CIC_{HH,LH})$ becomes strictly more difficult to satisfy than under M^D. Since collusion-proofness requires both constraints to be satisfied at the same time, M^D does better than any other optimal mechanism. Therefore, we have:

Proposition 5 *The principal can never implement the optimal grand-mechanism without side-contracting in a collusion-proof way if the following inequality holds;*

$$GC > \frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right].$$

Remark 2 (shutdown): When V_s is not large enough, shutdown can be optimal. There can be two kinds of shutdown policy: the public project will be undertaken either only when $(\beta_1, \beta_2) = (\underline{\beta}, \underline{\beta})$ or whenever $(\beta_1, \beta_2) \neq (\overline{\beta}, \overline{\beta})$. When the principal implements each shutdown policy by the optimal dominant-strategy mechanism without side-contracting, there is no room for collusion in the first case while, in the second case, room for collusion can exist only for downward manipulations of reports. Therefore, in the second case, a result similar to Proposition 4 holds. For instance, in the absence of transaction costs, an HH-coalition can achieve gains by reporting $(\underline{\beta}, \underline{\beta})$ and avoiding shutdown. However, when the gains are smaller than a threshold, the coalition fails to realize them under asymmetric information.

Remark 3 (comparison with Laffont and Martimort (1997)): A crucial difference between our setting and that of Laffont and Martimort (1997) consists in the fact that given the agents' reports, the regulator can specify each agent a different cost target in our setting while, in their setting, she specifies the same quantity target for both agents.[26] Therefore, in the optimal mechanism without side-contracting, an agent's effort level depends only on his own report in our setting while, in their setting, the quantity produced by an agent depends both on his report and on the other's report. Hence, when an agent manipulates his report, in our setting, he inflicts only pecuniary externalities on the other by affecting the transfer the latter receives while, in their setting, he inflicts not only pecuniary externalities but also production externalities by affecting the quantity the latter produces. In particular, in our setting (under M^D), the pecuniary externalities from an upward manipulation are negative and therefore no room for collusion exists for any upward manipulation: in contrast, in their setting, the negative pecuniary externalities can be compensated by positive production externalities such that room for collusion exists for an upward manipulation. However, since a high-cost type produces less quantity after an upward manipulation than without the manipulation, an upward manipulation decreases (instead of increasing) a low-cost type's incentive to pretend to be a high-cost type to the third-party. Therefore, in their setting, asymmetric information does not create any transaction cost in coalition formation.

6 The optimal collusion-proof grand-mechanism

In this section, we study the optimal collusion-proof grand-mechanism when the optimal grand-mechanism without side-contracting cannot be implemented in a collusion-proof way under asymmetric information. The principal's program (P) is defined as follows: to maximize social welfare under the individual incentive constraints (1) and (2), the individual rationality constraints (3) and (4), and the

[26]It is because they assume that the agents produce perfectly complementary goods.

coalition incentive constraints (8) to (14). Since there are many constraints, we proceed in the following way: we define a reduced program, find the condition under which the solution of the reduced program is equivalent to that of the original program and solve the reduced program.

The reduced program (RP) is defined as follows:

$$\max_{\epsilon, t(\beta_1,\beta_2), C_i(\beta_1,\beta_2)} SW,$$

subject to (BIC_L), (BIR_H), $(CIC_{LH,LL})$ and $(CIC_{HH,LH})$.

In the next proposition, we characterize the optimal collusion-proof grand-mechanism.

Proposition 6 *1. When the constraints in (RP) are satisfied with equality, all the constraints in (P) are satisfied if the cost schedule satisfies the following monotonicity condition:*

$$C(\underline{\beta},\underline{\beta}) \leq C_1(\underline{\beta},\overline{\beta}) \leq C_2(\underline{\beta},\overline{\beta}) \leq C(\overline{\beta},\overline{\beta}).$$

2. Suppose that the monotonicity condition holds and $\psi(e) = \frac{e^2}{2}$. The optimal collusion-proof grand-mechanism is characterized by:

a. All four constraints in (RP) are binding.

b. The Sup of the principal's payoff is obtained when $\epsilon = 1$.

c. The optimal collusion-proof effort schedule $\{e_i^{**}(\beta_1,\beta_2)\}$ is such that:

$$e^{**}(\underline{\beta},\underline{\beta}) > e^{*}(\underline{\beta},\underline{\beta}) > e^{*}(\overline{\beta},\overline{\beta}) > e^{**}(\overline{\beta},\overline{\beta}).$$

Proof. See Appendix 2.

Even though the principal cannot implement the optimal grand-mechanism without side-contracting in a collusion-proof way, she fully exploits asymmetric information between the agents since $\epsilon = 1$ in the optimal collusion-proof contract. The difference between the optimal collusion-proof effort schedule and the optimal effort schedule without side-contracting results from the two binding coalition incentive compatibility constraints. In particular, there are an upward distortion at the top and a downward distortion at the bottom with respect to the optimal effort schedule without side-contracting. To understand these distortions, it is important to notice that the coalition incentive constraints are binding for downward manipulations: an upward distortion (resp. a downward distortion) makes it more costly for an LH-coalition (resp. an HH-coalition) to report $(\underline{\beta},\underline{\beta})$ (resp. $(\underline{\beta},\overline{\beta})$ or $(\overline{\beta},\underline{\beta})$). Since the individual incentive constraint is binding for an upward manipulation, collusion creates countervailing incentives.

7 Uniform transfers and collusion-proofness principle

The collusion-proofness principle states that there is no loss of generality in restricting the principal to offer collusion-proof mechanisms. If there is no restriction on the set of grand-mechanisms, the collusion-proofness principle holds in our setting.[27] The idea of the proof can be given as follows. Suppose that the principal initially offers a mechanism M' and it is optimal for the third-party to offer a side-contract which is non-null $S'(\neq S^0)$. Then, the principal can design a new mechanism $M'' = M' \circ S'$. When the principal offers M'', it should be optimal for the third-party to offer S^0. Otherwise, there should exist a side-contract $S''(\neq S^0)$ that the third-party finds optimal. But this contradicts the interim efficiency of S': conditional on the offer of M', the third-party would strictly prefer offering $S' \circ S''$ to offering S'.

In the previous sketch of the proof, the principal is able to mimic any outcome of $M' \circ S'$ in a collusion-proof way by offering M''. However, if the principal is constrained to use uniform transfers, she is not able to mimic all the outcomes since S' may include non-uniform side-transfers. Therefore, letting the agents collude by using non-zero side-transfers could enlarge the set of implementable allocations.

More precisely, suppose that under the grand-mechanism initially offered by the principal, denoted by $M^c = \{t^c(\beta_1, \beta_2), C_i(\beta_1, \beta_2)\}$ with $C(\underline{\beta}, \underline{\beta}) \leq C_1(\overline{\beta}, \underline{\beta})$, there exists a unique gain from collusion for the manipulation of reports from $\left[(\underline{\beta}, \overline{\beta}), (\overline{\beta}, \underline{\beta})\right]$ to $\left(\underline{\beta}, \underline{\beta}\right)$. Suppose that the third-party can implement the manipulation through a side-mechanism S'. Since $C(\underline{\beta}, \underline{\beta}) \leq C_1(\overline{\beta}, \underline{\beta})$ holds, a high-cost type has to exert more effort after the manipulation than without the manipulation and therefore S' needs to specify a strictly positive side-transfer from a low-cost type to a high-cost type in order to induce the latter to accept the manipulation. Consider now the case in which the principal implements in a collusion-proof way the cost profile implemented by $M^c \circ S'$. In order to induce a high-cost type's participation, the transfer given to the high-cost type in the collusion-proof mechanism must be higher than the one in M^c by the amount equal to the side-transfer that he would receive from a low-cost type if the principal used M^c. If the principal were not constrained to use uniform transfers, she could mimic the outcome of $M^c \circ S'$ and therefore could reduce the transfer given to a low-cost type by the same amount. However, when she is constrained, she cannot mimic the non-uniform transfers implemented by S' and hence the increase in the transfer to a high-cost type implies also an increase in the transfer to a low-cost type. Therefore, the principal might end up paying more transfer when she uses collusion-proof mechanisms than when she lets collusion occur. In

[27] The proof is a straightforward application of Appendix 2 in Laffont and Martimort (1997).

fact, we can prove:

Proposition 7 *When the principal is constrained to use uniform transfers, the collusion-proofness principle does not hold.*

Proof. See Appendix 3.

Our result is contrary to Proposition 6 in Laffont and Martimort (1997), which states that there is no loss of generality in restricting the principal to offer anonymous contracts[28] which prevent collusion between a low-cost type and a high-cost type.[29] When the collusion-proofness principle does not hold, the optimal collusion-proof contract might be dominated by a mechanism which induces collusion to occur. It would be interesting to characterize the optimal mechanism in the set of the mechanisms which induce collusion to occur and to compare this with the optimal collusion-proof mechanism but this task is beyond the scope of our paper.

8 Case of $n(>2)$-agent

In this section, we briefly discuss the case of $n(>2)$-agent and show that our results hold in this case. A generic symmetric mechanism with uniform transfers is defined by $M_n = \{t_m, C_{Lm}, C_{Hm}\}$: as a function of the number of the reported high-cost types m (with $m \in \{0, 1, ..., n\}$), the mechanism specifies the transfer to each agent t_m, a low-cost type's cost target C_{Lm} and a high-cost type's cost target C_{Hm}. Let $M_n^D \equiv \{t_m^*, C_{Lm}^*, C_{Hm}^*\}$ represent the optimal dominant-strategy mechanism without side-contracting. First, in the mechanism, the effort level requested for an agent depends only on his own type and is equal to the optimal level in a one-agent setting, implying $C_{Lm}^* = \underline{\beta} - \underline{e}^*$ and $C_{Hm}^* = \overline{\beta} - \overline{e}^*$. Second, the transfers in the mechanism satisfy the incentive constraints in dominant strategies so that truth-telling is a dominant strategy. From the binding low-cost type's incentive constraints,[30] we have $t_m^* - t_{m+1}^* = \psi(\underline{e}^*) - \psi(\overline{e}^* - \Delta\beta)$.

Under M_n^D, room for collusion exists only for downward manipulations of reports: for an m-coalition (the coalition composed of m number of high-cost types), the gain from manipulating its report from $m'(\leq m)$ to $m' - 1$ is given by

$$GC(n) \equiv n\left[\psi(\underline{e}^*) - \psi(\overline{e}^* - \Delta\beta)\right] - \left[\psi(\underline{e}^* + \Delta\beta) - \psi(\overline{e}^*)\right].$$

[28]What they call anonymous transfers is equivalent to the uniform transfers in our setting.

[29]In fact, we can prove that the principle does not hold in their setting either.

[30]They are written as follows: $t_m^* - \psi(\underline{e}^*) \geq t_{m+1}^* - \psi(\overline{e}^* - \Delta\beta)$ for $m \in \{0, 1, ..., n-1\}$.

$GC(n)$ represents the gains from collusion per downward manipulation of reports. Since it is independent of m, when $GC(n) > 0$ and there is no transaction cost in coalition formation, an m-coalition has the incentive to report that all the agents have a low-cost type for any $m \in \{0, 1, ..., n\}$.

Consider now collusion under asymmetric information. We neglect the problem of subcoalitions and assume that the agents can form only one grand-coalition of size n. Then, the coalition incentive constraint preventing an m-coalition from reporting $m-1$ is written by:

$$(CIC_{m,m-1}) \quad \begin{array}{l} nt_m^* - (n-m)\psi(\underline{e}^*) - m\left[\psi(\overline{e}^*) + \epsilon\frac{\nu}{1-\nu}\Phi(\overline{e}^*)\right] \\ \geq nt_{m-1}^* - (n-m)\psi(\underline{e}^*) - \left[\psi(\underline{e}^* + \Delta\beta) + \epsilon\frac{\nu}{1-\nu}\Phi(\underline{e}^* + \Delta\beta)\right] \\ \quad -(m-1)\left[\psi(\overline{e}^*) + \epsilon\frac{\nu}{1-\nu}\Phi(\overline{e}^*)\right] \end{array}$$

where $\epsilon \in [0, 1)$. The Sup of the principal's payoff is obtained when $\epsilon = 1$. Then, $(CIC_{m,m-1})$ becomes:

$$\frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right] \geq GC(n). \tag{17}$$

If M_n^D satisfies (17), it satisfies all the other coalition incentive constraints as well. Therefore, if $GC(n)$ is smaller than the threshold $\frac{\nu}{1-\nu}\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right]$, the agents fail to realize the gains from collusion because of the transaction costs created by asymmetric information. However, as $GC(n)$ increases linearly with n, one can find n large enough such that the expected gains from collusion $(1-\nu)\,GC(n)$ are superior to the expected transaction costs $\nu\left[\Phi(\underline{e}^* + \Delta\beta) - \Phi(\overline{e}^*)\right]$ which do not depend on n. For the reasons given just before Proposition 5, M_n^D is better than any other mechanism in implementing the optimal effort schedule without side-contracting in a collusion-proof way. When the gains from collusion are larger than the threshold, we conjecture that (BIC_L), (BIR_H) and $(CIC_{m,m-.1})$ for $m \in \{1, 2..., n\}$ would bind in the optimal collusion-proof mechanism and the optimal collusion-proof effort schedule would exhibit an upward distortion at the top and a downward distortion at the bottom as in Proposition 6.

9 Conclusion

In this paper, we studied mechanism design under collusion and uniform transfers focusing on the transaction costs in coalition formation generated by asymmetric information. We found that the transaction costs can make the agents fail to realize gains from collusion and the principal can exploit the transaction costs to design the optimal collusion-proof mechanism. We also showed that the collusion-proofness principle does not hold when the principal is constrained to use uniform transfers. Although we studied a regulation setting, our model can be applied to other situations such as collusion inside organizations.[31] It would be interesting to extend the analysis to more general settings.[32] In particular, it seems challenging to allow subcoalitions to form when collusion involves more than two agents: since the grand-coalition has to deal with subcoalitions, the principal can take advantage of this conflict in designing her mechanism.[33]

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[31]For instance, the model can be adapted to collusion among workers who have to meet some performance targets since a worker's performance usually depends both on his ability and on his level of effort.

[32]However, characterizing the optimal collusion-proof mechanism when there are more than two types can be quite involved since it is often hard to determine which coalition incentive constraints are binding.

[33]When there are subcoalitions, the revelation principle does not apply to the problem of designing the side-contract for the grand-coalition.

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Appendix 1

The proof uses the methodology developed by Laffont and Martimort (2000).

The third-party maximizes the following objective:

$$\begin{aligned}
E(U_1+U_2) &= \nu^2 \sum_{\widetilde{\phi}\in\Theta^2} P^{\phi}(\underline{\beta},\underline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi})+t_2(\widetilde{\phi})-\psi(\underline{\beta}-C_1(\widetilde{\phi}))-\psi(\underline{\beta}-C_2(\widetilde{\phi}))] \\
&\quad +\nu(1-\nu)\sum_{\widetilde{\phi}\in\Theta^2} P^{\phi}(\underline{\beta},\overline{\beta},\widetilde{\phi})\left[t_1(\widetilde{\phi})+t_2(\widetilde{\phi})-\psi(\underline{\beta}-C_1(\widetilde{\phi}))-\psi(\overline{\beta}-C_2(\widetilde{\phi}))\right] \\
&\quad +\nu(1-\nu)\sum_{\widetilde{\phi}\in\Theta^2} P^{\phi}(\overline{\beta},\underline{\beta},\widetilde{\phi})\left[t_1(\widetilde{\phi})+t_2(\widetilde{\phi})-\psi(\overline{\beta}-C_1(\widetilde{\phi}))-\psi(\underline{\beta}-C_2(\widetilde{\phi}))\right] \\
&\quad +(1-\nu)^2\sum_{\widetilde{\phi}\in\Theta^2} P^{\phi}(\overline{\beta},\overline{\beta},\widetilde{\phi})\left[t_1(\widetilde{\phi})+t_2(\widetilde{\phi})-\psi(\overline{\beta}-C_1(\widetilde{\phi}))-\psi(\overline{\beta}-C_2(\widetilde{\phi}))\right]
\end{aligned}$$

subject to the following constraints.

- Budget balance constraints for the side transfers:

$$(BB) \qquad \sum_{i=1}^{2} y_i(\beta_1,\beta_2)=0, \text{ for any } (\beta_1,\beta_2)\in\Theta^2,$$

• Low-cost type's Bayesian incentive constraint for agent 1:

$$
\begin{aligned}
(BIC_1^S(\underline{\beta})) \qquad & \nu \sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\underline{\beta},\underline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - y_1(\underline{\beta},\underline{\beta})] \\
& +(1-\nu) \sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\underline{\beta},\overline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - y_1(\underline{\beta},\overline{\beta})] \\
\geq \; & \nu \sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\overline{\beta},\underline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - y_1(\overline{\beta},\underline{\beta})] \\
& +(1-\nu) \sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\overline{\beta},\overline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - y_1(\overline{\beta},\overline{\beta})],
\end{aligned}
$$

• Low-cost type's acceptance constraint for agent 1:

$$
\begin{aligned}
(BIR_1^S(\underline{\beta})) \quad & \nu \textstyle\sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\underline{\beta},\underline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - y_1(\underline{\beta},\underline{\beta})] \\
& +(1-\nu) \textstyle\sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\underline{\beta},\overline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - y_1(\underline{\beta},\overline{\beta})] \geq U^M(\underline{\beta}),
\end{aligned}
$$

• High-cost type's acceptance constraint for agent 1:

$$
\begin{aligned}
(BIR_1^S(\overline{\beta})) \quad & \nu \textstyle\sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\overline{\beta},\underline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\overline{\beta} - C_1(\widetilde{\phi})) - y_1(\overline{\beta},\underline{\beta})] \\
& +(1-\nu) \textstyle\sum_{\widetilde{\phi}\in\Theta^2} P^\phi(\overline{\beta},\overline{\beta},\widetilde{\phi})[t_1(\widetilde{\phi}) - \psi(\overline{\beta} - C_1(\widetilde{\phi})) - y_1(\overline{\beta},\overline{\beta})] \geq U^M(\overline{\beta}),
\end{aligned}
$$

• Low-cost type's Bayesian incentive constraint for agent 2 ($BIC_2^S(\underline{\beta})$):

• Low-cost type's acceptance constraint for agent 2 ($BIR_2^S(\underline{\beta})$):

• High-cost type's acceptance constraint for agent 2 ($BIR_2^S(\overline{\beta})$):

where ($BIC_2^S(\underline{\beta})$), ($BIR_2^S(\underline{\beta})$) and ($BIR_2^S(\overline{\beta})$) are defined similarly to ($BIC_1^S(\underline{\beta})$), ($BIR_1^S(\underline{\beta})$) and ($BIR_1^S(\overline{\beta})$).

We introduce the following multipliers:

• $\rho(\beta_1, \beta_2)$ for the budget-balance constraint for the side-transfers in state (β_1, β_2),

• δ_i for the low-cost type's Bayesian incentive constraint concerning agent i,

• $\underline{\nu}_i$ for the low-cost type's acceptance constraint concerning agent i,

• $\overline{\nu}_i$ for the high-cost type's acceptance constraint concerning agent i.

We define the Lagrangian function as follows:

$$
\begin{aligned}
L \;=\; & E(U_1+U_2) + \sum_{i=1,2} \delta_i (BIC_i^S(\underline{\beta})) + \sum_{i=1,2} \underline{\nu}_i (BIR_i^S(\underline{\beta})) + \sum_{i=1,2} \overline{\nu}_i (BIR_i^S(\overline{\beta})) \\
& + \sum_{(\beta_1,\beta_2)\in\Theta^2} \rho(\beta_1,\beta_2)(BB(\beta_1,\beta_2))
\end{aligned}
$$

Step 1: Optimizing with respect to $y_i(\beta_1, \beta_2)$

After optimizing with respect to $y_i(\underline{\beta}, \underline{\beta})$, we have:

$$\rho(\underline{\beta}, \underline{\beta}) - \delta_i \nu - \underline{\nu}_i \nu = 0, \text{ for } i = 1, 2.$$

After optimizing with respect to $y_1(\underline{\beta}, \overline{\beta})$ and $y_2(\underline{\beta}, \overline{\beta})$ respectively, we have:

$$\begin{aligned} \rho(\underline{\beta}, \overline{\beta}) - \delta_1(1-\nu) - \underline{\nu}_1(1-\nu) &= 0; \\ \rho(\underline{\beta}, \overline{\beta}) + \delta_2 \nu - \overline{\nu}_2 \nu &= 0. \end{aligned}$$

After optimizing with respect to $y_1(\overline{\beta}, \underline{\beta})$ and $y_2(\overline{\beta}, \underline{\beta})$ respectively, we have:

$$\begin{aligned} \rho(\overline{\beta}, \underline{\beta}) + \delta_1 \nu - \overline{\nu}_1 \nu &= 0; \\ \rho(\overline{\beta}, \underline{\beta}) - \delta_2(1-\nu) - \underline{\nu}_2(1-\nu) &= 0. \end{aligned}$$

After optimizing with respect to $y_i(\overline{\beta}, \overline{\beta})$, we have:

$$\rho(\overline{\beta}, \overline{\beta}) + \delta_i(1-\nu) - \overline{\nu}_i(1-\nu) = 0, \text{ for } i = 1, 2.$$

In what follows, without loss of generality, we restrict our attention to symmetric multipliers:

$$\delta \equiv \delta_1 = \delta_2, \quad \underline{\nu} \equiv \underline{\nu}_1 = \underline{\nu}_2, \quad \overline{\nu} \equiv \overline{\nu}_1 = \overline{\nu}_2$$

From the above equations, we have:

$$(1-\nu)(\delta + \underline{\nu}) = \nu(\overline{\nu} - \delta).$$

Step 2: Optimizing with respect to $\phi(\beta_1, \beta_2)$ (or $P^{\phi}(\beta_1, \beta_2, \widetilde{\phi})$)

We below give the conditions under which the third party finds it optimal to require any coalition to truthfully report: $P^{\phi}(\beta_1, \beta_2, \widetilde{\phi}) = 1$ for $\widetilde{\phi} = (\beta_1, \beta_2)$ for any $(\beta_1, \beta_2) \in \Theta^2$.

- When $(\beta_1, \beta_2) = (\underline{\beta}, \underline{\beta})$,

$$(\underline{\beta}, \underline{\beta}) \in \arg\max_{\tilde{\phi} \in \Theta^2} \left\{ t_1(\tilde{\phi}) + t_2(\tilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - \psi(\underline{\beta} - C_2(\widetilde{\phi})) \right\}.$$

- When $(\beta_1, \beta_2) = (\underline{\beta}, \overline{\beta})$,

$$(\underline{\beta}, \overline{\beta}) \in \arg\max_{\tilde{\phi} \in \Theta^2} \left\{ \begin{array}{c} t_1(\tilde{\phi}) + t_2(\tilde{\phi}) - \psi(\underline{\beta} - C_1(\widetilde{\phi})) - \psi(\overline{\beta} - C_2(\widetilde{\phi})) \\ -\epsilon \frac{\nu}{1-\nu} \Phi(\overline{\beta} - C_2(\widetilde{\phi})) \end{array} \right\},$$

where $\epsilon \equiv \frac{\delta}{\delta + v + \underline{v}}$.

- When $(\beta_1, \beta_2) = (\overline{\beta}, \underline{\beta})$

$$(\overline{\beta}, \underline{\beta}) \in \arg\max_{\tilde{\phi}\in\Theta^2} \left\{ \begin{array}{c} t_1(\tilde{\phi}) + t_2(\tilde{\phi}) - \psi(\overline{\beta} - C_1(\tilde{\phi})) - \psi(\underline{\beta} - C_2(\tilde{\phi})) \\ -\epsilon\frac{\nu}{1-\nu}\Phi(\overline{\beta} - C_1(\tilde{\phi})) \end{array} \right\}.$$

- When $(\beta_1, \beta_2) = (\overline{\beta}, \overline{\beta})$

$$(\overline{\beta}, \overline{\beta}) \in \arg\max_{\tilde{\phi}\in\Theta^2} \left\{ \begin{array}{c} t_1(\tilde{\phi}) + t_2(\tilde{\phi}) - \psi(\overline{\beta} - C_1(\tilde{\phi})) - \psi(\overline{\beta} - C_2(\tilde{\phi})) \\ -\epsilon\frac{\nu}{1-\nu}\Phi(\overline{\beta} - C_1(\tilde{\phi})) - \epsilon\frac{\nu}{1-\nu}\Phi(\overline{\beta} - C_2(\tilde{\phi})) \end{array} \right\}.$$

The above conditions are equivalent to the coalition incentive constraints stated in Proposition 3.

Appendix 2

As ϵ becomes larger, the two coalition incentive constraints in (RP) become more relaxed. Since we maximize the principal's payoff, we focus on the Sup of her payoff and therefore choose $\epsilon = 1$ in (RP). We introduce the following notation for simplicity:

$$\begin{aligned} t(\underline{\beta}, \underline{\beta}) &= \underline{t}; t(\underline{\beta}, \overline{\beta}) = t(\overline{\beta}, \underline{\beta}) = \widehat{t}; t(\overline{\beta}, \overline{\beta}) = \overline{t}; \\ e(\underline{\beta}, \underline{\beta}) &= \underline{e}; e_1(\underline{\beta}, \overline{\beta}) = e_2(\overline{\beta}, \underline{\beta}) = \widehat{e}_1; \\ e_1(\overline{\beta}, \underline{\beta}) &= e_2(\underline{\beta}, \overline{\beta}) = \widehat{e}_2, e(\overline{\beta}, \overline{\beta}) = \overline{e}. \end{aligned}$$

1. First, it is clear that (BIR_L) is satisfied when (BIC_L) and (BIR_H) are binding. Using (BIC_L), we can easily show that a sufficient condition for (BIC_H) to be satisfied is $\underline{e} + \Delta\beta \geq \widehat{e}_2$ and $\widehat{e}_1 + \Delta\beta \geq \overline{e}$. Second, the implications among the coalition incentive compatibility constraints are as follows. When $(CIC_{LH,LL})$ and $(CIC_{HH,LH})$ are binding, $(CIC_{LL,LH})$ is satisfied if $\underline{e}+\Delta\beta \geq \widehat{e}_2$, $(CIC_{LH,HH})$ is satisfied if $\widehat{e}_1 + \Delta\beta \geq \overline{e}$, $(CIC_{HH,LL})$ is satisfied if $\underline{e} \geq \widehat{e}_1$. When $(CIC_{LL,LH})$ and $(CIC_{LH,HH})$ are satisfied, $(CIC_{LL,HH})$ is satisfied if $\widehat{e}_2 \geq \overline{e}$. $(CIC_{LH,HL})$ is satisfied if $\widehat{e}_1 + \Delta\beta \geq \widehat{e}_2$. Last, $\underline{e} + \Delta\beta \geq \widehat{e}_1 + \Delta\beta \geq \widehat{e}_2 \geq \overline{e}$ is equivalent to the monotonicity condition on the costs in Proposition 6.

2. As all the constraints in (RP) are qualified, we can define the Lagrangian related to (RP) as follows:

$$L = SW+\mu_1(BIC_L)+\mu_2(BIR_H)+\mu_3(CIC_{LH,LL}(\epsilon = 1))+\mu_4 g_4(CIC_{HH,LH}(\epsilon = 1)).$$

When $\psi(e) = \frac{1}{2}e^2$, from the first order conditions with respect to transfers, we have:

$$\mu_2 = 2\lambda,\ \mu_3 = -\lambda\nu^2 + \frac{\nu}{2}\mu_1,\ \mu_4 = -\lambda\nu(1-\nu) + \frac{1-\nu}{2}\mu_1.$$

Obviously, $\mu_2 > 0$. If we know the value of μ_1, we can compute the other multipliers. If μ_1 is equal to zero, μ_3 and μ_4 become strictly negative. This is impossible since $\mu_i \geq 0$ for all i. Thus we have $\mu_1 > 0$. If the first and the second constraints are binding, there should be at least one other binding constraint: this is so because we supposed from the beginning the case in which the principal cannot implement the optimal grand-mechanism without side-contracting in a collusion-proof way. If $\mu_3 = 0$, we have $\mu_4 = 0$. If $\mu_4 = 0$, we have $\mu_3 = 0$. In both cases, the optimal grand-mechanism without side-contracting can be implemented. Thus, we must have $\mu_3 > 0$ and $\mu_4 > 0$.

The optimal effort schedule $\{e_i^{**}(\beta_1, \beta_2)\}$ is obtained from the first order conditions. In particular, we have:

$$\begin{aligned} \underline{e}^{**} &= 1 + \frac{\mu_3}{2\nu^2(1-\nu)(1+\lambda)}\Delta\beta > \underline{e}^*, \\ \overline{e}^{**} &= 1 - \frac{\lambda\nu\Delta\beta}{(1+\lambda)(1-\nu)} - \frac{\mu_3\Delta\beta}{(1+\lambda)\nu(1-\nu)} - \frac{\nu\mu_4\Delta\beta}{(1+\lambda)(1-\nu)^3} < \overline{e}^*. \end{aligned}$$

Appendix 3

It is enough to provide an example in which the principal can implement a given cost schedule with smaller transfers when she lets collusion occur than when she uses collusion-proof mechanisms. Consider the case in which $\nu = \frac{1}{2}$, $\underline{\beta} > 1$, $\Delta\beta = \frac{6}{10}$, $\psi(\cdot) = \frac{e^2}{2}$ hold. Suppose that the principal offers the following mechanism denoted by M^c:

$$\begin{aligned} t^c(\underline{\beta}, \underline{\beta}) &= \frac{116}{100}; t^c(\underline{\beta}, \overline{\beta}) = \frac{44}{100}; t^c(\overline{\beta}, \overline{\beta}) = \frac{20}{100}; \\ C(\underline{\beta}, \underline{\beta}) &= C_1(\underline{\beta}, \overline{\beta}) \equiv \underline{C} = \underline{\beta} - 1, C_2(\underline{\beta}, \overline{\beta}) = C(\overline{\beta}, \overline{\beta}) \equiv \overline{C} = \overline{\beta} - \frac{8}{10}. \end{aligned}$$

Then, (BIR_H) and (BIC_L) are binding in the absence of collusion and room for collusion exists only for the manipulation of reports from $\left[\left(\underline{\beta}, \overline{\beta}\right) \text{ or } \left(\overline{\beta}, \underline{\beta}\right)\right]$ to $\left(\underline{\beta}, \underline{\beta}\right)$. Let S' denote the side-contract in which the agents manipulate reports from $\left[\left(\underline{\beta}, \overline{\beta}\right) \text{ or } \left(\overline{\beta}, \underline{\beta}\right)\right]$ to $\left(\underline{\beta}, \underline{\beta}\right)$ with the side-transfer $\widehat{y} = \frac{24}{100}$ from a low-cost type to a high-cost type. S' allows the third-party to implement the manipulation of reports under asymmetric information. Therefore, if M^c is used, $\underline{C}$ will be realized by each agent when $(\beta_1, \beta_2) \neq (\overline{\beta}, \overline{\beta})$ and $\overline{C}$ will be realized by each agent when $(\beta_1, \beta_2) = (\overline{\beta}, \overline{\beta})$.

Consider now the case in which the principal implements in a collusion-proof way the cost schedule implemented by $M^c \circ S'$. In this case, (BIR_H) and $(CIC_{LH,HH})$ are binding and the optimal transfers are given by: $t^n(\underline{\beta}, \underline{\beta}) = \frac{116}{100}$,

$t^n(\overline{\beta},\overline{\beta}) = \frac{44}{100}$. Since we have $t^c(\underline{\beta},\underline{\beta}) = t^n(\underline{\beta},\underline{\beta})$ and $t^c(\overline{\beta},\overline{\beta}) < t^n(\overline{\beta},\overline{\beta})$, the total expected transfer to implement the above cost schedule is lower when the principal lets collusion occur than when she uses collusion-proof mechanisms.