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Abstract

The traditional theory of second-degree price discrimination tackles individual self-selection but does not address the possibility that buyers could form a coalition to conduct arbitrage, that is, to coordinate their purchases and to reallocate the goods. In this paper, we design the optimal sale mechanism which takes into account both individual and coalition incentive compatibility when buyers can form a coalition under asymmetric information. We show that the monopolist can achieve the same profit regardless of whether or not buyers can form a coalition. Although marginal rates of substitution are not equalized across buyers of different types in the optimal sale mechanism (hence there exists potential room for arbitrage), they fail to realize the gains from arbitrage because of the transaction costs in coalition formation generated by asymmetric information.

JEL Classification: D42, D82, L12

Key Words: Second-degree Price Discrimination, Arbitrage, Coalition Incentive Compatibility, Asymmetric Information, Transaction Costs.

1 Introduction

The theory of monopolistic screening[1] (second-degree price discrimination) studies a monopolist's optimal pricing scheme when she has incomplete information about buyers' individual preferences.[2] According to the theory, the monopolist can maximize her profit by using a menu of options which induces each type of buyer to select the option designed for the type. While the theory tackles the self-selection issue at the individual level, it assumes away the possibility that price discrimination might induce buyers to form coalitions to conduct arbitrage, that is, to coordinate their purchases and to reallocate the goods they bought among themselves. Since this might reduce the seller's profit, in this paper we study the optimal sales mechanism that takes into account not only individual incentive compatibility but also coalition incentive compatibility (i.e., buyers' incentive to collectively engage in arbitrage). In particular, in addressing this fundamental problem, we focus on the role of asymmetric information among buyers about each other's preferences.

In reality, there exists much evidence of (legal or illegal) coalitions among buyers. On the one hand, bidders' collusive behavior in auctions is well documented and auction literature has been devoting an increasing attention to the topic.[3] On the other hand, buyers often form cooperatives to jointly purchase goods.[4] One central question regarding buyer coalitions is how asymmetric information among the buyers affects coalition formation. Our major goal is to identify the transaction costs in coalition formation generated by asymmetric information and to find the sales mechanism which best exploits these transaction costs.

Consider for example the situation in which an upstream monopolist sells her goods to two downstream firms operating in separate markets. Given a menu of quantity-transfer pairs offered by the monopolist, the two downstream firms can employ two instruments to increase their joint payoffs. First, they can jointly decide which pair

[1]See, for instance, Maskin and Riley (1984) and Mussa and Rosen (1978) for an introduction and Rochet and Stole (2002) for a recent contribution dealing with random participation.

[2]We use 'she' to represent the monopolist and 'he' to represent a buyer or the third-party.

[3]For examples, see Caillaud and Jehiel (1998), Graham and Marshall (1987), McAfee and McMillan (1992) and Brusco and Lopomo (2002).

[4]There exist various forms of supply cooperatives to purchase some products together. For instance, Heflebower (1980) describes three types of supply cooperatives: farmers's cooperatives, consumer co-operatives and those run by urban businesses.

each buyer should choose. In our paper, this is modeled by manipulation of the reports which the buyers report to the sales mechanism. Second, they can reallocate among themselves the goods bought from the seller. We first show that under the standard optimal mechanism which neglects coalition incentive compatibility, buyers can increase their payoffs by engaging in arbitrage and this reduces the seller's profit. However, in our main result, we find an optimal mechanism which allows the monopolist to realize the same profit regardless of whether or not buyers can form a coalition to do arbitrage.

Consider for simplicity a two-buyer setting and suppose that the seller can produce any amount of a homogeneous product at a constant marginal cost and a buyer has either high valuation (H-type) or low valuation (L-type) for the product. Assume that types are independently and identically distributed and a buyer's type is his private information. It is well-known that in the optimal mechanism(s) without a buyer coalition, the quantity allocated to an H-type is equal to the first-best level while the quantity allocated to an L-type is distorted downward compared to the first-best level since the payment the seller receives from an H-type decreases in the quantity sold to an L-type. This implies that an L-type has a higher marginal surplus for the product than an H-type and, if there are no transaction costs in coalition formation, buyers can increase their payoffs by reallocating some quantity from an H-type to an L-type (with a suitable money transfer from the latter to the former) in the state of nature in which one buyer has an H-type and the other has an L-type. This may alter ex ante buyers' incentives to report truthfully and reduce the seller's expected profit.

Drawing on Laffont and Martimort (1997, 2000), we model coalition formation under asymmetric information by a side-contract offered to the buyers by a third-party who maximizes the sum of buyers' payoffs. The side-contract specifies both the manipulation of the reports made into the sale mechanism and the reallocation of the goods obtained from the seller. The side-contract must satisfy budget balance, participation and incentive constraints. The incentive constraints need to hold since the third-party does not know the buyers' types; the acceptance constraints are defined with respect to the utilities the buyers obtain when playing the sale mechanism non-cooperatively.

We first consider *simple* mechanisms in which both the quantity that a buyer receives and his payment do not depend on the other buyer's report. We show that if the seller uses the simple mechanism which is optimal without buyer coalition, buyers can realize strict gains at the seller's loss through arbitrage. For instance, when the both buyers have an H-type (HH-coalition) they have an incentive to report HL instead of truthtelling and to reallocate quantities and transfers. To see this, note that under the optimal *simple* mechanism, an H-type is indifferent between the quantity-transfer pair designed for an H-type and the pair for an L-type. This implies that if reallocation is impossible, an HH-coalition is indifferent between reporting HL and truth-telling. However, if reallocation is feasible, under standard convexity assumptions on buyers' preferences, each buyer's payoff conditional on reporting HL strictly increases since they can share equally the total quantity and transfers. In contrast, conditional on reporting HH, reallocation does not affect the payoffs since both buyers receive the same quantity from the seller. Therefore, an HH-coalition prefers to report HL rather than HH.

After studying simple mechanisms, we consider the mechanisms in which the seller makes the payment of a buyer depend on the report of the other buyer: in the mechanisms, the quantity profile and the buyers' expected payments are equal to the ones in the simple optimal mechanism. It turns out that there exists a transfer scheme which allows the seller to deter manipulation of reports and reallocation of goods at no cost, thus letting her realize the same profit as when there is no buyer coalition. In particular, even if the marginal rates of substitution are not equalized across buyers with different types, the third party is not able to implement any efficient reallocation between an H-type and an L-type in an HL-coalition because of the tension between incentive and participation constraints in the side-contract. The intuition for this result is as follows. Since the rent that an H-type obtains by pretending to be an L-type in the side mechanism increases in the quantity received by an L-type, if the third-party reallocates some quantity from an H-type to an L-type then he is forced to concede an H-type a higher rent in order to elicit a truthful report: the alternative of reducing an L-type's payoff is impossible since it would induce the L-type to reject the side-contract. This increase in the rent is defined as the transaction costs generated by asymmetric information. We quantify the transaction costs and show that they are larger than the gains from reallocating quantity from an H-type to an L-type; therefore the reallocation cannot be realized. We also show that this optimal outcome can be implemented by a menu of two-part tariffs. Finally, our main result that buyer coalition does not hurt the seller extends to more general settings: when the marginal cost is increasing, or there are n buyers, or there are three possible buyer types.

The literature about consumer coalitions mostly addresses issues different from the one we consider in this paper.[5] Alger (1999) is one exception: She studies the optimal menu of price-quantity pairs when (a continuum of) consumers are able to purchase multiple times and/or jointly in a two-type setting. She finds that with multiple purchases only, the monopolist offers strict quantity discounts while, with joint purchases only, discounts are infeasible. Her results are based on two following assumptions. First, consumer coalitions are formed under complete information among the consumers about each other's type and only consumers with the same type can form coalitions. Second, the set of mechanisms available to the seller is restricted by assuming that the quantity allocated to a consumer and his payment do not depend on the other consumers' choices. In contrast, in our model a coalition is formed under asymmetric information among buyers and the seller can use complete contracts such that the quantity sold to a buyer and his payment can depend on the others' choices.

Using a third-party to model collusion under asymmetric information was first introduced in auction literature – see the first three papers mentioned in footnote 3. While that literature studies the optimal auction in a restricted set of mechanisms, usually finding the optimal reserve price for a first or second price auction, Laffont and Martimort (1997, 2000) use a more general approach in that they characterize the set of collusion-proof mechanisms and optimize in this set. In their settings reallocation is infeasible[6] and they show that if the agents' types are independently distributed, then a dominant-strategy mechanism implements the second-best outcome and eliminates any gain from a joint manipulation of reports. Furthermore, this mechanism does not exploit the transaction costs created by asymmetric information. In our setting, the dominant-strategy mechanism is not collusion-proof since the coalition has the additional instrument of quantity reallocation, but the seller can still achieve the second-best profit by fully exploiting the transaction costs in coalition formation. We also note that Laffont

[5] For instance, Innes and Sexton (1993, 1994) analyze the case in which the monopolist is facing identical consumers who may form coalitions. They show that even though consumers' characteristics are homogeneous, the monopolist may price discriminate in order to deter the formation of coalitions, whereas price discrimination is unprofitable in the absence of the coalitions.

[6] In the first paper, they consider two regulated firms producing complementary inputs. The firms have independently distributed types and collusion has bite since an exogenous restriction on the set of the principal's mechanisms is imposed. In the second paper, they consider collusion between consumers of a public good with correlated types. Consumers have incentives to collude since the principal will fully extract their rents if they behave non-cooperatively.

and Martimort limit the analysis to the two-agent-two-type setting and do not consider implementation through indirect mechanisms.

Our paper is to some extent related to the papers studying auctions with resale. For instance, Ausubel and Cramton (1999) analyze the optimal auction when buyers can engage in resale after receiving goods from the seller and the resale is (assumed to be) always efficient. They prove that the seller maximizes his profit by allocating goods efficiently. In contrast, in our setting, buyers sign a binding side-contract before each buyer chooses how much to buy and they fail to achieve efficient reallocation because of the transaction costs.[7]

The rest of the paper is organized as follows. In Section 2, we introduce the model. In Section 3, we review as a benchmark the optimal sale mechanisms without buyer coalition and in Section 4 we prove that the simple optimal mechanism in which each buyer's allocation depends only on his own report leaves room for arbitrage such that buyer coalition reduces the seller's profit. In Section 5, we show however that the seller's profit is not reduced by arbitrage if she designs appropriately the sale mechanism – even though potential room for arbitrage is left. In Section 6, we extend the main result to more general settings. Concluding remarks are given in Section 7. The proofs of our first four propositions are provided in the appendix. The proofs of the other propositions are similar to those of Proposition 1 or 4 but they are considerably longer; therefore, they are omitted and can be found in Jeon and Menicucci (2002).

2 The model

2.1 Preferences, information and mechanisms

A seller (for instance, an upstream monopolist) can produce any amount $q \geq 0$ of homogeneous goods at cost $C(q)$ and sells the goods to $n \geq 2$ buyers (for instance, downstream firms operating in separate markets). Throughout the paper, we will interpret q as quantity except in Section 4, where we consider also the case in which q represents quality. Buyer i ($i = 1, ..., n$) obtains payoff $\mathcal{U}(q^i, \theta^i) - t^i$ from consuming quantity $q^i \geq 0$ of the goods and paying $t^i \in \mathbb{R}$ units of money to the seller. He privately observes his own

[7] Zheng (2002) allows resale in a one-good auction with asymmetrically distributed buyers' values and proves that an equilibrium exists which induces the same payoffs as if resale can be costlessly banned.

type $\theta^i \in \Theta \equiv \{\theta_L, \theta_H\}$, where $\Delta\theta \equiv \theta_H - \theta_L > 0$. The types θ^i and θ^j are identically and independently distributed for any $i \neq j$, with $p_L \equiv \Pr\left\{\theta^i = \theta_L\right\} \in (0,1)$ for $i = 1, ..., n$; the distribution of $(\theta^1, ..., \theta^n)$ is common knowledge. We suppose that $C(\cdot)$ and $\mathcal{U}(\cdot)$ are such that $C(0) = 0$, $C'(q) > 0$ and $C''(q) \geq 0$ for any $q \geq 0$; $\mathcal{U}(0, \theta) = 0$, $\mathcal{U}_1(q, \theta) > 0 > \mathcal{U}_{11}(q, \theta), \mathcal{U}_2(q, \theta) > 0$ and $\mathcal{U}_{12}(q, \theta) > 0$ for any (q, θ), where subscript denote partial derivatives and $\mathcal{U}_{12}(q, \theta) > 0$ is the standard Spence-Mirrlees single-crossing condition. Furthermore, we assume that $\frac{\mathcal{U}_1(0,\theta_L)}{p_L} - \frac{(1-p_L)\mathcal{U}_1(0,\theta_H)}{p_L} > C'(0)$ and that p_L is sufficiently large to make $\mathcal{U}(q, \theta_L) - (1 - p_L)\mathcal{U}(q, \theta_H)$ a concave function of q. While the first inequality guarantees that each type receives a positive quantity in the optimum without buyer coalition,[8] the second condition implies that the problem under collusion is well-behaved. The reservation utility of buyer i is given by 0, his payoff if he does not transact with the seller (i.e., if $(q^i, t^i) = (0, 0)$).

In what follows, for expositional simplicity, we focus on the case with $n = 2$, constant marginal cost $c(> 0)$ and $\mathcal{U}(q, \theta) = \theta u(q)$; in this case, the inequality $\frac{\mathcal{U}_1(0,\theta_L)}{p_L} - \frac{(1-p_L)\mathcal{U}_1(0,\theta_H)}{p_L} > c$ implies that $\mathcal{U}(q, \theta_L) - (1 - p_L)\mathcal{U}(q, \theta_H)$ is concave in q. However, our main result holds for any $n > 2$, any convex cost function and any $\mathcal{U}(q, \theta)$ with the properties described above and it also holds in the three-type setting with $\Theta \equiv \{\theta_L, \theta_M, \theta_H\}$. See Section 6 for all extensions.

The seller designs a sale mechanism to maximize her expected profit. A generic sale mechanism is denoted by M and, according to the revelation principle, we can restrict our attention to direct revelation mechanisms:

$$M = \left\{ q^i(\widehat{\theta}^1, \widehat{\theta}^2), t^i(\widehat{\theta}^1, \widehat{\theta}^2);\ i = 1, 2 \right\},$$

where $\widehat{\theta}^i \in \{\theta_L, \theta_H\}$ is buyer i's report, $q^i(\cdot)$ is the quantity he receives and $t^i(\cdot)$ is his payment to the seller. Since buyers are ex ante identical, without loss of generality we focus on symmetric mechanisms in which the quantity sold to a buyer and his payment depend only on the reports $(\widehat{\theta}^1, \widehat{\theta}^2)$ and not on his identity. Then, we can introduce the following notation to simplify the exposition: For quantities,

$$\begin{aligned} q_{HH} &= q^1(\theta_H, \theta_H) = q^2(\theta_H, \theta_H),\ q_{HL} = q^1(\theta_H, \theta_L) = q^2(\theta_L, \theta_H), \\ q_{LH} &= q^1(\theta_L, \theta_H) = q^2(\theta_H, \theta_L),\ q_{LL} = q^1(\theta_L, \theta_L) = q^2(\theta_L, \theta_L). \end{aligned}$$

[8] Our results below holds even when the seller finds it optimal to refuse to serve L-type.

$(t_{HH}, t_{HL}, t_{LH}, t_{LL}) \in \mathbb{R}^4$ are similarly defined. Let $\mathbf{q} \equiv (q_{HH}, q_{HL}, q_{LH}, q_{LL})$ denote the vector of quantities and $\mathbf{t} \equiv (t_{HH}, t_{HL}, t_{LH}, t_{LL})$ denote the vector of transfers.

The sale mechanisms we consider involve (second-degree) price discrimination. Although price discrimination can be illegal if it threatens to injure competition[9], in our context there is no such concern since the buyers operate in separate markets.

2.2 Buyer coalition

Drawing on Laffont and Martimort (1997, 2000), we model the buyers' coalition formation by a side-contract, denoted by S, offered by a benevolent third-party. The third party designs S in order to maximize the sum of buyers' expected payoffs subject to incentive compatibility (since he does not observe the types) and participation constraints written with respect to the utility a buyer obtains when M is played non-cooperatively.

We assume that the seller is the first mover and can commit not to serve a buyer if the other buyer refuses M. This limits the strategies available to the buyer coalition: in particular, the third-party cannot employ the strategy of making only one buyer buy from the seller and share the goods bought with the other buyer.[10] Precisely, the game of seller's mechanism offer cum buyer coalition formation has the following timing.

Stage 1. Nature draws buyers' types (θ^1, θ^2); buyer i privately observes θ^i, $i = 1, 2$.

Stage 2. The seller proposes a sale mechanism M.

Stage 3. Each buyer simultaneously accepts or rejects M. If at least one buyer refuses M, then each buyer realizes the reservation utility and the following stages do not occur.

Stage 4. If both buyers accept to play M, then the third party proposes a direct side-contract S that jointly manipulates their reports into M and reallocates between them the goods bought from the seller.[11]

[9] This is the purpose of the Robinson-Patman Act. Price discriminations are widely practiced as can be seen from Clark (1998)'s review of the Antitrust cases related to price discriminations.

[10] Alternatively, we may assume that if buyer 1 (say) does not accept M, then the seller can serve buyer 2 with a single-buyer mechanism. In this case, our results would still hold if the seller can observe whether or not a buyer uses her goods as in Rey and Tirole (1986). Since then the seller can induce buyer 2 not to resell to buyer 1 (part of) the goods he bought from the seller by specifying ex ante a high penalty for buyer 2, both buyers will buy from the seller in equilibrium.

[11] Actually, the Revelation Principle applies to the third-party's design of S but not to the seller's design of M. Thus, the seller may wish to propose non-direct sale mechanisms. Nevertheless, as Proposition 3 in Laffont and Martimort (2000) establishes, any perfect Bayesian equilibrium outcome

Stage 5. Each buyer simultaneously accepts or rejects S.

Stage 6. If at least one buyer refuses S, then M is played non-cooperatively. In this case, reports are directly made in M and stages 7 and 9 below do not occur. If instead S has been accepted by both buyers, then reports are made into S.

Stage 7. As a function of the reports in S, the third party enforces the manipulation of reports into M.

Stage 8. Quantities and transfers specified in M are enforced.

Stage 9. Quantity reallocation and side-transfers specified in S (if any) take place in the buyer coalition.

Formally, a side-contract S takes the following form:

$$S = \{\phi(\widetilde{\theta}^1, \widetilde{\theta}^2), x^i(\widetilde{\theta}^1, \widetilde{\theta}^2, \widetilde{\phi}), y^i(\widetilde{\theta}^1, \widetilde{\theta}^2);\ i = 1, 2\},$$

where $\widetilde{\theta}^i \in \{\theta_L, \theta_H\}$ is buyer i's report to the third-party. $\phi(\cdot)$ is the report manipulation function which maps any pair of reports $(\widetilde{\theta}^1, \widetilde{\theta}^2)$ made by the buyers to the third-party into a pair of reports to the seller. We assume that $\phi(\cdot)$ can specify stochastic manipulations, as this convexifies the third-party's feasible set. More precisely, let $\widetilde{\phi} \in \Theta^2$ denote an outcome of $\phi(\cdot)$. Then, $\phi(\cdot)$ specifies the probability $p^{\phi}(\widetilde{\theta}^1, \widetilde{\theta}^2, \widetilde{\phi})$ that the third party, after receiving reports $(\widetilde{\theta}^1, \widetilde{\theta}^2)$, requires the buyers to report $\widetilde{\phi}$ to the seller. When the manipulation is deterministic, i.e., $p^{\phi}(\widetilde{\theta}^1, \widetilde{\theta}^2, \widetilde{\phi}) = 1$ for a $\widetilde{\phi} \in \Theta^2$, we write $\phi(\widetilde{\theta}^1, \widetilde{\theta}^2) = \widetilde{\phi}$ with some abuse of notation.

After the buyers have bought goods from the seller, the third-party can reallocate them within the coalition. Let $x^i(\widetilde{\theta}^1, \widetilde{\theta}^2, \widetilde{\phi})$ represent the quantity of goods that buyer i receives from the third-party when $\widetilde{\phi}$ is reported to the seller. Finally, $y^i(\widetilde{\theta}^1, \widetilde{\theta}^2)$ denotes the monetary transfer from buyer i to the third-party; y^i does not need to depend on $\widetilde{\phi}$ because of quasi linearity of a buyer's payoff in money. Since we assume that the third party is not a source of goods or money, a side-contract should satisfy the ex post budget balance constraints for the reallocation of goods and for the side transfers:

$$\sum_{i=1}^{2} x^i(\theta^1, \theta^2, \widetilde{\phi}) = 0 \quad \text{and} \quad \sum_{i=1}^{2} y^i(\theta^1, \theta^2) = 0, \quad \text{for any } (\theta^1, \theta^2) \in \Theta^2 \text{ and any } \widetilde{\phi} \in \Theta^2.$$

arising from a non-direct sale mechanism can be obtained as a perfect Bayesian equilibrium outcome induced by a direct sale mechanism.

After a side-contract S is proposed, at stage 5 each buyer accepts or rejects S; at stage 6, the buyers report types either into M or into S depending on their decisions at stage 5. We are interested in (collusive continuation) equilibria in which both buyers accept S; thus, no learning about types occurs along the equilibrium path.[12] However, in order to decide whether to accept or reject S at stage 5, buyers need to have some beliefs on what is going to happen off-the-equilibrium-path. As in Laffont and Martimort (1997, 2000) we make the following assumption[13]:

$$\left\{ \begin{array}{l} \textbf{Assumption WCP:} \text{ Given an incentive compatible } M\text{, if buyer } i \\ \text{vetoes } S \text{ (an off-the-equilibrium-path event) then the other buyer still} \\ \text{has prior beliefs about } \theta^i \text{ and the truthful equilibrium is played in } M. \end{array} \right.$$

We let $U^M(\theta_j)$ $(j = L, H)$ denote the expected payoff of a j-type in the truthful equilibrium of M; thus $U^M(\theta_j)$ is the reservation utility for a j-type when deciding whether to accept S or not.

This coalition formation model may appear unrealistic as it may seem more natural to model coalition by considering a specific bargaining model. However, we point out an important property of the model we analyze: The revelation principle implies that, given a specific bargaining game G, any allocation achieved by a Bayesian equilibrium of G can be obtained by a side-contract offered by the third party. Since we let the third party maximize the sum of buyers' expected payoffs, we are describing the upper bound of what the coalition may achieve under asymmetric information. Furthermore, since we show that collusion does not hurt the seller, the property implies that specifying any particular bargaining game between the buyers would not change our main message.

3 The optimal mechanisms without buyer coalition

In this section, we characterize the profit maximizing mechanisms when there is no buyer coalition. The seller's expected profit with mechanism $M = \{\mathbf{q}, \mathbf{t}\}$ is

$$\Pi \equiv 2p_L^2(t_{LL} - cq_{LL}) + 2p_L(1-p_L)(t_{HL} + t_{LH} - cq_{HL} - cq_{LH}) + 2(1-p_L)^2(t_{HH} - cq_{HH})$$

[12] Notice, however, that there also exists an equilibrium in which both buyers refuse any side mechanism: If buyer i is vetoing any side mechanism, then rejecting is a best reply for buyer j.

[13] WCP means weakly collusion-proof. In Jeon and Menicucci (2002), we prove that our results are robust to removing assumption WCP.

M should satisfy the following Bayesian incentive compatibility constraints: for an H-type,

$$\begin{aligned}(BIC_H) \quad & p_L[\theta_H u(q_{HL}) - t_{HL}] + (1-p_L)[\theta_H u(q_{HH}) - t_{HH}] \\ & \geq p_L[\theta_H u(q_{LL}) - t_{LL}] + (1-p_L)[\theta_H u(q_{LH}) - t_{LH}];\end{aligned} \tag{1}$$

for an L-type,

$$\begin{aligned}(BIC_L) \quad & p_L[\theta_L u(q_{LL}) - t_{LL}] + (1-p_L)[\theta_L u(q_{LH}) - t_{LH}] \\ & \geq p_L[\theta_L u(q_{HL}) - t_{HL}] + (1-p_L)[\theta_L u(q_{HH}) - t_{HH}].\end{aligned} \tag{2}$$

M should also satisfy the following individual rationality constraints: for an H-type and an L-type, respectively

$$(BIR_H) \quad p_L[\theta_H u(q_{HL}) - t_{HL}] + (1-p_L)[\theta_H u(q_{HH}) - t_{HH}] \;\geq\; 0; \tag{3}$$

$$(BIR_L) \quad p_L[\theta_L u(q_{LL}) - t_{LL}] + (1-p_L)[\theta_L u(q_{LH}) - t_{LH}] \;\geq\; 0. \tag{4}$$

The seller designs M to maximize Π subject to (1) to (4). We characterize the optimal mechanisms in the next proposition:

Proposition 1 *The optimal mechanisms in the absence of buyer coalition are characterized as follows.*
(a) The optimal quantity schedule $\mathbf{q}^* = (q^*_{HH}, q^*_{HL}, q^*_{LH}, q^*_{LL})$ *is given by:*
(i) $q^*_{HH} = q^*_{HL} = q^*_H$, *where* $\theta_H u'(q^*_H) = c$;
(ii) $q^*_{LH} = q^*_{LL} = q^*_L$, *where* $(\theta_L - \frac{1-p_L}{p_L}\Delta\theta)u'(q^*_L) = c$.
(b) Transfers are such that the constraints (BIC_H) *and* (BIR_L) *are binding.*

In Proposition 1, q^*_H (q^*_L) is the optimal quantity allocated to an H-type (an L-type), when the seller faces a single buyer. Thus, Proposition 1 states that, in the optimal mechanisms for the two-buyer case, the quantity obtained by a buyer is equal to the quantity he would receive in the one-buyer setting, independently of the report of the other buyer. In the one-buyer case, it is well known that the payment the seller obtains from an H-type is decreasing in the quantity received by an L-type because of (BIC_H). This induces the seller to evaluate an L-type's surplus with the so-called virtual valuation $\theta^v_L \equiv \theta_L - \frac{1-p_L}{p_L}\Delta\theta < \theta_L$ instead of θ_L, and therefore to distort the quantity allocated to an L-type below the first-best level since she equalizes the L-type's marginal virtual surplus to marginal cost.

The facts that $q^*_{HH} = q^*_{HL} = q^*_H$, $q^*_{LH} = q^*_{LL} = q^*_L$ and (BIC_H), (BIR_L) bind imply that the expected payments of an L-type and an H-type, $\bar{t}_L \equiv p_L t_{LL} + (1 - p_L)t_{LH}$ and $\bar{t}_H \equiv p_L t_{HL} + (1-p_L)t_{HH}$ respectively, are equal to the payments of the two types in the one-buyer setting: $\bar{t}_L = t^*_L \equiv \theta_L u(q^*_L)$ and $\bar{t}_H = t^*_H \equiv \theta_H u(q^*_H) - (\Delta\theta)u(q^*_L)$. The seller has two degrees of freedom in the choice of transfers to satisfy $\bar{t}_L = t^*_L$ and $\bar{t}_H = t^*_H$. For instance, she can set $t_{LL} = t_{LH} = t^*_L$ and $t_{HL} = t_{HH} = t^*_H$, so that each buyer's payment does not depend on the other buyer's report. In what follows, we let $M^d \equiv \left\{\mathbf{q}^*, \mathbf{t}^d\right\}$ where $t^d_{LL} = t^d_{LH} = t^*_L$ and $t^d_{HL} = t^d_{HH} = t^*_H$. In M^d, truthtelling is a dominant strategy since each buyer's payoff depends only on his own report. Basically, with M^d the seller maximizes her profit by dealing with each buyer separately.

A simple intuition sheds light on the close relation between the optimal mechanism in one-buyer case and the ones in two-buyer case.[14] If there exists a mechanism $\{\mathbf{q}', \mathbf{t}'\}$ which is strictly better than the mechanisms characterized by Proposition 1, then we can find a menu of two (possibly stochastic) contracts[15] which is strictly better than (q^*_H, t^*_H) and (q^*_L, t^*_L) for the single-buyer model. However, this is impossible by definition.

Last, we make an obvious (but important) observation about the optimal mechanisms in the absence of buyer coalition.

Observation: In any optimal sale mechanism without a buyer coalition, an HL-coalition can increase its payoff by reallocating some quantity from an H-type to an L-type in the absence of transaction costs.

Since $\theta_H u'(q^*_H) = (\theta_L - \frac{1-p_L}{p_L}\Delta\theta)u'(q^*_L) = c$ implies that an L-type's marginal utility for goods is strictly larger than an H-type's, an HL-coalition has an incentive to reallocate some quantity from an H-type to an L-type if there exists no transaction costs in coalition formation. We emphasize that this incentive exists because the seller reduces the quantity consumed by an L-type below the socially efficient level in order to extract more rent from an H-type. In contrast, if she observed (θ^1, θ^2), there would be no room for arbitrage since the first-best quantity schedule (q^{FB}_H, q^{FB}_L) is such that

[14] We thank the editor Raymond Deneckere for pointing this out to us.

[15] The menu is such that conditional on the report of θ_H (θ_L), the buyer receives quantity q'_{HL} (q'_{LL}) with probability p_L and q'_{HH} (q'_{LH}) with probability $1 - p_L$ and pays $p_L t'_{HL} + (1 - p_L)t'_{HH}$ $(p_L t'_{LL} + (1 - p_L)t'_{LH})$.

$\theta_H u'(q_H^{FB}) = \theta_L u'(q_L^{FB}) = c.$

4 Coalition formation under asymmetric information

In this section we first introduce formally the third party's problem and then show that M^d characterized above leaves room for arbitrage, in the sense that buyers can increase their payoffs by manipulating reports and reallocating goods, at the expenses of the seller. Therefore, this section provides a motivation to look for a mechanism which performs better than M^d in the presence of buyer coalition, the issue that we will address in the next section.

Let $p(\theta^1, \theta^2)$ (respectively, $p(\theta^i)$ with $i = 1, 2$) denote the probability of having $(\theta^1, \theta^2) \in \Theta^2$ (respectively, the probability of having $\theta^i \in \Theta$). We recall that $p^{\phi}(\widetilde{\theta}^1, \widetilde{\theta}^2, \widetilde{\phi})$ denotes the probability that, after receiving reports $(\widetilde{\theta}^1, \widetilde{\theta}^2)$, the third party requires the buyers to report $\widetilde{\phi} \in \Theta^2$ to the seller. When $\widetilde{\phi}$ is reported to the seller, buyer i receives quantity $q^i(\widetilde{\phi})$ from the seller and pays $t^i(\widetilde{\phi})$ to her.

Definition 1 *A side-contract $S^* = \{\phi^*(\cdot), x^{i*}(\cdot), y^{i*}(\cdot)\}$ is coalition-interim-efficient with respect to an incentive compatible mechanism M providing the reservation utilities $\{U^M(\theta_L), U^M(\theta_H)\}$ if and only if it solves the following program:*

$$\max_{\phi(\cdot), x^i(\cdot), y^i(\cdot)} \sum_{(\theta^1,\theta^2)\in\Theta^2} p(\theta^1, \theta^2)[U^1(\theta^1) + U^2(\theta^2)]$$

subject to

$$U^i(\theta^i) = \sum_{\theta^j\in\Theta} p(\theta^j) \left\{ \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}(\theta^i, \theta^j, \widetilde{\phi})[\theta^i u(q^i(\widetilde{\phi}) + x^i(\theta^i, \theta^j, \widetilde{\phi})) - t^i(\widetilde{\phi})] - y^i(\theta^i, \theta^j) \right\},$$

for any $\theta^i \in \Theta$ and $i, j = 1, 2$ with $i \neq j$;

$$(BIC^S)\ U^i(\theta^i) \geq \sum_{\theta^j\in\Theta} p(\theta^j) \left\{ \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}(\widetilde{\theta}^i, \theta^j, \widetilde{\phi})[\theta^i u(q^i(\widetilde{\phi}) + x^i(\widetilde{\theta}^i, \theta^j, \widetilde{\phi})) - t^i(\widetilde{\phi})] - y^i(\widetilde{\theta}^i, \theta^j) \right\},$$

for any $(\theta^i, \widetilde{\theta}^i) \in \Theta^2$ and $i, j = 1, 2$ with $i \neq j$;

$$(BIR^S)\ U^i(\theta^i) \geq U^M(\theta^i),\ \textit{for any } \theta^i \in \Theta \textit{ and } i = 1, 2;$$

$$(BB\ :\ x)\ x^1(\theta^1, \theta^2, \widetilde{\phi}) + x^2(\theta^1, \theta^2, \widetilde{\phi}) = 0,\ \textit{for any } (\theta^1, \theta^2) \in \Theta^2 \textit{ and any } \widetilde{\phi} \in \Theta^2;$$
$$(BB\ :\ y)\ y^1(\theta^1, \theta^2) + y^2(\theta^1, \theta^2) = 0,\ \textit{for any } (\theta^1, \theta^2) \in \Theta^2.$$

In words, a side-contract is coalition-interim-efficient with respect to M if it maximizes the sum of the buyers' expected utilities subject to incentive, acceptance and budget balance constraints. Let $S^0 \equiv \{\phi(\cdot) = Id(\cdot), x^1(\cdot) = x^2(\cdot) = 0, y^1(\cdot) = y^2(\cdot) = 0\}$ denote the contract which implements no manipulation of reports, no reallocation of quantity and no side-transfer; S^0 is called the *null-side contract* and M is not affected by buyer coalition if the third-party proposes S^0. The next definition refers to this class of mechanisms.

Definition 2 *An incentive compatible mechanism M is weakly*[16] *collusion-proof if S^0 is coalition-interim-efficient with respect to M.*

In the rest of this section, we consider two interpretations of q, quality[17] or quantity, and examine whether or not M^d is weakly collusion-proof in each case. The next proposition states our result:

Proposition 2 *Suppose the seller offers M^d. Then*
(a) when q represents quality, M^d is weakly collusion-proof;
(b) when q represents quantity, there exists a side-contract S^d which increases the payoff of each type of buyer (and reduces the seller's profit) compared to when M^d is played truthfully; in S^d, an HH-coalition reports HL to the seller, an HL-coalition reports LL and then quantities are reallocated within the coalitions.

Consider first the case in which q represents quality and hence reallocation of q is impossible or the case in which q represents quantity but buyers cannot reallocate it (for instance, electricity, gas, water). In these cases, the only instrument of the coalition is manipulation of reports. Then, Proposition 2(a) establishes that M^d is

[16] The qualifier "weakly" comes from our assumption WCP in section 2.2.

[17] For instance, in Mussa and Rosen (1978), q represents quality. Alger (1999) considers both interpretations although she focus on the quantity interpretation.

weakly collusion-proof. This result easily follows from the property that in M^d a buyer's payoff is independent of the other buyer's report and no agent has an individual incentive to report untruthfully since (BIC_H) and (BIC_L) are satisfied. Therefore, the sum of the buyers' payoffs is maximized by truthtelling in every state of nature and the null side-contract satisfies (BIC^S), (BIR^S) and budget balance constraints; thus, S^0 is coalition-interim-efficient. Notice that collusion has no bite even though it occurs under symmetric information among buyers. We note that Laffont and Martimort (1997, 2000) obtain similar findings (Proposition 11 and Proposition 6, respectively) when they show that there exists a dominant-strategy optimal mechanism which eliminates any gain from joint manipulation of reports if the agents' types are independently distributed.

We now turn to the case in which q represents quantity and buyers can manipulate their reports and reallocate quantity. In what follows, for simplicity of discussion, we suppose that buyers have symmetric information at the time of collusion, which is equivalent to saying that the third party does not need to satisfy (BIC^S) or that there are no transaction costs in coalition formation. This simplification is innocuous since the underlying logic holds true even when buyers form the coalition under asymmetric information. One simple way to see why the possibility of reallocation overturns the result of Proposition 2(a) is to notice that actually – when reallocation is infeasible – an HH-coalition (an HL-coalition) is indifferent between truthtelling and reporting HL (LL) under M^d. Since reallocation makes the coalition more powerful, it is quite intuitive that now incentives to manipulate reports exist.

To be more clear, we graphically illustrate the result of Proposition 2(b). In Figure 1, points A and B represent the two quantity-transfer pairs (q_L^*, t_L^*) and (q_H^*, t_H^*) respectively in mechanism M^d. If an HH-coalition reports truthfully, each buyer will achieve B. If it reports HL and reallocates evenly the total quantity and the total payment, each buyer will obtain C, with $q^C = \frac{q_L^*+q_H^*}{2}$ and $t^C = \frac{t_L^*+t_H^*}{2}$. One can easily see from Figure 1 that each H-type strictly prefers C to B since C lies on a better indifference curve than B. Formally, C is preferred to A or B since C is a convex combination of A and B, H-type is indifferent between A and B and his preferences are strictly quasi-convex.

For an HL-coalition, if it reports LL and does not reallocate quantity, each buyer achieves A and obtains the same payoff as with truthtelling. However, since an H-type's marginal surplus for goods is higher than an L-type's one when both receive the same quantity, each buyer can achieve higher payoffs by reallocating some goods from an L-

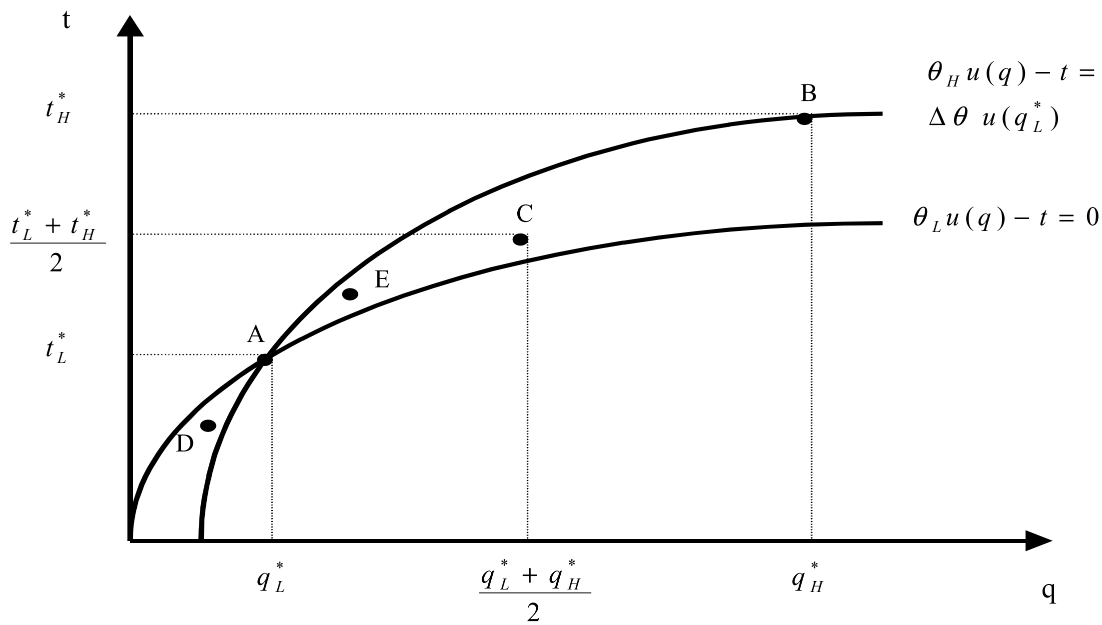

Figure 1: Gains from reallocation under the mechanism M^d

type to an H-type (with an appropriate money transfer from the H-type to the L-type): for instance, they can achieve D for the L-type and E for the H-type.

5 The optimal weakly collusion-proof mechanisms

In this section we present our main finding that the seller's profit is not affected by a buyer coalition. We first introduce a result which allows us to restrict attention to the set of weakly collusion-proof mechanisms.

Proposition 3 (weak collusion-proofness principle) *There is no loss of generality in restricting the seller to offer weakly collusion-proof mechanisms in order to characterize the outcome of any perfect Bayesian equilibrium of the game of seller's mechanism offer cum coalition formation such that a collusive equilibrium occurs on the equilibrium path.*

The idea behind Proposition 3 is that the third-party has no informational or instrumental advantage over the seller since he must satisfy the incentive, acceptance and budget balance constraints; therefore any outcome that can be implemented by allowing coalitions to manipulate reports and/or to reallocate goods can be mimicked by the seller in a collusion-proof way without loss.

Proposition 3 implies that the seller cannot realize more profit in the presence of collusion (i.e., buyer coalition) than in its absence. Indeed, when the third party proposes S^0, (i) the Bayesian incentive constraints (BIC^S) in the side mechanism reduce to (1)-(2); (ii) the acceptance constraints (BIR^S) are automatically satisfied with equality. Hence, under collusion the seller needs to choose M to maximize her profit subject to (1)-(4) *and* the constraints that make M weakly collusion-proof.[18] Therefore, she optimizes over a set which is smaller than the one in the absence of collusion and her profit under collusion cannot be higher than the one without collusion. From this argument we obtain the following corollary.

Corollary 1 *Under buyer coalition, the seller's profit is not larger than without buyer coalition.*

The next proposition states that even though buyers can form a coalition, the seller can achieve the same profit that she realizes in the absence of buyer coalition.

Proposition 4 *There exists a transfer scheme* $\mathbf{t}^{**}$ *– with* $t^{**}_{LH} < t^*_L < t^{**}_{LL}$ *and* $t^{**}_{HH} < t^*_H < t^{**}_{HL}$ *– such that* $M^{**} \equiv \{\mathbf{q}^*, \mathbf{t}^{**}\}$ *is an optimal mechanism in the absence of buyer coalition and is weakly collusion-proof.*[19]

Proposition 4 says that the seller can implement the quantity profile $\mathbf{q}^*$ as if there were no buyer coalition and can deter collusion at no cost, thus realizing the same profit as without collusion. Hence, under asymmetric information, the ability to form a coalition does not help the buyers to increase their payoffs. In particular, even though the third party aims at maximizing the buyers' payoffs and marginal rates of substitution are not equalized across buyers in an HL-coalition, no side mechanism can implement a desirable reallocation when the seller uses M^{**}. We now provide an intuition of the result of Proposition 4 in two steps.

[18] These are the constraints which induce no manipulation of reports and no reallocation of quantity. See Jeon and Menicucci (2002) for the explicit characterization of the constraints.

[19] The transfer scheme $\mathbf{t}^{**}$ is defined later on in this section, as well as in the proof of Proposition 4 in the appendix, since it involves terms still to be introduced.

No reallocation occurs if there is no manipulation of reports As a first step, suppose that the buyers do not manipulate their reports. Then, we can show that no reallocation of quantity occurs under M^{**}. Obviously, no room for reallocation exists within the coalitions HH and LL since the seller allocates the same quantity to each buyer in these homogenous coalitions. However, in the case of the HL-coalition, potential room for arbitrage exists since an L-type's marginal utility for the goods is larger than an H-type's. To understand why no reallocation occurs in this coalition, it is important to recall that under asymmetric information, a side mechanism needs to satisfy both (BIC^S) and (BIR^S). Since (BIC_H^S) binds in the side mechanism which is optimal with respect to M^{**} and the information rent an H-type obtains by pretending to be an L-type to the third-party increases in the quantity received by an L-type, the third party evaluates an L-type's surplus not with θ_L but with a virtual valuation, which is smaller than θ_L. Furthermore, since the third party has the same prior beliefs about the buyers' types as the seller and also (BIR_L^S) binds, an H-type's rent as a function of the quantity received by an L-type increases with the same slope both in the third-party's problem and in the seller's problem with no coalition. Therefore, the third party evaluates an L-type's surplus with the same virtual valuation θ_L^v as the seller[20] and consequently he has no incentive to modify the allocation $\mathbf{q}^*$ at which an H-type's marginal surplus is equal to an L-type's virtual marginal surplus.

Alternatively, we can explain the no-reallocation result by directly computing the transaction costs created by asymmetric information and showing that they are larger than the gains from reallocation.[21] Consider reallocating a quantity $\Delta q \in (0, q_H^*]$ from an H-type to an L-type within an HL-coalition. First, the gains from reallocation are given by $G \equiv \theta_L [u(q_L^* + \Delta q) - u(q_L^*)] - \theta_H [u(q_H^*) - u(q_H^* - \Delta q)]$, which is positive, at least for a small Δq, from the inequality $\theta_L u'(q_L^*) > \theta_H u'(q_H^*)$. Second, the reallocation also increases an H-type's rent since it increases the quantity consumed by an L-type; we define this increase in rent as the transaction costs TC created by asymmetric information. In order to compute TC, suppose that an H-type pretends to be an L-type to the third-party while the other buyer reports truthfully. Then, the expected surplus

[20] See the proof of Proposition 4 in Appendix for the formal derivation of L-type's virtual value from the third-party's point of view.

[21] Makowski and Mezzetti (1994) and Williams (1999) use an argument similar to ours to prove (non) existence of efficient mechanisms in environments which include Myerson-Satterthwaite (1983)'s one seller-one buyer setting as a special case.

of the former is equal to $(1-p_L)\theta_H u(q_L^*+\Delta q)+p_L\theta_H u(q_L^*)$ while his expected payment is equal to $(1-p_L)\theta_L u(q_L^*+\Delta q)+p_L\theta_L u(q_L^*)$, determined by the binding L-type's participation constraint in the side-mechanism. Therefore, an H-type's expected rent is $\Delta\theta\left[(1-p_L)u(q_L^*+\Delta q)+p_L u(q_L^*)\right]$, higher than his rent $\Delta\theta u(q_L^*)$ when $\Delta q=0$, and $TC=\Delta\theta(1-p_L)\left[u(q_L^*+\Delta q)-u(q_L^*)\right]$. Last, the third-party can implement the reallocation only if the expected gain from reallocation $2p_L(1-p_L)G$ are larger than the expected transaction costs $2(1-p_L)TC$. Since $2p_L(1-p_L)G<2(1-p_L)TC$ holds for any $\Delta q\in(0,q_H^*]$, we conclude that reallocation is infeasible.

No manipulation of reports is profitable In order to understand why no manipulation is implemented given M^{**}, it is useful to define $V_{jk}(x)$ as the total surplus that jk-coalition derives from a total quantity $x>0$ after optimally allocating x within the coalition. As we mentioned above, the third party evaluates an L-type's surplus with θ_L^v instead of θ_L. Therefore, we have $V_{HL}(x)\equiv\max_{z\in[0,x]}\theta_H u(z)+\theta_L^v u(x-z)$ and $V_{LL}(x)\equiv 2\theta_L^v u(\frac{x}{2})$, while $V_{HH}(x)\equiv 2\theta_H u(\frac{x}{2})$ as under symmetric information. jk-coalition prefers truthtelling to reporting $j'k'$ if and only if the following *coalition incentive constraint* is satisfied

$$(CIC_{jk,j'k'})\qquad V_{jk}(q_{jk}+q_{kj})-t_{jk}-t_{kj}\geq V_{jk}(q_{j'k'}+q_{k'j'})-t_{j'k'}-t_{k'j'}\tag{5}$$

As a first step, we below focus on the two downward manipulations which are mentioned in Proposition 2(b). When $\mathbf{q}=\mathbf{q}^*$, an HH-coalition prefers truthful report to reporting HL if and only if the following inequality holds:

$$V_{HH}(2q_H^*)-2t_{HH}\geq V_{HH}(q_H^*+q_L^*)-t_{HL}-t_{LH}\tag{6}$$

An HL-coalition reports truthfully rather than LL if and only if

$$V_{HL}(q_H^*+q_L^*)-t_{HL}-t_{LH}\geq V_{HL}(2q_L^*)-2t_{LL}\tag{7}$$

We notice that the transfers in M^d violate both (6) and (7), but the seller can find transfers which satisfy (6) and (7) and make (BIC_H) and (BIR_L) bind. On the one hand, a suitable decrease in t_{HH} and an increase in t_{HL}, both with respect to t_H^*, allow to satisfy (6) while keeping (BIC_H) binding, as it is necessary to achieve the same profit as without collusion. On the other hand, an increase in t_{LL} and a decrease in t_{LH}, both with respect to t_L^*, allow (7) to be satisfied while keeping (BIR_L) still binding. Formally, the

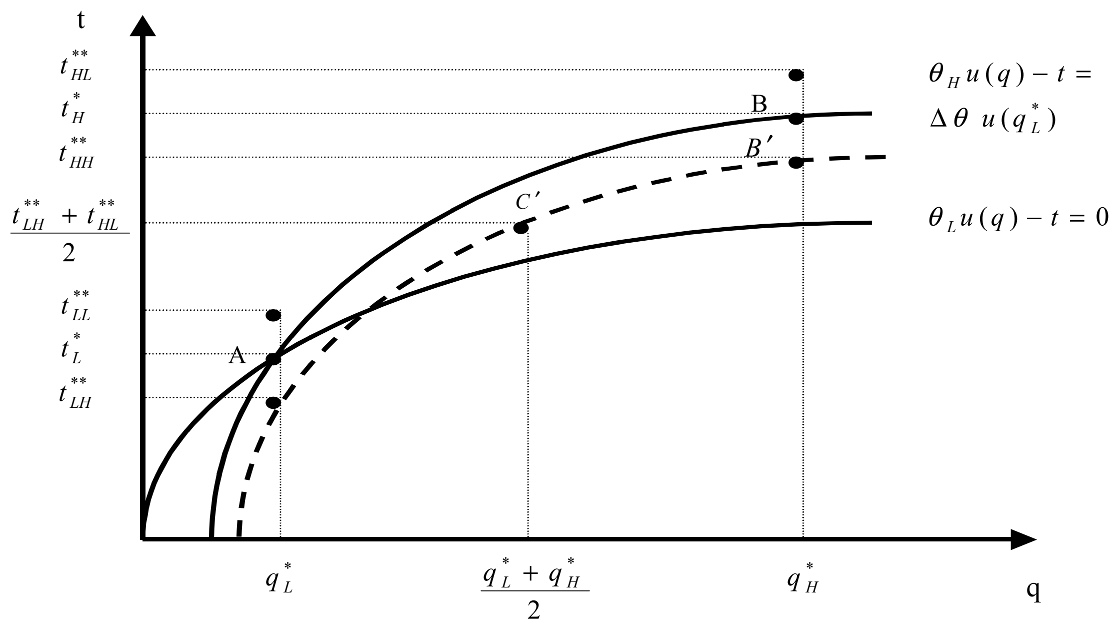

Figure 2: Transfers inducing an HH-coalition to report truthfully

seller can use two degrees of freedom in transfers to satisfy (6) and (7) at no cost while using the remaining two degrees freedom to leave (BIR_L) and (BIC_H) binding. Indeed, the transfers $\mathbf{t}^{**}$ in Proposition 4 are defined as the (unique) profile of transfers which satisfies all (BIR_L), (BIC_H), $(CIC_{HH,HL})$ and $(CIC_{HL,LL})$ with equality: Consistently with the intuition suggested above, we find $t^{**}_{LH} < t^{*}_{L} < t^{**}_{LL}$ and $t^{**}_{HH} < t^{*}_{H} < t^{**}_{HL}$.

We graphically explain how $\mathbf{t}^{**}$ deters an HH-coalition from reporting HL.[22] In figure 2, A and B are defined as in figure 1 and represent the two quantity-transfer pairs under M^d. Under M^{**}, after reporting truthfully, each buyer in an HH-coalition obtains the pair B', which is better than B since $t^{**}_{HH} < t^{*}_{H}$ holds while, after reporting HL and sharing equally the quantity and the transfer, each buyer obtains the pair C' i.e., $(\frac{q^*_L+q^*_H}{2}, \frac{t^{**}_{LH}+t^{**}_{HL}}{2})$. Since by construction H-type is indifferent between B' and C', an HH-coalition will report truthfully under M^{**}.

We now argue that also the coalition incentive constraints we neglected are satisfied by M^{**}. For this purpose, we note that (i) (6) and (7) (the local downward coalition

[22] We only examine $(CIC_{HH,HL})$ because representing $(CIC_{HL,LL})$ in Figure 2 is much more difficult; this is because, in an HL-coalition, goods are not reallocated evenly as in an HH-coalition.

incentive constraints) bind in M^{**} (ii) a single crossing condition for coalitions holds[23]: $V'_{HH}(x) > V'_{HL}(x) > V'_{LL}(x)$ for any $x > 0$ (iii) the quantity profile for coalitions is monotone: $2q^*_{HH} > q^*_{HL} + q^*_{LH} > 2q^*_{LL}$. Therefore, we can use a standard result from the theory of monopolistic screening [see Section 3 in Maskin and Riley (1984)] to conclude that (5) is satisfied for any jk and $j'k'$.[24]

It is interesting to notice that there exist infinitely many transfer schemes $\widehat{\mathbf{t}}$ such that $\left\{\mathbf{q}^*, \widehat{\mathbf{t}}\right\}$ is optimal under no coalition and weakly collusion-proof (for instance, it is possible to strictly satisfy (5) for any jk and $j'k'$ without reducing the profit). However, the following inequalities

$$\widehat{t}_{LH} < t^*_L < \widehat{t}_{LL} \qquad \text{and} \qquad \widehat{t}_{HH} < t^*_H < \widehat{t}_{HL} \tag{8}$$

must be satisfied by any such $\widehat{\mathbf{t}}$. The inequalities mean that upon reporting a type, each buyer faces a lottery which determines his payment as a function of the report of the other buyer. In particular, facing an L-type is bad news because then the payment is higher than when facing an H-type. This feature results from the seller's desire to deter coalitions' downward manipulation of reports, as we below argue in proving (8).

To show (8), let $\widehat{t}_{HL} = t^*_H + a$, $\widehat{t}_{LL} = t^*_L + b$, $\widehat{t}_{HH} = t^*_H - \frac{p_L a}{1-p_L}$ and $\widehat{t}_{LH} = t^*_L - \frac{p_L b}{1-p_L}$: therefore $\widehat{\mathbf{t}}$ satisfies (BIC_H) and (BIR_L) with equality. Define $\alpha \equiv V_{HH}(2q^*_H) - V_{HH}(q^*_H + q^*_L) - (t^*_H - t^*_L)$ and $\beta \equiv V_{HL}(q^*_H + q^*_L) - V_{HL}(2q^*_L) - (t^*_H - t^*_L)$. Then, $(CIC_{HH,HL})$ and $(CIC_{HL,LL})$ at $\mathbf{q} = \mathbf{q}^*$ reduce respectively to

$$(1-p_L)\alpha \geq -a - p_L(a-b) \qquad \text{and} \qquad (1-p_L)\beta \geq -b + (1-p_L)(a-b)$$

Therefore, the set of (a, b) satisfying $(CIC_{HH,HL})$ and $(CIC_{HL,LL})$ is given by $Z \equiv \left\{(a,b) \in R^2 \mid \frac{(1-p_L)\alpha + (1+p_L)a}{p_L} \geq b \geq \frac{(1-p_L)a - (1-p_L)\beta}{2-p_L}\right\}$ and the point at which $\frac{(1-p_L)\alpha+(1+p_L)a}{p_L} = b = \frac{(1-p_L)a-(1-p_L)\beta}{2-p_L}$ holds corresponds to the transfers $\mathbf{t}^{**}$ of M^{**}. Figure 3 represents Z graphically and, since $\alpha < 0$, $\beta < 0$[25] and $\frac{1+p_L}{p_L} > \frac{1-p_L}{2-p_L}$, any $(a, b) \in Z$ should be such that $a > 0$ and $b > 0$. Therefore, for any mechanism which is optimal under no coalition

[23] See the proof of Proposition 4 in the appendix for the details.

[24] An alternative strategy to prove that the seller is not hurt by a buyer coalition starts by characterizing the set of weakly collusion-proof mechanisms and then verifying that M^{**} is indeed weakly collusion-proof. This approach must be followed in particular when the second best outcome is not achievable under collusion, since then it is necessary to optimize within the set of weakly collusion-proof mechanisms.

[25] It is straightforward to verify that $\alpha < 0$. For the proof of $\beta < 0$ see the end of Appendix.

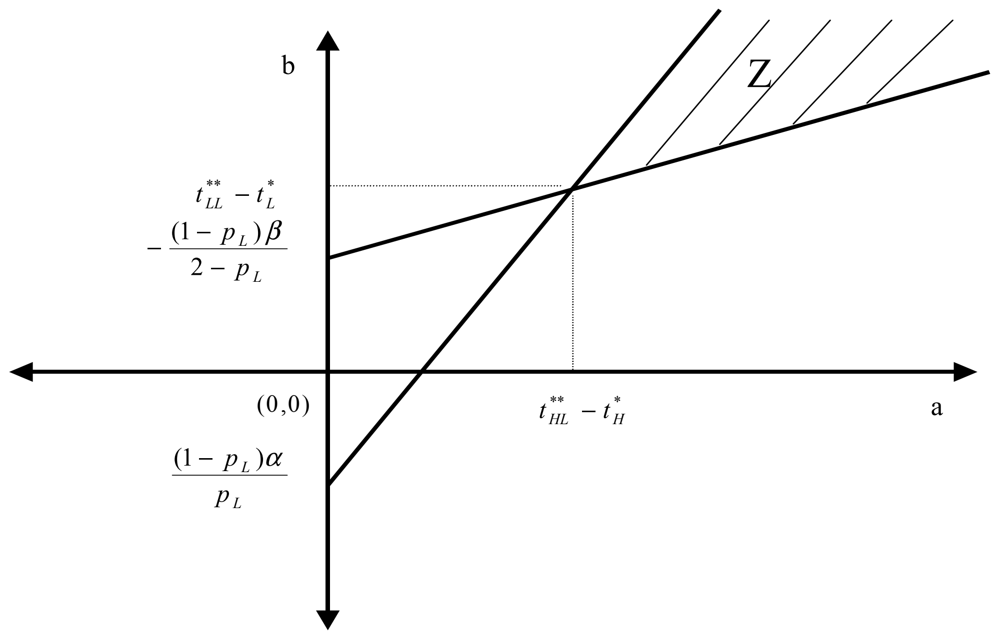

Figure 3: Transfers in the optimal collusion-proof mechanisms: a necessary condition

and satisfies (6)-(7) (necessary conditions for weak collusion-proofness), its transfers $\widehat{\mathbf{t}}$ must satisfy (8). This implies in particular that (i) M^d is not weakly collusion-proof since $a = b = 0$ in M^d (ii) ex post individual rationality is violated for L-type in any mechanism which is optimal under no coalition and weakly collusion-proof since $t^*_L = \theta_L u(q^*_L) < \widehat{t}_{LL}$ holds.

Remark 1 (symmetric information in the coalition): Even though we focus on the role of asymmetric information among buyers, it is interesting to look at the consequences of collusion taking place under symmetric information. For instance, suppose that the third party owns a technology that allows him to elicit credible reports from the buyers as in Baron and Besanko (1999).[26] In this case the side mechanism does not need to satisfy (BIC^S), implying that the third party evaluates an L-type's surplus with the real valuation θ_L rather than with the virtual value θ^v_L. The coalition incentive constraints under symmetric information are similar to those under asymmetric information except that now, in defining $V_{HL}(x)$ and $V_{LL}(x)$, θ_L is used instead of

[26]They assume that the third-party who organizes an informational alliance can verify the private information of each agent forming the alliance.

θ_L^v. Still, the seller can deter manipulation of reports at no cost as in the case of asymmetric information. However, reallocation within an HL-coalition takes place unless $\theta_H u'(q_{HL}) = \theta_L u'(q_{LH})$, a condition which reduces the seller's profit with respect to the case without buyer coalition.

Remark 2 (strategic robustness): It is possible to verify that in M^{**} truthtelling is a strictly dominant strategy for an L-type but iteratively weakly dominated for an H-type. This undermines the stability of truthtelling in M^{**}, but we can exploit the multiplicity of optimal transfers to find a mechanism in which truthtelling is strictly dominant for an L-type and serially weakly dominant for an H-type: See Jeon and Menicucci (2002). Since the definition of this mechanism is parameter-dependent and less intuitive with respect to M^{**}, we preferred to focus on the latter in Proposition 4.

Two-part tariffs Two-part tariffs are sometimes proposed as a simple way to implement non-linear tariffs, or as a "real-life" mechanism as opposed to abstract direct mechanisms. In the model with no buyer coalition, it is easy to see that the optimal outcome can be implemented by a menu of two-part tariffs such that each type of buyer chooses the tariff designed for his type and buys the quantity q_H^* or q_L^* according to his type. We note that the two-part tariff designed for an L-type needs a kink in order to prevent an H-type from choosing the tariff designed for an L-type and buying more than q_L^*.[27]

The next proposition states that a more complicated menu of two-part tariffs can be used to implement the optimal outcome when coalition formation is possible. We continue to assume that the seller can commit not to serve a buyer if the other buyer does not buy anything from the seller.[28] Let the seller offer tariffs $T_H = \{(A_{HH}, p_{HH}), (A_{HL}, p_{HL})\}$ and $T_L = \{(A_{LH}, p_{LH}), (A_{LL}, p_{LL})\}$ where, for instance, A_{HL} and p_{HL} represent the fixed fee and the marginal price that a buyer choosing T_H pays if the other buyer chooses T_L.

[27] The two-part tariff for an H-type takes the form $A_H + pq$ with $A_H = t_H^* - cq_H^*$ and $p = c$. Since the tariff for an L-type needs a kink at the point $q = q_L^*$, the seller has some discretion in choosing the marginal price. For instance, she can use $A_L + pq$ with $A_L = t_L^* - cq_L^*$ and $p = c$ for $q \leq q_L^*$, $p = \theta_H u'(q_L^*)$ for $q > q_L^*$.

[28] As we said in footnote 10, our results hold even if this assumption does not hold but the seller can observe whether or not a buyer uses her goods. This makes it impossible for a buyer to use a positive amount of the goods without paying any fixed fee to the seller as in Rey and Tirole (1986).

In particular, we consider the tariffs $\{T_H^{**}, T_L^{**}\}$ such that

$$\begin{cases} A_{jk}^{**} = t_{jk}^{**} - cq_j^*, \text{ for } j,k \in \{H,L\}, \\ p_{jk}^{**} = c \text{ for } q \leq q_j^* \text{ and } p_{jk}^{**} = \theta_H u'(q_L^*) \text{ for } q > q_j^* \text{ for } j,k \in \{H,L\}. \end{cases} \tag{9}$$

Proposition 5 *Suppose that the seller offers $\{T_H^{**}, T_L^{**}\}$ instead of M^{**}. Then, regardless of whether or not the buyers can form a coalition,*
(a) each buyer accepts the offer,
(b) j-type of buyer, with $j \in \{H,L\}$, chooses the tariff T_j^{**} and buys quantity q_j^*.

The menu (9) is such that (i) the fixed fee a buyer pays depends on the tariff chosen by the other buyer (which is necessary since t^{**} requires this sort of dependence) (ii) the tariff each buyer faces has a kink.[29] The kink is necessary in order to deter downward manipulation of reports. For instance, suppose there is no kink in T_H^{**}. Then, since $A_{HH}^{**} > A_{HL}^{**} + A_{LH}^{**}$ holds, an HH-coalition has an incentive to coordinate the buyers' purchases such that only one buyer chooses T_H^{**}, he buys more than q_H^* and shares it with the other buyer who chooses T_L^{**}.[30] This deviation is prevented by the increase in the marginal price at $q = q_H^*$ - the kink - from c to $\theta_H u'(q_L^*)$.

6 Extensions

In the previous sections, for simplicity we considered the two-buyer-two-type setting with $C(q) = cq$ and $\mathcal{U}(q,\theta) = \theta u(q)$. However, Proposition 4 can be extended to an environment with n buyers and two types.[31] We below show that it also can be extended to the setting with general cost and utility functions which satisfy the conditions introduced in subsection 2.1, or to the setting with two buyers and three types.

[29] Actually, no kink is needed when both buyers choose T_H^{**}: we can have $p_{HH} = c$ for all $q \geq 0$. However, in this case both the fixed fee *and* the marginal price paid by a buyer choosing T_H^{**} depends on the tariff chosen by the other buyer, while in (9) the marginal price depends only on his choice.

[30] Likewise, if there were no kink in T_L^{**}, the buyer who pretended to be an L-type may buy more than q_L^* and then share with the other buyer.

[31] When the seller faces $n > 2$ buyers, we obtain the result of Proposition 4 if we assume that the grand coalition (the one including all the buyers) is the only feasible coalition. More precisely, if at least one buyer rejects the side mechanism then no other attempt to organize a partial or complete coalition is pursued and the truthful equilibrium of the sale mechanism is played. The seller still has some residual degrees of freedom in the choice of transfers in the absence of collusion, which she can use to deter collusion at no cost as in the case of $n = 2$. For more details, see Jeon and Menicucci (2002).

6.1 General cost function C and utility function $\mathcal{U}$

We can show that Proposition 4 holds if (i) the cost function satisfies $C(0)=0$, $C'(q)>0$ and $C''(q)\geq 0$ for any $q\geq 0$; (ii) the utility function satisfies $\mathcal{U}_1(q,\theta)>0>\mathcal{U}_{11}(q,\theta)$, $\mathcal{U}(0,\theta)=0$, $\mathcal{U}_2(q,\theta)>0$, $\mathcal{U}_{12}(q,\theta)>0$ for any (q,θ) and $\mathcal{U}(q,\theta_L)-(1-p_L)\mathcal{U}(q,\theta_H)$ is concave in q. In this environment the optimal mechanisms in the absence of buyer coalition are still such that (BIR_L) and (BIC_H) bind; the optimal quantity profile $\mathbf{q}^*$ satisfies $\mathcal{U}_1(q^*_{HH},\theta_H)=C'(2q^*_{HH})$, $\mathcal{U}_1(q^*_{HL},\theta_H)=\frac{\mathcal{U}_1(q^*_{LH},\theta_L)}{p_L}-\frac{(1-p_L)\mathcal{U}_1(q^*_{LH},\theta_H)}{p_L}=C'(q^*_{HL}+q^*_{LH})$ and $\frac{\mathcal{U}_1(q^*_{LL},\theta_L)}{p_L}-\frac{(1-p_L)\mathcal{U}_1(q^*_{LL},\theta_H)}{p_L}=C'(2q^*_{LL})$; these conditions imply $q^*_{HL}>q^*_{HH}$ and $q^*_{LL}>q^*_{LH}$, but it is still true that $2q^*_{HH}>q^*_{HL}+q^*_{LH}>2q^*_{LL}$. We can show that (i) since the single crossing condition for coalitions still holds in this general environment, the seller can deter all reports manipulations at no cost by using the two residual degrees of freedom in transfers to satisfy $(CIC_{HH,HL})$ and $(CIC_{HL,LL})$ with equality (ii) given that the transfer scheme induces truthful reports, the third-party is unable to implement any efficient reallocation in an HL-coalition since the transaction costs created by asymmetric information are larger than the gains from reallocation.

Notice that, in particular, Proposition 4 holds in an auction setting in which a single object is up for sale: $q\in[0,1]$ is the probability to win the object and $\mathcal{U}(q,\theta)=\theta q$.[32]

6.2 The case of three types

Mechanism design problems under collusion often turn out to be qualitatively more complicated when there are more than two types than when there are only two types. For instance, Laffont and Martimort (1997, 2000) limit their analysis to the two-type setting since it is difficult to determine the binding coalition incentive constraints when there are more than two types. Here we briefly explain how – in our model – Proposition 4 extends to the three-type setting. The main difficulty is related to the fact that the single-crossing condition for coalitions holds only partially.

Now the valuation θ^i of buyer i lies in $\Theta\equiv\{\theta_L,\theta_M,\theta_H\}$, with $\theta_H>\theta_M>\theta_L>0$. The types θ^1 and θ^2 are identically and independently distributed with $p_L\equiv\Pr\left\{\theta^i=\theta_L\right\}>$

[32] In this environment the seller does not need to exploit the information asymmetry between the buyers. Indeed, under no buyer coalition, there exists no potential room for arbitrage in an HL-coalition because the marginal surplus of each type is constant and a corner solution achieves the first-best allocation and is optimal for the seller.

0, $p_M \equiv \Pr\left\{\theta^i = \theta_M\right\} > 0$ and $p_H \equiv \Pr\left\{\theta^i = \theta_H\right\} > 0$. In the absence of buyer coalition, the virtual values of an M-type and an L-type are given by:

$$\theta_M^v \equiv \theta_M - \frac{p_H}{p_M}(\theta_H - \theta_M) \qquad \theta_L^v \equiv \theta_L - \frac{p_H + p_M}{p_L}(\theta_M - \theta_L)$$

Clearly, $\theta_H > \max\{\theta_M^v, \theta_L^v\}$ but the order between θ_M^v and θ_L^v depends on the parameters; if $\theta_M^v \geq \theta_L^v$, then virtual values are said to be monotonic; if $\theta_M^v < \theta_L^v$, then let $\bar{\theta}_{ML}^v \equiv \frac{p_L\theta_L^v + p_M\theta_M^v}{p_L + p_M}$. In any case, we assume $\min\{\theta_M^v u'(0), \theta_L^v u'(0)\} > c$ so that each type receives a positive and bounded quantity in case of no coalition.

As in the previous sections, we can restrict attention to symmetric direct revelation mechanisms, hence a sale mechanism is $M = \{\mathbf{q}, \mathbf{t}\}$, with $\mathbf{q} \equiv \{q_{jk}\}_{j,k=L,M,H}$, $\mathbf{t} \equiv \{t_{jk}\}_{j,k=L,M,H}$ and q_{jk} (t_{jk}) is the quantity received by a buyer (his payment) if he reports j and the other buyer reports k. Let $\bar{t}_j \equiv p_L t_{jL} + p_M t_{jM} + p_H t_{jH}$ and $\bar{u}_j \equiv p_L u(q_{jL}) + p_M u(q_{jM}) + p_H u(q_{jH})$, $j = L, M, H$. Then, the expected profit is written as

$$\begin{aligned} \Pi &= 2(p_L\bar{t}_L + p_M\bar{t}_M + p_H\bar{t}_H) - 2c[p_L^2 q_{LL} + p_L p_M(q_{LM} + q_{ML}) + p_L p_H(q_{HL} + q_{LH})] \\ &\quad -2c[p_M^2 q_{MM} + p_M p_H(q_{MH} + q_{HM}) + p_H^2 q_{HH}] \end{aligned}$$

The Bayesian incentive compatibility and participation constraints are

$$\begin{aligned} (BIC) \qquad \theta_j\bar{u}_j - \bar{t}_j &\geq \theta_j\bar{u}_{j'} - \bar{t}_{j'}, \quad j, j' = L, M, H \\ (BIR) \qquad \theta_j\bar{u}_j - \bar{t}_j &\geq 0, \quad j = L, M, H \end{aligned}$$

The seller maximizes Π subject to (BIC) and (BIR). The next proposition characterizes the optimal mechanisms in the absence of buyer coalition.

Proposition 6 *The optimal mechanisms in the absence of buyer coalition are characterized as follows*
(a) The optimal quantity schedule $\mathbf{q}^*$ *is such that:*
i) $q_{Hj}^* = q_H^*$ *for* $j = L, M, H$, *where* $\theta_H u'(q_H^*) = c$;
ii) $q_{Mj}^* = q_M^*$, $q_{Lj}^* = q_L^*$ *for* $j = L, M, H$ *with* $\theta_M^v u'(q_M^*) = \theta_L^v u'(q_L^*) = c$ *if* $\theta_M^v \geq \theta_L^v$ *but* $q_M^* = q_L^*$ *with* $\bar{\theta}_{ML}^v u'(q_L^*) = c$ *if instead* $\theta_M^v < \theta_L^v$.
(b) Transfers are such that constraints (BIC_{HM}), (BIC_{ML}) *and* (BIR_L) *bind.*

As in the two-type case, the weak collusion-proofness principle holds; thus it is impossible for the seller to realize more profit under a buyer coalition than in its absence. However, we can still prove that buyer coalition does not hurt the seller.

Proposition 7 *There exists a transfer scheme* $\mathbf{t}^{**}$ *such that* $M^{**} \equiv \{\mathbf{q}^*, \mathbf{t}^{**}\}$ *is both an optimal mechanism in the absence of a buyer coalition and weakly collusion-proof.*

We below provide an intuition of the result: the intuition is similar to the one for the two-type case although some technical details of the proof are more complicated. Given M^{**}, the virtual values of an M-type and an L-type from the third party's viewpoint are equal to θ_M^v and θ_L^v, the virtual valuations from the seller's viewpoint; hence the third-party will not reallocate goods conditional on there being no manipulation of reports. If we let $\theta_H^v \equiv \theta_H$, the surplus a jk-coalition obtains from owning quantity $x > 0$ is $V_{jk}(x) \equiv \max_{z\in[0,x]} \theta_j^v u(z) + \theta_k^v u(x-z)$, $j,k = L, M, H$; the coalition incentive constraint preventing this coalition from reporting $j'k'$ is $V_{jk}(q_{jk} + q_{kj}) - t_{jk} - t_{kj} \geq V_{jk}(q_{j'k'} + q_{k'j'}) - t_{j'k'} - t_{k'j'}$. Since there are six degrees of freedom in transfers in the optimal mechanism(s) under no coalition, the seller can use them to satisfy all of the coalition incentive constraints. However, now the single crossing condition for coalitions holds only partially because it does not provide an order between coalitions HL and MM. This makes it more difficult to find the right transfers than in the two-type setting, although it is possible. We conjecture that our result will hold even when there are more than three types.

7 Concluding remarks

We found that if the seller uses simple sale mechanisms in which the quantity sold to a buyer and his payment depend solely on his own report, buyers can realize strict gains at the seller's loss by coordinating their purchases and reallocating the goods. However, we showed in various settings that when the seller judiciously designs her mechanism by exploiting the transaction costs in coalition formation, buyer coalition does not hurt her and, in particular, the buyers are unable to implement efficient arbitrage.

Our main result holds provided that there is no restriction on the set of mechanisms available to the seller, which seems to be a reasonable assumption for a situation in which the seller deals with a small number of buyers. For instance, when the marginal cost is constant, in the optimal collusion-proof mechanisms, a buyer's payment has to depend on the other buyer's report while the quantity he receives is independent of such a report.[33] Although this feature looks unnatural, we point out the fact that the same feature exists also in Vickrey auctions, where the price that a winner pays depends on other bidders' bids.

Our findings suggest that buyer coalitions are likely to emerge either when they share better information about each other's preferences than the seller has, or when the seller is constrained to use a restricted set of contracts. For instance, when there are a large number of buyers, the seller may have incomplete information about their number and identities. This would impose restrictions on the set of contracts available to the seller, as in Alger (1999). It would be interesting to study the case in which the seller can use only individual contracts: i.e., the quantity sold to a buyer and his payment do not depend on what other buyers do. In this setting, the collusion-proofness principle might not hold[34] and therefore the optimal mechanism might involve letting collusion occur.[35]

APPENDIX

Proof of Proposition 1

The arguments of the proof for the single-buyer model show that (BIC_H) and (BIR_L) bind in the optimum. After replacing in Π the transfers as obtained from (BIC_H) and (BIR_L) written with equality, (i)-(ii) emerge as necessary and sufficient conditions for the optimum and (BIC_L) and (BIR_H) are automatically satisfied. ■

Proof of Proposition 2(b)

[33] We note that when marginal cost is variable, (i) our main result holds, (ii) without buyer coalition, both the quantity received by a buyer and his payment depend on the other's report under dominant strategy implementation (iii) the optimal collusion-proof mechanisms also require such a dependence.

[34] The principle does not hold for instance if the third-party can make a buyer's (side) transfer dependent on the other buyers' reports while the seller cannnot.

[35] Another direction for extension is to consider different timing for buyer coalitions as Laffont and Martimort (1997) discuss. To focus on coordination of purchases and reallocation, we adopted the timing chosen by Laffont and Martimort (1997, 2000) but the analysis can be extended to a timing in which buyers can form a coalition after receiving the seller's offer and before deciding whether to accept or reject the offer. Independently, deQuiedt (2002) recently studied collusion with this timing in auctions.

The side mechanism $S^d = \left\{\phi^d(\theta^1, \theta^2), x^{id}(\theta^1, \theta^2, \widetilde{\phi}), y^{id}(\theta^1, \theta^2)\right\}$ mentioned in the statement of Proposition 2(b) is formally defined as follows. For simplicity, let $\phi^d_{jk} = \phi^d(\theta_j, \theta_k)$, $x^{id}_{jk,\widetilde{\phi}} = x^{id}(\theta_j, \theta_k, \widetilde{\phi})$ and $y^{id}_{jk} = y^{id}(\theta_j, \theta_k)$ with $j, k \in \{H, L\}$.

Reports manipulations: $\phi^d_{HH} = (\theta_H, \theta_L)$, $\phi^d_{HL} = \phi^d_{LH} = \phi^d_{LL} = (\theta_L, \theta_L)$.[36]

Reallocation of goods[37]: $x^{1d}_{HH} = -\frac{q^*_H - q^*_L}{2}$, $x^{2d}_{HH} = \frac{q^*_H - q^*_L}{2}$; $x^{1d}_{HL} = \hat{x} > 0$, with $\hat{x}$ close to 0, $x^{2d}_{HL} = -\hat{x}$; $x^{2d}_{LH} = -x^{1d}_{HL} = \hat{x}$; $x^{1d}_{LL} = x^{2d}_{LL} = 0$.

Side transfers: $y^{1d}_{HH} = -\frac{t^*_H - t^*_L}{2}$, $y^{2d}_{HH} = \frac{t^*_H - t^*_L}{2}$; $y^{1d}_{HL} = y^{2d}_{LH} = \hat{y}$, $y^{2d}_{HL} = y^{1d}_{LH} = -\hat{y}$; $y^{1d}_{LL} = y^{2d}_{LL} = 0$, where $\hat{y} > 0$ is still to be defined.

In words, an HH-coalition reports HL; then goods and payments are equally shared between the buyers. A coalition HL or LH reports LL; then goods are slightly reallocated from an L-type to an H-type and the H-type pays $\hat{y}$ to the L-type.

We prove that for a small $\hat{x} > 0$ there exists a $\hat{y} > 0$ such that (BIC^S) are satisfied and (BIR^S) are slack – $(BB : x)$ and $(BB : y)$ are satisfied by definition. Therefore, S^d is feasible and strictly increases the payoff of each buyer type with respect to playing M^d non-cooperatively.

Let $\widehat{q}_H \equiv q^*_L + \hat{x}$, $\widehat{q}_L \equiv q^*_L - \hat{x}$ and consider constraint (BIC^S_H):

$$\begin{aligned} & p_L[\theta_H u(\widehat{q}_H) - \theta_L u(q^*_L) - \hat{y}] + (1 - p_L)\{\theta_H u(\frac{q^*_L + q^*_H}{2}) - \theta_L u(q^*_L) - \frac{\theta_H}{2}[u(q^*_H) - u(q^*_L)]\} \\ \geq\ & p_L(\Delta\theta)u(q^*_L) + (1 - p_L)[\theta_H u(\widehat{q}_L) - \theta_L u(q^*_L) + \hat{y}] \end{aligned} \tag{10}$$

Let $\hat{y} = \tilde{y} \equiv \theta_H[u(q^*_L) - u(\widehat{q}_L)] > 0$, so that (i) the right hand side of (10) is equal to $U^{M^d}(\theta_H) = (\Delta\theta)u(q^*_L)$; (ii) if $\hat{x} = 0$, then (10) is strictly satisfied and therefore, when $\hat{x} > 0$ is close to 0, (10) is still strictly satisfied and (BIR^S_H) is strictly satisfied as well; (iii) (BIR^S_L) holds strictly. Given a small $\hat{x} > 0$, consider increasing $\hat{y}$ above $\tilde{y}$ until the point at which (10) binds. Then, (BIR^S_H) still holds strictly since the right hand side of (10) increased above $U^{M^d}(\theta_H)$; clearly, (BIR^S_L) holds strictly as well since now $\hat{y} > \tilde{y}$. In order to prove that (BIC^S_L) is satisfied, add up (BIC^S_L) and (BIC^S_H) (which binds) to obtain an inequality which holds strictly because $\widehat{q}_H > q^*_L$ and $\frac{q^*_L + q^*_H}{2} > \widehat{q}_L$. Therefore, S^d satisfies (BIC^S) and (BIR^S) and the payoff of each type of buyer is strictly larger than from playing M^d non-cooperatively.

[36] We recall that when the manipulation is deterministic, i.e., $p^\phi(\widetilde{\theta}^1, \widetilde{\theta}^2, \widetilde{\phi}) = 1$ for some $\widetilde{\phi} \in \Theta^2$, we write $\phi(\widetilde{\theta}^1, \widetilde{\theta}^2) = \widetilde{\phi}$ (see Section 2.2).

[37] Since the report manipulation is deterministic, we do not write $\widetilde{\phi}$ in $x^{id}_{jk,\widetilde{\phi}}$.

Thus, with M^d, the buyer coalition strictly reduces the seller's profit because (i) in the states of nature in which reports are manipulated, the quantity sold to buyers is smaller than under truthtelling, which reduces the surplus generated by the trade and (ii) each type of buyer obtains a higher payoff than with truthtelling.[38]

Proof of Proposition 3

The proof is omitted since it is a straightforward adaptation of the proof of Proposition 3 in Laffont and Martimort (2000). ■

Proof of Proposition 4

The transfers in M^{**} solve the linear system in $\mathbf{t}$ with (BIR_L), (BIC_H), $(CIC_{HH,HL})$ and $(CIC_{HL,LL})$ written as equalities and $\mathbf{q} = \mathbf{q}^*$. This four equations-four unknowns system has full rank; thus a unique solution $\mathbf{t}^{**}$ exists, which we report for completeness:

$$\begin{aligned} t_{HL}^{**} &= \frac{(1+p_L)\theta_L - (3-p_L^2)\theta_H}{2} u(q_L^*) + \theta_H \frac{p_L(3-p_L)}{2} u(q_H^*) \\ &\quad + (1-p_L)(2-p_L)\theta_H u(\frac{q_H^* + q_L^*}{2}) + \frac{p_L(1-p_L)}{2} V_{HL}(2q_L^*), \end{aligned}$$

$$\begin{aligned} t_{LH}^{**} &= \frac{(p_L+3)\theta_L + (2p_L + p_L^2 - 1)\theta_H}{2} u(q_L^*) + \theta_H \frac{p_L(1-p_L)}{2} u(q_H^*) \\ &\quad - p_L(1-p_L)\theta_H u(\frac{q_H^* + q_L^*}{2}) - \frac{p_L(1+p_L)}{2} V_{HL}(2q_L^*), \end{aligned}$$

$$\begin{aligned} t_{HH}^{**} &= \frac{(p_L+2)\theta_L - (1-p_L)(2+p_L)\theta_H}{2} u(q_L^*) + \theta_H \frac{2+2p_L - p_L^2}{2} u(q_H^*) \\ &\quad - p_L(2-p_L)\theta_H u(\frac{q_H^* + q_L^*}{2}) - \frac{p_L^2}{2} V_{HL}(2q_L^*), \end{aligned}$$

$$\begin{aligned} t_{LL}^{**} &= \frac{(p_L^2 + 2p_L - 1)\theta_L^1}{2} u(q_L^*) - \theta_H \frac{(1-p_L)^2}{2} u(q_H^*) + (1-p_L)^2 \theta_H u(\frac{q_H^* + q_L^*}{2}) \\ &\quad + \frac{1-p_L^2}{2} V_{HL}(2q_L^*). \end{aligned}$$

[38] Actually, S^d may not be the optimal side mechanism against M^d. In particular, goods are not efficiently reallocated within an HL-coalition since otherwise we are not sure of whether (BIR^S) and (BIC^S) can all be satisfied. However, if the third party chooses the optimal side mechanism against M^d, then still the profit is smaller than if M^d is played non-cooperatively.

By Proposition 1, M^{**} is optimal under no coalition. We prove that M^{**} is also weakly collusion-proof, which means that S^0 is optimal for the third party given M^{**}. Since L-type's incentive constraint is not binding in M^{**}, the incentive constraint of L-type will be slack in the side mechanism as well. In what follows, for the sake of brevity, let $x^i_{jk,\widetilde{\phi}}$ denote $x^i(\theta_j, \theta_k, \widetilde{\phi})$ with $j,k \in \{H, L\}$. Likewise, $p^{\phi}_{jk,\widetilde{\phi}}$ denotes $p^{\phi}(\theta_j, \theta_k, \widetilde{\phi})$.

The third-party maximizes the following objective,

$$\begin{aligned}
&(1-p_L)^2 \textstyle\sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{HH,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{HH,\widetilde{\phi}}) - t^1(\widetilde{\phi}) + \theta_H u(q^2(\widetilde{\phi}) + x^2_{HH,\widetilde{\phi}}) - t^2(\widetilde{\phi})] \\
&+p_L(1-p_L) \textstyle\sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{LH,\widetilde{\phi}}[\theta_L u(q^1(\widetilde{\phi}) + x^1_{LH,\widetilde{\phi}}) - t^1(\widetilde{\phi}) + \theta_H u(q^2(\widetilde{\phi}) + x^2_{LH,\widetilde{\phi}}) - t^2(\widetilde{\phi})] \\
&+p_L(1-p_L) \textstyle\sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{HL,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{HL,\widetilde{\phi}}) - t^1(\widetilde{\phi}) + \theta_L u(q^2(\widetilde{\phi}) + x^2_{HL,\widetilde{\phi}}) - t^2(\widetilde{\phi})] \\
&+p_L^2 \textstyle\sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{LL,\widetilde{\phi}}[\theta_L u(q^1(\widetilde{\phi}) + x^1_{LL,\widetilde{\phi}}) - t^1(\widetilde{\phi}) + \theta_L u(q^2(\widetilde{\phi}) + x^2_{LL,\widetilde{\phi}}) - t^2(\widetilde{\phi})]
\end{aligned}$$

subject to the following constraints.

- Budget balance constraints: for the quantity reallocation

$$\sum_{i=1}^{2} x^i(\theta^1, \theta^2, \widetilde{\phi}) = 0, \text{ for any } (\theta^1, \theta^2) \in \Theta^2 \text{ and any } \widetilde{\phi} \in \Theta^2;$$

for the side transfers

$$\sum_{i=1}^{2} y^i(\theta^1, \theta^2) = 0, \text{ for any } (\theta^1, \theta^2) \in \Theta^2,$$

- H-type's Bayesian incentive constraint for buyer 1: $(BIC^S_1(\theta_H))$

$$\begin{aligned}
& p_L \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{HL,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{HL,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{HL}] \\
& +(1-p_L) \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{HH,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{HH,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{HH}] \\
\geq\ & p_L \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{LL,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{LL,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{LL}] \\
& +(1-p_L) \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{LH,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{LH,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{LH}],
\end{aligned}$$

- H-type's acceptance constraint for buyer 1: $(BIR^S_1(\theta_H))$

$$\begin{aligned}
& p_L \textstyle\sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{HL,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{HL,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{HL}] \\
& +(1-p_L) \textstyle\sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{HH,\widetilde{\phi}}[\theta_H u(q^1(\widetilde{\phi}) + x^1_{HH,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{HH}] \geq U^M(\theta_H),
\end{aligned}$$

• L-type's acceptance constraint for buyer 1: $(BIR_1^S(\theta_L))$

$$\begin{array}{l} p_L \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{LL,\widetilde{\phi}}[\theta_L u(q^1(\widetilde{\phi}) + x^1_{LL,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{LL}] \\ +(1-p_L) \sum_{\widetilde{\phi}\in\Theta^2} p^{\phi}_{LH,\widetilde{\phi}}[\theta_L u(q^1(\widetilde{\phi}) + x^1_{LH,\widetilde{\phi}}) - t^1(\widetilde{\phi}) - y^1_{LH}] \geq U^M(\theta_L), \end{array}$$

• H-type's Bayesian incentive constraint for buyer 2 : $(BIC_2^S(\theta_H))$

• H-type's acceptance constraint for buyer 2: $(BIR_2^S(\theta_H))$

• L-type's acceptance constraint for buyer 2: $(BIR_2^S(\theta_L))$,

where $(BIC_2^S(\theta_H))$, $(BIR_2^S(\theta_H))$, $(BIR_2^S(\theta_L))$ are in the same way as $(BIC_1^S(\theta_H))$, $(BIR_1^S(\theta_H))$, $(BIR_1^S(\theta_L))$ are defined.

We introduce the following multipliers:

• $\rho^x(\theta^1, \theta^2, \widetilde{\phi})$ for the budget-balance constraint for the quantity reallocation in state $(\theta^1, \theta^2, \widetilde{\phi})$,

• $\rho^y(\theta^1, \theta^2)$ for the budget-balance constraint for the side-transfers in state (θ^1, θ^2),

• δ^i for the H-type's Bayesian incentive constraint concerning buyer i,

• v^i_H for the H-type's acceptance constraint concerning buyer i,

• v^i_L for the L-type's acceptance constraint concerning buyer i.

We define the Lagrangian function $\mathcal{L}$ as follows:

$$\mathcal{L} = E(U_1 + U_2) + \sum_{i=1,2} \delta^i (BIC_i^S)(\theta_H) + \sum_{i=1,2} v^i_H (BIR_i^S)(\theta_H) + \sum_{i=1,2} v^i_L (BIR_i^S)(\theta_L)$$

$$+ \sum_{(\theta^1,\theta^2)\in\Theta^2} \sum_{\widetilde{\phi}\in\Theta^2} \rho^x(\theta^1, \theta^2, \widetilde{\phi})(BB:x)(\theta^1, \theta^2, \widetilde{\phi}) + \sum_{(\theta^1,\theta^2)\in\Theta^2} \rho^y(\theta^1, \theta^2)(BB:y)(\theta^1, \theta^2)$$

Step 1: First order conditions for $y^i(\theta^1, \theta^2)$

After optimizing with respect to y^i_{HH}, we have:

$$\rho^y_{HH} - \delta^i(1-p_L) - v^i_H(1-p_L) = 0, \text{ for } i = 1, 2.$$

After optimizing with respect to y^1_{HL} and y^2_{HL} respectively, we have:

$$\begin{aligned} \rho^y_{HL} - \delta^1 p_L - v^1_H p_L &= 0; \\ \rho^y_{HL} + \delta^2(1-p_L) - v^2_L(1-p_L) &= 0 \end{aligned}$$

After optimizing with respect to y_{LH}^1 and y_{LH}^2 respectively, we have:

$$\begin{aligned}\rho_{LH}^y + \delta^1(1-p_L) - v_L^1(1-p_L) &= 0;\\ \rho_{LH}^y - \delta^2 p_L - v_H^2 p_L &= 0\end{aligned}$$

After optimizing with respect to y_{LL}^i, we have:

$$\rho_{LL}^y + \delta^i p_L - v_L^i p_L = 0, \text{ for } i = 1, 2.$$

In what follows, without loss of generality, we consider symmetric multipliers:

$$\delta \equiv \delta^1 = \delta^2, \quad v_H \equiv v_H^1 = v_H^2, \quad v_L \equiv v_L^1 = v_L^2$$

From the above equations, we have:

$$p_L(\delta + v_H) = (1-p_L)(v_L - \delta) \tag{11}$$

Step 2: The optimal reallocation given the manipulation $p^\phi(\theta^1, \theta^2, \widetilde{\phi})$
For simplicity, let $\rho_{jk,\widetilde{\phi}}^x = \rho^x(\theta_j, \theta_k, \widetilde{\phi})$. After optimizing with respect to $x_{HH,\widetilde{\phi}}^i$, we have:[39]

$$\rho_{HH,\widetilde{\phi}}^x + p_{HH,\widetilde{\phi}}^\phi(1 - p_L + \delta + v_H)(1-p_L)\theta_H u'(q^i(\widetilde{\phi}) + x_{HH,\widetilde{\phi}}^i) = 0, \text{ for } i = 1, 2, \text{ and any } \widetilde{\phi} \in \Theta^2.$$

These equations imply $q^1(\widetilde{\phi}) + x_{HH,\widetilde{\phi}}^1 = q^2(\widetilde{\phi}) + x_{HH,\widetilde{\phi}}^2$ for any $\widetilde{\phi} \in \Theta^2$. Since $x_{HH,\widetilde{\phi}}^1 + x_{HH,\widetilde{\phi}}^2 = 0$ from the budget balance constraint, we have $q^i(\widetilde{\phi}) + x_{HH,\widetilde{\phi}}^i = \frac{q^1(\widetilde{\phi}) + q^2(\widetilde{\phi})}{2}$ for each $\widetilde{\phi}$. Hence, any total quantity which is available to an HH-coalition is always split evenly between the two buyers. A similar result holds for an LL-coalition, since after optimizing with respect to $x_{LL,\widetilde{\phi}}^i$, we have:

$$\rho_{LL,\widetilde{\phi}}^x + p_{LL,\widetilde{\phi}}^\phi(p_L\theta_L - \delta\theta_H + v_L\theta_L)p_L u'(q^i(\widetilde{\phi}) + x_{LL,\widetilde{\phi}}^i) = 0, \text{ for } i = 1, 2 \text{ and any } \widetilde{\phi} \in \Theta^2.$$

[39] In homogeneous coalitions such as an HH or an LL- coalition, the reallocation cannot lead to corner solutions. In an $HL-$coalition, instead, this is conceivable but it is not going to occur when the seller designs the sale mechanism optimally. Hence, we only consider interior solutions for the reallocation problem.

Summarizing, we find

$$\begin{cases} q^i(\widetilde{\phi}) + x^i_{HH,\widetilde{\phi}} = \frac{q^1(\widetilde{\phi})+q^2(\widetilde{\phi})}{2}, \text{ for } i=1,2 \text{ and any } \widetilde{\phi}\in\Theta^2 \\ q^i(\widetilde{\phi}) + x^i_{LL,\widetilde{\phi}} = \frac{q^1(\widetilde{\phi})+q^2(\widetilde{\phi})}{2}, \text{ for } i=1,2 \text{ and any } \widetilde{\phi}\in\Theta^2 \end{cases} \tag{12}$$

After optimizing with respect to $x^1_{HL,\widetilde{\phi}}$ and $x^2_{HL,\widetilde{\phi}}$ respectively, we have:

$$\begin{aligned} \rho^x_{HL,\widetilde{\phi}} + p^{\phi}_{HL,\widetilde{\phi}}(1-p_L+\delta+v_H)p_L\theta_H u'(q^1(\widetilde{\phi})+x^1_{HL,\widetilde{\phi}}) &= 0, \text{ for any } \widetilde{\phi}\in\Theta^2, \\ \rho^x_{HL,\widetilde{\phi}} + p^{\phi}_{HL,\widetilde{\phi}}(p_L\theta_L-\delta\theta_H+v_L\theta_L)(1-p_L)u'(q^2(\widetilde{\phi})+x^2_{HL,\widetilde{\phi}}) &= 0, \text{ for any } \widetilde{\phi}\in\Theta^2. \end{aligned}$$

By using (11), we obtain from the two above equations:

$$\theta_H u'(q^1(\widetilde{\phi})+x^1_{HL,\widetilde{\phi}}) = \left(\theta_L - \frac{1-p_L}{p_L}(\Delta\theta)\epsilon\right) u'(q^2(\widetilde{\phi})+x^2_{HL,\widetilde{\phi}}), \text{ for any } \widetilde{\phi}\in\Theta^2, \tag{13}$$

where $\epsilon \equiv \frac{\delta}{1-p_L+\delta+v_H} \in [0,1)$. Thus, any total quantity available to an HL-coalition is split according to (13). Because of symmetry, a similar condition holds for an LH-coalition.

Step 3: Given M^{**}, no reallocation is implemented conditional on truthful reports and no manipulation of reports is profitable.
From (12) follows that no reallocation is implemented within an HH-coalition or an LL-coalition if no manipulation occurs (formally, (12) is satisfied with $x^1_{HH,HH} = x^2_{HH,HH} = x^1_{LL,LL} = x^2_{LL,LL} = 0$) since M^{**} is symmetric. From (13) we see that – if an HL-coalition reports truthfully – good are not reallocated in the coalition if $\epsilon = 1$ because $\theta_L - \frac{1-p_L}{p_L}(\Delta\theta)\epsilon = \theta^v_L$ when $\epsilon = 1$ and $\theta_H u'(q^*_H) = \theta^v_L u'(q^*_L)$ from Proposition 1. In other terms, the third-party does not modify the allocation $\mathbf{q}^*$ decided by the seller since he evaluates an L-type's surplus with the same virtual valuation as the seller. Note that the seller has some flexibility in choosing $\epsilon\in[0,1)$ since S^0 is optimal for the third party if and only if it satisfies the necessary and sufficient conditions for optimality in the third party's problem for at least one ϵ in $[0,1)$.[40]
By examining how the Lagrangian function $\mathcal{L}$ depends on $p^{\phi}_{HH,\widetilde{\phi}}$ and recalling (12), we see that an HH-coalition reports $\tilde{\phi}\in\Theta^2$ to the seller in order to maximize $2\theta_H u(\frac{q^1(\tilde{\phi})+q^2(\tilde{\phi})}{2}) - t^1(\tilde{\phi}) - t^2(\tilde{\phi})$, or $V_{HH}(q^1(\tilde{\phi})+q^2(\tilde{\phi})) - t^1(\tilde{\phi}) - t^2(\tilde{\phi})$ after defining $V_{HH}(x) \equiv 2\theta_H u(\frac{x}{2})$.

[40] Although ϵ belongs to $[0,1)$, we allow ϵ to take the value equal to one since we are interested in the Sup of the seller's profit.

Likewise, if we let $V_{HL}(x) \equiv \max_{z\in[0,x]} \theta_H u(z) + \theta_L^v u(x-z)$ and $V_{LL}(x) \equiv 2\theta_L^v u(\frac{x}{2})$, the reports of an HL-coalition and an LL-coalition are chosen to maximize $V_{HL}[q^1(\tilde{\phi}) + q^2(\tilde{\phi})] - t^1(\tilde{\phi}) - t^2(\tilde{\phi})$ and $V_{LL}[q^1(\tilde{\phi}) + q^2(\tilde{\phi})] - t^1(\tilde{\phi}) - t^2(\tilde{\phi})$, respectively. The coalition incentive constraint (5) emerges from requiring that a jk-coalition reports truthfully rather than $j'k'$.

In order to show that M^{**} satisfies all the coalition incentive constraints, we first need to prove the following single crossing condition for coalitions:

$$V'_{HH}(x) > V'_{HL}(x) > V'_{LL}(x) \text{ for any } x > 0 \tag{14}$$

We have $V'_{HH}(x) = \theta_H u'(\frac{x}{2})$ and $V'_{LL}(x) = \theta_L^v u'(\frac{x}{2})$. For an HL-coalition, consider for simplicity interior allocations (but the proof is easily adapted to the non-interior case). Then $q_H(x)$ and $q_L(x)$ are such that $q_H(x) + q_L(x) = x$, $\theta_H u'[q_H(x)] = \theta_L^v u'[q_L(x)]$ and the envelope theorem implies $V'_{HL}(x) = \theta_H u'[q_H(x)] = \theta_L^v u'[q_L(x)]$. Since u' is strictly decreasing and $\theta_H > \theta_L^v$, $q_H(x) > \frac{x}{2} > q_L(x)$ must hold; hence $V'_{HH}(x) = \theta_H u'(\frac{x}{2}) > \theta_H u'[q_H(x)] = \theta_L^v u'[q_L(x)] > \theta_L^v u(\frac{x}{2}) = V'_{LL}(x)$.

Armed with (14) we recall that (i) the transfers in M^{**} are such that the local downward coalition incentive constrains ($CIC_{HH,HL}$) and ($CIC_{HL,LL}$) bind; (ii) $2q^*_{HH} > q^*_{HL} + q^*_{LH} > 2q^*_{LL}$. These properties suffice to invoke a result from the theory of monopolistic screening [see Section 3 in Maskin and Riley (1984)] to conclude that (5) is satisfied for any jk and $j'k'$. Thus, M^{**} is weakly collusion-proof. ■

Proof that $\beta < 0$

Let $g(z) \equiv V_{HL}(2q^*_L+z) - V_{HL}(2q^*_L) - \theta_H\left[u(q^*_L+z) - u(q^*_L)\right]$; we want to show that $g(q^*_H - q^*_L) < 0$ because $\beta = g(q^*_H - q^*_L)$. Since $g(0) = 0$ and we can prove that $g'(z) < 0$ $\forall z \in [0, q^*_H - q^*_L)$, we obtain $g(q^*_H - q^*_L) < 0$. We find $g'(z) \equiv \theta_H u'\left[q_H(2q^*_L + z)\right] - \theta_H u'(q^*_L + z)$ where $q_H(x)$ – as in the proof of proposition 4, step 3, – is the quantity the third-party gives to an H-type in an HL-coalition given the total quantity x available to the coalition, hence $\theta_H u'[q_H(x)] = \theta_L^v u'[q_L(x)]$ holds. $g'(z) < 0$ is equivalent to $q_H(2q^*_L + z) > q^*_L + z$, which holds for $\forall z \in [0, q^*_H - q^*_L)$ since otherwise we have $\theta_H u'\left[q_H(2q^*_L + z)\right] \geq \theta_H u'(q^*_L + z) > \theta_H u'(q^*_H) = \theta_L^v u'(q^*_L) > \theta_L^v u'\left[q_L(2q^*_L + z)\right]$, a violation of $\theta_H u'[q_H(x)] = \theta_L^v u'[q_L(x)]$ for $x = 2q^*_L + z$.

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