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Nominal Debt as a Burden on Monetary Policy

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Abstract

We study the effects of nominal debt on the optimal sequential choice of monetary policy. When the stock of debt is nominal, the incentive to generate unanticipated inflation increases the cost of the outstanding debt even if no unanticipated inflation episodes occur in equilibrium. Without full commitment, the optimal sequential policy is to deplete the outstanding stock of debt progressively until these extra costs disappear. Nominal debt is therefore a burden on monetary policy, not only because it must be serviced, but also because it creates a time inconsistency problem that distorts interest rates. The introduction of alternative forms of taxation may lessen this burden, if there is enough commitment to fiscal policy. Full commitment for the fiscal authority can override any commitment problem of the monetary authority.

JEL Classification Numbers: E40, E50, E58, and E60

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1 Introduction

Fiscal discipline has often been seen as a precondition to sustain price stability. Such is, for example, the rationale behind the Growth and Stability Pact in Europe. The policy discussion seems to be fueled by the awareness of a time inconsistency problem likely to be associated with high levels of nominal debt. While this discussion is certainly worth undertaking, it is usually misguided by insufficient theoretical discipline. In this paper we analyze the implications of nominal debt for optimal sequential policy, using as framework a very simple model but still one that provides powerful insights into optimal dynamic monetary policy and accumulation of debt in environments without commitment.

We show how optimal monetary policy differs depending on whether there is nominal or indexed debt and on the degree of commitment of monetary authorities. Because debt is nominal there is an ex-post incentive to deplete it, so that future distortionary taxes may be lower. There is also a cost. Agents must use money balances that are predetermined and therefore a depletion of the real value of nominal debt also means lower real balances and therefore lower consumption. This incentive to deplete the real value of debt depends on the level of debt. It is a feature of optimal policy in an equilibrium without commitment that the amount of debt is reduced to the point where those incentives disappear. Rational governments, even when noncommitted, optimally manipulate the incentives of future governments by choosing the level of debt as the state variable for the optimal choice of their successors.

We start by computing the optimal policy that obtains when the stock of debt is nominal and there is full commitment to monetary policy. We show that, in this case, it is optimal to monetize part of the initial stock of nominal debt. Consumption is constant from the second period onwards, and in the initial period it is lower. This incentive to deplete the real value of nominal debt is present every period if there is no commitment.

We proceed to study optimal policy in an environment where the stock of debt is nominal and there is no commitment to monetary policy. We restrict attention to the Markov perfect equilibrium. We call this equilibrium recursive as in Cole and Kehoe (1996), and in Obstfeld (1997). Two interesting features of the optimal policy that obtains in this economy are that the optimal inflation tax is non-stationary, and that it converges to the inflation tax that obtains when there is no government debt. This last result arises because, in the recursive equilibrium, it is optimal for the government to reduce the value of the stock of nominal government debt until it is asymptotically zero. We show that nominal debt is indeed a burden for monetary policy not only because it has to be serviced, but also because of the dynamic distortion associated with the time inconsistency. Not surprisingly, when the initial nominal liabilities are the same, welfare under full commitment is higher than welfare under no commitment.

As a benchmark we characterize the optimal policy that obtains when the stock of government debt is indexed. Our results build on those of Nicolini (1998), who shows that, when the utility function is logarithmic in consumption and linear in leisure, and the stock of government debt is indexed, the optimal monetary policy is to abstain from inflation surprises. This result follows from applying optimal taxation principles, and it means that the solution to the Ramsey problem is time consistent in this model economy. Furthermore, this solution is also stationary, and there is a unique interest rate that balances the government budget.

We proceed to compare the equilibrium allocations in the two economies without commitment, with and without indexed debt. We discuss in detail how to make the welfare comparisons meaningful, taking into account that in the nominal economies the initial conditions are in nominal terms and that in the indexed economy the initial condition for debt is in real terms.

We finally ask whether the results are robust to the introduction of additional taxes. This is important since, in advanced economies, seigniorage is a minor source of tax revenues, and we want to know if our results still hold when government outlays are financed with other taxes. Specifically, we study the case of consumption taxes. First, we impose the natural assumption that taxes are chosen before the monetary policy decisions are made. In particular they are assumed to be chosen one period in advance. We find that the same equilibria result when there are both seigniorage and consumption taxes than when there is only seigniorage, provided that the optimal monetary policy distortions can be supported with strictly positive nominal interest rates.

When, instead, there is enough fiscal commitment, the fiscal authority can constrain the monetary authority to follow the Friedman rule, of zero nominal rates, from the outset. In this case, since negative interest rates cannot be sustained in equilibrium, the monetary authority has no incentive to monetize the debt and, as a result, it implements the optimal equilibrium with commitment.[1]

The relationship between fiscal and monetary policy has been addressed in the unpleasant monetarist arithmetic literature of Sargent and Wallace (1981), and in the fiscal theory of the price level of Sims (1994) and Woodford (1996). In these approaches, however, policies are taken to be exogenous. This is not the case in our analysis, nor in the related work of Chari and Kehoe (1999), Ellison and Rankin (2005), and Obstfeld (1997). These last two papers are the closest to ours. Both, however, assume that debt is real, and they focus only on monetary policy. They aim at characterizing the Markov perfect equilibrium when the source of the time inconsistency of monetary policy is related to the depletion of the real value of money balances. This source of time inconsistency is ambiguous: while in Lucas and Stokey (1983) the government would want to completely deplete the outstanding money balances, in Svensson (1985)'s set up, as was shown in Nicolini (1998), under certain elasticity conditions, the government problem would be time consistent. This ambiguity led Obstfeld (1997) to consider an ad-hoc cost of a surprise inflation. Our analysis differs from Obstfeld's both because we consider nominal debt, and because, in our model economy, the cost of unanticipated

[1] See also Marimon, Nicolini and Teles, 2003.

inflation arises from the timing of the cash-in-advance constraint, rather than being imposed ad-hoc. In a similar framework, Ellison and Rankin (2005) show that with alternative preferences to the ones in Nicolini (1998) the level of real debt matters for the direction of the time inconsistency problem. They show that there can be a value of real debt such that the elasticity is unitary and, therefore, there exists a steady state with positive debt.

An additional contribution of this paper is the full characterization and the computation of the optimal policy in a recursive equilibrium with a state variable. In this respect, our work is closely related to the recent work of Krusell, Martín and Ríos-Rull (2003) who characterize the recursive equilibria that obtain in an optimal labor taxation problem.

2 The model economy

In our model economy there is a representative household and a government. In each period $t \geq 0$, the government issues currency M^g_{t+1} and nominal debt B^g_{t+1}, to finance an exogenous and constant level of public consumption g.[2] Initially, we abstract from all other sources of public revenues. The sequence of government budget constraints is the following:

$$M^g_{t+1} + B^g_{t+1} \geq M^g_t + B^g_t(1 + i_t) + p_t g,\ \ t \geq 0 \tag{1}$$

where i_t is the nominal interest rate paid on debt issued by the government at time $t-1$, and p_t is the price of one unit of the date t composite good in units of money. The initial stock of currency, M^g_0, and initial debt liabilities, $B^g_0(1 + i_0)$, are given. A government policy is, therefore, a specification of $\{M^g_{t+1}, B^g_{t+1}, g\}$ for $t \geq 0$.

We assume that the household's preferences over consumption and labor can be represented by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t] \tag{2}$$

where $c_t > 0$ denotes consumption at time t, n_t denotes labor at time t, and $0 < \beta < 1$ is the time discount factor. We assume that the utility of consumption satisfies the standard assumptions of being strictly increasing and strictly concave. For reasons that will become clear below, in most of this article we assume that the utility is logarithmic in consumption, i.e., $u(c) = \log(c)$.

We assume that consumption in period t must be purchased using currency carried over from period $t-1$ as in Svensson (1985). This timing of the cash-in-advance

[2] We assume that government expenditures, g, are given, although our analysis can easily be extended to the case of endogenous government expenditures.

constraint implies that the representative household takes both M_0 and $B_0(1+i_0)$ as given when solving its maximization problem, and it is crucial to obtain the results that we report here. The specific form of the cash-in-advance constraint faced by the representative household is:

$$p_t c_t \leq M_t \tag{3}$$

for every $t \geq 0$.

To simplify the production side of this economy, we assume that labor can be transformed into either the private consumption good or the public consumption good on a one-to-one basis. The economy's resource constraint is:

$$c_t + g \leq n_t \tag{4}$$

for every $t \geq 0$.

Each period the representative household faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - p_t c_t + B_t(1+i_t) + p_t n_t \tag{5}$$

where M_{t+1} and B_{t+1} denote, respectively, the stock of money and the stock of nominal government debt that the household carries over from period t to period $t+1$. Finally, we assume that the representative household faces a no-Ponzi games condition:

$$\lim_{T \longrightarrow \infty} \beta^T \frac{B_{T+1}}{p_T} \geq 0 \tag{6}$$

2.1 A competitive equilibrium

Definition 1 *A competitive equilibrium for an economy with nominal debt is a government policy, $\{M^g_{t+1},\ B^g_{t+1}, g, \}_{t=0}^{\infty}$, an allocation $\{M_{t+1}, B_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, and a price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, such that:*

(i) given M^g_0 and $B^g_0(1+i_0)$, the government policy and the price vector satisfy the government budget constraints described in expression (1);

(ii) when households take M_0, $\mathrm{B}_0(1+\mathrm{i}_0)$ and the price vector as given, the allocation maximizes the problem described in expression (2), subject to the cash-in-advance constraints described in expression (3), the household budget constraints described in expression (5), and the no-Ponzi games condition described in expression (6); and

(iii) all markets clear, that is: $M^g_{t+1} = M_{t+1}$, $B^g_{t+1} = B_{t+1}$, and g and $\{c_t, n_t\}_{t=0}^{\infty}$ satisfy the economy's resource constraint described in expression (4), for every $t \geq 0$.

Given our assumptions on the utility of consumption u, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies both the economy's resource constraint (4) and the household's budget constraint (5) with equality, and that the first order conditions of the Lagrangian of the household's problem are both necessary and sufficient to characterize the solution to the household's problem. The cash in advance constraint, (3), will be binding in every period $t \geq 0$ if $\frac{u'(c_t)}{\alpha} > 1$. Since $\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}$, $t \geq 0$, this will be the case whenever $i_{t+1} > 0$. In period zero the cash in advance constraint will be binding whenever $c_0 \leq c_{t+1}$, $t \geq 0$. This will be a feature of the equilibria that we characterize.

The competitive equilibrium allocation of an economy with nominal debt can be completely characterized by the following conditions that must hold for every $t \geq 0$:

$$\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}, \tag{7}$$

$$1 + i_{t+1} = \beta^{-1} \frac{p_{t+1}}{p_t}, \tag{8}$$

$$c_t = \frac{M_t}{p_t}, \tag{9}$$

the government budget constraints (1), the resource constraints (4), and the transversality condition (6)

$$\lim_{T \longrightarrow \infty} \beta^T \left(\frac{M_{T+1} + B_{T+1}}{p_T} \right) = 0 \tag{10}$$

2.2 Implementability

When choosing its policy the government takes into account the above equilibrium conditions. These conditions can be summarized with implementability conditions in terms of the real allocations. In particular, the government budget constraint (1) with

equality can be written as the implementability condition

$$c_{t+1} u'(c_{t+1}) \frac{\beta}{\alpha} + \beta z_{t+1} c_{t+1} = c_t + z_t c_t + g, \;\; t \geq 0 \tag{11}$$

where

$$z_t \equiv \frac{B_t^g (1 + i_t)}{M_t^g}$$

To see this, notice that the constraint (1) with equality can be written in real terms as

$$\frac{M_{t+1}^g}{p_t}+\frac{B_{t+1}^g}{p_t}=\frac{M_t^g}{p_t}+\frac{B_t^g(1+i_t)}{p_t}+g$$

and, using the first order conditions of the households problem, (7), (8) and (9), $\frac{M_t^g}{p_t}=c_t$; $\frac{M_{t+1}^g}{p_t}=\frac{M_{t+1}^g}{p_{t+1}}\frac{p_{t+1}}{p_t}=c_{t+1}\beta(1+i_{t+1})=c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha}$; $\frac{B_t^g(1+i_t)}{p_t}=\frac{B_t^g(1+i_t)}{M_t^g}\frac{M_t^g}{p_t}=z_tc_t$, and $\frac{B_{t+1}^g}{p_t}=\frac{B_{t+1}^g(1+i_{t+1})}{M_{t+1}^g}\frac{M_{t+1}^g/p_{t+1}}{p_t(1+i_{t+1})/p_{t+1}}=\beta z_{t+1}c_{t+1}$.

From the transversality condition (10) we have that $\lim_{T\longrightarrow\infty}\beta^T(c_{T+1}u'(c_{T+1})\frac{\beta}{\alpha}+$ $+\beta z_{T+1}c_{T+1})=0$, which implies that the present value government budget constraint takes the form

$$\sum_{t=0}^{\infty}\beta^t\left(c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha}-(c_t+g)\right)=z_0c_0 \tag{12}$$

In the log case, this reduces to

$$\sum_{t=0}^{\infty}\beta^t\left(\frac{\beta}{\alpha}-g-c_t\right)=z_0c_0 \tag{13}$$

This condition summarizes the competitive equilibrium restrictions on the sequence of consumption $\{c_t\}_{t=0}^{\infty}$.

3 Optimal policy with nominal debt

3.1 The case with full commitment

We start the analysis by studying the optimal monetary policy that obtains when the stock of government debt is nominal and the government can fully commit. When debt is not indexed a higher initial price level depletes the real value of both money balances and debt, which is a seigniorage tax at $t=0$ that can be levied without affecting the commitment to future interest rates. The fact that consumption must be purchased with currency carried over from the previous period means that there is a cost in reducing the real value of money balances. Even if this cost is present, there may be an incentive for a Ramsey government to increase the initial seigniorage tax at $t=0$, and to use its proceeds to reduce the need for future distortionary taxation.

Definition 2 *A **full commitment Ramsey equilibrium** with nominal debt is a competitive equilibrium such that $\{c_t\}$ solves the following problem:*

$$Max\ \sum_{t=0}^{\infty}\beta^t\left[u(c_t)-\alpha(c_t+g)\right] \tag{14}$$

subject to the implementability condition (12).

The other competitive equilibrium variables which are the government policy $\{M_{t+1}^g, B_{t+1}^g, g, \}_{t=0}^{\infty}$, the allocation $\{M_{t+1}, B_{t+1}, n_t\}_{t=0}^{\infty}$, and the price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, are obtained using the competitive equilibrium conditions.

In the log case where $u(c_t) = \ln(c_t)$, the full commitment Ramsey equilibrium is characterized by the following conditions:

$$\frac{1}{c_0^F} - \alpha = \left[\frac{1}{c_1^F} - \alpha\right][1 + z_0] \tag{15}$$

$$c_{t+1} = c_1^F, \text{ for } t \geq 1. \tag{16}$$

Notice that, as long as $z_0 > 0$, c_0^F is smaller than c_1^F. The initial outstanding nominal debt is depleted at the expense of a lower real stock of money, and, therefore, lower consumption.

3.2 Optimal policy with nominal debt and no commitment

When the government cannot commit to its monetary policy, the incentive to monetize part of the debt discussed in the previous section arises every period. In other words, the special features of 'period zero' in the full commitment economy with nominal debt become the recurrent features of the economy without commitment.

In each period $t \geq 0$, the government chooses c_t and is not able to commit. We look at Markov perfect equilibria where c_t does not depend on the whole history up to period t but may only depend on the pay-off relevant state variable z_t. That is, the government policy is a function C such that the consumption path is sequentially defined by $c_t = C(z_t)$.

Definition 3 *A* ***recursive monetary equilibrium*** *with nominal debt is a value function V(z) and policy functions C(z) and Z(z) such that $c = C(z)$ and $z' = Z(z)$ solve*

$$V(z) = \max_{\{c,z'\}}\{u(c) - \alpha(c+g) + \beta V(z')\} \tag{17}$$

subject to

$$C(z')u'(C(z'))\frac{\beta}{\alpha} + \beta z' C(z') = zc + c \tag{18}$$

The recursive equilibrium can be obtained by solving the following dynamic programming problem

$$V(z) = \max_{\{c,z'\}}\{u(c) - \alpha(c+g) + \beta V(z')\} \tag{19}$$

s.t.

$$\overline{C}(z')u'(\overline{C}(z')) + \beta z'\overline{C}(z') = c(1+z) \tag{20}$$

for an arbitrary function $\overline{C}$. As a solution to this problem there is an optimal policy $\overline{\overline{C}} = \mathcal{F}(\overline{C})$, where $\mathcal{F}$ maps the –exogenously given– functions into optimal policy functions. A recursive equilibrium is then a fixed point $C = \mathcal{F}(C)$.

In the log case, this simplifies to finding a function C that is the optimal policy to the problem

$$V(z) = \max\{\log(c) - \alpha(c+g) + \beta V(z')\} \tag{21}$$

s.t.

$$\frac{\beta}{\alpha} - g + \beta z' C(z') = c(1+z) \tag{22}$$

To characterize the recursive monetary equilibrium, notice that the first order conditions of the problem described in expressions (21) and (22) are

$$\frac{1}{c} - \alpha = \lambda(1+z) \tag{23}$$

and

$$V'(z') = -\lambda C(z')[1 + \varepsilon_c(z')] \tag{24}$$

$\varepsilon_c(z) = \frac{zC'(z)}{C(z)}$ is the elasticity of $C(z)$. This implies

$$\frac{\frac{1}{c} - \alpha}{1+z} = -\frac{V'(z')}{C(z')}[1 + \varepsilon_c(z')]^{-1} \tag{25}$$

This condition equates the 'reduced' marginal gain of one additional unit of consumption to its 'amplified' marginal cost associated with higher debt needed to finance this consumption. The gain is 'reduced', by the factor $1+z$, since increasing consumption, by decreasing the current price level, increases the real value of the outstanding stock of debt. Alternatively, lowering consumption, with an increase in the current price level, has the benefit of partially monetizing the current stock of debt. The cost of the debt is 'amplified' since increasing the stock of debt makes future consumption more costly by facing the trade-off of monetizing a higher stock of debt. Notice that the first distortion ('reduction') is a price level effect, already present in the period zero of the

Ramsey equilibrium (15), while the second distortion ('amplification') is an interest rate effect, not present in the Ramsey equilibrium, reflecting the intertemporal distortions that appear along a recursive equilibrium path when nominal debts can be sequentially monetized.

Using the envelope condition,

$$V'(z) = -\lambda c$$

for $\frac{V'(z')}{c'}$, equation (25), translates into the following intertemporal condition

$$\frac{\frac{1}{c} - \alpha}{1+z} = \frac{\frac{1}{c'} - \alpha}{1+z'} [1 + \varepsilon_c(z')]^{-1} \tag{26}$$

which can be rewritten as

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{(1+i)B}{M}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{(1+i')B'}{M'}\right]} \left[1 + \varepsilon_c \left(\frac{(1+i')\, B'}{M'}\right)\right]^{-1} \tag{27}$$

This intertemporal equation reflects the different distortions present as a result of debt being nominal and policy decisions being sequential. In the next section we solve for the optimal policy with indexed debt and show that the terms $\left[1 + \frac{(1+i)B}{M}\right]$ and $\left[1 + \varepsilon_c \left(\frac{(1+i')B'}{M'}\right)\right]$ are not present in the corresponding intertemporal condition. Neither the discretionary incentive to reduce the real value of debt, nor the dynamic effect on these incentives associated with the accumulation of debt are there if debt is indexed.

The recursive equilibrium is less efficient than the full commitment Ramsey equilibrium. The full commitment Ramsey solution is the choice of a sequence of consumption $\{c_t\}_{t=0}^{\infty}$ that maximizes welfare in the set of competitive equilibrium sequences defined by (13). Since the utility function is strictly concave and the set of restrictions is convex, there is a single maximum. The recursive equilibrium is a competitive equilibrium, and therefore, being different, must give lower welfare.

The point is clear when we think of the recursive equilibrium as the equilibrium of a game between successive governments. The recursive equilibrium, imposes as additional restrictions the optimality of decisions by future governments. More formally,

Proposition 1 *Assume that $u(c) = \log(c)$. Consider two identical economies with the same initial conditions, $B_0(1+i_0)$ and M_0. The full commitment Ramsey equilibrium gives higher welfare than the no-commitment Recursive equilibrium.*

4 Optimal policy with indexed debt and welfare comparison

4.1 Indexed debt

In this section we study the optimal policy when the stock of government debt is indexed. This is the benchmark against which we compare the optimal policy that obtains when the stock of government debt is nominal —that is, not indexed— which is the main focus of this article.

In the economies analyzed in the previous two sections, the real value of the outstanding debt, b_t, is given by $b_t \equiv \frac{B_t(1+i_t)}{p_t} = \frac{B_t(1+i_t)}{M_t}\frac{M_t}{p_t} = z_t c_t$, where z_t is predetermined. In the economy where debt is indexed to the price level, b_t is predetermined. The nominal interest rate adjusts to movements in the price level, as to keep the real interest rate unchanged at $\beta^{-1} - 1$, which is equivalent to z_t adjusting to movements in c_t, as to keep b_t unchanged.

A competitive equilibrium for an economy with indexed debt is defined as a government policy, $\{M^g_{t+1}, b^g_{t+1}, g, \}_{t=0}^{\infty}$, an allocation $\{M_{t+1}, b_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, and a price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, such that the conditions (i), (ii) and (iii) of Definition 1 are satisfied when nominal liabilities are replaced by real liabilities, according to $\frac{B_t(1+i_t)}{p_t} = b_t$, where b_t is predetermined.

The design of the government optimal policy consists in solving the problem:

$$\max_{\{c_t,n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha(c_t + g)]$$

$$c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha} + \beta b_{t+1} = c_t + g + b_t, \quad t \geq 0$$

This problem is recursive when $u(c) = \ln(c)$. In this case, the price elasticity is unitary and, as Nicolini (1998) has shown, the optimal monetary policy is time-consistent. This problem can be written recursively as follows:

$$V(b) = \max_{c,b'} \{\log(c) - \alpha(c+g) + \beta V(b')\} \tag{28}$$

subject to:

$$\frac{\beta}{\alpha} + \beta b' = c + g + b \tag{29}$$

The first order condition for c is:

$$\frac{1}{c} - \alpha = -\beta V'(b') \tag{30}$$

That is, the marginal gain of increasing consumption is equated to the marginal cost of increasing future debt, without the price distortions present in (25). Using the envelope theorem, we have

$$V'(b) = V'(b'), \tag{31}$$

and, substituting expression (30) into this expression, we obtain that

$$\frac{1}{c} - \alpha = \frac{1}{c'} - \alpha \tag{32}$$

which implies that the optimal level of consumption, c^I, is constant and equal to:

$$c^I = \frac{\beta}{\alpha} - g - (1-\beta)\, b_0. \tag{33}$$

Notice that expression (31) implies that the real value of the government debt is stationary and, consequently, that $b^I = b_0$.

4.2 Welfare comparisons

As we have seen in Proposition 1, if we compare two economies with identical initial nominal liabilities, M_0 and $B_0(1+i_0)$, welfare is higher in the economy with full commitment. In this section we extend this comparison to economies with indexed debt.

Indexed debt can be viewed as an extreme form of commitment. Therefore, we want to compare the full commitment Ramsey equilibrium allocation in the economy with nominal debt with the equilibrium allocation that obtains in a comparable economy when debt is indexed. Notice that in order for this comparison to be meaningful, we must choose the appropriate initial conditions. To do this, we require that the initial money holdings, M_0, and the real value of the initial debt liabilities in both economies must be the same. This is not completely straightforward because while the initial real liabilities, b_0, are exogenous in the indexed debt economy, they are endogenous in the nominal debt economies.

If we start with the nominal economy with full commitment, then the real value of the initial nominal liabilities is $b_0 = z_0 c_0^F(z_0) = \frac{B_0(1+i_0)}{p_0^F}$. Then, we can make the following welfare comparison:

Proposition 2 *Assume that $u(c) = \log(c)$. Consider two economies with initial money stock M_0. One of them has initial nominal debt $B_0(1+i_0)$, and the other has initial indexed debt b_0. If $b_0 = \frac{B_0(1+i_0)}{p_0^F}$, then the welfare in the economy with indexed debt is higher than in the economy with nominal debt.*

Proof: In the case with indexed debt we have that consumption is constant over time $c^I = \frac{\beta}{\alpha} - g - (1-\beta) b_0$. In the case with nominal debt, using the implementability condition (13), we have

$$(1-\beta) c_0^F + \beta c_1^F = \frac{\beta}{\alpha} - g - (1-\beta) z_0 c_0^F \tag{34}$$

where

$$\frac{1}{c_0^F} - \alpha = \left[\frac{1}{c_1^F} - \alpha \right] [1 + z_0] \tag{35}$$

If $b_0 = z_0 c_0^F$, then, by Jensen's inequality, the welfare in the case with indexed debt is higher than in the case with nominal debt.■

In the economy with nominal debt it is optimal to reduce the initial level of consumption because monetizing part of this debt creates no distortions, since there is no time zero indexation to internalize, and because it reduces the amount of future seigniorage. By imposing that $b_0 = \frac{B_0(1+i_0)}{p_0^F}$, we restrict our attention to the distortion introduced by the incentive to monetize the nominal debt, and we abstract from the gain introduced by the reduction in its real value.

In the discussion above we started with a nominal economy with initial condition z_0 and we compared it to an indexed economy with initial condition $b_0^F = b_F(z_0) \equiv z_0 c_0^F(z_0)$. Alternatively, we can start with an indexed economy with initial condition b_0 and compare it with a nominal economy with full commitment and initial condition $z_0^F = b_F^{-1}(b_0)$. In this economy, the Ramsey equilibrium has the property that the real value of the initial debt is $\frac{B_0(1+i_0)}{p_0^F} = b_0$. Furthermore, since we consider economies with the same M_0, choosing z_0 is equivalent to choosing $B_0(1+i_0) = z_0 M_0$.[3]

Likewise, we can compare a nominal debt economy with no commitment with an indexed debt economy. In this comparison, if the initial condition of the nominal economy is z_0, then the initial condition of the indexed economy should be $b_0^N = b_N(z_0) \equiv z_0 C(z_0)$, where $C(z)$ is the policy function of the recursive monetary equilibrium. Alternatively, we could start with an indexed economy with initial condition b_0 and compare it with a nominal economy with no commitment and initial condition $z_0^N = b_N^{-1}(b_0)$.

In the following section we compute the equilibrium paths of economies with the same initial real liabilities b_0. In the indexed economy the exogenous initial state is this b_0. In the nominal economies the exogenous initial states are $z_0^F = b_F^{-1}(b_0)$ and $z_0^N = b_N^{-1}(b_0)$.

[3]In the computations in next section we look at economies where $1 + i_0 = u'(c_0^F)/\alpha$ since this are economies in which the realized real return in period $t = 0$ is $1 + r_0 = \beta^{-1}$. This is equivalent to choosing $B_0 = \frac{z_0 M_0}{1+i_0}$.

5 Numerical solutions

Our model economies are characterized by three parameters: α, β and g. To make them comparable, we follow the strategy of equating the initial values of real debt, b_0. The economy with indexed debt is computed directly using expression 33. The economy with nominal debt and full commitment is computed solving a system of two equations in two unknowns. The equations are (34) and (35) and the unknowns are c_0^F and c_1^F. To find the initial condition z_0^F, we define $b_F(z) \equiv zc_0^F(z)$. This function is invertible and we use it to obtain $z_0^F = b_F^{-1}(b_0)$. As we have discussed in the previous section, in this economy, we define $1 + i_0^F \equiv 1/(\alpha c_0^F)$ and we have $1 + i_1^F = 1/(\alpha c_1^F)$. By equation (35) this implies that $i_0^F = i_1^F \left[1 + z_0^F\right]$.

The algorithm that we use to compute the recursive competitive equilibrium is described in the Appendix. The output of this algorithm is the policy function $C(z)$ that we use to define $b_N(z) \equiv zC(z)$. This function is invertible and we use it to obtain the new initial condition $z_0^N = b_N^{-1}(b_0)$. Since $c_0^F(z_0) > c_0^N(z_0)$ for any z_0, it turns out that the implied $z_0^F < z_0^N$. In this economy, $1 + i_t^N = 1/(\alpha C(z_t^N)$ for all $t \geq 0$.

The values that we use in our computational exercise to identify the parameters of the model economies are $\alpha = 0.45$, $\beta = 0.98$, and $g = 0.00822$. To compute the time paths we choose the initial condition $b_0 = 0.17865$. Since our period corresponds to a year, this value of b_0 is intentionally chosen to be very high. Specifically, the value of the debt to government expenditures ratio is $b_0/g \simeq 22$. The results that we obtain for the time paths of real debts, nominal interest rates, and consumption are reported in Figures 1, 2, and 3.

Figure 1 shows that in the economies with indexed debt is to optimal to keep its value stationary, and that in the economies with nominal debt it is optimal to reduce its real value. Under full commitment this reduction is carried out only in the first period, while under no commitment, the real value of debt is depleted progressively until, asymptotically, it is completely monetized. Recall that in these economies the time paths of real debt are given by $b_t^F = z_t^F c_t^F$ and by $b_t^N = z_t^N c_t^N$.

We also find that the long-run nominal interest rate that obtains when debt is indexed is higher than the nominal interest rates that obtain when debt is nominal. Moreover, the long-run interest rate under full commitment is higher than the one that obtains than under no commitment (see Figure 2). Notice that in the three economies the nominal interest rates in period $t = 0$ satisfy the condition $1 + i_0 = 1/(\alpha c_0)$.

Finally, Figure 3 compares the optimal consumption paths. Using these paths and the corresponding optimal labor paths we compute the utility value of the optimal equilibrium paths. We find that this value is highest in the economy with indexed debt and that it is smallest in the economy with nominal debt and no commitment. Specifically, in the economy with nominal debt and full commitment the value of the optimal path is 0.012 percent smaller than in the economy with indexed debt, and in the economy with nominal debt and no commitment the value of the optimal path is

0.133 percent smaller.

6 Additional taxes

In most advanced economies, seigniorage is a minor source of revenue, and government liabilities are financed mostly with consumption and income taxes. In this section we show two basic results, regarding the introduction of taxes in our economy. First, we show that the introduction of taxes, while it will reduce the need to raise revenues through seigniorage, may not change the characterization of equilibria with respect to the economies with monetary policy only, analyzed in the previous sections. This is the case if the fiscal authority simply sets taxes one period in advance and, subsequently, the monetary authority sets its policy. This adds realism to the analysis of the interaction of fiscal and monetary policies. Second, we show that, in contrast, if there is full commitment on the part of the fiscal authority that makes its policy choices in the initial period, before the monetary authority does, then the full commitment outcome can be achieved even if there is no commitment on the part of the monetary authority. We show that it is part of such policy to finance all the outstanding government liabilities with the consumption tax, and to constrain the monetary authority to implement a zero nominal interest rate.

We show these results introducing consumption taxes, $\{\tau_t\}_{t=0}^{\infty}$. The analysis easily generalizes to the introduction of other taxes. However, it is not our intention to provide a complete characterization of all possible fiscal instruments, neither to consider all different games that fiscal and monetary authorities can play.

6.1 The model economy with consumption taxes

When the government levies consumption taxes, the household problem becomes:

$$\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha n_t\right] \tag{36}$$

subject to:

$$p_t(1+\tau_t)c_t \leq M_t \tag{37}$$

$$M_{t+1} + B_{t+1} \leq M_t - p_t(1+\tau_t)c_t + B_t(1+i_t) + p_t n_t \tag{38}$$

and to:

$$\lim_{T \longrightarrow \infty} \beta^T \frac{B_{T+1}}{p_T} \geq 0 \tag{39}$$

Now, expressions (7), (8) and (9), that characterize the households's optimal choice, become:

$$\frac{u'(c_{t+1})}{\alpha} = (1 + i_{t+1})(1 + \tau_{t+1}) \tag{40}$$

$$1 + i_{t+1} = \beta^{-1}\frac{p_{t+1}}{p_t} \tag{41}$$

and

$$c_t = \frac{M_t}{p_t(1 + \tau_t)} \tag{42}$$

These conditions must hold for every $t \geq 0$. Notice that expression (40) reflects the fact that the household makes its plans based on its expectations about both interest rates and taxes. The intertemporal condition (41) is exactly the same as expression (40) and the cash-in-advance constraint (42) now includes consumption taxes.

The sequence of government budget constraints in this economy is now given by:

$$p_t g + M_t^g + B_t^g(1 + i_t) \leq p_t \tau_t c_t + M_{t+1}^g + B_{t+1}^g \tag{43}$$

and the feasibility conditions (4) do not change.

The implementability conditions can be written as

$$u'(c_{t+1})c_{t+1}\frac{\beta}{\alpha} + z_{t+1}c_{t+1}\beta(1 + \tau_{t+1}) = z_t c_t(1 + \tau_t) + c_t + g \tag{44}$$

together with the terminal condition

$$\lim_{T \longrightarrow \infty} \beta^T c_{T+1}u'(c_{T+1})\frac{\beta}{\alpha} + \beta z_{T+1}c_{T+1}(1 + \tau_{T+1})) = 0 \tag{45}$$

6.2 Optimal monetary policy when the fiscal authority moves one period in advance

We now consider the case where tax decisions for some period t must be made one period in advance, and may depend only on the state at $t-1$. In this case we can define the new state variable $\widehat{z}_t \equiv z_t(1 + \tau_t)$, and the problems are isomorphic to the problems in the previous sections, since the implementability condition (44) reduces to

$$u'(c_{t+1})c_{t+1}\frac{\beta}{\alpha} + \widehat{z}_{t+1}c_{t+1}\beta = \widehat{z}_t c_t + c_t + g \tag{46}$$

which is formally identical to (11).

There is an additional restriction that the nominal interest rate must be nonnegative. This constraint was satisfied when seigniorage was the only source of revenue, but it is not necessarily satisfied in this case.

We can model the interaction between the monetary and the fiscal authority in two ways. One way is to assume that they decide jointly as a single authority. In this case the analysis goes through as before and we obtain the same results in terms of allocations. The alternative is to assume some given strategy for taxes as a function of the state. As long the nominal interest rates are away from the lower bound of zero nominal interest rates, the problem for the monetary authority has the same structure as before, and therefore the same results go through, even if part of the government liabilities are financed with taxes.

In summary, the monetary authority faces the same problem with consumption taxes than the one faced when there was only seigniorage, for any degree of monetary commitment. Therefore, the allocations that obtain for the various types of debt and monetary policy commitment technologies are exactly the same as those that obtained before. This result is established in the following subsections:

Consumption taxes, nominal debt and full commitment to monetary policy. In this case we obtain the Ramsey equilibrium allocation, characterized by

$$\frac{1}{c_0^F} - \alpha = \left[\frac{1}{c_1^F} - \alpha\right] [1 + z_0 (1 + \tau_0)] \tag{47}$$

$$c_{t+1} = c_1^F, \text{ for } t \geq 1. \tag{48}$$

where τ_0 is predetermined.

Consumption taxes, nominal debt and no commitment to monetary policy. In this case the recursive equilibrium allocation must satisfy the following intertemporal condition

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{B(1+i_M)}{M}(1+\tau)\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{B'\left(1+i'_M\right)}{M'}(1+\tau')\right]} \left[1 + \epsilon_c(\widehat{z}')\right]^{-1} \tag{49}$$

It follows that in the economy with nominal debt and no commitment to monetary policy, the path of depletion of the stock of debt in real terms coincides with the one characterized in Section 3.2 and computed in Section 5.

Consumption taxes and indexed debt. In this case policies are stationary and we obtain the stationary equilibrium allocation $c^I = \frac{\beta}{\alpha} - g - (1-\beta)\,\widehat{b}_0$ where $\widehat{b}_0 = b_0\,(1+\tau_0)$.

6.3 Optimal fiscal policy with commitment

In the three regimes discussed in the previous section exactly how the equilibrium allocations are supported is not determined since the household only cares about the effective nominal rate of return, $(1+i)\,(1+\tau)$. For instance, it is always possible to set taxes in a way that the resulting monetary policy follows the Friedman rule of zero nominal interest rates, even though in our economy there is no efficiency gain from following such a rule.[4]

To see this, suppose that the stock of debt is nominal and that there is full commitment to fiscal policy. Let the fiscal authority set, for $t \geq 0, \tau_{t+1} = \tau(\widehat{z}_t) = \tau(\widehat{z}_0)$, where $\tau(\widehat{z}_0)$ corresponds to the tax rate that fully finances the government liabilities in the allocation that obtains with full commitment, from period one on. That is,

$$(1+\tau(\widehat{z}_0)) = \frac{u'(c^F)}{\alpha} \tag{50}$$

If, at any $t > 0$, the monetary authority tries to monetize part of the existing stock of nominal debt and to use the resulting revenues to increase future consumption —say, maintaining a constant $\overline{c}$ from then on— then, it must be the case that $c_t < c^F < \overline{c}$. Given that

$$(1+\tau(\widehat{z}_0))(1+\overline{i}) = \frac{u'(\overline{c})}{\alpha} \tag{51}$$

and that (50) must be satisfied, the interest rate would have to be negative, $\overline{i} < 0$. Negative interest rates can not be an equilibrium in this economy since then the household would like to borrow unboundedly. Therefore, given that it is not possible to raise future consumption with negative taxes, there is no gain in partially monetizing the stock of nominal debt in period zero. In this case monetary policy is time consistent. Therefore if there is no commitment to monetary policy, a fully committed fiscal authority that wants to maximize expression (2), will set $\tau_{t+1} = \tau(\widehat{z}_0)$, $t \geq 0$.[5] The following proposition summarizes this result:

Proposition 3 *Assume that fiscal authorities maximize the welfare of the representative household and can fully commit to their policies. Then the equilibrium allocation is*

[4] This may not be true in a more general model economy. For instance, this is not true if we introduce a distinction between cash and credit goods. In this case, the Friedman rule would eliminate the distortion between cash and credit goods created by the cash-in-advance constraint. This notwithstanding, the distortions introduced by the presence of a positive stock of nominal debt would still be there, just as in the economy with only cash goods.

[5] Marimon, Nicolini, and Teles (2003) make a similar argument.

the optimal equilibrium allocation that obtains when there is a single Ramsey planner, regardless of the degree of commitment of the monetary authority.

7 Concluding comments

This paper discusses the different ways in which nominal and indexed debt affect the sequential choice of optimal monetary and debt policies. To this purpose, we study a general equilibrium monetary model where the costs of an unanticipated inflation arise from a cash-in-advance constraint with the timing as in Svensson (1985), and where government expenditures are exogenous. In our environment, as in Nicolini (1998), when the utility function is logarithmic in consumption and linear in leisure and debt is indexed, there is no time-inconsistency problem. In this case, the optimal monetary policy is to maintain the initial level of indexed debt, independently of the level of commitment of a Ramsey government.

In contrast, for the same specification of preferences, when the initial stock of government debt is nominally denominated, a time inconsistency problem arises. In this case, the government is tempted to inflate away its nominal debt liabilities. When the government cannot commit to its planned policies, the optimal sequential policy consists in progressively depleting the outstanding stock of debt, so that it converges asymptotically to zero. Optimal nominal interest rates in this case are also decreasing and converge asymptotically. Hence, the optimal monetary policy in this economy coincides in the long term with the one that obtains in an economy which has no outstanding debt, and from which these time-inconsistency distortions are obviously absent.

Such equilibrium path is not chosen when the initial stock of government debt is nominally denominated and the government can fully commit to its planned policies. In this case, it is optimal to increase the inflation tax in the first period, and to keep a lower and constant inflation tax for the rest of the future.

In the rational expectations equilibria of our economies there are no surprise inflations. Still, for a given initial real value of outstanding debt, the most efficient equilibrium is the one that obtains when debt is indexed, the equilibrium with nominal debt and full commitment comes second, and the equilibrium with nominal debt and no commitment is the least efficient. This result highlights the sense in which nominal debt is indeed a burden on optimal monetary policy.

It should be noted that the source of the inefficiencies and of the monetary policy distortions discussed in this paper is not the desire to run a soft budgetary policy that increases the debt liabilities of the government. Every policy discussed in this article is an optimal policy, subject to the appropriate institutional and commitment constraints, and it is implemented by a benevolent and far-sighted government who does not face either uncertainty or the need for public investment, and who would, therefore, prefer to reduce debt liabilities. The source of the inefficiencies is the distortion created by the lack of commitment that results from the mere existence of an outstanding stock of nominal debt. Therefore, our results highlight the need to implement policy and institutional arrangements that either guarantee high commitment levels, or that reduce the allowed levels of nominal debt. This notwithstanding, our results also show that a constraint on deficits may be ineffective to reduce the distortions created by nominal debt since they are independent of the size of the deficits.

The introduction of additional forms of taxation further clarifies the interplay between the various forms of debt and commitment possibilities. Under the natural assumption that fiscal policy choices are predetermined, we show that the optimal policy problem has the same characterization, provided that the revenues levied through seigniorage are enough to allow for an optimal monetary policy with non-negative interest rates. Instead, as in Marimon, Nicolini and Teles (2003), if there is full commitment to an optimal fiscal policy, the fiscal authorities, anticipating monetary policy distortions, choose to fully finance government liabilities, and the resulting monetary policy is the Friedman rule of zero nominal interest rates. Moreover, this policy results in the equilibrium that obtains in the economy with full commitment.

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Appendix: Computation

To compute the recursive monetary equilibrium defined in Section 3.2, we solve the following dynamic program:

$$V(z) = \max\{\log(c) - \alpha(c+g) + \beta V(z')\} \tag{52}$$

s.t.

$$\frac{\beta}{\alpha} - g + \beta z' C(z') = c(1+z) \tag{53}$$

To solve this problem, we use the following algorithm:

- Step 1: Define a discrete grid on z
- Step 2: Define a decreasing discrete function $\bar{C}(z)$
- Step 3: Iterate on the Bellman operator described in equation (52) until we find the converged $V^*(z)$, $Z'^*(z)$,$C^*(z)$
- Step 4: If $C^*(z) = \bar{C}(z)$, we are done. Else, let $\bar{C}(z) = C^*(z)$ and go to Step 3.

Figure 1: The optimal stocks of indexed debt and of nominal debt with full commitment and with no commitment

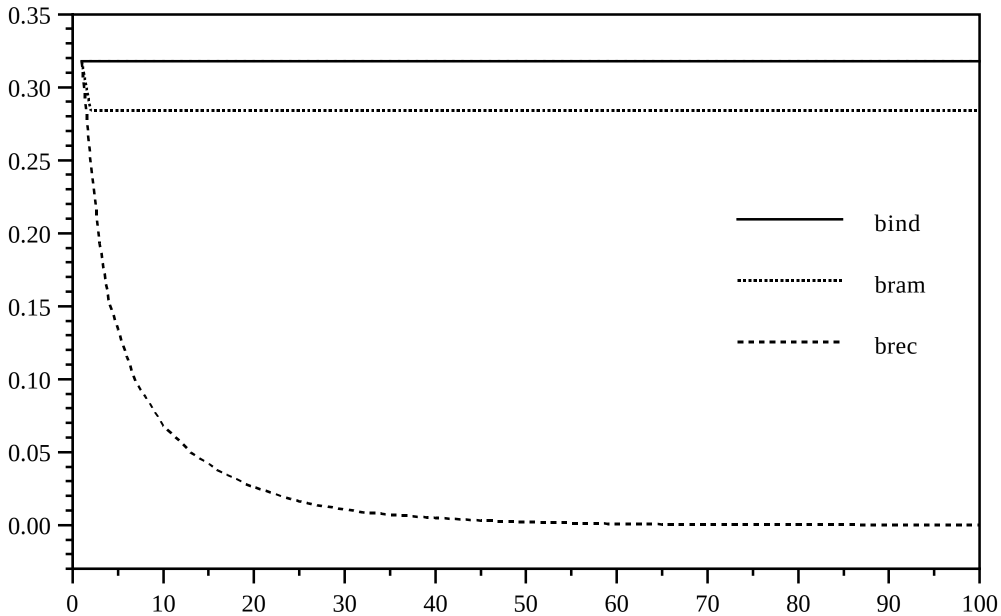

Figure 2: The optimal paths of nominal interest rates with indexed debt and with nominal debt with full commitment and with no commitment

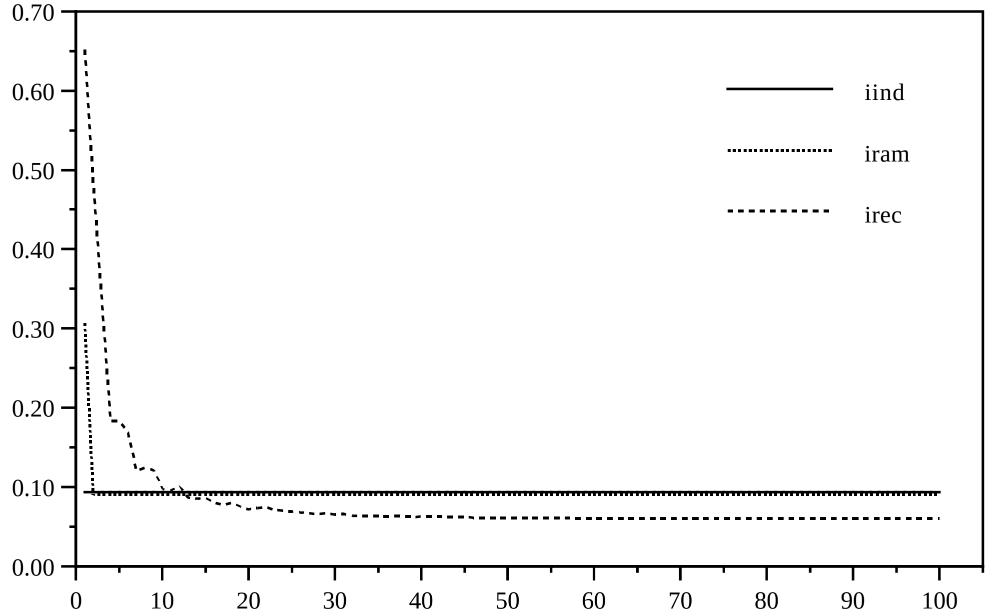

Figure 3: The optimal paths of consumption with indexed debt and with nominal debt with full commitment and with no commitment