Simultaneous Elections

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Abstract

This paper analyzes the possible electoral advantages and disadvantages of a unique party that competes in two simultaneous elections with respect a those obtained when it competes as two different parties. I assume that a unified party has a larger strategy set but it is required to choose the same policy in both elections. I analyze different scenarios depending on the features of the electorates and of the party configuration that it faces. In all cases I show that a unified party is more likely to fare worse than two independent parties when facing simultaneous elections. A unified party can only obtain a gain when the distribution of the voters' preferences of the two electorates are favorable to the opponent.

Keywords: simultaneous elections, state-wide parties, sub-national parties

JEL classification: D72

1 Introduction

There are many instances in which several elections are held at the same time. Different regional elections in a given country, different state elections in a federation, elections for the European Union parliament in each one of the countries. In addition, we can also observe that elections for different government levels take place simultaneously: elections for a central government and for regional governments in a given country, elections for a federal government and for state governments in a federation, and in some instances even the elections for the European Union parliament coincide with other elections, such as regional or municipal, in some countries (Callander 2005 and Fabre 2010).

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In all these cases we observe a great variety of party configurations. In particular, we see that in some cases there are large parties that are defined as a unique institution over all the territories of a given country (state-wide parties), and that as such they compete in all elections (Libbrecht *et al.* 2011). And at the same time, we observe that in some cases these large parties coexist with smaller parties (sub-national parties) that are defined only in a certain part of the territory (Brancati 2008, Muller 2012 and Toubeau 2011). The aim of this paper is to shed some light on the possible gains and losses that a party may derive from competing as a single institution in several simultaneous elections relative to the payoffs that the party could expect if it were split into different parties that specialize their competition on a given territory.

It is clear that parties may enjoy advantages from their larger sizes derived from the economies of scale and scope. The larger the number of territories that a party may cover the more efficiently that it can overcome the fixed costs of its internal organization. This advantage is also relevant in terms of the visibility of the party in the media and the possible electoral gains that this visibility may induce. However, we argue that when competing in simultaneous elections large party institutions involve strategic complications that may end up representing an electoral cost and the characterization of this electoral cost is precisely the main goal of this paper.

The fact that elections take place at the same time, or within a short period of time, may imply that the most relevant political dimension in all of them is the same, that is all parties are competing on the same issue in all elections. When this is the case, the main difference among the elections is that they face different electorates because the distribution of the voters' preferences is bound to be different in each one of the elections. And they also may face different party configurations depending on whether the opponent is a state-wide party or a subnational one. In particular, we will study different scenarios that a party may encounter in terms of the configuration of the party competition when facing two elections simultaneously. On the one hand, we will evaluate the electoral benefits or losses from party unification when facing different parties that compete as separate entities in each election; and on the other hand, we will evaluate the electoral benefits or losses from party unification when competing against a unified party that runs as a unique entity in all elections.

The design of the best policy proposal for a party that faces an electoral competition depends mostly on the policy preferences of the electorate that it is facing, on the characteristics of the opponent or opponents that it is confronting, and on the own party's objectives. Accordingly, differences in the policy preferences of the voters will imply differences in the policy proposal decision. And changes in the characteristics of the opponent will also be reflected in changes in the determination of the best policy platform (Muller 2012).

Therefore, when a party is facing two different elections, with possibly two different electorates and two different sets of competitors, the best policy proposal in each one of the competitions is bound to be different, because the best policy proposal in any given election is independent of the features and conditions of the other election. Thus, the design of the best policy proposal should take into account only the features of the corresponding electorate and competitors (Libbrecht *et al.* 2011). For purely ideological parties, the simultaneous competition in more than one election does not affect the trade-off that they have to solve in a regular single election because its best policy proposal depends mostly on its own ideology. However, for parties that care about their chances of winning elections the analysis of the party's strategic behaviour becomes more relevant.

If a party is facing two different elections that take place at the same time, the choice of a different policy proposal in each case may induce a problem of credibility that may end up causing large electoral losses. As mentioned before, the fact that the two elections are taking place at the same time implies that the policy dimension that is relevant in each one of the elections is likely to be the same. That is, the most salient issue at a given time can be considered to be common in the different electorates, and more so given the current intense information globalization. And this implies that if a party sends a different message to each electorate at the same time, voters may interpret it as a weakness in terms of credibility and the party's electoral prospects in both elections may be jeopardized. From the point of view of the voters, a party that claims that a given policy is the best one in front of an electorate and at the same time proposes a different policy as the best one in another election may not be entirely convincing. This phenomenon is known in the literature as ideological inconsistency (Kreps et al. 2017, Andreottola 2021, Stone and Simas 2010). It affects negatively the party reputation and it induces voters to punish those parties that show ideological inconsistency over time. It has been analized in the literature about primaries (Hummel 2010, Agranov 2016). However, when elections are held simultaneously this effect can only thought to be much stronger.

If voters from both electorates lose confidence in a party's proposals when it exhibits ideological inconsistency, a unified party that proposes two different policies in two different simultaneous elections could suffer a loss in its electoral support from both elections. The electoral cost implied by the loss of credibility may end up being very large as this credibility loss can be attached as a stigma to the party and carried on by voters to future electoral contests. This argument is bound to be at work even if the two elections are not simultaneous but very close to each other in terms of time.

In order to avoid this large future electoral cost, a party that faces two simultaneous elections may decide to choose a unique policy proposal for both of them. Competing in the two elections with a unique policy platform, a party would not be able to confront each election with the best policy choice, but it would be able to avoid future electoral costs due to its loss of credibility. Thus, it would be sacrificing some current electoral benefits instead of engaging in some large electoral costs in the future. The current electoral losses are determined by the difference between the expected payoffs of a party that competes in the two simultaneous elections with a unique policy proposal, and the sum of the expected payoffs from two different parties that compete separately in each election which choose different policy proposal without jeopardizing any future electoral costs. The aim of this paper is to characterize the current electoral gains or losses that a party may suffer from facing two different elections at the same time, when it is forced to choose the same policy proposal for both of them. These losses are going to depend on the differences between the voters' policy preferences in the two electorates and also on the differences between the two sets of competitors.

This paper proposes a model of two simultaneous electoral competitions. In each election two parties are competing on the same one-dimensional policy space. The difference between the two electorates is determined by the expected location of the ideal point of the median voter in each case. All parties have policy preferences, thus in each election we have a leftist party competing with a rightist party. We consider different scenarios depending on the party configuration. First, we suppose that a party is facing a different opponent in each election, and then we consider another scenario in which a party is facing the same opponent in both elections, that is, the opponent is a unique party and thus is forced to choose the same policy proposal in both elections. In each setup we compare the expected payoffs of a party when it competes as a unique (unified) party in the two elections with those that the same party would obtain if it was split in two, and competed as different parties in each election.

Parties care about the policy outcome and they maximize their expected utility. The preferences of the median voter are represented by a probability distribution that assigns some probability to each one of the parties' ideal points (at the extremes of the policy space) and to some moderate policy. This reduced model is enough to reproduce the main trade-off that parties face in any electoral competition, that is, the party's strategic choice between the expected ideal point of the median voter and the party's own ideal point.

The strategy set of each party is restricted to the policies that are contained in the support of the probability distribution that represents the electorate that the party is facing. The strategy set of a unified party that competes in the two elections is given by the union of the supports of each one of the electorates. Notice that even though the restriction that a unified party faces with respect to its policy choice having to be unique for both elections represents a disadvantage for a unified party, it is compensated by the fact that strategy set of a unified party is larger than any of the strategy sets of the parties that compete in a single election. Thus, it is not obvious ex ante whether this combination would produce a net gain or a net loss for a unified party. We consider two different scenarios in terms of the features of the electorate: one in which the ideal points of the expected medians of both electorates are ideologically aligned (and in turn ideologically aligned with one the parties), and one in which the ideal points of the expected medians of both electorates are not ideologically aligned (each one of them is ideologically aligned with a different party). We analyze two different scenarios in terms of the party configuration: one in which a party is facing a different independent party in each election, and one in which a party is facing the same (a unified) party in the two elections. And in each case we find the Nash equilibrium strategies and compare the equilibrium payoffs obtained when the party is unified (has to choose the same policy in both elections) and when it is represented by two independent parties (it can choose a different policy in each election).

Overall we find that in all cases the equilibrium strategies show some degree of polarization, as in Callander (2005). With respect to the comparison of the parties payoffs we observe that a unified party performs better than two independent parties in simultaneous elections whenever the ideal points of the expected medians are ideologically aligned with it to some extend. On the other hand, if the ideal points of the expected medians are extremely different from the party's ideal point, its best reponse is to forgo the competition and propose its own ideal point, and thus party unification does not producce any change in its payoffs. Otherwise, the unified party exhibits worse payoffs than the sum of two independent parties, and this case becomes more significant when the electoral competition intensifies.

In particular, we show that if both medians are ideologically aligned, the payoffs of the parties ideologically aligned with the medians are not affected by a possible party unification. Instead, the parties that are not ideologically aligned with the medians obtain a benefit from the party unification when both medians are moderate. The payoffs of these parties do not change with the union if both medians are very extreme, otherwise unification produces a loss. These results hold independently of whether the opponent competes as a single party or as two independent parties.

If the two medians are not ideologically aligned party unification produces a gain only if both medians are rather ideologically aligned with its ideal point and this case becomes less significant when electoral competition is more intense. The party's payoffs are not affected by the unification if the median that is ideologically opposed to the party is rather extreme. Otherwise, the unification exhibits losses and this last case becomes more significant when the electoral competition is more intense.

The gains of a unified party that competes against a two independent parties are obtained only for very extreme values of the median that is ideologically aligned with the party. Instead the gains of a unified party that competes against a unique party are obtained for more moderate values of the median that is ideologically aligned with the party, and the required moderation increases with increasing electoral competition. The possibility of obtaining gains from the party unification disappears when electoral competition is strong enough. Thus in both cases, increasing the intensity of the electoral competition renders the likelihood of obtaining losses from the party unification.

The rest of the paper is organized as follows: first I describe and solve the basic model of a unique electoral competition. In section 3 I analyze the case of two simultaneous elections and characterize the gains and losses of a unified party. Section 4 offers a discussion of the comparison among the different cases analyzed. Finally, section 5 contains some concluding remarks and possible extensions.

2 The basic model

There are two parties L and R that compete on a unidimensional policy space represented by $X = [0, 1]$. First, they propose policies simultaneously then voters vote and the winner implements the proposed policy.

Voters have a utility function characterized by an ideal point with utility of the alternatives given by the negative distance between the voter's ideal point and the location of the proposed policy. Let $x \in X$ denote the policy position chosen by a given party. Then, the utility that a voter with ideal point $x_i \in X$ attaches to that party is given by $U_i(x) = -|x_i - x|$. Voters vote for the party whose proposed policy offers the highest utility. In case of indifference, a voter votes for each party with equal probability. Thus, we have that given two policies x_L and x_R with $x_L < x_R$ all voters with ideal point $x_i < \frac{x_L + x_R}{2}$ vote for party L and all voters with ideal point $x_i > \frac{x_L + x_R}{2}$ vote for party R.

The location of the median voter's ideal point is unknown to both parties and they have the same beliefs about its distribution denoted by $f(x)$. These beliefs are common knowledge and they are represented by a probability distribution function. The support of this distribution is given by a set of three alternatives $S = \{0, m, 1\} \subset X$ with $m \in X$ and $0 < m < 1$. And the probability function is parametrized by $\alpha \in (0, 1)$ and such that the ideal point of the median voter is equal to m with probability α, it is equal to 0 with probability $\frac{1-\alpha}{2}$, and it is equal to 1 with probability $\frac{1-\alpha}{2}$:

$$f(x) = \begin{cases} 0 & w.p. \quad \frac{1-\alpha}{2} \\ m & w.p. \quad \alpha \\ 1 & w.p. \quad \frac{1-\alpha}{2} \end{cases}$$

Parties have ideal points in the policy space that are equal to 0 for L and 1 for R and they evaluate policies as voters do, that is, the utility that they derive from policy x is given by $u_L(x) = -x$ and $u_R(x) = -(1-x)$ for L and R respectively. Parties maximize the utility derived from the expected policy, thus their payoff functions are represented by:

$$U_L(x_L, x_R) = -P_L(x_L, x_R)\, x_L - (1 - P_L(x_L, x_R))\, x_R$$

$$U_R(x_L, x_R) = -P_L(x_L, x_R)(1 - x_L) - (1 - P_L(x_L, x_R))(1 - x_R)$$

where $0 \leq P_L(x_L, x_R) \leq 1$ denotes the probability of winning the election for party L which implies that $0 \leq 1 - P_L(x_L, x_R) \leq 1$ denotes the probability of winning the election for party R. Notice that $U_L(x_L, x_R) + U_R(x_L, x_R) = -1$ for all x_L and x_R.

We assume that both parties have the same set of strategies and it coincides with the support of the parties' beliefs about the location of the median voter which is $S = \{0, m, 1\} \subset X$. This strategy set is enough to represent the main tradeoff of the parties between pandering to the ideal point of the median voter or choosing its own ideal point. We solve for the Nash Equilibrium of this complete information game between the two parties.

First we consider the case that $0 < m < \frac{1}{2}$ and we look for the best responses of the two parties. We find that party L has a dominant strategy and party R has to solve a tradeoff between the ideal point of the expected median voter and its own ideal point. In equilibrium party R panders to the expected median when it is moderate (rightist) enough. Otherwise, when the median is extreme (leftist) party R prefers to choose its ideal point.

Proposition 1 *If $0 < m < \frac{1}{2}$, in equilibrium $x_L^* (\alpha, m) = 0$ and*

$$x_R^* (\alpha, m) = \begin{cases} m & if \quad m \geq \frac{1-\alpha}{1+\alpha} \\ 1 & if \quad m \leq \frac{1-\alpha}{1+\alpha} \end{cases} .$$

All proofs are relegated to the appendix.

If the expected median voter is leftist, $0 < m < \frac{1}{2}$, we find that the leftist party has a dominant strategy equal to its ideal point. For moderate median voters, $m \geq \frac{1-\alpha}{1+\alpha}$, in equilibrium the rightist party panders to the median voter $(x_L^*, x_R^*) = (0, m)$, and for extreme median voters, $m \leq \frac{1-\alpha}{1+\alpha}$, the equilibrium exhibits full polarization $(x_L^*, x_R^*) = (0, 1)$. If $m = \frac{1-\alpha}{1+\alpha}$ both equilibria coexist. Notice that the equilibrium polarization depends on the value of α : the smaller is α, then more likely is the full polarization equilibrium. In fact, for $\alpha < \frac{1}{3}$ we have $\frac{1-\alpha}{1+\alpha} > \frac{1}{2}$ and the equilibrium is $(x_L^*, x_R^*) = (0, 1)$ for all $0 < m < \frac{1}{2}$.

In the full polarization equilibrium $(x_L^*, x_R^*) = (0, 1)$ the expected policy $\left(\frac{1-\alpha}{2}\right)$ decreases with α and the expected utility of the parties ($U_L (0, 1) = -\frac{1-\alpha}{2}, U_R (0, 1) = -\frac{1+\alpha}{2}$) increases (decreases) with α for party L (R). In the less polarized equilibrium $(x_L^*, x_R^*) = (0, m)$, the expected policy $\left(\frac{1+\alpha}{2} m\right)$ increases with α and with m and the expected utility of the parties ($U_L (0, m) = -\frac{1+\alpha}{2} m, U_R (0, m) = \frac{1+\alpha}{2} m - 1$) decreases (increases) with α and with m for party L (R). Notice that L's expected payoffs are larger than R's in any equilibrium.

The analysis of the case $\frac{1}{2} < m < 1$ is analogous to the previous one: party R has a dominant strategy $x_R^* (\alpha, m) = 1$ and in equilibrium party L panders to the expected median when it is moderate (leftist) enough $\left(m \leq \frac{2\alpha}{1+\alpha}\right)$. Otherwise, when the median is extreme (rightist) party L prefers to choose its ideal point. In this case R's payoffs are larger than L's in any equilibrium. Finally, suppose that $m = \frac{1}{2}$. In this case the full polarization equilibrium is the unique equilibrium, the expected policy in equilibrium is $\frac{1}{2}$, and the expected utility of the parties in equilibrium is $U_L (0, 1) = U_R (0, 1) = -\frac{1}{2}$.

Overall, we obtain that for extreme values of the ideal point of the expected median voter the party whose ideal point is aligned with it obtains a payoff that increases with the intensity of the electoral competition (α). Instead for moderate values of the ideal point of the expected median voter the party whose ideal point is aligned with it obtains a payoff that decreases with the intensity of the electoral competition (α) and it also decreases when the expected median voter becomes more moderate.

3 Simultaneous elections

We extend the basic model in order to consider two different elections that take place at the same time. Without loss of generality, suppose that we have two different electorates represented by the expected medians m_1 and m_2 with $m_1 < m_2$ and in both cases the probability distribution function of the ideal point of the median voter is as described in the basic model and parametrized by $\alpha \in (0, 1)$. Let's denote by (L_1, L_2, R_1, R_2) the scenario in which all competing parties are different; (L, R_1, R_2) and (L_1, L_2, R) represent the scenarios in which a unified party competes in the two elections agains two different parties; and (L, R) denotes the scenario in which two unified parties compete in the two elections.

The set of strategies of each party depends on the scenario they are competing. In particular, at (L_1, L_2, R_1, R_2) party L_1 with $S_1 = \{0, m_1, 1\}$ competes against R_1 with $S_1 = \{0, m_1, 1\}$ in election 1, and party L_2 with $S_2 = \{0, m_2, 1\}$ competes against R_2 with $S_2 = \{0, m_2, 1\}$ in election 2. At (L, R_1, R_2) party L with $S_{1,2} = \{0, m_1, m_2, 1\}$ competes against R_1 with $S_1 = \{0, m_1, 1\}$ in election 1, and against R_2 with $S_2 = \{0, m_2, 1\}$ in election 2. At (L_1, L_2, R) party R with $S_{1,2} = \{0, m_1, m_2, 1\}$ competes against L_1 with $S_1 = \{0, m_1, 1\}$ in election 1, and against L_2 with $S_2 = \{0, m_2, 1\}$ in election 2. And finally, at (L, R) party L with $S_{1,2} = \{0, m_1, m_2, 1\}$ competes against party R with $S_{1,2} = \{0, m_1, m_2, 1\}$ in each one of the elections.

If a party competes simultaneously in the two elections, its strategy set $S_{1,2} = \{0, m_1, m_2, 1\}$ is larger than it would be if the party was split, which ex ante enlarges its strategic options. However, a unified party has a unique policy choice, that is, it is forced to choose the same policy in both elections, which in principle would reduce its strategic options. The payoffs for the unified party are defined as the sum of the payoffs that it obtains in each one of the elections. Let $P_L^t(x_L, x_R)$ denote the probability of winning for party L in election t with $t \in \{1, 2\}$. Then if party L competes in both elections its payoffs can be written as:

$$U_L(x_L, x_{R_1}, x_{R_2}) = U_L^1(x_L, x_{R_1}) + U_L^2(x_L, x_{R_2})$$

where $U_L^1(x_L, x_{R_1}) = -P_L^1(x_L, x_{R_1})\, x_L - \left(1 - P_L^1(x_L, x_{R_1})\right) x_{R_1}$ and $U_L^2(x_L, x_{R_2}) = -P_L^2(x_L, x_{R_2})\, x_L - \left(1 - P_L^2(x_L, x_{R_2})\right) x_{R_2}$. Similarly, if party R competes in both elections its payoffs can be written as:

$$U_R(x_{L_1}, x_{L_2}, x_R) = U_R^1(x_{L_1}, x_R) + U_R^2(x_{L_2}, x_R)$$

where $U_R^1(x_{L_1}, x_R) = -P_L^1(x_{L_1}, x_R)(1 - x_{L_1}) - \left(1 - P_L^1(x_{L_1}, x_R)\right)(1 - x_R)$ and $U_R^2(x_{L_2}, x_R) = -P_L^2(x_{L_2}, x_R)(1 - x_{L_2}) - \left(1 - P_L^2(x_{L_2}, x_R)\right)(1 - x_R)$.

The Nash equilibrium strategies in the case of two simultaneous elections are denoted by $\left(x_{L_1}^*, x_{L_2}^*, x_{R_1}^*, x_{R_2}^*\right)$ for (L_1, L_2, R_1, R_2) ; $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right)$ for (L_1, L_2, R); $\left(x_L^*, x_{R_1}^*, x_{R_2}^*\right)$ for (L, R_1, R_2); and (x_L^*, x_R^*) for (L, R) .

In order to evaluate the advantage or disadvantage for a unified party that competes in the two elections simultaneously we will analyze two different scenarios. First, we will solve the case in which a unified party competes against two different parties in the two different elections. This scenario is denoted by either (L_1, L_2, R) or (L, R_1, R_2) depending on which party is unified. And then we will compare the equilibrium results with those produced when there are no unified parties, that is, (L_1, L_2, R_1, R_2) . Notice that the equilibrium results of this last scenario coincide with the ones obtained in the previous section. This case will allow us to evaluate the conditions under which two different parties decide to unify or to split when competing against two independent parties. Then, we will analyze the competition between two unified parties (L, R) and we will compare the equilibrium results to those produced when only one party is unified, that is, (L_1, L_2, R) or (L, R_1, R_2) . This case will allow us to evaluate the conditions under which a party may decide to unify or to split when competing against a unified party.

Let Δ_R (Δ_L) denote the difference between the payoffs of party R (L) when competing as two different parties and the payoffs obtained when competing as a unique party. For instance

$$\Delta_R \left(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R\right) = U_{R_1}\left(x_{L_1}, x_{R_1}\right) + U_{R_2}\left(x_{L_1}, x_{R_2}\right) - U_R\left(x_{L_1}, x_{L_2}, x_R\right)$$

represents the difference between the payoffs of party R when competing as two different parties and the payoffs obtained when competing as a unique party always against two different leftist parties. If $\Delta_R > 0$ we have that party R suffers a loss from competing as a unified party in the two elections relative to what it obtains if it competes as two separate parties. And if $\Delta_R < 0$ the unified party obtains a gain relative to what it obtains if it competes as two separate parties. Notice that $\Delta_R = -\Delta_L$ always holds.

3.1 Competing against two different parties

In this section we first solve the case in which a unified party competes against two different parties in the two different elections. This scenario may be represented by either (L_1, L_2, R) or (L, R_1, R_2) depending on which party is unified. Then we will compare the equilibrium results obtained with those produced when there are no unified parties, that is, (L_1, L_2, R_1, R_2) . This analysis will allow us to evaluate the conditions under which two different parties may decide to unify or to split when competing against two independent parties.

We divide the analysis into two different cases, depending on whether the expected medians of the two electorates are ideologically aligned with each other: both medians are ideologically aligned if we have either $0 < m_1 < m_2 < \frac{1}{2}$ or $\frac{1}{2} < m_1 < m_2 < 1$; the expected medians are not ideologically aligned if we have $0 < m_1 < \frac{1}{2} < m_2 < 1$.

First consider the case $0 < m_1 < m_2 < \frac{1}{2}$. From the previous analysis we know that the best response of the leftist party is always $x_L = 0$ for all x_R, α, m_1, and m_2 in each of the elections. Thus if a unified leftist party weas to compete simultaneously in the two elections against two different rightist parties the strategic analysis presents no tradeoff: since the leftist parties have the same dominant strategy in all cases, a unified leftist party would alsohave the same dominant strategy. Therefore, the equilibrium strategies for the scenarios (L_1, L_2, R_1, R_2) and (L, R_1, R_2) coincide and $\Delta_L (L_1, L_2, R_1, R_2 \setminus L, R_1, R_2) = 0$.

If instead we consider a unified rightist party that competes in the two elections we have to analyze the scenario denoted by (L_1, L_2, R). In this case have that R competes with $S_{1,2} = \{0, m_1, m_2, 1\}$ against party L_1 with $S_1 = \{0, m_1, 1\}$ in election 1 and it competes against party L_2 with $S_2 = \{0, m_2, 1\}$ in election 2. And the equilibrium is determined by the dominant strategy of the leftist parties $x^*_{L_1}(\alpha, m_1, m_2) = x^*_{L_2}(\alpha, m_1, m_2) = 0$ and a unique policy choice by party R for both elections that is characterized in the next theorem.

Theorem 1 *If $0 < m_1 < m_2 < \frac{1}{2}$ and (L_1, L_2, R) in equilibrium $x^*_{L_1}(\alpha, m_1, m_2) = x^*_{L_2}(\alpha, m_1, m_2) = 0$ and*

$$x^*_R(\alpha, m_1, m_2) = \begin{cases} m_2 & if \quad \frac{1-\alpha}{1+\alpha} \leq m_2 \leq 2m_1 \text{ or } \max\{2m_1, 1-\alpha\} \leq m_2 \\ 1 & \qquad\qquad otherwise \end{cases}$$

Notice that m_1 is never a best response for party R, and party R only panders if m_2 is moderate enough and α is large enough. In fact, for $\alpha < \frac{1}{3}$ only the full polarization equilibrium holds, and as α increases the set of parameter values for which party R panders increases. In particular, the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ if both m_1 and m_2 are small enough and otherwise it is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$. See figure 1.

Figure 1 about here

The relation between the values of m_1 and $\frac{m_2}{2}$ is very important in this case, because for $m_1 > \frac{m_2}{2}$ the choice of m_2 implies a larger probability of wining for party R in both elections. This is because when the two medians are close enough to each other choosing m_2 allows party R to obtain the moderate vote also in election 1. In this case, party R chooses to pander for smaller values of m_2 and of α. For $2m_1 < m_2$ the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ if $m_2 < 1-\alpha$ and $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ otherwise. And for $2m_1 > m_2$ the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ if $m_2 < \frac{1-\alpha}{1+\alpha}$ and $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ otherwise.

Equilibrium outcomes for (L_1, L_2, R)

strategy	expected policy	U_{L_1}	U_{L_2}	$U_{R_1+R_2}$
$x_R^* = 1$	$\frac{1-\alpha}{2}$	$-\frac{1-\alpha}{2}$	$-\frac{1-\alpha}{2}$	$-(1+\alpha)$
$x_R^* = m_2\,(m_2 > 2m_1)$	$\frac{1-\alpha}{2}m_2, \frac{1+\alpha}{2}m_2$	$-\frac{1-\alpha}{2}m_2$	$-\frac{1+\alpha}{2}m_2$	$m_2 - 2$
$x_R^* = m_2\,(m_2 < 2m_1)$	$\frac{1+\alpha}{2}m_2$	$-\frac{1-\alpha}{2}m_2$	$-\frac{1-\alpha}{2}m_2$	$(1+\alpha)\,m_2 - 2$

In the full polarization equilibrium $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (0,0,1)$ the expected policy and the payoffs are independent of the values of m_1 and m_2, and the expected policy and the payoff of party R, in each election and overall, decreases with α. In the less polarized equilibrium $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (0,0,m_2)$ the expected policy in both elections increases with m_2, and the payoff of party R increases with m_2 in each election and overall because its own policy choice is more favorable. In this case, the comparative statics with respect to α are a bit more complex. If $m_2 < 2m_1$ both the expected policy and the payoff of party R in all elections and overall increase with α because party R obtaines the vote of the moderate voters in all elections. If $m_2 > 2m_1$ in election 1 both the expected policy and the payoff of party R decrease with α because party R does not obtain the vote of the moderates; and in election 2 both the expected policy and the payoff of party R increase with α because party R obtains the vote of the moderates; and the overall payoff of R does not change with α.

We want to compare the payoffs obtained in this equilibrium with those that would obtain if party R was competing as two different parties. If $0 < m_1 < m_2 < \frac{1}{2}$ and all parties compete separately in the two elections the full polarization equilibrium obtains if $m_1 < m_2 < \frac{1-\alpha}{1+\alpha}$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(0,1) = -\frac{1-\alpha}{2}$; the equilibrium is $\left(x_{L_1}^*, x_{L_2}^*, x_{R_1}^*, x_{R_2}^*\right) = (0,0,m_1,m_2)$ if $\frac{1-\alpha}{1+\alpha} < m_1 < m_2$, with $U_{R_1}(0,m_1) = \frac{1+\alpha}{2}m_1 - 1$ and $U_{R_2}(0,m_2) = \frac{1+\alpha}{2}m_2 - 1$; otherwise the equilibrium is $\left(x_{L_1}^*, x_{L_2}^*, x_{R_1}^*, x_{R_2}^*\right) = (0,0,1,m_2)$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(0,m_2) = \frac{1+\alpha}{2}m_2 - 1$. We now compare these payoffs to the ones obtained by party R when competing as a unified party in the two elections (see figure 1).

Proposition 2: *If* $0 < m_1 < m_2 < \frac{1}{2}$:

$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = 0$ *if* $m_1 < m_2 < \frac{1-\alpha}{1+\alpha}$

$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = \frac{1+\alpha}{2}(m_1 - m_2) < 0$ *if* $\frac{1-\alpha}{1+\alpha} < m_1 < m_2$

$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = \frac{1-\alpha}{2} - \frac{1+\alpha}{2}m_2 < 0$ *if* $m_1 < \frac{1-\alpha}{1+\alpha} < m_2 < 2m_1$

$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = \frac{1-\alpha}{2}(1 - m_2) > 0$ *if* $2m_1 < m_2$ *and* $1 - \alpha < m_2$

$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = \frac{1+\alpha}{2}m_2 - \frac{1-\alpha}{2} > 0$ *if* $2m_1 < m_2$ *and* $\frac{1-\alpha}{1+\alpha} < m_2 < 1 - \alpha$

If the two medians are extreme (leftist) the rightist parties do not have any interest in pandering in any scenario, thus a unified rightist party does not produce any gains nor losses relative to two independent rightist parties. This result becomes less likely for larger values of α. Otherwise, if the two medians are close enough to each other ($m_2 < 2m_1$), all rightist parties have an interest in pandering in all scenarios. When party R competes as a united party it can obtain the vote of all moderates (from both elections) by choosing m_2 in the two elections, because it is better than pandering with m_1 in election 1, which is its best response if competing as two separate parties. This implies a utility gain for the union of party R. This gain increases with α and with m_2. The intuition follows easily. As electoral competition increases it becomes more valuable for party R to choose m_2 because it allows it obtain the vote of the moderate in the two elections. And for larger values of m_2 the expected policy becomes more favorable to party R ideological interests. This result becomes more likely for larger values of α.

When the values of the two medians are very different ($m_2 > 2m_1$), party R only moderates its policy for large enough values of m_2. In this case, moderation allows party R to obtain the moderate vote only in election 2, and thus the restriction of having to choose a unique policy for both elections implies a utility loss for the unified party with respect to competing separately in the two elections. This loss decreases with α and with m_2. The intuition is as follows: larger values of α imply that the choice of m_2 becomes more valuable for party R, and as before larger values of m_2 imply that the expected policy becomes more favorable to party R ideological interests. This result becomes more likely for larger values of α.

Finally, when the values of the two medians are very different but m_2 is not large enough, party R cannot pander only in election 2 when it competes as a unified party and the full polarization equilibrium obtains. This implies a utility loss for the union of party R. This loss increases with α and with m_2. The intuition in this case is as follows: increasing the intensity of the electoral competition renders the full polarization equilibrium less valuable for party R and larger values of m_2 imply that the payoffs of an independent rightist party increase in election 2. This result becomes less likely for larger values of α (the proof of proposition 2 contains the details of this statement).

A similar analysis could be performed for $\frac{1}{2} < m_1 < m_2 < 1$. In this case, we would find that the best response of the rightist party is always $x_R = 1$ in each of the elections and thus $\Delta_R (L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = 0$.

If instead we consider the case of a unified leftist party competing simultaneously in the two elections against two different rightist parties (L, R_1, R_2) we can replicate the argument of Theorem 1 and obtain symmetric results: the full polarization equilibrium obtains when both expected medians are extreme (large enough), otherwise party L best responds with $x_L = m_1$. We also have that when the two medians are close enough to each other ($m_2 < \frac{1+m_1}{2}$) choosing m_1 allows party L to obtain the moderate vote in both elections. In this case, party L chooses to pander for larger values of m_1 and of α and it obtains a gain from the unification, relative to competing with two different parties.

Now we turn to the case in which the two expected medians are not ideologically aligned, that is, $0 < m_1 < \frac{1}{2} < m_2 < 1$. Since in this case no party has a dominant strategy, we have to consider the possibility that any of the two parties competes as a unique party in both elections. We focus on the scenario (L_1, L_2, R), that is, party R with $S_{1,2} = \{0, m_1, m_2, 1\}$ is competing as a unique party in the two elections against L_1 with $S_1 = \{0, m_1, 1\}$ and L_2 with $S_2 = \{0, m_2, 1\}$ respectively. The case (L, R_1, R_2) can be solved with a symmetric argument. First we look for the best responses of each one of the leftist parties and then we complete the equilibrium with the corresponding best responses of party R. Notice that in addition to the results that we have obtained for independent parties in section 2 we have to compute the best response of party L_1 to $x_R = m_2$ and the best response of party L_2 to $x_R = m_1$.

Proposition 3 *If* $0 < m_1 < \frac{1}{2} < m_2 < 1$ *and* (L_1, L_2, R), *the best responses of* L_1 *are* $x_{L_1}(0) = x_{L_1}(m_1) = x_{L_1}(1) = 0$ *and* $x_{L_1}(m_2) = \begin{cases} m_1 & if \quad \frac{m_2}{2} < m_1 < \frac{2\alpha}{1+\alpha} m_2 \\ 0 & otherwise \end{cases}$ *and the best responses of* L_2 *are* $x_{L_2}(m_1) = x_{L_2}(m_2) = x_{L_2}(0) = 0$ *and* $x_{L_2}(1) = \begin{cases} m_2 & if \quad m_2 < \frac{2\alpha}{1+\alpha} \\ 0 & otherwise \end{cases}$.

Notice that for $\alpha < \frac{1}{3}$ both leftist parties have a dominant strategy $x_{L_1} = x_{L_2} = 0$. For larger values of α, party L_1 always best responds with its ideal point except against $x_R = m_2$ whenever R's choice of m_2 allows it to obtain the moderate vote also in election 1, in which case L_1 best responds with m_1, as long as m_1 is not too large (not too costly for L_1). In this case L_1 uses a more aggressive strategy in order to avoid losing the vote of the moderate voters in both elections. And party L_2 always best responds with its ideal point except against $x_R = 1$ whenever m_2 is not too large, in which case L_2 best responds with m_2. Given the best responses of the leftist parties, we look for the corresponding best responses of party R and we find the equilibrium strategies as stated in the next theorem.

Theorem 2 *If* $0 < m_1 < \frac{1}{2} < m_2 < 1$ *and* (L_1, L_2, R), *then in equilibrium:*
for $\alpha < \frac{1}{3}$: $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = \begin{cases} (0, 0, m_2) & if \quad \frac{1}{1+\alpha} < m_2 < 2m_1 \\ (0, 0, 1) & otherwise \end{cases}$
and for $\alpha > \frac{1}{3}$:
$\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ *if* $m_2 < \min\left\{\frac{2\alpha}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\}$ or $\frac{2(1-\alpha)}{1+\alpha} < m_2 < \min\left\{\frac{2\alpha}{1+\alpha}, 1 - \frac{1+\alpha}{2} m_1, 2 - \frac{1+\alpha}{1-\alpha} m_1\right\}$
$\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$ *if* $\frac{1+\alpha}{2\alpha} m_1 < m_2 < 2m_1$ and $m_2 > \min\left\{1 - \frac{1+\alpha}{2} m_1, 2 - \frac{1+\alpha}{1-\alpha} m_1\right\}$
$\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ *if* $\max\left\{\frac{1}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\} < m_2 < \frac{1+\alpha}{2\alpha} m_1$
$\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ *if* $\max\left\{2m_1, \frac{2\alpha}{1+\alpha}\right\} < m_2$ or $\frac{2\alpha}{1+\alpha} < m_2 < 1 - \frac{1+\alpha}{2} m_1$ or $1 - \frac{1+\alpha}{2} m_1 < m_2 < \frac{1}{1+\alpha}$

The best responses of party R (developed in the proof of Theorem 2) are as follows. The best response of R is $x_R(\alpha, m_1, m_2) = 1$ except if the two medians are close enough $(m_2 < 2m_1)$ and m_2 is large enough in which case the best response is $x_R(\alpha, m_1, m_2) = m_2$. The set of parameter values (m_1, m_2) for which party R prefers $x_R(\alpha, m_1, m_2) = m_2$ increases with α, that is, for larger values of α party R's best respondes with $x_R(\alpha, m_1, m_2) = m_2$ even for moderate values of m_2.

Since for very small values of α $\left(\alpha < \frac{1}{3}\right)$ both leftist parties have a dominant strategy $x_{L_1} = x_{L_2} = 0$, the equilibrium strategies follow easily: party R chooses $x_R = m_2$ whenever it is not too costly (large enough) and it allows to obtain the moderate vote also in election 1. Otherwise,party R chooses its ideal point.

For larger values of α the equilibrium strategies are characterized by the four different regions in the parameter space (m_1, m_2) related in Theorem 2 and represented in figure 2. These areas are defined as follows:

$whitedotted: m_2 < \min\left\{\frac{2\alpha}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\}$ or $\frac{2(1-\alpha)}{1+\alpha} < m_2 < \min\left\{\frac{2\alpha}{1+\alpha}, 1 - \frac{1+\alpha}{2}m_1, 2 - \frac{1+\alpha}{1-\alpha}m_1\right\}$

$greydotted: \frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1$ and $m_2 > \min\left\{1 - \frac{1+\alpha}{2}m_1, 2 - \frac{1+\alpha}{1-\alpha}m_1\right\}$

$grey: \max\left\{\frac{1}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\} < m_2 < \frac{1+\alpha}{2\alpha}m_1$

$white: \max\left\{2m_1, \frac{2\alpha}{1+\alpha}\right\} < m_2$ or $\frac{2\alpha}{1+\alpha} < m_2 < 1 - \frac{1+\alpha}{2}m_1$ or $1 - \frac{1+\alpha}{2}m_1 < m_2 < \frac{1}{1+\alpha}$

In all the white areas the equilibrium strategy of party R is $x_R = 1$, and in all the grey areas the equilibrium strategy of party R is $x_R = m_2$. In all the areas with a plain color the equilibrium strategy of the leftist parties is $x_{L_1} = x_{L_2} = 0$. Finally, in the white dotted area the equilibrium strategies of the leftist parties are $x_{L_1} = 0$ and $x_{L_2} = m_2$; and in the grey dotted area the equilibrium strategies of the leftist parties are $x_{L_1} = m_1$ and $x_{L_2} = 0$.

Figure 2 about here

For very small values of α $\left(\alpha < \frac{1}{3}\right)$ only the plain areas grey and white hold, because the leftist parties always choose their ideal point. Instead for larger values of α the leftist parties find it profitable to be more aggressive: party L_2 in election 2 chooses m_2 against $x_R = 1$ if m_2 is small enough (white dotted area), and party L_1 in election 1 chooses m_1 whenever the two medians are close enough so that by choosing m_1 it prevents party R from obtaining the moderate vote (grey dotted area). The size of these two areas increases with the value of α. Thus the larger the probability that the median voter is moderate the more likely the leftist parties will use aggressive strategies.

We want to compare the payoffs obtained in this equilibrium with those that would obtain if party R was competing as two different parties. If $0 < m_1 < \frac{1}{2} < m_2 < 1$ and all parties compete separately in the two elections the full polarization equilibrium obtains if $m_1 < \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(0,1) = -\frac{1-\alpha}{2}$; the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, 1, 1)$ if $m_1 < \frac{1-\alpha}{1+\alpha}$ and $m_2 < \frac{2\alpha}{1+\alpha}$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(m_2, 1) = -\frac{1+\alpha}{2}(1 - m_2)$;

the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, 0, m_1, 1)$ if $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$ with $U_{R_1}(0, m_1) = \frac{1+\alpha}{2} m_1 - 1$ and $U_{R_2}(0, 1) = -\frac{1-\alpha}{2}$; otherwise, if $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 < \frac{2\alpha}{1+\alpha}$, the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, m_1, 1)$ with $U_{R_1}(0, m_1) = \frac{1+\alpha}{2} m_1 - 1$ and $U_{R_2}(m_2, 1) = -\frac{1+\alpha}{2}(1 - m_2)$.

We now compare these payoffs to the ones obtained by party R when competing as a unified party in the two elections.

Proposition 4: *If* $0 < m_1 < \frac{1}{2} < m_2 < 1$
for $\alpha < \frac{1}{3}$:
$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = 1 - (1+\alpha) m_2 < 0$ *if* $\frac{1}{1+\alpha} < m_2 < 2m_1$
$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = 0$ *otherwise*
for $\alpha > \frac{1}{2}$:
$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = 0$ *if* $m_1 < \frac{1+\alpha}{1-\alpha}$ in [*white*] or [*whitedotted*]
$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) = \frac{1+\alpha}{2} - m_2 < 0$ *if* $\frac{1+\alpha}{2} < m_2 < 2m_1$ in [*greydotted*]
$\Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R) > 0$ *if otherwise*

For $\alpha < \frac{1}{3}$ unification represents a gain for party R whenever both m_1 and m_2 are large enough and close enough to each other, and the gain in this area is mantained for values of α beyond $\frac{1}{3}$. This gain is due to the fact that party R obtains the vote of the moderate voters in both elections. Thus it increases with α and m_2; and this area increases with α. Otherwise, for $\alpha < \frac{1}{3}$ unification does not make a difference because the full polarization equilibrium holds in both cases.

For $\alpha > \frac{1}{2}$ the gains and losses produced by the unification of party R are represented in the different areas of the parameter space (m_1, m_2) described in figure 2. The subareas included in the white and whitedotted areas that exhibit no gains nor loses are concentrated around small values of m_1 $(m_1 < \frac{1-\alpha}{1+\alpha})$ and these areas become smaller when α increases and disappear as α approaches 1. In the rest of grey and greydotted subareas the unified party exhibits losses. These losses are generated by the restriction that the unified party faces having to propose the same policy in both elections. Since in this case the medians are not aligned, the party could do better choosing a different policy for each election, which is what it does when competing as two independent parties. These losses $(\frac{1+\alpha}{2} m_1 - \frac{1-\alpha}{2})$ increase with α and with m_1, because the restriction of having to choose a unqiue policy for both elections drives party R to choose its ideal point, and its loss increases with more intense electoral competition.

For intermediate values of α $\left(\frac{1}{3} < \alpha < \frac{1}{2}\right)$ unification produces increasing gains in the grey dotted area when α increases. The unification also produces gains in some subareas of the grey area. These become smaller when α increases and they disappear for $\alpha > \frac{1}{2}$. These results are formally described in the proof and the intuition follows the arguments described in this section for the cases of smaller and larger values of α.

The set of parameter values that produce a gain for the unified party are concentrated around large values of m_1 and m_2. It is interesting to note that

$m_2 < 2m_1$ always holds in this set, which means that unification can only produce gains whenever the choice of m_2 guarantees the vote of all the moderate voters in both elections, however this condition is not sufficient to produce gains for the union. This set is also contained in the grey dotted area, which implies more aggressive strategies of the leftist party in election 1. These gains increase with α and decrease with m_2. The intuition is as follows: as electoral competition increases it becomes more valuable for party R to choose m_2 because it allows it to obtain the vote of the moderate voters in the two elections. And for larger values of m_2 the expected policy becomes more favorable to party R ideological interests. The size of this area becomes smaller for larger values of α, which implies that as electoral competition becomes more intense it becomes less likely that the unification of the rightist party produces a gain and the possibility of gains disappear as α approaches 1.

Thus we find that the unification exhibits increasing areas of losses for larger values of α. Therefore, we have that as electoral competition becomes more intense it becomes more likely that the unification of the rightist party produces a loss. These losses take different forms in the different subareas and they are fully described in the proof of proposition 4. They decrease with α and m_2 in the grey dotted area and they increase with α and m_1 in the white and white dotted areas. The intuition is as follows: in the grey dotted area smaller values of α imply that the choice of m_2 becomes less valuable for party R, and smaller values of m_2 imply that the expected policy becomes less favorable to party R ideological interests. This area becomes larger for larger values of α. In the white area larger values of α and m_1 render the full polarization equilibrium less worthy for the rightist party, and in the white dotted area larger values of α and m_1 render the aggressive strategy of the leftist parties more effective. Recall that for larger values of α we also have larger grey dotted areas and larger white dotted areas, that is, the leftist parties use more widely aggressive strategies. At the same time, for larger values of α full polarization becomes less likely when parties compete separately.

Overall, we have shown that when competing against two different parties if both medians are ideologically aligned, the parties that are ideologically aligned with medians have no incentive to unite since there are no expected gains from it (because they have a dominant strategy which is equal to their ideal point). Instead the parties that are not ideologically aligned with the medians have an incentive to unite only if both medians are close to each other, so that the united party has a strategy that guarantees the vote of the moderate voters from both elections. If there is only one moderate median the unification produces a loss to the unified party because it cannot reach the moderate votes of one of the elections, and thus it is harmed by the restriction of having to choose the same policy in both election. If both medians are rather extrem, unification does not affect the payoffs of these parties because in all cases the party is better off proposing his ideal point and forgoing the chance to convince any moderate voter. As the electoral competition becomes more intense (larger values of α) this last case becomes less significant (only holds for more very extreme medians) and thus the possibilities of obtaining gains or losses for the united party become more likely.

We have also seen that when competing against two different parties if both medians are not ideologically aligned, again a party has incentives to unite only if the two medians are close enough to each other (so that the party can ontain the vote of the moderate voters of both elections) and it also needs one of the medians to be closely aligned with the ideology of the party. However, as opposed to the previous case, as the intensity of the electoral competition increases, it becomes less likely that the unified party obtains gains and the possibility of gains disappear as α approaches 1.

If the median that is aligned with the opponent is rather extreme, unification does not affect the payoffs of these parties because the party is better off forgoing the chance to convince any moderate voter in that election, and the party prefers to choose its own ideal point. This condition holds for more extreme medians when α increases. Therefore, unfication is more likely to produce losses when medians are not aligned with each other, and as opposed to the case of ideologically aligned medians, in this case the losses caused by the party unification becomes a general result for intense electoral competition.

3.2 Competing against a unified party

In this section we first analyze the competition between two unified parties (L, R) when they compete in two simultaneous elections: party L with $S_{1,2} = \{0, m_1, m_2, 1\}$ competes against party R with $S_{1,2} = \{0, m_1, m_2, 1\}$ in each one of the two elections, and each party is forced to choose a unique strategy for both elections. And then we will compare the equilibrium results obtained to those produced when only one party is unified, that is, we want to characterize $\Delta_L (L_1, L_2, R \setminus L, R)$ and $\Delta_R (L, R_1, R_2 \setminus L, R)$. This case will allow us to evaluate the conditions under which a party may decide to split or unify when competing against a unified party.

Again we divide the analysis into different cases, depending on the values of the expected medians of the two electorates. Recall that for $0 < m_1 < m_2 < \frac{1}{2}$ the leftist parties have a dominant strategy equal to their ideal point in all elections, which implies that if we consider a unified leftist party in this setup we have that its best response will coincide with this dominant strategy. This implies that if $0 < m_1 < m_2 < \frac{1}{2}$ the equilibrium for (L, R) coincides with the equilibrium found before for (L_1, L_2, R) and described in Theorem 1, because the leftist parties have a dominant strategy equal to their ideal point in all cases, and thus party R has the same best response in both scenarios. Therefore we must have that $\Delta_L (L_1, L_2, R \setminus L, R) = 0$ which implies that the leftist parties have no incentives to unite when competing against a unified rightist party, exactly as it happened when they were competing against two different rightist parties.

For the same reason we have that the equilibrium for (L, R_1, R_2) coincides with the equilibrium found before for (L_1, L_2, R_1, R_2) and we must have that $\Delta_R (L, R_1, R_2 \setminus L, R) = \Delta_R (L_1, L_2, R_1, R_2 \setminus L, R)$. Since we have already seen that the equilibrium for (L, R) coincides with the equilibrium found for

(L_1, L_2, R) we obtain

$$\Delta_R(L, R_1, R_2 \setminus L, R) = \Delta_R(L_1, L_2, R_1, R_2 \setminus L, R) = \Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R)$$

which is exactly the result described in proposition 2. Thus if the two medians are ideologically aligned (either $0 < m_1 < m_2 < \frac{1}{2}$ or $\frac{1}{2} < m_1 < m_2 < 1$) the benefits of the unified party do not depend on the party configuration that it is facing because the equilibrium results are the same whether it faces a united party or two different parties.

Corollary: *If* $0 < m_1 < \frac{1}{2} < m_2 < 1$, $\Delta_L(L_1, L_2, R \setminus L, R) = 0$ *and* $\Delta_R(L, R_1, R_2 \setminus L, R) = \Delta_R(L_1, L_2, R_1, R_2 \setminus L_1, L_2, R)$ *as described in proposition 2.*

However when the two medians are not aligned $(0 < m_1 < \frac{1}{2} < m_2 < 1)$ no party has a dominant strategy, and in order to compute the gains and losses of a unified party we first have to calculate the equilibrium strategies corresponding to the scenario (L, R).

Theorem 3 *If* $0 < m_1 < \frac{1}{2} < m_2 < 1$, *and* (L, R) *in equilibrium:*

for $\alpha < \frac{1}{2} : (x_L^*, x_R^*) = \begin{cases} (0, m_2) & if \quad \frac{1}{1+\alpha} < m_2 < 2m_1 \\ (m_1, 1) & if \quad 2m_2 - 1 < m_1 < \frac{\alpha}{1+\alpha} \\ (0,1) & \quad otherwise \end{cases}$

for $\alpha > \frac{1}{2}$:

$(x_L^*, x_R^*) = (0, m_2)$ *if* $\frac{1}{1+\alpha} < m_2 < \frac{m_1}{\alpha}$

$(x_L^*, x_R^*) = (m_1, 1)$ *if* $\frac{m_2}{\alpha} - \frac{1-\alpha}{\alpha} < m_1 < \frac{\alpha}{1+\alpha}$

$(x_L^*, x_R^*) = (0, 1)$ *if* $\frac{1}{1+\alpha} > m_2$ *and* $m_1 > \frac{\alpha}{1+\alpha}$

$(x_L^*, x_R^*) = (0, 1)$ *if* $m_2 > \max\left\{2m_1, \frac{1+m_1}{2}\right\}$

$(x_L^*, x_R^*) = (m_1, m_2)$ *if* $\max\left\{\frac{m_1}{\alpha}, 1 - \alpha + \alpha m_1\right\} < m_2 < \min\left\{2m_1, \frac{1+m_1}{2}\right\}$

otherwise there is no pure strategy equilibrium.

This case produces symmetric equilibria in which the strategies of the parties are in $\{0, m_1\}$ for L and in $\{1, m_2\}$ for R, thus as before all equilibria involve some degree of polarization. We find that full polarization equilibrium exists for a large set of parameter values. In fact, for very small values of α $\left(\alpha < \frac{1}{2}\right)$ the full polarization equilibrium holds except when both m_1 and m_2 are very large in which case the equilibrium is $(x_L^*, x_R^*) = (0, m_2)$, and when both m_1 and m_2 are very small in which case the equilibrium is $(x_L^*, x_R^*) = (m_1, 1)$. Notice in each one of these cases we have that one party can guarantee the vote of the moderate in both elections. When the two medians are very large we have that m_2 is very extreme and m_1 is very moderate, and it is cheap for party R to obtain the vote of the moderate in both elections. Simmetrically, when the two medians are very small we have that m_1 is very extreme and m_2 is very moderate, and it is cheap for party L to obtain the vote of the moderate in both elections. It is only when one party can guarantee the vote of the moderate in both elections that the full polarization equilibrium breaks, and this condition becomes more likely for larger values of α as long as $\alpha < \frac{1}{2}$.

Figure 3 about here

For larger values of α $\left(\alpha > \frac{1}{2}\right)$ an additional equilibrium appears in which parties choose $(x_L^*, x_R^*) = (m_1, m_2)$, and there is a region of the parameter space where no pure strategy equilibrium exist. The equilibrium strategies are characterized by the five different regions in the parameter space (m_1, m_2) related in Theorem 3 and represented in figure 3. These areas are defined as follows:

dashed : $\frac{1}{1+\alpha} < m_2 < \frac{m_1}{\alpha}$
dotted : $(x_L^*, x_R^*) = (m_1, 1)$ *if* $\frac{m_2}{\alpha} - \frac{1-\alpha}{\alpha} < m_1 < \frac{\alpha}{1+\alpha}$
grey : $\frac{1}{1+\alpha} > m_2$ *and* $m_1 > \frac{\alpha}{1+\alpha}$
grey : $m_2 > \max\left\{2m_1, \frac{1+m_1}{2}\right\}$
black : $\max\left\{\frac{m_1}{\alpha}, 1-\alpha+\alpha m_1\right\} < m_2 < \min\left\{2m_1, \frac{1+m_1}{2}\right\}$
white : *no pure strategy equilibrium*

The dashed and dotted regions correspond to the equilibria described already when one party can guarantee the vote of the moderate in both elections. For small values of α these areas lie clsoe to the corners in the space (m_1, m_2) as described before. However as α increases beyond $\frac{1}{2}$ these areas move closer to the center of the policy space and become smaller. The grey areas represent the full polarization equilibrium. The large grey area does not change with α but the small grey area becomes smaller for larger values of α. The black area represents the parameter values for which the new equilibrium $(x_L^*, x_R^*) = (m_1, m_2)$ exists, and it becomes larger for larger values of α. Finally, in the white area we have no existence of pure strategy equilibrium and this area increases with α.

We now compare the payoffs obtained in this equilibrium where only two parties compete in the two elections with those obtained if one of the parties was split into two different ones. Without loss of generality suppose it is the leftist party. Thus we compare the equilibrium just obtained for (L, R) with the one obtained for (L_1, L_2, R) and described in Theorem 2.

Proposition 5: *If* $0 < m_1 < \frac{1}{2} < m_2 < 1$

if $\alpha < \frac{1}{3}$: $\Delta_L(L_1, L_2, R \setminus L, R) = \begin{cases} (1+\alpha)m_1 - \alpha < 0 & if \quad 2m_2 - 1 < m_1 < \frac{\alpha}{1+\alpha} \\ 0 & otherwise \end{cases}$

if $\alpha > \frac{1}{3}$:

$\Delta_L(L_1, L_2, R \setminus L, R) = 0$ *if* $\frac{2\alpha}{1+\alpha} < m_2 < 2m_1$ or $\frac{1}{1+\alpha} < m_2 < \frac{1+\alpha}{2\alpha} m_1$

$\Delta_L(L_1, L_2, R \setminus L, R) = (1+\alpha)m_1 - \alpha < 0$ if $\frac{1}{3} < \alpha < \frac{1}{2}, \frac{2\alpha}{1+\alpha} < m_2 < \frac{1+m_1}{2}, m_1 < \frac{\alpha}{1+\alpha}$

$\Delta_L(L_1, L_2, R \setminus L, R) = (1+\alpha)\left(m_1 - \frac{m_2}{2}\right) < 0$ if $\left[\frac{1}{3} < \alpha < \frac{1}{2}, 2m_1 < m_2 < \min\left\{\frac{1+m_1}{2}, \frac{2\alpha}{1+\alpha}\right\}, m_1 <\right.$ or $\left[\frac{1}{2} < \alpha < \frac{2}{3}, 2m_1 < m_2 < \min\left\{1-\alpha+\alpha m_1, 2 - \frac{1+\alpha}{1-\alpha}m_1\right\}, m_1 < \frac{\alpha}{1+\alpha}\right]$

$\Delta_L(L_1, L_2, R \setminus L, R) > 0$ *otherwise.*

This proposition shows that the unified party L obtains gains only in the dotted area (figure 3). Notice that as always gains from unification only occur whenever the party L has a strategy that guarantees the vote of the moderate voters from both elections, which in this case is $m_2 < \frac{1+m_1}{2}$.

For $\alpha < \frac{1}{3}$ gains are obtained in all the dotted area which holds for extreme values of m_1 and moderate values of m_2 and this area increases with α. Otherwise unification makes no difference in the equilibria. For $\alpha > \frac{1}{3}$ gains for the unified party L are obtained in a subarea of the dotted area which holds for moderate values of m_2 and for values of m_1 that are required to be more moderate for larger values of α. The subarea that produces gains is also restricted by $m_2 > 2m_1$ which implies that the party R does not have a strategy that guarantees the vote of the moderate voters in both elections. The size of the area that produces gains increases with α for $\alpha < \frac{1}{2}$, it decreases with α for $\alpha > \frac{1}{2}$, and it approaches zero when α approaches $\frac{2}{3}$. In all cases, the gains of party L increase with α and decrease with m_1. The intuition is as follows: larger values of α imply larger probabilities of winning for party L because it obtains the vote of all moderate voters; and for larger values of m_1 party L has to propose a a policy that is further away from its ideal point.

In the dashed area the unified party shows a loss for the less moderate values of m_1 and m_2. This loss increases with α and m_2. The intuition is explained by the fact that party R always proposes m_2 when the two parties compete as united. Otherwise, for more moderate values of m_1 and m_2 the unification of party L does not make a difference when competing against a unified party R. The size of the dashed area increases with α for $\alpha < \frac{1}{2}$, it decreases with α for $\alpha > \frac{1}{2}$, and it approaches zero when α approaches 1.

The black area shows a loss for the union. This loss increases with m_1, because it is the policy chosen by party L in this case. This loss decreases with α whenever m_2 is large enough because the choice of m_1 becomes more beneficial for the leftist party. And this loss increases with α whenever m_2 is small enough because party R is responding with 1 when the leftist are competing as two different parties and the payoffs of the leftist parties rely more on the vote of the moderates. The size of the black area increases with α.

For $\frac{1}{3} < \alpha < \frac{1}{2}$ the grey area shows a loss for the union for moderate values of m_2. This loss increases with α and decreases with m_2 because in this case the equilibrium strategy chosen by party L_2 is the aggressive one: m_2. And the union makes no difference for more extreme values of m_2 because the full polarization equilibrium holds in both cases. This area decreases with α. For $\alpha > \frac{1}{2}$ we have two grey areas: a small one for values of m_1 and m_2 that are very close to $\frac{1}{2}$, and a large one (see figure 3). In the large one the unification of party L does not have any effect on its payoffs for large values of m_2 and it produces a loss otherwise. The large grey area does not change its size with α. In the small grey area the unification of party L only produces losses and this area decreases its size with α and it approaches zero when α approaches 1.

In this case we have that for values of α larger than $\frac{2}{3}$, the possibility of obtaining gains from the unification vanishes and for values of α that approach 1 the size of the set of parameter values for which unification has no-effect also approaches zero. However, we also have that the set of parameter values for which there is no pure strategy equilibrium for (L,R) becomes larger with larger values of α. Thus in this case we cannot conclude that the losses from unification become the only possible prediction when electoral competition becomes more intense.

Overall, we have shown that if both medians are ideologically aligned, the advantages and disadvantages of a unified party are the same independently of whether the party is competing against a unique party or two independent parties. Thus, as we showed in the previous section: the payoffs of the parties ideologically aligned with the medians do no change whether they are united or they are independent. Instead, the payoffs of the parties that are not ideologically aligned with the medians are larger if the party is unified when both medians are moderate, and they are larger for independent parties if one median is extreme and the other one is moderate. The payoffs of these parties are not affected by the union only if both medians are very extreme.

And we have also shown that if both medians are not ideologically aligned, when competing against a unified party, unification produces a gain only for very moderated values of the non aligned median, and for increasing (with α) moderate values of the median aligned with the party. The possibility of gains disapears with increasing electoral competition. Thus the loss for the united party becomes the best prediction for most parameter values.

4 Discussion

This paper has analyzed a model of two simultaneous elections considering different scenarios with the aim of evaluating the benefits of each possible party structure: a unified party competing in the two elections at the same time or two different parties each competing in one of the elections. The different scenarios analyzed are defined according to the feaures that the electorates of each election can exhibit and also according to the party configurations that the party under consideration is facing.

In particular, with respect ot the features of the two electorates we have taken into account the possibility that the expected medians of the two electorates are aligned with each other, and the possibility that they are not aligned. And with respect to the party configurations we have analyzed the case in which the party competes against a unique party in both elections and also the case in which it competes with a different party in each election.

All these scenarios are studied for a wide range of parameter values. With respect to the expected ideal points of the median voter the results depend on whether they are rather extreme or they are rather moderate. Regarding the parameter that represents the distribution of the voters' preferences the results rely on the degree of the intensity of the electoral competition. This parameter can also be interpreted as the proportion of moderate voters that an electorate exhibits. This paper shows the importance of the value of this parameter. Low intensity of the electoral competition represents a polarized distribution of the voters' preferences and induces a highly polarized equilibrium in all scenarios. Instead high intensity of the electoral competition represents a unimodal distribution of the voters' preferences and implies the choice of more aggressive strategies by all parties in competition.

In particular, for extremely low intensity of the electoral competition ($\alpha < \frac{1}{3}$) the full polarization equilibrium obtains except when the two expected medians are not aligned, one of them is very moderate and the other one is very extreme, and there is at least one unified party competing. This implies that when the probability that the median voters are moderate is very low, all parties have incentives to forgo the competition for the moderate voters whether they compete as a unified party of they compete as different independent parties. This is always the case for parties that compete in just one of the elections. Instead a unified party may decide to choose a moderate more aggressive policy when one of the medians is aligned with its ideal point and with that choice the party can guarantee the vote of the moderate voters in both elections. In this case the payoffs of the unified party are higher than those obtained by the sum of two parties that compete indepedently. The amount of these benefits from unification decreases with the intensity of the electoral competition and it also decreases when the median becomes more extreme. However these gains from unification hold for less extreme values of the median when the intensity of the electoral competition increases. This result holds for all scenarios analyzed under the provision that the medians are not aligned with each other, that is, independently of whether the party is facing a unified party or two different independent parties. Therefore, for low intensity of the electoral competition the decision about whether to unify or to split a party is irrelevant except when one of the electorates is very ideologically aligned with the party and the other one is moderate.

For more intense electoral competition the equilibrium strategies exhibit a larger variety of features. First consider the case of two electorates with expected medians that are aligned with each other. The party whose ideal point is also aligned with the medians has no incentives to choose anything that is not its own ideal point, and thus for this party the decision of unifying or spliting becomes irrelevant in all scenarios. Instead the party whose ideal point is not aligned with those of the expected medians only forgoes the competition for the vote of the moderate voters if the two medians are very extreme, and it is only in that case that the decision of unifying or spliting becomes irrelevant for this party. This case becomes less relevant when the intensity of the electoral competition increases. Instead the party will have strong incentives to unite when the two medians are rather moderate and close to each other, and this restriction becomes more relaxed when the intensity of the electoral competition increases. Otherwise, when the two medians are aligned with each other but they are not very close to each other the unification of the party that is not aligned with them will produce losses. In this case, two independent parties will fare better, because they can choose a different policy for each election. The results described for the case of two medians that are ideologically aligned hold independently of the party configuration, that is, whether the party under analysis is facing a unique party in the two elections or two different independent parties.

When the expected medians of the two electorates are not aligned with each other we find that a unified is more likely to produce losses than gains. When competing against two different parties if the median that is aligned with the opponent is rather extreme the party is better off forgoing the chance to convince any moderate voter in that election and prefers to choose its own ideal point. A party competing against two different parties has incentives to unite only if the medians are close enough (so that the party can obtain the vote of the moderate voters of both elections) and it also needs one of the medians closely aligned with the ideology of the party. As the intensity of the electoral competition increases, it becomes less likely that the unified party obtains gains and the possibility of gains disappear as α approaches 1.

If both medians are not ideologically aligned, when competing against a unified party, unification produces a gain only for very moderated values of the non aligned median, and for moderate values of the median aligned with the party, that are required to be more moderate when the intensity of the electoral competition increases. The possibility of gains disappears with increasing electoral competition. Therefore, independently of the party configuration, unification is more likely to produce losses when medians are not aligned with each other and these losses become a general result for intense electoral competition independently of the party configuration.

In any scenario the necessary condition to obtain gains from a unified party is that the two expected medians are close enough to each other so that they guarantee the existence of a strategy that allows the unified party to obtain the vote of the moderate voters in both elections.

5 Concluding remarks

This paper has presented a framework to study the possible electoral advantages and disadvantages of a unified party in order to face simultaneous elections. It concludes that unification can only produce gains with respect to competing with an independent party in each election under very particular conditions, and that the most general prediction is that a unified party performs worse that independent parties in simultaneous elections.

These results are based on the analysis of a theoretical model that relies on two main assumptions. On the one hand, a unified party is supposed to use a unique policy proposal in the two elections that it is facing. This assumption represents a restriction for the unified party with respect to two independent parties that can choose different policy proposals in the different elections. On the other hand, a unified party has a larger strategy set from which to choose its policy proposal, and each independent party has a strategy set which is only a proper subset of it. This assumption offers an advantage to the unified party. In particular, it allows the unified party to be able to obtain the vote of all the moderates with a preferable policy proposal under certain conditions. The analysis provided here solves this tradeoff and predicts that even though the larger strategy set allows the unified party to improve upon the results of the competition with two different parties in some cases, the result that should be expected is that independent parties perform better in simultaneous elections.

The implications of this result can be twofold. On the one hand it provides of arguments as to when two different parties may decide to unite in order to enhance their electoral success, or as to when a party may decide to split into different sub-territorial parties in order to increase its overall electoral support-the. On the other hand, this analysis also provides arguments as to whether parties should support the call for simultaneous elections or they should rather induce a time delay between elections in order to guarantee themselves a better electoral performance. In particular, the results obtained from the analysis of the proposed model offer support for the argument that nation-wide parties should always try to avoid simultaneous elections, while sub-national parties should not care much about having to face elections that are close to each other in terms of time.

The model presented here can be extended in several ways. There are two obvious ways to generalize it that imply a relaxation of the two main assumptions: the restriction of the unique policy choice by a unified party and the definition of the composition of the strategy sets. I argue that these generalizations cannot provide new insights.

The restriction of the unique policy choice can be understood as a reduced form expression of the punishment induced by the ideological inconsistency that voters could apply to parties that propose different policies. Thus, this assumption can be introduced in the model in a more elaborated form. However, the reduced form adopted here includes the effects found in the models that analyze parties' ideological consistency. Thus, a more complicated model should produce the same kind of predictions.

The assumption about the size of the strategy sets can be relaxed by considering that the strategy space is equal to the policy space for all parties. In such a model parties would have to deal with a well known main tradeoff: whether to pander to the expected median in order to increase its chances of winning or to implement a policy close to the party's ideal point in order to increase the value of its chances of winning. And this is precisely the tradeoff that presents the reduced form game analyzed in this paper. Thus the results from this generalization are expected to be qualitatively equal to the ones obtained here.

There are at least two more ways to extend the present model, that imply the introduction of a new tradeoff in the analysis: the assumption that different electorates might have different probabilities of having a moderate expected median voter, that is, the assumption of different electoral intensities (α) for each election; and the consideration that a unified party values differently its success in each of the two elections, that is, the assumption of different weights for each election results in the formulation of the payoff function of a unified party. These two instances represent proper extensions to the present analysis because each one of them implies an additional tradeoff in the parties' decision problem.

If we assume that each electorate has a different probability that the median voter is a moderate voter we are increasing the internal conflict of a unified party because it becomes more likely that two different policies are going to perform better than a unified one. This extension would complicate the formal analysis because it would require to analyze many more combinations of parameter values. But at the same time it would include more realistic features of the simultaneous electoral competition, because when we consider two different elections it is plausible that we have to deal with two completely different distributions of the preferences of the voters. Here we have assumed that the two expected medians of the two electorates are different. However one should also expect that shape of the distribution of the voters' preferences is different in each case. Thus, a different probability of a moderate expected median voter, which can also be interpreted as a different proportion of moderate voters would represent a proper generalization and bring the model closer to a real life situation.

If a unified party values the payoffs obtained from the two elections with different weights, the tradeoff that the party has to solve is clearly affected and the results are bound to be different from the ones obtained here. This extension also brings the model closer to representing a real world case. Indeed, many of the elections that take place simultaneous involve elections for different levels of government: national an regional, regional and municipal, supranational and national,... In all this cases, it makes sense to consider that a party may be much more interested in its electoral success in the election that is held for the higher government level. Thus the strategies chosen by a unified party are going to lean more towards the maximization of the payoffs that it derives from the higher level election, and may even imply a complete disregard of its results derived from the lower level election.

6 References

Agranov, M., (2016) "Flip-flopping, primary visibility, and the selection of candidates" *American Economic Journal: Microeconomics*, 8(2): 61-85.

Andreottola, G. (2021) "Flip-Flopping and Electoral Concerns" *The Journal of Politics* 83 (4): 1669–1680. https://doi.org/10.1086/715070

Brancati, D. (2007) "The Origins and Strengths of Regional Parties" *British Journal of Political Science* 38(1): 135 - 159. https://doi.org/10.1017/S0007123408000070

Callander, S. (2005) "Electoral Competition in Heterogeneous Districts" *Journal of Political Economy* 113(5): 1116–1145. https://doi.org/10.1086/444405

Fabre, E. (2010) "Multi-level Election Timing—A Comparative Overview" *Regional & Federal Studies* 20(2): 175-199. https://doi.org/10.1080/13597561003729897

Hummel, P. (2010) "Flip-flopping from primaries to general elections" *Journal of Public Economics*, 94(11-12): 1020-1027.

Kreps, T.A., Laurin, K., and Merritt, A.C. (2017) "Hypocritical flip-flop, or courageous evolution? When leaders change their moral minds" *Journal of*

Personality and Social Psychology, 113(5): 730-752.

Libbrecht, L., Maddens, B., and Swenden, W. (2011) "Party competition in regional elections: The strategies of state-wide parties in Spain and the United Kingdom" *Party Politics* 19(4): 624-640. https://doi.org/10.1177/1354068811407602

Müller, J. (2012) "On a Short Leash? Sub-National Party Positions between Regional Context and National Party Unity" *Journal of Elections, Public Opinion and Parties* 23(2): 177-199. https://doi.org/10.1080/17457289.2012.742092

Stone, W.J. and Simas, E.N. (2010) "Candidate valence and ideological positions in the uS House elections" *American Journal of Political Science* 54(2): 371-388.

Toubeau, S. (2011) "Regional Nationalist Parties and Constitutional Change in Parliamentary Democracies: A Framework for Analysis" *Regional & Federal Studies* 21(4-5): 427-446. https://doi.org/10.1080/13597566.2011.578929

7 Appendix

Proof of Proposition 1

Suppose $0 < m < \frac{1}{2}$. We have that:
$U_L(0,1) = -\frac{1-\alpha}{2} > -\frac{1+\alpha}{2}m - \frac{1-\alpha}{2} = U_L(m,1)$
$U_L(0,1) = -\frac{1-\alpha}{2} > -1 = U_L(1,1)$
and the best response of L against $x_R = 1$ is $x_L(1) = 0$.
We also have that:
$U_L(0,m) = -\frac{1+\alpha}{2}m > -m = U_L(m,m)$
$U_L(0,m) = -\frac{1+\alpha}{2}m > -\frac{1-\alpha}{2} - \frac{1+\alpha}{2}m = U_L(1,m)$
and the best response of L against $x_R = m$.
Finally,we have that:
$U_L(0,0) = 0 > -\frac{1+\alpha}{2}m = U_L(m,0)$
$U_L(0,0) = 0 > -\frac{1-\alpha}{2} = U_L(1,0)$
and the best response of L against $x_R = 0$ is $x_L(0) = 0$.

Therefore, $x_L = 0$ is a dominant strategy for L for all $0 < \alpha < 1$ and all $0 < m < \frac{1}{2}$.

Next we need to find the best responses of party R to $x_L = 0$ and we have that:
$U_R(0,1) = -\frac{1+\alpha}{2} > -1 = U_R(0,0)$
$U_R(0,m) = -\frac{1-\alpha}{2} - \frac{1+\alpha}{2}(1-m) > -\frac{1+\alpha}{2} = U_R(0,1)$ iff $m > \frac{1-\alpha}{1+\alpha}$
and the best response of R against $x_L = 0$ is

$$x_R(0) = \begin{cases} m & if \quad m \geq \frac{1-\alpha}{1+\alpha} \\ 1 & if \quad m \leq \frac{1-\alpha}{1+\alpha} \end{cases} \quad \blacksquare$$

Proof of Theorem 1

Suppose that $0 < m_1 < m_2 < \frac{1}{2}$ and (L_1, L_2, R)

We know from proposition 1 that the best response of parties $L_1$and L_2 is always $x_L(x_R) = 0$ for all x_R, for all $0 < \alpha < 1$ and all $0 < m_1 < m_2 < \frac{1}{2}$. The strategy space for party R is $S_{1,2} = \{0, m_1, m_2, 1\}$. The payoffs for party R are given by:

$U_R(0,0,1) = 2\left(-\frac{1+\alpha}{2}\right) = -(1+\alpha)$

$U_R(0,0,m_1) = 2\left(-\frac{1-\alpha}{2} - \frac{1+\alpha}{2}(1-m_1)\right) = (1+\alpha)m_1 - 2$

$U_R(0,0,m_2) = 2\left(-\frac{1-\alpha}{2} - \frac{1+\alpha}{2}(1-m_2)\right) = (1+\alpha)m_2 - 2$ if $2m_1 > m_2$

$U_R(0,0,m_2) = -\frac{1+\alpha}{2} - \frac{1-\alpha}{2}(1-m_2) - \frac{1-\alpha}{2} - \frac{1+\alpha}{2}(1-m_2) = m_2 - 2$ if $2m_1 < m_2$

$U_R(0,0,0) = -2$

Notice that $U_R(0,0,0) = -2 < -(1+\alpha) = U_R(0,0,1)$. Thus, $x_R = 0$ is a dominated strategy for all $0 < \alpha < 1$ and all $0 < m_1 < m_2 < \frac{1}{2}$.

Now we compare the remaining payoffs considering different cases:

1) If $\alpha \leq \frac{1}{3}$ we have that $\frac{1-\alpha}{1+\alpha} \geq \frac{1}{2}$ and thus $m_1 < m_2 < \frac{1-\alpha}{1+\alpha}$ which implies that the best response of the rightist party in both electoral competitions is $x_R(0) = 1$.

2) If $\alpha > \frac{1}{3}$ and $m_1 < m_2 < \frac{1-\alpha}{1+\alpha} < \frac{1}{2}$ again the best response of the rightist party in both electoral competitions is $x_R(0) = 1$.

3) If $\alpha > \frac{1}{3}$ and $m_1 < \frac{1-\alpha}{1+\alpha} < m_2 < 2m_1$ then we have that the best response of the rightist party in election 1 is $x_{R_1}(0) = 1$ and the best response of the rightist party in election 2 is $x_{R_2}(0) = m_2$. However in this case the rightist party is required to choose the same policy in both elections.

Observe that since:

$U_R(0,0,1) > U_R(0,0,m_1)$ iff $-(1+\alpha) > (1+\alpha)m_1 - 2$ iff $\frac{1-\alpha}{1+\alpha} > m_1$ which always holds in this case, and

$U_R(0,0,1) < U_R(0,0,m_2)$ iff $-(1+\alpha) < (1+\alpha)m_2 - 2$ iff $\frac{1-\alpha}{1+\alpha} < m_2$ which always holds in this case.

Thus $U_R(0,0,m_1) < U_R(0,0,1) < U_R(0,0,m_2)$ implies that the common best response of the rightist party is $x_R(0) = m_2$.

4) If $\alpha > \frac{1}{3}$ and $m_1 < \frac{1-\alpha}{1+\alpha} < m_2$ and $2m_1 < m_2$ then we have that the best response of the rightist party in election 1 is $x_{R_1}(0) = 1$ and the best response of the rightist party in election 2 is $x_{R_2}(0) = m_2$. However in this case the rightist party is required to choose the same policy in both elections.

Observe that since:

$U_R(0,0,1) > U_R(0,0,m_1)$ iff $-(1+\alpha) > (1+\alpha)m_1 - 2$ iff $\frac{1-\alpha}{1+\alpha} > m_1$ which always holds in this case, and

$U_R(0,0,1) < U_R(0,0,m_2)$ iff $-(1+\alpha) < m_2 - 2$ iff $m_2 > 1 - \alpha$

we have that the common best response of the rightist party is $x_R(0) = 1$ if $\frac{1-\alpha}{1+\alpha} < m_2 \leq 1 - \alpha$; and $x_R(0) = m_2$ if $1 - \alpha \leq m_2 < \frac{1}{2}$.

Notice that for $\frac{1}{3} < \alpha < \frac{1}{2}$ we have that $1 - \alpha > \frac{1}{2}$ and thus $m_2 < 1 - \alpha$. This implies that in this case $x_R(0) = 1$.

5) If $\alpha > \frac{1}{3}$ and $\frac{1-\alpha}{1+\alpha} < m_1 < m_2 < 2m_1$ then we have that the best response of the rightist party in election 1 is $x_{R_1}(0) = m_1$ and the best response of the rightist party in election 2 is $x_{R_2}(0) = m_2$. However, the rightist party is required to choose the same policy in both elections.

Observe that since

$U_R(0,0,m_1) > U_R(0,0,1)$ iff $(1+\alpha)\,m_1 - 2 > -(1+\alpha)$ iff $m_1 > \frac{1-\alpha}{1+\alpha}$, which always holds in this case, and

$U_R(0,0,m_1) < U_R(0,0,m_2)$ iff $(1+\alpha)\,m_1 - 2 < (1+\alpha)\,m_2 - 2$ iff $m_1 < m_2$ which always holds in this case.

Thus $U_R(0,0,1) < U_R(0,0,m_1) < U_R(0,0,m_2)$ the common best response of the rightist party is $x_R(0) = m_2$.

6) If $\alpha > \frac{1}{3}$ and $\frac{1-\alpha}{1+\alpha} < m_1 < m_2$ and $2m_1 < m_2$ then we have that the best response of the rightist party in election 1 is $x_{R_1}(0) = m_1$ and the best response of the rightist party in election 2 is $x_{R_2}(0) = m_2$. However, the rightist party is required to choose the same policy in both elections.

Observe that since

$U_R(0,0,m_1) > U_R(0,0,1)$ iff $(1+\alpha)\,m_1 - 2 > -(1+\alpha)$ iff $m_1 > \frac{1-\alpha}{1+\alpha}$, which always holds in this case, and

$U_R(0,0,m_1) < U_R(0,0,m_2)$ iff $(1+\alpha)\,m_1 - 2 < m_2 - 2$ iff $(1+\alpha)\,m_1 < m_2$.

Notice that since $2m_1 < m_2$ and $(1+\alpha)\,m_1 < 2m_1$ we must have that $(1+\alpha)\,m_1 < m_2$, and $U_R(0,0,m_1) < U_R(0,0,m_2)$ always holds in this case.

Thus $U_R(0,0,1) < U_R(0,0,m_1) < U_R(0,0,m_2)$ implies that the common best response of the rightist party is $x_R(0) = m_2$. Notice that this case only holds for $\alpha > \frac{3}{5}$.

Overall we have that 1) and 2) imply that for $m_1 < m_2 < \frac{1-\alpha}{1+\alpha}$ the best response is $x_{R_1}(0) = x_{R_2}(0) = 1$; 5) and 6) imply that for $\frac{1-\alpha}{1+\alpha} < m_1 < m_2$ the best response is $x_{R_1}(0) = x_{R_2}(0) = m_2$; and 3) and 4) imply that for $m_1 < \frac{1-\alpha}{1+\alpha} < m_2$ the best response is:

$x_{R_1}(0) = x_{R_2}(0) = m_2$ if $2m_1 > m_2$

$x_{R_1}(0) = x_{R_2}(0) = m_2$ if $2m_1 < m_2$ and $1-\alpha < m_2$

$x_{R_1}(0) = x_{R_2}(0) = 1$ if $2m_1 < m_2$ and $1-\alpha > m_2$

Equilibrium strategies:

$$x_L^*\,(\alpha, m_1, m_2) = 0$$

$$x_R^*\,(\alpha, m_1, m_2) = \begin{cases} 1 & if \quad \alpha \leq \frac{1}{3} \\ 1 & if \quad m_1 < m_2 \leq \frac{1-\alpha}{1+\alpha} \\ 1 & if \quad m_1 < \frac{1-\alpha}{1+\alpha} < m_2 \wedge 2m_1 < m_2 < 1-\alpha \\ m_2 & if \quad m_1 < \frac{1-\alpha}{1+\alpha} < m_2 \wedge 2m_1 < m_2 \wedge 1-\alpha < m_2 \\ m_2 & if \quad m_1 < \frac{1-\alpha}{1+\alpha} < m_2 < 2m_1 \\ m_2 & if \quad \frac{1-\alpha}{1+\alpha} < m_1 < m_2 \end{cases} \quad \blacksquare$$

Proof of Proposition 2

Suppose $0 < m_1 < m_2 < \frac{1}{2}$. If all parties compete separately in the two elections (L_1, L_2, R_1, R_2) the full polarization equilibrium obtains if $m_1 <$

$m_2 < \frac{1-\alpha}{1+\alpha}$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(0,1) = -\frac{1-\alpha}{2}$; the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0,0,m_1,m_2)$ if $\frac{1-\alpha}{1+\alpha} < m_1 < m_2$, with $U_{R_1}(0,m_1) = \frac{1+\alpha}{2}m_1 - 1$ and $U_{R_2}(0,m_2) = \frac{1+\alpha}{2}m_2 - 1$; otherwise the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0,0,1,m_2)$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(0,m_2) = \frac{1+\alpha}{2}m_2 - 1$.

We compare these payoffs to the ones obtained by party R when competing as a united party (L_1, L_2, R) in the two elections described in theorem 1 in order to obtain $\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R)$.

First of all notice that if both medians are extreme, $m_1 < m_2 < \frac{1-\alpha}{1+\alpha}$, in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R}\right) = (0,0,1)$ which is exactly the same that obtains if the rightist parties compete separately $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0,0,1,1)$, thus $\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = 0$.

If both medians are moderate, $\frac{1-\alpha}{1+\alpha} < m_1 < m_2$, in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R}\right) = (0,0,m_2)$. If the rightist parties were competing separately we would have $x_{R_1} = m_1$ and $x_{R_2} = m_2$, and the payoff difference is given by

$$\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1}(0,m_1) + U_{R_2}(0,m_2) - U_R(0,0,m_2) = \frac{1+\alpha}{2}(m_1 - m_2) < 0$$

which implies a gain from the union. This gain $\left|\frac{1+\alpha}{2}(m_2 - m_1)\right|$ increases with α and increases with the distance between the two medians. The gain comes from the fact that as a unique party R may obtain the moderate vote in both cases with m_2 which is a more favorable policy.

For intermediate medians, $m_1 < \frac{1-\alpha}{1+\alpha} < m_2$, if the rightist parties were competing separately we would have $x_{R_1} = 1$ and $x_{R_2} = m_2$. If the medians are separated enough and m_2 is not too large, $2m_1 < m_2 < 1-\alpha$, in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R}\right) = (0,0,1)$ and the payoff difference is given by

$$\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1}(0,1) + U_{R_2}(0,m_2) - U_R(0,0,1) = \frac{1+\alpha}{2}m_2 - \frac{1-\alpha}{2} > 0$$

iff $m_2 > \frac{1-\alpha}{1+\alpha}$ which holds in this case and implies a loss from the union. This loss $\left|\frac{1+\alpha}{2}m_2 - \frac{1-\alpha}{2}\right|$ increases with α and with m_2. Notice that the size of this area decreases with α because it can be represented as: $\frac{(1-\alpha)^2}{1+\alpha} + \frac{1}{4}\left(1 - \alpha - \frac{1-\alpha}{1+\alpha}\right)^2 = \frac{1-\alpha}{1+\alpha}\left(1 - \frac{3\alpha}{4}\right)$ which decreases with α.

For intermediate medians, $m_1 < \frac{1-\alpha}{1+\alpha} < m_2$, if the medians are separated enough and m_2 is large enough $(m_2 > 1-\alpha)$ in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R}\right) = (0,0,m_2)$ and the difference between their payoffs is given by

$$\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1}(0,1) + U_{R_2}(0,m_2) - U_R(0,0,m_2 : 2m_1 < m_2) = \frac{1-\alpha}{2}(1 - m_2) >$$

which implies a loss from the union. This loss $\left|\frac{1-\alpha}{2}(1 - m_2)\right|$ decreases with α and decreases with m_2.

For intermediate medians, $m_1 < \frac{1-\alpha}{1+\alpha} < m_2$, if the medians are close enough, $2m_1 > m_2$, in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ and the difference between their payoffs is given by

$$\Delta_R\left(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R\right) = U_{R_1}(0,1) + U_{R_2}(0, m_2) - U_R(0, 0, m_2 : 2m_1 > m_2) = \frac{1-\alpha}{2} - \frac{1+\alpha}{2} m_2 \cdot$$

whenever $m_2 > \frac{1-\alpha}{1+\alpha}$ which holds in this case and it implies a gain from the union. This gain $\left|\frac{1+\alpha}{2} m_2 - \frac{1-\alpha}{2}\right|$ increases with α and increases with m_2.

Notice that in the last two cases, the parties strategies are the same but the payoffs are different, because in the last case choosing m_2 in the first election allows party R to obtain the moderate vote, which only happens when the medians are close enough to each other. Because of this, party R obtains a gain from competing as a unique party. ■

Proof of Proposition 3

Suppose that $0 < m_1 < \frac{1}{2} < m_2 < 1$.
For L_1 we know from proposition 1 that $x_{L_1}(0) = x_{L_1}(m_1) = x_{L_1}(1) = 0$ and for $x_{L_1}(m_2)$ we have:
$U_{L_1}(0, m_2) = -\frac{1+\alpha}{2} m_2$ if $m_1 > \frac{m_2}{2}$
$U_{L_1}(0, m_2) = -\frac{1-\alpha}{2} m_2$ if $m_1 < \frac{m_2}{2}$
$U_{L_1}(m_1, m_2) = -\frac{1+\alpha}{2} m_1 - \frac{1-\alpha}{2} m_2$
$U_{L_1}(1, m_2) = -\frac{1-\alpha}{2} - \frac{1+\alpha}{2} m_2$
If $m_1 > \frac{m_2}{2}$ we have that:
$U_{L_1}(0, m_2) > U_{L_1}(1, m_2)$
$U_{L_1}(m_1, m_2) > U_{L_1}(0, m_2)$ iff $\frac{2\alpha}{1+\alpha} m_2 > m_1$
If $m_1 < \frac{m_2}{2}$ we have that:
$U_{L_1}(0, m_2) > U_{L_1}(1, m_2)$
$U_{L_1}(0, m_2) > U_{L_1}(m_1, m_2)$
Thus the best response of L_1 to m_2 is $x_{L_1}(m_2) = \begin{cases} m_1 & if \quad \frac{m_2}{2} < m_1 < \frac{2\alpha}{1+\alpha} m_2 \\ 0 & \qquad otherwise \end{cases}$

For L_2 we know from proposition 1 that $x_{L_2}(0) = x_{L_2}(m_2) = 0$, and $x_{L_2}(1) = \begin{cases} m_2 & if \quad m_2 < \frac{2\alpha}{1+\alpha} \\ 0 & \quad otherwise \end{cases}$
and for $x_{L_2}(m_1)$ we have:
$U_{L_2}(0, m_1) = -\frac{1+\alpha}{2} m_1$
$U_{L_2}(m_2, m_1) = -\frac{1+\alpha}{2} m_2 - \frac{1-\alpha}{2} m_1$
$U_{L_2}(1, m_1) = -\frac{1-\alpha}{2} - \frac{1+\alpha}{2} m_1$ if $m_2 < \frac{1+m_1}{2}$
$U_{L_2}(1, m_1) = -\frac{1+\alpha}{2} - \frac{1-\alpha}{2} m_1$ if $m_2 > \frac{1+m_1}{2}$
and
$U_{L_2}(0, m_1) = -\frac{1+\alpha}{2} m_1 > -\frac{1+\alpha}{2} m_2 - \frac{1-\alpha}{2} m_1 = U_{L_2}(m_2, m_1)$ iff $\frac{2\alpha}{1+\alpha} m_1 < m_2$
which always holds since $\frac{2\alpha}{1+\alpha} < 1$ for $\alpha < 1$
$U_{L_2}(0, m_1) = -\frac{1+\alpha}{2} m_1 > -\frac{1-\alpha}{2} - \frac{1+\alpha}{2} m_1 = U_{L_2}(1, m_1)$ if $m_2 < \frac{1+m_1}{2}$

$U_{L_2}(0, m_1) = -\frac{1+\alpha}{2} m_1 > -\frac{1+\alpha}{2} - \frac{1-\alpha}{2} m_1 = U_{L_2}(1, m_1)$ if $m_2 > \frac{1+m_1}{2}$

Thus the best response of L_2 to m_1 is $x_{L_2}(m_1) = 0$. ■

Proof of Theorem 2

Suppose that $0 < m_1 < \frac{1}{2} < m_2 < 1$ and (L_1, L_2, R). From proposition 3 we have the best responses of L_1 and L_2.

First we consider $\alpha < \frac{1}{3}$ and we look for the best responses of R to $x_{L_1} = x_{L_2} = 0$. We have that:

$U_R(0, 0, 1) = -\frac{1+\alpha}{2} - \frac{1-\alpha}{2} = -1$

$U_R(0, 0, m_1) = (1 + \alpha) m_1 - 2$

$U_R(0, 0, m_2) = (1 + \alpha) m_2 - 2$ if $2m_1 > m_2$

$U_R(0, 0, m_2) = m_2 - 2$ if $2m_1 < m_2$

$U_R(0, 0, 0) = -2$

Notice that $U_R(0, 0, 0) = -2 < -1 = U_R(0, 0, 1)$ thus $x_R = 0$ is a dominated strategy.

Now we compare the rest of the payoffs:

If $2m_1 > m_2$:

$U_R(0, 0, m_2) > U_{R_1+R_2}(0, 0, m_1)$ iff $(1 + \alpha) m_2 - 2 > (1 + \alpha) m_1 - 2$ iff $m_2 > m_1$ which always holds.

$U_R(0, 0, m_2) > U_{R_1+R_2}(0, 0, 1)$ iff $(1 + \alpha) m_2 - 2 > -1$ iff $m_2 > \frac{1}{1+\alpha} > \frac{1}{2}$ iff $1 > \alpha$ thus the best responses of party R to $(x_{L_1}, x_{L_2}) = x_R$ are

$$x_R(0, 0) = \begin{cases} m_2 & if \quad m_2 > \frac{1}{1+\alpha} \\ 1 & \quad otherwise \end{cases}$$

If $2m_1 < m_2$:

$U_R(0, 0, 1) > U_R(0, 0, m_2)$ iff $-1 > m_2 - 2$ iff $1 > m_2$ which always holds.

$U_R(0, 0, 1) > U_R(0, 0, m_1)$ iff $-1 > (1 + \alpha) m_1 - 2$ iff $\frac{1}{1+\alpha} > m_1$ which always holds thus the best response of party R to $(x_{L_1}, x_{L_2}) = x_R$ is $x_R = 1$

Next we analyze the case of $\alpha > \frac{1}{3}$. The payoffs for R are as follows:

$U_R(0, 0, 1) = -\frac{1+\alpha}{2} - \frac{1-\alpha}{2} = -1$

$U_R(0, 0, m_1) = (1 + \alpha) m_1 - 2$

$U_R(0, 0, m_2) = (1 + \alpha) m_2 - 2$ if $2m_1 > m_2$

$U_R(0, 0, m_2) = m_2 - 2$ if $2m_1 < m_2$

$U_R(0, 0, 0) = -2$

and in addition:

$U_R(0, m_2, 1) = \frac{1+\alpha}{2} (m_2 - 2)$ for $m_2 < \frac{2\alpha}{1+\alpha}$

$U_R(m_1, 0, m_2) = \frac{1+\alpha}{2} m_1 + m_2 - 2$ for $\frac{m_2}{2} < m_1 < \frac{2\alpha}{1+\alpha} m_2$

Notice that $U_R(0, 0, 0) = -2 < -1 = U_R(0, 0, 1)$ thus $x_R = 0$ is a dominated strategy.

Now we compare the remaining payoffs considering different cases:

1) If $m_2 < \frac{2\alpha}{1+\alpha}$ and $\frac{1+\alpha}{2\alpha} m_1 < m_2 < 2m_1$ the payoffs of R are:

$U_R(0, m_2, 1) = \frac{1+\alpha}{2} (m_2 - 2)$

$U_R(0, 0, m_1) = (1 + \alpha) m_1 - 2$

$U_R(m_1, 0, m_2) = \frac{1+\alpha}{2} m_1 + m_2 - 2$

and comparing payoffs we have:

$U_R(0,0,m_1) < U_R(m_1,0,m_2)$ iff $(1+\alpha)m_1 - 2 < \frac{1+\alpha}{2}m_1 + m_2 - 2$ iff $\frac{1+\alpha}{2}m_1 < m_2$ which always holds

$U_R(0,m_2,1) < U_R(m_1,0,m_2)$ iff $\frac{1+\alpha}{2}(m_2-2) < \frac{1+\alpha}{2}m_1 + m_2 - 2$ iff $m_2 > 2 - \frac{1+\alpha}{1-\alpha}m_1$

Notice that $2 - \frac{1+\alpha}{1-\alpha}m_1 = \frac{1}{2}$ iff $m_1 = \frac{3(1-\alpha)}{2(1+\alpha)}$, and $\frac{3(1-\alpha)}{2(1+\alpha)} < \frac{1}{4}$ iff $\frac{5}{7} < \alpha$. In this case we have that $m_2 > 2 - \frac{1+\alpha}{1-\alpha}m_1$ for all values and thus $x_R = m_2$.

Since $2 - \frac{1+\alpha}{1-\alpha}m_1 = \frac{2\alpha}{1+\alpha}$ iff $m_1 = \frac{2(1-\alpha)}{(1+\alpha)^2}$, and $\frac{1+\alpha}{2\alpha}m_1 = \frac{2\alpha}{1+\alpha}$ iff $m_1 = \frac{4\alpha^2}{(1+\alpha)^2}$ we have that $\frac{2(1-\alpha)}{(1+\alpha)^2} > \frac{4\alpha^2}{(1+\alpha)^2}$ (iff $0 > 2\alpha^2 + \alpha - 1$ iff $\alpha < \frac{1}{2}$) implies $m_2 < 2 - \frac{1+\alpha}{1-\alpha}m_1$ for all values. Thus for $\alpha < \frac{1}{2}$ we have that $x_R = 1$.

And for $\frac{1}{2} < \alpha < \frac{5}{7}$, we must have that.

$x_R = m_2$ if $m_2 > 2 - \frac{1+\alpha}{1-\alpha}m_1$

$x_R = 1$ if $m_2 < 2 - \frac{1+\alpha}{1-\alpha}m_1$

Therefore the best responses of party R in this case are:

$$x_R = \begin{cases} 1 & if \quad \alpha < \frac{1}{2} \\ 1 & if \quad \frac{1}{2} < \alpha < \frac{5}{7} \text{ and } m_2 \leq 2 - \frac{1+\alpha}{1-\alpha}m_1 \\ m_2 & if \quad \frac{1}{2} < \alpha < \frac{5}{7} \text{ and } m_2 \geq 2 - \frac{1+\alpha}{1-\alpha}m_1 \\ m_2 & if \quad \alpha > \frac{5}{7} \end{cases}$$

2) If $m_2 > \frac{2\alpha}{1+\alpha}$ and $\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1$ the payoffs of R are:

$U_R(0,0,1) = -1$

$U_R(0,0,m_1) = (1+\alpha)m_1 - 2$

$U_R(m_1,0,m_2) = \frac{1+\alpha}{2}m_1 + m_2 - 2$

and comparing payoffs we have:

$U_R(0,0,m_1) < U_R(m_1,0,m_2)$ iff $(1+\alpha)m_1 - 2 < \frac{1+\alpha}{2}m_1 + m_2 - 2$ iff $\frac{1+\alpha}{2}m_1 < m_2$ which always holds

$U_R(0,0,1) > U_R(m_1,0,m_2)$ iff $-1 > \frac{1+\alpha}{2}m_1 + m_2 - 2$ iff $1 - \frac{1+\alpha}{2}m_1 > m_2$

Notice that $1 - \frac{1+\alpha}{2}m_1 = 2m_1$ iff $m_1 = \frac{2}{5+\alpha} < \frac{1}{2}$; $2m_1 = \frac{2\alpha}{1+\alpha}$ iff $m_1 = \frac{\alpha}{1+\alpha}$ and $\frac{2}{5+\alpha} < \frac{\alpha}{1+\alpha}$ iff $0 < \alpha^2 + 3\alpha - 2$ iff $\alpha > \frac{-3\pm\sqrt{17}}{2} = 0,56$. In this case $1 - \frac{1+\alpha}{2}m_1 < m_2$ for all values and $x_R = m_2$.

Otherwise for $\alpha < 0,56$ we have that:

$x_R = 1$ if $1 - \frac{1+\alpha}{2}m_1 > m_2$

$x_R = m_2$ if $1 - \frac{1+\alpha}{2}m_1 < m_2$

Therefore the best responses of party R in this case are:

$$x_R = \begin{cases} 1 & if \quad \alpha < 0,56 \text{ and } m_2 \leq 1 - \frac{1+\alpha}{2}m_1 \\ m_2 & if \quad \alpha < 0,56 \text{ and } m_2 \geq 1 - \frac{1+\alpha}{2}m_1 \\ m_2 & if \quad \alpha > 0,56 \end{cases}$$

Notice that $1 - \frac{1+\alpha}{2}m_1 > 2m_1$ iff $\frac{2}{5+2\alpha} > m_1$; and $\frac{2}{5+2\alpha} < \frac{1-\alpha}{1+\alpha}$ iff $0 > -3 + 5\alpha + 2\alpha^2$ iff $\alpha = \frac{-5\pm7}{4} < \frac{1}{2}$.

Notice that $1 - \frac{1+\alpha}{2}m_1 > \frac{1}{1+\alpha}$ iff $\frac{2\alpha}{(1+\alpha)^2} > m_1$

3) If $2m_1 < m_2 < \frac{2\alpha}{1+\alpha}$ the payoffs of R are:

$U_R(0,m_2,1) = \frac{1+\alpha}{2}(m_2-2)$

$U_R(0,0,m_1) = (1+\alpha)m_1 - 2$

$U_R(0,0,m_2) = m_2 - 2$ if $2m_1 < m_2$

and comparing payoffs we have:

$U_R(0, m_2, 1) > U_R(0, 0, m_2)$ iff $\frac{1+\alpha}{2}(m_2 - 2) < m_2 - 2$ iff $\frac{1+\alpha}{2} < 1$ which always holds

$U_R(0, m_2, 1) > U_R(0, 0, m_1)$ iff $\frac{1+\alpha}{2}(m_2 - 2) > (1+\alpha)m_1 - 2$ iff $\frac{m_2}{2} + \frac{1-\alpha}{1+\alpha} > m_1$ which always holds. Thus the best response of party R is $x_R = 1$

4) If $m_2 < \frac{2\alpha}{1+\alpha}$ and $m_2 < \frac{1+\alpha}{2\alpha}m_1$ the payoffs of R are:

$U_R(0, m_2, 1) = \frac{1+\alpha}{2}(m_2 - 2)$

$U_R(0, 0, m_1) = (1+\alpha)m_1 - 2$

$U_R(0, 0, m_2) = (1+\alpha)m_2 - 2$ if $2m_1 > m_2$

and comparing payoffs we have:

$U_R(0, 0, m_2) > U_R(0, 0, m_1)$ iff $(1+\alpha)m_2 - 2 > (1+\alpha)m_1 - 2$ which always holds.

$U_R(0, m_2, 1) > U_R(0, 0, m_2)$ iff $\frac{1+\alpha}{2}(m_2 - 2) > (1+\alpha)m_2 - 2$ iff $\frac{2(1-\alpha)}{1+\alpha} > m_2$.

If $\alpha > \frac{3}{5}$ we have that $\frac{2(1-\alpha)}{1+\alpha} < \frac{1}{2}$ and $\frac{2(1-\alpha)}{1+\alpha} < m_2$ which implies $x_R = m_2$.

We have that $\frac{2(1-\alpha)}{1+\alpha} > \frac{2\alpha}{1+\alpha}$ iff $\frac{1}{2} > \alpha$. Thus for $\alpha < \frac{1}{2}$ we must have $x_R = 1$.

And for $\frac{1}{2} < \alpha < \frac{3}{5}$ we have $x_R = m_2$ for $\frac{2(1-\alpha)}{1+\alpha} < m_2 < \frac{2\alpha}{1+\alpha}$;

and $x_R = 1$ for $\frac{1}{2} < m_2 < \frac{2(1-\alpha)}{1+\alpha}$.

Therefore the best responses of party R in this case are:

$$x_R = \begin{cases} 1 & if & \alpha < \frac{1}{2} \\ 1 & if & \frac{1}{2} < \alpha < \frac{3}{5} \text{ and } m_2 \leq \frac{2(1-\alpha)}{1+\alpha} \\ m_2 & if & \frac{1}{2} < \alpha < \frac{3}{5} \text{ and } m_2 \geq \frac{2(1-\alpha)}{1+\alpha} \\ m_2 & if & \alpha > \frac{3}{5} \end{cases}$$

5) If $m_2 > \frac{2\alpha}{1+\alpha}$ and $m_2 > 2m_1$ the payoffs of R are:

$U_R(0, 0, 1) = -1$

$U_R(0, 0, m_1) = (1+\alpha)m_1 - 2$

$U_R(0, 0, m_2) = m_2 - 2$ if $2m_1 < m_2$

and comparing payoffs we have:

$U_R(0, 0, 1) > U_R(0, 0, m_1)$ iff $-1 > (1+\alpha)m_1 - 2$ iff $\frac{1}{1+\alpha} > m_1$ which always holds.

$U_R(0, 0, 1) > U_R(0, 0, m_2)$ iff $-1 > m_2 - 2$ iff $1 > m_2$ which always holds.

Thus the best response of party R is $x_R = 1$

6) If $\frac{2\alpha}{1+\alpha} < m_2 < \frac{1+\alpha}{2\alpha}m_1$ the payoffs of R are:

$U_R(0, 0, 1) = -1$

$U_R(0, 0, m_1) = (1+\alpha)m_1 - 2$

$U_R(0, 0, m_2) = (1+\alpha)m_2 - 2$ if $2m_1 > m_2$

and

$U_R(0, 0, m_2) > U_R(0, 0, m_1)$ iff $(1+\alpha)m_2 - 2 > (1+\alpha)m_1 - 2$ which always holds.

$U_R(0, 0, m_2) > U_R(0, 0, 1)$ iff $(1+\alpha)m_2 - 2 > -1$ iff $m_2 > \frac{1}{1+\alpha}$.

Since $\frac{1+\alpha}{2\alpha}\left(\frac{1}{2}\right) = \frac{1+\alpha}{4\alpha}$, for this case to hold we need $\frac{2\alpha}{1+\alpha} < \frac{1+\alpha}{4\alpha}$ iff $7\alpha^2 - 2\alpha - 1 < 0$ iff $\alpha < \frac{2\pm\sqrt{32}}{14} = 0,54$.

Since $\frac{2\alpha}{1+\alpha} > \frac{1}{1+\alpha}$ iff $\frac{1}{2} < \alpha < 0,54$ we have $x_R = m_2$ for $\alpha > \frac{1}{2}$.

For $\alpha < \frac{1}{2}$ we have $x_R = m_2$ if $\frac{2\alpha}{1+\alpha} < \frac{1}{1+\alpha} < m_2$;
and we have $x_R = 1$ if $\frac{2\alpha}{1+\alpha} < m_2 < \frac{1}{1+\alpha}$.
Therefore the best responses of party R in this case are:

$$x_R = \begin{cases} 1 & if \quad \alpha < \frac{1}{2} \text{ and } m_2 \leq \frac{1}{1+\alpha} \\ m_2 & if \quad \alpha < \frac{1}{2} \text{ and } m_2 \geq \frac{1}{1+\alpha} \\ m_2 & if \quad \frac{1}{2} < \alpha < 0,54 \end{cases}$$

Combining cases 3 and 5, we find that for $m_2 > 2m_1$ the best response of R is $x_R = 1$.

Combining cases 4 and 6 we find that for $m_2 < \frac{1+\alpha}{2\alpha}m_1$ the best responses of party R in this case are:

$$x_R = \begin{cases} 1 & if \quad \alpha < \frac{1}{2} \text{ and } m_2 \leq \frac{1}{1+\alpha} \\ m_2 & if \quad \alpha < \frac{1}{2} \text{ and } m_2 \geq \frac{1}{1+\alpha} \\ 1 & if \quad \frac{1}{2} < \alpha < \frac{3}{5} \text{ and } m_2 \leq \frac{2(1-\alpha)}{1+\alpha} \\ m_2 & if \quad \frac{1}{2} < \alpha < \frac{3}{5} \text{ and } m_2 \geq \frac{2(1-\alpha)}{1+\alpha} \\ m_2 & if \quad \alpha > \frac{3}{5} \end{cases}$$

Combining cases 1 and 2 we have that for $\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1$:

if $\alpha > \frac{5}{7} : x_R = m_2$ for both areas

if $0,56 < \alpha < \frac{5}{7} : x_R = m_2$ for all area 2 and for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ in area 1; and $x_R = 1$ for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ in area 1.

if $\frac{1}{2} < \alpha < 0,56 : x_R = m_2$ for $1 - \frac{1+\alpha}{2}m_1 < m_2$ in area 2 and for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ in area 1; $x_R = 1$ for $1 - \frac{1+\alpha}{2}m_1 > m_2$ in area 2 and for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ in area 1.

Notice that $2 - \frac{1+\alpha}{1-\alpha}m_1 = 1 - \frac{1+\alpha}{2}m_1 = \frac{2\alpha}{1+\alpha}$ iff $m_1 = \frac{2(1-\alpha)}{(1+\alpha)^2}$.

if $\frac{1}{2} > \alpha : x_R = 1$ in all area 1, and for $1 - \frac{1+\alpha}{2}m_1 > m_2$ in area 2; and $x_R = m_2$ for $1 - \frac{1+\alpha}{2}m_1 > m_2$ in area 2.

We also have that $2 - \frac{1+\alpha}{1-\alpha}m_1 < 1$ iff $m_1 > \frac{1-\alpha}{1+\alpha}$.

Combining cases 1, 2, 4, and 6 we have that:

if $\alpha > 0,71 : x_R = m_2$ for $0 < m_2 < \frac{1}{2}$

if $\frac{3}{5} < \alpha < 0,71 : x_R = m_2$ for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ and $x_R = 1$ for $2 - \frac{1+\alpha}{1-\alpha}m_1 > m_2$. Notice that in this case $2 - \frac{1+\alpha}{1-\alpha}m_1 > m_2$ does not cross area 4.

if $0,56 < \alpha < \frac{3}{5} : x_R = m_2$ for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ and $x_R = 1$ for $2 - \frac{1+\alpha}{1-\alpha}m_1 > m_2$. Notice that in this case $2 - \frac{1+\alpha}{1-\alpha}m_1 > m_2$ does not cross area 4.

if $\frac{1}{2} < \alpha < 0,56 : x_R = m_2$ for $1 - \frac{1+\alpha}{2}m_1 < m_2$ and $m_2 > \frac{2\alpha}{1+\alpha}$;

$x_R = m_2$ for $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ and $\frac{1+\alpha}{2\alpha}m_1 < m_2$;

$x_R = m_2$ for $\frac{2(1-\alpha)}{1+\alpha} < m_2 < \frac{1+\alpha}{2\alpha}m_1$;

$x_R = 1$ otherwise.

Notice that $2 - \frac{1+\alpha}{1-\alpha}m_1 = \frac{1+\alpha}{2\alpha}m_1$ iff $m_1 = \frac{4\alpha(1-\alpha)}{(1+\alpha)^2}$ and $\frac{1+\alpha}{2\alpha}m_1 = \frac{2(1-\alpha)}{1+\alpha}$ iff $m_1 = \frac{4\alpha(1-\alpha)}{(1+\alpha)^2}$.

if $\frac{1}{2} > \alpha : x_R = m_2$ for $1 - \frac{1+\alpha}{2}m_1 < m_2$ and $m_2 > \frac{2(1-\alpha)}{1+\alpha}$; $x_L = 0$ otherwise.

Summarizing the previous results, the best responses of party R, given the corresponding best responses of the leftist parties are as follows:

$x_R(\alpha, m_1, m_2) = m_2$ if $m_2 < 2m_1$ and $m_2 > \min\left\{2 - \frac{1+\alpha}{1-\alpha}m_1, 1 - \frac{1+\alpha}{2}m_1\right\}$ and $m_2 > \max\left\{\frac{1}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\}$,

otherwise $x_R(\alpha, m_1, m_2) = 1$.

And the equilibrium is given by:

$\alpha < \frac{1}{3} : x_{L_1} = x_{L_2} = 0$ and $x_R = m_2$ iff $\frac{1}{1+\alpha} < m_2 < 2m_1$; otherwise $x_R = 1$.

$\frac{1}{3} < \alpha < \frac{1}{2}$:

$m_2 < \frac{2\alpha}{1+\alpha} : x_R = 1, x_{L_1} = 0, x_{L_2} = m_2$

$\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1 \wedge m_2 > 1 - \frac{1+\alpha}{2}m_1 \Longrightarrow x_R = m_2, x_{L_1} = m_1, x_{L_2} = 0$

$\frac{1+\alpha}{2\alpha}m_1 > m_2 \wedge \frac{1}{1+\alpha} < m_2 \Longrightarrow x_R = m_2, x_{L_1} = x_{L_2} = 0$

otherwise $x_R = 1, x_{L_1} = x_{L_2} = 0$

$\frac{1}{2} < \alpha < 0.56$:

$m_2 < \frac{2(1-\alpha)}{1+\alpha} \vee \left[\frac{2(1-\alpha)}{1+\alpha} < m_2 < \frac{2\alpha}{1+\alpha} \wedge 2 - \frac{1+\alpha}{1-\alpha}m_1 > m_2\right] : x_R = 1, x_{L_1} = 0, x_{L_2} = m_2$

$\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1 \wedge m_2 > \min\left\{1 - \frac{1+\alpha}{2}m_1, 2 - \frac{1+\alpha}{1-\alpha}m_1\right\} \Longrightarrow x_R = m_2, x_{L_1} = m_1, x_{L_2} = 0$

$\frac{1+\alpha}{2\alpha}m_1 > m_2 \wedge \frac{2(1-\alpha)}{1+\alpha} < m_2 \Longrightarrow x_R = m_2, x_{L_1} = x_{L_2} = 0$

otherwise $x_R = 1, x_{L_1} = x_{L_2} = 0$

$0.56 < \alpha < \frac{3}{5}$:

$2m_1 < m_2 \wedge \frac{2\alpha}{1+\alpha} < m_2 \Longrightarrow x_R = 1, x_{L_1} = x_{L_2} = 0$

$\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1 \wedge m_2 > 2 - \frac{1-\alpha}{1+\alpha}m_1 \Longrightarrow x_R = m_2, x_{L_1} = m_1, x_{L_2} = 0$

$\frac{1+\alpha}{2\alpha}m_1 > m_2 \wedge \frac{2(1-\alpha)}{1+\alpha} < m_2 \Longrightarrow x_R = m_2, x_{L_1} = x_{L_2} = 0$

otherwise $x_R = 1, x_{L_1} = 0, x_{L_2} = m_2$

$\frac{3}{5} < \alpha < \frac{5}{7}$:

$2m_1 < m_2 \wedge \frac{2\alpha}{1+\alpha} < m_2 \Longrightarrow x_R = 1, x_{L_1} = x_{L_2} = 0$

$2m_1 < m_2 \wedge m_2 < \frac{2\alpha}{1+\alpha} \wedge m_2 < 2 - \frac{1-\alpha}{1+\alpha}m_1 \Longrightarrow x_R = 1, x_{L_1} = 0, x_{L_2} = m_2$

$\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1 \wedge m_2 > 2 - \frac{1-\alpha}{1+\alpha}m_1 \Longrightarrow x_R = m_2, x_{L_1} = m_1, x_{L_2} = 0$

$\frac{1+\alpha}{2\alpha}m_1 > m_2 \Longrightarrow x_R = m_2, x_{L_1} = x_{L_2} = 0$

$\alpha > \frac{5}{7}$:

$2m_1 < m_2 \wedge \frac{2\alpha}{1+\alpha} < m_2 \Longrightarrow x_R = 1, x_{L_1} = x_{L_2} = 0$

$2m_1 < m_2 \wedge m_2 < \frac{2\alpha}{1+\alpha} \Longrightarrow x_R = 1, x_{L_1} = 0, x_{L_2} = m_2$

$\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1 \Longrightarrow x_R = m_2, x_{L_1} = m_1, x_{L_2} = 0$

$\frac{1+\alpha}{2\alpha}m_1 > m_2 \Longrightarrow x_R = m_2, x_{L_1} = x_{L_2} = 0$. ■

Proof of Proposition 4

Suppose that $0 < m_1 < \frac{1}{2} < m_2 < 1$. If all parties compete separately in the two elections (L_1, L_2, R_1, R_2) the full polarization equilibrium obtains if $m_1 < \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(0,1) = -\frac{1-\alpha}{2}$; the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, 1, 1)$ if $m_1 < \frac{1-\alpha}{1+\alpha}$ and $m_2 < \frac{2\alpha}{1+\alpha}$ with $U_{R_1}(0,1) = -\frac{1+\alpha}{2}$ and $U_{R_2}(m_2, 1) = -\frac{1+\alpha}{2}(1 - m_2)$; the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, 0, m_1, 1)$ if $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$ with

$U_{R_1}(0,m_1) = \frac{1+\alpha}{2}m_1 - 1$ and $U_{R_2}(0,1) = -\frac{1-\alpha}{2}$; otherwise, if $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 < \frac{2\alpha}{1+\alpha}$, the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, m_1, 1)$ with $U_{R_1}(0,m_1) = \frac{1+\alpha}{2}m_1 - 1$ and $U_{R_2}(m_2,1) = -\frac{1+\alpha}{2}(1-m_2)$.

We will compare these payoffs to the ones obtained by party R when competing as a unified party in the two elections (L_1, L_2, R) described in Theorem 2 in order to obtain $\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R)$.

First of all notice that if $\alpha < \frac{1}{3}$ in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0,0,1)$ except for $\frac{1}{1+\alpha} < m_2 < 2m_1$ in which case we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0,0,m_2)$. Thus we have that R obtains the same payoff that he would obtain if he competes separately and $\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = 0$, except if $\frac{1}{1+\alpha} < m_2 < 2m_1$. Here the difference between payoffs is

$$\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1}(0,1) + U_{R_2}(0,1) - U_R(0,0,m_2) = 1 - (1+\alpha)m_2 < 0$$

for $\frac{1}{1+\alpha} < m_2$ which holds in this case and it implies a gain from the union. This gain increases with α and m_2. This area increases with α.

For $\alpha > \frac{1}{3}$ we analyze the gains and losses of R in the different areas of the parameter space (m_1, m_2) represented in figure 2.

In the **white area** in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0,0,1)$ and $m_2 > \frac{2\alpha}{1+\alpha}$ always holds. If $m_1 < \frac{1-\alpha}{1+\alpha}$ the equilibrium coincides with the parties' choices if they compete separately and thus $\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = 0$. If $m_1 > \frac{1-\alpha}{1+\alpha}$ and parties compete separately in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0,0,m_1,1)$ and the difference between payoffs is

$$\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1}(0,m_1) + U_{R_2}(0,1) - U_R(0,0,1) = \frac{1+\alpha}{2}m_1 - \frac{1-\alpha}{2} > 0$$

iff $m_1 > \frac{1-\alpha}{1+\alpha}$ which holds in this case and it implies a loss from the union. This loss increases with α and m_1. This area decreases with α.

In the **white dotted area** in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ and $m_2 < \frac{2\alpha}{1+\alpha}$ always holds. If $m_1 < \frac{1-\alpha}{1+\alpha}$ the equilibrium coincides with the parties' choices if they compete separately and thus $\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = 0$. If $m_1 > \frac{1-\alpha}{1+\alpha}$ and parties compete separately in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, m_1, 1)$ and the difference between payoffs is

$$\Delta_R(L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1}(0,m_1) + U_{R_2}(m_2,1) - U_R(0,m_2,1) = \frac{1+\alpha}{2}m_1 - \frac{1-\alpha}{2} > 0$$

iff $m_1 > \frac{1-\alpha}{1+\alpha}$ which holds in this case and it implies a loss from the union. This loss increases with α and m_1. This area increases with α.

In the **grey dotted area** in equilibrium we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$ and $\frac{1+\alpha}{2\alpha}m_1 < m_2 < 2m_1$ always holds. First suppose that $\alpha > \frac{1}{2}$. This implies that $m_1 > \frac{1-\alpha}{1+\alpha}$ and we have to consider two cases depending on whether $m_2 > \frac{2\alpha}{1+\alpha}$ or $m_2 < \frac{2\alpha}{1+\alpha}$. If $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 < \frac{2\alpha}{1+\alpha}$ and parties compete separately we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, m_1, 1)$ and the difference between payoffs is

$$\Delta_R (L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1} (0, m_1) + U_{R_2} (m_2, 1) - U_R (m_1, 0, m_2) = \frac{1-\alpha}{2} (1 - m_2) > 0$$

which implies a loss from the union. This loss decreases with α and m_2. This area increases with α.

If $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$ and parties compete separately we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, 0, m_1, 1)$ and the difference between payoffs is

$$\Delta_R (L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1} (0, m_1) + U_{R_2} (0, 1) - U_R (m_1, 0, m_2) = \frac{1+\alpha}{2} - m_2 > 0$$

iff $\frac{1+\alpha}{2} > m_2$. Thus for $\frac{2\alpha}{1+\alpha} < m_2 < \frac{1+\alpha}{2}$ we have a loss from the union, and for $\frac{1+\alpha}{2} < m_2 < 1$ we have a gain from the union. This loss increases with α and decreases with m_2 and this area increases with α. This gain decreases with α and increases with m_2 and this area decreases with α.

The last result also holds for $0,46 < \alpha < \frac{1}{2}$. Becasue in this case we have that $1 - \frac{1+\alpha}{2} m_1 = 2m_1$ iff $m_1 = \frac{2}{5+\alpha}$, and $\frac{2}{5+\alpha} > \frac{1-\alpha}{1+\alpha}$ iff $\alpha > 0,46$, which implies that $m_2 > \frac{2\alpha}{1+\alpha}$ and $m_1 > \frac{1-\alpha}{1+\alpha}$.

Next suppose that $\alpha < 0,46$. We have that $m_2 > \frac{2\alpha}{1+\alpha}$ and we have to consider two cases depending on whether $m_1 > \frac{1-\alpha}{1+\alpha}$ or $m_1 < \frac{1-\alpha}{1+\alpha}$. If $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$, as before if parties compete separately we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, 0, m_1, 1)$ and the difference between payoffs is

$$\Delta_R (L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1} (0, m_1) + U_{R_2} (0, 1) - U_R (m_1, 0, m_2) = \frac{1+\alpha}{2} - m_2 > 0$$

iff $\frac{1+\alpha}{2} > m_2$. Thus for $\frac{2\alpha}{1+\alpha} < m_2 < \frac{1+\alpha}{2}$ we have a loss from the union, and for $\frac{1+\alpha}{2} < m_2 < 1$ we have a gain from the union. This loss increases with α and decreases with m_2 and this area increases with α. This gain decreases with α and increases with m_2 and this area decreases with α.

If $m_1 < \frac{1-\alpha}{1+\alpha}$ and $m_2 > \frac{2\alpha}{1+\alpha}$ and parties compete separately we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, 0, 1, 1)$ and the difference between payoffs is

$$\Delta_R (L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1} (0, 1) + U_{R_2} (0, 1) - U_R (m_1, 0, m_2) = 1 - \frac{1+\alpha}{2} m_1 - m_2 < 0$$

iff $1 - \frac{1+\alpha}{2} m_1 < m_2$ which holds in this case and it implies a gain for the union. This gain increases with α, m_1 and m_2. This area decreases with α.

In the **grey area** the equilibrium is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ and $\frac{1+\alpha}{2\alpha} m_1 > m_2$ always holds. First suppose that $\alpha > 0,54$. In this case we have that $m_1 > \frac{1-\alpha}{1+\alpha}$ and $m_2 < \frac{2\alpha}{1+\alpha}$ because: $\frac{1+\alpha}{2\alpha} \frac{1}{2} < \frac{2\alpha}{1+\alpha}$ iff $\alpha > 0,54$; $\frac{1+\alpha}{2\alpha} m_1 = \frac{1}{1+\alpha}$ iff $m_1 = \frac{2\alpha}{(1+\alpha)^2}$, and $\frac{2\alpha}{(1+\alpha)^2} > \frac{1-\alpha}{1+\alpha}$ iff $\alpha > 0,41$. If parties compete separately we have $\left(x^*_{L_1}, x^*_{L_2}, x^*_{R_1}, x^*_{R_2}\right) = (0, m_2, m_1, 1)$ and the difference between payoffs is

$$\Delta_R (L_1, L_2, R_1, R_2 \diagdown L_1, L_2, R) = U_{R_1} (0, m_1) + U_{R_2} (m_2, 1) - U_R (0, 0, m_2) = \frac{1+\alpha}{2} (m_1 - m_2) + \frac{1-\alpha}{2} > 0$$

iff $\frac{1-\alpha}{1+\alpha}+m_1>m_2$. Notice that $\frac{1-\alpha}{1+\alpha}+m_1>\frac{1+\alpha}{2\alpha}m_1$ iff $\frac{2\alpha}{1+\alpha}>m_1$ which holds in this case (since $\frac{1+\alpha}{2\alpha}m_1>m_2$ and $\frac{2\alpha}{1+\alpha}>\frac{1}{2}$) and it implies a loss from the union. This loss decreases with α and m_2 and it increases with m_1. This area decreases with α.

Now suppose that $\frac{1}{2}<\alpha<0,54$. In this case we have that $m_1>\frac{1-\alpha}{1+\alpha}$. If $m_1>\frac{1-\alpha}{1+\alpha}$ and $m_2<\frac{2\alpha}{1+\alpha}$ we have as before that $\Delta_R\left(L_1,L_2,R_1,R_2\diagdown L_1,L_2,R\right)=\frac{1+\alpha}{2}\left(m_1-m_2\right)+\frac{1-\alpha}{2}>0$ and it implies a loss from the union. This loss decreases with α and m_2 and it increases with m_1. This area increases with α.

And if $m_1>\frac{1-\alpha}{1+\alpha}$ and $m_2>\frac{2\alpha}{1+\alpha}$ and parties compete separately we have $\left(x^*_{L_1},x^*_{L_2},x^*_{R_1},x^*_{R_2}\right)=(0,0,m_1,1)$ and the difference between payoffs is

$$\Delta_R\left(L_1,L_2,R_1,R_2\diagdown L_1,L_2,R\right)=U_{R_1}\left(0,m_1\right)+U_{R_2}\left(0,1\right)-U_R\left(0,0,m_2\right)=\left[\frac{1+m_1}{2}-m_2\right](1+\alpha)>0$$

iff $\frac{1+m_1}{2}>m_2$ which holds because for $\alpha>\frac{1}{2}$ we have that $\frac{1+\alpha}{2\alpha}m_1<\frac{1+m_1}{2}$. Thus $\frac{1+m_1}{2}>\frac{1+\alpha}{2\alpha}m_1>m_2$ and it implies a loss from the union. This loss increases with α and m_1 and it decreases with m_2. This area decreases with α.

Next suppose that $0,41<\alpha<\frac{1}{2}$. In this case we have that $m_1>\frac{1-\alpha}{1+\alpha}$ and $m_2>\frac{2\alpha}{1+\alpha}$, because $\alpha>0,41$ implies $m_1>\frac{1-\alpha}{1+\alpha}$ and $\alpha<\frac{1}{2}$ implies $\frac{1}{1+\alpha}>\frac{2\alpha}{1+\alpha}$. Thus as before we have $\Delta_R\left(L_1,L_2,R_1,R_2\diagdown L_1,L_2,R\right)=\left[\frac{1+m_1}{2}-m_2\right](1+\alpha)>0$ iff $\frac{1+m_1}{2}>m_2$. Since $\alpha<\frac{1}{2}$ we have that for $\frac{1+m_1}{2}>m_2$ it represents a loss for the union; and for $\frac{1+m_1}{2}<m_2$ it represents a gain for the union. This loss increases with α and m_1 and it decreases with m_2; this area increases with α. This gain decreases with α and m_1 and it increases with m_2; this area decreases with α.

Finally, suppose that $\frac{1}{3}<\alpha<0,41$. In this case we have that $m_2>\frac{2\alpha}{1+\alpha}$. If $m_1>\frac{1-\alpha}{1+\alpha}$ and $m_2>\frac{2\alpha}{1+\alpha}$, as before we have that $\Delta_R\left(L_1,L_2,R_1,R_2\diagdown L_1,L_2,R\right)=\left[\frac{1+m_1}{2}-m_2\right](1+\alpha)>0$ iff $\frac{1+m_1}{2}>m_2$. Since $\alpha<\frac{1}{2}$ we have that for $\frac{1+m_1}{2}>m_2$ it represents a loss for the union; and for $\frac{1+m_1}{2}<m_2$ it represents a gain for the union. This loss increases with α and m_1 and it decreases with m_2; this area increases with α. This gain decreases with α and m_1 and it increases with m_2; this area decreases with α.

If instead $m_1<\frac{1-\alpha}{1+\alpha}$ and $m_2>\frac{2\alpha}{1+\alpha}$ and parties compete separately we have $\left(x^*_{L_1},x^*_{L_2},x^*_{R_1},x^*_{R_2}\right)=(0,0,1,1)$ and the difference between payoffs is

$$\Delta_R\left(L_1,L_2,R_1,R_2\diagdown L_1,L_2,R\right)=U_{R_1}\left(0,1\right)+U_{R_2}\left(0,1\right)-U_R\left(0,0,m_2\right)=1-(1+\alpha)\,m_2<0$$

iff $\frac{1}{1+\alpha}<m_2$ which holds in this case and it implies a gain for the union. This gain increases with α and m_2. This area decreases with α. ■

Proof of Theorem 3

Suppose that $0<m_1<\frac{1}{2}<m_2<1$ and (L,R).

1) Suppose that $m_2 > \frac{1+m_1}{2}$ and $m_2 > 2m_1$.

First we look for the best responses of party R:

$U_R(0,1) = -1 > -2 = U_R(0,0)$

$U_R(0,1) = -1 > m_2 - 2 = U_R(0,m_2)$

$U_R(0,1) = -1 > -2 + (1+\alpha)m_1 = U_R(0,m_1)$ iff $m_1 < \frac{1}{1+\alpha}$ which always holds, because $m_1 < \frac{1}{2} < \frac{1}{1+\alpha}$.

Thus the best response of party R against $x_L = 0$ is $x_R(0) = 1$.

$U_R(m_1,1) = -(1-m_1) > -2 + (1+\alpha)m_1 = U_R(m_1,0)$

$U_R(m_1,1) = -(1-m_1) > -2(1-m_1) = U_R(m_1,m_1)$

$U_R(m_1,1) = -(1-m_1) > m_1 + m_2 - 2 = U_R(m_1,m_2)$

Thus the best response of party R against $x_L = m_1$ is $x_R(m_1) = 1$.

$U_R(m_2,1) = -(1+\alpha)(1-m_2) > m_2 - 2 = U_R(m_2,0)$

$U_R(m_2,1) = -(1+\alpha)(1-m_2) > -2(1-m_2) = U_R(m_2,m_2) > -(1-m_1) - (1-m_2) = U_R(m_2,m_1)$

Thus the best response of party R against $x_L = m_2$ is $x_R(m_2) = 1$.

Finally $U_R(1,1) = 0 > U_R(x,1)$ for any $x \in \{0,m_1,m_2\}$, and the best response of party R against $x_L = 1$ is $x_R(1) = 1$.

Thus the best response of party R is $x_R = 1$ for all x_L.

Now we look for the best responses of party L to $x_R = 1$:

$U_L(0,1) = -1 > -2 = U_L(1,1)$

$U_L(0,1) = -1 > -1 - m_1 = U_L(m_1,1)$

$U_L(0,1) = -1 > -(1-\alpha) - (1+\alpha)m_2 = U_L(m_2,1)$ iff $m_2 > \frac{\alpha}{1+\alpha}$ which holds in this case.

Thus, the best response of party L to $x_R = 1$ is $x_L(1) = 0$ and in equilibrium we must have $(x_L^*, x_R^*) = (0,1)$.

2) Next suppose that $m_2 < \frac{1+m_1}{2}$ and $m_2 < 2m_1$.

We first look for the best responses of party R:

$U_R(0,1) = -1 > -2 = U_R(0,0)$

$U_R(0,m_2) > -2 + (1+\alpha)m_1 = U_R(0,m_1)$

$U_R(0,1) = -1 > -2 + (1+\alpha)m_2 = U_R(0,m_2)$ iff $m_2 < \frac{1}{1+\alpha}$

Thus the best response of party R to $x_L = 0$ is $x_R(0) = \begin{cases} m_2 & if \quad m_2 > \frac{1}{1+\alpha} \\ 1 & otherwise \end{cases}$

$U_R(m_1,1) = -(1+\alpha)(1-m_1) > -2 + (1+\alpha)m_1 = U_R(m_1,0)$

$U_R(m_1,1) = -(1+\alpha)(1-m_1) > -2(1-m_1) = U_R(m_1,m_1)$

$U_R(m_1,1) = -(1+\alpha)(1-m_1) > m_1 + m_2 - 2 = U_R(m_1,m_2)$ iff $1 - \alpha + \alpha m_1 > m_2$

Thus the best response of party R to $x_L = m_1$ is $x_R(m_1) = \begin{cases} m_2 & if \quad 1-\alpha+\alpha m_1 < m_2 \\ 1 & otherwise \end{cases}$

$U_R(m_2,1) = -(1+\alpha)(1-m_2) > -2 + (1+\alpha)m_2 = U_R(m_2,0)$

$U_R(m_2,m_2) = -2(1-m_2) > -(1-m_1) - (1-m_2) = U_R(m_2,m_1)$

$U_R(m_2,1) = -(1+\alpha)(1-m_2) > -2(1-m_2) = U_R(m_2,m_2)$

Thus the best response of party R to $x_L = m_2$ is $x_R(m_2) = 1$.

Finally $U_R(1,1) = 0 > U_R(x,1)$ for any $x \in \{0,m_1,m_2\}$, and the best response of party R to $x_L = 1$ is $x_R(1) = 1$.

Thus the best responses of party R are:

$x_R(1) = x_R(m_2) = 1$

$x_R(m_1) = \begin{cases} m_2 & if \quad 1-\alpha+\alpha m_1 < m_2 \\ 1 & otherwise \end{cases}$

$x_R(0) = \begin{cases} m_2 & if \quad \frac{1}{1+\alpha} < m_2 \\ 1 & otherwise \end{cases}.$

And the corresponding best responses of party L are:

$U_L(0,1) = -1 > -2 = U_L(1,1)$

$U_L(m_1,1) = -(1-\alpha) - (1+\alpha)m_1 > -(1-\alpha) - (1+\alpha)m_2 = U_L(m_2,1)$

$U_L(0,1) = -1 > -(1-\alpha) - (1+\alpha)m_1 = U_L(m_1,1)$ iff $m_1 > \frac{\alpha}{1+\alpha}$

Thus the best responses of party L to $x_R = 1$ are $x_L(1) = \begin{cases} m_1 & if \quad \frac{1}{1+\alpha} > m_1 \\ 0 & otherwise \end{cases}$

$U_L(0,m_2) = -(1+\alpha)m_2 > -(1-\alpha) - (1+\alpha)m_2 = U_L(1,m_2)$

$U_L(m_1,m_2) = -m_1 - m_2 > -2m_2 = U_L(m_2,m_2)$

$U_L(0,m_2) = -(1+\alpha)m_2 > -m_1 - m_2 = U_L(m_1,m_2)$ iff $\alpha m_2 < m_1$

Thus the best responses of party L to $x_R = m_2$ are $x_L(m_2) = \begin{cases} m_1 & if \quad m_1 < \alpha m_2 \\ 0 & otherwise \end{cases}$

Thus, the best responses of party L are:

$x_L(1) = \begin{cases} 0 & if \quad m_1 > \frac{\alpha}{1+\alpha} \\ m_1 & otherwise \end{cases}$

$x_L(m_2) = \begin{cases} m_1 & if \quad m_1 < \alpha m_2 \\ 0 & otherwise \end{cases}$

And in equilibrium we must have:

for $\alpha < \frac{1}{3} : (x_L, x_R) = (0,1)$.

for $\frac{1}{3} < \alpha < \frac{1}{2} : (x_L, x_R) = \begin{cases} (0,1) & if \quad m_1 > \frac{\alpha}{1+\alpha} \text{ and } m_2 < \frac{1}{1+\alpha} \\ (0,m_2) & if \quad m_2 > \frac{1}{1+\alpha} \\ (m_1,1) & if \quad m_1 < \frac{\alpha}{1+\alpha} \end{cases}$

for $\alpha > \frac{1}{2} : (x_L, x_R) = \begin{cases} (0,1) & if \quad m_1 > \frac{\alpha}{1+\alpha} \text{ and } m_2 < \frac{1}{1+\alpha} \\ (0,m_2) & if \quad m_2 > \frac{1}{1+\alpha} \text{ and } \alpha m_2 < m_1 \\ (m_1,1) & if \quad m_1 < \frac{\alpha}{1+\alpha} \text{ and } m_2 < 1-\alpha+\alpha m_1 \\ (m_1,m_2) & otherwise \end{cases}$

3) Suppose that $m_2 > \frac{1+m_1}{2}$ and $m_2 < 2m_1$.

We first look for the best responses of party R:

$U_R(0,1) = -1 > -2 = U_R(0,0)$

$U_R(0,m_2) = -2 + (1+\alpha)m_2 > -2 + (1+\alpha)m_1 = U_R(0,m_1)$

$U_R(0,1) = -1 > -2 + (1+\alpha)m_2 = U_R(0,m_2)$ iff $m_2 < \frac{1}{1+\alpha}$

Thus the best responses of party R to $x_L = 0$ are $x_R(0) = \begin{cases} m_2 & if \quad m_2 > \frac{1}{1+\alpha} \\ 1 & otherwise \end{cases}$

$U_R(m_1,1) = m_1 - 1 > -2 + (1+\alpha)m_1 = U_R(m_1,0)$

$U_R(m_1,1) = m_1 - 1 > -2(1-m_1) = U_R(m_1,m_1)$

$U_R(m_1,1) = m_1 - 1 > m_1 + m_2 - 2 = U_R(m_1,m_2)$

Thus the best response of party R to $x_L = m_1$ is $x_R(m_1) = 1$.

$U_R(m_2,1) = -(1-\alpha) - (1+\alpha)m_2 > -2 + (1+\alpha)m_2 = U_R(m_2,0)$

$U_R(m_2,m_2) = -2(1-m_2) > -(1-m_1) - (1-m_2) = U_R(m_2,m_1)$

$U_R(m_2,1) = -(1-\alpha) - (1+\alpha)m_2 > -2(1-m_2) = U_R(m_2,m_2)$

Thus the best response of party R to $x_L = m_2$ is $x_R(m_2) = 1$.

Finally $U_R(1,1) = 0 > U_R(x,1)$ for any $x \in \{0, m_1, m_2\}$ and the best response of party R to $x_L = 1$ is $x_R(1) = 1$.

Thus the best responses of party R are:

$x_R(1) = x_R(m_2) = x_R(m_1) = 1$

$x_R(0) = \begin{cases} m_2 & if \quad \frac{1}{1+\alpha} < m_2 \\ 1 & otherwise \end{cases}$

And the corresponding best responses of party L :

$U_L(0,1) = -1 > -2 = U_L(1,1)$

$U_L(0,1) = -1 > -m_1 - 1 = U_L(m_1,1)$

$U_L(0,1) = -1 > -(1-\alpha) - (1+\alpha) m_2 = U_L(m_2,1)$ iff $m_2 > \frac{\alpha}{1+\alpha}$ which holds in this case.

Thus the best response of party L to $x_R = 1$ is $x_L(1) = 0$.

$U_L(0,m_2) = -(1+\alpha) m_2 > -(1-\alpha) - (1+\alpha) m_2 = U_L(1,m_2)$

$U_L(m_1,m_2) = -m_1 - m_2 > -2m_2 = U_L(m_2,m_2)$

$U_L(0,m_2) = -(1+\alpha) m_2 > -m_1 - m_2 = U_L(m_1,m_2)$ iff $\alpha m_2 < m_1$

Thus the best responses of party L to $x_R = m_2$ are $x_L(m_2) = \begin{cases} m_1 & if \quad m_1 < \alpha m_2 \\ 0 & otherwise \end{cases}$

And the best responses of party L are:

$x_L(1) = 0$

$x_L(m_2) = \begin{cases} m_1 & if \quad m_1 < \alpha m_2 \\ 0 & otherwise \end{cases}$

And in equilibrium we must have:

for $\alpha < \frac{1}{2} : (x_L, x_R) = \begin{cases} (0, m_2) & if \quad m_2 > \frac{1}{1+\alpha} \\ (0,1) & otherwise \end{cases}$

for $\alpha > \frac{1}{2} : (x_L, x_R) = (0, m_2)$ if $\alpha m_2 < m_1$ and no pure strategy equilibrium otherwise.

4) Suppose that $m_2 < \frac{1+m_1}{2}$ and $m_2 > 2m_1$.

We first look for the best responses of party R:

$U_R(0,1) = -1 > -2 = U_R(0,0)$

$U_R(0,1) = -1 > m_2 - 2 = U_R(0,m_2)$

$U_R(0,1) = -1 > -2 + (1+\alpha) m_1 = U_R(0,m_1)$ iff $m_1 < \frac{1}{1+\alpha}$ which holds in this case

Thus the best response of party R to $x_L = 0$ is $x_R(0) = 1$.

$U_R(m_1,1) = -(1+\alpha)(1-m_1) > -2 + (1+\alpha) m_1 = U_R(m_1,0)$

$U_R(m_1,1) = -(1+\alpha)(1-m_1) > -2(1-m_1) = U_R(m_1,m_1)$

$U_R(m_1,1) = -(1+\alpha)(1-m_1) > m_1 + m_2 - 2 = U_R(m_1,m_2)$ iff $1 - \alpha + \alpha m_1 > m_2$

Thus the best responses of party R to $x_L = m_1$ are $x_R(m_1) = \begin{cases} m_2 & if \quad 1-\alpha+\alpha m_1 < m_2 \\ 1 & otherwise \end{cases}$

$U_R(m_2,m_1) = -2 + m_1 + m_2 > -2 + m_2 = U_R(m_2,0)$

$U_R(m_2,m_2) = -2(1-m_2) > -2 + m_1 + m_2 = U_R(m_2,m_1)$

$U_R(m_2,1) = -(1+\alpha)(1-m_2) > -2(1-m_2) = U_R(m_2,m_2)$

Thus the best response of party R to $x_L = m_2$ is $x_R(m_2) = 1$.

Finally $U_R(1,1) = 0 > U_R(x,1)$ for any $x \in \{0, m_1, m_2\}$ and the best response of party R to $x_L = 1$ is $x_R(1) = 1$.

Thus the best responses of party R are:
$x_R(1) = x_R(m_2) = x_R(0) = 1$
$x_R(m_1) = \begin{cases} m_2 & if \quad 1-\alpha+\alpha m_1 < m_2 \\ 1 & otherwise \end{cases}$
And the corresponding best responses of party L :
$U_L(0,1) = -1 > -2 = U_L(1,1)$
$U_L(m_1,1) = -(1-\alpha) - (1+\alpha) m_1 > -(1-\alpha) - (1+\alpha) m_2 = U_L(m_2,1)$
$U_L(0,1) = -1 > -(1-\alpha) - (1+\alpha) m_1 = U_L(m_1,1)$ iff $m_1 > \frac{\alpha}{1+\alpha}$
Thus the best responses of party L to $x_R = 1$ are $x_L(1) = \begin{cases} 0 & if \quad m_1 > \frac{\alpha}{1+\alpha} \\ m_1 & otherwise \end{cases}$
$U_L(0,m_2) = -m_2 > -(1-\alpha) - (1+\alpha) m_2 = U_L(1,m_2)$
$U_L(0,m_2) = -m_2 > -2m_2 = U_L(m_2,m_2)$
$U_L(0,m_2) = -m_2 > -m_1 - m_2 = U_L(m_1,m_2)$
Thus the best response of party L to $x_R = m_2$ is $x_L(m_2) = 0$.
Thus, the best responses of party L are:
$x_L(m_2) = 0$
$x_L(1) = \begin{cases} 0 & if \quad m_1 > \frac{\alpha}{1+\alpha} \\ m_1 & otherwise \end{cases}$
And in equilibrium we must have:
for $\alpha < \frac{1}{2} : (x_L, x_R) = \begin{cases} (m_1,1) & if \quad m_1 < \frac{\alpha}{1+\alpha} \\ (0,1) & otherwise \end{cases}$

for $\alpha > \frac{1}{2}$: $(x_L, x_R) = (m_1, 1)$ if $1-\alpha+\alpha m_1 > m_2$ and no pure strategy equilibrium otherwise.

Overall the equilibrium strategies are:

for $\alpha < \frac{1}{2}$: $(x_L, x_R) = (0, m_2)$ *if* $\frac{1}{1+\alpha} < m_2 < 2m_1$; $(x_L, x_R) = (m_1, 1)$ *if* $m_1 < \frac{\alpha}{1+\alpha}$ *and* $m_2 < \frac{1+m_1}{2}$; *otherwise* $(x_L, x_R) = (0,1)$

for $\alpha > \frac{1}{2}$: $(x_L, x_R) = (0, m_2)$ *if* $\frac{1}{1+\alpha} < m_2 < \frac{m_1}{\alpha}$; $(x_L, x_R) = (m_1, 1)$ *if* $m_1 < \frac{\alpha}{1+\alpha}$ *and* $m_2 < 1-\alpha+\alpha m_1$; $(x_L, x_R) = (0,1)$ *if* $\frac{1}{1+\alpha} > m_2$ *and* $m_1 > \frac{\alpha}{1+\alpha}$; $(x_L, x_R) = (0,1)$ *if* $m_2 > 2m_1$ *and* $m_2 > \frac{1+m_1}{2}$;

$(x_L, x_R) = (m_1, m_2)$ *if* $\frac{m_1}{\alpha} < m_2 < 2m_1$ *and* $1-\alpha+\alpha m_1 < m_2 < \frac{1+m_1}{2}$; *otherwise there is no pure strategy equilibrium.* ■

Proof of Proposition 5

Suppose $0 < m_1 < \frac{1}{2} < m_2 < 1$. If the leftist parties compete separately in the two elections (L_1, L_2, R) from Theorem 2 (figure 2) we have that in equilibrium:

for $\alpha < \frac{1}{3}$: $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0,0,m_2)$ *with* $U_{L_1}(0,m_2) + U_{L_2}(0,m_2) = -(1+\alpha) m_2$ *if* $\frac{1}{1+\alpha} < m_2 < 2m_1$; *otherwise* $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0,0,1)$ *with* $U_{L_1}(0,1) + U_{L_2}(0,1) = -1$

and for $\alpha > \frac{1}{3}$:

[*whitedotted*] $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ *with* $U_{L_1}(0,1) + U_{L_2}(m_2,1) = -(1-\alpha) - \frac{1+\alpha}{2} m_2$ *if* $m_2 < \min\left\{\frac{2\alpha}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\}$ or

$\left[\frac{2(1-\alpha)}{1+\alpha} < m_2 < \frac{2\alpha}{1+\alpha} \text{ and } m_2 < \min\left\{1 - \frac{1+\alpha}{2} m_1, 2 - \frac{1+\alpha}{1-\alpha} m_1\right\}\right]$

[*greydotted*] $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (m_1, 0, m_2)$ *with* $U_{L_1}(m_1, m_2) + U_{L_2}(0, m_2) = -m_2 - \frac{1+\alpha}{2} m_1$ *if* $\frac{1+\alpha}{2\alpha} m_1 < m_2 < 2m_1$ and $m_2 > \min\left\{1 - \frac{1+\alpha}{2} m_1, 2 - \frac{1+\alpha}{1-\alpha} m_1\right\}$

[*grey*] $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (0, 0, m_2)$ *with* $U_{L_1}(0, m_2) + U_{L_2}(0, m_2) = -(1+\alpha) m_2$ *if* $\max\left\{\frac{1}{1+\alpha}, \frac{2(1-\alpha)}{1+\alpha}\right\} < m_2 < \frac{1+\alpha}{2\alpha} m_1$

[*white*] $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (0, 0, 1)$ *with* $U_{L_1}(0, 1) + U_{L_2}(0, 1) = -1$ *otherwise.*

We will compare these payoffs for (L_1, L_2, R) to the ones obtained when both parties compete as united parties in the two elections (L, R) described in theorem 3 in order to obtain $\Delta_L(L_1, L_2, R \setminus L, R)$.

First of all notice that if $\alpha < \frac{1}{3}$ the equilibrium in both cases coincides except if $m_2 < \frac{1+m_1}{2}$ and $m_1 < \frac{\alpha}{1+\alpha}$ in which case

$$\Delta_L(L_1, L_2, R \setminus L, R) = U_{L_1}(0, 1) + U_{L_2}(0, 1) - U_L(m_1, 1) = (1+\alpha) m_1 - \alpha < 0$$

iff $m_1 < \frac{\alpha}{1+\alpha}$ which holds in this case and it implies a gain for the union. These gains increase with α and decrease with m_1. Otherwise, we have $\Delta_L = 0$.

The remain of the proof proceeds to analyze each one the areas represented in figure 3 for $\frac{1}{3} < \alpha$. We have that the size of the black area and the size of the area with no equilibrium for (L,R) both increase with α. The size of the dotted and dashed areas increase with α for $\alpha < \frac{1}{2}$, they decrease with α for $\alpha > \frac{1}{2}$, and they approach zero when α approaches 1. Finally, the grey area decreases with α for $\alpha < \frac{1}{2}$. For $\alpha > \frac{1}{2}$ we have two grey areas: the large one does not change size with α and the small one decreases with α and it approaches zero when α approaches 1.

Dotted area:

The equilibrium for (L, R) in the dotted area is $(x_L^*, x_R^*) = (m_1, 1)$.

For $\frac{1}{3} < \alpha < \frac{1}{2}$ the dotted area is defined by $m_2 < \frac{1+m_1}{2}$ and $m_1 < \frac{\alpha}{1+\alpha}$.

If $m_2 < \frac{2\alpha}{1+\alpha}$ the equilibrium for (L_1, L_2, R) is $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (0, m_2, 1)$ and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \setminus L, R) = U_{L_1}(0, 1) + U_{L_2}(m_2, 1) - U_L(m_1, 1) = (1+\alpha)\left(m_1 - \frac{m_2}{2}\right) > 0$$

iff $m_2 < 2m_1$. Thus we have a loss for the union for $m_2 < 2m_1$ and a gain for the union for $m_2 > 2m_1$. This loss increases with α and m_1 and it decreases with m_2. This gain increases with α and m_2 and it decreases with m_1.

If $\frac{2\alpha}{1+\alpha} < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x_{L_1}^*, x_{L_2}^*, x_R^*\right) = (0, 0, 1)$ and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \setminus L, R) = U_{L_1}(0, 1) + U_{L_2}(0, 1) - U_L(m_1, 1) = (1+\alpha) m_1 - \alpha < 0$$

iff $m_1 < \frac{\alpha}{1+\alpha}$ which holds in this case and it implies a gain for the union. This gain increases with α and decreases m_1.

For $\alpha > \frac{1}{2}$ the dotted area is defined by $m_2 < 1 - \alpha + \alpha m_1$ and $m_1 < \frac{\alpha}{1+\alpha}$.

For $m_2 < 2 - \frac{1+\alpha}{1-\alpha}m_1$, the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \diagdown L, R) = U_{L_1}(0,1) + U_{L_2}(m_2, 1) - U_L(m_1, 1) = (1+\alpha)\left(m_1 - \frac{m_2}{2}\right) > 0$$

iff $m_2 < 2m_1$. Thus we have a loss for the union for $m_2 < 2m_1$ and a gain for the union for $m_2 > 2m_1$. This loss increases with α and m_1 and it decreases with m_2. This gain increases with α and m_2 and it increases with m_1. Notice that for $\alpha > \frac{2}{3}$ we have that $m_2 < 2m_1$ always holds in the dotted area, and thus we have a loss for the union in this case.

For $2 - \frac{1+\alpha}{1-\alpha}m_1 < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$, and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \diagdown L, R) = U_{L_1}(m_1, m_2) + U_{L_2}(0, m_2) - U_L(m_1, 1) = 1 - m_2 + \frac{1+\alpha}{2}m_1 > 0$$

which holds in this case and it implies a loss for the union. This loss increases with α and m_1 and it decreases with m_2. This result holds for $\frac{3}{5} < \alpha$.

Dashed area:

The equilibrium for (L, R) in the dashed area is $(x^*_L, x^*_R) = (0, m_2)$.

For $\frac{1}{3} < \alpha < \frac{1}{2}$ the dashed area is defined by $\frac{1}{1+\alpha} < m_2 < 2m_1$.

If $\max\left\{\frac{1+\alpha}{2\alpha}m_1, 1 - \frac{1+\alpha}{2}m_1\right\} < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$ and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \diagdown L, R) = U_{L_1}(m_1, m_2) + U_{L_2}(0, m_2) - U_L(0, m_2) = \alpha m_2 - \frac{1+\alpha}{2}m_1 > 0$$

iff $m_2 > \frac{1+\alpha}{2\alpha}m_1$ which holds in this case and it implies a loss for the union. This loss increases with α and m_2 and it decreases with m_1.

If $m_2 < 1 - \frac{1+\alpha}{2}m_1$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \diagdown L, R) = U_{L_1}(0,1) + U_{L_2}(0,1) - U_L(0, m_2) = (1+\alpha)m_2 - 1 > 0$$

iff $m_2 > \frac{1}{1+\alpha}$ which holds in this case and it implies a loss for the union. This loss increases with α and m_2.

If $m_2 < \frac{1+\alpha}{2\alpha}m_1$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ and the difference between payoffs is $\Delta_L(L_1, L_2, R \diagdown L, R) = U_{L_1}(0, m_2) + U_{L_2}(0, m_2) - U_L(0, m_2) = 0$.

For $\alpha > \frac{1}{2}$ the dashed area is defined by $m_2 < \frac{m_1}{\alpha}$ and $m_2 > \frac{1}{1+\alpha}$.

If $\max\left\{2 - \frac{1+\alpha}{1-\alpha}m_1, \frac{1+\alpha}{2\alpha}m_1\right\} < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$ and the difference between payoffs is

$$\Delta_L(L_1, L_2, R \diagdown L, R) = U_{L_1}(m_1, m_2) + U_{L_2}(0, m_2) - U_L(0, m_2) = \alpha m_2 - \frac{1+\alpha}{2}m_1 > 0$$

iff $m_2 > \frac{1+\alpha}{2\alpha} m_1$ which holds in this case and it implies a loss for the union. This loss increases with α and m_2 and it decreases with m_1.

If $m_2 < \frac{1+\alpha}{2\alpha} m_1$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ and the difference between payoffs is $\Delta_L \left(L_1, L_2, R \diagdown L, R\right) = U_{L_1}(0, m_2) + U_{L_2}(0, m_2) - U_L(0, m_2) = 0$.

Black area:

The black area is defined by $\max\left\{1 - \alpha + \alpha m_1, \frac{m_1}{\alpha}\right\} < m_2 < \min\left\{2m_1, \frac{1+m_1}{2}\right\}$ and this area is empty for $\alpha < \frac{1}{2}$. The equilibrium for (L, R) in the black area is $(x^*_L, x^*_R) = (m_1, m_2)$

If $2 - \frac{1+\alpha}{1-\alpha} m_1 < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$ and the difference between payoffs is

$$\Delta_L \left(L_1, L_2, R \diagdown L, R\right) = U_{L_1}(m_1, m_2) + U_{L_2}(0, m_2) - U_L(m_1, m_2) = \frac{1-\alpha}{2} m_1 > 0$$

which implies a loss for the union. This loss decreases with α and increases with m_1.

If $m_2 < 2 - \frac{1+\alpha}{1-\alpha} m_1$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ and the difference between payoffs is

$$\Delta_L \left(L_1, L_2, R \diagdown L, R\right) = U_{L_1}(0, 1) + U_{L_2}(m_2, 1) - U_L(m_1, m_2) = \frac{1-\alpha}{2} m_2 + m_1 - (1-\alpha) > 0$$

iff $m_2 > 2 - \frac{2}{1-\alpha} m_1$ which holds in this case (because $2 - \frac{2}{1-\alpha} m_1 > 1 - \alpha + \alpha m_1$) and it implies a loss for the union. This loss increases with α, m_1 and m_2.This result holds for $\frac{1}{2} < \alpha < \frac{2}{3}$.

Grey area:

The equilibrium for (L, R) in the grey area is $(x^*_L, x^*_R) = (0, 1)$.

For $\frac{1}{3} < \alpha < \frac{1}{2}$ the grey area is defined by the complementary of the dotted and dashed areas, that is, all parameter values except $m_2 < \frac{1+m_1}{2}$ and $m_1 < \frac{\alpha}{1+\alpha}$; and $\frac{1}{1+\alpha} < m_2 < 2m_1$.

If $m_2 < \frac{2\alpha}{1+\alpha}$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ and the difference between payoffs is

$$\Delta_L \left(L_1, L_2, R \diagdown L, R\right) = U_{L_1}(0, 1) + U_{L_2}(m_2, 1) - U_L(0, 1) = \alpha - \frac{1+\alpha}{2} m_2 > 0$$

iff $m_2 < \frac{2\alpha}{1+\alpha}$ which holds in this case and it implies a loss for the union. This loss increases with α and decreases with m_2.

Otherwise, the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ and the difference between payoffs is $\Delta_L \left(L_1, L_2, R \diagdown L, R\right) = U_{L_1}(0, 1) + U_{L_2}(0, 1) - U_L(0, 1) = 0$.

For $\alpha > \frac{1}{2}$ the grey area is defined on two separated areas: a large one defined by $m_2 > \left\{\frac{1+m_1}{2}, 2m_1\right\}$ and a small one defined by $m_1 > \frac{\alpha}{1+\alpha}$ and $m_2 < \frac{1}{1+\alpha}$.

On the large grey area:

If $\frac{2\alpha}{1+\alpha} < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, 1)$ and the difference between payoffs is $\Delta_L (L_1, L_2, R \diagdown L, R) = U_{L_1}(0, 1) + U_{L_2}(0, 1) - U_L(0, 1) = 0$.

If $m_2 < \frac{2\alpha}{1+\alpha}$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ and the difference between payoffs is

$$\Delta_L (L_1, L_2, R \diagdown L, R) = U_{L_1}(0, 1) + U_{L_2}(m_2, 1) - U_L(0, 1) = \alpha - \frac{1+\alpha}{2} m_2 > 0$$

iff $m_2 < \frac{2\alpha}{1+\alpha}$ which holds in this case and it implies a loss for the union. This loss increases with α and decreases with m_2.

On the small grey area:

If $\max\left\{2 - \frac{1+\alpha}{1-\alpha} m_1, \frac{1+\alpha}{2\alpha} m_1\right\} < m_2$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (m_1, 0, m_2)$ and the difference between payoffs is

$$\Delta_L (L_1, L_2, R \diagdown L, R) = U_{L_1}(m_1, m_2) + U_{L_2}(0, m_2) - U_L(0, 1) = 1 - \frac{1+\alpha}{2} m_1 - m_2 > 0$$

iff $m_2 < 1 - \frac{1+\alpha}{2} m_1$ which holds in this case and it implies a loss for the union. This loss decreases with α, m_1 and m_2.

If $\frac{2(1-\alpha)}{1+\alpha} < m_2 < \frac{1+\alpha}{2\alpha} m_1$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, 0, m_2)$ and the difference between payoffs is

$$\Delta_L (L_1, L_2, R \diagdown L, R) = U_{L_1}(0, m_2) + U_{L_2}(0, m_2) - U_L(0, 1) = 1 - (1+\alpha) m_2 > 0$$

iff $m_2 < \frac{1}{1+\alpha}$ which holds in this case and it implies a loss for the union. This loss decreases with α and m_2.

If $m_2 < 2 - \frac{1+\alpha}{1-\alpha} m_1$ the equilibrium for (L_1, L_2, R) is $\left(x^*_{L_1}, x^*_{L_2}, x^*_R\right) = (0, m_2, 1)$ and the difference between payoffs is

$$\Delta_L (L_1, L_2, R \diagdown L, R) = U_{L_1}(0, 1) + U_{L_2}(m_2, 1) - U_L(0, 1) = \alpha - \frac{1+\alpha}{2} m_2 > 0$$

iff $m_2 < \frac{2\alpha}{1+\alpha}$ which holds in this case and it implies a loss for the union. This loss increases with α and decreases with m_2.This result only holds for $\frac{1}{2} < \alpha < \frac{3}{5}$.

Overall for $\alpha > \frac{1}{3}$ we have that the gains of the union are obtained only if:

$\frac{1}{3} < \alpha < \frac{1}{2}, 2m_1 < m_2 < \min\left\{\frac{1+m_1}{2}, \frac{2\alpha}{1+\alpha}\right\}, m_1 < \frac{\alpha}{1+\alpha}$

or

$\frac{1}{3} < \alpha < \frac{1}{2}, \max\left\{2m_1, \frac{2\alpha}{1+\alpha}\right\} < m_2 < \frac{1+m_1}{2}, m_1 < \frac{\alpha}{1+\alpha}$

or

$\alpha > \frac{1}{2}, m_2 < 1 - \alpha + \alpha m_1, m_1 < \frac{\alpha}{1+\alpha}, m_2 < 2 - \frac{1+\alpha}{1-\alpha} m_1$

This area increases with α for $\alpha < \frac{1}{2}$ and it decreases with α for $\alpha > \frac{1}{2}$.

There are no losses nor gains from the union if $\max\left\{\frac{2\alpha}{1+\alpha}, 2m_1\right\} < m_2$ or $\frac{1}{1+\alpha} < m_2 < \frac{1+\alpha}{2\alpha} m_1$.

Otherwise the union obtains a loss. ■

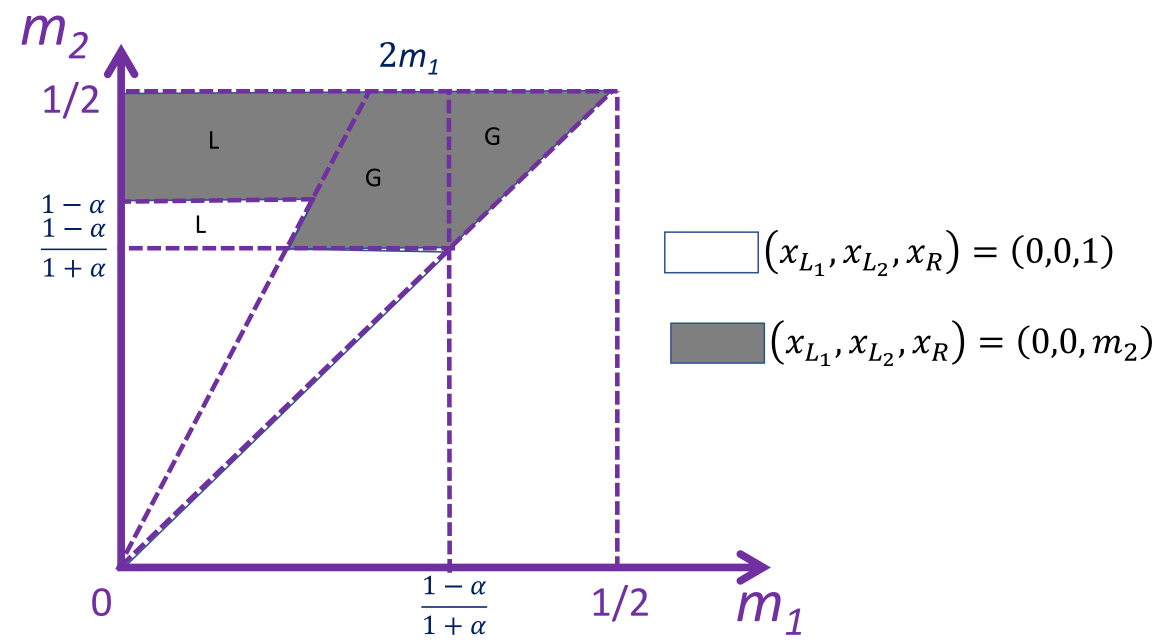

Figure 1: Equilibrium outcome for (L_1, L_2, R), $0 < m_1 < m_2 < 1/2$ and $1/2 < \boldsymbol{\alpha} < 3/5$.

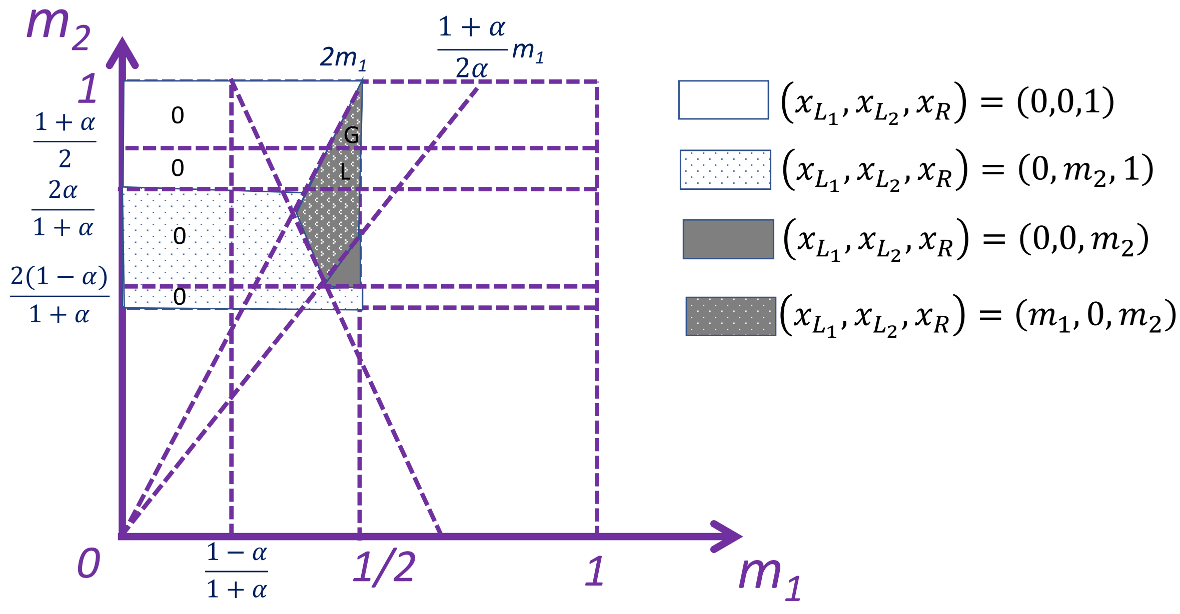

Figure 2: Equilibrium outcome for (L_1, L_2, R), $m_1 < 1/2 < m_2$ and $1/2 < \boldsymbol{\alpha} < 3/5$.

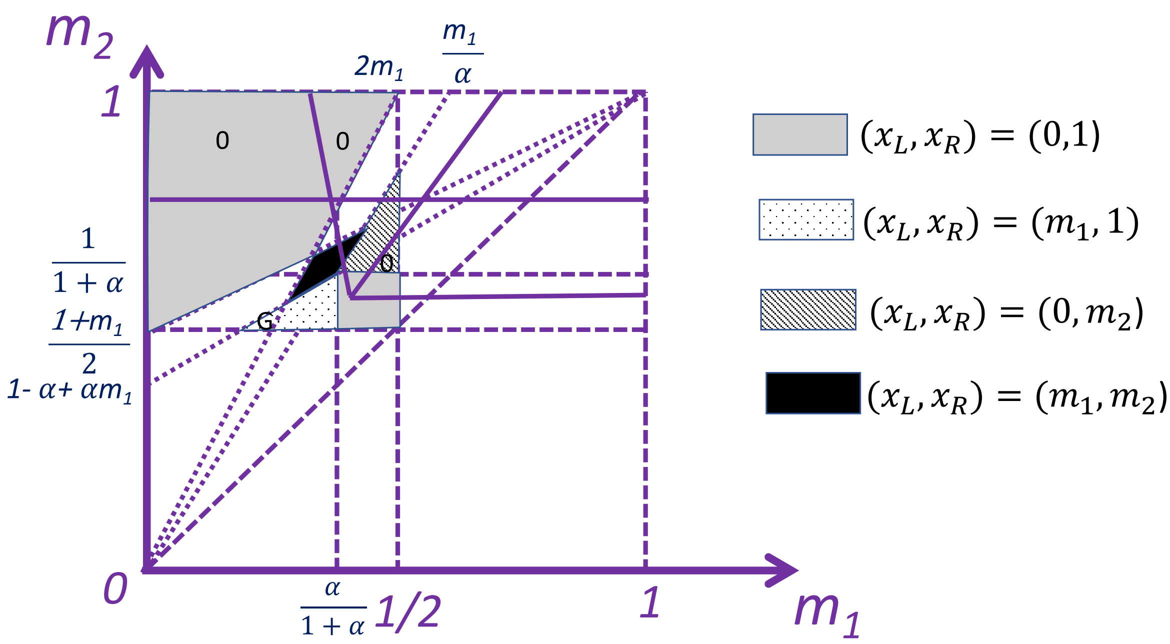

Figure 3: Equilibrium outcome for (L, R), $m_1 < 1/2 < m_2$ and $1/2 < \boldsymbol{\alpha} < 3/5$.