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Does Paternity Leave Promote Gender Equality within Households?

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DOES PATERNITY LEAVE PROMOTE GENDER EQUALITY WITHIN HOUSEHOLDS?*

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Abstract

We provide a theory of paternity leave and a comprehensive empirical analysis covering a range of outcomes including take up of paternity leave, employment, time use, fertility, and divorce. Our theory predicts that paternity leave has heterogeneous effects for low, intermediate, and high wage gap couples, such that a quota for fathers can break traditional specialization agreements in couples with an intermediate gender gap in wages between the spouses. Using Spanish data and a regression discontinuity design, we first identify the three groups empirically using the model's predictions regarding the effect of paternity leave on fathers' leave length. Then we test our model's predictions on a range of outcomes. We don't find systematic effects of paternity leave on low or high wage gap couples, while we document that, among intermediate gap couples, the two-week paternity leave introduced in Spain in 2007 led to a 3 percentage-point drop in the fraction having another child, a 4 percentage-point increase in the divorce rate, a persistent increase in fathers' housework and childcare time of more than an hour per day each, and an increase of 8 percentage points in maternal employment two years after childbirth. Our theory and empirical analysis strongly suggest that small or zero aggregate effects may hide significant heterogeneity. Our results also suggest that paternity leave pushes some couples to become more egalitarian, with women working more and men sharing childcare. Thus, more generous paternity leave policies have the potential to be an instrumental tool in promoting gender equality.

Keywords: Gender equality, specialization, fertility, divorce, time allocation
JEL: D13, J12, J13, J16

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1 Introduction

Large gender inequalities in labor market outcomes persist across countries, and recent evidence shows that to a large extent they are driven by within-household specialization, which becomes more pronounced after parenthood (Kleven et al., 2019a,b). After having children, women tend to specialize in childcare and home production, working less and earning less in the paid labor market, while men tend to specialize in paid work. These patterns of specialization may increase the household's surplus when the wage gap between the partners is sufficiently high.

Recent research suggests that popular public policies, such as subsidized childcare and paid parental leave, may not be very successful in reducing these gender disparities (Kleven et al., 2022). An additional policy tool that has become increasingly popular in recent years is paternity leave. A common design is a paid parental leave that reserves a number of non-transferable weeks for fathers (a "quota"). These subsidies have the explicit goal of reducing gender inequalities at home and at work by encouraging higher involvement of men in childcare.

A number of rigorous empirical studies have evaluated the effects of paternity leave policies on a variety of outcomes, including fathers' time in home production as well as labor market outcomes and even fertility and divorce. However, the evidence is quite mixed, spanning from some meaningful impacts to small or zero effects, depending on the setting.[1] This inconclusive evidence invites a unified view about the effects of paternity leave on households' choices concerning employment, time-use, fertility, and divorce. There is so far no over-arching theoretical framework that allows us to understand and reconcile these seemingly

[1]Several papers find that paternity leave increases fathers' involvement in childcare and/or household work persistently (Patnaik, 2019; Farré and González, 2019), while some others find zero effects (Ekberg et al., 2013; Kluve and Tamm, 2013). A number of studies document no impact on fathers' labor supply (Ekberg et al., 2013; Dahl et al., 2014; Patnaik, 2019; Farré and González, 2019), but Rege and Solli (2013) and Avdic and Karimi (2018) find some evidence of negative effects. Patnaik (2019), Farré and González (2019), and Dunatchik and Özcan (2021) document positive effects on mothers' employment, while Ekberg et al. (2013), Rege and Solli (2013), and Dahl et al. (2014) find no effect. Regarding fertility, Dahl et al. (2014) and Cools et al. (2015) find zero effects, while Farré and González (2019) find a reduction. Avdic and Karimi (2018) find an increase in parental separation following paternity leave, while Several studies find no effect (Dahl et al., 2014; Cools et al., 2015; Farré and González, 2019). See Canaan et al. (2022) for a recent survey.

inconsistent documented effects and the mechanisms that drive them. This paper provides such a framework.

Our model assumes that couples can choose between interacting non-cooperatively, which defines their outside option within marriage, or reaching an informal agreement, in which they trade time for consumption between themselves. Specifically, in couples with a positive gender wage gap (the husband out-earns the wife), the wife specializes in raising kids (or home production) and the husband specializes in market work and transfers consumption for an ex-ante agreed price.[2] The agreement to specialize has two separate benefits. First, it increases productivity in raising kids and thus reduces their cost. Second, it increases the household's potential income due to the gender wage gap and returns on experience. However, the agreement also has a cost. We assume limited commitment between spouses regarding the redistribution of consumption. Specifically, we assume two types of husbands: A fair (or honest) husband, who makes the transfer ex-post, and an unfair (or dishonest) one, who shirks and does not transfer the agreed-upon consumption to his wife. We show that, under the assumption that the wife does not know the type of her spouse at the time of the agreement, the unfair husband can always mimic the fair one and proposes to his wife the same agreement proposed by a fair husband. The model thus generates a pooling equilibrium.[3]

The model predicts that paternity leave introduction (or expansion) decreases the net benefits from the agreement. Therefore, for the marginal couple, the agreement stops being optimal and both partners optimally choose not to specialize but to share childcare somewhat equally.[4]

[2] Our model is symmetric and can account for a reversed specialization when the wife out-earns her husband. However, we limit our theoretical analysis to the more common case of positive gender wage gap. The literature on gender norms finds that couples tend to avoid a situation in which the wife out-earns her husband (Bertrand et al., 2015). Finally, the share of such households is only 11% in Spain in 2007 (Survey of Income and Living Conditions).

[3] The details of the agreement (costs and benefits) have not been tested empirically, but our empirical results, especially the one on time-use, strongly suggest a discrete change in couples' choices, which our model captures as breaking an agreement. Thus, although we believe that our model captures reasonable costs (limited commitment of redistributing consumption) and benefits (returns to experience and learning by doing), other types of costs and benefits may lead to similar results.

[4] The intuition for the dissolution of agreements for the marginal couples is as follows. For cou-

Interestingly, our model predicts that this effect is not homogeneous across all households but depends on the gender potential wage gap between the spouses. The model distinguishes among three distinct groups. First, couples with a sufficiently low gender wage gap, who never reach an agreement to specialize, and thus always choose the outside option in which they share childcare costs. Henceforth, we will label this group as "egalitarian" couples. Although an expansion in paternity leave has small continuous effects on their time allocation, fertility, and consumption, these couples will keep sharing childcare. Second, for a sufficiently high gender wage gap the range of prices (compensation) that sustains an agreement in equilibrium is very large, such that a paternity leave expansion changes the agreement's conditions but will keep its existence optimal. Henceforth, we will label this group as "high-gap" couples. Third, in between these two corner groups comes the group that is most affected by the paternity leave policy. For this group, the range of prices that sustains an agreement in equilibrium is small enough that small expansions in paternity leave will make this range empty. Namely, couples cannot find a price of time that will make at least one of them better off. In this case, couples will move from a state of agreement to their outside option. Henceforth, we will label this group as "intermediate-gap" couples.

Therefore, paternity leave induces some couples to share childcare, which yields higher equality within households. However, for a positive gender wage gap couple, sharing childcare implies a higher cost of raising children compared to the specialization case, which reduces fertility. Thus our model predicts that paternity leave reduces fertility, which is counter intuitive since paternity leave is a subsidy: Money received by couples conditional on the birth of a child. Moreover, assuming that divorce yields a loss of utility from kids, a reduction in fertility increases the

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ples who have an agreement in which the husband does not take paternity leave, an increase in paternity leave, which is a subsidy for raising children, ceteris paribus, pushes the household's solution towards more children. Given an agreement in which the wife is specializing in raising kids, the price of compensation in terms of consumption that the wife requests increases by more than the amount that the fair husband is willing to pay, due to the risk of shirking by unfair husbands. As a result, they don't agree on the price, and the agreement to specialize stops being optimal. In case the husband does take paternity leave, his increase in productivity due to learning by doing in raising kids and his loss of experience at work act as additional forces negatively affecting his willingness to compensate his wife.

probability of divorce.[5] Put differently, the transition from a state of agreement to a state of disagreement triggers separation, which works in our model through the under provision of the public good, namely, fertility. This also reflects the notion that children act as a friction preventing divorce.

Thus, while paternity leave will merely lead to small income and price (substitution) effects for egalitarian and high-gap couples as the effect is continuous, it is discrete and discontinuous for the intermediate ones. Thus, the model predicts that for intermediate-gap couples, an expansion of paternity leave leads to (1) an increase in the effective length of leave taken by fathers; (2) a reduction in fertility; (3) an increase in the probability to divorce; (4) an increase in women's employment at the expense of childcare time; and (5) an increase in men's childcare time at the expense of their employment.

We test the predictions of the model in the setting of Spain, which introduced two weeks of paternity leave in 2007. Eligibility was based on the date of birth of the child. We study the effect of paternity leave on household outcomes following a regression discontinuity design, such that we compare families who had children very close to the cutoff date, and are thus very similar on average along all dimensions except paternity leave eligibility.

A non-trivial question is how to classify families as egalitarian, intermediate-gap and high-gap couples. Since potential wages are unobserved (and actual wages are endogenous), the literature uses the gap in age and/or educational attainment between the spouses to proxy or predict potential wages (Folke and Rickne, 2020; Bertrand et al., 2015). Therefore, one possibility is to use the gap in age and/or education between partners to proxy for the three distinct groups that our model predicts. However, any exogenous choice will be somewhat arbitrary.

We take advantage of the fact that our model has a clear prediction for fathers' response in terms of time at home (or time-off from work) as a result of paternity leave introduction. Since fathers in egalitarian couples share a major part of childcare regardless of the policy, paternity leave will not change their behavior much.

[5] See Weiss and Willis (1985) and Browning et al. (2014).

Neither does the policy affect fathers in high-gap couples, who always have an agreement to outsource their part in childcare to their wives. However, fathers' responses in intermediate-gap couples are discrete and significant, as these fathers move from an agreement where they specialize in market work, to sharing childcare, as egalitarian couples do.

This model's prediction allows us to run a sort of first stage, in which we calibrate the age-gap and education-gap thresholds, that define the borders between the three groups of couples, by targeting some moments in the take-up data. Specifically, we target a very small time-off response to paternity leave introduction for fathers in the corner groups while maximizing the effect for the intermediate group. That is, we sacrifice the take-up data to endogenize the classification of couples into the three groups. Thus, not only does our model predict three groups, but it also allows us to identify them. We find that, indeed, the take-up data gives rise to such a pattern: Very small and insignificant effects of paternity leave in the corners, and a large effect in the middle, which represents a first test of the model's predictions. In our application, about 22% of couples are in this middle group.

Then, in a second stage, we use this endogenous classification to test our model's predictions with regard to couples' fertility, employment, time-use, and divorce. Specifically, while we don't find systematic effects of the paternity leave expansion on low- and high-gap couples, we do find that, among intermediate-gap couples, eligibility for the two-week paternity leave led to a 3 percentage-point lower fraction of couples having another child, a 4 percentage-point increase in the proportion of couples getting divorced, a persistent increase in fathers' housework and childcare time of more than an hour per day each, and an increase in maternal employment of 8 percentage-points (and a 10% increase in earnings) two years after the birth of the child. Our empirical results thus strongly support the main predictions of the model.

Our theory relates to several strands of research. The literature on family economics has established a strong role for the gender wage gap in household choices (Galor and Weil, 1996).[6] Our model contributes to this important literature by

[6]Later contributions examine the role of the gender wage gap in a variety of important out-

showing that the gender wage gap can predict the type of interaction that different couples have: Some couples have an agreement to specialize while others don't. Efficiency in our model is reached through an agreement to trade time for consumption. Our model thus assumes a non-cooperative outside option (Lundberg and Pollak, 1993), which describes the final decision among egalitarian couples.[7] Moreover, our model shows that the lack of commitment, which produces the cost of specialization, sometimes prevents couples reaching efficiency.[8] To the best of our knowledge, we are the first to model optimal specialization as an agreement between spouses, which gives rise to heterogeneous effects.[9] Finally, while the focus of our model is on paternity leave policy, it can be a useful tool for analyzing family policies in general.

We contribute to the empirical literature analyzing the effects of family policies in four ways. First, we document the heterogeneous effects of paternity leave across different types of couples, and we are able to identify the marginal group: the one with an intermediate-gap in potential earnings between the partners. We provide evidence confirming that paternity leave has no effect on couples that are either very egalitarian (who never specialize), or ones where there is a large difference in potential earnings (who always specialize). Instead, there is a middle group of couples for whom even small extensions can have large effects across a number of dimensions. Second, we examine a range of outcomes, which so far have been analyzed separately in empirical studies. We document and rationalize effects on

comes such as women's empowerment (Duflo, 2012; Doepke and Tertilt, 2019); the marketization of childcare (Hazan and Zoabi, 2015a; Gobbi et al., 2018; Bar et al., 2018); international trade (Sauré and Zoabi, 2014; Do et al., 2016) and the gender educational gap (Chiappori et al., 2009; Becker et al., 2010; Hazan and Zoabi, 2015b)

[7]For a deep discussion about the validity of the non-cooperative game and the separate budget constraints within the family, see Doepke and Tertilt (2019), who analyze the effect of mandated transfers on the public good provision.

[8]Within a dynamic setting, Voena (2015) argues that a unilateral divorce yields lack of commitment, which produces distortion in household asset accumulation, Gobbi (2018) finds that limited commitment between spouses produces an underinvestment in childcare, and Doepke and Kindermann (2019) argue that disagreement over having children stems from lack of commitment.

[9]Meier and Rainer (2017) study some theoretical aspects of paternity leave and argue that paternity leave solves the hold-up problem and may increase fertility and Carrer (2021) focuses on gender norms to examine the efficiency of parental leave policies.

take-up,[10] time-use (childcare and housework),[11] labor supply,[12], fertility,[13], and divorce,[14] all of which are connected via our theory. Our model proposes a mechanism consistent with the declines in subsequent fertility found in several countries (Farré and González, 2019; Fontenay and Tojerow, 2020; Lee, 2022). Moreover, our theory links the existing evidence on fertility and parental divorce (Dahl et al., 2014; Avdic and Karimi, 2018; Olafsson and Steingrimsdottir, 2020). Finally, our results provide a potential explanation for why some papers have found small or zero effects of paternity leave on some of the outcomes of interest. We find small effects overall, but strong effects for a subset of households, which may have been "washed out" in previous studies that do not incorporate the type of heterogeneity predicted by our model.

Finally, our paper has a clear policy implication. We show that a quota for fathers as part of any parental leave policy increases the bargaining power of the wife and pushes some couples from preserving traditional roles (the husband is the breadwinner and the wife is homemaker) to more egalitarian ones.[15] Our paper argues that paternity leave expands the egalitarian group, thus promoting gender equality.[16]

[10] See Avdic and Karimi (2018), Bartel et al. (2018), Cools et al. (2015), Dahl et al. (2014), Ekberg et al. (2013), Olafsson and Steingrimsdottir (2020), Patnaik (2019) and Rege and Solli (2013).

[11] Several papers find that paternity leave increases fathers' involvement in childcare and/or household work persistently (Farré and González, 2019; Kotsadam and Finseraas, 2011; Patnaik, 2019; Rege and Solli, 2013; Tamm, 2019), while others find zero effects (Ekberg et al., 2013; Kluve and Tamm, 2013).

[12] A number of studies document no impact on fathers' labor supply (Cools et al., 2015; Dahl et al., 2014; Ekberg et al., 2013; Patnaik, 2019; Rege and Solli, 2013; Farré and González, 2019), but Rege and Solli (2013) and Avdic and Karimi (2018) find some evidence of small negative effects. Regarding mothers, Patnaik (2019), Farré and González (2019), and Dunatchik and Özcan (2021) document positive effects on maternal employment, while Dahl et al. (2014), Ekberg et al. (2013) and Rege and Solli (2013) find no effect.

[13] Cools et al. (2015), and Dahl et al. (2014) find no effects of paternity leave extensions on fertility in Norway, while Farré and González (2019), Fontenay and Tojerow (2020) and Lee (2022) find small negative effects on fertility in Spain, Belgium, and Korea.

[14] Avdic and Karimi (2018) find that an increase in fathers' share of parental leave led to higher marital separation rates in Sweden, while Dahl et al. (2014) and Cools et al. (2015) report no effect of paternity leave on marital stability in Norway. Finally, Olafsson and Steingrimsdottir (2020) find heterogenous effects in Iceland, with paternity leave increasing marital stability in more egalitarian couples but decreasing it in those where the father is more educated than the mother.

[15] Recent papers argue that labor market penalties associated with motherhood are the main obstacle for closing the gender earnings gap (Kleven et al., 2019a,b; Bertrand, 2020; Titan et al., 2021).

[16] While gender equality is important per se, it is found to be an important driver for development

The paper proceeds as follows. Section 2 presents the setup of the model. Section 3 characterizes the equilibrium and provides the main results. Section 4 presents our empirical analysis and Section 5 concludes. Our Appendix presents the conditions for optimality and provides proofs for all propositions, lemmas and corollaries.

2 THE MODEL

Consider a married couple, which is composed of a man (father), m and woman (mother), f. Each agent derives utility from private consumption, the number of children and a non-monetary value that reflects the match quality of the relationship. They face exogenous wages and decide on their labor employment and the allocation of time in raising their kids.[17] The utility of agent $i \in \{m, f\}$ is given by

$$U_i(c_i, n, \theta) = \log c_i + \log n + \theta \tag{1}$$

where $c_i \geq 0$ is the private consumption of an agent i, and $n \geq 0$ is the couple's number of children. θ is a non-monetary shock to a couple's relationship that is revealed after living together for some time and is assumed to be similar for both spouses (Weiss and Willis, 1993, 1997; Browning et al., 2014). θ is drawn from a given uniform distribution with zero mean and positive variance: $\theta \sim U(0, \sigma^2)$.

Figure I sketches the sequence of events that the couple faces. They make their economic choices in two stages. In the first stage, each agent decides non-cooperatively on their own labor supply, which implies the remaining time that each one will allocate for raising children. Each one can also decide whether to take parental leave. This will define the outside option of each agent. The couple may choose the outside option or reach an agreement, in which they trade time for consumption between themselves. Specifically, the husband proposes to reduce the time for raising kids, which may include avoiding paternity leave, and make

(Doepke and Tertilt, 2009; Hazan et al., 2019, 2021).

[17] As will be explained below, the hourly wage is endogenous, yet the hourly wage per unit of experience is exogenous.

a transfer to his wife. Once they make their first stage choices - labor supply and childcare time - they are committed to this time allocation as the labor market commitment is binding. In the second stage the quality of the relationship is revealed. At this stage, couples may divorce or save their marriage by redistributing consumption again.

Raising children requires parents' time, and the number of children n is given by:

$$n(t_m + \tau_m, t_f + \tau_f) = (t_m + \tau_m)^a + (t_f + \tau_f)^a \tag{2}$$

We assume that $a > 1$, which reflects "*learning by doing*" in raising children. $\tau_i \in [0, \bar{\tau}_i]$ is the parental leave that the agents take ($\bar{\tau}_i$ is the maximum parental leave that the government provides), $t_i \geq 0$ is the time individual i spends on raising children.

Agents accumulate experience at work. The budget constraint of an individual i is given by

$$c_i(t_i, \tau_i) = w_i(1 - t_i)(1 - \tau_i - t_i) \tag{3}$$

where w_i is an exogenous wage per unit of time and experience. $(1 - t_i)$ is the labor supply and $(1 - t_i - \tau_i)$ is the return to experience. We assume that $\bar{\tau}_i$ is a non-transferable parental leave, which is paid to compensate fully for the forgone labor income. The full wage compensation is just a simplifying assumption. Any partial payment delivers the same qualitative results. We also assume that the wage of the father is more than the wage of the mother.

2.1 Outside option

In the outside option in the first stage parents choose the amount of time that they spend for raising children and the parental leave that they take. It is assumed that in this case of non-cooperation, the solution is given by a Nash-Cournot game, i.e. the couple solves the maximization problems:

$$\max_{t_m,\tau_m} \mathbb{E}(U_m)$$
$$\max_{t_f,\tau_f} \mathbb{E}(U_f)$$

with the constraints $t_i \geq 0$, $\bar{\tau}_i \geq \tau_i \geq 0$, $i \in \{m, f\}$ and also the time constraint $\tau_i + t_i \leq 1$.

We denote the optimal outside option choices by $t_m^0,\ t_f^0,\ \tau_m^0,\ \tau_f^0$ and the indirect utilities by U_m^0, U_f^0.

2.2 First stage

In the first stage of the game, the father chooses an optimal amount of time which he wants to "buy" from his spouse $(\tilde{t}_m,\ \tilde{\tau}_m)$.[18] Then the couple bargains over redistributing consumption which is given by a transfer T.

An *agreement* exists if there exists a non-zero set $(\tilde{t}_m, \tilde{\tau}_m, T)$ that both agents gain from agreeing upon transferring time for consumption, i.e. the expected utility from trading is greater than the outside option for both parents. If it exists, we call this set $(\tilde{t}_m, \tilde{\tau}_m, T)$ an *agreement*. By having an agreement, the father increases his working time at the market by $\tilde{t}_m$ or $\tilde{t}_m + \tilde{\tau}_m$, and agrees to pay the transfer, T to his spouse. The mother thus increases her time spent on raising children by $\tilde{t}_m$, or $\tilde{t}_m + \tilde{\tau}_m$.[19]

We assume two types of men: fair, f and unfair, u. The fair father follows the rules of the agreement, and transfers the agreed redistribution, T in full. The unfair father shirks and does not send any transfer, namely $T = 0$.[20]

[18] Transferring τ means that the husband does not take paternity leave but buys this time from his wife as well.

[19] As will be analyzed below, our model allows for two types of agreements: one in which the fathers take a paternity leave and this is the only time they spend raising children and the other in which they do not take paternity leave and make a higher compensation to their spouses.

[20] Assuming that the unfair father sends a partial transfer does not change any of the model's results.

Let c_{fj} be the mother's consumption and c_{mj} the father's consumption for the type of a father $j \in \{f, u\}$ - fair or unfair. Since $T_f = T$ and $T_u = 0$, the budget constraints for both spouses are:

$$c_{mj} = w_m(1 - t_m^0 + \tilde{t}_m)(1 - t_m^0 - \tau_m^0 + \tilde{t}_m + \tilde{\tau}_m) - T_j$$
$$c_{fj} = w_f(1 - t_f^0 - \tilde{t}_m - \tilde{\tau}_m)(1 - t_f^0 - \tau_f^0 - \tilde{t}_m - \tilde{\tau}_m) + T_j$$

Thus, the number of kids is:

$$n = (t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^a + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^a$$

In the second stage, which will be discussed below, the spouses can decide whether to get a divorce upon the realization of a non-monetary shock θ.[21] Denote the ex-ante probability of divorce in the second stage by p_d. Then each spouse maximizes the expected utility from both stages.

The expected utility of a father of type $j \in \{f, u\}$:

$$\mathbb{E}(U_{mj}(\tilde{t}_m, \tilde{\tau}_m, T)) = (1 - p_d)U_{mj}^M + p_d U_{mj}^D$$

Where U_{mj}^M is the utility in case of marriage, which is given by:

$$U_{mj}^M = \log c_{mj} + \log n + \theta$$

And U_i^D is the utility in case of divorce, which is given by:

$$U_{mj}^D = \log c_{mj} + d \log n$$

We follow Browning et al. (2014) and assume that in case of a divorce the spouses do not suffer from non monetary shock, but their utility from children is depreciated, as captured by $d < 1$.[22]

Note that it is always profitable for an unfair father to pursue an agreement,

[21]We follow Weiss and Willis (1985) and assume that the shock is the same for both spouses.

[22]This drop in utility from children comes to capture the idea that since parents are not living together after divorce, they lose part of the control they had over children. Weiss and Willis (1985) argue that "divorce causes [parents] to reveal a reduced interest in the welfare of their children".

as he benefits from specialization, and does not send back anything. A fair father cannot separate himself from an unfair one. Since a mother would not make an agreement with an unfair father, the optimal strategy for an unfair father is to mimic a fair one. Thus, the model generates a pooling equilibrium. We consider the problem of a fair father. We also assume that all agents know the distribution of fathers' types - $\beta \in (0,1)$ share of fathers are of type f, and $1-\beta$ are of type u - and that every mother believes that her husband is a fair one with probability β. Therefore, the mother's expected utility:

$$\mathbb{E}(U_f(\tilde{t}_m, \tilde{\tau}_m, T)) = \beta \left[(1-p_d)U^M_{ff} + p_d U^D_{ff}\right] + (1-\beta)\left[(1-p_d)U^M_{fu} + p_d U^D_{fu}\right]$$

where U_{ff} is the mother's utility in the case of a realized "fair" spouse, and U_{fu} stands for an "unfair" one.

For any given $\tilde{t}_m$ and $\tilde{\tau}_m$ we calculate the maximum price - in terms of consumption redistribution - that a father is willing to pay by equalizing his utility to one he gets in the outside option. This maximum price is denoted by T_m. Similarly, by equalizing the utility of the mother to her outside option, we calculate the minimum price that she is willing to accept. This minimum price is denoted by T_f. We assume that the equilibrium price is a weighted average between T_m and T_f. specifically:

$$T = \alpha T_m + (1-\alpha)T_f,\ \alpha \in [0,1] \tag{4}$$

We assume that the father proposes an agreement to the mother considering this equilibrium price, T.[23] Thus, the father maximizes:

$$\max_{\tilde{t}_m, \tilde{\tau}_m} \mathbb{E}(U_m(\tilde{t}_m, \tilde{\tau}_m, T))$$

[23]It does not matter who proposes the agreement's conditions. See Proposition 10 and its proof in the appendix

$$
\begin{aligned}
\textbf{s.t. } T &= \alpha T_m + (1-\alpha) T_f \\
\mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T_m) &= \mathbb{E}U_m^0 \\
\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T_f) &= \mathbb{E}U_f^0 \\
T_m &\geq T_f
\end{aligned}
$$

where U_i^0 is the utility of individual i in the outside option.

Lemma 1. *The agreement exists if and only if there exist $\tilde{t}_m, \tilde{\tau}_m, T$ such that having non-zero trade in time is profitable:*

$$
\begin{aligned}
\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T) &\geq \mathbb{E}U_f^0 \\
\mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T) &\geq \mathbb{E}U_m^0
\end{aligned}
$$

That is, the agreement exists if and only if for the chosen $\tilde{t}_m$ and $\tilde{\tau}_m$, $T_m > T_f$ is satisfied. Put differently, if there exists a surplus from specialization.[24]

Let the optimal utility levels at this stage be denote by $U_i^1(\tilde{t}_m, \tilde{\tau}_m, T)$, $i \in \{m, f\}$.

2.3 Second stage

In the second stage, the non-monetary shock θ is realized and the individuals may divorce. Assume a uniform distribution such that, $\theta \in [x_1, x_2]$ s.t. $x_1 < (d-1)\log n_0 < x_2$, individuals choose to divorce if their utility from staying married (defined in the first stage) is less than that from divorce. The utility of an agent $i \in \{m, f\}$ of a type of father $j \in \{f, u\}$ in case of divorce is:

$$U_{ij}^D = \log c_{ij} + d \log n$$

The time spent for children t is fixed due to the commitment in the labor market. That is, even in the case of divorce, the agents spend the same amount of time for work as in marriage. Moreover, we assume that in the case of divorce, fair fathers keep sending the same transfers. This implies that the private consumption is also fixed. At this stage, the agents can choose another transfer T_2 to prevent divorce. The divorce does not occur if there exists T_2 such that both agents find it

[24]Clearly, equation 4 expresses the idea that the model is silent about the way this surplus is divided as this division does not play any role for any of our results.

more appealing to keep the marriage:

$$U_m^M(\tilde{t}_m, \tilde{\tau}_m, T, T_2) > U_m^D$$
$$U_f^M(\tilde{t}_m, \tilde{\tau}_m, T, T_2) > U_f^D$$

where U_m^M and U_f^M are the utilities of a father and a mother in case of keeping their marriage:

$$U_m^M = \log(c_{mj} + T_2) + \log n + \theta$$
$$U_f^M = \log(c_{fj} - T_2) + \log n + \theta,\ j \in \{f, u\}$$

3 Solution

Firstly, we solve for the outside option:

$$\max_{t_m, \tau_m} \mathbb{E}(U_m)$$
$$\max_{t_f, \tau_f} \mathbb{E}(U_f)$$

with the constraints $\bar{\tau}_i \geq \tau_i \geq 0,\ t_i \geq 0,\ \tau_i + t_i \leq 1$ for $i = \{m, f\}$.

Lemma 2. *In any agreement* $T \geq 0$

It immediately follows that whenever a mother can observe the unfair spouse, she chooses the outside option. We now show that in any agreement an unfair father mimics a fair one, thus constituting a pooling equilibrium.

Proposition 1. *In any agreement, the weakly dominant strategy of an unfair father is to imitate a fair father.*

In the outside option, the utilities of fair and unfair fathers are the same. In the case of an agreement, an unfair father can increase his consumption compared to a fair one. Moreover, no mother will make an agreement with an unfair father. Thus, imitating a fair father is the dominant strategy.

This result allows us to consider solely a problem of a fair father for the solution of the first stage of the game.

Proposition 2. *In equilibrium in the outside option, agents always prefer a full paid leave.*

The outside option solution is the set: $(t_m^0, \tau_m^0, t_f^0, \tau_f^0)$.

In other words, since in the outside option t_m^0 is interior and since it is assumed that the time required for raising children is far above paternity leave, this policy becomes a free subsidy granted to fathers as they can reduce the voluntary unpaid t_m^0 by the size of the publicly financed $\bar{\tau}_m$.

Now we solve the first stage of the game for the case of an agreement. We show that it is sufficient to solve the model for the case where $\theta = 0$.

Lemma 3. *The problems*

$$\max_{t_m, \tau_m} \mathbb{E}(U_m)$$
$$\max_{t_f, \tau_f} \mathbb{E}(U_f)$$

and

$$\max_{t_m, \tau_m} U_m(\theta = 0)$$
$$\max_{t_f, \tau_f} U_f(\theta = 0)$$

subject to identical constraints:

$$\tau_i \geq 0$$
$$t_i \geq 0$$
$$\tau_i + t_i \leq 1$$

for $i = f, m$
have coinciding solutions

Proof.

$$\mathbb{E}(U_m) = \log w_m(1 - t_m)(1 - \tau_m - t_m) + \log n + \mathbb{E}(\theta) = U_m|_{\theta=0}$$
$$\mathbb{E}(U_f) = \log w_f(1 - t_f)(1 - \tau_f - t_f) + \log n + \mathbb{E}(\theta) = U_f|_{\theta=0}$$

$\blacksquare$

First, we define the utilities from the second stage over the choice between getting divorced and keeping the marriage. The divorce does not occur if there exists T_2 such that:

$$U_i^M > U_i^D \quad i \in \{m, f\}$$

where U_i^M and U_i^D are the utilities for sustaining marriage and divorce, respectively, for the individual i.

Proposition 3. *The stability of the marriage does not depend on the first stage transfer, T. The agents choose to divorce if and only if*

$$\theta < (d-1)\log n$$

In fact, the first-stage transfer does not affect the probability of divorce, since in the second stage, couples can preserve their marriage by another redistribution.

Corollary. *The probability of divorce is*

$$p_d = \frac{x_1 - (d-1)\log n}{x_1 - x_2}$$

Having obtained these results, we can now find the solution for the first stage.

Proposition 4. *If it is optimal to transfer a single unit of time, then it is optimal to transfer the full amount of time. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$t_m^0 \geq \tilde{t}_m$$
$$\tilde{t}_m \geq 0$$

Proposition 5. *If it is optimal to transfer a single unit of parental leave, then it is optimal to transfer the full parental leave. Formally, one of the following constraints always binds* *in the maximization problem in the first stage:*

$$\tau_m^0 \geq \tilde{\tau}_m$$
$$\tilde{\tau}_m \geq 0$$

Intuitively, both propositions state that whenever it is optimal to transfer a single unit of time, it is optimal to transfer the full amount of time. This is because once the mother devotes more time to childcare, she becomes better by the assumption of learning by doing. Similarly, the more the father specializes at work, the larger his wage becomes due to the assumption of positive return to experience. both assumptions increase the benefit from an agreement the larger the traded amount of time is, which leads to corner solutions.

Hence, it suffices to compare 4 cases:

- Agents choose an outside option
- Agents choose an agreement in both t and τ: $(\tilde{t}_m = t_m^0,\ t\tilde{a}u_m = \tau_m^0, T)$
- Agents choose an agreement in t: $(\tilde{t}_m = t_m^0, \tilde{\tau}_m = 0, T)$
- Agents choose an agreement in τ: $(\tilde{t}_m = 0, \tilde{\tau}_m = \tau_m^0, T)$

Proposition 6. *The agreement* $(\tilde{t}_m = 0, \tilde{\tau}_m = \tau_m^0, T)$ *is never optimal*

The intuition is that since t^0 is interior and since τ is paid by the government it is always cheaper to trade in t before τ.

Thus, we consider the other three cases. The first is when we do not have an agreement at all. That is, for any T, the agents derive higher utility from the outside option:[25]

$$U_m(c_m(0, \tau_m^0 - \tilde{\tau}_m) - T, n(\tau_m^0 - \tilde{\tau}_m, \tau_f^0 + t_m^0 + t_f^0 + \tilde{\tau}_m), \theta) < U_m(c_m(t_m^0, \tau_m^0), n(\tau_m^0 + t_m^0, \tau_f^0 + t_f^0), \theta)$$
$$\beta(U_f(c_{ff}(t_f^0 + t_m^0, \tau_f^0 + \tilde{\tau}_m) + T, n(\tau_m^0 - \tilde{\tau}_m, \tau_f^0 + t_m^0 + t_f^0 + \tilde{\tau}_m), \theta))+$$
$$(1-\beta)(U_f(c_{fu}(t_f^0 + t_m^0, \tau_f^0 + \tilde{\tau}_m), n(\tau_m^0 - \tilde{\tau}_m, \tau_f^0 + t_m^0 + t_f^0 + \tilde{\tau}_m), \theta) <$$
$$U_f(c_f(t_f^0, \tau_f^0), n(\tau_m^0 + t_m^0, \tau_f^0 + t_f^0), \theta)$$

[25]See Appendix I.A for the conditions for having the outside option as an optimal choice.

where $\tilde{\tau}_m \in \{\tau_m^0, 0\}$, i.e. none of the two possible agreements is profitable.[26]

Secondly, if the previous conditions are not satisfied, we have an agreement. Moreover, the optimal agreement is the one with both, $\tilde{t}_m$ and $\tilde{\tau}_m$, if for any T there exists T_1:

$$U_m(c_m(0,\tau_m^0) - T, n(\tau_m^0, \tau_f^0 + t_m^0 + t_f^0), \theta) < U_m(c_m(0,0) - T_1, n(0, \tau_f^0 + t_f^0 + \tau_m^0 + t_m^0), \theta)$$
$$U_f(c_f(t_f^0 + t_m^0, \tau_f^0) + T, n(\tau_m^0, \tau_f^0 + t_f^0 + t_m^0), \theta) < U_f(c_f(t_f^0 + t_m^0, \tau_f + \tau_m^0) + T_1, n(0, \tau_f^0 + t_f^0 + \tau_m^0 + t_m^0), \theta)$$

Otherwise an optimal agreement will be an agreement in $\tilde{t}_m$ only.[27]

These conditions conclude the solution of the model.

3.1 Comparative statics

We now use the results of the model for a comparative statics analysis. We start with how the wage gap between spouses affects the optimality of an agreement. Then we show that an increase in paternity leave, $\bar{\tau}_m$, can only move some couples from a state of agreement to a state of no agreement. Finally, we summarize the model's results.

Proposition 7. *For a sufficiently high gender wage gap, there always exists an agreement in both t and τ. For a sufficiently low gender wage gap, there always exists a range of parameters under which there is no agreement. As the gender wage gap increases, there can be a switch only from no agreement to some agreements.*

The intuition for this result is that for a sufficiently low gender wage gap, the ability of the father to redistribute consumption by transferring consumption to his spouse is rather low. Given that the mother is unaware of the type of her husband and takes into consideration that with probability $1 - \beta$ he is an unfair spouse, the compensation that she requires in any agreement is higher than what the husband is willing to pay. This makes the range of prices for any potential agreement empty. On the contrary, for a sufficiently high gender wage gap, the ability of the husband

[26] Remember that the model produces two possible agreements: an agreement on $\tilde{t}_m$ or one on both $\tilde{t}_m$, and $\tilde{\tau}_m$.

[27] See Appendix I.A for the conditions for having an agreement in both t, τ.

to redistribute consumption is relatively high. Therefore, couples can always find a price to agree on.

Proposition 8. *Considering the parameters of the model ($\bar{\tau}_m$, $\bar{\tau}_f$, a, α, β, w_m, w_f) and divorce parameters s.t., there is no agreement. Then if $\bar{\tau}_m$ increases, then*
(i) the agents cannot switch to an agreement in $\tilde{t}_m$.
(ii) the agents cannot switch to an agreement in $\tilde{t}_m$ and $\tilde{\tau}_m$.

Propositions 7 and 8 summarize the main results of the model. While Proposition 7 identifies different types of families in equilibrium, Proposition 8 examines the effect of an introduction or extension of paternity leave policy on that equilibrium. Specifically, proposition 8 states that as paternity leave increases, there can be a switch only from some agreements to no agreement.

We thus conclude our model's results by the following proposition.

Proposition 9. *A switch from some agreements to no agreement leads to:*
(i) an increase in take-up of paternity leave.
(ii) a reduction in fertility.
(iii) an increase in the probability of divorce.
(iv) an increase in women's employment at the expense of childcare time.
(v) an increase in men's childcare time at the expense of their employment.

These results, summarized in Proposition 9, can be elaborated using a numerical example depicted in Figure II. The figure shows the gender wage gap (w_m/w_f) on the horizontal axis and the size of paternity leave in terms of time ($\bar{\tau}$) on the vertical axis. A dot expresses that an agreement exists, while in every empty (white) coordinate, partners do not have an agreement to specialize and thus share childcare.[28]

The figure shows three different regimes that correspond to three groups of couples. Couples with a sufficiently small gender wage gap will always share

[28] The orange dot stands for an agreement in both t_m and τ_m and the blue dot stands for an agreement in t_m only. This is a second order effect and does not account for any of our model's results

childcare, regardless of the size of paternity leave. Couples with a sufficiently high gender wage gap will always have an agreement to specialize. In between the two corner groups comes the one most affected by paternity leave: intermediate wage-gap couples. For couples in this group, a change in the policy will move them from an agreement to specialize to sharing childcare. Only for this group does our model predict a discontinuous change.

Thus, the model predicts that an expansion of paternity leave leads to (1) an increase in take-up of paternity leave; (2) a reduction in fertility; (3) an increase in the probability to divorce; (4), an increase in women's employment at the expense of childcare time; and (5) an increase in men's childcare time at the expense of their employment.

Before moving to our empirical examination to test these predictions, it is important to note that although the large effect reaches only the intermediate group, our model is not completely silent about the two corner groups. While egalitarian couples always share childcare and high-gap couples always specialize, our model predicts small changes in these groups. Specifically, paternity leave increases women's bargaining power across the board, and thus allows those from the egalitarian group to better balance their investment at home vis-a-vis their husbands, and those from the high wage gap group to extract a larger share of the household's surplus.

4 EMPIRICAL ANALYSIS

In order to test the predictions of our theory, we exploit the introduction of thirteen days of paternity leave in Spain in March 2007. This reform did not affect the length or generosity of maternity leave.[29] The new permit was voluntary and non-transferable, and it replaced 100% of earnings (financed by Social Security). All new fathers were eligible, provided that they held a formal job at the time of birth (and had worked in the formal sector for at least 180 days during the previous seven years).

[29] Maternity leave was 16 weeks both before and after the reform.

The model predicts heterogeneous effects of a paternity leave extension, depending on the gap in potential wages within the couple. In particular, we expect that paternity leave extensions will not affect behavior in couples that are either egalitarian (who would already have been sharing market and household work before the extension) or high potential wage-gap couples (who would have been specializing before and who would continue to specialize after the reform). However, we expect a decrease in specialization (division of labor within the couple in terms of home production and market work) in an intermediate group of couples that are neither egalitarian nor high wage-gap (in terms of comparative advantage in market work). More specifically, we expect that this middle group, which we label as intermediate wage-gap couples, will react to extensions in paternity leave with increases in the length of leave taken by the father, an increase in fathers' involvement in childcare and housework beyond paternity leave, as well as an increase in maternal labor supply. The model also predicts a decrease in subsequent fertility, and a potential increase in divorce.

Fathers were eligible for the longer paternity leave if their child was born after March 23, 2007. Our population of interest is composed of couples who had a child in a close neighborhood of the date of the policy change. Our regression discontinuity design compares couples who had a child shortly before the threshold date with those who had a child shortly after, using several data sources to measure the different outcomes of interest. The exact bandwidth around the threshold varies across data sets due to sample size considerations. We allow the effect of paternity leave to vary as a function of the characteristics of couples.

4.1 Data and descriptive statistics

4.1.1 Take-up

To study the take-up of paternity leave among eligible families, we use the *Survey on the Use of Parental Leave and their Labor Consequences* (which we will refer to as the *Madrid Survey* or MS), which was conducted between January and June 2012 in the metropolitan area of Madrid (Fernández-Cornejo et al., 2012). The survey targeted parents living in Madrid with a child aged 3 to 7 at the time of the survey. The MS provides information on the month and year of birth of the youngest child, as well as data on parental leave take-up, socio-demographic characteristics of the family, labor supply, and child-related time-use of both parents, for 1,130 children. Out of these 1,130 observations, there are 1,101 observations that have information on month and year of birth of the child, and 94.5% of the children were born between January 2005 and December 2008. Our final sample includes 1,094 observations. In this sample, overall take-up of paternity leave among eligible fathers was 66%.

We use this data set to analyze the effect of paternity leave introduction on the total number of leave days taken by fathers surrounding childbirth. Before the introduction of paternity leave in 2007, fathers could take 2 days of paid leave after the birth of a child. They could also take vacation days, unpaid leave, or even use up some of the maternity leave time. After March 23, 2007, fathers were offered an additional two weeks (13 days) of paternity leave (with 100% wage replacement).

Our main dependent variable is the number of days off taken around the birth of a child (including paternity leave as well as additional days off: vacation and other). We consider 3 bandwidths around the introduction of paternity leave: 12, 15 and 18 months. Total leave length is 13 days on average.

4.1.2 Fertility

To analyze the effects of paternity leave on subsequent fertility, we use administrative micro data on the universe of births taking place in Spain between 2007 and 2013. The data are made available publicly by the Spanish Statistical Institute and come from birth certificates. We requested as additional variables the exact date of birth of each child as well as the previous child born to the same mother.

The high quality of the data allows us to restrict the sample to a close neighborhood of the threshold. Our main sample includes women having a child between January and June of 2007. We observe all of them, with their precise date of birth, in the 2007 birth certificates. We refer to this one as the "reference child".

Using the data for later years combined with the information on the date of later births as well as the previous birth to the same mother, we can construct our main dependent variables, which are individual-level indicators for whether each mother had another child within 2, 4 and 6 years after the date of birth of the reference child. About 6% of mothers had another child within two years of the birth of the reference child, while 22% had another child within 4 years, and close to a third within 6 years.

4.1.3 Childcare and housework time

To analyze the effects of paternity leave on fathers' time-use, and in particular the time devoted to childcare and housework, we use the Spanish Time-Use Survey, conducted between October 2009 and September 2010, i.e. about three years after the birth of the reference child (for parents whose child was born close to the introduction of paternity leave). We restrict the sample to include only different-sex parents living in a couple (married or cohabiting) whose youngest child was born 3 years before or after the reform (2004-2010). The final sample includes 941 fathers and 1,047 mothers (the survey interviews only one adult per household).

The survey includes detailed information on the daily minutes devoted to different activities, including childcare and housework, as well as household socio-economic characteristics, and the month and year of birth of all the interviewee's children. We use the number of daily minutes devoted to each task as dependent variables. On average, fathers in our full sample devote about 100 minutes per day to both childcare and household chores.

4.1.4 Maternal employment

To analyze the effects of paternity leave on maternal employment, we use longitudinal Social Security data ("Muestra Continua de Vidas Laborales"). This data set provides information on the working histories of a representative sample of 4% of people affiliated with Social Security in a given year. Our sample includes women having a child in a 9-month window around the introduction of paternity leave (i.e., between July 2006 and December 2007).

Our main dependent variables are indicators for maternal employment 12 and

24 months after the birth of the reference child (born close to the date of paternity leave introduction), as well as their accumulated earnings over the 24 months following the birth of the reference child. About 68% of the women in our sample were employed two years after the birth of the reference child, and their accumulated earnings over those initial two years were close to 13,000 euro.

4.1.5 Divorce

Finally, to analyze the effects of the 2007 extension on parental divorce, we merge Labor Force Survey (LFS) data for all quarters of 2008-2010 (i.e. between 4 and 15 quarters after the policy change). We select all women with a child born between November 2006 and August 2007 (5 months before and after the policy change). The main outcome is an indicator for parental separation. About 8.3% of women reported being separated or divorced at the time of the survey.

4.2 Empirical strategy

We follow a regression discontinuity approach, where the running variable is the month (or exact date) of birth of each couple's child, and the threshold is the date of birth that determines eligibility for paternity leave. The identifying assumption is that, close enough to the threshold, control and treated families are comparable in all dimensions but paternity leave eligibility, or at least there is no discontinuous jump for other reasons exactly at the threshold. We estimate the following equation:

$$Y_{i\tau} = \alpha + \beta T_{\tau} + \delta_1 m + \delta_2 I[T = 1]m + \gamma X_{i\tau} + \varepsilon_{i\tau} \tag{5}$$

where Y is the dependent variable of interest (e.g. subsequent fertility) for family i who had a child in month τ, T is an indicator for paternity leave eligibility (i.e. the couple having had a child after the paternity leave introduction), m is the running variable (month of birth of the child in most data sets, normalized so it takes value 0 in April 2007, -1 in March 2007, etc.), and X are control variables (such as mother's age and educational attainment). We allow for a linear trend in the running variable, which is allowed to vary at the threshold. The coefficient of interest is β, which captures a discrete jump in the outcome variable coinciding with paternity leave eligibility.

We estimate this equation in the full sample (which will give us the average intent-to-treat effect), and also separately for couples that vary in terms of the gap in potential wages between the partners. We classify couples in terms of the age and education gap between the partners, which serves as a proxy for the gap in potential earnings. In order to detect potential heterogeneous effects, we split couples into three groups: egalitarian, intermediate and high wage gap couples.

We provide support for our identifying assumption by testing for balance in covariates across the threshold in all of our different data sets, and separately for each group of couples. Our results show that families are comparable in their observable characteristics on both sides of the RD cutoff, which assuages potential seasonality concerns. Some of our outcomes are observed during the recession period that followed after 2007. Our research design relies on comparing couples close to the threshold, i.e. who had a child close to March 2007, and who are observed at the same point in time afterwards. Thus, the recession does not pose a threat to our identification strategy per se. Note also that our focus on heterogeneous effects across groups helps alleviate most of these types of concerns.

4.3 Results

4.3.1 Validity checks

We test for balance in covariates across the threshold for the different data sets, both in the full sample and by family type. We run regressions of the form of equation 5 without controls, where we use the control variables one by one as the dependent variable, to detect any possible discontinuities in family characteristics coinciding with the policy cutoff.[30]

[30] The full regression results are available upon request. Farré and González (2019) also show that there was no sorting across the threshold in terms of number of births.

4.3.2 Take-up and identifying the marginal group

Table I presents the results for the effect of the introduction of paternity leave in 2007 on the number of leave days taken by fathers surrounding the birth of their child.[31] We report the results for three different bandwidths: 12, 15, and 18 months around the threshold. The first panel shows the results for the full sample. We find that the introduction of paternity leave led to an average increase in the number of leave days actually taken by fathers of about 8 days. Considering that the take-up of paternity leave was 63-64% in our sample, this corresponds to a full length (13 days) for those who actually take it.

Our model predicts heterogeneous treatment effects of paternity leave eligibility depending on the gap in potential wages between the spouses. In particular, the corner solution summarized in propositions 4 and 5 gives rise to a clear distinction in fathers' responses in terms of time at home (or time off from work). This implies that paternity leave will discontinuously increase fathers' effective time off only for the intermediate group of couples, while it will have no discontinuous effect on families with a larger or smaller wage gap between the partners. We use this prediction about take-up to identify the marginal group of couples.

We proxy the gap in potential wages with the gap in age and educational attainment between the partners (the age and education gaps are calculated as husband's age or schooling minus wife's age or schooling, in years). In our take-up sample, the man is 2 years older than the woman in the average couple, while she has about 0.6 more years of education.

We then calibrate the age and education gap thresholds that define the three groups by targeting the difference in the effects of paternity leave introduction on fathers' time off across the groups. More specifically, we target a zero take-up response to paternity leave introduction for the corner groups while maximizing the

[31] The results of our tests for balance in covariates are available upon request. We consider 18 family characteristics. In the full sample, we find a discontinuity in two of them (at the 95% confidence level): education of the father, and education gap between the partners. In the three subsamples, only one coefficient is significant at 95%, out of 54.

difference with the intermediate one.[32] Thus, we sacrifice the take-up data in order to calibrate and endogenize the classification of couples into the three groups. The resulting classification is depicted in Figure III.

Figure IV shows the coefficients in the leave length regressions for the three groups. We find, as targeted, a positive significant effect of paternity leave eligibility on total leave length by fathers in the intermediate group of couples, while the effect is smaller and not statistically significant for egalitarian and high wage-gap couples. The difference between the middle group and the other two is also significant. Although these are targeted moments, the fact that we do find an intermediate group with large effects while the effects are close to zero in the other groups is a first piece of evidence consistent with the model predictions (Proposition 5).

The full regression results for the three groups are shown in Table I.[33] Our results show that the increase in total time off by fathers in the full sample is driven mainly by the intermediate families. Fathers in egalitarian couples increase their leave by 1 to 4 days in response to paternity leave introduction, while the effect is between 4 and 9 days for high wage-gap couples. The middle group of intermediate couples, on the other hand, increase the father's time off by almost a month (22 to 27 days). Note that this intermediate group includes 20-23% of all couples. Our estimates thus suggest that fathers in this group not only take the full 13 days of paid leave, but they increase their time off by an additional 9 to 14 days, as a result of the reform. This is consistent with our predictions that the reform leads these couples to switch from a specialization regime to one where childcare is shared.

[32]Denote egalitarian, intermediate and high-gap groups as 1, 2 and 3. The maximization problem is then $\max\min\{t_{21}, t_{23}\}$ where t_{21} and t_{23} are the t-statistics of the treatment effect differences between 1) intermediate and egalitarian groups; 2) intermediate and high gap groups, i.e. $t_{21} = \frac{\beta_2-\beta_1}{\sqrt{s.e._2^2+s.e._1^2}}$ and $t_{23} = \frac{\beta_2-\beta_3}{\sqrt{s.e._2^2+s.e._3^2}}$. The maximization is done subject to insignificance of treatment effect for the corner groups, so that the calibration exercise allows us to choose, from all classifications with zero response in the corner groups, the one that maximizes the difference between the intermediate and the corner groups.

[33]The results for the test of balance in covariates in each group of families are available upon request.

4.3.3 Fertility

Next, we estimate the effect of paternity leave on couples' subsequent fertility. Dependent variables are now individual-level indicators for the mother having another child within 2, 4 and 6 years after the date of birth of the reference child in the neighborhood of March 24, 2007. Because the birth data cover the universe of children born in Spain, and since we observe the exact date of birth, we can focus on parents of children born very close to the relevant threshold. We consider two bandwidths: children born within 8 and 12 weeks around March 23, 2007. Results for the full sample in Table II indicate that paternity leave eligibility may have led to lower subsequent fertility after 2, 4 and 6 years, as already documented in Farré and González (2019). However, the coefficient is small, not stable across specifications, and mostly insignificant.

We next apply the endogenous classification of families calibrated in the take-up sample, and show that the negative effect on subsequent fertility is driven by the group of couples with an intermediate wage gap. Figure V shows the effects on subsequent fertility (after 6 years) for the three groups of couples (and the 12-week bandwidth). We find a significant decrease in the fraction of parents having another child within 6 years for intermediate couples, while we find small and insignificant effects for egalitarian and high wage gap couples.

The full regression results are shown in Table II.[34] Two years after having a child in 2007, egalitarian couples that were eligible for paternity leave are slightly less likely to have had another child, but the effect is small and insignificant. For high wage-gap couples, on the other hand, we find small positive coefficients (also insignificant). However, eligible intermediate couples are 1.4 to 1.6 percentage points less likely to have another child within 2 years, a 23% reduction (compared to the mean in the control group, shown in the table). This significant effect for the intermediate group persists after 4 years (2.1-3.4). By 2013 (six years after paternity leave introduction), we find no effect on additional fertility for egalitarian and high wage-gap couples, while intermediate couples are 2.5 to 4 percentage points

[34]We find no significant discontinuity at the threshold in the covariates considered (results are available upon request). Farré and González (2019) report balance in a rich set of covariates for the full sample.

less likely to have had another child, suggesting a 7-11% reduction in completed fertility. The fact that the effect persists after 6 years supports the prediction of the model that intermediate wage gap families have switched regimes due to the introduction of paternity leave.

4.3.4 Childcare and housework time

We next analyze the effect of paternity leave introduction on fathers' contribution to childcare and housework beyond the paternity leave period (about 3 years after the birth of the child). We estimate regressions for total daily minutes of childcare time, housework, and market work by fathers. Table III presents the results for two different bandwidths around the birth-date determining eligibility (20 and 28 months).[35]

Results for the full sample in Table III show no significant effects on fathers' time-use. We do find positive coefficients for childcare and housework time, and negative ones for market work, but they are imprecisely estimated.

We next explore heterogeneity across types of families using our endogenous classification of couples. We find that, as a result of paternity leave introduction, fathers in intermediate couples increase childcare and housework time (see Figure VI), unlike fathers in egalitarian and high wage-gap couples. The results are presented separately for each of the groups in Table III. We find no significant effect of paternity leave introduction on fathers' time devoted to childcare or housework for egalitarian and high wage-gap families. The coefficients for egalitarian and high-wage couples are unstable across different bandwidths, and never significantly different from zero.

We do find significant positive effects for fathers in the intermediate group. Eligible fathers in intermediate couples spend significantly more time on childcare and housework after the introduction of paternity leave. The increase in daily childcare and housework time is more than one hour each.

[35] The results of the balance in covariates tests are available upon request.

In terms of magnitude, fathers increase their childcare time to a level that roughly corresponds to mothers' pre-treatment mean in the relevant group of couples. Paternity leave eligibility appears to lead to about equal contributions from fathers and mothers to total childcare time. As for housework time, the increase by 69-103 daily minutes (from a baseline of 102-105) also leads to men spending roughly the same amount of time as mothers on housework in the control group of intermediate couples.

The estimated effect of paternity leave introduction on the time fathers spend on paid work is negative in intermediate couples, although the coefficients are not significantly different from zero for any of the bandwidths.

4.3.5 Maternal employment

Next, we study how paternity leave affected mothers' labor market outcomes. Our main dependent variables are indicators for maternal employment 12 and 24 months after the birth of the reference child (born close to the date of paternity leave introduction), and accumulated earnings over the 24 months following the birth of the reference child. Results for the full sample are shown in the first panel of Table IV. We find that mothers whose partners were eligible for paternity leave are about 3 percentage points more likely to be employed a year after childbirth, an effect that seems to persist after two years. This has a small impact on earnings, of about 300 euros (or 3%).

We then explore heterogeneity of this effect across types of couples.[36] Table IV presents the results for two different bandwidths around the birth-date determining paternity leave eligibility: 3 and 6 months.[37] Once again, our results show that

[36]The Social Security data do not provide information on the educational attainment of the partner. Thus, we classify couples based on the age difference and maternal education level, using the main age difference thresholds as determined by the endogenous classification of couples used in the previous sets of outcomes. Intermediate couples are those with an age gap of 2-3 and a mother with medium education level, or an age gap >3 and maternal education is high. Couples are classified as high wage gap if mother's education level is low OR the age gap is >3 years and mother's education level is medium/low. Couples are egalitarian if the age gap is <2 and mother's education level is medium/high OR age gap 2-3 but mother's education level is high.

[37]We show that there was no significant discontinuity in covariates at the threshold (results available upon request).

the aggregate effect is driven by women in intermediate couples (Figure VII). In these marginal households, paternity leave eligibility leads to large, significant increases in maternal employment and earnings. Two years after childbirth, women in eligible families in the intermediate group are 8-9% more likely to be employed, and their accumulated earnings are higher by 7-9%. We find no such effect among mothers in egalitarian or high wage-gap couples.

4.3.6 Divorce results

Finally, we study the effect of the introduction of paternity leave on marriage dissolution. The sample is now composed of all women living with a child born close to the threshold, including those who were not living with a partner at the time of the survey. 8.3% of women living with a child born close to the cutoff were separated when surveyed in 2008-10.

We find (Table V) no overall effect on the probability of divorce for the full sample of women, as also found in Farré and González (2019). The coefficients are all very small in magnitude, and none are significant at the 95% confidence level.

For the analysis of heterogeneous treatment effects, we cannot apply the same classification of couples as before because we do not observe husband characteristics for separated women. We thus now classify mothers based on their own age and educational attainment only (i.e. high vs. low predicted potential wage of women).[38]

Figure VIII (using a 3-month bandwidth around the date of paternity leave introduction and the classification based on both age and schooling) shows that the effect of paternity leave introduction on the probability of divorce is heterogeneous across the three groups of women: couples in the intermediate group experience an increase in divorce probability, while we find no significant effect on egalitarian

[38]The following classifications are used: Based on mother's schooling: college, high school, less than high school. Based on mother's age and education: college education and age 40+ at first childbirth, high school graduates aged 22-39 at first childbirth, and less than high school and <40 at first childbirth OR high school and <27 at first childbirth.

or high wage gap couples. As shown in Table V, the increase in the probability of divorce for women from the intermediate group is between 3 and 6 percentage points, depending on the classification and the bandwidth.[39] We find no significant increase in the divorce probability for the high wage-gap and egalitarian groups (in fact, we find negative effects among high-wage mothers in the egalitarian group, consistent with Olafsson and Steingrimsdottir (2020)).

5 Conclusions

Our model suggests one possible mechanism through which husbands and wives in heterosexual couples decide on parental leave and the allocation of time between home production and the labor market. They can choose the non-cooperative outside option, or they can reach an agreement of traditional gender roles, in which the wife specializes in home production (raising kids) while the husband works for pay and transfers consumption to his wife. The model shows that egalitarian couples (with a sufficiently small gender wage gap) do not specialize and play the outside option, while intermediate-gap (with a medium gender wage gap) and high-gap couples do have such an agreement. An expansion in paternity leave reduces the net benefits from the agreement and moves intermediate-gap couples to their outside option, where women work more and men do more home production. As a result, the cost of raising children increases and fertility declines. Assuming that children act as friction preventing divorce, lower fertility increases the probability of divorce.

Using Spanish data and a regression discontinuity design, we confirm our model's predictions. In a first stage, using the model's predictions about fathers' take-up responses, we calibrate the thresholds for age and education gaps between the spouses to endogenously classify couples into egalitarian, intermediate-gap and high-gap couples. In a second stage, we use this endogenous classification to examine the impact of paternity leave on subsequent fertility, time-use, employment, and marital stability. While we don't find systematic effects of paternity leave expansion for egalitarian or high-gap families, we find a reduction of 3 percentage

[39] The results of testing for balance in covariates around the threshold are available upon request.

points in the fraction of intermediate couples going on to have another child, a 4 percentage-point increase in the proportion of intermediate couples getting divorced, a persistent increase in fathers' housework and childcare time of more than an hour per day each, and an increase in maternal employment of 8 percentage points two years after childbirth.

Our theory and empirical results show that introducing or expanding paternity leave produces more equality within couples by pushing some couples to the egalitarian group, where fathers start sharing more time in different home production activities. The discontinuous change in the equilibrium for intermediate-gap couples expresses something deep that has changed in the relationships within couples. Our model suggests that some couples abandon the agreement of traditional gender roles.

A number of studies have analyzed the effects of family policies on gender inequality in labor market outcomes. A recent paper by Kleven et al. (2022) analyzes parental leave and subsidized childcare policies in Austria, and finds that they did not lead to any substantial improvements in the gap in earnings between men and women with children. While our paper shares this view at the aggregate level, our model suggests that the small or zero aggregate effects may hide significant impacts in the marginal group of couples, defined by the (intermediate) gap in potential wages between the spouses. Moreover, our framework also indicates that more generous maternity leave may actually lead to increased specialization, as it increases women's relative advantage in home production. A growing literature argues that labor market penalties associated with motherhood are the main remaining obstacle for closing the gender gap in earnings (Kleven et al., 2019a,b; Bertrand, 2020; Titan et al., 2021). Our model shows that paternity leave leads some couples (with an intermediate gap in earnings between the spouses) to become more egalitarian, with women working more and men sharing childcare. We thus argue that more generous paternity leave policies have the potential to be an instrumental tool in promoting gender equality.

Online Appendix

Appendix

A — The Outside Option and Optimality Conditions

In this appendix we derive the conditions for not having an agreement. The idea of the proof is to find $\tilde{t}_m$ and T_f such that $T_m < T_f$, meaning that the maximum price the husband is ready to pay is less than the minimum that the wife is ready to accept. We will show the conditions under which neither an agreement in both $\tilde{t}_m, \tilde{\tau}_m$ nor an agreement only in $\tilde{t}_m$ are preferred to an outside option.

Firstly, let us find T_f. It is defined by $\mathbb{E}(U_f) = U_f^0$, so that the wife is indifferent between an agreement and the outside option. After substitution of utilities and simplification we obtain:

$$T_f = \left[\frac{c_f^0 \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta} \cdot n^{1-p_d+p_d d}}\right]^{1/\beta} - w_f(1 - t_f^0 - \tilde{t}_m - \tilde{\tau}_m)(1 - t_f^0 - \tau_f^0 - \tilde{t}_m - \tilde{\tau}_m)$$

Let us now find $\tilde{t}_m$. Similarly, it is defined by $\mathbb{E}(U_m) = U_m^0$. After substitution of utilities and simplification we obtain:

$$\tilde{t}_m = w_m(1 - t_m^0 + \tilde{t}_m)(1 - t_m^0 - \tau_m^0 + \tilde{t}_m + \tilde{\tau}_m) - \frac{c_m^0 \cdot n_0^{1-p_d+p_d d}}{n^{1-p_d+p_d d}}$$

The agreement does not exist whenever $T_m < T_f$, i.e. when the lower bound of the interval for price of the agreement exceeds the upper bound.

Condition 1: agreement only on $\tilde{t}_m$ is not profitable:

$$w_m(1-\tau_m^0) - \frac{w_m(1-t_m^0)(1-t_m^0-\tau_m^0)\cdot\Big((t_m^0+\tau_m^0)^a+(t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{\Big((\tau_m^0)^a+(t_f^0+\tau_f^0+t_m^0)^a\Big)^{1-p_d+p_d d}} <$$

$$\left[\frac{(w_f(1-t_f^0)(1-t_f^0-\tau_f^0))\cdot\Big((t_m^0+\tau_m^0)^a+(t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{(w_f(1-t_f^0-t_m^0)(1-t_f^0-\tau_f^0-t_m^0))^{1-\beta}\cdot\Big((\tau_m^0)^a+(t_f^0+\tau_f^0+t_m^0)^a\Big)^{1-p_d+p_d d}}\right]^{1/\beta} -$$

$$-\,w_f(1-t_f^0-t_m^0)(1-t_f^0-\tau_f^0-t_m^0)$$

Condition 2: agreement on both $\tilde{t}_m$ and $\tilde{\tau}_m$ is not profitable:

$$w_m - \frac{w_m(1-t_m^0)(1-t_m^0-\tau_m^0)\cdot\Big((t_m^0+\tau_m^0)^a+(t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{\Big((t_f^0+\tau_f^0+t_m^0+\tau_m^0)^a\Big)^{1-p_d+p_d d}} <$$

$$\left[\frac{(w_f(1-t_f^0)(1-t_f^0-\tau_f^0))\cdot\Big((t_m^0+\tau_m^0)^a+(t_f^0+\tau_f^0)^a\Big)^{1-p_d+p_d d}}{(w_f(1-t_f^0-t_m^0-\tau_m^0)(1-t_f^0-\tau_f^0-t_m^0-\tau_m^0))^{1-\beta}\cdot\Big((t_f^0+\tau_f^0+t_m^0+\tau_m^0)^a\Big)^{1-p_d+p_d d}}\right]^{1/\beta} -$$

$$-\,w_f(1-t_f^0-t_m^0-\tau_m^0)(1-t_f^0-\tau_f^0-t_m^0-\tau_m^0)$$

Conditions for the optimality of the agreement in both $(t,\ \tau)$

In this appendix we provide the conditions for which an agreement in both $t,\ \tau$ is profitable.

The agreement on both $\tilde{t}_m,\ \tilde{\tau}_m$ should be a Pareto-improvement for any other agreement. That is, it should be at least as good as the outside option:

$$T^0_{mt,\tau} > T^0_{ft,\tau}$$

Where $T^0_{mt,\tau}$ and $T^0_{ft,\tau}$ are the upper and lower bounds of the interval for prices in case when we compare an agreement in both $\tilde{t}_m,\ \tilde{\tau}_m$ with the absence of an agreement as the outside option.

And this agreement should be at least as good as the agreement only on $\tilde{t}_m$:

$$T^1_{mt,\tau} > T^1_{mt,\tau}$$

Where $T^1_{mt,\tau}$ and $T^1_{ft,\tau}$ are the upper and lower bounds of the interval for prices in case when we compare an agreement in both $\tilde{t}_m,\ \tilde{\tau}_m$ with an agreement only in $\tilde{t}_m$ as the outside option.

Calculate $T^0_{ft,\tau}$. It is such that $\mathbb{E}(U_f) = U^0_f$ (we take it from Appendix IA):

$$T^0_f = \left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta} \cdot n_{t,\tau}^{1-p_d+p_d d}} \right]^{1/\beta} - w_f(1 - t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m)$$

Calculate T^0_m. It is such that $\mathbb{E}(U_m) = U^0_m$ (we take it from Appendix IA as well):

$$T^0_m = w_m - \frac{c^0 \cdot_0^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Where $n_{t,\tau} = (t^0_m + \tau^0_m + t^0_f + \tau^0_f)^a$ is the number of kids in case of agreement in both $\tilde{t}_m,\ \tilde{\tau}_m$

Calculate $T^1_{ft,\tau}$. It is such that $\mathbb{E}U_f(t^0_m, \tau^0_m) = \mathbb{E}U_f(t^0_m, 0)$, i.e. when an agreement in $\tilde{t}_m$ is an outside option. Using calculations from Appendix IA we found that:

$$T^1_{ft,\tau} = \left[\frac{(c_{f2} + T^0_{ft})^\beta \cdot c_{f2}^{1-\beta} \cdot n_t^{1-p_d+p_d d}}{c_{f1}^{1-\beta} n_{t,\tau}^{1-p_d+p_d d}} \right]^{1/\beta} - c_{f1}$$

Calculate $T^1_{mt,\tau}$. It is such that $\mathbb{E}U_m(t^0_m, \tau^0_m) = U_m(t^0_m, 0)$:

$$\log\left(w_m - T^1_{mt,\tau}\right) + (1 - p_d + p_d d)\log n_{t,\tau} = \log\left(w_m(1-\tau^0_m) - T^0_{mt}\right) + (1 - p_d + p_d d)\log n_t$$

$$T^1_{mt,\tau} = w_m - \frac{\left(w_m(1-\tau^0_m) - T^0_{mt}\right) n_t^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Then the agreement on $(\tilde{t}_m, \tilde{\tau}_m)$ is an equilibrium agreement when two conditions are satisfied:

$$\begin{cases} T^0_{mt,\tau} > T^0_{ft,\tau} \\ T^1_{mt,\tau} > T^1_{ft,\tau} \end{cases}$$

B — Proofs

Proof of lemma 1

Lemma.1 The agreement exists if and only if there exist $\tilde{t}_m, \tilde{\tau}_m, T$ such that having non-zero transfers of time is profitable:

$$\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_f^0; \;\; \mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_m^0$$

Proof. If there is an agreement, than

$$T = \alpha T_m + (1-\alpha)T_f; \;\; 0 \leq a \leq 1; \;\; \mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T_m) = \mathbb{E}U_m^0;$$
$$\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T_f) = \mathbb{E}U_f^0; \quad T_m \geq T_f$$

Thus:

$$T_m \geq T \geq T_f$$

Thus, because U_m and U_f is increasing in T:

$$\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_f^0; \quad \mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_m^0$$

If exist such $\tilde{t}_m, \;\tilde{\tau}_m, T$ that:

$$\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_f^0; \quad \mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_m^0$$

Because because U_m and U_f is increasing in T, exist such $T_m, \;T_f$ that:

$$\mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T_m) = \mathbb{E}U_m^0; \quad \mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T_f) = \mathbb{E}U_f^0; \quad T_m \geq T \geq T_f$$

Thus there exist $\tilde{t}_m, \;\tilde{\tau}_m$ such that the utility of the father subject to optimization constrain is greater than under no agreement. Thus, there is an agreement. ■

Proof of Lemma 2

Lemma. In any agreement $T \geq 0$

Proof. It follows from an assumption that neither husband nor wife exhaust all of their time on raising children ($t_m^0 + t_f^0 + \tau_m^0 + \tau_f^0 < 1$). Assume by contradiction that there exists a agreement with $T < 0$. That is, given optimally chosen (t_f^0, τ_f^0), the wife is willing to give up some of her private consumption to dedicate more time to childcare. If so, then initial t_f^0, τ_f^0 were chosen suboptimally: in the outside option, an increase in t_f resulting in an equivalent loss of private consumption would be beneficial, as it is beneficial in an agreement. This is a contradiction, and t_f^0 is not optimally chosen. As we assumed arbitrary $T < 0$, this suggests that in any agreement it must be the case that $T \geq 0$. ■

Proof of proposition 1

Proposition. *In any agreement, a weakly dominant strategy of an unfair father is to imitate a fair father.*

Proof. Assuming that the wife always specializes, we get $T > 0$ in any equilibrium. Note that there never exists a separating equilibrium. Any separating equilibrium would mean that the wife is able to distinguish between the fair and unfair agent. And she would not agree to have an agreement with an unfair father, as she would not receive a transfer. Thus, it is only rational for an unfair father to mimic a fair one.

Now consider a pooling equilibrium. The utility of the fair father is:

$$U_{m,f} = \log(c_m - T) + \log(n) + \theta$$

The outside option utility is:

$$U^0_{m,f} = U^0_{m,u} = \log(c^0_m) + \log(n^0) + \theta$$

The utility of the unfair father in case of an agreement is:

$$U_{m,u} = \log(c_m) + \log(n) + \theta$$

If it is profitable to have an agreement then:

$$U_{m,f} > U^0_{m,f}$$

Note that

$$U_{m,u} > U_{m,f} > U^0_{m,f} = U^0_{m,u}$$

Then it is profitable for an unfair father to imitate a fair one.

When there is no agreement, the unfair father is indifferent.

Hence, it is a weakly dominant strategy to imitate a fair father.

■

Proof of proposition 2

Proposition. *In equilibrium in the outside option, agents always prefer full paid leave.*

Proof. The agents can choose the paid parental leave of $\tau_i \leq \bar{\tau}_i$. Assume by contradiction that $\tau < \bar{\tau}$. Consider $\tau_i = \tau_i + \Delta$, $t_i = t_i - \Delta$. Then:

$$\mathbb{E}\tilde{U}_i = \log(1 - t_i + \Delta) + C$$

where C includes all the terms which do not depend on Δ. It is clear that for positive Δ the expected utility increases, so it is profitable to choose $\tau_i = \bar{\tau}_i$. ■

Proof of proposition 3

Proposition. *The stability of the marriage does not depend on the transfer T. The agents choose to divorce if and only if*

$$\theta < (d-1)\log n$$

Proof. The no-divorce condition is that given some T_2:

$$U_m^M(T + T_2) \geq U_m^D; \quad U_f^M(T_r + T_2) \geq U_f^D$$

where T_r can be either T or 0.

Expand it using the definition of the utility function and rearrange the terms and simplify:

$$\log(w_m(1 - t_m^0 + \tilde{t}_m)(1 - t_m^0 + \tilde{t}_m + \tilde{\tau}_m - \tau_m^0) - T - T_2) + \theta \geq$$
$$\log(w_m(1 - t_m^0 + \tilde{t}_m)(1 - t_m^0 + \tilde{t}_m + \tilde{\tau}_m - \tau_m^0) - T) + (d-1)\log(n)$$

$$\log(w_f(1-t_f^0-\tilde{t}_m-\tilde{\tau}_m)(1-t_f^0-\tilde{t}_m-\tilde{\tau}_m-\tau_f^0)+T_r+T_2)+\theta \geq$$
$$\log(w_f(1-t_f^0-\tilde{t}_m-\tilde{\tau}_m)(1-t_f^0-\tilde{t}_m-\tilde{\tau}_m-\tau_f^0)+T_r)+(d-1)\log(n)$$

Which is equivalent to:

$$-c_f(1-\frac{n^{d-1}}{e^\theta}) \leq T_2 \leq c_m(1-\frac{n^{d-1}}{e^\theta})$$

Case $1-\frac{n^{d-1}}{e^\theta} \geq 0$:

$$-c_f(1-\frac{n^{d-1}}{e^\theta}) \leq T_2 \leq c_m(1-\frac{n^{d-1}}{e^\theta}) \Rightarrow -c_f(1-\frac{n^{d-1}}{e^\theta}) \leq 0 \Rightarrow c_m(1-\frac{n^{d-1}}{e^\theta}) \geq 0$$

Case $1-\frac{n^{d-1}}{e^\theta} < 0$:

$$0 > c_m(1-\frac{n^{d-1}}{e^\theta}) \geq T_2 \Rightarrow 0 < -c_f(1-\frac{n^{d-1}}{e^\theta}) \leq T_2$$

There does not exist such T_2 that the marriage is stable for the second case. As for the first case, for $T_2 = 0$ the marriage is stable. Hence, the marriage is not stable if and only if $1-\frac{n^{d-1}}{e^\theta} > 0$.

$$1-\frac{n^{d-1}}{e^\theta} < 0 \ <=> \ \theta < (d-1)\log n$$

■

Proof of corollary

Corollary. *Assume the uniform distribution of $\theta \in [x_1, x_2]$ s.t. $x_1 < (d-1)\log n_0 < x_2$. Then the probability of divorce is*

$$p_d = \frac{x_1-(d-1)\log n}{x_1-x_2}$$

Proof. In the first stage the agents have rational beliefs regarding the probability of divorce. Assuming a uniform distribution $\theta \sim U[x_1, x_2]$ s.t. $x_1 < (d-1)\log n_0 < x_2$ we have

$$\Pr(\theta < (d-1)\log n) = \frac{x_1-(d-1)\log n_1}{x_1-x_2}$$

Where $\theta < (d-1)\log n_1$ is the condition for having divorce.

■

Proofs of propositions 4 and 5

Proposition. *If it is optimal to transfer a single unit of time, then it is optimal to transfer the full amount of time. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$t_m^0 \geq \tilde{t}_m$$
$$\tilde{t}_m \geq 0$$

Proof. The utility of father:

$$\begin{aligned} U_m = &\log((1 - t_m^0 + \tilde{t}_m)(1 - t_m^0 - \tau_m^0 + \tilde{t}_m + \tilde{\tau}_m) - T) + \\ &\log((t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^a + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^a) \end{aligned}$$

Where the first term is logarithm of private consumption and the second term is logarithm of number of kids. Both terms under the logarithms are convex w.r.t $\tilde{t}_m$. Their product is convex as well.The optimal solution of the convex problem is the corner solution: either $\tilde{t}_m = 0$ or $\tilde{t}_m = t_m^0$. Monotonic transformation does not change the optimal solution.

The utility of the mother:

$$\begin{aligned} U_f = &\mathbb{E}\log((1 - t_f^0 - \tilde{t}_m - \tilde{\tau}_m)(1 - t_f^0 - \tau_f^0 - \tilde{t}_m - \tilde{\tau}_m) + T) + \\ &\log((t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^a + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^a) \end{aligned}$$

Apply the exponentiation to the utility function. The resulting problem is convex w.r.t. $\tilde{t}_m$, If the mother agrees to a non-zero $\tilde{t}_m$, meaning that for a given $\tilde{t}_m$ there exists T_f s.t. $T_f < \tilde{t}_m$, then due to convexity of the utility function w.r.t. $\tilde{t}_m$ every additional $\Delta > 0$ costs less for the mother in terms of utility. At the same time every additional $\Delta > 0$ brings more utility to the father. Thus if they can agree upon a non-zero $\tilde{t}_m$ (so that there exists T_f s.t. $T_f < \tilde{t}_m$), then they can agree upon $\tilde{t}_m + \Delta$, making an interior solution not optimal. The monotonic logarithmic transformation does not change the optimal solution.

■

Proposition. *If it is optimal to transfer a single unit of parental leave, then it is optimal to transfer the full parental leave. Formally, one of the following constraints always binds in the maximization problem in the first stage:*

$$\tau_m^0 \geq \tilde{\tau}_m$$
$$\tilde{\tau}_m \geq 0$$

The proof is identical to the previous proposition.

Proof of proposition 6

Proposition. *The agreement $(\tilde{t}_m = 0, \tau_m = \tau_m^0, T)$ is not optimal*

Proof. The utility of father:

$$U_m = \log((1 - t_m^0)(1 - t_m^0) - T) + \log((t_m^0)^a + (t_f^0 + \tau_f^0 + \tau_m^0)^a)$$

To prove that the agreement is not optimal it suffices to show that a little deviation is profitable. Consider $\tilde{t}_m = \tilde{t}_m + \Delta$, $\tilde{\tau}_m = \tilde{\tau}_m - \Delta$

$$\begin{aligned}\tilde{U}_m =& \log((1 - t_m^0 + \Delta)(1 - t_m^0 + \Delta - \Delta) - T) + \log((t_m^0 + \Delta - \Delta)^a + (t_f^0 + \tau_f^0 + \tau_m^0 + \Delta - \Delta)^a) = \\ & \log((1 - t_m^0 + \Delta)(1 - t_m^0) - T) + \log((t_m^0)^a + (t_f^0 + \tau_f^0 + \tau_m^0)^a) > U_m\end{aligned}$$

The utility of mother:

$$\begin{aligned}U_f = & \mathbb{E}\log((1 - t_f^0 - \tilde{t}_m - \tilde{\tau}_m)(1 - t_f^0 - \tau_f^0 - \tilde{t}_m - \tilde{\tau}_m) + T_r) + \\ & + \log((t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^a + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^a)\end{aligned}$$

It is easy to see that $\tilde{U}_f = U_f$. Then there is Pareto-improvement, and the initial agreement is not optimal.

■

Proof of Proposition 7

Proposition. *For sufficiently high wage gap there always exists an agreement in both t, τ. For sufficiently low gender wage gap there exist some parameters under which there is no* *agreement. As the gender wage gap increases, there can be a switch only from no agreement to some agreement.*

Proof. We start the proof with Lemma 3.

Lemma 4. *For sufficiently high wage gap there always exists an agreement in both t, τ*

Proof: Let us use the results from Appendices I and II in the proof.

$(T^0_{mt,\tau}, T^0_{ft,\tau})$, $(T^1_{mt,\tau}, T^1_{ft,\tau})$ are the sets of upper and lower bounds of the interval for prices for an agreement in both $(\tilde{t}_m, \ \tilde{\tau}_m)$ in case when 1) lack of agreement is the outside option; 2) the agreement only on $\tilde{t}_m$ is the outside option. Then an agreement in both $(\tilde{t}_m, \ \tilde{\tau}_m)$ is the equilibrium choice if the following conditions are satisfied:

$$\begin{cases} T^0_{mt,\tau} > T^0_{ft,\tau} \\ T^1_{mt,\tau} > T^1_{ft,\tau} \end{cases}$$

Where $n_0 = (t^0_m + \tau^0_m)^a + (t^0_f + \tau^0_f)^a$ is the number of kids without an agreement, $n_{t,\tau} = (t^0_m + \tau^0_m + t^0_f + \tau^0_f)^a$ is the number of kids with an agreement in both $(\tilde{t}_m, \ \tilde{\tau}_m)$,

$$T^0_{ft,\tau} = \left[\frac{c^0_f \cdot n_0^{1-p_d+p_d d}}{c_{fu}^{1-\beta} \cdot n_{t,\tau}^{1-p_d+p_d d}} \right]^{1/\beta} - w_f(1 - t^0_f - t^0_m - \tau^0_m)(1 - t^0_f - \tau^0_f - t^0_m - \tau^0_m)$$

and

$$T^0_{mt,\tau} = w_m - \frac{w_m(1-\tau^0_m)(1-t^0_m-\tau^0_m) \cdot n_0^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

The other variables are defined in the Appendix IA.

Note that due to loglinearity of the utility function, the w_m and w_f do not affect the outside option solutions of the husband's and wife's problems. As the number of kids with an agreement is always greater than without it (due to specialization in kids, $a > 1$), we have $n_t > n_0$, $n_{t,\tau} > n_0$. Also $(1-\tau^0_m)(1-t^0_m-\tau^0_m) < 1$. Hence $T^0_{mt,\tau}$ is increasing in w_m. At the same time an increase in w_m does not affect $T^0_{ft,\tau}$. Thus for any given parameters we can always find w_m s.t. $T^0_{mt,\tau} > T^0_{ft,\tau}$.

Rewrite $T^1_{mt,\tau}$:

$$T^1_{mt,\tau} = w_m \left(1 - \frac{\left(1-\tau^0_m\right) n_t^{1-p_d+p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}\right) + \frac{T^0_{mt} n_t^{1-p_d p_d d}}{n_{t,\tau}^{1-p_d+p_d d}}$$

Note that due to specialization in raising kids $n_{t,\tau} > n_t$. Also $(1-\tau^0_m) < 1$, hence the first term is increasing in w_m. Note that T^0_{mt} is also increasing in w_m. Thus $T^1_{mt,\tau}$ is increasing in w_m. An increase in w_m does not affect $T^1_{ft,\tau}$. Then for any given set of parameters there exists such w_m that $T^1_{mt,\tau} > T^1_{ft,\tau}$.

□

Lemma 5. *For sufficiently low gender wage gap there exist some parameters under which there is no agreement.*

Proof: Let us find the condition on β s.t. there is no agreement in $(\tilde{t}_m,\ \tilde{\tau}_m)$: $T^0_{mt,\tau} < T^0_{ft,\tau}$. Rearranging the terms, the condition on β is:

$$\tilde{\beta} < \frac{\log(c^0_f n_0^{1-p_d+p_d d}) - \log(c_{fu,t,\tau} n_{t,\tau}^{1-p_d+p_d d})}{log[w_m(1-(1-\tau^0_m)(1-t^0_m-\tau^0_m)(n_0/n_{t,\tau})^{1-p_d+p_d d}) + c_{fu,t,\tau}] - \log c_{fu,t,\tau}}$$

Let us now find the condition on β s.t. there is no agreement in $\tilde{t}_m$: $T^0_{mt} < T^0_{ft}$. Rearranging in terms of β:

$$\bar{\beta} < \frac{\log(c^0_f n_0^{1-p_d+p_d d}) - \log(c_{fu,t} n_t^{1-p_d+p_d d})}{log[w_m((1-\tau^0_m)-(1-\tau^0_m)(1-t^0_m-\tau^0_m)(n_0/n_1)^{1-p_d+p_d d}) + c_{fu,t}] - \log c_{fu,t}}$$

Then there is no agreement if $\beta \leq \min[\tilde{\beta}, \bar{\beta}]$

It is easy to see that for $a = 1$ $\tilde{\beta}$ and $\bar{\beta}$ are positive. For $a = 1$ $n_0 = n_t = n_{t,\tau}$ because there is no specialization in kids. And the outside option consumption is higher than consumption of a wife married to an unfair husband.

Both $\tilde{\beta}$ and $\bar{\beta}$ are decreasing in w_m. Hence, lower w_m is associated with higher threshold for fair fathers in the population for the agreement to be profitable for the mother.

□

Lemma 6. *There can only be a switch from no agreement to any type of agreement as the gender wage gap increases*

Proof: The agreement is not profitable to make iff $T^0_{mt,\tau} < T^0_{ft,\tau}$ and $T^0_{mt} < T^0_{ft}$. Thus to show that the "switch" from no agreement to an agreement may occur only in one direction, we need to show that the ranges $T^0_{mt,\tau} - T^0_{ft,\tau}$ and $T^0_{mt,\tau} - T^0_{ft,\tau}$ are expanding in w_m/w_f. Normalize $w_f = 1$ without loss of generality.

As previously analyzed in Lemma 5, the range $T^0_{mt,\tau} - T^0_{ft,\tau}$ is expanding in w_m. Using similar arguments, it is easy to see that T^0_{ft} does not depend on w_m, while T^0_{mt} is increasing in w_m. Hence the range $T^0_{mt} - T^0_{ft}$ is expanding in w_m as well.

As a result of an increase in w_m, the "switch" can occur only in one direction: from no agreement to some agreements. We abstain from further analysis of switches between the two types of agreements, as the case of switch from no agreement is of importance here. □

This concludes the proof. ■

Proof of Proposition 8

Proposition. *Consider the parameters of the model ($\tilde{\tau}_m$, τ_f, a, α, β, w_m, w_f) and divorce parameters s.t. there is no agreement. Then if $\tilde{\tau}_m$ increases, then*
(i) the agents cannot switch to an agreement in $\tilde{t}_m$.
(ii) the agents cannot switch to an agreement in $\tilde{t}_m$ and $\tilde{\tau}_m$.

Proof. ***(i) the agents cannot switch to an agreement in $\tilde{t}_m$.***

We start the proof with the following lemma:

Lemma 7. *The choice between having an agreement in $\tilde{t}_m$ and having no agreement at all does not depend on the bargaining power, α.*

Proof: This follows directly from the conditions of having an agreement: the agents refuse to engage in any sort of agreements iff $T_{m,t} < T_{f,t}$ and $T_{m,t,\tau} < T_{f,t,\tau}$, i.e. the maximum price the father is ready to pay is lower than the minimum price the mother is ready to accept. As these prices do not depend on α, then α does not affect the choice of the agents. □

As the type of an agreement does not depend on α, we can assume it to be 1 without loss of generality, i.e. if there is an agreement, the transfer is such that the father is indifferent between the agreement and the outside option.

As the agreement exists if it is a Pareto-improvement of the outside-option, it suffices to show that

$$\frac{\partial(U_f - U_f^0)}{\partial\tilde{\tau}_m} \leq 0$$

As the father in case of agreement is indifferent, this condition will ensure that an increase in paternity leave will not lead to switch to agreement from the outside option.

By definition of the utility function this is equivalent to:

$$\frac{\partial\left(f(n) + \log(c_f) - f(n^0) - \log(c_f^0)\right)}{\partial\tilde{\tau}_m} \leq 0$$

Lemma 8. *If Proposition 8 is true for $a = 1$, then it is also true for any $a \geq 1$.*

Proof:

Outside option number of kids: $n^0 = (t_m^0 + \tau_m^0)^a + (t_f^0 + \tau_f^0)^a$
Number of kids for agreement in t_m: $n = (\tau_m^0)^a + (t_m^0 + t_f^0 + \tau_f^0)^a$

Consider the difference in number of kids under an agreement and in the outside option and take its derivative with respect to τ_m^0:

$$\begin{aligned}\frac{\partial(n - n^0)}{\partial\tau_m^0} &= a((\tau_m^0)^{a-1} - (t_m^0 + \tau_m^0)^{a-1}) + a\frac{\partial t_m^0}{\partial\tau_m^0}((t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (t_m^0 + \tau_m^0)^{a-1}) \\ &+ a\frac{\partial t_f^0}{\partial\tau_m^0}((t_m^0 + t_f^0 + \tau_f^0)^{a-1} - (t_f^0 + \tau_f^0)^{a-1})\end{aligned}$$

Note that $\frac{\partial t_f^0}{\partial\tau_m^0} < 0$ and $\frac{\partial t_m^0}{\partial\tau_m^0} < 0$.

If $a > 1$ than for any $x > 0$, x^{a-1} is an increasing function. Note also that $\tau_f \geq \tau_m$.

Hence, we have

$$(t_m^0 + t_f^0 + \tau_f^0)^{a-1} > (t_f^0 + \tau_f^0)^{a-1}); \quad (t_m^0 + t_f^0 + \tau_f^0)^{a-1} > (t_m^0 + \tau_m^0)^{a-1}); \quad (\tau_m^0)^{a-1} < (t_m^0 + \tau_m^0)^{a-1}$$

Which implies that

$$
\begin{aligned}
&a\frac{\partial t_f^0}{\partial \tau_m^0}((t_m^0+t_f^0+\tau_f^0)^{a-1}-(t_f^0+\tau_f^0)^{a-1})<0\\
&a\frac{\partial t_m^0}{\partial \tau_m^0}((t_m^0+t_f^0+\tau_f^0)^{a-1}-(t_m^0+\tau_m^0)^{a-1})<0\\
&a((\tau_m^0)^{a-1}-(t_m^0+\tau_m^0)^{a-1}-(t_f^0+\tau_f^0)^{a-1})<0
\end{aligned}
$$

Hence, for $a > 1$ the difference in number of kids is declining in τ_m^0:

$$\frac{\partial(n-n^0)}{\partial \tau_m^0}<0$$

Now consider the difference in utilities for $a = 1$:

$$n-n^0=(\tau_m^0)+(t_m^0+t_f^0+\tau_f^0)-(t_m^0+\tau_m^0)-(t_f^0+\tau_f^0)=0$$

Partial derivative of this difference with respect to τ_m^0 is also 0.

Utility of the agent is defined by the number of kids and consumption:

$$U_i^j=\log(c_i^j)+\log(n_i^j)$$

Where $i \in \{m, f\}$ defines whether the agent is father or mother, j defines whenever there is an agreement or the outside option.

The proposition is true for $a = 1$ (as the first derivative wrt τ_m^0 is 0), thus when τ_m increases there is no switch to an agreement.

Denote by 'A' parameters before the increase in τ_m and by 'B' parameters after the increase. So if initially there was no agreement then: $U_f^A \leq U_f^{0A}$, that is $\log(c_f^{0A}) > \log(c_f^A)$ (because $n = n^0$) and there is an agreement after the increase, that is $U_f^B \geq U_f^{0B}$, or $\log(c_f^B) > \log(c_f^{0B})$.

We already know that for $a > 1$: $\frac{\partial(n-n^0)}{\partial \tau_m^0} < 0$ so because τ_m increases when we moving from 'A' to 'B', hence $(n^B - n^A) > (n^{0B} - n^{0A})$.

If there was no agreement before the increase (f is some increasing and mono-

tonic function. Note that this is not logarithm because of the divorce): $f(n1) + \log(c_f 1) < f(n^0 1) + \log(c_f^0 1)$
So we have:

$$f(n^A) + \log(c_{fA}) < f(n^{0A}) + \log(c_{fA}^0); \quad (n^B - n^A) > (n^{0B} - n^{0A})$$

$$\log(c_f^B) > \log(c_f^{0B}); \quad \log(c_f^{0A}) > \log(c_f^A)$$

Hence $\log(n^B) + \log(c_{fB}) < \log(n^{0B}) + \log(c_{f2}^0)$ that is there is no agreement after increase. So we cannot switch to agreement in t_m. This concludes the proof.
$\square$

Hence, we can consider $a = 1$ without loss of generality. Then $n = n^0$ and our condition is equivalent to:

$$\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} \leq 0$$

By definition:

$$c_f = (w_f(1 - t_f - t_m)(1 - t_f - \tau_f - t_m) + T)^\beta (w_f(1 - t_f - t_m)(1 - t_f - \tau_f^0 - t_m))^{1-\beta}$$

$$c_f^0 = w_f(1 - t_f)(1 - t_f - \tau_f)$$

T is defined from the fact that father is indifferent between agreement and no agreement:

$$U_m = \log(c_m) + f(n) = \log(c_m^0) + f(n^0)$$

Because $n = n^0$ and by definition of consumption of father it is equivalent to:

$$w_m(1 - t_m)(1 - \tau_m - t_m) = w_m(1 - \tau_m) - T$$

Then:

$$\log c_f - \log c_f^0 = \beta \log(w_f(1 - t_f - t_m)(1 - t_f - \tau_f - t_m) - w_m(1 - t_m)(1 - \tau_m - t_m) +$$
$$+ w_m(1 - \tau_m)) + (1 - \beta)\log(w_f(1 - t_f - t_m)(1 - t_f - \tau_f^0 - t_m)) - \log(w_f(1 - t_f)(1 - t_f - \tau_f^0))$$

Let us take the derivative and normalize the wage of mother to 1 (by definition $t'_m = \frac{\partial t_m}{\partial \tau_m}$, $t'_f = \frac{\partial t_f}{\partial \tau_m}$):

$$\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} = (\beta)((1-t_f-t_m)(1-t_f-\tau_f-t_m) - w_m(1-t_m)(1-\tau_m-t_m)$$
$$+ w_m(1-\tau_m))^\beta + ((1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{-1}\left(-w_m + w_m(1-\tau_m-t_m)t_m' + \right.$$
$$\left. + w_m(1-t_m)(1+t_m') + \frac{\partial((1-t_f-t_m)(1-t_f-\tau_f-t_m))}{\partial \tau_m}\right)$$
$$- \frac{\partial((1-\beta)\log(w_f(1-t_f-t_m)(1-t_f-\tau_f^0-t_m)) - \log(w_f(1-t_f)(1-t_f-\tau_f^0)))}{\partial \tau_m}$$

Note that:

$$-1 + (1-\tau_m-t_m)t_m' + (1-t_m)(1+t_m') = 2(1-t_m)t_m' - t_m - \tau_m t_m' < 0$$

So $(-w_m + w_m(1-\tau_m-t_m)t_m' + w_m(1-t_m)(1+t_m'))$ is decreasing in w_m.
Also:

$$(1-t_m)(1-\tau_m-t_m) < (1-\tau_m)$$

So $((1-t_f-t_m)(1-t_f-\tau_f-t_m) - w_m(1-t_m)(1-\tau_m-t_m) + w_m(1-\tau_m))^\beta + ((1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{-\beta}$ is also decreasing in w_m (because $0 < \beta < 1$).
Thus, we have that $\frac{\partial(c_f - c_f^0)}{\partial \tau_m}$ is decreasing in w_m. So if we show that $\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} < 0$ for $w_m = 1$, that is the minimum wage we can have (because $1 = w_f \leq w_m$), then for any wage $w_m > 1$ we also have $\frac{\partial(c_f - c_f^0)}{\partial \tau_m} < 0$.
So to prove our statement it is sufficient to show that:

$$\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} \leq 0$$

if $c_f = c_f^0$ and $w_m = w_f = 1$
Up to now we have:

$$((1-t_f-t_m)(1-t_f-\tau_f-t_m) - (1-t_m)(1-\tau_m-t_m)$$

$$+(1-\tau_m))^\beta((1-t_f-t_m)(1-t_f-\tau_f^0-t_m))^{1-\beta} - ((1-t_f)(1-t_f-\tau_f^0)) = 0$$

From this condition we find β and put it into the equation $\frac{\partial(\log c_f - \log c_f^0)}{\partial \tau_m} \leq 0$. After taking the derivative we receive the following:

$$-(1-\log\left(\frac{(1-\tau_f)(1-\tau_f-t_f)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)})\right)$$

$$\frac{t'_m(1-\tau_f-t_f-t_m)-(1-\tau_f-t_m)(t'_f+t'_m)+1-(1-\tau_m-t_m)-(1-\tau_m)(1-t'_m)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)+(1-\tau_m)-(1-\tau_m)(1-\tau_m-t_m)}-$$

$$-\log\left(\frac{(1-\tau_f)(1-\tau_f-t_f)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)}\right)$$

$$\frac{t'_m(1-\tau_f-t_f-t_m)-(1-\tau_f-t_m)(t'_f+t'_m)}{(1-\tau_f-t_m)(1-\tau_f-t_f-t_m)}-\frac{(1-\tau_f)t'_f}{(1-\tau_f)(1-\tau_f-t_f)}\geq 0 \tag{1}$$

If we can show that (1) is true then we are done with the prove. But for that we have to find t_m, t_f, t'_m, t'_f. We find them in the outside option:

$$\max_{t_m}(1-t_m)(1-\tau_m-t_m)(t_m+\tau_m+t_f+\tau_f)$$

$$\max_{t_m}(1-t_f)(1-\tau_f-t_f)(t_m+\tau_m+t_f+\tau_f)$$

Taking the first order conditions and rewriting them as:

$$15t_m^4+(38\tau_m+8\tau_f-28)t_m^3+(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)t_m^2+$$
$$(10\tau_m^3+7\tau_m^2\tau_f-36\tau_m^2-20\tau_m\tau_f+18\tau_m+8\tau_f+4)t_m+\tau_m^4+\tau_m^3\tau_f-6\tau_m^3-5\tau_m^2\tau_f+8\tau_m^2+$$
$$6\tau_m\tau_f+2\tau_m-1=0$$

$$15t_f^4+(38\tau_f+8\tau_m-28)t_f^3+(32\tau_f^2+14\tau_f\tau_m-58\tau_f-16\tau_m+10)t_f^2+$$
$$(10\tau_f^3+7\tau_f^2\tau_m-36\tau_f^2-20\tau_f\tau_m+18\tau_f+8\tau_m+4)t_f+\tau_f^4+\tau_f^3\tau_m-6\tau_f^3-5\tau_f^2\tau_m+8\tau_f^2+$$
$$6\tau_f\tau_m+2\tau_f-1=0$$

We can solve these for t_m by the following algorithm:

$$a1=-\frac{(38\tau_m+8\tau_f-28)^2}{600}+\frac{(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)}{15}$$

$$b1=\frac{(38\tau_m+8\tau_f-28)^3}{27000}-\frac{(38\tau_m+8\tau_f-28)(32\tau_m^2+14\tau_m\tau_f-58\tau_m-16\tau_f+10)}{450}+$$
$$\frac{(10\tau_m^3+7\tau_m^2\tau_f-36\tau_m^2-20\tau_m\tau_f+18\tau_m+8\tau_f+4)}{15}$$

$$g1 = -\frac{(38\tau_m + 8\tau_f - 28)^4}{4320000} + \frac{(38\tau_m + 8\tau_f - 28)^2(32\tau_m^2 + 14\tau_m\tau_f - 58\tau_m - 16\tau_f + 10)}{27000}$$
$$-\frac{(38\tau_m + 8\tau_f - 28)(10\tau_m^3 + 7\tau_m^2\tau_f - 36\tau_m^2 - 20\tau_m\tau_f + 18\tau_m + 8\tau_f + 4)}{900} +$$
$$\frac{\tau_m^4 + \tau_m^3\tau_f - 6\tau_m^3 - 5\tau_m^2\tau_f + 8\tau_m^2 + 6\tau_m\tau_f + 2\tau_m - 1}{15}$$
$$p = -\frac{a1^2}{12} - g1$$
$$q = -\frac{a1^3}{108} + \frac{a1g1}{3} - \frac{b1^2}{8}$$
$$r = -\frac{q}{2} - (\frac{q^2}{4} + \frac{p^3}{27})^{\frac{1}{2}}$$

$u = r^{\frac{1}{3}}$ Note that here any root is suitable(they give the same results).

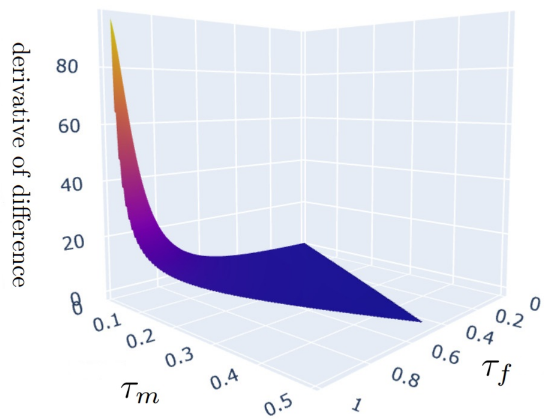
$$y = -\frac{5a1}{6} + u - \frac{p}{3u}$$
$$w = (a1 + 2y)^{\frac{1}{2}}$$
$$t_m = -\frac{(38\tau_m + 8\tau_f - 28)}{120} + \frac{(\pm w \pm (-(3a1 + 2y + \frac{2b1}{w}))^{0.5})}{2}$$

Here we have 4 roots we take the root that is real. If it is less then 0, $t_m = 0$. The solution for t_f is the same the only change is that instead of τ_m we have τ_f and instead of τ_f we have τ_m (because of symmetry). So up to now we find the t_m and t_f and the only unknowns are t'_m and t'_f.
If $t_m = 0$ then $t'_m = 0$ and if $t_f = 0$ then $t'_f = 0$ (because unbounded t_m and t_f is continuous). If t_m and t_f are not 0 by implicit derivative theorem:

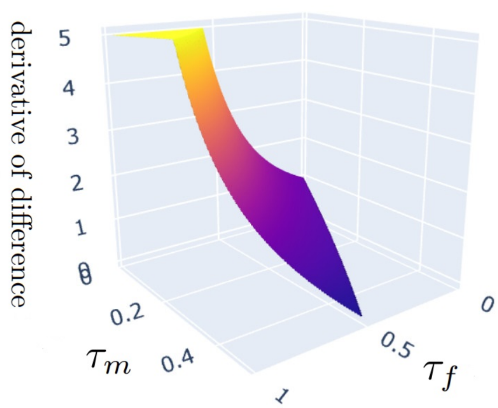
$$t'_m = \frac{38t_m^3 + (-58 + 64\tau_m + 14\tau_f)t_m^2 + 2(9 + 15\tau_m^2 - 10\tau_f + \tau_m(-36 + 7\tau_f))t_m + 2 + 4\tau_m^3 + 3\tau_m^2(-6 + \tau_f) + 6\tau_f - 2\tau_m(-8 + 5\tau_f)}{60t_m^3 + 3(38\tau_m + 8\tau_f - 28)t_m^2 + 2(32\tau_m^2 + 14\tau_m\tau_f - 58\tau_m - 16\tau_f + 10)t_m + 10\tau_m^3 + 7\tau_m^2\tau_f - 36\tau_m^2 - 20\tau_m\tau_f + 18\tau_m + 8\tau_f + 4}$$

$$t'_f = \frac{8t_f^3 + 2(-8 + 7\tau_f)t_f^2 + (8 - 20\tau_f + 7\tau_f^2)t_f + \tau_f(6 - 5\tau_f + \tau_f^2)}{60t_f^3 + 3(38\tau_f + 8\tau_m - 28)t_f^2 + 2(32\tau_f^2 + 14\tau_f\tau_m - 58\tau_f - 16\tau_m + 10)t_f + 10\tau_f^3 + 7\tau_f^2\tau_m - 36\tau_f^2 - 20\tau_f\tau_m + 18\tau_f + 8\tau_m + 4}$$

So given τ_m and τ_f we found t_m, t_f and then found t'_m and t'_f which can be plugged in (1). Then the plot is as follows:



The same plot but where all values that are more than 5 are set to 5:



It can be seen that for any values of τ_m and τ_f (1) holds. This concludes the proof.

(ii) the agents cannot switch to an agreement in t_m and τ_m.

According to Lemma 6, α does not affect the regime, so we can set it to 1 (mother has full bargaining power). So if there is agreement, father is always indifferent between agreement and no agreement.

Now consider utility of the father in case of agreement, U_m, and utility of mother in case of agreement, U_f; utility of father in case of no agreement, U_m^0, and utility of mother in case of no agreement, U_f^0. The agents have an agreement if the utility from agreement for both agents is more or equal than the utility from no agreement.

The statement from the proposition is equivalent to the following "statement 2": if there is no agreement for some values of parameters, the agreement for the same parameters except bigger τ_m is impossible.

Note that to prove the statement 2 it is sufficient to show: if there exist some set of parameters such that $U_f = U_f^0$, then for that set $\frac{\partial(U_f - U_f^0)}{\partial \tau_m} \leq 0$.

If there is no such set when τ_m increases there is no change in the regime of the agreement because of the continuity of the utilities in τ_m. More strictly, if there is a change in regime, that is for some τ_{m1} there is a agreement ($U_f > U_f^0$), and for another τ_{m2} there is no agreement ($U_f < U_f^0$), then there exists some $\tau_m \in (\tau_{m1}, \tau_{m2})$ such that $U_f = U_f^0$ by the continuity theorem. Note that here we need U_f and U_f^0 to be continuous in τ_m (that is obviously true). By the same theorem, if $\tau_{m1} > \tau_{m2}$ (there is a switch from agreement to no agreement), then there exist τ_m such that $U_f = U_f^0$ and $\frac{\partial(U_f - U_f^0)}{\partial \tau_m} > 0$. So under "statement 2", the switch from no agreement to agreement is impossible.

Hence, it remains to prove the following:

$$\frac{\partial(U_f - U_f^0)}{\partial \tau_m} < 0$$

if $U_f = U_f^0$.

Denote c_f the consumption of the mother under agreement, c_f^0 — consumption of mother under no agreement, n number of kids under agreement, n^0 number of kids without agreement.

In the outside option the maximization is as follows:

$$\max_{t_f, t_m} (1 - t_f - \tau_f)(1 - t_f)(1 - t_m - \tau_m)(1 - t_m)((t_f + \tau_f)^a + (t_m + \tau_m)^a)$$

The first order conditions for this maximization problem are:

$$\frac{1}{1 - t_f - \tau_f} + \frac{1}{1 - t_f} = \frac{a(t_f + \tau_f)^{a-1}}{(t_f + \tau_f)^a + (t_m + \tau_m)^a}$$

$$\frac{1}{1 - t_m - \tau_m} + \frac{1}{1 - t_m} = \frac{a(t_m + \tau_m)^{a-1}}{(t_f + \tau_f)^a + (t_m + \tau_m)^a}$$

Solving the two equations numerically, t_m and t_f can be obtained. Let us find the derivative of the t_m and t_f with respect to τ_m. Denote $t'_m = \frac{\partial t_m}{\partial \tau_m}$ and $t'_f = \frac{\partial t_f}{\partial \tau_m}$. Taking the derivative of first order conditions with respect to τ_m, we get:

$$\frac{t'_f}{(1-t_f-\tau_f)^2} + \frac{t'_f}{(1-t_f)^2} =$$
$$= \frac{t'_f a(a-1)(t_f+\tau_f)^{a-2}}{(t_f+\tau_f)^a + (t_m+\tau_m)^a} - \frac{a^2(t_f+\tau_f)^{a-1}(t'_f(t_f+\tau_f)^{a-1} + (t'_m+1)(\tau_m+t_m)^{a-1})}{((t_f+\tau_f)^a + (t_m+\tau_m)^a)^2}$$
$$\frac{t'_m+1}{(1-t_m-\tau_m)^2} + \frac{t'_m}{(1-t_m)^2} =$$
$$= \frac{(t'_m+1)a(a-1)(t_m+\tau_m)^{a-2}}{(t_f+\tau_f)^a + (t_m+\tau_m)^a} - \frac{a^2(t_m+\tau_m)^{a-1}(t'_f(t_f+\tau_f)^{a-1} + (t'_m+1)(\tau_m+t_m)^{a-1})}{((t_f+\tau_f)^a + (t_m+\tau_m)^a)^2}$$

Note that these derivatives are linear in t'_m and t'_f. To find t'_m and t'_f, denote:

$$x = \frac{1}{(1-t_f-\tau_f)^2} + \frac{1}{(1-t_f)^2} - \frac{a(a-1)(t_f+\tau_f)^{a-2}}{(t_f+\tau_f)^a + (t_m+\tau_m)^a} + \frac{a^2(t_f+\tau_f)^{2a-2}}{((t_f+\tau_f)^a + (t_m+\tau_m)^a)^2}$$

$$y = \frac{a^2(t_f+\tau_f)^{a-1}(\tau_m+t_m)^{a-1}}{((t_f+\tau_f)^a + (t_m+\tau_m)^a)^2}$$

$$z = \frac{1}{(1-t_m-\tau_m)^2} + \frac{1}{(1-t_m)^2} - \frac{a(a-1)(t_m+\tau_m)^{a-2}}{(t_f+\tau_f)^a + (t_m+\tau_m)^a} + \frac{a^2(t_m+\tau_m)^{2a-2}}{((t_f+\tau_f)^a + (t_m+\tau_m)^a)^2}$$

$$f = \frac{1}{(1-t_m-\tau_m)^2} - \frac{a(a-1)(t_m+\tau_m)^{a-2}}{(t_f+\tau_f)^a + (t_m+\tau_m)^a} + \frac{a^2(t_m+\tau_m)^{2a-2}}{((t_f+\tau_f)^a + (t_m+\tau_m)^a)^2}$$

Rewriting the conditions for t'_m and t'_f as: $xt'_f + yt'_m + y = 0$ and $yt'_f + zt'_m + f = 0$ and solving them, we obtain:

$$t'_m = \frac{fx-y^2}{y^2-zx}; \quad t'_f = \frac{zy-fy}{y^2-zx}$$

Also note that if $t_m = 0$, then $t'_m = 0$ and if $t_f = 0$, then $t'_f = 0$ (no change for a small increase in τ_m). Hence, t'_m and t'_f are determined by the equations above.
Let us now return to the derivative of the difference in utility.

$$\frac{\partial(U_f - U_f^0)}{\partial \tau_m} < 0$$

Note that by definition of U_f and U_f^0:

$$\frac{\partial(U_f - U_f^0)}{\partial \tau_m} = \frac{\partial((c_f + T)^c (c_f)^{1-c} n - c_f^0 n^0)}{\partial \tau_m} = \frac{\partial((1 + \frac{T}{c_f})^c c_f n - c_f^0 n^0)}{\partial \tau_m}$$

Taking the derivative, we obtain:

$$\frac{\partial(c_f n)}{\partial \tau_m}(1 + \frac{T}{c_f})^c - \frac{\partial(c_f^0 n^0)}{\partial \tau_m} + c_f n c \left(1 + \frac{T}{c_f}\right)^{c-1} \frac{\partial(\frac{T}{c_f})}{\partial \tau_m} < 0$$

This equation is equivalent to:

$$\frac{\partial(\frac{c_f}{w_f} n)}{\partial \tau_m}(1 + \frac{T}{c_f})^c - \frac{\partial(\frac{c_f^0}{w_f} n^0)}{\partial \tau_m} + \frac{c_f}{w_f} n c (1 + \frac{T}{c_f})^{c-1} \frac{\partial(\frac{T}{c_f})}{\partial \tau_m} < 0 \tag{1}$$

c is such that mother is indifferent between agreement and no agreement. So $U_f = U_f^0$. From it we found that:

$$c = \frac{\log(\frac{c_f^0 n^0}{c_f n})}{\log(1 + \frac{T}{c_f})}$$

The value of T is such that father is indifferent between agreement and no agreement:

$$(c_m - T) n = c_m^0 n^0$$

Plugging c and T in the equation (1):

$$\frac{\partial(\frac{c_f}{w_f}n)}{\partial\tau_m}\frac{c_f^0 n^0}{c_f n} - \frac{\partial(\frac{c_f^0}{w_f}n^0)}{\partial\tau_m} + \frac{c_f^0}{w_f}n^0 \frac{\log(\frac{c_f^0 n^0}{c_f n})}{\log(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})^{-1}\frac{\partial(\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}{\partial\tau_m} < 0 \tag{3}$$

Let us now find the relationship between the left hand side of (3) and w_f or w_m. Note that $\frac{c_f}{w_f}$, $\frac{c_f^0}{c_f}$, n^0, n do not depend on w_m or w_f. So the only part of (3) that depend on w_f or w_m is

$$\frac{\frac{\partial(\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}{\partial\tau_m}}{\log(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}$$

Note that $\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f}$ is linear in $\frac{w_m}{w_f}$. Let $d = \frac{w_f c_m}{w_m c_f} - \frac{w_f c_m^0 n^0}{w_m n c_f}$. Note that d does not depend on w_f or w_m. Then we can rewrite previous equation as:

$$\frac{w_m}{w_f}\frac{\frac{\partial(a)}{\partial\tau_m}}{\log(1+\frac{w_m}{w_f}a)(1+\frac{w_m}{w_f}a)} \tag{4}$$

Note that $a > 0$ because $a = \frac{w_f T}{w_m c_f}$ and w_m, w_f, T, c_f are positive. Also note that previous equation is monotone in $\frac{w_m}{w_f}$. It can be proved by dividing by constant $\frac{\partial(a)}{\partial\tau_m}$ (it does not depend on $\frac{w_m}{w_f}$) and taking log (monotonic transformation). Then taking a derivative with the respect to $\frac{w_m}{w_f}$, we obtain:

$$\frac{1}{\frac{w_m}{w_f}} + \frac{a}{(1+\frac{w_m}{w_f}a)} + \frac{a}{\log(1+\frac{w_m}{w_f}a)(1+\frac{w_m}{w_f}a)}$$

Note that this equation is positive, motononicity of (4) is proved.
Note also that $\frac{w_m}{w_f} \in [1, +\infty)$. So the minimum and maximum (with the respect to $\frac{w_m}{w_f}$) of (4) and (3) are achieved for either $\frac{w_m}{w_f} = 1$ or $\frac{w_m}{w_f} \to +\infty$.
For $\frac{w_m}{w_f} = 1$ (4) is equal to

$$\frac{\frac{\partial(a)}{\partial\tau_m}}{\log(1+a)(1+a)}$$

For $\frac{w_m}{w_f} \to +\infty$ (4) is equal to 0.
To prove the proposition, we need to prove that (3) is less than 0 for all possible $\frac{w_m}{w_f}$.
Because (4) is linear in (3) it is sufficient to show that (3) is less than 0 for maximum and minimum of (4). Hence, inequality (3) is equivalent to:

$$\frac{\partial(\frac{c_f}{w_f}n)}{\partial\tau_m}\frac{c_f^0 n^0}{c_f n} - \frac{\partial(\frac{c_f^0}{w_f}n^0)}{\partial\tau_m} + \frac{c_f^0}{w_f}n^0 q < 0 \tag{5}$$

where

$$q = \max\{0, \log(\frac{c_f^0 n^0}{c_f n})\frac{\frac{\partial(\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}{\partial\tau_m}}{\log(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})(1+\frac{c_m}{c_f}-\frac{c_m^0 n^0}{nc_f})}\text{if } \frac{w_m}{w_f} = 1\}$$

Note that (5) does not depend on $\frac{w_m}{w_f}$. Using the definition of c_f^0, n^0, n, c_m^0, c_f, c_m, we obtain:

$$\frac{\partial((1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a)}{\partial\tau_m}\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a} -$$
$$-\frac{\partial((1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a))}{\partial\tau_m} + (1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)\max\{0,K_1\} < 0$$

where

$$K_1 = \log\left(\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a}\right)$$
$$\partial\log\left(\log\left(1+\frac{1}{(1-\tau_f-\tau_m-t_m-t_f)}-\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)\right)\Big/\partial\tau_m$$

This inequality comes down to

$$(a(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^{a-1}(1+t'_m+t'_f)-(1+t'_m+t'_f)(\tau_m+t_m+\tau_f+t_f)^a)$$
$$\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a} + t'_f((\tau_m+t_m)^a+(\tau_f+t_f)^a)-(1-\tau_f-t_f)$$
$$a((\tau_m+t_m)^{a-1}(1+t'_m)+t'_f(\tau_f+t_f)^{a-1}) + (1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)\max\{0,K_2\} < 0$$

where

$$K_2 = \log\left(\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(1-\tau_f-\tau_m-t_m-t_f)(\tau_m+t_m+\tau_f+t_f)^a}\right)\left(-\frac{1+t'_m+t'_f}{(1-\tau_f-\tau_m-t_m-t_f)^2}+\right.$$
$$+(1-\tau_m-t_m)((\tau_m+t_m)^a+(\tau_f+t_f)^a)$$
$$\frac{((\tau_m+t_m+\tau_f+t_f)^{a-1}(1+t'_m+t'_f)(1-\tau_f-\tau_m-t_m-t_f))-(\tau_m+t_m+\tau_f+t_f)^a(1+t'_m+t'_f)}{(\tau_m+t_m+\tau_f+t_f)^{2a}(1-\tau_f-\tau_m-t_m-t_f)^2}-$$
$$\left.-\frac{(1-\tau_m-t_m)((\tau_m+t_m)^{a-1}(1+t'_m)+(\tau_f+t_f)^{a-1}t'_f)-(1+t'_m)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)$$
$$\Bigg/\left(\log\left(1+\frac{1}{(1-\tau_f-\tau_m-t_m-t_f)}-\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)\right)$$
$$\left(1+\frac{1}{(1-\tau_f-\tau_m-t_m-t_f)}-\frac{(1-\tau_f-t_f)((\tau_m+t_m)^a+(\tau_f+t_f)^a)}{(\tau_m+t_m+\tau_f+t_f)^a(1-\tau_f-\tau_m-t_m-t_f)}\right)$$

Note that this inequality depends only on a, τ_m, τ_f, t_m, t_f, t'_m, t'_f. We previously showed that t_m, t_f, t'_m, t'_f can be expressed as a function of a, τ_m, τ_f. Recall that we should have $t_m + t_f + \tau_m + \tau_f < 1$. For any given values of a, τ_m, τ_f, it can be shown that the above inequality hold

Hence, for any fixed a, τ_m, τ_f there cannot be a switch from no agreement to agreement in both t_m and τ_m. ■

Proof of proposition 9

Proposition. *A switch from some agreements to no agreement leads to:*
(i) An increase in take-up of paternity leave
(ii) A reduction in fertility
(iii) An increase in the probability to divorce
(iv) An increase in women's employment at the expense of childcare time
(v) An increase in men's childcare time at the expense of their employment

Proof. (i) It was proved earlier that there are only 3 cases:
- no agreement: $\tilde{t}_m = 0,\ \tilde{\tau}_m = 0$
- agreement in $\tilde{t}_m$: $\tilde{t}_m = t^0_m,\ \tilde{\tau}_m = 0$
- agreement in $\tilde{t}_m$ and in $\tilde{\tau}_m$: $\tilde{t}_m = t^0_m,\ \tilde{\tau}_m = \bar{\tau}_m$

Thus, when there is a switch from some agreement, that is agreement in $\tilde{t}_m$ or agreement in $\tilde{t}_m$ and in $\tilde{\tau}_m$ to no agreement, $\tilde{\tau}_m$ decreases or does not change. Since the take-up of paternity leave is $\bar{\tau}_m - \tilde{\tau}_m$ a switch from some agreements to no

agreement leads to its increase.
(ii) By defenition the number of kids is: $n = (t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^a + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^a$
take a derivative of number of kids with the respect to traded time and with the respect to traded leave:

$$\frac{\partial n}{\partial \tilde{t}_m} = a(-(t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^{a-1} + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^{a-1})$$

$$\frac{\partial n}{\partial \tilde{\tau}_m} = a(-(t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m)^{a-1} + (t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m)^{a-1})$$

From $\tau_m < \tau_f$, $t_m^0 \leq t_f^0$ and $\tilde{t}_m \geq 0$ follows that:

$$t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m < t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m$$

Thus, because $a > 1$, $\frac{\partial n}{\partial \tilde{t}_m} > 0$ and $\frac{\partial n}{\partial \tilde{\tau}_m} > 0$. So if $\tilde{t}_m$ or $\tilde{\tau}_m$ decreases the number of kids decreases.
When there is a switch from some agreement, that is agreement in $\tilde{t}_m$ or agreement in $\tilde{t}_m$ and in $\tilde{\tau}_m$ to no agreement t_m^0, t_f^0, τ_f^0 and τ_m^0 does not change while $\tilde{t}_m$ and $\tilde{\tau}_m$ decreases. Thus, number of kids decreases.
(iii) The probability of divorce is

$$\mathrm{p}_d = \frac{x_1 - (d-1)\log n}{x_1 - x_2}$$

Because $d < 1$ and $x_2 > x_1$: $\frac{\partial p_d}{\partial n} = \dfrac{(1-d)}{(x_1 - x_2)n} < 0$
When there is a switch from some agreement to no agreement x_1, x_2, d does not change while n decreases. Thus, the probability of divorce, p_d, increases.
(iv) women's childcare time is $t_f^0 + \tau_f^0 + \tilde{t}_m + \tilde{\tau}_m$ while women's employment is $1 - t_f^0 - \tau_f^0 - \tilde{t}_m - \tilde{\tau}_m$.
When there is a switch from some agreement to no agreement t_f^0, τ_f^0 does not change while $\tilde{t}_m + \tilde{\tau}_m$ decreases. Thus, women's childcare time decreases and women's employment increases.
(v) men's childcare time is $t_m^0 + \tau_m^0 - \tilde{t}_m - \tilde{\tau}_m$ while men's employment is $1 - t_m^0 - \tau_m^0 + \tilde{t}_m + \tilde{\tau}_m$.

When there is a switch from some agreement to no agreement t_m^0, τ_m^0 does not change while $\tilde{t}_m + \tilde{\tau}_m$ decreases. Thus, men's childcare time increases and men's employment decreases.

■

Proposition 10. *It does not matter who proposes the agreement. If the mother does, then T, $\tilde{t}_m$ and $\tilde{\tau}_m$ are equal to the case when the father does propose the agreement.*

Proof. If the mother proposes an agreement, the optimal $\tilde{t}_m$ and $\tilde{\tau}_m$ are defined as:

$$\max_{\tilde{t}_m, \tilde{\tau}_m} \mathbb{E}(U_f(\tilde{t}_m, \tilde{\tau}_m, T))$$

$$T = \alpha T_m + (1-\alpha)T_f; \quad \mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T_m) = \mathbb{E}U_m^0; \quad \mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T_f) = \mathbb{E}U_f^0; \quad T_m > T_f$$

Now we will prove that the optimal $\tilde{t}_m$ and $\tilde{\tau}_m$ from this optimization coincide with the one where the father proposes.

Lemma 1 holds for this problem (the prove is the same but instead of father utility we have mother utility). Thus the agreement exists if and only if there exist $\tilde{t}_m, \tilde{\tau}_m, T$ such that having non-zero traded time is profitable:

$$\mathbb{E}U_f(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_f^0; \quad \mathbb{E}U_m(\tilde{t}_m, \tilde{\tau}_m, T) \geq \mathbb{E}U_m^0$$

It was proved earlier that there are only 3 cases:
- no agreement: $\tilde{t}_m = 0,\ \tilde{\tau}_m = 0$
- agreement in $\tilde{t}_m$: $\tilde{t}_m = t_m^0,\ \tilde{\tau}_m = 0$
- agreement in $\tilde{t}_m$ and in $\tilde{\tau}_m$: $\tilde{t}_m = t_m^0,\ \tilde{\tau}_m = \bar{\tau}_m$

Thus, for the specified mother's problem there is an agreement with the same parameters as for the father's problem. Thus, the values of $\tilde{t}_m$ and $\tilde{\tau}_m$ are intact. ■

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TABLES

TABLE I: Effect of paternity leave introduction on total leave length

	Total leave days		
	(1)	(2)	(3)
		Full sample	
Total leave in days	8.185***	9.594***	6.509***
	(1.464)	(2.514)	(2.260)
Mean control	7.650	8.351	8.535
N	459	595	724
		Egalitarian couples	
Total leave in days	3.714	3.931*	0.526
	(2.247)	(2.168)	(3.772)
Mean control	8.494	8.413	8.052
N	231	289	352
		Intermediate couples	
Total leave in days	24.90***	27.25***	21.99***
	(6.079)	(6.606)	(5.690)
Mean control	7.373	8.806	9.608
N	96	133	163
		High wage gap couples	
Total leave in days	4.132	9.051*	7.019**
	(2.521)	(4.969)	(3.133)
Mean	9.811	10.41	11.20
N	132	173	209
Bandwidth	12 months	15 months	18 months

Note: Each coefficient comes from a different regression. Robust standard errors are shown in parentheses. The dependent variable is the number of days off taken by the father after the birth of the reference child (including paternity leave, vacation days, etc). The main explanatory variable is an indicator for paternity leave eligibility (reference child born after March 23, 2007). We always control for a linear trend in month of birth that is allowed to change after the threshold. The data source is the Madrid Survey. The sample includes fathers who had a child 12 to 18 months before or after March 2007. Controls include a dummy for first births, marital status of the mother, and mother's and father's age at birth, immigrant and labor market status, and educational attainment. *** p<0.01, ** p<0.05, * p<0.1

TABLE II: Effect of paternity leave introduction on subsequent fertility

	Two years		Four years		Six years	
	(1)	(2)	(3)	(4)	(5)	(6)
			Full sample			
Subseq. fertility	-0.00431	-0.00592*	-0.000275	-0.0116*	0.00177	-0.0155*
	(0.00299)	(0.00355)	(0.00467)	(0.00604)	(0.00591)	(0.00804)
Mean control	0.0620	0.0628	0.226	0.235	0.324	0.337
N	126,051	182,305	126,051	182,305	126,051	182,305
			Egalitarian couples			
Subseq. fertility	-0.00383	-0.00658	0.00668	-0.00844	0.00725	-0.0119
	(0.00333)	(0.00401)	(0.00670)	(0.00851)	(0.00816)	(0.0106)
Mean control	0.0633	0.0635	0.232	0.240	0.329	0.343
N	55,269	79,860	55,269	79,860	55,269	79,860
			Intermediate couples			
Subseq. fertility	-0.0141**	-0.0156**	-0.0214**	-0.0344**	-0.0246**	-0.0394**
	(0.00574)	(0.00678)	(0.0107)	(0.0139)	(0.0122)	(0.0164)
Mean control	0.0632	0.0659	0.244	0.256	0.352	0.370
N	29,023	41,984	29,023	41,984	29,023	41,984
			High wage gap couples			
Subseq. fertility	0.00185	0.00174	0.00506	-9.57e-07	0.00640	-0.00928
	(0.00497)	(0.00598)	(0.00850)	(0.0110)	(0.0110)	(0.0145)
Mean control	0.0595	0.0596	0.207	0.214	0.296	0.308
N	41,759	60,461	41,759	60,461	41,759	60,461
Bandwidth	8 weeks	12 weeks	8 weeks	12 weeks	8 weeks	12 weeks
Linear trend	Y	Y	Y	Y	Y	Y
Quadratic trend	N	Y	N	Y	N	Y

Each coefficient comes from a different regression. Robust standard errors are shown in parentheses. The dependent variable is an indicator for whether a woman had another child within 2 to 6 years of the date of birth of the reference child. The main explanatory variable is an indicator for paternity leave eligibility (reference child born after March 23, 2007). The data source is Spanish birth certificates (Spanish Statistical Institute), 2006-2013. The sample includes women who had a child 8 to 12 weeks before or after March 2007. Controls include fixed effects for day of the week (of birth). *** p<0.01, ** p<0.05, * p<0.1

TABLE III: Effect of paternity leave introduction on the time use by fathers

	Childcare		**Housework**		**Paid work**	
	(1)	(2)	(3)	(4)	(5)	(6)
			Full sample			
Minutes (per day)	26.03	17.05	15.37	8.171	-13.12	-28.75
	(20.18)	(16.05)	(20.99)	(17.61)	(41.51)	(33.75)
Mean control	97.08	94.42	107.4	105.8	258.1	264.7
Mean (mothers)	178.5	183.3	213.5	215.0	140.1	139.2
Observations	525	730	525	730	525	730
			Egalitarian couples			
Minutes (per day)	15.50	-0.481	5.574	-1.575	-24.68	-43.93
	(35.94)	(26.59)	(32.28)	(27.00)	(62.82)	(51.83)
Mean control	104.8	97.25	102.8	104.7	269.2	273.2
Mean (mothers)	185.5	186.4	203.8	205.6	143.5	146.6
Observations	241	345	241	345	241	345
			Intermediate couples			
Minutes (per day)	90.71**	71.36**	102.6**	68.78*	-63.11	-83.86
	(36.57)	(31.48)	(49.30)	(40.57)	(78.72)	(71.37)
Mean control	79.62	90.72	101.5	104.8	277.3	259.0
Mean (mothers)	170.3	175.0	207.3	210.4	141.5	136.0
Observations	117	160	117	160	117	160
			High wage gap couples			
Minutes (per day)	27.54	10.35	-11.82	-13.72	64.51	49.63
	(31.92)	(27.44)	(37.55)	(32.72)	(78.11)	(61.83)
Mean control	98.57	93.19	115.9	107.8	235.1	258.0
Mean (mothers)	175.3	184.0	226.9	227.8	135.8	132.8
Observations	164	224	164	224	164	224
Bandwidth	20 months	28 months	20 months	28 months	20 months	28 months

Each coefficient comes from a different regression. Robust standard errors are shown in parentheses. The dependent variable is the number of minutes per day that a father devotes to childcare, housework or paid work. The main explanatory variable is an indicator for paternity leave eligibility (reference child born after March 2007). We always control for a linear trend in month of birth that is allowed to change after the threshold. The data source is the Spanish Time-Use Survey of 2009-10. The sample includes men living in a couple at the time of the survey who had a child 20 to 28 months before or after March 2007. Controls include gender of the child, a dummy for first births, marital status of the mother, an indicator for weekdays (vs. weekend), as well as age, educational attainment, and migrant status of mother and father. *** p<0.01, ** p<0.05, * p<0.1

TABLE IV: Effect of paternity leave introduction on mother's employment

	Employment after 12 months		**Employment after 24 months**		**Accum. earnings (24 months)**	
	(1)	(2)	(1)	(2)	(1)	(2)
			Full sample			
Labor outcome	0.025***	0.031**	0.013	0.029**	335.82**	334.13
	(0.009)	(0.013)	(0.009)	(0.014)	(156.15)	(226.63)
Mean control	0.602	0.605	0.638	0.651	11,500	11,526
Observations	7,548	15,636	7,548	15,636	7,653	15,885
			Egalitarian couples			
Labor outcome	0.035*	0.049*	0.003	0.016	406.61	234.07
	(0.019)	(0.028)	(0.018)	(0.026)	(539.95)	(778.84)
Mean control	0.804	0.810	0.847	0.848	19,608	19,405
Observations	1,311	2,561	1,311	2,561	1,049	2,234
			Intermediate couples			
Labor outcome	0.065**	0.088**	0.079***	0.092**	1,254.21**	1,755.61**
	(0.030)	(0.042)	(0.026)	(0.040)	(608.92)	(881.78)
Mean control	0.770	0.786	0.811	0.814	18,663	18,655
Observations	595	1,225	595	1,225	704	1,352
			High wage gap couples			
Labor outcome	0.024	0.032	0.001	0.024	254.55	338.99
	(0.016)	(0.023)	(0.016)	(0.023)	(222.56)	(318.71)
Mean control	0.542	0.542	0.577	0.595	9,536	9,443
Observations	3,108	6,531	3,108	6,531	3,241	6,690
Bandwidth	3 months	6 months	3 months	6 months	3 months	6 months
Linear trend	N	Y	N	Y	N	Y
Quadratic trend	N	N	N	N	N	N

Each coefficient comes from a different regression. Robust standard errors are shown in parentheses. The dependent variables are: An indicator for maternal employment 12 (24) months after the birth of the reference child, and her accumulated earnings over the 24 months following the birth of the reference child. The main explanatory variable is an indicator for paternity leave eligibility (reference child born after March 2007). The data source is Spanish Social Security administrative data (2015 sample). The sample includes women who had a child 3 to 6 months before or after March 2007. Controls include age and educational attainment of the mother, birth order, and mother's employment status 3 months before the birth of the reference child. *** p<0.01, ** p<0.05, * p<0.1

TABLE V: Effect of paternity leave introduction on parents' separation

	Schooling			**Schooling & Age**		
	(1)	(2)	(3)	(1)	(2)	(3)
			Full sample			
Separation	0.00828	0.00460	0.00345	0.00828	0.00460	0.00345
	(0.00933)	(0.0116)	(0.00885)	(0.00933)	(0.0116)	(0.00885)
Mean control	0.0703	0.0795	0.0858	0.0703	0.0795	0.0858
Observations	3,006	9,168	15,471	3,006	9,168	15,471
			Egalitarian couples			
Separation	-0.0318***	-0.0284*	-0.0116	-0.0327***	-0.0176	-0.0117
	(0.0109)	(0.0150)	(0.0112)	(0.0103)	(0.0146)	(0.0111)
Mean control	0.126	0.129	0.124	0.0442	0.0479	0.0558
Observations	994	2,857	4,680	1,107	3,180	5,222
			Intermediate couples			
Separation	0.0451***	0.0581***	0.0298**	0.0473***	0.0388**	0.0286**
	(0.0147)	(0.0178)	(0.0140)	(0.0163)	(0.0187)	(0.0145)
Mean control	0.0425	0.0650	0.0710	0.0455	0.0594	0.0692
Observations	1,043	3,166	5,282	956	2,927	4,916
			High wage gap couples			
Separation	0.00708	-0.0170	-0.0130	0.0171	-0.00754	-0.00656
	(0.0208)	(0.0255)	(0.0189)	(0.0214)	(0.0259)	(0.0193)
Mean control	0.0447	0.0411	0.0546	0.126	0.131	0.128
Observations	969	3,145	5,509	943	3,061	5,333
Bandwidth	1 month	3 months	5 months	1 month	3 months	5 months

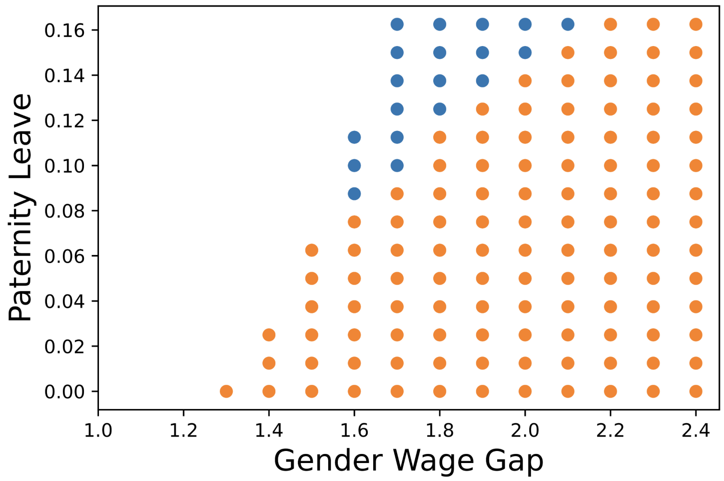
Each coefficient comes from a different regression. Robust standard errors are shown in parentheses. The dependent variable is an indicator for maternal separation (the mother not residing with a partner at the time of the survey). The main explanatory variable is an indicator for paternity leave eligibility (reference child born after March 2007). The data source is the Spanish Labor Force Survey (2008-10). The sample includes women who had a child 1 to 5 months before or after March 2007. Controls include age and education level of the mother, and quarter fixed effects.
*** p<0.01, ** p<0.05, * p<0.1

Figures

Figure I: The Sequence of Events

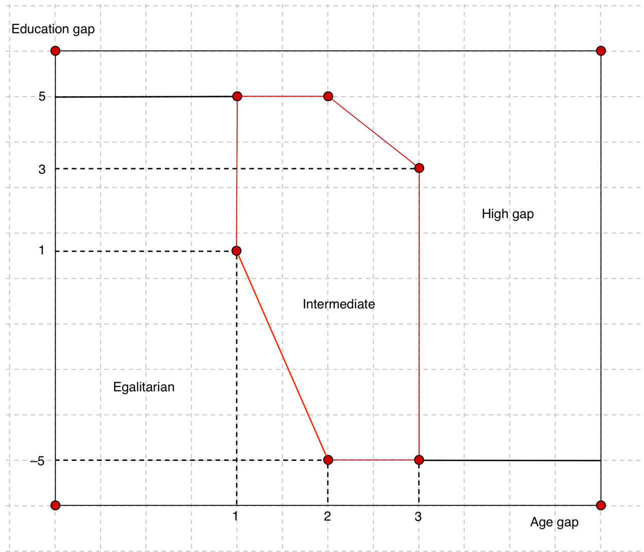
Stage 1	Stage 2
Allocating Time (Non-cooperatively)	Match Quality (θ)
Trading Time-Consumption	Divorce
(Avoiding Paternity Leave)	Consumption Redistribution

Figure II: A Numerical Example



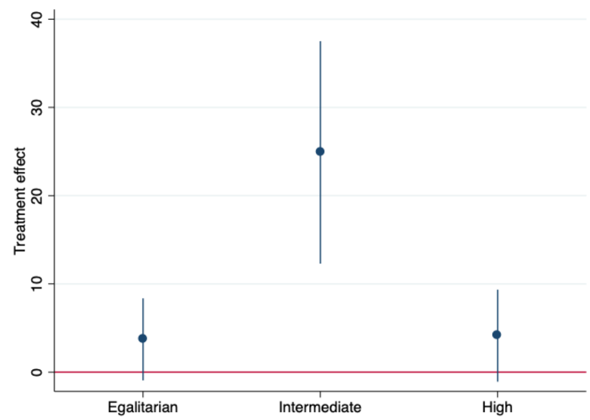
Note: The Figures depicts the gender wage gap on the x-axis and the paternity leave policy as a fraction of fathers' unit time on the y-axis. The parameters' values are $x_1 = -x_2 = 0.5$, $d = 0.7$, $\bar{\tau}_f = 0.2$, $\beta = 0.55$, $a = 1.1$.

Figure III: Endogenous classification of couples based on age and education gap



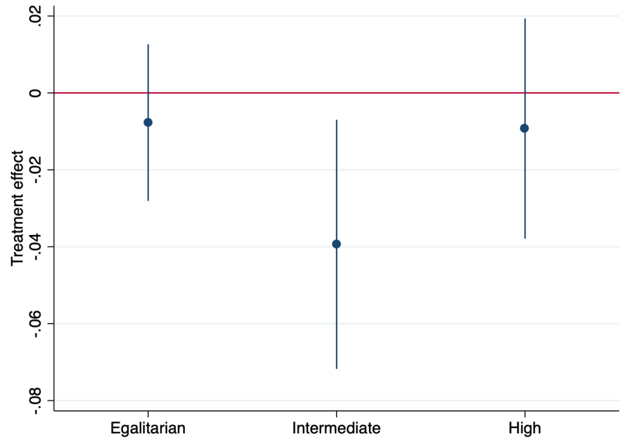
Note: The age and education gap between the partners are measured in years. The figure shows our classification of couples into groups, based on our calibration exercise using the predictions of the model regarding take-up of paternity leave.

Figure IV: Effect of of paternity leave eligibility on total time off by fathers in days



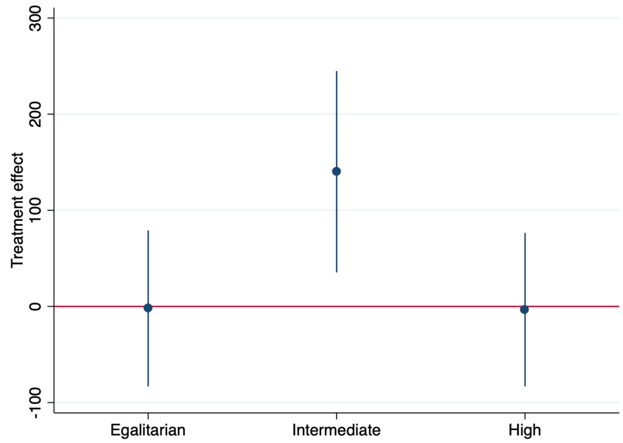
Note: The figure shows the coefficients and 95% confidence intervals from three separate RD regressions that estimate the effect of the introduction of paternity leave on days off taken by fathers, for our three groups of families (based on the gap in potential wages between the partners). The data source is the Madrid survey. The sample is limited to a 12-month bandwidth before and after the reform.

Figure V: Effect of paternity leave eligibility on 6-years subsequent fertility



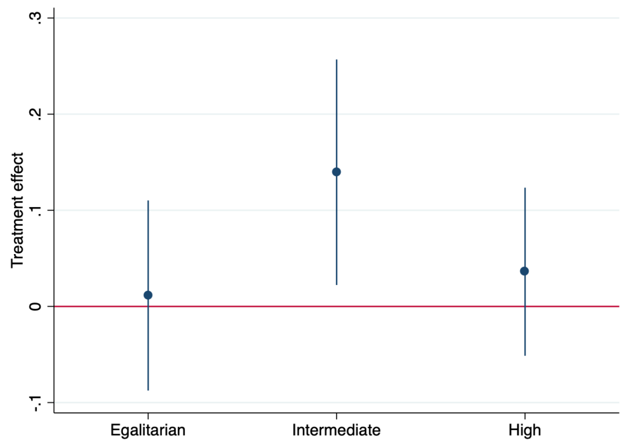
Note: The figure shows the coefficients and 95% confidence intervals from three separate RD regressions that estimate the effect of the introduction of paternity leave on subsequent fertility, for our three groups of families (based on the gap in potential wages between the partners). The data source is birth certificates. The sample is limited to a 12-week bandwidth before and after the reform.

Figure VI: Effect of paternity leave eligibility on fathers' daily childcare and housework time in minutes



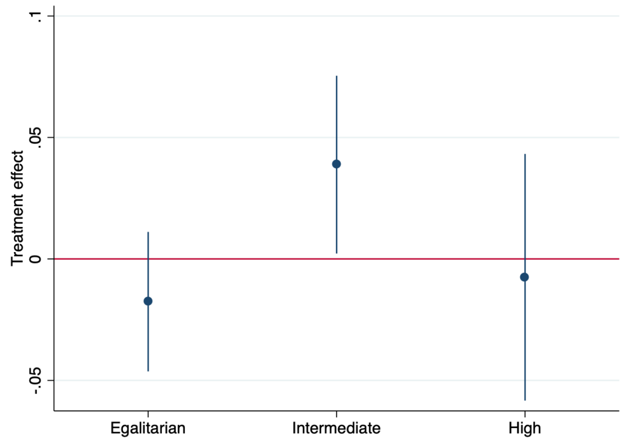
Note: The figure shows the coefficients and 95% confidence intervals from three separate RD regressions that estimate the effect of the introduction of paternity leave on daily minutes of childcare and housework by fathers, for our three groups of families (based on the gap in potential wages between the partners). The data source is the Spanish Time Use Survey of 2010. The sample is limited to a 28-month bandwidth before and after the reform.

Figure VII: Effect of paternity leave eligibility on mothers' accumulated earnings in the following 24 months after giving birth



Note: The figure shows the coefficients and 95% confidence intervals from three separate RD regressions that estimate the effect of the introduction of paternity leave on the earnings of mothers, for our three groups of families (based on the gap in potential wages between the partners). The coefficients are divided by mean earnings in the respective (control) group. The data source is Spanish Social Security data. The sample is limited to a 9-month bandwidth before and after the reform.

Figure VIII: Effect of paternity leave on parents’ separation



Note: The figure shows the coefficients and 95% confidence intervals from three separate RD regressions that estimate the effect of the introduction of paternity leave on parental separation, for our three groups of families. The data source is the Spanish Labor Force Survey of 2008-10. The sample is limited to a 3-month bandwidth before and after the reform.